

## The Value of Information in Container Transport: Leveraging the Triple Bottom Line

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# The Value of Information in Container Transport: Leveraging the Triple Bottom Line\*

Rob A. Zuidwijk, Albert Veenstra<sup>†</sup>

October 12, 2010

## Abstract

Planning the transport of maritime containers from the sea port to final destinations while using multiple transport modes is challenged by uncertainties regarding the time the container is released for further transport or the transit time from the port to its final destination. This paper assesses the value of information in container transport in terms of multiple performance dimensions, i.e. logistics costs, reliability, security, and emissions. The analysis is done using a single period model where a decision maker allocates arriving containers to two transport modes (slow, low price, no flexible departure times, versus fast, high price, flexible departure times). We construct a frontier of Pareto optimal decisions under each of the information scenarios and show that these frontiers move in a favorable direction when the level of information progresses. Each of the Pareto frontiers help strike the balance between the aforementioned performance dimensions. The mathematical results are illustrated using two numerical examples involving barge transport and train transport.

*Keywords:* value of information, Pareto frontier, collateral benefits, maritime container, intermodal transport

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\*This paper is dedicated to the memory of prof. dr. ir. J.A.E.E. van Nunen.

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To relieve congestion at the deep sea container terminal and in the port area, a number of sea ports, such as the ports of Los Angeles and Long Beach, and the port of Rotterdam, have implemented so-called 'dry ports', i.e. hub terminals positioned inland (Roso et al., 2009). Instead of stacking import containers at the deep sea terminal until they can be forwarded to their final destinations, containers are being pushed in bulk to these dry ports. Congestion is reduced at the terminal as containers remain at the terminal for a shorter period of time. Transport of large volumes of containers permits the use of modes of transport other than truck without compromising frequency of service, which relieves the road infrastructure in the port area.

Compared to truck transport, however, river barges and trains usually feature longer transit times, and the river and train networks do not connect directly to any final destination. The limited availability of scheduled departures of river and rail connections creates another disadvantage: late arrivals of containers may force ad hoc shipment by truck to avoid any further delay, which could compromise the use of certified carriers and henceforth could deteriorate the guaranteed level of security (Sheffi, 2001).

Since barge and train have less negative environmental and social impacts than road transport, there is a need to address these issues. This can be done by offering co-modal transport services (Groothedde et al., 2005), which involve both slow and less costly transport modes such as barge and rail to carry the bulk of containers, and the fast and flexible trucking option to execute shipments under time pressure.

The planning of co-modal transport services is challenged due to the uncertain-

ties in, for example, the times the containers are available for transport and the transit times. As a result, the planning of containers is nowadays characterized by a conservative approach where slack times are built in to avoid late arrivals at the customer.

The performance of the transport of maritime containers is relevant to a variety of stakeholders such as organizations in the global supply chains involved and external governmental and non-governmental organizations that represent environmental and social interests. As a result, performance need to be accounted for across economic, social, and environmental dimensions, i.e. the 'triple bottom line' (Elkington, 1994). In particular, there is a need to incorporate the external costs caused by intermodal transport (Liao et al., 2009) in the analysis of new concepts such as the dry port. Systemic improvements enable enhanced trade-offs between various performance dimensions and create so-called "collateral benefits", for example in balancing logistics performance and level of security (Lee and Whang, 2005).

This paper contributes to the existing literature by providing a single period model where the allocation of containers to truck or barge, and the barge departure time can be tuned to strike the balance between operational costs, reliability, emissions, and security. We study the value of information by comparing performance of the transport system under a number of information scenarios. Since we need to evaluate along multiple performance dimensions, we identify a frontier of Pareto optimal decisions under each of the information scenarios and show that these frontiers move in a favorable direction when the level of information progresses. Here we are able to formalize the notion of collateral benefits mentioned above. The balancing

of multiple objectives for single period models has also been considered in (Parlar and Weng, 2003), but in our decision problem, the efficiency frontiers need not be convex.

We formulate the problem in Section 1 and model the performance criteria logistics costs, reliability, emissions, and security, in a stylized way. In Section 2, we describe a number of progressive information scenarios. For each level of information, a frontier of Pareto optimal decisions is defined, and we show that, as more information becomes available, the frontier moves into a favorable direction. In Section 3, we construct the Pareto frontiers and identify the Pareto optimal decisions for each of the information scenarios while considering two performance dimensions. By means of numerical examples, we discuss the results. In Section 4, we draw some conclusions and discuss opportunities for further research. Mathematical statements are proven in Appendix A.

## 1 Problem Formulation

In this section, we formulate the decision problem and introduce the model parameters. We consider the following decision situation from the viewpoint of the shipper (or forwarder) involved in the global transport of maritime containers. The shipper has consigned a number of containers that will be released at the port of destination. The shipper needs to decide on the appropriate combination of land transport modes in order to get the containers to their final destinations at the customers in time against minimum costs and emissions, while taking into account security

aspects.

The release time of the container is defined here as the earliest possible pick-up time for further transport to the final destination, and is assumed to follow a probability distribution  $f$ . We assume that all containers are due at a single customer destination before a single deadline  $T$ . The transport can be done either by barge, i.e. river vessel, or truck. We will consistently refer to barge, but train could be considered as well by using different parameter values, which is done in the illustrative numerical examples in Section 3.3. Transportation by barge involves a planned departure time  $t_0$ . Depending on the volume of the container flow consigned by the shipper, this departure time can be influenced by the shipper or he can pick a departure time from a fixed barge schedule. We assume here that the shipper can set the departure time, so that  $t_0$  is a continuous decision variable. Transportation by truck can be initiated as soon as the container has been released. The transit times by truck  $\tau_{\text{truck}}$  and barge  $\tau_{\text{barge}}$  are governed by probability distributions  $g_{\text{truck}}$  and  $g_{\text{barge}}$ , respectively. The transit times represent the throughput times between container release and arrival at the final destination, so they usually include waiting and handling times. The associated transit costs for barge and truck are denoted by  $c_{\text{barge}}$  and  $c_{\text{truck}}$ , respectively.

The probability that a container is released too late in order to be transported by barge equals  $\alpha = 1 - F(t_0)$ , where  $F$  is the cumulative probability density function associated with  $f$ . The parameter  $\alpha$  can be interpreted as a tolerance parameter which the shipper takes into account while setting the planned departure time  $t_0$  of the barge. In such a manner, the departure time  $t_0$  is related to the amount of

slack time built-in to increase the probability that containers arrive in time to be transported by barge. In the case a container was planned to be transported by barge and is released only after the departure (cut-off) time  $t_0$ , it will be transported by recourse truck, i.e. in an ad hoc fashion. The dispatch of a recourse truck will result in a longer transit time  $\tau_{\text{rctruck}}$ , governed by the probability distribution  $g_{\text{rctruck}}$ , and higher transit costs  $c_{\text{rctruck}}$ , as compared to a planned truck. The main cause of longer transit time and higher costs of a recourse truck shipment is the allocation of resources in an ad hoc way.

The transit costs are ordered as follows:  $c_{\text{barge}} < c_{\text{truck}} < c_{\text{rctruck}}$ , i.e. barge is the cheapest transport mode, while a recourse dispatch of a truck is the most expensive alternative. The transit times should be modeled in such a way that truck is faster than barge and that recourse truck shipment is slower than planned truck shipment. We do so by first order stochastic dominance, i.e. by stating that  $G_{\text{truck}} > G_{\text{rctruck}} > G_{\text{barge}}$ ; see Rothschild and Stiglitz (1970). We also consider emissions associated with barge and truck and observe that the emissions per container are ordered similar to costs:  $e_{\text{barge}} < e_{\text{truck}} < e_{\text{rctruck}}$ , as both emissions and costs are predominantly driven by fuel consumption (McKinnon, 2007).

We study the performance of the transport system at hand along the following performance measures: transit costs, fraction of containers in time at final destination, emissions, and security. The level of security  $s$  will be measured in terms of fraction of containers shipped as planned, referring to the fact that ad hoc shipments by recourse truck may result in compromised security levels. All performance measures are normalized in such a way that they attain values in the interval  $[0, 1]$  and



symbol	description
$T$	deadline at the customer
$t$	release time container
$\tau_{\text{mode}}$	transit time transport mode
$c_{\text{mode}}, e_{\text{mode}}$	costs and emissions transport mode per container
$f, F$	probability distribution function, cumulative distribution function of $t$
$g_{\text{mode}}, G_{\text{mode}}$	probability distribution function, cumulative distribution function of $\tau_{\text{mode}}$
$\sigma_k, \varepsilon_k, s_k, \rho_k$	normalized costs, normalized emissions, security, and expected fraction of timely arrivals under Information Scenario $k$

Table 1: Model parameters

that 1 is the optimal value. For the costs  $c$  this requires a linear transformation given by

$$\sigma = \frac{c_{\text{rctruck}} - c}{c_{\text{rctruck}} - c_{\text{barge}}}. \quad (1)$$

When  $p_{\text{mode}}$  denotes the fraction of containers transported by a certain mode, and  $p_{\text{barge}} + p_{\text{truck}} + p_{\text{rctruck}} = 1$ , and  $c_{\text{barge}} < c_{\text{truck}} < c_{\text{rctruck}}$ , then  $\sigma = p_{\text{barge}} + (1 - \theta)p_{\text{truck}}$ , where

$$\theta = \frac{c_{\text{truck}} - c_{\text{barge}}}{c_{\text{rctruck}} - c_{\text{barge}}} \in (0, 1). \quad (2)$$

The parameter  $\theta$  expresses the relative performances of the various modes. The normalized emissions and normalized cost functions have the same structure but different parameter values  $\theta$ . It turns out that the security parameter  $s$  can also be written in a similar way by putting  $\theta = 0$ .

