Essays on Consumer Search and Interlocking Directorates

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Essays on Consumer Search and Interlocking Directorates

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Preface

In 1999 I started my undergraduate studies at the Erasmus University Rotterdam. It did not take long for me to get increasingly fascinated by the idea of doing research and working in academia. Therefore, after obtaining my MSc degree, it was an easy choice to do a PhD. And now that I almost finished my dissertation I must say that the past four and a half years have only increased my fascination. I am afraid that I am hooked on forever. Still, though this all sounds as a smooth process, writing a PhD thesis almost never is, and the writing of my thesis was no exception to this. I am indebted to many people who helped me with my research, inspired me to pursue my dreams and lent me a listening ear now and then. I would like to take this opportunity to give my thanks to them.

First of all, thanks to my promotor Maarten Janssen for offering me a PhD position and guiding me through the process of writing a PhD thesis. Somehow you always seemed to have time for me, even when I knew that you were busy. Also many thanks for a short extension of my contract. Thanks also go to Philip Hans Franses for being my co-promotor and the co-author of Chapter 5 of this thesis. My thanks are extended to the members of the small committee, who were willing to evaluate this thesis and agreed to take part in the defense. Also thanks to the members of the large committee, who are willing to take part in the defense as well. One member of the small committee should get a special mention. Jose Luis Moraga offered me to come to Groningen to do a postdoc there. Thank you so much for the opportunity you gave me, and I am looking forward to work together!

Writing a PhD thesis can be lonely experience from time to time, but I have been blessed with wonderful colleagues at the 9th floor. The first to mention are my roommates over the years: Michiel, Matthijs and Marcel. Michiel, although I still do not share your taste in music, I have always enjoyed your company. The time I spent together with Matthijs and Marcel has been shorter, but it was always a pleasure to have you around. Many dim research days were brightened by the 12.15 lunch group. Thank you Chen, Diana, Francesco, Gus, Josse, Milan, Nuno, Sandra and Silvia for very nice breaks. Also

thanks to Bram, Joost, Merel, Remco, Rene and Robin. Although you were in the 'competing' 12.00 lunch group, I enjoyed our chats, and even an occasional lunch. Nita, you introduced me to the joys of sports. By distracting me from my research from time to time you probably contributed more to this thesis than any of my other colleagues. Also many thanks to Jan Brinkhuis for restoring my faith in myself from time to time.

Two persons should get a special mention. Haikun, I am truly blessed with having a friend like you. Thank you so much for being one of my paranimphs. Haris, we stumbled through TI's master program together, and were colleagues for several more years. You progressed through the PhD process at an amazing speed, and even now you are working incredibly hard. You set a good example for me. Thank you for your friendship, and for letting me be paranimph on your special day. It is a pity Indonesia is too far away to drop by at my defense, but luckily there always is email and chat.

Many practical and administrative tasks were handled by the TI secretariat. Thanks to Carien, Carine, Dave, Ine and Gerald for their assistance. Several conference visits were made possible by financial contributions of the Vereniging Trustfonds EUR.

Living close to my parents and other family means that they were closely involved in my PhD project. My parents were always interested in the topics of my research, and especially my mother was a great 'guinea pig' to try to explain my research at a non-scientific level. Thank you for all your support. Two years ago my oldest brother also decided to do a PhD in micro-economics. Sometimes it was a bit weird to have your brother doing the same thing as you did, a few office doors away from you. But it also made for very nice discussions at home. Arjan, I hope you will enjoy writing your thesis as much as I did! My youngest brother has decided that two economists in one family is more than enough and opted for a completely different field. Wilco, I am sorry for all the economists discussions you had and still have to cope with. Luckily you have enough humor to even distract Arjan and me from our serious discussions. Many thanks also go to my uncle for being one of my paranimphs.

Finally, I would like to express my gratitude to God, my Father and Creator. He gave his Son so I might live and guides me through his Spirit. This thesis could only be written because He gave me the talents and abilities. He alone deserves all glory and praise.

Groningen, September 2008

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Chapter 1

Introduction

1.1 Motivation

For all economic agents, from the consumer to the multinational, information is crucial to make good decisions. A consumer who, for instance, wants to buy a new camera seeks information on the characteristics of the cameras that are available on the market, needs to know which shops sell the type of camera he is interested in and wants to know the prices and the conditions of sale to find the best deal. On the other side of the market, firms seek information on for instance competitor strategies and market conditions to optimize their own strategies. Moreover, if a firm decides on a major strategy change it is very useful if one or more members of the management board have inside information on and experience with the new strategy.

Information, however, is costly to obtain. For a consumer it takes time and effort to find out which of the available cameras best suits his taste and where he can buy this camera at the lowest price and under the best conditions of sale. Firms likewise need to invest time and money in gaining market knowledge, probably hiring an external consultant to obtain information and to implement changes. Every economic actor therefore has to balance the costs and the benefits of obtaining (additional) information. This consideration is the main theme in this thesis.

This thesis consists of two parts. The first part uses theoretical models to investigate how the search for information by consumers affects the strategy of retail shops. In this part of the thesis, information flows from shops to consumers. Consumers incur costs when searching for this information and shops can help the consumers by providing information or by reducing the consumers' search costs. The shops need to balance the benefits and costs of information provision, while the consumers need to balance the benefits and costs of searching for information. The next section of this introduction will discuss this part of the thesis in more depth.

The second part of this thesis empirically investigates the information flows between large firms brought about by director ties. A director with multiple board memberships generally brings more information and more experience to the board, but this information is costly since a director with multiple board memberships is also more busy. The final effect on firm performance is therefore uncertain. Section 3 of this introduction discusses this part of the thesis in more depth.

1.2 Part I. Consumer search

The notion of consumer search dates back to Stiglers 1961 seminal paper on the economics of information. Before Stigler, economists mostly assumed that consumers have perfect knowledge of all prices and varieties in the market. As Stigler stated it, 'knowledge occupies a slum dwelling in the town of economics' (p. 213). In his paper, he notes that in real life, information plays an important role in consumer markets and that partly informed consumers have a profound effect on the pricing strategy of shops. The famous paper by Bertrand (1883) had shown that when all shops sell homogenous goods and all consumers are fully informed of the prices, price competition drives all the prices down to marginal costs. But, as Stigler argues, when consumers are incompletely informed the price competition between shops is softened and this leads to both above cost pricing and price dispersion.

Stigler points to two distinct mechanisms that help to bridge the information gap between shops and consumers: consumer search and (informative) advertising. After Stigler's seminal work, both mechanisms have been analyzed extensively. In the consumer search literature some seminal papers are Diamond (1971), Burdett and Judd (1983) and Stahl (1989). In the informative advertising literature seminal papers are for instance Butters (1977) and Stahl (1994). As in the paper by Stigler, most of these papers analyze search and advertising in isolation. Consumer search and advertising, however, also interact with each other in providing information to the consumers. The search by consumers will have an effect on the advertising strategy of shops and advertisements sent by the shops will, in turn, affect the search strategy of consumers. In such a setting several questions arise. For instance, who will take the main burden of information provision? Will the shops invest in advertising such that the consumers are provided with free information and do not need to search, will the consumers search so much that shops do not need to advertise, or will there be some balance with both consumer search and advertising? And what will be the final effect on the prices? Will there be sufficient information in the market to drive prices down to production costs? Butters (1977) makes a first attempt to analyze these questions, but, due to 'certain unpalatable complications', in his model consumers do not choose an optimal search strategy. The only other paper jointly analyzing consumer search and (informative) advertising is Robert and Stahl (1993). In Chapters 2 and 3 of this thesis I add to this literature by once again analyzing the interaction between search, informative advertising and pricing. Whereas Robert and Stahl assume that advertising does not lower the search costs of consumers, in Chapter 2 I assume that advertising reduces the search costs to zero. This seemingly small change in assumptions has a large influence on the results. To analyze this in more detail, in Chapter 3 I consider a model where the reduction of search costs by advertising is endogenously determined, reinforcing some of the results of Chapter 2.

Advertising is not the only way in which shops can influence the search costs of consumers. Chapter 4 of this thesis considers firms' location choice, where firms can either choose to locate in a mall with no direct competitors or in a mall with existing direct competitors such that consumers' costs of searching the firm and its rivals are reduced. As with advertising, the search strategy of consumers will influence the location choice of shops and the location choice of shops will, in turn, affect the search strategy of consumers. Because of this interaction, questions arise on the final division of information provision and the ultimate pricing and locating strategy. For instance, could locating close to a competitor be profitable? And if there is a cluster of competitors located closely together, will then all consumers prefer to go to that cluster? And will prices in a shopping mall be lower than the price in an isolated shop? These questions have been analyzed before, see e.g. Wolinsky (1983), Dudey (1990) and Fischer and Harrington (1996). The model in Chapter 4 is, however, the first to explicitly analyze the role of prices in a model combining consumer search and location choice.

Chapters 2, 3 and 4 together form Part I of this thesis. These chapters and their contribution to the literature will now be discussed in more detail.

In Chapter 2, which is based on Janssen and Non (2008), a homogenous good duopoly is considered where consumers search sequentially and advertising decreases the costs to search the advertising shop to zero. For many consumer goods the search behavior of consumers is best described by sequential search, where after each search the consumer decides whether to search another shop or to stop searching. The threat of continued search that is present in a sequential search model has a huge impact on the pricing behavior of shops. The amount of advertising, in turn, depends on the prices that can be asked. The analysis shows that when the search costs are low, and consequently the threat of continued search is important, the prices are so low that advertising is not profitable. When the search costs are relatively high, however, advertising is profitable. The combination of search and advertising in general does not lead to fully informed consumers. As a consequence, as long as both the search costs and the advertising costs are strictly positive, prices are always above marginal costs.

The combination of sequential search and advertising leads to three new conclusions. First, Chapter 2 shows that advertising and search are substitutes to each other. For low enough search costs or high enough advertising costs consumers search and shops do not advertise while for high enough search costs shops advertise but consumers do not search at all. For intermediate search costs shops advertise and consumers search. In this case search and advertising are substitutes in the sense that when the advertising intensity decreases because of, say, increasing advertising costs, the search intensity increases. Moreover, when the search intensity decreases because of, say, increasing search costs, the advertising intensity increases. It does not come as a surprise that advertising and search are substitutes since they are both aimed at bridging the information gap between shops and consumers. The model in Chapter 2 of this thesis is, however, the first that finds this relation between search and advertising.

Second, prices become non-monotonic in search costs. Without advertising, prices increase in search costs since higher search costs make consumers less willing to continue search. When shops, however, have the possibility to advertise, an increase in search costs not only makes consumers less willing to continue search but also increases the advertising intensity. Search and advertising therefore have opposite effects on prices, and the final effect can go both ways.

Third, advertised prices can be higher than non-advertised prices. This is caused by the assumption that advertising lowers the search costs of the consumers and can best be explained by an example. Suppose a shop advertises a price of 10 euros, while its competitor does not advertise and sets a price of 9 euros. At first sight, a consumer could best visit the non-advertising shop, since there the price is lower. But once a consumer incurs at least one euro of search costs when visiting the non-advertising shop, he is better off when visiting the advertising shop at no search costs. As this example suggests, an advertising shop can set a higher price than a non-advertising shop and still attract consumers.

The result that advertised prices can be higher than non-advertised prices is further examined in Chapter 3 of this thesis, which is based on Janssen and Non (forthcoming). In Chapter 2 the result on advertised prices was obtained by exogenously imposing that advertising lowers the search costs. In Chapter 3 this assumption is endogenized by adding an initial phase in the model in which shops decide on whether or not to shelve the product the consumer wants to buy. Shops who do not shelve the product will never invest in advertising, and so once a consumer has received an advertisement he is certain that the shop sells the product. When visiting a shop that has not advertised, there is a probability that the shop does not sell the product at all. The costs of visiting a shop are always the same, independent of whether the shop has advertised or not. When not all shops shelve the product, however, the expected costs of finding a shop that shelves the product are lower when an advertising shop is visited than when a non-advertising shop is visited. The difference in expected costs depends on the probability that a non-advertising shop has shelved the product. If this probability is low the difference in expected costs is large and an advertising shop could ask a higher price than a non-advertising shop. In this chapter it is shown that indeed the search, advertising and shelving costs can be chosen in such a way that the probability that a shop shelves the product is low and high advertised prices are a unique equilibrium outcome.

Chapter 4 of this thesis combines sequential consumer search with the location choice of homogenous shops. The main question in this chapter is why shopping malls exist. Shopping malls are a widespread phenomenon, but everyday experience also suggests that malls reduce the search costs of consumers. One consequence of these lower search costs is that the competition between shops in a mall is increased and prices are lower. The model in this thesis captures this feature of shopping malls, but nevertheless shows that shops can have an incentive to form a shopping mall.

To capture the effect of shopping malls on search costs, the search costs in the model consist of two different elements. First, consumers incur costs when entering a shop. These costs can be interpreted as the costs of walking through the shop to find the right product, finding the price of the product and the conditions of sale, possibly waiting for a shop assistant to help, etc. Second, consumers incur travel costs when travelling between different malls or isolated shops. This cost structure ensures that visiting two or more shops in the same mall comes at less search costs than visiting the same number of isolated shops.

As mentioned before, one of the main findings is that shops can have an incentive to locate together and form a shopping mall. The main reason for this is that a shopping mall not only increases competition and decreases prices, but also increases the sales of mall shops. The analysis of the model shows that there are two distinct mechanisms behind the increase in sales. First, the lower prices make consumers active who otherwise would have stayed at home. And, second, shops in a mall attract more consumers than isolated shops. Simulations show that the final effect of the lower prices and higher sales on profits can be positive, making the formation of a mall a dominant strategy.

The finding that locating together in a shopping mall can be a dominant strategy is not new. Previous papers combining consumer search and location choice in a pricing model, however, all assume heterogenous goods, while the model in Chapter 4 assumes homogenous goods. In a model with heterogenous goods consumers have an additional reason to visit a shopping mall. When visiting a mall, a consumer has a larger probability of finding a product that closely matches his taste. This has a large effect on the sales of a mall shop. Chapter 4 shows that this additional effect is not necessary to obtain an equilibrium with a shopping mall. The uneven distribution of consumers over mall shops and isolated shops and the increase in active consumers together increase the sales enough to offset the decrease in prices. Moreover, most of the previous research on consumer search and location choice mentions only the uneven distribution of consumers or mentions only the increase in active consumers. Chapter 4 of this thesis is the first paper to rigorously investigate both effects.

The analysis also gives a striking result on the pricing strategy of isolated shops when there also is a mall. In that case an isolated shop will either set a price strictly above the highest mall price or will set a price below the lowest mall price. This can be interpreted as an isolated shop generally pricing above the mall prices, but sometimes offering a large discount. Such a type of price distribution is seldom found in models where shops compete in prices. The result is driven mostly by the travel costs of the consumers. Once a consumer is in an isolated shop he has to incur travel costs to leave this shop and this makes the consumer willing to pay a somewhat higher price once he is at the isolated shop. To ensure that some consumers visit an isolated shop in the first place, these shops sometimes offer a discount. This will attract consumers to the isolated shops who hope to be lucky and find a discounted price.

1.3 Part II. Director ties

Director ties, or interlocks, occur when a director has board positions in two or more firms. Interlocks have a long history, and have sparked public interest ever since they exist, until the present day. A common concern among the public is the conflict of interests when a director sits in the boards of competing firms. Moreover, many people are concerned about a possible concentration of power in the hands of only a small group of directors. These general concerns are reflected in several laws and regulations, for instance in Section 8 of the 1914 U.S. Clayton Antitrust Act. The Clayton Act prohibited several practices that were deemed anticompetitive and interlocks between competing corporations were one of those practices. A more recent example of a regulation of interlocks is the Dutch 2004 corporate governance code of good conduct, the so called code Tabaksblat, that limits the number of board positions a director might have.

Although the public concerns mentioned above are interesting in their own right, most economic and business research on interlocks has concentrated on the effects of interlocks on firm performance. The fact that virtually all large firms are interlocked suggests that the existence of interlocks is not just a coincidence and that there is some reason why firms choose to interlock with each other. Many researchers have suggested that interlocks have a positive influence on firm performance and several reasons have been proposed why this could be so. Mizruchi (1996) provides an overview. Most of this research presumes that interlocking directors have more experience and information than directors who have only one directorship, and interlocking therefore will help firms to coopt and monitor each other. In a sense, interlocking directors provide a network between firms along which information can flow. In this Introduction I will refer to this effect as the network effect. One strong example of this is provided by Davis (1991). In his paper, Davis examines the adoption of the poison pill, or "shareholder rights plan", a (by that time) new type of takeover defense that makes a hostile takeover more difficult by greatly increasing the costs a potential acquirer would have to pay. His research shows that the adoption of the poison pill indeed follows the interlock network, with more central firms adopting the poison pill earlier. Moreover, firms also adopt the poison pill earlier if they have ties with other firms that have already adopted the poison pill.

Recently, economists have started to investigate a possible negative side effect of interlocks: busyness. As Ferris et al. (2003) state it, directors with multiple directorships can be 'too busy to mind the business'. This effect counters the potential network effect of interlocks. Another negative side effect of interlocks that is sometimes mentioned in the literature (see, e.g., Useem (1984)) but has not yet been investigated empirically, is upper class cohesion. Interlocking directors often have the same background and work experience, making the board of directors a cohesive group, potentially suffering from groupthink and a lack of diversity.

In Chapter 5 of this thesis, which is based on Non and Franses (2007), I empirically investigate the effect of interlocks on firm performance in The Netherlands. As mentioned before, the main theme of this thesis is the balance between benefits and costs of information. In the case of interlocks, benefits are provided by the network effect and costs are provided by the busyness and cohesive group effects. The total effect of interlocks on firm performance is determined by all three effects. Untangling these effects is complicated, but the total effect of interlocks on firm performance will show which of the effects is strongest.