We now summarize the formulation of the single period model. Two decisions are made under uncertainty. The first decision variable is  $t_0$ , the planned departure time of the barge, or alternatively  $\alpha = 1 - F(t_0)$ , the probability that containers do not

arrive in time to be transported by barge. The second decision variable  $\gamma$  represents the planned fraction of containers to be shipped by barge. The uncertainty in the arrival times of the containers and the transit times of the various transport modes are represented by stochastic distributions. In the next section, we shall elaborate on four information scenarios under which decisions are made. The performance of decisions under Information Scenario  $k \in \{1, 2, 3, 4\}$  are presented as normalized costs  $\sigma_k$ , normalized emissions  $\epsilon_k$ , security  $s_k$ , and reliability, i.e. expected fraction of timely arrivals at the customer  $\rho_k$ . We present the model parameters in Table 1.

## 2 Information Scenarios

A number of progressive information scenarios and the corresponding decision situations will be described. We consider (1) the case of no information known to the shipper, (2) the case where the probability distributions of the container release times and transit times are known to the shipper, (3) the case when the probability distributions of the container release times are further specified for a number of categories of containers, and (4) the ideal case where the actual arrival times are known to the shipper beforehand. The decision situation associated with information scenario 2 coincides with the problem formulation in Section 1. The decision situations associated with the other information scenarios are modifications of the decision situation presented in Section 1, as will be explained in the subsections below. In particular, for each information scenarios, decision policies are described, together with expressions of their performance in terms of cost, security, and reliability.

## 2.1 No Information

The shipper has no information on the distributions of the arrival of containers and the transit times, so has no basis to make any decisions before the containers arrive. In line with common practice, we will assume that the shipper uses truck as transport mode when the risk of late arrival at the customer is too large or unknown, i.e. the shipper will source for trucks as soon as containers have arrived. The sourcing of a truck is ad hoc and therefore recourse truck costs and transit times are applicable here.

**Lemma 1** *Under Information Scenario 1, the normalized cost per container is given by  $\sigma_1 = 0$ , the security level by  $s_1 = 0$ , and the expected fraction of containers that reach their destination in time satisfies  $\rho_1 = \int_0^T G_{\text{rctruck}}(T - t)f(t) dt$ .*

## 2.2 Information on container arrival and transit time distributions

The shipper has the probability distributions of container arrivals and transit times at his disposal. Therefore, he will be able to evaluate decisions on the planned departure time of the barge and the amount of containers to be shipped by barge before the containers actually arrive. We model this decision situation as a single period model, as explained in Section 1. The shipper plans the departure time  $t_0$  of the barge and the fraction  $0 \leq \gamma \leq 1$  of containers to be transported by barge. As a consequence, the fraction  $1 - \gamma$  of the containers are planned to be shipped by truck. After these decisions have been made, the release times of the containers are

realized. For containers planned to be transported by truck, the arrival time at the final destination is given by  $t + \tau_{\text{truck}}$ . For containers planned to be transported by barge and that are released before  $t_0$ , the arrival time at the final destination equals  $t_0 + \tau_{\text{barge}}$ . The arrival times of late containers transported by a recourse truck read  $t + \tau_{\text{rctruck}}$ . We remark that containers are identified by a container number and that the planning specifies for each specific container a transport mode. We now consider the performance under this policy.

**Lemma 2** *Under Information Scenario 2, the normalized costs read*

$$\sigma_2(\alpha, \gamma) = 1 - \theta + \gamma(\theta - \alpha), \quad (3)$$

where  $\theta$  is defined as in (2). The fraction of containers delivered in time at the final destination is given by

$$\begin{aligned} \rho_2(\alpha, \gamma) = & \gamma \int_{t_0}^T G_{\text{rctruck}}(T - t) f(t) dt + \\ & \gamma(1 - \alpha) G_{\text{barge}}(T - t_0) + (1 - \gamma) \int_0^T G_{\text{truck}}(T - t) f(t) dt. \end{aligned} \quad (4)$$

The level of security satisfies  $s_2(\alpha, \gamma) = 1 - \gamma\alpha$ .

### 2.3 Specific information on container arrival time distributions for categories of containers

This information scenario incorporates the situation where the shipper is able to categorize the containers based on features such as port of origin, cargo type, and

consigner, that will help him to assess the release time probability distributions more specifically. In other words, the decision maker is now able to determine more specific probability distributions  $f_k$  for categories of containers  $k = 1, \dots, n$  in such a way that

$$f(t) = \sum_{k=1}^n w_k f_k(t). \quad (5)$$

The weights satisfy  $w_k > 0$  and  $\sum_{k=1}^n w_k = 1$ , and they represent the relative sizes of the categories. The shipper decides on barge departure time  $t_0$  and fractions of container categories to be shipped by barge, i.e.  $\gamma_k$  for category  $k$ . We shall write  $\vec{\gamma} = (\gamma_1, \dots, \gamma_n)$ . If we set  $\alpha_k = 1 - F_k(t_0)$ , then we may write  $\alpha = 1 - F(t_0) = \sum_{k=1}^n w_k \alpha_k$ .

**Lemma 3** *Under Information Scenario 3, the normalized costs read*

$$\sigma_3(\alpha, \vec{\gamma}) = 1 - \theta + \sum_{k=1}^n w_k \gamma_k (\theta - \alpha_k). \quad (6)$$

*The expected fraction of containers that arrives before the deadline  $T$  is equal to*

$$\begin{aligned} \rho_3(\alpha, \vec{\gamma}) &= \sum_{k=1}^n w_k \gamma_k \int_{t_0}^T G_{\text{rctruck}}(T-t) f_k(t) dt + \\ & \sum_{k=1}^n w_k \gamma_k G_{\text{barge}}(T-t_0) F_k(t_0) + \sum_{k=1}^n w_k (1-\gamma_k) \int_0^T G_{\text{truck}}(T-t) f_k(t) dt. \end{aligned} \quad (7)$$

*The level of security equals  $s_3(\alpha, \vec{\gamma}) = 1 - \sum_{k=1}^n w_k \gamma_k \alpha_k$ .*

## 2.4 Information on actual container arrival times

The decision situation is now as follows. The shipper observes the actual arrival times of the containers beforehand, and plans the transport modes accordingly. Once the barge departure time  $t_0$  has been set, a fraction  $\gamma$  of containers that will arrive before  $t_0$  will be shipped by barge, and the remainder  $1 - \gamma$  fraction of containers that arrive before  $t_0$  will be shipped by truck. It is optimum to ship by barge the last  $\gamma$  fraction of containers that arrive before the departure of the barge. We may formalize this policy by defining

$$t_\gamma = F^{-1}((1 - \gamma)F(t_0)), \quad 0 \leq \gamma \leq 1,$$

and plan all containers that arrive before  $t_\gamma$  by barge, and plan the remainder of the containers by truck.

**Lemma 4** *Under Information Scenario 4, the normalized expected costs are equal to*

$$\sigma_4(\alpha, \gamma) = 1 - \theta + \theta\gamma(1 - \alpha). \quad (8)$$

*The expected fraction of containers that arrive in time at the customer reads*

$$\rho_4(\alpha, \gamma) = \gamma(1 - \alpha)G_{\text{barge}}(T - t_0) + \int_0^{t_\gamma} G_{\text{truck}}(T - t)f(t) dt + \int_{t_0}^T G_{\text{truck}}(T - t)f(t) dt. \quad (9)$$

*The level of security satisfies  $s_4(\alpha, \gamma) = 1$ .*

## 2.5 Relative positioning of the Pareto frontiers

For each of the information scenarios, a policy has been defined that incorporates the decision variables  $\alpha$  and  $\gamma$  (or  $\vec{\gamma}$ ) in a certain way, which generated a set  $\mathcal{A}_k = \{(\rho_k(\alpha, \gamma), \sigma_k(\alpha, \gamma), \varepsilon_k(\alpha, \gamma), s_k(\alpha, \gamma)) : (\alpha, \gamma) \in [0, 1] \times [0, 1]\}$  of performance levels for  $k = 1, 2, 4$ , while  $\mathcal{A}_3 = \{(\rho_3(\alpha, \vec{\gamma}), \sigma_3(\alpha, \vec{\gamma}), \varepsilon_3(\alpha, \vec{\gamma}), s_3(\alpha, \vec{\gamma})) : (\alpha, \vec{\gamma}) \in [0, 1] \times [0, 1]^n\}$ .

In order to compare the performance of the transport system under the various information scenarios, we describe the notion of a Pareto frontier. The Pareto frontier  $\mathcal{E}$  of a set  $\mathcal{A}$ , given a partial ordering  $\leq$ , consists of those elements  $u \in \mathcal{A}$  for which there exist no element  $v \in \mathcal{A}$  such that  $u < v$ . Consequently, a Pareto frontier  $\mathcal{E}$  consists of maximal elements only. An element  $u \in \mathcal{A}$  is maximal when for each  $v \in \mathcal{A}$  with  $u \leq v$ , it holds true that  $u = v$ . A Pareto frontier  $\mathcal{E}$  majorizes another Pareto frontier  $\mathcal{F}$ , and we write  $\mathcal{E} \preceq \mathcal{F}$ , if for each  $u \in \mathcal{E}$ , there exists  $v \in \mathcal{F}$  such that  $u \leq v$ . The relation  $\preceq$  constitutes a partial ordering on the set of Pareto frontiers. We denote the Pareto frontier of  $\mathcal{A}_k$  by  $\mathcal{E}_k$  for  $k \in \{1, 2, 3, 4\}$ , and we establish the following.

**Theorem 5** *Let  $\mathcal{E}_k$  denote the Pareto frontier of performance levels under Information Scenario  $k = 1, 2, 3, 4$ . It holds true that*

$$\mathcal{E}_1 \preceq \mathcal{E}_2 \preceq \mathcal{E}_3 \preceq \mathcal{E}_4. \quad (10)$$

### 3 Construction of the Pareto Frontiers

In the previous section, we have identified decision policies under the various information scenarios such that (10) holds true. In order to analyze the Pareto optimal solutions, we need more specific information about the characteristics of the Pareto frontiers, and we achieve this by constructing the frontiers. We restrict the explicit construction of the Pareto frontiers to two dimensions for tractability reasons. As normalized costs  $\sigma$ , normalized emissions  $\epsilon$ , and security  $s$  differ only in the parameter values of  $\theta$ , we focus on Pareto optimal decisions with respect to reliability and normalized costs. Further, we interpret the decision policies that give rise to these Pareto frontiers in the various decision situations as explained in Section 2. We omit the proof of the following lemma.

**Lemma 6** *In the case when  $\theta = 0$ , i.e. in the case when we consider Pareto optimal solutions with respect to reliability and security, it can be seen that the Pareto frontiers collapse into single points:  $\mathcal{E}_1 = \{(\rho_1, 0)\}$  and  $\mathcal{E}_2 = \mathcal{E}_3 = \mathcal{E}_4 = \{(\rho_0, 1)\}$ .*

We now proceed with the analysis of the case when  $\theta > 0$ . First of all, we observe that the Pareto frontier  $\mathcal{E}_1$  consists of the single point  $(\rho_1, \theta)$ .

#### 3.1 The Pareto frontier $\mathcal{E}_2$

In this subsection, we construct the Pareto frontier  $\mathcal{E}_2$ , and we interpret the Pareto optimal decisions associated with the frontier. The construction goes along the following lines. First, we observe that we can write (4) as  $\rho_2(\alpha, \gamma) = \rho_0 - \gamma\rho(\alpha)$ ,



where

$$\rho_0 = \int_0^T G_{\text{truck}}(T-t)f(t) dt, \quad (11)$$

and

$$\rho(\alpha) = \rho_0 - \int_{t_0}^T G_{\text{retruck}}(T-t)f(t) dt - G_{\text{barge}}(T-t_0)F(t_0). \quad (12)$$

Observe that  $0 \leq \rho(\alpha) \leq \rho_0$  and  $\rho(\alpha)$  is decreasing in  $\alpha$ . Further, recall that (3) provides  $\sigma_2(\alpha, \gamma) = 1 - \theta + \gamma(\theta - \alpha)$ . In other words, for fixed  $\alpha$ , the functions  $\gamma \mapsto \rho_2(\alpha, \gamma)$  and  $\gamma \mapsto \sigma_2(\alpha, \gamma)$  are linear. As a consequence, the function  $(\rho_2, \sigma_2)$  maps the line segment  $L_\alpha = \{(\alpha, \gamma) \mid 0 \leq \gamma \leq 1\}$  onto another line segment

$$M_\alpha = \{(\rho_0, 1 - \theta) + \gamma(-\rho(\alpha), \theta - \alpha) \mid 0 \leq \gamma \leq 1\}. \quad (13)$$

Since  $[0, 1]^2 = \bigcup_\alpha L_\alpha$ , we obtain  $\mathcal{A}_2 = \bigcup_\alpha M_\alpha$ . We proceed to construct  $\mathcal{E}_2$  based on the line segments  $M_\alpha$ . First, we observe that  $M_\alpha$  for  $\alpha > \theta$  do not contribute to  $\mathcal{E}_2$ , so we may restrict our analysis of  $M_\alpha$  to  $0 \leq \alpha \leq \theta$ . Next, write the negative slope of the line  $M_\alpha$  as

$$\Phi(\alpha) = \frac{\theta - \alpha}{\rho(\alpha)}. \quad (14)$$

Note that  $\Phi(0) = \frac{\theta}{\rho_0} > 0$  with  $\rho(0) = \rho_0 > 0$ . On the other hand,  $\rho(\theta) \geq \rho(1) = \rho_0 - \rho_1 > 0$ , which implies  $\Phi(\theta) = 0$ . An expression of the Pareto frontier  $\mathcal{E}_2$  and a description of the Pareto optimal solutions are given by the following theorem.

**Theorem 7** *Define the function*

$$\Psi(\lambda) = \max\{\Phi(\alpha) \mid 0 \leq \alpha \leq \lambda\}, \quad (15)$$

then  $\Psi(\lambda)$  is increasing,  $\Psi(0) = \frac{\theta}{\rho_0} > 0$ , and

$$\mathcal{E}_2 = \left\{ \left( \rho_0 - \frac{\theta - \lambda}{\Psi(\lambda)}, 1 - \lambda \right) \mid 0 \leq \lambda \leq \theta \right\}. \quad (16)$$

For each fixed  $0 \leq \lambda \leq \theta$ , given that  $\Psi(\lambda) = \Phi(\alpha)$  for some  $0 \leq \alpha \leq \lambda$ , the associated Pareto optimal decision reads as follows: We assign fraction  $\gamma = \frac{\theta - \lambda}{\theta - \alpha}$  of the containers to barge and set the barge departure time equal to  $t_0 = F^{-1}(1 - \alpha)$ .

### 3.2 The Pareto frontiers $\mathcal{E}_3$ and $\mathcal{E}_4$

Similar as in the previous section, we construct the Pareto frontiers  $\mathcal{E}_3$  and  $\mathcal{E}_4$ , and interpret the set of Pareto optimal decisions in the respective decision contexts. We construct the Pareto frontier  $\mathcal{E}_3$  by first establishing the following proposition.

**Proposition 8** *The mapping  $(\rho_3, \sigma_3)$  as defined by (7) and (6) is an affine mapping. It maps the  $n$ -dimensional cube  $L_\alpha = \{(\alpha, \vec{\gamma}) : \vec{\gamma} \in [0, 1]^n\}$  onto the polygon*

$$M_\alpha = \{(\rho_3(\alpha, \vec{\gamma}), \sigma_3(\alpha, \vec{\gamma})) : \vec{\gamma} \in [0, 1]^n\}. \quad (17)$$

A mapping  $(\tilde{\sigma}_3, \tilde{\sigma}_3)$  can be constructed which maps  $\{(\alpha, \gamma) : 0 \leq \gamma \leq 1\}$  onto the Pareto frontier of  $M_\alpha$  for each  $0 \leq \alpha \leq \theta$ .

This result is useful for the construction of the Pareto frontier  $\mathcal{E}_3$  as only Pareto optimal points of  $M_\alpha$  for  $0 \leq \alpha \leq \theta$  will constitute  $\mathcal{E}_3$ . We now use the following theorem to establish the Pareto frontier  $\mathcal{E}_3$ .