For the analysis, I use a panel data set on 101 Dutch listed firms in the period 1994-2004. This data set is unique in two respects. First, even though Dutch firms since a long time have formed a dense interlocking network, empirical research on The Netherlands is scarce. Second, most empirical research on the influence of interlocks on firm performance uses cross-sectional data. Cross-sectional data leads to a causality problem: a positive (negative) relation between number of interlocks and performance could exist because interlocks influence performance, but could also exist because a good (bad) performance attracts more interlocks. A panel data set helps to solve this problem, because it allows an investigation of the long term effects of interlocks.

The analysis in Chapter 5 uses five different measures of firm performance. The first part of the analysis addresses the question whether the number of interlocks of a firm has a predominantly positive or predominantly negative effect on performance. We find that for two of the performance measures the estimated effect is negative and significant, while the other three performance measures do not give significant results. This negative effect is not unexpected. Dutch directors meet each other regularly outside the boardroom in selective informal meetings and therefore the interlock network is not expected to provide much new information to a firm. Busyness and a lack of diversity, however, are serious concerns in The Netherlands.

Given this weak evidence of a negative relationship, the second part of the analysis concerns the effects of busyness and group cohesion. Since we only have data on board composition and firm performance, it is not easy to find good measures for these variables. Based on the advice in the Code Tabaksblat, which limits the number of directorships to five, we define a director as busy when he has four or more directorships. The reason to deviate slightly from the advice in the Code Tabaksblat is that this code counts a chair position twice while we count it once. Moreover, we only have data on listed firms, while a director could also have a board position in non-listed firms or government organizations. We define a director as upper-class if he has at least two directorships. This definition is not based on any scientific evidence, but several books and articles in the Dutch media suggest that directors with two or more directorships can be seen as member of a cohesive group of upper-class directors, see, e.g., Van Hezewijk (1986, 1988).

We measure the busyness of the board of the firm by the percentage of busy directors in the board. Regressions show that all performance measures are negatively affected by busyness of the board, but for only two of the performance measures the estimates are significant. Similarly, the cohesiveness of the board is measured by the percentage of upper-class directors in the board. The most likely effect of this variable on performance is an inverse u-shape. Having one or two upper-class directors on the board should help to increase performance, while the board still is diverse enough to prevent groupthink. When the number of upper-class directors however grows too large, the board is likely to turn into a cohesive group and performance should be affected negatively. The data show limited support for this hypothesis. For two of the performance measures the estimates are significant and take the hypothesized signs, but the other performance measures the estimates are nonsignificant and sometimes take the wrong sign as well.

Concluding, we do not find any evidence that in The Netherlands interlocks influence firm performance positively. This, however, does not necessarily imply that there is no positive network effect. The analysis in Chapter 5 only shows that the possible network effect is outweighed by busyness and upper class cohesion. In fact, the results on upper class cohesion do suggest that there is some network effect: there is weak evidence that having a small number of experienced directors on the board increases performance.

Part I

Consumer search

Chapter 2

Advertising and consumer search in a duopoly model

2.1 Introduction

Consumers and firms do not find each other costlessly in the marketplace. Consumers spend time, money and effort searching to find firms that offer the price and quality that best suits their tastes. Firms, on the other hand, spend money on trying to attract (new) consumers. There is a considerable amount of economic literature on both advertising (see, e.g., the seminal paper by Butters (1977)) and search (see, e.g., seminal papers by Diamond (1971), Burdett and Judd (1983) and Stahl (1989)) separately, but very little on the interaction between advertising and search activities.

In this chapter we study a homogeneous goods duopoly model where firms choose an advertising intensity and a price they charge for their product. We consider a linear advertising technology where the cost of reaching an additional consumer is independent of the fraction of consumers already reached. This allows us to get closed-form solutions. After (not) having observed firms' advertisements, consumers decide whether or not to search for (more) prices in a sequential way, i.e., they first choose whether or not to obtain one price quotation and after having seen the result of this price search they decide whether to continue searching or not. As some consumers want to search despite the fact that they are already informed about one firm's price, we allow consumers to be heterogeneous in their search costs: one group of consumers has a positive search cost, whereas another group has zero search cost either because they enjoy shopping or have a negligibly small opportunity cost of time or because they use search engines.

In the context where one allows for both search and advertising, the interpretation of the search cost parameter deserves some attention. In general, search costs consist of (at least) two components: the *cost of visiting* a shop knowing that the shop carries the product the consumer wants to buy and the *cost of finding* or searching for a shop that carries the product. A firm's advertisement not only informs a consumer of the price the firm charges, it also informs the consumer that the firm carries the product, *i.e.*, it (importantly) eliminates the second component of search costs. As this general formulation is difficult to analyze in detail, we consider the extreme situation, where the costs of visiting a shop are negligibly small compared to the costs of finding a shop that sells the product. The role of advertising is to inform consumers that the shop sells the product and at which price.

We arrive at the following results. First, there is no consistent relationship between advertised and non-advertised prices. In particular, contrary to conventional wisdom advertised prices may be higher than non-advertised prices. The intuition for this result may be understood as follows. In our model, receiving an ad means that a consumer does not have to pay search costs, and so advertised prices can be fairly high. Furthermore, the existence of shoppers is relatively more important for non-advertising firms than for advertising firms as advertising firms reach out for many more consumers, whereas non-advertising firms only receive those consumers that actively search themselves. As shoppers create competitive pressure, this creates a tendency for non-advertised prices to be lower.

Second, in our model advertising and search are 'substitutes' meaning that, roughly speaking, if for a certain parameter constellation the equilibrium has firms advertising a lot, then consumers search relatively little, and vice versa. This result is quite intuitive as the main goal of search and of advertising is the same, namely to overcome the problem that without information, there cannot be any beneficial form of exchange. If one side of the market is willing to pay for overcoming the information gap, then the other side of the market can free ride.

Third, expected prices are non-monotonic in changes in the search cost parameter, but when search costs approach zero, equilibrium prices converge to the competitive price level. To understand the reason why expected prices are non-monotonic, it is important to understand the reason why there is price dispersion in this model. Price dispersion arises from the fact that in equilibrium there are different types of consumers: those that are informed of all prices (or at least more than one firm's price) and those that have only observed one price (either through advertisement or through search). Over the last group of consumers, firms have monopoly power, but there is competition for the first group of consumers. Price dispersion is the way firms balance these two forces. When search cost decline, it is natural for consumers to search more *ceteris paribus*. This forces firms to lower their prices. On the other hand, when consumers search more, and with lower prices, firms have an incentive to lower their advertising intensity and thereby to increase prices (as *ceteris paribus* there are fewer consumers who make price comparisons). In some cases, the first tendency is larger than the second; in other cases, the second tendency is larger.

Bakos (1997) argues that firms have an incentive to raise search costs since this will lead to higher prices and profits. The result above shows that this need not be the case if one takes advertising into account. The underlying mechanism of two contrasting forces, less search and more advertising, has some similarity to the literature on switching costs. Initially, one would argue that higher switching costs give firms more market power and so raise prices. Firms, however, also have an incentive to compete for market shares, since consumers who are 'locked in' can be profitably exploited. The result of these competing forces can go both ways: switching costs can lead to both higher and lower prices. See e.g. Klemperer (1987) and Dube et al. (2006).

Robert and Stahl (1993) is the first[1] paper analyzing the interaction between search and advertising incentives. Their model differs in two important ways. First, all consumers are *ex ante* identical even with respect to their search costs. Second, Robert and Stahl interpret the search cost parameter as the cost of visiting a firm. In their model therefore consumers also have to pay search costs when buying from a firm they got an advertisement from, neglecting the fact that receiving an ad reduces the cost of finding a shop that carries the product. In their case, the role of advertising is to inform consumers of the price only.[2] There are also many important qualitative differences in results obtained. For example, in Robert and Stahl's model search and advertising turn out to be complements to each other in the sense that for those parameter values for which firms do not advertise, consumers do not search either (autarky). Moreover, whenever firms do advertise, consumers also are engaged in search activities. Above, we explained why we think it is more natural to (partly) think of search and advertising as substitutes for each other. Second, lower prices are unambiguously more heavily advertised than higher prices in Robert and Stahl (1993). Finally, our model shows that perfect competition is a limiting case when search cost approach zero. Robert and Stahl arrive at the surprising conclusion that pricing behavior does not converge to competitive pricing when search cost vanish.

The remainder of this Chapter is organized as follows. Section 2.2 presents the model and Section 2.3 gives a full characterization of the equilibrium configurations possible. The most important comparative static results are presented in Section 2.4 and Section 2.5 concludes. Some proofs are given in the appendix to this chapter; the remaining proofs are given in Janssen and Non (2005).

2.2 Model

Our model deals with a homogeneous good market with two active firms. The production costs of the good are constant and equal across firms. We will normalize the production costs to 0. There are no capacity constraints. Firms have the possibility to advertise. The per consumer advertising costs are A. Advertising is an 'all-or-nothing' decision, that is, a firm either does not advertise or it advertises to the complete market. In an ad the firm informs the consumers that it exists and sells the product and it mentions the price it charges. It is assumed firms have to stick to the price they announce in their ad, that is, ads never lie, and that they have to set a single price to all consumers.

At the demand side of the market there is a unit mass of consumers. Each consumer demands a single unit of the good and has valuation $\theta > 0$ for the good. We will con-

[1]Although Butters (1977) considers a combination of search and advertising, in his model search is not optimal. The only other theoretical paper is Stahl (2000). This paper builds on the model by Robert and Stahl (1993) and analyzes some specific cases. Recently, Cason and Datta (2006) reported on an experimental study of the Robert and Stahl model with a fixed advertising intensity. Overall, the experiment supports the predictions of the (modified) Robert and Stahl model.

[2]Other differences are that Robert and Stahl (1993) consider the general oligopoly case with convex advertising technology, i.e., it requires more and more money to reach one more consumer. This level of generality implies that they cannot get a closed-form solution for the equilibrium strategies.

centrate on the case of $\theta > A$, otherwise it is clear firms will not advertise. A fraction γ, with $0 < \gamma < 0.5$[3], of the consumers is called shoppers. These consumers are assumed to know the prices charged by both firms and they will buy at the firm with the lowest price (provided this price is lower than θ). The other $1-\gamma$ consumers a priori do not know the prices charged by the two firms. Sometimes they will get an ad from one or both firms, depending on the advertising strategy of the firms. Consumers can also decide to search one or both firms for prices. This search is costly: each search action costs c, where $c < \theta$. Consumers have perfect memory; they know which firms they already searched and also remember which price they found there. Furthermore consumers receive all advertisements that are sent by the firms before they start to search. In our model search is sequential: after each search action the consumer decides whether or not to continue searching.

The timing of the game is as follows. First the firms simultaneously decide on their advertising and pricing strategies. With probability α^j a firm j advertises and chooses a price from price distribution $F_1^j(p)$, where $F_1^j(p)$ denotes the probability that a price smaller than or equal to p is chosen. With probability $1-\alpha^j$ a firm does not advertise and chooses a price from price distribution $F_0^j(p)$. So a firm j's strategy is given by $\{\alpha^j, F_0^j(p), F_1^j(p)\}$.

After the firms have decided on their strategy, advertisements are realized according to α^j and prices are drawn from $F_0^j(p)$ or $F_1^j(p)$. We will denote by $\overline{p}_0^j$ and $\overline{p}_1^j$ the upper bounds of the supports of price distributions $F_0^j(p)$ and $F_1^j(p)$ respectively. In the same way, $\underline{p}_0^j$ and $\underline{p}_1^j$ denote the respective lower bounds. The shoppers now buy at the lowest-priced firm (provided the price is lower than θ). The non-shoppers first see the advertisements and then decide on their search strategy. If they decide not to search, they buy at the firm with the lowest advertised price lower than θ (or, if there are no ads, they do not buy at all). If they decide to search they pick a non-advertising firm at random and obtain a price quotation from that firm and decide whether to search further or to stop searching. If they decide to stop searching, the product is bought from the firm with the lowest price lower than θ in the set of already obtained price quotations.

We analyze the symmetric perfect Bayesian equilibrium of the game described above. In the remainder we will therefore drop the index j. The profit $\pi_0(p)$ denotes the profit when a firm does not advertise and charges price p and $\pi_1(p)$ denotes the expected profit when advertising and charging price p. We define $\pi_0 = \pi_0(p)$ for all p in the support of $F_0(p)$ and $\pi_1 = \pi_1(p)$ for all p in the support of $F_1(p)$, Whenever $\alpha > 0$, $\pi_0 = \pi_1$.

The assumption made above that advertising is an 'all-or-nothing' decision may seem somewhat unrealistic. Other models incorporating advertising (eg. Butters (1977), Stahl

[3] The equilibrium analysis is dependent on whether γ is smaller than or larger than 0.5. Denote by μ the probability that a consumer searches. One of our equilibria needs $\mu > 2\gamma$, which is impossible for $\gamma > 0.5$. Therefore, for $\gamma > 0.5$ this equilibrium disappears. On the other hand, one of our equilibria needs $\mu < 2\gamma$. This restriction holds automatically for $\gamma > 0.5$. We decided to analyze the case of $\gamma < 0.5$, since Moraga-Gonzalez and Wildenbeest (forthcoming) estimate the fraction of shoppers to be clearly below 0.5, but significantly above 0.

(1994), Robert and Stahl (1993)) assume that firms choose an advertising intensity, indicating the fraction of consumers who are informed by an advertisement. This advertising intensity generally depends on the price chosen, and so a firm's strategy in this context can be denoted by a price distribution $F(p)$, indicating the probability that the price chosen by the firm is below p and an advertising function $\kappa(p)$, indicating the advertising intensity conditional on a price p. When the advertising costs are linear and given by $A\kappa(p)$ it can be shown[4] that the two formulations are equivalent, meaning that if in the 'all-or-nothing' model α, $F_0(p)$ and $F_1(p)$ are part of an equilibrium, then $F(p) = (1-\alpha)F_0(p) + \alpha F_1(p)$ and $\kappa(p) = \frac{\alpha f_1(p)}{(1-\alpha)f_0(p)+\alpha f_1(p)}$ form an equilibrium in the 'advertising reach' model. On the other hand, when $F(p)$ and $\kappa(p)$ are part of an equilibrium in the 'advertising reach' model then $\alpha = \int_{\underline{p}}^{\overline{p}} \kappa(p)dF(p)$, $F_0(p) = \frac{\int_{\underline{p}}^{p} 1-\kappa(p)dF(p)}{1-\alpha}$ and $F_1(p) = \frac{\int_{\underline{p}}^{p} \kappa(p)dF(p)}{\alpha}$ form an equilibrium in the 'all-or-nothing' model. Since the two formulations are equivalent and the 'all-or-nothing' model is easier to analyze, we will use this formulation throughout the Chapter.

2.3 Equilibria

Before focussing on equilibrium behavior, we discuss optimal search behavior and optimal firm behavior. By doing so, we use the fact that firms will never set a price equal to 0 or above θ as this yields nonpositive profits, while setting a positive price below θ and not advertising always yields strictly positive profits.

Consumer behavior

As in all sequential search models, the optimal search behavior is characterized by a reservation price r_0. To define the reservation price assume the lowest price already observed is given by $\hat{p}$. The expected gain from searching once more is then given by

$$\int_{\underline{p}_0}^{\hat{p}} (\hat{p} - p)dF_0(p). \tag{2.1}$$

This expression shows that the expected gain arises when the price found is below $\hat{p}$. Note that $F_0(p)$ is used: only non-advertising firms are searched.

The above expression can be integrated in parts and simplifies to

$$\int_{\underline{p}_0}^{\hat{p}} F_0(p)dp.$$

This defines r_0 as $\int_{\underline{p}_0}^{r_0} F_0(p)dp = c$. One can easily see it is profitable to search if observed prices are above r_0.

We implicitly assumed that the consumer already observed a price quotation. If this is not the case, consumers will search for sure if $r_0 < \theta$. If $r_0 = \theta$ they will search with

[4] Details can be found in the appendix to this chapter.

probability $\mu \leq 1$, and if $r_0 > \theta$ they will not search at all. To see this, note that the expected gain from searching when no price has yet been obtained is given by expression (2.1) where $\hat{p}$ is replaced by θ. This implies that searching is profitable if and only if $\theta > r_0$. Hence, $\mu = 1$ when $r_0 < \theta$, $0 \leq \mu \leq 1$ when $r_0 = \theta$ and $\mu = 0$ when $r_0 > \theta$.

Firm behavior

In this part we will derive some general results on firm behavior that will be helpful in the next part where we derive the equilibria of our model.

Lemma 2.3.1 *In a symmetric equilibrium, $\alpha < 1$.*

The main idea behind this lemma can be explained as follows. If both firms advertise for sure, all consumers will be aware of all prices in the market. Price will therefore be driven down to 0 (Bertrand outcome). The firms obtain negative payoffs $-A$ and so have an incentive not to advertise.

A second observation is that like many search papers, but unlike the paper by Robert and Stahl (1993), the price distributions are atomless. Also, prices that are chosen will never be larger than the reservation price of non-shoppers. A third observation is that for all prices between $\underline{p}_0$ and $\overline{p}_0$ it should be that $\pi_0(p) = \pi_0$. The same holds for $\underline{p}_1$, $\overline{p}_1$ and $\pi_1(p)$. This implies that whether the price distributions have a gap or not, $\pi_0(p) = \pi_1(p)$ in the region $[\max\{\underline{p}_0, \underline{p}_1\}, \min\{\overline{p}_0, \overline{p}_1\}]$. We will use this in the derivation of our equilibria.

Lemma 2.3.2 *$F_0(p)$ and $F_1(p)$ are atomless and $F_0(r_0) = F_1(r_0) = 1$. Furthermore, for $\underline{p}_0 \leq p \leq \overline{p}_0$ $\pi_0(p) = \pi_0$ and for $\underline{p}_1 \leq p \leq \overline{p}_1$ $\pi_1(p) = \pi_1$.*

Characterization of Equilibria

The model has four exogenous parameters: θ, c, A and γ. It can be shown that if θ, c and A are all multiplied by a scalar $x \neq 1$ the equilibria do not change except that all prices (and r_0) and profits are multiplied by x. For convenience, we can thus normalize with respect to θ, and set $\theta = 1$.