**Theorem 9** Given the mapping  $\tilde{\sigma}_3 : (\alpha, \gamma) \mapsto 1 - \theta + \gamma(\theta - \alpha)$  and the mapping  $\tilde{\rho}_3 : (\alpha, \gamma) \mapsto \tilde{\rho}_3(\alpha, \gamma)$  as provided by Proposition 8, with  $\gamma \mapsto \tilde{\rho}_3(\alpha, \gamma)$  strictly decreasing for each  $\alpha \in [0, \theta]$ , define

$$\rho_3^*(\lambda) = \max \left\{ \tilde{\rho}_3 \left( \alpha, \frac{\theta - \lambda}{\theta - \alpha} \right) : 0 \leq \alpha \leq \lambda \right\}, \quad (18)$$

then  $\mathcal{E}_3 = \{(\rho_3^*(\lambda), 1 - \lambda) : 0 \leq \lambda \leq \theta\}$ .

We interpret the Pareto optimal decisions associated with  $\mathcal{E}_3$  in the decision context by considering the proofs of Proposition 8 and Theorem 9. We summarize the findings in the following proposition.

**Proposition 10** The element  $(\rho_3^*(\lambda), 1 - \lambda)$  of the Pareto optimal frontier  $\mathcal{E}_3$  gives rise to the Pareto optimal decision  $(\alpha, \vec{\gamma})$ , where the value of  $\alpha \in [0, \lambda]$  corresponds with  $\rho_3^*(\lambda) = \tilde{\rho}_3(\alpha, \frac{\theta - \lambda}{\theta - \alpha})$ , and  $\vec{\gamma}$  is given by

$$\begin{cases} \gamma_{k_i} = 1, & 1 \leq i \leq p \\ \gamma_{k_{p+1}} = \frac{\lambda - \lambda_p}{\lambda_{p+1} - \lambda_p}, & \\ \gamma_{k_i} = 0, & p + 1 < i \leq n \end{cases}, \quad (19)$$

where  $\lambda_p = 1 - \theta + \sum_{i=1}^p w_{k_i}(\theta - \alpha_{k_i})$ ,  $\lambda_p \leq \lambda < \lambda_{p+1}$ , and  $k_1, \dots, k_n$  distinct such that  $\Phi_{k_1}(\alpha) \geq \Phi_{k_2}(\alpha) \geq \dots \geq \Phi_{k_n}(\alpha)$ , with  $\Phi_k(\alpha) = \frac{\theta - \alpha_k}{\rho_k(\alpha)}$  and

$$\rho_k(\alpha) = \int_0^T G_{\text{truck}}(T - t)f_k(t) dt - F_k(t_0)G_{\text{barge}}(T - t_0) - \int_{t_0}^T G_{\text{rctruck}}(T - t)f_k(t) dt. \quad (20)$$

Proposition 10 clarifies that Pareto optimal decisions seek to assign container categories to barge for which the trade-off between reliability and cost are favorable i.e for which  $\Phi_k(\alpha)$  is large. Note that the function  $\Phi_k(\alpha)$  is similar to the function  $\Phi(\alpha)$  as defined in (14), which provided the negative slope of the trade-off line  $M_\alpha$ . We find in particular that the fraction of containers of category  $k$  planned for barge is equal to  $\gamma_k$  as given in Proposition 10, while the departure time of the barge is given by  $t_0 = F^{-1}(1 - \alpha)$ .

We construct the Pareto frontier  $\mathcal{E}_4$  using the following theorem.

**Theorem 11** *Given the mapping  $\sigma_4 : (\alpha, \gamma) \mapsto 1 - \theta + \gamma\theta(1 - \alpha)$  and the mapping  $\rho_4 : (\alpha, \gamma) \mapsto \rho_4(\alpha, \gamma)$  as defined in (9) which is strictly decreasing in  $\gamma$ , define*

$$\rho_4^*(\lambda) = \max \left\{ \rho_4 \left( \alpha, \frac{\theta - \lambda}{\theta(1 - \alpha)} \right) : 0 \leq \alpha \leq \frac{\lambda}{\theta} \right\}, \quad (21)$$

then  $\mathcal{E}_4 = \{(\rho_4^*(\lambda), 1 - \lambda) : 0 \leq \lambda \leq \theta\}$ .

The proof of this theorem is similar to the proof of Theorem 9 and is not provided in the appendix. For Scenario 4, a similar interpretation can be given for the Pareto optimal decisions associated with  $\mathcal{E}_4$  as in Scenario 2. In this case, however, the fraction  $\gamma$  of containers assigned to barge are the ones with the arrival times as late as possible before barge departure, as discussed in Section 1.

### 3.3 Numerical Example and Discussion of Results

In this subsection, we provide two numerical examples to illustrate the results obtained in the previous subsections. We now providede the parameter choices in the

symbol	barge	train	truck	recourse truck
$e_{\text{mode}}$ (kg/ton CO <sub>2</sub> )	6	6	23	25
$c_{\text{mode}}$ (€/container)	35	70	100	110
$\tau_{\text{mode}}$ (hrs)	12	8	4	5

Table 2: Data used in the numerical examples for train and barge (distance of 100 miles)

examples in detail. One example concerns barge transport, the other concerns train transport. The numerical examples also allow us to interpret the mathematical results of the previous sections in terms of the problem formulation more concretely. Table 2 displays the data used in the numerical example. We denote the symmetric triangular probability distribution with mean  $\bar{t}$  and support  $[\bar{t} - \delta, \bar{t} + \delta]$  by  $\Delta(\bar{t}, \delta)$ . The release time distribution is assumed equal to  $f = \Delta(\bar{t}, \delta)$ , where we assume  $\delta = \bar{t}$ . In Information Scenario 3, we assume three container categories ( $n = 3$ ) where  $f_1 = \Delta(\bar{t} - \frac{1}{2}\delta, \frac{1}{2}\delta)$ ,  $f_2 = \Delta(\bar{t}, \frac{1}{2}\delta)$ ,  $f_3 = \Delta(\bar{t} + \frac{1}{2}\delta, \frac{1}{2}\delta)$ . As weights we put  $w_1 = \frac{1}{4}$ ,  $w_2 = \frac{1}{2}$ , and  $w_3 = \frac{1}{4}$  in order to get  $f = \sum_{k=1}^3 w_k f_k$ . The transit time distributions are given by  $g_{\text{mode}} = \Delta(\tau_{\text{mode}}, \frac{1}{2}\tau_{\text{mode}})$ . The distributions used in the numerical example do not satisfy the requirement that the cumulative distribution is an invertible function. However, this can be achieved by a small perturbation and this does not seriously effect the numerical results. The numerical results are obtained using the computer algebra package Maple 13.

We now discuss the method used to determine the Pareto frontier  $\mathcal{E}_2$  in detail. In Figure 1, we show how the unit square of decisions  $(\alpha, \gamma) \in [0, 1]^2$  is mapped onto  $\mathcal{A}_2$  as described in Section 2. Observe that  $\mathcal{A}_2$  is not convex and that the description of the Pareto frontier will be intricate.

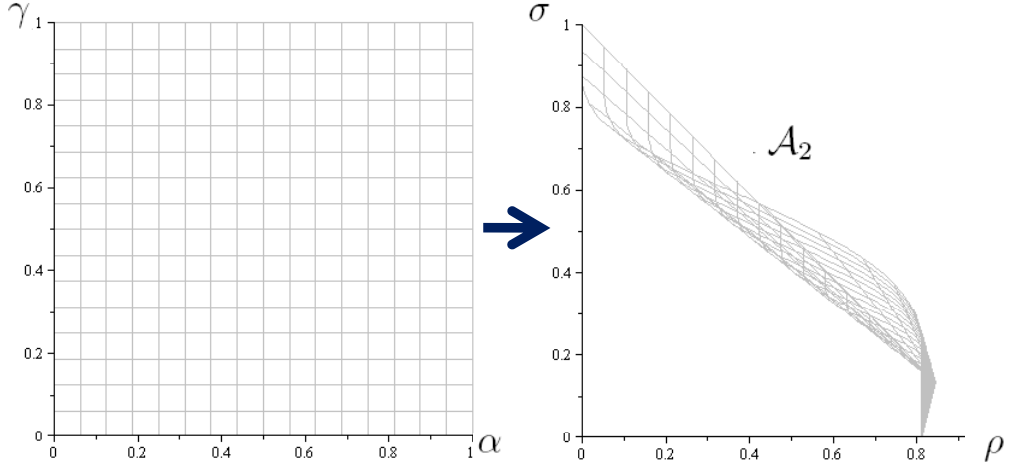


Figure 1: Mapping of  $(\alpha, \gamma)$  plane to  $(\rho, \sigma)$  plane

Figure 2 shows how the (rotated) graph of  $\Psi(\lambda)$ , as obtained from the graph of  $\Phi(\lambda)$  which is also depicted, gives rise to certain zones on the Pareto frontier of  $\mathcal{E}_2$ . This relates to the definition of  $\Psi(\lambda) = \max\{\Phi(\alpha) : 0 \leq \alpha \leq \lambda\}$  in Theorem 7. Zone (A) corresponds with the case when  $\Psi(\lambda) = \Phi(0)$  for  $\lambda \in [0, \lambda_1)$ . Here the Pareto frontier consists of points  $(\rho(0, \gamma), \sigma(0, \gamma))$  where  $\gamma = \frac{\theta - \lambda}{\theta}$  with  $\lambda \in [0, \lambda_1)$ . Zone (A) describes Pareto optimal solutions where the barge departure time is set at the deadline  $T$  and the fraction of containers planned for barge is equal to  $\frac{\theta - \lambda}{\theta}$  with  $\lambda \in [0, \lambda_1)$ . As a result, the reliability  $\rho$  is low, while the normalized cost performance  $\sigma$  is high. Zone (B) corresponds with the case when  $\Psi(\lambda) = \Phi(\lambda)$  with  $\lambda \in [\lambda_1, \lambda_2)$ . In this zone, the Pareto frontier consists of points  $(\rho(\lambda, 1), \rho(\lambda, 1))$  i.e. where all containers are planned for barge, while the departure time  $t_0 = F^{-1}(1 - \lambda)$  of the barge varies. The zone (C) has a similar structure as zone (A). Zone (C) describes

Pareto optimal solutions where the barge departure time is set at  $t_0 = F^{-1}(1 - \lambda_2)$  and the fraction of containers planned for barge is equal to  $\frac{\theta - \lambda}{\theta - \lambda_2}$  with  $\lambda \in [\lambda_2, \theta]$ . Here the reliability is high while the cost performance is low, due to the fraction of containers planned for truck. The Pareto optimal decisions as described above are also indicated in Figure 3; note that the zones (A)-(C) can be distinguished clearly in this graph. Observe that the breakpoints  $0 < \lambda_1 < \lambda_2 < 1$  are not

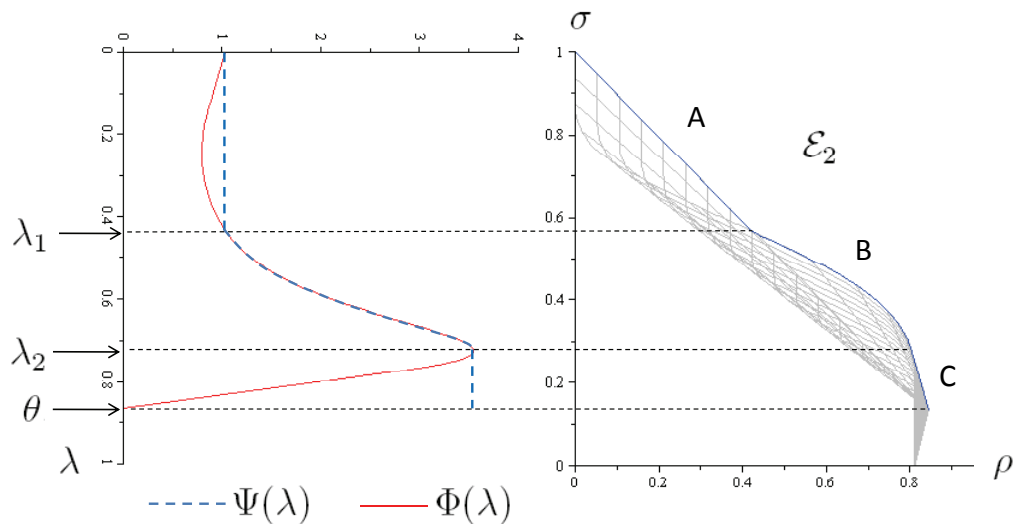


Figure 2: Construction of  $\mathcal{E}_2$  along the lines of Theorem 7

explicitly indicated in Theorem 7. Indeed, this theorem holds true in general and does not make assumptions about the shape of  $\Psi$ . However, the theorem implicitly recognizes the aforementioned zones by the definition of  $\Psi$ . We refrain from giving the details regarding the construction of Pareto frontiers  $\mathcal{E}_3$  and  $\mathcal{E}_4$ . The Pareto frontiers corresponding to the four information scenarios are depicted in Figure 4.

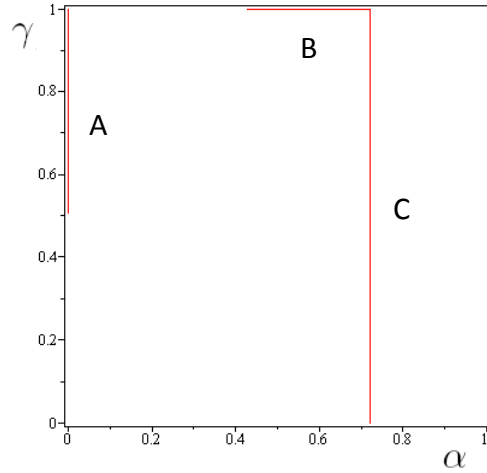


Figure 3: Pareto optimal decisions associated with  $\mathcal{E}_2$

In the numerical examples, we have used the data on barge, train, truck, and recourse truck as depicted in Table 2, together with mean arrival time  $\bar{t} = 18$  hrs, and spread  $\delta = 18$  hrs. For both the example involving the mode barge and the example involving train, the deadline parameter has been set equal to  $T = \bar{t} + \tau_{\text{mode}}$ . It is obvious that the results are sensitive to the deadline as it determines the importance of balancing the use of the two modes of transport. So for the barge numerical example, we used as deadline  $T = 30$  hrs, and for the train numerical example, we used  $T = 26$  hrs.

The values of the normalized cost parameter read  $\theta = 0.87$  for the barge example, and  $\theta = 0.75$  for the train example. Figure 5 depicts the Pareto frontiers for the numerical example involving train transport. Observe that the Pareto frontiers need not be convex. Therefore, the quantification of the value of information using



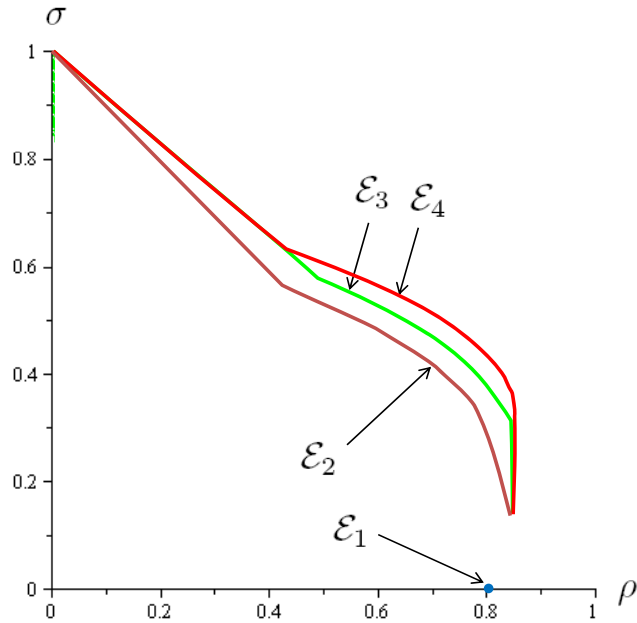


Figure 4: Pareto frontiers information scenarios for barge

scalarization, as in (Tammer and Goepfert, 2002), may not capture the value of information as depicted in Figures 4 and 5. The notion of "collateral benefit" may be more helpful here. This comes down to the identification of room for improvement in single or multiple performance directions; see Figure 5 for an illustration: performance  $a$  under Information Scenario 2 can be improved to performances  $b_1$ ,  $b_2$ , or  $b_3$  under Information Scenario 3, where  $a \leq b_j$  for  $j = 1, 2, 3$ .

## 4 Conclusions

This paper provides a method to support tactical decisions on the transport of containers from the sea port to final destinations inland by a combination of transport

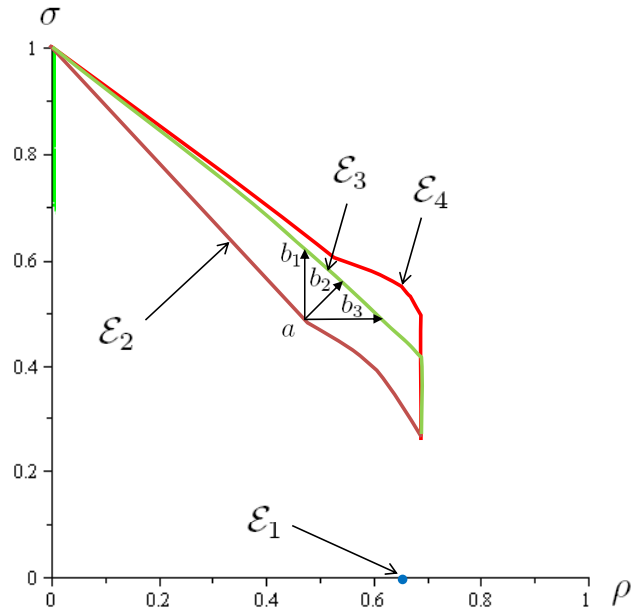


Figure 5: Pareto frontiers information scenarios for train

modes. By studying a number of information scenarios, the value of information is quantified. More specifically, Pareto frontiers have been constructed to assess the value of information while taking into account multiple objectives simultaneously. As the Pareto frontiers are not concave, the use of scalarization, i.e. the creation of a single objective by weighing multiple objectives, may be considered less applicable than the approach of collateral benefits, where for each point on the Pareto frontier under a given information scenario, other points on the Pareto frontier under a more progressive information scenario are identified that majorize the previous point.