We first provide a classification of the different types of equilibria that may arise in our model and then characterize the equilibrium strategies of firms and consumers in each of these cases. The following theorem shows that three types of equilibria may arise in our model.

Theorem 2.3.1 *Each symmetric equilibrium can be classified in one of three different types. Type I has firms not advertising at all ($\alpha = 0$). Type II has firms advertising with strictly positive probability ($0 < \alpha < 1$) and overlapping supports of the price distributions with $\overline{p}_0 = \overline{p}_1$. Type III has firms advertising with strictly positive probability ($0 < \alpha < 1$) and price distributions that do not overlap. In particular, $\overline{p}_0 = \underline{p}_1$ and so advertised prices are higher than non-advertised prices.*

Note that in type III equilibria, advertised prices are always higher than non-advertised prices. The reverse, advertised prices always being below non-advertised prices, cannot arise in equilibrium. In the appendix to this chapter we show that if advertised prices are always lower than non-advertised prices, an advertising firm has an incentive to deviate and advertise the highest non-advertised price. This leads to somewhat less sales, but at a much higher price.

Each of the three types of equilibria has a corresponding parameter region where the equilibrium exists. These regions will be expressed in terms of A and c, taking γ constant. The derivation of the equilibria and corresponding parameter regions will show that each parameter set $[A, c, \gamma]$ has a unique corresponding symmetric equilibrium. Each symmetric equilibrium of a certain type can be further classified in one of two cases, dependent on the value of μ, the probability that uninformed consumers search. Case a has $0 < \mu < 1$ and case b has μ equal to 1 (for type I and II) or 0 (for type III); see Janssen and Non (2005) for more details. For each of the three types of equilibrium, we will now derive case a. The derivation of case b is very similar in nature.

Equilibrium type I: no advertising $(\alpha = 0)$.

As indicated above when characterizing optimal search behavior, $0 < \mu < 1$ implies $r_0 = 1$. As the no advertising case is identical to the search model of Janssen et al. (2005), we can use the same arguments to show that

$$F_0(p) = 1 - \frac{(r_0 - p)\frac{1}{2}\mu(1-\gamma)}{\gamma p}.$$

Since $r_0 = 1$, $\int_{\underline{p}_0}^{r_0} F_0(p)dp = c$ is an expression in γ, c and μ. This expression provides an implicit definition of μ:

$$\mu \ln \frac{\mu(1-\gamma)}{2\gamma + \mu(1-\gamma)} = (c-1)\frac{2\gamma}{1-\gamma}. \tag{2.2}$$

Note that $\mu \ln \frac{\mu(1-\gamma)}{2\gamma+\mu(1-\gamma)}$ is strictly decreasing from 0 to $\ln \frac{1-\gamma}{1+\gamma}$ for $0 \leq \mu \leq 1$ so that μ is uniquely defined.

For $\alpha = 0$ to hold, advertising should not be profitable. The expected profit from advertising a price p is given by $p[(1-\gamma)+\gamma(1-F_0(p))] - A = (1-\gamma)(p + \frac{1}{2}\mu r_0 - \frac{1}{2}p\mu) - A$. This expected profit is maximized for $p = r_0$, giving profit $r_0(1-\gamma) - A$. Advertising is not profitable when $r_0(1-\gamma) - A < r_0(1-\gamma)\frac{1}{2}\mu$. Rearranging terms and using $r_0 = 1$ gives

$$\mu > 2(1 - \frac{A}{1-\gamma}).$$

The two assumptions $0 < \mu < 1$ and $\alpha = 0$ together hold when

$$\max\{0, 2(1 - \frac{A}{1-\gamma})\} < \mu < 1.$$

Using the definition of μ given by (2.2) and the fact that $\mu \ln \frac{\mu(1-\gamma)}{2\gamma+\mu(1-\gamma)}$ is strictly decreasing for $0 < \mu < 1$, this gives rise to the following parameter restrictions:

$$\beta_1 \equiv 1 + \frac{1-\gamma}{2\gamma} \ln \frac{1-\gamma}{1+\gamma} < c < \min\{1, 1 + \frac{1-\gamma-A}{\gamma} \ln \frac{1-\gamma-A}{1-A}\}. \tag{2.3}$$

As $0 < \beta_1 < 1$, it is clear that this type of equilibrium exists whenever $\frac{1-\gamma}{2} < A$.

The above discussion, and the derivation of equilibrium *Ib* can be summarized as follows.

Proposition 2.3.1 *An equilibrium with* $\alpha = 0$ *has*

$$F_0(p) = 1 - \frac{(r_0 - p)\frac{1}{2}\mu(1-\gamma)}{\gamma p}.$$

If (2.3) holds, $r_0 = 1$ *and* μ *is implicitly defined by (2.2). If* $c < \min\{\beta_1, \frac{2A\beta_1}{1-\gamma}\}$*,* $\mu = 1$ *and* $r_0 = \frac{c}{\beta_1}$*.*

Equilibrium type II: some advertising $(0 < \alpha < 1)$ *and partially overlapping price distributions* $(\overline{p}_0 = \overline{p}_1)$*.*
First note that since $0 < \mu < 1$ the reservation price for non-shoppers should be equal to the consumers' valuation in this case, i.e., $r_0 = 1$. Furthermore, the profit equations in the case of non-advertising and advertising are equal to

$$\pi_0(p) = p[\gamma\alpha(1 - F_1(p)) + \gamma(1-\alpha)(1 - F_0(p)) + (1-\gamma)(1-\alpha)\frac{1}{2}\mu],$$

respectively,

$$\pi_1(p) = p[\alpha(1 - F_1(p)) + \gamma(1-\alpha)(1 - F_0(p)) + (1-\gamma)(1-\alpha)] - A.$$

These equations can be interpreted as follows. A firm only attracts all shoppers if it has the lowest price taking into account that the competitor may charge different prices depending on whether or not it advertises. In case the firm does not advertise, it attracts only half of the non-shoppers that do search themselves and only when the competitor does not advertise. The advertising firm attracts more consumers, namely all non-shoppers, if the competitor does not advertise or if the competitor advertises a higher price, but has to pay the advertising cost A.

Whenever the upper bounds of the two price distributions are equal we can use standard arguments to show that the upper bound then has to be equal to the reservation price, i.e., $\overline{p}_0 = \overline{p}_1 = r_0$, and therefore $\pi_0(r_0)$ has to be equal to $\pi_1(r_0)$, which gives the condition

$$r_0(1-\gamma)(1-\alpha)(1 - \frac{1}{2}\mu) = A. \tag{2.4}$$

For all prices larger than $\max\{\underline{p}_0, \underline{p}_1\}$, we can use $\pi_0(p) = \pi_1(p)$ (see lemma 3.2) to derive

$$F_1(p) = 1 - \frac{(r_0 - p)(1-\alpha)(1-\frac{1}{2}\mu)}{\alpha p}. \tag{2.5}$$

Using $\pi_0(p) = \pi_0(r_0)$ and the above expression for $F_1(p)$ we can also derive that

$$F_0(p) = 1 - \frac{(r_0 - p)(\frac{1}{2}\mu - \gamma)}{\gamma p}. \tag{2.6}$$

We stress that these price distributions only hold for prices larger than or equal to $\max\{\underline{p}_0, \underline{p}_1\}$. Thus, we distinguish two cases: (i), $\underline{p}_0 < \underline{p}_1$ and (ii), $\underline{p}_0 > \underline{p}_1$.
In case (i) we can use $\pi_0(p) = \pi_0(r_0)$ to get the price distribution

$$F_0(p) = 1 - \frac{(r_0 - p)(1-\gamma)(1-\alpha)\frac{1}{2}\mu - p\gamma\alpha}{p\gamma(1-\alpha)}, \tag{2.7}$$

for all $p < \underline{p}_1$. Similarly, in case (ii) we get that for all $p < \underline{p}_0$

$$F_1(p) = 1 - \frac{r_0(1-\gamma)(1-\alpha) - p(1-\alpha)}{\alpha p}. \tag{2.8}$$

To check under which conditions an equilibrium of type IIa exists, we first note that μ should be between 0 and 1. Furthermore, α as defined by (2.4) should also be between 0 and 1. This gives rise to condition $\mu < 2 - \frac{2A}{1-\gamma}$. Finally, it should be that $\mu > 2\gamma$, since otherwise $F_0(p)$ is decreasing in p. Thus, we have that μ should satisfy $2\gamma < \mu < \min\{1, 2 - 2A/(1-\gamma)\}$.
For equilibrium $IIai$, $\int_{\underline{p}_0}^{r_0} F_0(p)dp = c$ and $r_0 = 1$ gives

$$\begin{aligned} f(\mu, A, \gamma) \equiv 1 + \frac{1-\gamma}{2\gamma}\mu \ln \frac{\frac{1}{2}\mu A}{\frac{1}{2}\mu A + (1-\frac{1}{2}\mu)\gamma} \\ -(1-\frac{1}{2}\mu) \ln \frac{A(1-\frac{1}{2}\mu)}{(1-\gamma)(1-\frac{1}{2}\mu) - A\frac{1}{2}\mu} = c, \end{aligned} \tag{2.9}$$

which implicitly defines μ as a function of A, c and γ. For equilibrium $IIaii$, $\int_{\underline{p}_0}^{r_0} F_0(p)dp = c$ and $r_0 = 1$ gives

$$g(\mu, \gamma) \equiv 1 - \frac{\gamma - \frac{1}{2}\mu}{\gamma} \ln \frac{\mu - 2\gamma}{\mu} = c, \tag{2.10}$$

again implicitly defining μ as function of c and γ. The L.H.S. of expressions (2.9) and (2.10) are both decreasing in μ and so $2\gamma < \mu < \min\{1, 2 - 2A/(1-\gamma)\}$ can be rewritten as

$$\max\{f(1,A,\gamma), f(2-\frac{2A}{1-\gamma},A,\gamma)\} < c < f(2\gamma,A,\gamma) \tag{2.11}$$

for case (i), and

$$\max\{g(1,\gamma), g(2-\frac{2A}{1-\gamma},\gamma)\} < c < g(2\gamma,\gamma) \tag{2.12}$$

for case (ii). For equilibrium type IIa to hold either restriction (2.11) or restriction (2.12) should hold. However, note that for restriction (2.11) to be relevant, $\underline{p}_0$ should be smaller than $\underline{p}_1$, while for restriction (2.12), $\underline{p}_0$ should be larger than $\underline{p}_1$. It can be shown that the resulting conditions can be simplified to the ones mentioned in Proposition 2.3.2.

Proposition 2.3.2 *An equilibrium with* $0 < \alpha < 1$ *and* $\overline{p}_0 = \overline{p}_1$ *has* $\alpha = 1 - \frac{A}{r_0(1-\gamma)(1-\frac{1}{2}\mu)}$ *and* $F_0(p)$ *and* $F_1(p)$ *being defined by (2.6) and (2.5) respectively in the common support* $[\max\{\underline{p}_0, \underline{p}_1\}, r_0]$ *and (2.7) and (2.8) respectively for* $p < \underline{p}_1$ *and* $p < \underline{p}_0$.

If either $A > \frac{1-2\gamma}{2}$ *and (2.11) hold or* $A < \frac{1-2\gamma}{2}$ *and* $g(1,\gamma) < c < f(2\gamma, A,\gamma)$ *hold, where* $f(\mu, A,\gamma)$ *and* $g(\mu,\gamma)$ *are as defined in (2.9) and (2.10), then* $0 < \mu < 1$, $r_0 = 1$ *and* μ *is defined by equations (2.9) for the case where* $\underline{p}_0 < \underline{p}_1$ *and by (2.10) for the case where* $\underline{p}_1 < \underline{p}_0$.

If $A < \frac{1-2\gamma}{2}$ *and* $\frac{2A\beta_1}{1-\gamma} < c < \beta_2$ *or* $A > \frac{1-2\gamma}{2}$ *and* $\frac{2A\beta_1}{1-\gamma} < c < f(2-\frac{2A}{1-\gamma}, A, \gamma)$, $\mu = 1$ *and* r_0 *is implicitly defined by* $r_0 f(1, \frac{A}{r_0}, \gamma) = c$ *for the case where* $\underline{p}_0 < \underline{p}_1$ *and by* $r_0 = \frac{c}{\beta_2}$ *for the case where* $\underline{p}_0 > \underline{p}_1$.

Equilibrium type III : some advertising $(0 < \alpha < 1)$ *and price distributions that do not overlap* $(\overline{p}_0 = \underline{p}_1)$.

Finally, we will turn to the last type of equilibrium, namely the one where firms spend some money on advertising and when they do advertise, they set consistently higher prices than when they don't advertise, i.e., $\overline{p}_0 = \underline{p}_1$. Using standard arguments, we first observe that $\overline{p}_1 = r_0 = 1$. Profits in case of advertising are now given by the following expression

$$\pi_1(p) = p\alpha(1-F_1(p)) + p(1-\alpha)(1-\gamma) - A.$$

This expression can be understood as follows. When a firm advertises, it knows it gets all consumers when its competitor also advertises and sets a higher price. When the competitor does not advertise, he always asks a lower price and so shoppers will buy from him. Non-shoppers however do not search after receiving an ad and buy from the advertising firm. In order to derive the equilibrium price distribution in case of advertising, we equate this expression with the profits the firm gets when advertising the reservation price: $\pi_1(1) = (1-\alpha)(1-\gamma) - A$. This yields the following expression

$$F_1(p) = 1 - \frac{(1-p)(1-\alpha)(1-\gamma)}{p\alpha}, \tag{2.13}$$

The profits a firm gets from not advertising are given by

$$\pi_0(p) = p\gamma\alpha + p\gamma(1-\alpha)(1-F_0(p)) + p(1-\gamma)(1-\alpha)\frac{1}{2}\mu.$$

A non-advertising firm in this case gets shoppers if, and only if, the other firm advertises or the other firm does not advertise and sets a higher price. Non-shoppers will come to the shop only when the competitor also does not advertise and in that case, both firms receive half of the non-shoppers. It is easy to see that setting the highest non-advertised price yields a profit equal to $\pi_0(\overline{p}_0) = \overline{p}_0\gamma\alpha + \overline{p}_0(1-\gamma)(1-\alpha)\frac{1}{2}\mu$. Equating $\pi_0(p)$ and $\pi_0(\overline{p}_0)$ gives

$$F_0(p) = 1 - \frac{(\overline{p}_0 - p)(\gamma\alpha + (1-\gamma)(1-\alpha)\frac{1}{2}\mu)}{p\gamma(1-\alpha)}. \tag{2.14}$$

It is easy to see that advertising a price below $\underline{p}_1$ is never profitable.[5] It also should not be profitable to refrain from advertising and charge a price above $\overline{p}_0$. Note that for $p \geq \overline{p}_0$, $\pi_0(p) = p(\gamma\alpha(1-F_1(p)) + (1-\alpha)(1-\gamma)\frac{1}{2}\mu)$. Substituting the expression for $F_1(p)$ given in (2.13) yields an expression that is decreasing in p whenever $\mu < 2\gamma$. Hence, this is a necessary condition for this equilibrium to exist.

For the above equilibrium to hold, there are some other parameter restrictions as well. First, $0 < \alpha < 1$ requires $\pi_0 = \pi_1$, which gives

$$\alpha^2\frac{1}{2}\mu(1-\gamma) + \alpha((1-\gamma)(1-\mu) + \frac{\gamma}{1-\gamma}A) + A - (1-\gamma)(1-\frac{1}{2}\mu) = 0. \tag{2.15}$$

Furthermore, $0 < \mu < 1$ gives $\int_{\underline{p}_0}^{1} F_0(p)dp = c$. Substituting $F_0(p)$ gives

$$1 + \frac{(1-\alpha)(1-\gamma) - A}{\gamma(1-\alpha)} \ln \frac{\gamma\alpha + (1-\alpha)(1-\gamma)\frac{1}{2}\mu}{\gamma + (1-\alpha)(1-\gamma)\frac{1}{2}\mu} = c. \tag{2.16}$$

Equations (2.15) and (2.16) together define α and μ as equations of c, A and γ. For the equilibrium to hold, $0 < \alpha < 1$ and $0 < \mu < 2\gamma$.

Note that equation (2.15) is an increasing function of α, that reaches $A-(1-\gamma)(1-\frac{1}{2}\mu)$ for $\alpha = 0$ and $\frac{A}{1-\gamma}$ for $\alpha = 1$. The restriction $0 < \alpha < 1$ therefore reduces to $A - (1-\gamma)(1-\frac{1}{2}\mu) < 0$, which gives $\mu < 2 - \frac{2A}{1-\gamma}$.

It can be shown that expression (2.16) is decreasing in μ and so the restrictions $0 < \mu < \min\{2\gamma, 2 - \frac{2A}{1-\gamma}\}$ can be written as

$$\max\{1 + (1-\gamma)\ln\frac{(1-\gamma)^2 - A\gamma}{(1-\gamma)^2 + A(1-\gamma)}, 1 + \frac{(1-\gamma)-A}{\gamma}\ln\frac{(1-\gamma)-A}{1-A}\} \quad <$$

[5] To see this, write down $\pi_1(p)$ for $p \leq \underline{p}_1$ using the above expression for $F_0(p)$ and note that $\pi_1(p)$ is increasing in p.

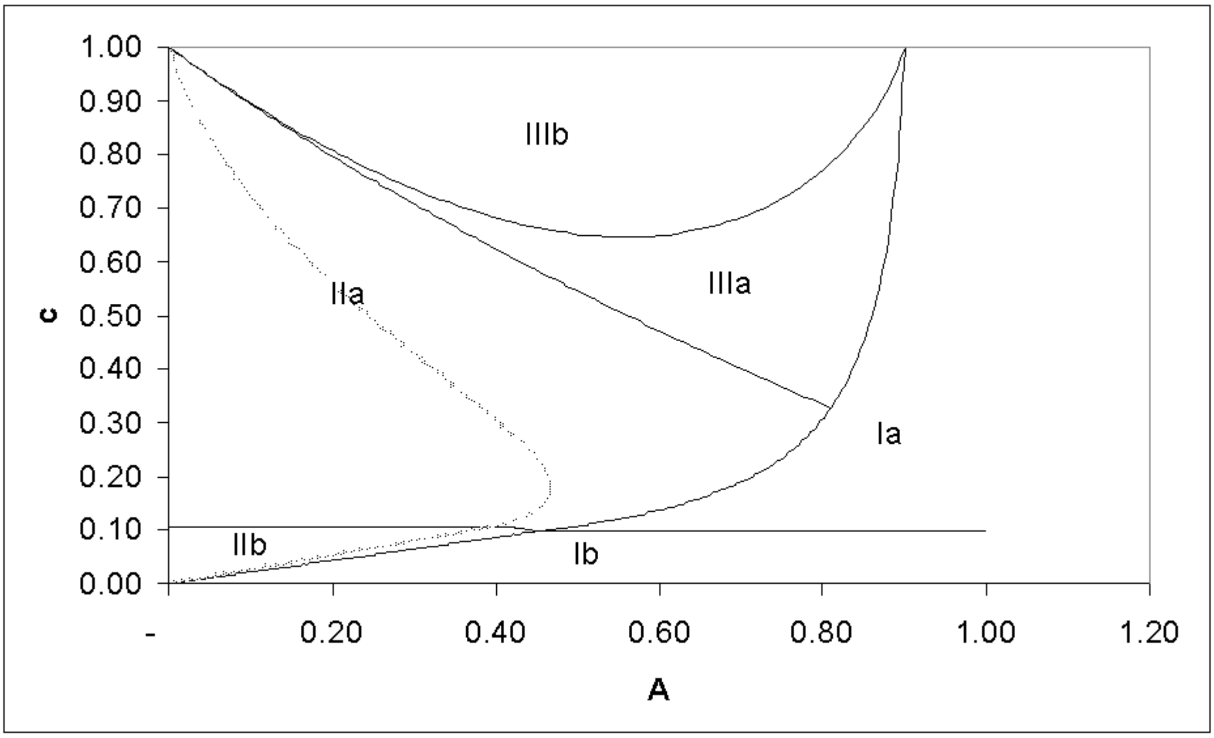


Figure 2.1: The parameter regions in terms of A and c where the different equilibria exist, assuming $\gamma = 0.1$.

$$c < 1 + ((1-\gamma) - A) \ln \frac{(1-\gamma)^2 - A(1-\gamma)}{(1-\gamma)^2 + A\gamma} \equiv \beta_3. \qquad (2.17)$$

We summarize as follows:

Proposition 2.3.3 *Equilibria with* $0 < \alpha < 1$ *and* $\overline{p}_0 = \underline{p}_1$ *have* $F_1(p)$ *as defined in (2.13) and* $F_0(p)$ *as defined in (2.14) with* $\overline{p}_0 = \frac{(1-\alpha)(1-\gamma)}{\alpha+(1-\alpha)(1-\gamma)}$. *If (2.17) holds we have* $0 < \mu < 1$ *and* α *and* μ *are implicitly defined by (2.15) and (2.16). If* $c > \beta_3$, *with* β_3 *as defined in (2.17),* $\mu = 0$ *and* α *is given by* $\frac{(1-\gamma)^2 - A(1-\gamma)}{(1-\gamma)^2 + A\gamma}$.

Discussion

We will now take a closer look at the parameter regions in which the different types of equilibria exist. Figure 2.3 depicts for the case where $\gamma = 0.1$ for each equilibrium the region where it exists in terms of A and c. We note that the regions do not overlap and that they together fill the complete parameter space. Equilibria IIa and IIb are both divided by a dotted line. Left of this line, $\underline{p}_1 < \underline{p}_0$, while right of this line $\underline{p}_0 < \underline{p}_1$.

We note that for the equilibria with no advertising (equilibrium type I) to exist, the search costs c should be low or the advertising costs A should be high. The intuition is fairly simple: for high A advertising is too expensive to be profitable. For low search cost, firms can not ask high prices since otherwise the consumers will search on and so advertising is too expensive relative to the prices that can be asked.

The equilibria with full consumer search (equilibrium types Ib and IIb where $\mu = 1$) are in a region with low search costs, whereas their counterparts with partial consumer search ($0 < \mu < 1$) are in a region with higher search cost. The equilibrium with no consumer search (equilibrium type $IIIb$) only exists for c relatively high and A not too low or high. It is clear that higher search cost lead to less search. The intuition for the restriction on A is as follows. For high A it is not profitable to advertise and so firms only sell to the shoppers and searching consumers. Note that if consumers do not search at all, prices and profits will be driven to 0, giving consumers an incentive to search. For low A firms have a large incentive to advertise, which drives prices down. This leads to a higher payoff from search and therefore to some consumer search even if the search costs are large.

We note that if firms do not advertise ($\alpha = 0$), the non-shoppers search with strictly positive probability as long as the search cost c are below the valuation for the product (see also Janssen et al. (2005)). On the other hand, if the non-shoppers do not search ($\mu = 0$), the firms advertise with strictly positive probability as long as the advertising costs are below $1 - \gamma$. Thus, certainly in these regions advertising and search act as substitutes for each other.

2.4 Comparative Statics

In this section we will give some asymptotic and comparative static results. We are interested in the impact of changes in c and A on the variables that are of main interest such as prices, profits and welfare. We start the discussion with some limiting results.

Theorem 2.4.1 a *When c becomes arbitrarily small, firms do not advertise and there is full consumer search ($\mu = 1$). The maximum price charged approaches 0.*

b *When A becomes arbitrarily small, the advertising intensity α converges to 1 and the expected advertised price Ep_1 converges to 0.*

These results can be understood as follows. When the search costs approach 0, the Bertrand result arises. This asymptotic result also occurs in the pure search models of Stahl (1989) and Janssen et al. (2005). The intuition is simple: if the search costs are very low, consumers are almost always willing to search and to prevent further search, firms lower their prices, in that way preventing advertising. This Bertrand-like result however does not arise in Robert and Stahl's model. In their model, when search costs approach zero, advertising diminishes, but the minimum price is strictly above zero. Without shoppers, Robert and Stahl obtain a Diamond type of result, namely that when c vanishes, the equilibrium price distribution converges to a degenerate distribution where all firms charge one particular, strictly positive, price for sure. A fraction of shoppers, however small, causes a breakdown of this result and the traditional Bertrand-type result emerges.

When the advertising costs approach 0 we get the intuitive result that the advertising probability rises to 1 and advertised prices decrease to 0.

We next proceed to some comparative statics results. The analysis provides an assessment of the impact of a change in exogenous parameter values on the economic variables

	Ia	Ib	IIa	IIb	$IIIa$	$IIIb$
α	0	0	+	+	+	0
μ	-	0	-	0	-	0
Ep_0	-	+	-	+/-	-	0
Ep_1			-*	+	-	0
π	-	+	-	0	-	0
Welfare	-	-	-	-	-*	0

Table 2.1: Local comparative statics results with respect to changes in c.

of interest α, μ, the expected non-advertised price Ep_0, the expected advertised price Ep_1, the profit π and welfare conditional on a certain type of equilibrium to be present in the economy. We define expected welfare as the value of all transactions taking place minus the search and advertising costs that have been incurred, i.e, expected welfare equals

$$1-(1-\gamma)(1-\mu)(1-\alpha)^2-2\alpha A-(1-\gamma)\mu(1-\alpha)^2c.$$

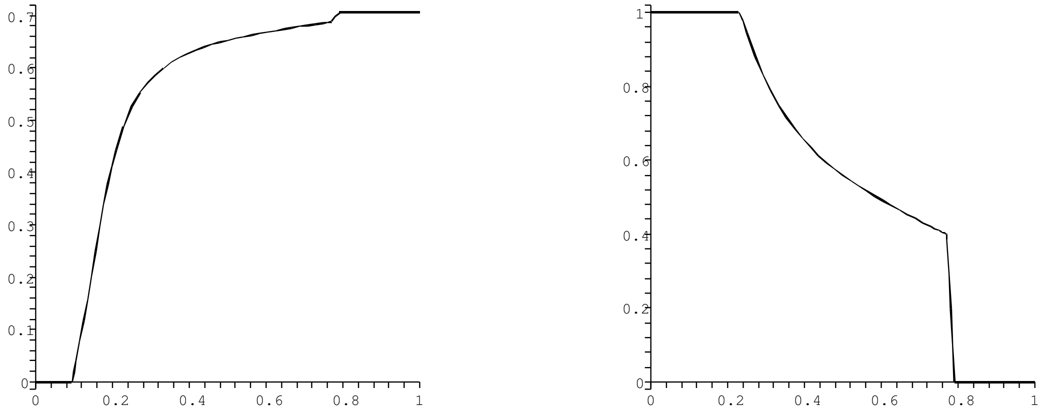
In Figures 2.4 and 2.4 we give a description of how for example, expected price depends on search cost across equilibria for specific values of the other exogenous parameters. Tables 2.1 and 2.22 provide an overview of the local comparative statics.

We will first discuss the comparative statics results with respect to changes in search costs. The local comparative statics are given in Table 2.1.[6]

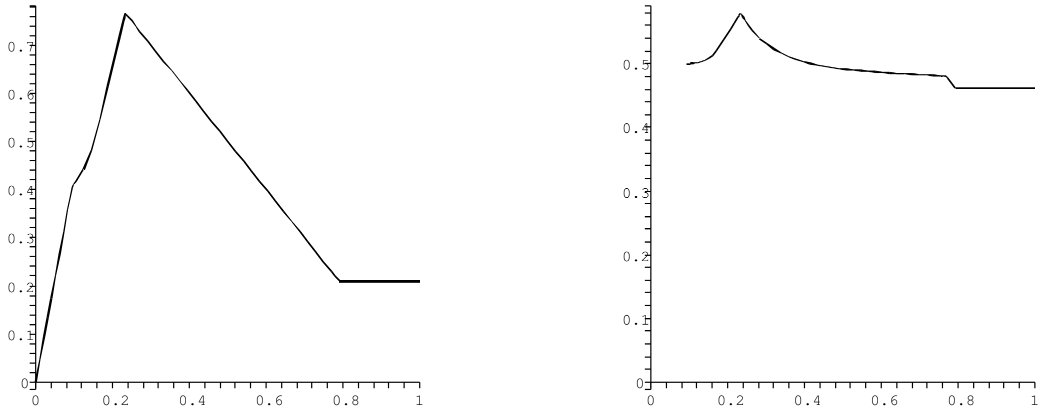
We note that the primary effect of a change in search costs is twofold. In equilibria with full consumer search, the reservation price increases in the search costs, while in equilibria with partial consumer search ($0<\mu<1$) the primary effect of a change in search costs is in reducing the amount of search. In the equilibrium with no search there is no effect of a change in search costs. This comes back in Table 2.1 as the comparative statics results in equilibria Ia, IIa and $IIIa$ are clearly different from the results in equilibria Ib and IIb. In equilibria of type a, the decreasing search intensity makes not-advertising less attractive (a non-advertising firm only attracts those non-shoppers that are searching) and so firms advertise more. It is intuitively clear that more advertising leads to lower prices. In equilibria b, the reservation price increases in a reaction to increasing search costs. This leads to more advertising. The effect on prices is twofold: a higher reservation price means higher prices can be asked, but on the other hand there is more competition and so a decreasing effect on prices. These opposite effects are nicely illustrated by the '+/-' for Ep_0 in equilibrium IIb.

An overview of these results is given in Figure 2.4, where we depict α, μ, Ep_0, Ep_1, π and welfare as a function of c, taking γ and A fixed. The figures are drawn for $\gamma =$

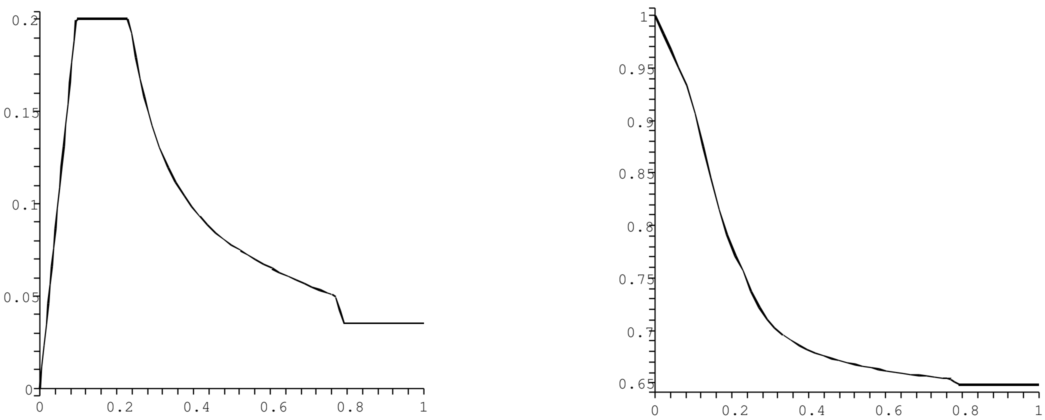
[6] A $'+'$ in these tables denotes a positive impact, a $'-'$ a negative impact, $'0'$ no impact and $'+/-'$ denotes that the impact can be positive as well as negative, depending on specific parameter values. In some cases we have been unable to analytically evaluate the sign. In these cases we resort to a numerical analysis and evaluate the impact by taking γ fixed to $0.01, 0.05, 0.1, 0.15$, and so on. These cases are marked by a *.



(a) Plots of α (left) and μ (right) as a function of c.

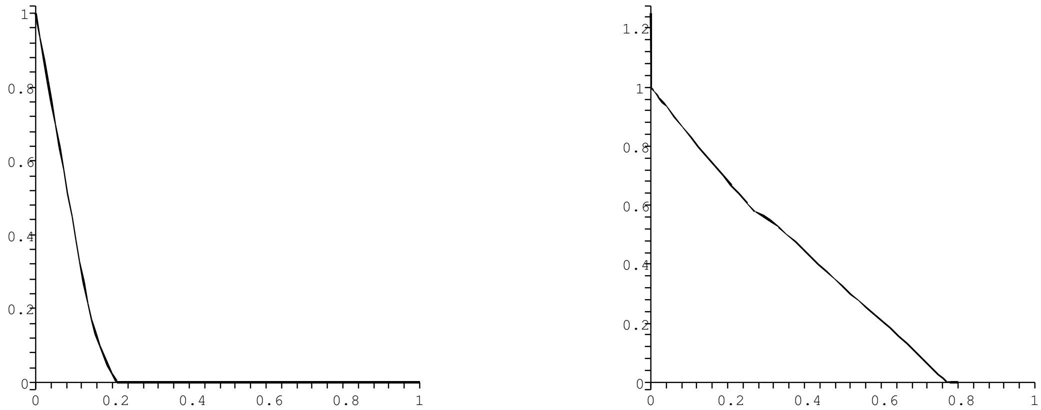


(b) Plots of Ep_0 (left) and Ep_1 (right) as a function of c.

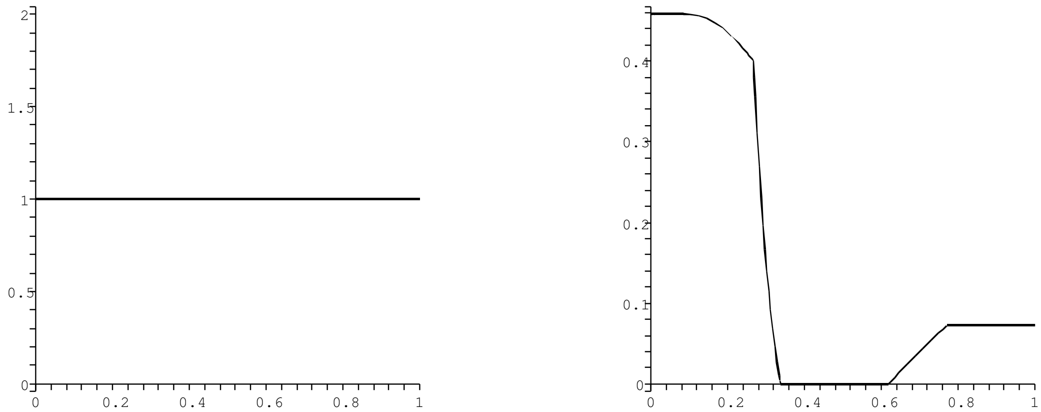


(c) Plots of π (left) and welfare (right) as a function of c.

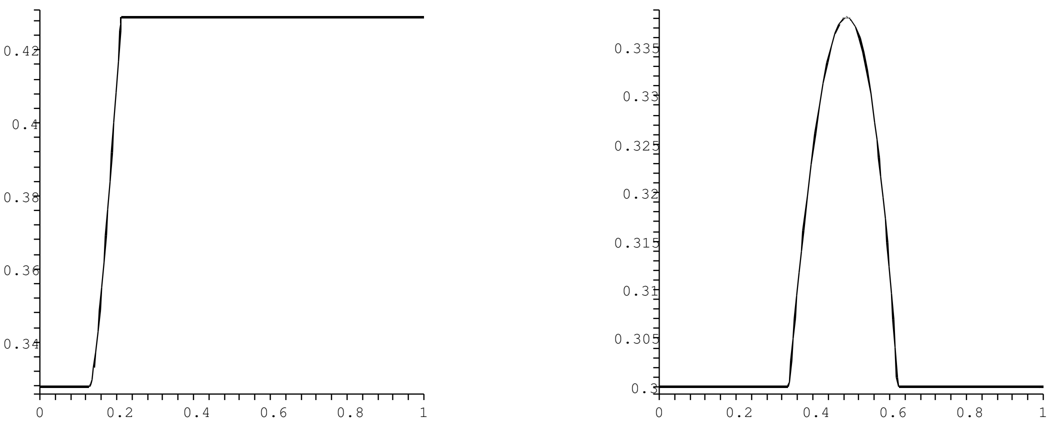
Figure 2.2: Plots of several variables as a function of c. The figures are drawn for $\gamma = 0.2$ and $A = 0.2$.



(a) Plots of α as a function of A.