The model incorporates the barge departure time as a decision variable, and this variable is equivalent to the slack time reserved to mitigate the risk of containers

planned for barge arriving after its departure time. In this manner, progressive information may be used to reduce slack times, similar to the reduction of safety stock in supply chains under information sharing (Lee et al., 2000).

Pareto optimal decision range from decisions where barge is deployed at a very late stage to reduce costs while accepting a low level of reliability, to decisions where containers are planned for truck to enhance reliability while accepting higher costs. The Pareto frontiers describe how optimal balances can be struck between reliability and operational costs, and the other performance dimensions.

There are some limitations to the study presented in this paper. First of all, only a single destination and a single deadline are considered. The introduction of multiple routes and multiple deadlines would extend the decision problem to a network design problem. In particular, such a study would address the optimum design of services in a transport network under different information scenarios. These network analyses may also involve more than two transport modes, i.e. truck, barge, and train simultaneously.

Further, the opportunities to consolidate container flows in the network, as discussed in (Trip and Bontekoning, 2002; Tyan et al., 2003), could be taken into account as well. The dry port concept as discussed in this paper does not elaborate on the impacts on the supply chain inventories. Elements of the existing work on the management of floating stock (Teulings and Vlist, 2001; Pourakbar et al., 2009), where routing and inventory management are considered in combination, would be helpful to extend the results from this paper in that direction.

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## A Mathematical Proofs

### A.1 Proof of Lemma 1

When the shipper has no information, then the costs per container are equal to  $c_{\text{rctruck}}$ , so  $\sigma_1 = 0$ . Since all containers are transported ad hoc, the security level  $s_1 = 0$ , and the expected fraction of containers that reach their destination in time is equal to

$$\rho_1 = \text{Prob}(t + \tau_{\text{rctruck}} \leq T) = \int_0^T G_{\text{rctruck}}(T - t)f(t) dt. \quad (22)$$

This proves the lemma.  $\square$

### A.2 Proof of Lemma 2

Given  $\gamma$  and  $t_0$ , the expected transit costs are given by

$$\begin{aligned} \gamma \text{Prob}(t \leq t_0)c_{\text{barge}} + \gamma \text{Prob}(t > t_0)c_{\text{rctruck}} + (1 - \gamma)c_{\text{truck}} = \\ \gamma(1 - \alpha)c_{\text{barge}} + \gamma\alpha c_{\text{rctruck}} + (1 - \gamma)c_{\text{truck}}. \end{aligned} \quad (23)$$

By using (1), we obtain the normalized costs (3) where  $\theta$  is defined as in (2). The percentage of containers delivered in time at the final destination is given by

$$\begin{aligned}
\rho_2(\alpha, \gamma) &= \gamma \text{Prob}(t + \tau_{\text{rctruck}} \leq T \wedge t > t_0) + \\
&\quad \gamma \text{Prob}(t_0 + \tau_{\text{barge}} \leq T \wedge t \leq t_0) + \\
&\quad (1 - \gamma) \text{Prob}(t + \tau_{\text{truck}} \leq T) = \\
&\quad \gamma \int_{t_0}^T G_{\text{rctruck}}(T - t) f(t) dt + \gamma(1 - \alpha) G_{\text{barge}}(T - t_0) + \\
&\quad (1 - \gamma) \int_0^T G_{\text{truck}}(T - t) f(t) dt.
\end{aligned}$$

The level of security will be measured in terms of percentage of containers shipped as planned, which is equal to  $s_2(\alpha, \gamma) = 1 - \gamma\alpha$ . Observe that this expression coincides with  $\sigma_2(\alpha, \gamma)$  when  $\theta$  is set equal to zero. This proves the lemma.  $\square$

### A.3 Proof of Lemma 3

First observe that if we substitute (5) in (4), we get

$$\begin{aligned}
\rho_2(\alpha, \gamma) &= \gamma \int_{t_0}^T G_{\text{rctruck}}(T - t) f(t) dt + \gamma G_{\text{barge}}(T - t_0) F(t_0) + \\
&\quad (1 - \gamma) \int_0^T G_{\text{truck}}(T - t) f(t) dt = \\
&\quad \sum_{k=1}^n w_k \gamma \int_{t_0}^T G_{\text{rctruck}}(T - t) f_k(t) dt + \sum_{k=1}^n w_k \gamma G_{\text{barge}}(T - t_0) F_k(t_0) +
\end{aligned}$$

$$\sum_{k=1}^n w_k(1 - \gamma) \int_0^T G_{\text{truck}}(T - t) f_k(t) dt.$$

The decision maker is able to plan the amount of containers to be shipped by barge per category of containers. In other words, we may introduce decision parameters  $\vec{\gamma} = (\gamma_1, \dots, \gamma_n)$  and define

$$\begin{aligned} \rho_3(\alpha, \vec{\gamma}) &= \sum_{k=1}^n w_k \gamma_k \int_{t_0}^T G_{\text{rctruck}}(T - t) f_k(t) dt + \\ &\quad \sum_{k=1}^n w_k \gamma_k G_{\text{barge}}(T - t_0) F_k(t_0) + \\ &\quad \sum_{k=1}^n w_k(1 - \gamma_k) \int_0^T G_{\text{truck}}(T - t) f_k(t) dt. \end{aligned}$$

By substituting (5) in (3), we may now write

$$\sigma_2(\alpha, \gamma) = 1 - \theta + \sum_{k=1}^n w_k \gamma (\theta - \alpha_k),$$

and

$$\sigma_3(\alpha, \vec{\gamma}) = 1 - \theta + \sum_{k=1}^n w_k \gamma_k (\theta - \alpha_k).$$

Similarly, the level of security equals  $s_3(\alpha, \vec{\gamma}) = 1 - \sum_{k=1}^n w_k \gamma_k \alpha_k$ , which is again equal to  $\sigma_3(\alpha, \vec{\gamma})$  when  $\theta$  is set equal to zero. This proves the lemma.  $\square$

## A.4 Proof of Lemma 4

The expected costs read

$$\gamma(1 - \alpha)c_{\text{barge}} + (1 - \gamma(1 - \alpha))c_{\text{truck}}, \quad (24)$$

so normalized expected cost equal

$$\sigma_4(\alpha, \gamma) = 1 - \theta + \theta\gamma(1 - \alpha).$$

The expected fraction of containers that arrive in time at the customer reads

$$\begin{aligned} \rho_4(\alpha, \gamma) &= \gamma(1 - \alpha)G_{\text{barge}}(T - t_0) + \\ &\int_0^{t_\gamma} G_{\text{truck}}(T - t)f(t) dt + \int_{t_0}^T G_{\text{truck}}(T - t)f(t) dt. \end{aligned}$$

The level of security satisfies  $s_4(\alpha, \gamma) = 1$  as no ad hoc shipments need to be made, and it is equal to  $\sigma_4(\alpha, \gamma)$  for  $\theta = 0$ . This proves the lemma.  $\square$

## A.5 Proof of Theorem 5

We first prove two lemmas.

**Lemma 12** *Let  $g(t) \geq 0$  be decreasing and assume that  $M(t) = \int_0^t \mu(x) dx \geq 0$  for all  $0 \leq t \leq t_0$  and that  $M(t_0) = 0$ . Then*

$$\int_0^{t_0} g(x)\mu(x) dx \geq 0.$$

**Proof.** We may approximate  $g(t)$  by nonnegative decreasing step functions, so it suffices to show the lemma for  $g(t) = \sum_{k=1}^m g_k E_k(t)$ , where  $0 = a_0 < a_1 < \dots < a_m = t_0$  is a partition and  $E_k(t)$  equals 1 for  $a_{k-1} \leq t < a_k$  and equals 0 elsewhere. As a result,

$$\int_0^{t_0} g(x)\mu(x) dx = \sum_{k=1}^m g_k \{M(a_k) - M(a_{k-1})\} = g_m M(a_m) + \sum_{k=1}^{m-1} (g_k - g_{k+1})M(a_k) - g_1 M(0) \geq 0,$$

since  $g_k \geq g_{k+1} \geq 0$ ,  $M(a_k) \geq 0$ , and  $M(0) = 0$ . This proves the lemma.  $\square$

**Lemma 13** *If we put*

$$\gamma = \frac{1}{1-\alpha} \sum_{k=1}^n w_k \gamma_k (1-\alpha_k), \quad (25)$$