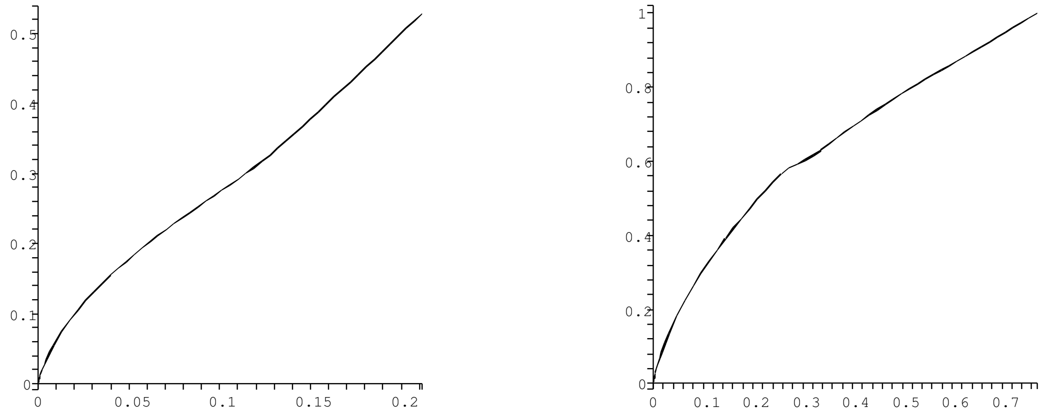


(b) Plots of μ as a function of A.

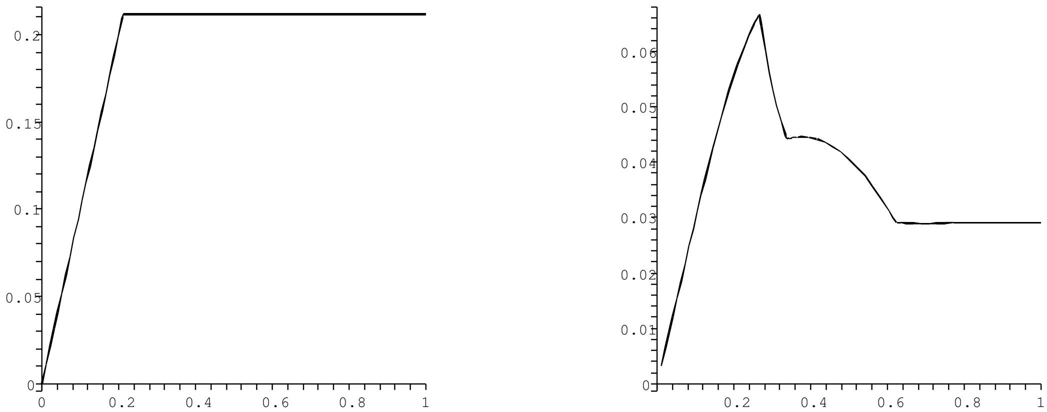


(c) Plots of Ep_0 as a function of A.

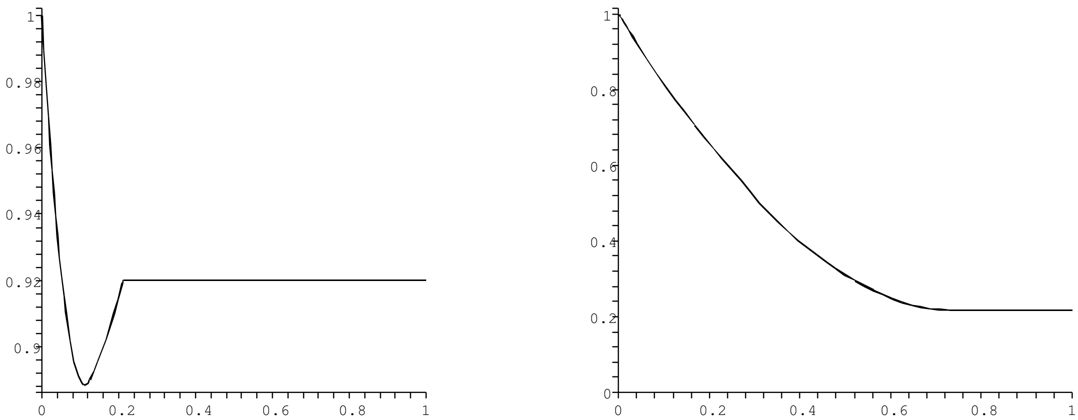
Figure 2.3: Plots of several variables as a function of A. The left figures are drawn for $\gamma = 0.2$ and $c = 0.1$, the right figures are drawn for $\gamma = 0.2$ and $c = 0.7$.



(d) Plots of Ep_1 as a function of A.



(e) Plots of π as a function of A.



(f) Plots of welfare as a function of A.

Figure 2.3: (continued) Plots of several variables as a function of A. The left figures are drawn for $\gamma = 0.2$ and $c = 0.1$, the right figures are drawn for $\gamma = 0.2$ and $c = 0.7$.

	Ia	Ib	IIa	IIb	$IIIa$	$IIIb$
α	0	0	-*	-	-*	-
μ	0	0	0/-	0	+/-	0
Ep_0	0	0	0	+/0	0	+/-
Ep_1			+*	+	+*	+
π	0	0	+/-	+	+/-	+/-
Welfare	0	0	+/-	+/-	+/-	-

Table 2.2: Local comparative statics results with respect to changes in A.

0.2 and $A = 0.2$. Letting c increase from 0 to 1 means that equilibria $Ib, IIbi, IIbii$, $IIaii, IIai, IIIa$ and $IIIb$ hold in this order. The figure clearly shows how the monotonicity of both α and μ with respect to changes in c may give rise to non-monotonic behavior of both the expected advertised and non-advertised price with respect to c. The figures also show that the profits are non-monotonic with respect to c, which is quite intuitive given the behavior of prices and advertising.

It is also possible to look at the effects of a change in advertising costs A. The impact of a change in advertising costs on the economy is through a change in advertising intensity: higher advertising costs leads to less advertising and therefore to higher advertised prices.

An overview of the results is depicted in Figure 2.4, where $\alpha, \mu, Ep_0, Ep_1, \pi$ and welfare are represented as a function of A, taking γ and c fixed. In the left graphs, we take $\gamma = 0.2$ and $c = 0.1$, while in the right graphs $\gamma = 0.2$ and $c = 0.7$. When $c = 0.1$, letting A increase from 0 to 1 means that equilibria $IIbii, IIbi$ and Ib hold in this order. For $c = 0.7$ equilibria $IIaii, IIai$, $IIIa, IIIb, IIIa$ and Ia hold in this order. Note that for $c = 0.1$ we plot Ep_1 only for $A < 0.21$. For higher values of A Ep_1 is not defined. The figures show that although Ep_0 is non-monotonic in A, the expected non-advertised price has only little variation. This is in contrast with the expected advertised price, which monotonically increases from 0 to 0.5 - 1.

Profits and welfare can both both increase and decrease in the advertising costs. The effect on profits can be explained by the fact that an increase in advertising costs implies on one hand less competition and therefore higher advertised prices, but on the other hand also leads to more expenditure on advertising if firms decide to advertise.

2.5 Conclusion

In this Chapter we have analyzed the interaction between two information transferring technologies: advertising and search. In particular we have focused on the interaction between the incentives for firms to advertise and for consumers to search in relation to the parameters underlying the search and advertising technology.

We reach three important conclusions. First, if the cost of one of these technologies becomes arbitrary small, the model's outcome get very close to the fully competitive outcome where price equals marginal cost. The fully competitive outcome occurs when consumers are somehow fully informed of all prices. This will necessarily happen if one of the technologies becomes arbitrarily cheap. Second, there are important non-monotonicities in the relations between expected prices on one hand and search and advertising cost on the other hand. An important part of the explanation for this phenomenon to occur is that search and advertising are 'substitutes' over an important domain of the model's parameters in the sense that if consumers (initially) start searching more (because of a decrease in search cost) firms will advertise less. In the aggregate this may mean that consumers actually are less informed about prices when search cost are lower and this gives firms an incentive to raise prices. Third, advertising firms have an advantage since consumers who receive an advertisement do not have to pay search costs. Therefore, advertised prices may actually be higher than non-advertised prices.

2.A Proofs

2.A.1 Proof that two specifications of advertising are equivalent

In this section we will show that the 'all-or-nothing' advertising specification and the 'advertising reach with linear advertising costs' specification are equivalent. We will prove this for the case of two identical firms, i and j. The general case of n firms can be proven using the same arguments but at the cost of a more complicated notation.

The proof is given for the case where the equilibrium price distributions are atomless. The same line of thought as outlined in the proofs below can be used to prove equivalence in the case of distributions with atoms, but again at the cost of a more complicated notation.

In the proof we will regularly use the expression 'support of $F(p)$', where $F(p)$ is the cdf of a price distribution. With this expression we mean all prices p where the derivative of $F(p)$ is strictly positive, so all prices that occur with a strictly positive probability.

The proof is for a slightly more advanced model, that is used in Chapter 3. The proof can easily be rewritten to the model used in this chapter.

In model I ('all-or-nothing') firms have to make three decisions: they have to decide on whether or not to be active, given that a firm decides to be active it has to decide on whether or not to advertise to the complete market and conditional on the advertising decision it has to decide on its price.

Before we specify the profit functions of model I we will introduce some general notation. We will use $\pi(p_i, a_i|p_j, a_j)$ to denote the expected profits that firm i can make from a single randomly chosen consumer given that firm j is active. Here $a_i \in 0, 1$ and $a_j \in 0, 1$ denote whether the consumer sees an advertisement from firm i (if so $a_i = 1$, else $a_i = 0$) and whether the consumer sees an advertisement from firm j. The expected profits also depend on the prices of the firms: p_i and p_j. In the expected profits $\pi(p_i, a_i|p_j, a_j)$ all costs, like shelving and production costs, are incorporated, except advertising costs. Since these form one of the differences between the two models we will explicitly take these into account. We will use $\pi(p_i, a_i|0)$ to denote the expected profits that firm i can make from a single randomly chosen consumer when firm j is not active. Again a_i denotes whether the consumer sees an advertisement from firm i and p_i is the price firm i sets. And again, in $\pi(p_i, a_i|0)$ all costs are incorporated, except advertising costs.

Using this notation we note that in model I the expected profits per consumer for firm i of being active, not advertising and asking a price p_i are

$$\pi^I(p_i, 0) = (1 - \alpha_j)\beta_j \int_0^1 \pi(p_i, 0|p, 0)dF_{0j}(p) + \alpha_j\beta_j \int_0^1 \pi(p_i, 0|p, 1)dF_{1j}(p) +$$

$$(1 - \beta_j)\pi(p_i, 0|0)$$

where β_j denotes the probability that the competitor, firm j is active, α_j denotes the probability that firm j advertises given that it is active, $F_{0j}(p)$ is the price distribution firm j chooses its price from when it does not advertise and $F_{1j}(p)$ is the price distribution firm j chooses its price from when it advertises. Note that with this notation α_j is the probability that $a_j = 1$.

We also note that in model I the expected profits of firm i per consumer of being active, advertising and asking a price p_i are

$$\pi^I(p_i,1) = (1-\alpha_j)\beta_j \int_0^1 \pi(p_i,1|p,0)dF_{0j}(p) + \alpha_j\beta_j \int_0^1 \pi(p_i,1|p,1)dF_{1j}(p) +$$

$$(1-\beta_j)\pi(p_i,1|0) - A$$

where A denotes the costs of advertising to the complete market. Without loss of generality we will set the profits of being inactive equal to 0.

In model II firms also first decide on whether or not to be active. An active firm however then decides on its price and conditional on its price it decides on the advertising reach, which can be anything between 0 (reaching no consumers) and 1 (reaching all consumers). The advertising costs are linear in the reach, with the costs of reaching no consumers being 0 and the costs of reaching all consumers being A. In this model the expected profits of firm i per consumer of being active, setting a price p_i and choosing a reach $\alpha_i(p_i)$ are

$$\begin{aligned}\pi^{II}(p_i,\alpha_i(p_i)) \quad = \quad & \beta_j[(1-\alpha_i(p_i)) \int_0^1 \pi(p_i,0|p,0)(1-\alpha_j(p))dF_j(p) + \\ & (1-\alpha_i(p_i)) \int_0^1 \pi(p_i,0|p,1)\alpha_j(p)dF_j(p) + \\ & \alpha_i(p_i) \int_0^1 \pi(p_i,1|p,0)(1-\alpha_j(p))dF_j(p) + \\ & \alpha_i(p_i) \int_0^1 \pi(p_i,1|p,1)\alpha_j(p)dF_j(p)] + \\ & (1-\beta_j)[(1-\alpha_i(p_i))\pi(p_i,0|0) + \alpha_i(p_i)\pi(p_i,1|0)] - \alpha_i(p_i)A.\end{aligned}$$

In the expression above β_j denotes the probability that the competitor, firm j is active, $\alpha_j(p)$ denotes the advertising reach of an active competitor that asks price p and $F_j(p)$ is the price distribution the competitor chooses its price from. As in model I, without loss of generality we set the profits of an inactive firm equal to 0.

In both models we will look for a symmetric equilibrium, where both firms are active with probability β and choose identical price distributions and advertising strategies. This means that in an equilibrium situation we will drop the subscripts i and j. We will show that if in model I β, α, $F_0(p)$ and $F_1(p)$ form an equilibrium then β, $F(p) = (1-\alpha)F_0(p) + \alpha F_1(p)$ and $\alpha(p) = \frac{\alpha f_1(p)}{(1-\alpha)f_0(p)+\alpha f_1(p)}$ form an equilibrium in model II. On the other hand, when β, $F(p)$ and $\alpha(p)$ form an equilibrium in model II then β, $\alpha = \int_0^1 \alpha(p)dF(p)$, $F_0(p) = \frac{\int_{\underline{p}}^p (1-\alpha(p^*))f(p^*)dp^*}{1-\alpha}$ and $F_1(p) = \frac{\int_{\underline{p}}^p \alpha(p^*)f(p^*)dp^*}{\alpha}$ form an equilibrium in model II.

We will first give a sequence of theorems on the conversion from model I to model II.

Theorem 2.A.1 *Suppose that in model I it is an equilibrium to have $\beta = 0$, so all firms are inactive. Then in model II $\beta = 0$ also is an equilibrium.*

Proof
Since $\beta = 0$ in model I it should be that for all p_i $\pi^I(p_i,0) = \pi(p_i,0|0) < 0$ and $\pi^I(p_i,1) =$

$\pi(p_i, 1|0) - A < 0$. Now suppose that firm j in model II sets $\beta_j = 0$. If firm i is active it has profits $\pi^{II}(p_i, \alpha_i(p_i)) = (1 - \alpha_i(p_i))\pi(p_i, 0|0) + \alpha_i(p_i)(\pi(p_i, 1|0) - A) < 0$. Since these profits are negative this firm also chooses to be inactive and therefore $\beta = 0$ is an equilibrium strategy in model II as well.

□

Theorem 2.A.2 *Suppose that in model I it is an equilibrium to have $\beta = 1$, $\alpha = 0$ and $F_0(p)$. Then in model II it is an equilibrium to have $\beta = 1$, $\alpha(p) = 0 = \frac{\alpha f_1(p)}{\alpha f_1(p)+(1-\alpha)f_0(p)}$ for all p in the support of $F(p)$ and $F(p) = F_0(p) = \alpha F_1(p) + (1 - \alpha)F_0(p)$.*

Proof

If $\beta = 1$ and $\alpha = 0$ is an equilibrium in model I, we have that

$$\pi^I(p_i, 0) = \int_0^1 \pi(p_i, 0|p, 0)dF_0(p)$$

and

$$\pi^I(p_i, 1) = \int_0^1 \pi(p_i, 1|p, 0)dF_0(p) - A.$$

We also know that $\pi^I(p_i, 0) = \pi > 0$ for all p_i in the support of $F_0(p)$. For all other p_i, $\pi^I(p_i, 0) \leq \pi$. And for all p_i, $\pi^I(p_i, 1) \leq \pi$.

Now look at a firm deciding on $(\beta, F(p), \alpha(p))$ and suppose firm j chooses $\beta_j = 1$, $\alpha_j(p) = 0$ for all p in the support of $F_j(p)$ and $F_j(p) = F_0(p)$. In that case

$$\pi^{II}(p_i, \alpha_i(p_i)) = (1 - \alpha_i(p_i)) \int_0^1 \pi(p_i, 0|p, 0)dF_0(p) +$$

$$\alpha_i(p_i)(\int_0^1 \pi(p_i, 1|p, 0)dF_0(p) - A).$$

Note that $\pi^{II}(p_i, \alpha_i(p_i)) = (1 - \alpha_i(p_i))\pi^I(p_i, 0) + \alpha_i(p_i)\pi^I(p_i, 1)$. So for all p_i outside the support of $F_0(p)$, $\pi(p_i, \alpha_i(p_i)) \leq \pi$. Furthermore, for all p_i inside the support of $F_0(p)$ with $\alpha_i(p_i) > 0$, $\pi(p_i, \alpha_i(p_i)) \leq \pi$. And lastly, for all p_i inside the support of $F_0(p)$ with $\alpha_i(p_i) = 0$, $\pi(p_i, \alpha_i(p_i)) = \pi > 0$. This shows that in case II, it is an equilibrium to have $\beta = 1$, $\alpha(p) = 0$ for all p in the support of $F(p)$, and to have $F(p) = F_0(p)$.

□

Theorem 2.A.3 *Suppose that in model I it is an equilibrium to have $0 < \beta^I < 1$, $\alpha = 0$ and $F_0(p)$. Then in model II it is an equilibrium to have $\beta = \beta^I$, $\alpha(p) = 0 = \frac{\alpha f_1(p)}{\alpha f_1(p)+(1-\alpha)f_0(p)}$ for all p in the support of $F(p)$ and $F(p) = F_0(p) = \alpha F_1(p) + (1 - \alpha)F_0(p)$.*

Proof

If $\beta = \beta^I$ and $\alpha = 0$ is an equilibrium in model I, we know that $\pi^I(p_i, 0) = 0$ for all p_i in the support of $F_0(p)$. For all other p_i, $\pi^I(p_i, 0) \leq 0$. And for all p_i, $\pi^I(p_i, 1) \leq 0$.

Now look at a firm deciding on $(\beta, F(p), \alpha(p))$ and suppose firm j chooses $\beta_j = 1$, $\alpha_j(p) = 0$ for all p in the support of $F_j(p)$ and $F_j(p) = F_0(p)$. In that case one can easily show that

$\pi^{II}(p_i, \alpha_i(p_i)) = (1-\alpha_i(p_i))\pi^I(p_i,0) + \alpha_i(p_i)\pi^I(p_i,1)$. So for all p_i outside the support of $F_0(p)$, $\pi(p_i,\alpha_i(p_i)) \leq 0$. Furthermore, for all p_i inside the support of $F_0(p)$ with $\alpha_i(p_i) > 0$, $\pi(p_i,\alpha_i(p_i)) \leq 0$. And lastly, for all p_i inside the support of $F_0(p)$ with $\alpha_i(p_i) = 0$, $\pi(p_i,\alpha_i(p_i)) = 0$. This shows that in case II, it is an equilibrium to have $\beta = \beta^I$, $\alpha(p) = 0$ for all p in the support of $F(p)$, and to have $F(p) = F_0(p)$.

□

Theorem 2.A.4 *Suppose that in model I it is an equilibrium to have $\beta = 1$, $\alpha = 1$ and $F_1(p)$. Then in model II it is an equilibrium to have $\beta = 1$, $\alpha(p) = 1 = \frac{\alpha f_1(p)}{\alpha f_1(p)+(1-\alpha)f_0(p)}$ for all p in the support of $F(p)$ and $F(p) = F_1(p) = \alpha F_1(p) + (1-\alpha)F_0(p)$.*

Proof

If $\beta = 1$ and $\alpha = 1$ is an equilibrium in model I, we know that $\pi^I(p_i,1) = \pi > 0$ for all p_i in the support of $F_1(p)$. For all other p_i, $\pi^I(p_i,1) \leq \pi$. And for all p_i, $\pi^I(p_i,0) \leq \pi$.

Now look at a firm deciding on $(\beta, F(p), \alpha(p))$ and suppose firm j chooses $\beta_j = 1$, $\alpha_j(p) = 1$ for all p in the support of $F_j(p)$ and $F_j(p) = F_1(p)$. In that case it is easy to show that $\pi^{II}(p_i, \alpha_i(p_i)) = (1-\alpha_i(p_i))\pi^I(p_i,0) + \alpha_i(p_i)\pi^I(p_i,1)$. So for all p_i outside the support of $F_1(p)$, $\pi(p_i,\alpha_i(p_i)) \leq \pi$. Furthermore, for all p_i inside the support of $F_1(p)$ with $\alpha_i(p_i) < 1$, $\pi(p_i,\alpha_i(p_i)) \leq \pi$. And lastly, for all p_i inside the support of $F_1(p)$ with $\alpha_i(p_i) = 1$, $\pi(p_i,\alpha_i(p_i)) = \pi > 0$. This shows that in case II, it is an equilibrium to have $\beta = 1$, $\alpha(p) = 1$ for all p in the support of $F(p)$, and to have $F(p) = F_1(p)$.

□

Theorem 2.A.5 *Suppose that in model I it is an equilibrium to have $0 < \beta^I < 1$, $\alpha = 1$ and $F_1(p)$. Then in model II it is an equilibrium to have $\beta = \beta^I$, $\alpha(p) = 1 = \frac{\alpha f_1(p)}{\alpha f_1(p)+(1-\alpha)f_0(p)}$ for all p in the support of $F(p)$ and $F(p) = F_1(p) = \alpha F_1(p) + (1-\alpha)F_0(p)$.*

Proof

The proof is the same as for Theorem 4, except that now $\pi = 0$ and in model II firm j chooses $\beta_j = \beta^I$, equilibrium profits are 0 and therefore $0 < \beta < 1$ is an optimal strategy.

□

Theorem 2.A.6 *Suppose that in model I it is an equilibrium to have $\beta = 1$, $0 < \alpha < 1$ and $F_0(p)$ and $F_1(p)$. Then in model II it is an equilibrium to have $\beta = 1$,$\alpha(p) = \frac{\alpha f_1(p)}{\alpha f_1(p)+(1-\alpha)f_0(p)}$ for p in the support of $F(p)$ and $F(p) = \alpha F_1(p) + (1-\alpha)F_0(p)$.*

Proof

If $0 < \alpha < 1$ and $\beta = 1$ is an equilibrium in model I we have that

$$\pi^I(p_i,0) = (1-\alpha)\int_0^1 \pi(p_i,0|p,0)dF_0(p) + \alpha\int_0^1 \pi(p_i,0|p,1)dF_1(p)$$

and

$$\pi^I(p_i,1) = (1-\alpha)\int_0^1 \pi(p_i,1|p,0)dF_0(p) + \alpha_j\int_0^1 \pi(p_i,1|p,1)dF_1(p) - A.$$

We also know that $\pi^I(p_i, 0) = \pi > 0$ for all p_i in the support of $F_0(p)$. For all other p_i, $\pi^I(p_i, 0) \leq \pi$. Furthermore, $\pi^I(p_i, 1) = \pi$ for all p_i in the support of $F_1(p)$. For all other p_i, $\pi^I(p_i, 1) \leq \pi$.

Now look at a firm deciding on $(\beta, F(p), \alpha(p))$ and suppose firm j chooses $\beta_j = 1$,$\alpha_j(p) = \frac{\alpha f_1(p)}{\alpha f_1(p)+(1-\alpha)f_0(p)}$ and $F_j(p) = \alpha F_1(p) + (1-\alpha)F_0(p)$. We then have that

$$\begin{aligned}
\pi^{II}(p_i, \alpha_i(p_i)) &= (1-\alpha_i(p_i)) \int_0^1 \pi(p_i, 0|p, 0)(1-\alpha)dF_0(p) + \\
&\quad (1-\alpha_i(p_i)) \int_0^1 \pi(p_i, 0|p, 1)\alpha dF_1(p) + \\
&\quad \alpha_i(p_i) \int_0^1 \pi(p_i, 1|p, 0)(1-\alpha)dF_0(p) + \\
&\quad \alpha_i(p_i) \int_0^1 \pi(p_i, 1|p, 1)\alpha dF_1(p) - \alpha_i(p_i)A.
\end{aligned}$$

Note that $\pi^{II}(p_i, \alpha_i(p_i)) = \alpha_i(p_i)\pi^I(p_i, 1) + (1-\alpha_i(p_i))\pi^I(p_i, 0)$.

For p_i in the support of both $F_0(p)$ and $F_1(p)$, $\pi^{II}(p_i, \alpha_i(p_i)) = \alpha_i(p_i)\pi + (1-\alpha_i(p_i))\pi$ and so the firm is indifferent about $\alpha_i(p_i)$. Assume it chooses $\alpha_i(p_i) = \frac{\alpha f_1(p_i)}{\alpha f_1(p_i)+(1-\alpha)f_0(p_i)}$.

For p_i in the support of $F_0(p)$ but not in the support of $F_1(p)$, $\pi^I(p_i, 1) \leq \pi^I(p_i, 0) = \pi$ and so $\alpha_i(p_i) = 0 = \frac{\alpha f_1(p_i)}{\alpha f_1(p_i)+(1-\alpha)f_0(p_i)}$ since $f_1(p_i) = 0$. Furthermore, $\pi^{II}(p_i, \alpha_i(p_i)) = \pi$.

For p_i in the support of $F_1(p)$ but not in the support of $F_0(p)$, $\pi^I(p_i, 0) \leq \pi^I(p_i, 1) = \pi$ and so $\alpha_i(p_i) = 1 = \frac{\alpha f_1(p_i)}{\alpha f_1(p_i)+(1-\alpha)f_0(p_i)}$ since $f_0(p_i) = 0$. Furthermore, $\pi^{II}(p_i, \alpha_i(p_i)) = \pi$.

For p_i not in the support of $F_0(p)$ and not in the support of $F_1(p)$, $\pi^I(p_i, 0) \leq \pi$ and $\pi^I(p_i, 1) \leq \pi$, and so $\pi^{II}(p_i, \alpha_i(p_i)) \leq \pi$.

Combining these observations we conclude that prices in the support of $F(p) = (1-\alpha)F_0(p) + \alpha F_1(p)$ earn $\pi > 0$ while other prices earn an amount $\leq \pi$. Furthermore, for p_i in the support of $F(p)$ the optimal advertising reach is $\alpha_i(p_i) = \frac{\alpha f_1(p_i)}{\alpha f_1(p_i)+(1-\alpha)f_0(p_i)}$. We conclude that $(\beta = 1, \alpha(p) = \frac{\alpha f_1(p)}{\alpha f_1(p)+(1-\alpha)f_0(p)}, F(p) = (1-\alpha)F_0(p) + \alpha F_1(p))$ is an equilibrium strategy.

□

Theorem 2.A.7 *Suppose that in model I it is an equilibrium to have* $0 < \beta^I < 1$, $0 < \alpha < 1$ *and* $F_0(p)$ *and* $F_1(p)$. *Then in model II it is an equilibrium to have* $\beta = \beta^I$,$\alpha(p) = \frac{\alpha f_1(p)}{\alpha f_1(p)+(1-\alpha)f_0(p)}$ *for* p *in the support of* $F(p)$ *and* $F(p) = \alpha F_1(p) + (1-\alpha)F_0(p)$.

Proof

The proof is identical to the proof of theorem 6 with two modifications. First, the expressions for $\pi^I(p_i, 0)$, $\pi^I(p_i, 1)$ and $\pi^{II}(p_i, \alpha(p_i))$ are slightly different, but the central equality $\pi^{II}(p_i, \alpha_i(p_i)) = (1-\alpha_i(p_i))\pi^I(p_i, 0) + \alpha_i(p_i)\pi^I(p_i, 1)$ still holds. And, second, $\pi = 0$ and in model I firm j chooses $\beta = \beta^{II}$, equilibrium profits are 0 and therefore $0 < \beta < 1$ is an optimal strategy.

□

The seven theorems above show that every equilibrium in model I can be translated into an equilibrium in model II. We will now show that every equilibrium in model II can be translated into an equilibrium in model I.

Theorem 2.A.8 *Suppose that in model II it is an equilibrium to have $\beta = 0$. Then in model I $\beta = 0$ also is an equilibrium.*

Proof

If $\beta = 0$ is an equilibrium in model II we have that

$$\pi^{II}(p_i, \alpha_i(p_i)) = (1 - \alpha_i(p_i))\pi(p_i, 0|0) + \alpha_i(p_i)(\pi(p_i, 1|0) - A).$$

For $\beta = 0$ to be an equilibrium it should be that for all possible prices p_i and advertising reaches $\alpha_i(p_i)$ $\pi^{II}(p_i, \alpha_i(p_i)) < 0$, implying that $\pi(p_i, 0|0) < 0$ and $\pi(p_i, 1|0) - A < 0$.

Now suppose that in model I firm j chooses $\beta_j = 0$. Then $\pi^I(p_i, 0) = \pi(p_i, 0|0) < 0$ and $\pi^I(p_i, 1) = \pi(p_i, 1|0) - A < 0$, implying that also for firm i being inactive is the optimal strategy. □

Theorem 2.A.9 *Suppose that in model II it is an equilibrium to have $\beta = 1$, $\alpha(p)$ and $F(p)$. Then in model I it is an equilibrium to have $\beta = 1$, $\alpha = \int_{\underline{p}}^{\overline{p}} \alpha(p)dF(p)$, $F_0(p) = \frac{\int_{\underline{p}}^{p}(1-\alpha(p))dF(p)}{1-\alpha}$ and $F_1(p) = \frac{\int_{\underline{p}}^{p}\alpha(p)dF(p)}{\alpha}$.*

Proof

We have that in model II

$$\begin{aligned}\pi^{II}(p_i, \alpha_i(p_i)) \;\; = \;\; & (1 - \alpha_i(p_i))[\int_0^1 \pi(p_i, 0|p, 0)(1 - \alpha(p))dF(p) + \\ & \int_0^1 \pi(p_i, 0|p, 1)\alpha(p)dF(p)] + \\ & \alpha_i(p_i)[\int_0^1 \pi(p_i, 1|p, 0)(1 - \alpha(p))dF(p) + \\ & \int_0^1 \pi(p_i, 1|p, 1)\alpha(p)dF(p) - A].\end{aligned}$$

We will denote the equilibrium profits in model II by π, with $\pi > 0$ since $\beta = 1$.

Now look at model I and suppose firm j chooses $\beta_j = 1$, $\alpha_j = \int_{\underline{p}}^{\overline{p}} \alpha(p)dF(p)$, $F_{0j}(p) = \frac{\int_{\underline{p}}^{p}(1-\alpha(p))dF(p)}{1-\alpha_j}$ and $F_{1j}(p) = \frac{\int_{\underline{p}}^{p}\alpha(p)dF(p)}{\alpha_j}$. Note that the cdf's $F_{0j}(p)$ and $F_{1j}(p)$ can be differentiated to give $f_{0j}(p) = \frac{(1-\alpha(p))f(p)}{1-\alpha_j}$ and $f_{1j}(p) = \frac{\alpha(p)f(p)}{\alpha_j}$. This gives that $\int_0^1 g(p)dF_{0j}(p) = \int_0^1 g(p)\frac{1-\alpha(p)}{1-\alpha_j}dF(p)$ and $\int_0^1 g(p)dF_{1j}(p) = \int_0^1 g(p)\frac{\alpha(p)}{\alpha_j}dF(p)$, where $g(p)$ is an arbitrary function of p. Using this information we find that

$$\pi^I(p_i, 0) = int_0^1\pi(p_i, 0|p, 0)(1 - \alpha(p))dF(p) + \int_0^1 \pi(p_i, 0|p, 1)\alpha(p)dF(p)$$

and

$$\pi^I(p_i, 1) = \int_0^1 \pi(p_i, 1|p, 0)(1 - \alpha(p))dF(p) + \int_0^1 \pi(p_i, 1|p, 1)\alpha(p)dF(p) - A.$$

Note that $\pi^{II}(p_i, \alpha_i(p_i)) = (1-\alpha_i(p_i))\pi^I(p_i, 0) + \alpha_i(p_i)\pi^I(p_i, 1)$. This leads to the following observations:

If p_i is not in the support of $F(p)$, $\pi^I(p_i, 0) \leq \pi$ and $\pi^I(p_i, 1) \leq \pi$.

If p_i is in the support of $F(p)$ and $\alpha(p_i) = 0$, $\pi^I(p_i, 0) = \pi \geq \pi^I(p_i, 1)$.

If p_i is in the support of $F(p)$ and $\alpha(p_i) = 1$, $\pi^I(p_i, 1) = \pi \geq \pi^I(p_i, 0)$.

If p_i is in the support of $F(p)$ and $0 < \alpha(p_i) < 1$, $\pi^I(p_i, 0) = \pi^I(p_i, 1) = \pi$.

Now suppose in model II it is an equilibrium that $\alpha(p) = 0$ for all p in the support of $F(p)$. Then for all p_i in the support of $F(p)$ $\pi = \pi^I(p_i, 0) \geq \pi^I(p_i, 1)$, while for all p_i not in the support of $F(p)$ $\pi^I(p_i, 0) \leq \pi$ and $\pi^I(p_i, 1) \leq \pi$. Consequently, it is an equilibrium in model I to have $\beta = 1$, $\alpha = 0$ and $F_0(p) = F(p)$.

The case that it is an equilibrium in model II to have $\alpha(p) = 1$ for all p in the support of $F(p)$ can be analyzed similarly and gives that it is an equilibrium in model I to have $\beta = 1$, $\alpha = 1$ and $F_1(p) = F(p)$.

Lastly, suppose that in model II it is an equilibrium to have both $\alpha(p) < 1$ for some p in the support of $F(p)$ and $\alpha(p) > 0$ for some p in the support of $F(p)$. Now look at model I and suppose firm i decides to advertise. When he chooses a price p_i such that p_i is in the support of $F(p)$ and $\alpha(p_i) > 0$ he obtains profits $\pi^I(p_i, 1) = \pi$ while for all other p_i $\pi^I(p_i, 1) \leq \pi$. This implies that for p_i in the support of $F_1(p) = \frac{\int_{\underline{p}}^{p} \alpha(p) dF(p)}{\alpha}$ advertising p_i gives profits of π while for other prices profits are smaller than or equal to π.

Now suppose firm i decides not to advertise. When he chooses a price p_i such that p_i is in the support of $F(p)$ and $\alpha(p_i) < 1$ he obtains profits $\pi^I(p_i, 0) = \pi$ while for all other p_i $\pi^I(p_i, 0) \leq \pi$. This implies that for p_i in the support of $F_0(p) = \frac{\int_{\underline{p}}^{p}(1-\alpha(p))dF(p)}{1-\alpha}$ not advertising p_i gives profits of π while for other prices profits are smaller than or equal to π.

Since both advertising and not advertising lead to profits $\pi > 0$ it is optimal for firm i to choose $\beta = 1$ and firm i is indifferent between advertising and not advertising, making $\alpha = \int_{\underline{p}}^{\overline{p}} \alpha(p) dF(p)$ part of an optimal strategy. This shows that the strategy given in the theorem is indeed optimal for firm i and therefore is an equilibrium in model I.

□

Theorem 2.A.10 *Suppose that in model II it is an equilibrium to have* $0 < \beta^{II} < 1$, $\alpha(p)$ *and* $F(p)$. *Then in model I it is an equilibrium to have* $\beta = \beta^{II}$, $\alpha = \int_{\underline{p}}^{\overline{p}} \alpha(p) dF(p)$, $F_0(p) = \frac{\int_{\underline{p}}^{p}(1-\alpha(p))dF(p)}{1-\alpha}$ *and* $F_1(p) = \frac{\int_{\underline{p}}^{p} \alpha(p) dF(p)}{\alpha}$.