*then  $\rho_3(\alpha, \vec{\gamma}) \leq \rho_4(\alpha, \gamma)$ ,  $\sigma_3(\alpha, \vec{\gamma}) \leq \sigma_4(\alpha, \gamma)$ , and  $s_3(\alpha, \vec{\gamma}) \leq s_4(\alpha, \gamma)$ .*

**Proof.** If we assume that  $\gamma$  is defined as in (25), then

$$\sigma_4(\alpha, \gamma) - \sigma_3(\alpha, \vec{\gamma}) = \gamma\theta(1-\alpha) - \sum_{k=1}^n w_k \gamma_k (\theta - \alpha_k) \geq \theta \sum_{k=1}^n w_k (\gamma - \gamma_k) (1 - \alpha_k) = 0.$$

We now consider, using the expression for  $\gamma$  as in (25),

$$\begin{aligned} \rho_4(\alpha, \gamma) - \rho_3(\alpha, \vec{\gamma}) &= \int_0^{t_\gamma} G_{\text{truck}}(T-t) f(t) dt + \gamma(1-\alpha) G_{\text{barge}}(T-t_0) + \int_{t_0}^T G_{\text{truck}}(T-t) f(t) dt \\ &\quad - \sum_{k=1}^n w_k \gamma_k \int_{t_0}^T G_{\text{rctruck}}(T-t) f_k(t) dt - \sum_{k=1}^n w_k \gamma_k (1-\alpha_k) G_{\text{barge}}(T-t_0) \end{aligned}$$



$$\begin{aligned}
& - \sum_{k=1}^n w_k (1 - \gamma_k) \int_0^T G_{\text{truck}}(T - t) f_k(t) dt = \\
& \sum_{k=1}^n w_k \left\{ \int_0^{t_\gamma} G_{\text{truck}}(T - t) f_k(t) dt - (1 - \gamma_k) \int_0^{t_0} G_{\text{truck}}(T - t) f_k(t) dt \right\} \\
& + \sum_{k=1}^n w_k \gamma_k \int_{t_0}^T \{G_{\text{truck}}(T - t) - G_{\text{retruck}}(T - t)\} f_k(t) dt \geq \\
& \sum_{k=1}^n w_k \left\{ \int_0^{t_\gamma} G_{\text{truck}}(T - t) f_k(t) dt - (1 - \gamma_k) \int_0^{t_0} G_{\text{truck}}(T - t) f_k(t) dt \right\}.
\end{aligned}$$

If we put

$$h(t) = \sum_{k=1}^n w_k (1 - \gamma_k) f_k(t), \quad (26)$$

then  $0 \leq h(t) \leq f(t)$  and we get

$$\rho_4(\alpha, \gamma) - \rho_3(\alpha, \vec{\gamma}) \geq \int_0^{t_\gamma} G_{\text{truck}}(T - t) f(t) dt - \int_0^{t_0} G_{\text{truck}}(T - t) h(t) dt.$$

Let  $E_\gamma(t)$  be the function which equals 1 for  $0 \leq t < t_\gamma$  and equals 0 elsewhere, and define  $\mu(t) = f(t)E_\gamma(t) - h(t)$ . Then we obtain

$$M(t) = \int_0^t \mu(x) dx \geq 0, \quad 0 \leq t \leq t_0, \quad (27)$$

and that  $M(t_0) = 0$ . Indeed,

$$M(t_0) = \int_0^{t_\gamma} f(t) dt - \int_0^{t_0} h(t) dt = F(t_\gamma) - \sum_{k=1}^n w_k (1 - \gamma_k) F_k(t_0) =$$

$$(1 - \gamma)(1 - \alpha) - \sum_{k=1}^n w_k(1 - \gamma_k)(1 - \alpha_k) = 0.$$

Consequently we get for  $t_\gamma \leq t \leq t_0$

$$M(t) = \int_0^{t_\gamma} \mu(x) dx - \int_{t_\gamma}^t h(x) dx \geq \int_0^{t_\gamma} \mu(x) dx - \int_{t_\gamma}^{t_0} h(x) dx = M(t_0) = 0.$$

If  $0 \leq t \leq t_\gamma$ , we arrive at

$$M(t) = \int_0^t \mu(x) dx = \int_0^t \{f(x) - h(x)\} dx \geq 0.$$

The lemma is proved applying Lemma 12 to

$$\rho_4(\alpha, \vec{\gamma}) - \rho_3(\alpha, \gamma) \geq \int_0^{t_0} G_{\text{truck}}(T - t)\mu(t) dt. \quad \square$$

We now proceed with the proof of Theorem 5.

We may verify directly that  $\mathcal{E}_1 \preceq \mathcal{E}_2$  by showing that  $\rho_1 \leq \rho_2(\alpha, 0)$ ,  $\sigma_1 \leq \sigma(\alpha, 0)$ , and  $s_1 \leq s_2(\alpha, 0)$ . Observe that  $\rho_1 \leq \rho_2(\alpha, \gamma)$  need not hold true for  $\gamma > 0$ . Further,  $\mathcal{E}_2 \preceq \mathcal{E}_3$  follows trivially from  $\mathcal{A}_2 \subseteq \mathcal{A}_3$ . The set  $\mathcal{A}_2$  emerges as a subset of  $\mathcal{A}_3$  by considering the special case  $\vec{\gamma} = (\gamma, \gamma, \dots, \gamma)$ . The assertion  $\mathcal{E}_3 \preceq \mathcal{E}_4$  follows from Lemma 13. This proves the theorem.  $\square$

## A.6 Proof of Theorem 7

Fix  $0 \leq \lambda \leq \theta$  and write  $z(\lambda) = (\rho_0 - \frac{\theta - \lambda}{\Psi(\lambda)}, 1 - \lambda)$ . To prove the theorem, we need to show that  $z(\lambda)$  for  $0 \leq \lambda \leq \theta$  constitute all maximal elements in  $\mathcal{A}_2$ . There are two

cases to be discerned.

*Case 1:*  $\Psi(\lambda) = \Phi(\lambda)$ . In this case,

$$z(\lambda) = (\rho_2(\lambda, 1), \sigma_2(\lambda, 1)) \in \mathcal{A}_2$$

Next, write  $x = (\rho_0 - \gamma\rho(\alpha), 1 - \theta + \gamma(\theta - \alpha))$  and assume that  $x \geq z(\lambda)$ . This implies  $1 - \theta + \gamma(\theta - \alpha) \geq 1 - \lambda$ , so  $\lambda \geq \gamma\alpha + (1 - \gamma)\theta$ . Further,  $\rho_0 - \gamma\rho(\alpha) \geq \rho_0 - \frac{\theta - \lambda}{\Psi(\lambda)}$  or  $\gamma\rho(\alpha) \leq \frac{\theta - \lambda}{\Psi(\lambda)}$  or  $\gamma\Psi(\lambda) \leq \frac{\theta - \lambda}{\rho(\alpha)} \leq \frac{\gamma(\theta - \alpha)}{\rho(\alpha)}$ . In other words,  $\Psi(\lambda) \leq \Phi(\alpha)$  which can only be true when  $\lambda = \alpha$  and  $\gamma = 1$ , so  $x = z(\lambda)$ . This implies that  $z(\lambda)$  is a maximal element in  $\mathcal{A}_2$ .

*Case 2:*  $\Psi(\lambda) > \Phi(\lambda)$ . We may write  $\Psi(\lambda) = \Phi(\alpha)$  for some  $0 \leq \alpha < \lambda$ . Write  $\lambda = \gamma\alpha + (1 - \gamma)\theta$  for some  $0 < \gamma < 1$ . Then, with  $1 - \lambda = 1 - \theta + \gamma(\theta - \alpha)$ , we get

$$z(\lambda) = \left(\rho_0 - \frac{\theta - \lambda}{\Psi(\lambda)}, 1 - \lambda\right) = \left(\rho_0 - \frac{\theta - \lambda}{\Phi(\alpha)}, 1 - \theta + \gamma(\theta - \alpha)\right) \in M_\alpha \subseteq \mathcal{A}_2.$$

Write  $x = (\rho_0 - \gamma\rho(\beta), 1 - \theta + \gamma(\theta - \beta))$  and  $x \geq z(\lambda)$ . This implies  $\rho(\alpha) \geq \rho(\beta)$  and hence  $\alpha \leq \beta$  since  $\rho$  is decreasing. On the other hand,  $1 - \theta + \gamma(\theta - \beta) \geq 1 - \theta + \gamma(\theta - \alpha)$  so  $\alpha \geq \beta$ . This implies  $\alpha = \beta$  and that  $z(\lambda)$  is a maximal element in  $\mathcal{A}_2$ .