Proof

The proof is identical to the proof of theorem 9 with two modifications. First, the expressions for $\pi^I(p_i, 0)$, $\pi^I(p_i, 1)$ and $\pi^{II}(p_i, \alpha(p_i))$ are slightly different, but the central equality $\pi^{II}(p_i, \alpha_i(p_i)) = (1-\alpha_i(p_i))\pi^I(p_i, 0) + \alpha_i(p_i)\pi^I(p_i, 1)$ still holds. And, second, $\pi = 0$ and in model I firm j chooses $\beta = \beta^{II}$, equilibrium profits are 0 and therefore $0 < \beta < 1$ is an optimal strategy.

□

These theorems show that both formulations are equivalent.

2.A.2 Proof of Theorem 2.3.1

There are three possible types of equilibria not mentioned in the theorem. We will show that each of these possibilities cannot be part of an equilibrium.

(i) $0 < \alpha < 1$ and $\overline{p}_0 > \overline{p}_1$. Using standard techniques, we note that in this case $F_0(p) = 1 - \frac{(\overline{p}_0 - p)\frac{1}{2}\mu(1-\gamma)}{p\gamma}$ for $p \geq \overline{p}_1$. Whenever $p \geq \overline{p}_1$, the profit for an advertising firm equals $\pi_1(p) = p[\gamma(1-\alpha)(1-F_0(p)) + (1-\gamma)(1-\alpha)] - A$. Using the expression for $F_0(p)$ it can be shown that

$$\pi_1(p) = (1-\alpha)\frac{1}{2}\mu(1-\gamma)(\overline{p}_0 - p) + (1-\gamma)(1-\alpha)p - A,$$

which is increasing in p for $\overline{p}_1 \leq p \leq \overline{p}_0$, showing that advertising firms have an incentive to deviate.

(ii) $0 < \alpha < 1$ and $\underline{p}_1 < \overline{p}_0 < \overline{p}_1$. In this case, using $\pi_0(p) = \pi_1(p)$ in the price region $[\max\{\underline{p}_0, \underline{p}_1\}, \overline{p}_0]$, yields the following expression for $F_1(p)$:

$$F_1(p) = 1 - \frac{A - (1 - \frac{1}{2}\mu)p(1-\alpha)(1-\gamma)}{\alpha(1-\gamma)p}.$$

Moreover, using this and $\pi_0(\overline{p}_0) = \pi_0(p)$ we get $F_0(p) = 1 - \frac{(\overline{p}_0 - p)(\frac{1}{2}\mu - \gamma)}{p\gamma}$ on $[\max\{\underline{p}_0, \underline{p}_1\}, \overline{p}_0]$. To make sure $F_0(p)$ is below 1, $\frac{1}{2}\mu - \gamma$ should be positive. We furthermore can derive an expression for $F_1(p)$ for $p \geq \overline{p}_0$ by using $\pi_1(p) = \pi_1(\overline{p}_0)$. This gives for $p \geq \overline{p}_0$,

$$1 - F_1(p) = \frac{\alpha\overline{p}_0[(1 - F_1(\overline{p}_0)] - (1-\alpha)(1-\gamma)(p - \overline{p}_0)}{\alpha p}$$

Plugging this expression into $\pi_0(p)$ for $p \geq \overline{p}_0$ gives $\pi_0(p) = \gamma\overline{p}_1(1-\alpha)(1-\gamma) + p(1-\alpha)(1-\gamma)(\frac{1}{2}\mu - \gamma)$, which is increasing in p since $\frac{1}{2}\mu - \gamma$ is positive. This shows that non-advertising firms have an incentive to deviate.

(iii) $0 < \alpha < 1$ and $\overline{p}_0 < \underline{p}_1$. In this case, $\pi_0(p) = p(\alpha\gamma + (1-\alpha)(1-\gamma)\frac{1}{2}\mu)$ for $\overline{p}_0 \leq p \leq \underline{p}_1$, which is increasing in p and so non-advertising firms have an incentive to deviate.

$\square$

2.A.3 Proof of Theorem 2.4.1

a. Observe that when $c \to 0$, the equilibrium is in region Ib with $\alpha = 0$ and $\mu = 1$. Furthermore, the maximum price $r_0 = \frac{c}{\beta_1} \to 0$.

$\square$

b. Observe that when $A \to 0$, either equilibrium $IIaii$ or equilibrium $IIbii$ holds. In equilibrium $IIaii$, μ does not depend on A, and as $A \to 0$, $\alpha = 1 - \frac{A}{(1-\gamma)(1-\frac{1}{2}\mu)} \to 1$ and $Ep_1 = A(\frac{\frac{1}{2}\mu-\gamma}{\alpha(1-\gamma)(1-\frac{1}{2}\mu)} \ln \frac{\mu-2\gamma}{\mu} + \frac{1}{\alpha(1-\frac{1}{2}\mu)} \ln \frac{1-\frac{1}{2}\mu}{A}) \to 0$.

In equilibrium $IIbii$, $\alpha = 1 - \frac{2A}{(1-\gamma)r_0}$ and $Ep_1 = A\frac{2}{\alpha} \ln \frac{(1-2\gamma)r_0}{2A} - \frac{A}{(1-\gamma)\alpha} \ln(1-2\gamma)$, while r_0 does not depend on A. As $A \to 0$, this yields $\alpha \to 1$ and $Ep_1 \to 0$.

□

Chapter 3

Going where the ad leads you: on high advertised prices and searching where to buy

3.1 Introduction

Imagine yourself attending a conference in a foreign country. You sit with your laptop in a hotel room and realize that when your battery expires you cannot charge it because of the different electricity outlet system. You decide to go out in town to search for an electricity converter and enter a first electronics shop where the charming sales representative tells you that they unfortunately do not carry such an item in their store. The same story repeats in a number of stores, after which you disappointingly go back to the hotel. In a last desperate attempt you ask at the hotel lobby whether they, by any chance, would know a shop where they carry the item you are looking for. Triumphantly, the clerk at the desk tells you that a firm has left an advertisement behind informing people that they carry all different types of electronic converters one may ever wish to use (possibly with the prices at which they sell). You are very happy for this piece of information, immediately go to the shop and are prepared to buy at any (somewhat reasonable) price.

This story contains an element that we believe is important in many markets, not just when hanging around in far away destinations: namely that part of the search activities of people is not about "searching for firms with the lowest price", but rather about "searching for firms that sell the product". This distinction has not been made before. The typical search model only considers situations where all firms in the market carry the product and the only reason for consumers to search (further) is to look for a price-quality combination that better fits the individual's preferences. That is, the literature on consumer search is not about the "real" search activity of consumers when they are uncertain about which firms carry the product.

Another important aspect of the story above is that a potentially important role of advertising is simply to inform consumers about the fact that the advertising firm carries the product, thereby helping the consumer to save on expected search costs. If the uncertainty about which firm carries the product is very large, then the reduction in "real" search costs may be quite significant. This, so this Chapter argues, may lead to advertised prices being higher than non-advertised prices. This is contrary to conventional wisdom expressed in the literature on informative advertising, according to which informative price advertising leads to better-informed consumers and therefore to more competition and lower prices (see, e.g., Farris and Albion (1980) and Tirole (1998, Section 7.3)). Thus, this Chapter contributes to the strategic literature on advertising by arguing that in the presence of uncertainty about which firms sell the product, informative price advertising may lead to higher prices compared to the non-advertised prices. This insight is also important in empirical work on advertising. It shows that one should be careful to conclude from an observed positive correlation between advertising and prices that advertising is persuasive, see e.g. Boulding et al. (1994) and Clark (2005).

The mechanism we uncover is potentially also important in understanding emerging markets that use multimedia technologies. A first issue in such markets concerns discussions one can find in the popular press and in online discussion groups on personalized advertising.[1] Google's CEO Eric Schmidt has claimed that over the next few years he wants to be able to send personalized ads on GPS-based in-car communication systems to consumers to help them find the shops they are looking for. One interpretation of this would be that personalized advertising helps consumers to get products that are really close to their tastes, i.e., in a world where product heterogeneity is important. Another interpretation of personalized advertising is one where these ads help consumers to economize on their search costs by directing their search activities on the firm(s) from which they received an ad. This second interpretation comes very close to what we analyze in this Chapter.

A second aspect of these markets relates to the commercial advertisements one sees when using websites, like http://www.allbookstores.com/, where book prices of different online bookshops are quoted. These websites offer the option to search for the lowest price or to buy directly at one particular online bookseller. Apparently, this bookseller is willing to pay for this link suggesting that a significant fraction of consumers does not continue the search, but simply buys immediately at the firm where they know they can buy the book. Something similar happens at well-known search engines like Google and Yahoo, where firms pay to get their firm's website listed, knowing that (a fraction of) consumers are likely to click on one of the first-listed sites first.[2] Firms may exploit this search behavior by charging higher prices to compensate for the advertising expenditures.[3]

This Chapter studies "searching for the product" and "high prices through informative advertising" in relation to each other. To this end, we develop a simple three-stage

[1] See, e.g., http://blogs.zdnet.com/micro-markets/index.php?p=131.

[2] In this sense, this Chapter is related to the modeling of page views on the internet (see, e.g., Danaher (2007)).

[3] The main difference between this example and the model in this Chapter is that prices are typically not mentioned in these advertisements. Without price advertisement it is, however, easier to see that advertising firms may set higher prices (see Section 3.6 for more details on this) and the model in this Chapter basically makes the additional point that even if prices were advertised, they still can be higher and attract consumers.

model.[4] In the first stage, firms decide whether or not they want to allocate shelf-space to a particular type of product. Doing so has an opportunity cost of not using that space for having some other commodity on display. We call all firms who decide to carry the product "active firms". In the second stage, active firms decide on their price and on whether they will advertise the fact that they carry the product (and the price at which they sell it) by sending an advertisement to consumers. In the third stage, after potentially having received some ads, consumers decide on whether or not to search for a firm that carries the product, with a potentially lower price than the firms that advertised. However, if the firm has not advertised, the consumer does not know whether or not the firm carries the product in the first place.

The simplest search model we can imagine that makes the point that advertised prices can be higher than non-advertised prices, even under informative price advertisement, has two types of consumers: low-valuation consumers having low search costs c_L and high-valuation consumers having higher search costs c_H. For simplicity, we normalize c_L to be equal to 0. One may think of the high-demand consumers as having high income from demanding jobs to justify the positive correlation between the size of consumers' search cost and their willingness to pay. In previous literature this positive relation between a consumer's willingness to pay and her search costs has been used by, e.g., Iyer (1998) and Coughlan and Soberman (2005).

In such a model, there may be many different types of equilibria, depending on the parameter configurations. We focus on equilibria where advertised prices are higher than non-advertised prices.[5] The simplest such equilibrium has the following structure. Low-valuation consumers search all firms and then buy at the lowest-priced firm. As soon as high-valuation consumers receive an advertisement, they buy at the advertising firm. High-demand consumers who get no advertisements do not search for an active firm as they find the probability that these firms do not carry the product too high compared to the search costs they have to make. Non-advertising firms therefore completely concentrate on the low-valuation consumers, whereas advertising firms concentrate on high-valuation consumers. The probability that firms are active and the intensity with which active firms advertise are determined endogenously in such a way that firms are indifferent between being inactive, advertising a high price and not-advertising a low price. We show for which parameter values such an equilibrium exists and, in addition, we show for which subset of these parameter values the equilibrium is unique. The main result is thus best interpreted as a screening result: high-valuation consumers buy at the advertising firms (if any) at high prices and the low-valuation consumers buy at the non-advertising firms (if any) at low prices.

This Chapter is, of course, related to the large literature on consumer search and

[4] The model we develop abstracts from some of the features that are relevant to the examples mentioned above. For example, we abstract from product heterogeneity (that probably is important even in the book example when we consider that the book sale comes with a certain level of additional services) and also from nonprice advertisement (see also the concluding remarks).

[5] In this context, this Chapter may contribute to an understanding of performance regimes (as in Pauwels and Hanssens (2007)) or the implications of different advertising themes (cf., Bass et al. (2007))

advertising (see, e.g., the seminal papers by Stigler (1961), Diamond (1971), Stahl (1989, 1994) and Butters (1977)). The main difference with the consumer search literature is, as we mentioned, the uncertainty consumers face of not finding the product at a shop they visit. As Diamond (1971) has shown, the price uncertainty in search models can lead to monopoly prices. Rao and Syam (2001) obtain the same type of result in an advertising model. Here uncertain prices in the sense of unadvertised prices are above advertised prices. Our results are exactly the reverse: known, advertised prices are higher than unknown, unadvertised prices. One of the reasons why the Diamond effect does not play a role in our model is that there is a fraction of consumers with zero search costs.

In the advertising literature the main distinction is between persuasive and informative advertising. As indicated above, the main role of advertising in our model is to inform consumers of the availability of the product at the firm. In this way, advertising creates value to consumers and this value is evaluated differently by different consumers. The value that is created is different from that studied in the literature on persuasive advertising (see, e.g., Dixit and Norman (1978)), however. This persuasive advertising literature basically argues that advertising changes the preferences of individuals and the demand effect that emerges is mainly dependent on psychological factors (exogenous to the model) determining how much people's preferences have been affected. In this Chapter, preferences of individuals are unaffected and the only effect of advertising, namely the reduction in expected search costs, is endogenously determined.

In the informative advertising literature (see Meurer and Stahl (1994) and Soberman (2004)), price differentiation sometimes arises from horizontal product differentiation. Informative advertising plays two roles in this context. First, it creates awareness of products so that consumers know what best fits their tastes. This strengthens the product differentiation aspect, giving firms an incentive to raise prices. On the other hand, it also leads to more consumers with full information giving firms an incentive to reduce prices. When product differentiation is important enough, the first aspect dominates the second so that advertising can lead to price increases. In our model, however, all firms are *ex ante* identical and any form of differentiation is thus endogenously created.

The literature that combines consumer search and advertising is much more limited.[6] Robert and Stahl (1993) is the first paper where consumers' ignorance about prices can be resolved by consumers searching optimally for prices or by firms informing consumers about the prices they charge through advertising. Following on their work, Stahl (2000) and Chapter 2 of this thesis check the robustness of the model by investigating the properties of different modeling assumptions. Chapter 2 of this thesis shows that in some equilibria advertised prices may be higher than non-advertised prices. This result is driven by the assumption that less-informed consumers can buy at firms that advertise without

[6]Apart from the literature mentioned here, there is also a recent paper by Stivers and Tremblay (2005). Their model is, however, very different from the standard search models as they model search costs as the wedge between producer prices and consumer prices, very much like the analysis in traditional tax studies. Moreover, they assume that advertising lowers the search costs of consumers. In such a world, they show that it is possible that advertising raises the price the firms ask, while at the same time decreasing the price (including search costs) that consumers have to pay.

incurring search cost, giving advertising firms an advantage above non-advertising firms that have to be searched for. In this Chapter we formally explain how this difference in search cost to buy from an advertising and a non-advertising firm can emerge out of the uncertainty consumers face when they visit a shop that did not advertise.

Finally, this version of the Chapter - with the assumption of low valuation consumers having zero search cost - is close in spirit to papers understanding the nature of online price competition, modeling the importance of shopbots and search engines (see, e.g. Chen and Sudhir (2004), Iyer and Pazgal (2003) and Janssen et al. (2007), and most notably He and Chen (2006)). He and Chen (2006) analyze behavior of e-marketplaces where consumers have the possibility to use the embedded search engine or they can shop directly at a featured store. They show that prices at the featured stores can be higher, but that consumers with relatively high search cost may still want to shop there as by doing so they economize on their time. Insofar as advertising is similar in nature to featuring, the results of this Chapter reinforce the insight obtained by He and Chen (2006). This Chapter shows that this insight is not confined to e-marketplaces, and that it may hold under a very different set of assumptions. For example, we employ sequential instead of simultaneous search, we use price advertising instead of nonprice advertising, and consumers in our model are not loyal to any of the firms. Moreover, advertising in our model is a signal of product availability, an aspect that is absent in previous models. Since the gains of buying from an advertising firm depend on the product availability in non-advertising firms and product availability is endogenously determined, in our model the gains of buying from an advertising firm are endogenous. In previous models these gains are exogenously given. Finally, in the working paper on which this Chapter is based (Janssen and Non (2007)), we show that even if both groups of consumers have positive search costs, the result that advertised equilibrium prices are higher than non-advertised prices does hold.

The rest of this Chapter is organized as follows. Section 3.2 presents the model. In Section 3.3 we analyze a simplified version of the model where we show that the result of high advertised prices does not depend on price uncertainty but instead is caused by uncertainty about whether the firm carries the product. Section 3.4 formally characterizes one equilibrium where advertised prices are higher than non-advertised prices. In Section 3.5 we will elaborate on the existence of other equilibria and we will show that the equilibrium with high advertised prices is unique in some parameter region. Section 3.6 concludes.

3.2 The model

Consider a homogeneous goods market where (at most) two firms produce without incurring production costs.[7] The only cost relevant for our analysis is the opportunity cost, S, firms face for shelving the product. The decisions firms take are modeled as a three-stage game. In the first stage, firms decide on whether or not to carry the product. We will

[7]Extending the model to more than two firms does not change the results but only complicates the analysis. Production costs can be easily introduced, but they are simply normalized to be equal to 0.

denote the probability of a firm being active by β. If a firm decides to be inactive, it makes no profits or losses.