Finally, we need to show that all maximal elements have been described. This follows from the fact that for  $(x_1, x_2) \in \mathcal{A}_2$ , we have the following assertions. First of all, if  $x_2 \leq 1 - \theta$ , then  $x_1 \leq \rho_0$ , so  $(x_1, x_2) \leq (\rho_0, 1 - \theta)$ . Secondly, if  $1 - \theta \leq x_2 \leq 1$ , then according to the analysis given above,  $(x_1, x_2) \leq z(\lambda)$  with  $\lambda = 1 - x_2$ . This proves the theorem.  $\square$

## A.7 Proof of Proposition 8

It can readily be verified that the mapping  $(\sigma_3, \rho_3)$  is affine with  $\rho_3$  as given by (7) and  $\sigma_3$  as given by (6). For  $k = 1, \dots, n$ , we put

$$\rho_k(\alpha) = \int_0^T G_{\text{truck}}(T-t)f_k(t) dt - F_k(t_0)G_{\text{barge}}(T-t_0) - \int_{t_0}^T G_{\text{rctruck}}(T-t)f_k(t) dt,$$

which is decreasing in  $\alpha$  and  $\rho_k(\theta) > 0$ . Write  $\vec{e} = (1, 1, \dots, 1)$  and consider the unit  $n$ -cube  $\{\vec{x} \in \mathbb{R}^n : \vec{0} \leq \vec{x} \leq \vec{e}\}$ . As the polygon  $M_\alpha = \{(\rho_3(\alpha, \vec{\gamma}), \sigma_3(\alpha, \vec{\gamma})) : \vec{0} \leq \vec{\gamma} \leq \vec{e}\}$  is the image of an affine transformation of the  $n$ -cube, the extreme points of the polygon are the images under the mapping of the extreme points of the  $n$ -cube. Moreover, the edges of the polygon are the images of edges of the  $n$ -cube. Consequently, we may construct the Pareto frontier of  $M_\alpha$  by considering the slopes of the polygon edges

$$\Phi_k(\alpha) = \frac{\theta - \alpha_k}{\rho_k(\alpha)},$$

and establish distinct  $k_1, k_2, \dots, k_n$  such that

$$\Phi_{k_1}(\alpha) \geq \Phi_{k_2}(\alpha) \geq \dots \geq \Phi_{k_n}(\alpha).$$

The extreme points that define the piecewise linear Pareto frontier of  $M_\alpha$  are given by

$$\begin{aligned} \vec{a}_0 &= (\rho_0, 1 - \theta), \\ \vec{a}_p &= \left( \rho_0 - \sum_{i=1}^p w_{k_i} \rho_{k_i}(\alpha), 1 - \theta + \sum_{i=1}^p w_{k_i} (\theta - \alpha_{k_i}) \right), \quad p = 1, \dots, n, \\ \vec{a}_n &= (\rho_0 - \rho(\alpha), 1 - \alpha). \end{aligned} \quad (28)$$

We therefore define the function

$$\tilde{\sigma}_3(\alpha, \gamma) = 1 - \theta + \gamma(\theta - \alpha), \quad (29)$$

and with  $r_p = \rho_0 - \sum_{i=1}^p w_{k_i} \rho_{k_i}(\alpha)$  for  $p = 1, \dots, n$ , we define

$$\tilde{\rho}_3(\alpha, \gamma) = r_p + \frac{\gamma - g_p}{g_{p+1} - g_p} (r_{p+1} - r_p), \quad g_p \leq \gamma \leq g_{p+1}, \quad (30)$$

where  $0 = g_0 < g_1 < \dots < g_n = 1$  are given by

$$g_p = \frac{\sum_{i=1}^p w_{k_i} (\theta - \alpha_{k_i})}{\sum_{i=1}^n w_{k_i} (\theta - \alpha_{k_i})} = \frac{\sum_{i=1}^p w_{k_i} (\theta - \alpha_{k_i})}{\theta - \alpha}. \quad (31)$$

In this manner, the Pareto frontier of  $M_\alpha$  is given by  $\{(\tilde{\rho}_3(\alpha, \gamma), \tilde{\sigma}_3(\alpha, \gamma)) : 0 \leq \gamma \leq 1\}$  for each  $0 \leq \alpha \leq \theta$ . The proposition is proved.  $\square$

## A.8 Proof of Theorem 9

Fix  $0 \leq \lambda \leq \theta$  and put  $z(\lambda) = (\rho_3^*(\lambda), 1 - \lambda)$ .

*Case 1:*  $\rho_3^*(\lambda) = \tilde{\rho}_3(\lambda, 1)$

Then  $z(\lambda) = (\tilde{\rho}_3(\lambda, 1), \tilde{\sigma}_3(\lambda, 1)) \in M_\lambda \subseteq \mathcal{A}_3$ . If  $x = (\tilde{\rho}_3(\alpha, \gamma), \tilde{\sigma}_3(\alpha, \gamma))$  satisfies  $x \geq z(\lambda)$ , then  $1 - \theta + \gamma(\theta - \alpha) \geq 1 - \lambda$ , so  $\lambda \geq \gamma\alpha + (1 - \gamma)\theta$ . Further,  $\tilde{\rho}_3(\alpha, \gamma) \geq \tilde{\rho}_3(\lambda, 1) = \rho_3^*(\lambda)$ . On the other hand, since  $\gamma \geq \frac{\theta - \lambda}{\theta - \alpha}$ , we get  $\tilde{\rho}_3(\alpha, \gamma) \leq \tilde{\rho}_3(\alpha, \frac{\theta - \lambda}{\theta - \alpha}) \leq \rho_3^*(\lambda)$  since  $\alpha \leq \lambda$ . This implies  $\tilde{\rho}_3(\alpha, \gamma) = \tilde{\rho}_3(\lambda, 1) = \rho_3^*(\lambda)$  and  $\gamma = \frac{\theta - \lambda}{\theta - \alpha}$ , so  $\tilde{\sigma}_3(\alpha, \gamma) = 1 - \lambda$  and hence  $x = z(\lambda)$ . We have obtained that  $z(\lambda) \in \mathcal{A}_3$  is a maximal element.

*Case 2:*  $\rho_3^*(\lambda) = \tilde{\rho}_3(\alpha, \frac{\theta - \lambda}{\theta - \alpha})$  for some  $0 \leq \alpha \leq \lambda$

Then  $z(\lambda) = (\tilde{\rho}_3(\alpha, \gamma), \tilde{\sigma}_3(\alpha, \gamma)) \in M_\lambda \subseteq \mathcal{A}_3$  with  $\gamma = \frac{\theta - \lambda}{\theta - \alpha}$ . If  $x = (\tilde{\rho}_3(\beta, \delta), \tilde{\sigma}_3(\beta, \delta))$  satisfies  $x \geq z(\lambda)$ , then  $1 - \theta + \delta(\theta - \beta) \geq 1 - \lambda$ , so  $\lambda \geq \delta\beta + (1 - \delta)\theta$ , and  $\tilde{\rho}_3(\beta, \delta) \geq \tilde{\rho}_3(\alpha, \gamma) = \rho_3^*(\lambda)$ . On the other hand, since  $\delta \geq \frac{\theta - \lambda}{\theta - \beta}$ , we get  $\tilde{\rho}_3(\beta, \delta) \leq \tilde{\rho}_3(\beta, \frac{\theta - \lambda}{\theta - \beta}) \leq \rho_3^*(\lambda) = \tilde{\rho}_3(\alpha, \frac{\theta - \lambda}{\theta - \alpha})$  since  $\beta \leq \lambda$ . This implies  $\tilde{\rho}_3(\beta, \delta) = \rho_3^*(\lambda) = \tilde{\rho}_3(\alpha, \gamma)$ , and  $\tilde{\sigma}_3(\beta, \delta) = 1 - \lambda = \tilde{\sigma}_3(\alpha, \gamma)$  and hence  $x = z(\lambda)$ . We have obtained that  $z(\lambda) \in \mathcal{A}_3$  is a maximal element. We may argue as in Theorem 7 that we have captured all maximal elements in  $\mathcal{A}_3$  in this manner.  $\square$

## A.9 Proof of Proposition 10

In case  $(\rho_3^*(\lambda), 1 - \lambda) \in \mathcal{E}_3$ , is given, we construct the corresponding Pareto optimal solution  $(\alpha, \vec{\gamma})$  as follows. First of all, we determine  $\alpha$  by  $(\rho_3^*(\lambda), 1 - \lambda) = \tilde{\rho}_3(\alpha, \gamma)$  with  $\gamma = \frac{\theta - \lambda}{\theta - \alpha}$ . We then construct  $\vec{\gamma} = (\gamma_1, \dots, \gamma_n)$  by either identifying  $0 \leq p < n$  such that  $g_p \leq \gamma < g_{p+1}$  or by identifying that  $\lambda = \alpha$ , i.e.  $\gamma = 1$ . Based on the proof of Proposition 8, we may conclude that  $\vec{\gamma}$  can be established as identified in the statement of the proposition, with the special case of  $\gamma = 1$  giving rise to  $\vec{\gamma} = (1, \dots, 1)$ .  $\square$

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