In the second stage of the model, firms decide simultaneously on their advertising strategy and price. Firms do not know the outcome of the first stage (the decision to be active) and therefore, it is just as if firms play these two stages simultaneously. An active firm can decide to advertise that it sells the product and at which price. Note here that advertising is purely informative: an advertisement informs about existence and price. Advertising is an 'all-or-nothing' decision, that is, a firm either advertises to the complete market or does not advertise at all.[8] The cost of advertising is A. We will denote the probability with which a firm advertises by α. The pricing strategy depends on whether a shop advertises or not. We will therefore specify a pricing strategy conditional on advertising and a pricing strategy conditional on not advertising. Denote by $F_1(p)$ the price distribution conditional on advertising and let $F_0(p)$ denote the price distribution conditional on not advertising. We use $\overline{p}_1$ to indicate the highest price and $\underline{p}_1$ to indicate the lowest price in the support of $F_1(p)$. Similarly, $\overline{p}_0$ and $\underline{p}_0$ denote the highest and lowest price in the support of $F_0(p)$.[9]

In the third stage of the game, consumers receive the advertisements that are sent and decide on their search strategy. There is a unit mass of consumers with unit demand. Consumers come into two types. A fraction γ has a low valuation θ_L for the product and zero search costs.[10] A fraction $1-\gamma$ has a high valuation θ_H for the product and strictly positive search costs c. We assume $\theta_H - c > \theta_L$. Consumers search sequentially, conditional on the advertisements that are sent. Sequential search means that the consumers first look at the advertisements they have received. They then decide on whether to visit one additional firm, to buy immediately from the cheapest advertising firm, or not to buy at all. After searching one firm they again decide on whether to visit a second firm, to buy from the cheapest known firm or not to buy at all. Note that consumers only search non-advertising firms, since they already know that advertising firms are active and the ad also tells at which price the active firm sells. We assume that for the high-valuation consumers every first visit to a firm costs c.[11] Visiting an advertising firm, a consumer has to bear the search costs, but is sure to find the product. Visiting a non-advertising

[8]This advertising technology may seem unrealistic at first sight, but as is shown in Chapter 2 of this thesis it is exactly equivalent to an advertising technology where firms choose an advertising reach.

[9]Note that the price distributions $F_0(p)$ and $F_1(p)$ can describe randomized price strategies as well as pure strategies. For instance, if advertising firms use the pure strategy of always setting price p^* then $F_1(p) = 0$ for $p < p^*$ and $F_1(p) = 1$ for $p \geq p^*$. It is, however, easy to see that in equilibrium advertising firms always use a randomized price strategy. For an advertising firm there is a strictly positive probability $\alpha\beta$ that the competitor advertises as well and so a standard undercutting argument can be used to show there are no atoms in $F_1(p)$.

[10]Note that because of the zero search costs these consumers know all active firms and prices. Firms compete for these consumers and therefore an undercutting argument shows there are no atoms in $F_0(p)$ for $p \leq \theta_L$. In an earlier version, we analyzed a model where the low-valuation consumers had a strictly positive search cost. The equilibrium with high advertised prices is also an equilibrium in this modified model, but we have not been able to prove uniqueness.

[11]For simplicity, we follow the search literature in assuming "free recall", i.e., consumers do not bear any costs for return visits to firms they already visited once.

firm, a consumer incurs the same search costs, but in this case is not sure to find the product. This implies that the 'real' search cost for searching a non-advertising firm is higher than the search cost for searching an advertising firm.

We will solve this model for a symmetric perfect Bayesian equilibrium, the standard equilibrium notion for games with asymmetric information. Symmetry implies that all *ex ante* identical players play the same strategy. Bayesian updating plays a role when consumers form expectations about the probability a firm is active given that it did not advertise.

3.3 Equilibrium under price certainty

For didactical purposes, and to highlight that not price uncertainty, but uncertainty about product availability, is the key element driving the result mentioned in the introduction, we first consider a version of the model where firms do not freely choose prices, but instead choose between an exogenously given high price p_H and low price p_L, with $\theta_H - c \geq p_H > \theta_L > p_L$. The question we address is whether there exists an equilibrium with the same features as the equilibrium mentioned in the introduction: the high price is advertised, the low price is not advertised and high-valuation consumers do not search. Such an equilibrium has price certainty for searching consumers as all firms that do not advertise set the same price. We concentrate on a symmetric equilibrium and in this case that means that both firms are indifferent between choosing the high price p_H and advertising it and choosing the low price and not advertising it. It is clear that for a symmetric equilibrium to exist, it must be the case that the probability with which a firm is active is strictly smaller than 1 as otherwise a consumer who did not receive any advertisements or received only one advertisement will infer that the non-advertising firm(s) is (are) active and charge(s) a low price, making it profitable to search. Thus, in such an equilibrium a firm has to be indifferent between being active and not being active implying that the equilibrium pay-off should equal 0.

We pose that there exists a symmetric equilibrium where advertising firms charge price p_H and non-advertising firms charge a price p_L, where high-valuation consumers buy at an advertising firm without searching and where high-valuation consumers who get no advertisements do not search at all. Low-valuation consumers observe all prices and only buy if a firm sells at p_L.

We now have to check that this is indeed an equilibrium, i.e., that none of the players has an incentive to deviate. We first look at consumer behavior and note that if it is not optimal to search after not receiving any advertisements, then it certainly is not optimal to search after receiving one advertisement as the utility of following the ad is positive. We therefore concentrate on the case where high-valuation consumers did not receive any advertisements. If their first search leads to an active firm, they will stop searching since a lower price cannot be found. If they search once and do not find an active firm, the utility of a second search is $-c+\frac{(1-\alpha)\beta}{1-\alpha\beta}(\theta_H - p_L)$.[12] If this expression is below zero, the utility of not searching, consumers will not search a second time and the utility of the first search also is $-c+\frac{(1-\alpha)\beta}{1-\alpha\beta}(\theta_H - p_L) < 0$. Therefore if this inequality holds, consumers will not search at all.

Given the strategy of the high-valuation consumers, the pay-offs to a firm choosing p_H and advertising are given by

$$\pi_H = p_H(1-\gamma)(1-\frac{1}{2}\beta\alpha) - A - S$$

and the pay-offs to a firm choosing p_L and not advertising are given by

$$\pi_L = p_L\gamma(1-\frac{1}{2}\beta(1-\alpha)) - S.$$

As $\pi_H = \pi_L = 0$ in the equilibrium we are looking for, we can determine α and β endogenously. It follows that

$$\beta = 4 - 2\frac{A+S}{(1-\gamma)p_H} - 2\frac{S}{\gamma p_L}$$

and

$$\alpha = \frac{2}{\beta} - \frac{2}{\beta}\frac{A+S}{(1-\gamma)p_H}$$

.

As $0 < \alpha < 1$ and $0 < \beta < 1$ this gives restrictions

$$\frac{A+S}{p_H(1-\gamma)} < 1,$$

$$\frac{S}{p_L\gamma} < 1$$

and

$$3 - 2\frac{A+S}{p_H(1-\gamma)} - 2\frac{S}{p_L\gamma} < 0.$$

Firms should also not have an incentive to deviate. If a firm would set p_H and not advertise, its profits are $-S$, which is clearly below the equilibrium profits of zero. If it sets a price p_L and advertises this price, its profits are given by

[12] Note that consumers only search non-advertising firms and therefore the probability of finding an active firm is given by $\frac{(1-\alpha)\beta}{(1-\beta\alpha)}$.

$$p_L(1-\gamma) + p_L\gamma(1 - \frac{1}{2}\beta(1-\alpha)) - A - S.$$

Using the expressions for α and β that are given above we can rewrite these profits as $p_L(1-\gamma)-A$ and so for this deviation not to be profitable it should be that $p_L(1-\gamma) < A$.

It is easy to find parameter values such that all restrictions are satisfied. For example, take $\theta_L = 1, p_L = 0.9, \gamma = 0.5, A = 0.75, S = 0.375$ and $p_H = 2.5$. Then if we choose θ_H and the search cost parameter c such that $2.5 + c < \theta_H < 0.9 + 2.4c$, all conditions are satisfied.

3.4 Existence of an equilibrium with high advertised prices

The previous section has shown that the result of high advertised prices is not driven by price uncertainty but instead by uncertainty about the availability of the product. In this section we will construct a symmetric equilibrium of the full model specified in Section 3.2 where firms are free to choose any price. In this equilibrium, again, advertising firms set higher prices than non-advertising firms and high-valuation consumers do not search. The equilibrium is symmetric in the sense that both firms choose the same (ex ante) strategy. Their ex post behavior (whether or not to advertise and which price to choose) may be different because of different realizations of the mixed strategy chosen.

Intuitively, when high-valuation consumers do not search the non-advertising firms compete for the low-valuation consumers. Advertising firms, however, also sell to the high-valuation consumers and therefore have an incentive to ask higher prices. Even in such a case high-valuation consumers do not want to search for the lower non-advertised prices as long as the probability of a firm being active, β, is low enough.

Proposition 3.4.1 *If the following four conditions are satisfied*

(i) $(1-\gamma)\theta_L < A < (1-\gamma)(\theta_H - c) - S$

(ii) $S < \gamma\theta_L$

(iii) $1 - \frac{S}{\gamma\theta_L} - \frac{A+S}{(1-\gamma)(\theta_H - c)} < 0$

(iv) $\theta_H(1 - \frac{S}{\gamma\theta_L}) - \frac{S}{\gamma}\ln\frac{\gamma\theta_L}{S} < \frac{A+S}{(1-\gamma)(\theta_H-c)}c,$

then there exists a symmetric equilibrium with $\underline{p}_1 > \overline{p}_0$. *In this equilibrium consumer search behavior is characterized by*

(i) *Low-valuation consumers buy at the cheapest active firm out of all firms, provided the price is not above* θ_L.

(ii) *High-valuation consumers who receive at least one advertisement buy immediately at the the advertising firm with the lowest price, provided the price is not above* $\theta_H - c$*.*

(iii) *High-valuation consumers who do not receive an ad do not search.*

Firm behavior is characterized by

(i) *Firms are active with probability* $\beta = 2 - \frac{S}{\theta_L \gamma} - \frac{A+S}{(1-\gamma)(\theta_H - c)}$*.*

(ii) *Active firms advertise with probability* $\alpha = \frac{1 - \frac{A+S}{(1-\gamma)(\theta_H - c)}}{2 - \frac{S}{\theta_L \gamma} - \frac{A+S}{(1-\gamma)(\theta_H - c)}}$*.*

(iii) *Non-advertising firms choose prices according to distribution*

$$F_0(p) = 1 - \frac{(\theta_L - p)(1 - \beta(1-\alpha))}{p\beta(1-\alpha)},$$

with $\overline{p}_0 = \theta_L$ *and* $\underline{p}_0 = \theta_L(1 - \beta(1-\alpha))$*.*

(iv) *Advertising firms choose prices according to distribution*

$$F_1(p) = 1 - \frac{(\theta_H - c - p)(1 - \beta\alpha)}{p\beta\alpha},$$

with $\overline{p}_1 = \theta_H - c$ *and* $\underline{p}_1 = (\theta_H - c)(1 - \beta\alpha)$*.*

The proof can be found in the appendix to this chapter and basically shows that under the four conditions specified in the proposition no player has an incentive to deviate from his strategy.

The interpretation of the four restrictions used in Proposition 3.4.1 is fairly straightforward. The first part of the first restriction tells us that the advertising costs A should be so high that advertising a price at or below θ_L is not profitable. This makes sure that advertising firms concentrate completely on the high-valuation consumers and enables our screening result. The second part of the first restriction states that the advertising costs A should be low enough to make advertising profitable[13] while the second restriction states that the shelving costs S should be low enough to make shelving the product a profitable strategy. On the other hand, the third restriction states that both advertising and shelving costs should be high enough to guarantee that firms do not always want to be active, which in turn is needed as otherwise high-valuation consumers want to continue searching after receiving one advertisement with a relatively high price. Finally, the last

[13] Note that the profits of an advertising firm are equal to $(\theta_H - c)(1-\gamma)(1-\alpha\beta) - A - S$ and that when $A + S$ is close to $(\theta_H - c)(1-\gamma)$, α is close to 0.

restriction basically gives a lower bound on the search costs c such that consumers do not want to search.

The first restriction implies that $\theta_H - c > \theta_L$, which can be interpreted as that the difference in valuations between the two types of consumers should be high enough. This restriction is necessary for any equilibrium where advertised prices are higher. To make this clear, suppose to the contrary that $\theta_H - c < \theta_L$. The maximum price that an advertising firm will then ask is still $\theta_H - c$. This is because if an advertising firm sets a price between $\theta_H - c$ and θ_L it would only sell to low-valuation consumers and so there would be no reason to advertise. For non-advertising firms it could, however, be profitable to set a price above $\theta_H - c$. There are two reasons for this. First, low-valuation consumers are willing to pay θ_L and so a non-advertising firm setting a price θ_L has a probability of $1 - \beta$ to be the only firm in the market and to sell to the low-valuation consumers. Second, searching high-valuation consumers are willing to pay θ_H when they find an active non-advertising firm. This difference in willingness to pay between an advertising firm and a non-advertising firm comes from the search costs being sunk when visiting a non-advertising firm. It can be shown that when $\theta_H - c < \theta_L$, it is indeed always profitable for a non-advertising firm to ask a higher price than an advertising firm and so an equilibrium where advertising firms charge higher prices than non-advertising firms is not feasible anymore.

We now want to check whether all four restrictions can jointly hold, that is, whether there really exists a parameter region where the above equilibrium exists. To do so we set, without loss of generality, $\theta_L = 1$.[14] It is easily checked that the four restrictions can jointly hold when, for instance, $\gamma = 0.1$, $S = 0.099$, $A = 1$, $c = 0.04$ and $1.3 < \theta_H < 2.5$. Moreover, we would like to establish the following result, arguing that the existence conditions become easier to satisfy when both θ_H and c are relatively large.

Proposition 3.4.2 *If for some values A^*, S^*, θ_H^*, c^* and γ^* the equilibrium of Proposition 3.4.1 exists, then the equilibrium also exist for values A^*, S^*, $\theta_H^* + x$, $c^* + x$ and γ^*, where x is an arbitrary positive number.*

The proof can be found in the appendix to this chapter. Below we provide an intuitive explanation for the result. The expressions for the strategy of the firms, $(F_0(p), F_1(p), \alpha, \beta)$, given in Proposition 3.4.1 do not depend on θ_H or c in isolation, but instead only depend on $\theta_H - c$, since this is the maximum price that advertising firms can ask. As the first three restrictions are imposed to guarantee that α and β are in between 0 and 1, these restrictions remain unaffected as long as $\theta_H - c$ is unaffected.

The fourth restriction ensures that consumers do not want to search. Consumers who do not receive any advertisement have to make a tradeoff between incurring search costs c and having a probability of finding an active firm, which would give an uncertain payoff

[14] Looking at the equilibrium shows that if the parameters θ_L, θ_H, A, S and c are multiplied by a scalar x, the same equilibrium emerges, except that the prices are multiplied by x. To ease the analysis we decided to normalize the parameters such that $\theta_L = 1$.

of $\theta_H - p$, where p is the uncertain price being found. Suppose that for some parameters θ_H^* and c^* it is optimal not to search, so the search costs are above the expected payoff. Then if both c^* and θ_H^* increase by x, the expected payoff, given by $\frac{\beta(1-\alpha)}{1-\alpha\beta}(\theta_H - p)$, with $\frac{\beta(1-\alpha)}{1-\alpha\beta} < 1$, also increases but with less than x. Therefore, if both c^* and θ_H^* increase by x, it is still not optimal to search and the fourth restriction is still satisfied.

3.5 Other equilibria

The model we specified in Section 3.2 has many possible equilibria. It is natural to inquire whether for some parameter values the equilibrium we have identified in the previous section is the unique symmetric perfect Bayesian equilibrium. If the equilibrium is unique, we can legitimately argue that there is a parameter region where advertised prices are higher than non-advertised prices. If such a parameter region does not exist, then there is a possibility that advertised prices are higher, but this cannot be guaranteed. In this section, we do three things. First, we provide some observations on how the reservation price characterizing consumer search behavior is determined. This reservation price is needed in the remainder of this section and the exact derivation of it is in the appendix to this chapter. Next, we illustrate by means of an example that there are parameter values where the equilibrium described in the previous section exists, but it is not unique. Finally, we prove that when θ_H and c are relatively large, the equilibrium described in the previous section is unique.

As in any consumer search model, we can define a reservation price r by $\int_{\underline{p}_0}^{r+c} F_0(p)dp = \frac{1-\alpha\beta}{(1-\alpha)\beta}c$, where $\int_{\underline{p}_0}^{r+c} F_0(p)dp$ is the benefit of an additional search provided the firm found is active and did not advertise; $\frac{(1-\alpha)\beta}{1-\alpha\beta}$ is the probability a firm is active conditional on that it does not advertise. When high-valuation consumers receive an advertisement, they buy immediately when the lowest advertised price is at or below r, while for higher prices they search until they find an active non-advertising firm asking a price at or below $r + c$ or until all firms have been searched. When high-valuation consumers have not received any advertisement they search when $r+c < \theta_H$, they do not search when $r+c > \theta_H$, and they are indifferent between searching and not searching if $r+c = \theta_H$. When they search, they continue to search until they find an active firm asking a price at or below $r+c$ or until all firms have been searched. The derivation of this result follows the usual lines, except that we need to take into account that consumers have to pay search costs when visiting any firm, independent of whether it advertises or not. Moreover, there is a difference between advertising and non-advertising firms in the sense that when a consumer observes the price set by a non-advertising firm, its search costs are already sunk, while it still has to make the search costs just after having observed the advertised price.

Example (Pure Consumer Search). In the previous section we noted that the equilibrium with high advertised prices exists for $\gamma = 0.1$, $S = 0.099$, $A = 1$, $c = 0.04$ and $1.3 < \theta_H < 2.5$. We now show that for these very same parameter values a 'pure search equilibrium' exists where firms do not advertise ($\alpha = 0$) and where all firms are active ($\beta = 1$). In this example we look at the case where $r + c < \theta_L < \theta_H - c$, implying that all high-valuation consumers search. A traditional undercutting argument shows that when $p > r + c$ there are no atoms and so $\pi_0(\overline{p}_0) = -S$ for any $\overline{p}_0 > r + c$. On the other hand, $\overline{p}_0$ cannot be smaller than $r + c$ either, since deviation to $r + c$ would be profitable. For any price $p \leq \overline{p}_0 = r + c$ profits are given by

$$\pi_0(p) = p\gamma(1 - F_0(p)) + p(1 - \gamma)\frac{1}{2} - S.$$

Equating this to $\pi_0(r + c) = \frac{1}{2}(1 - \gamma)(r + c) - S$ gives

$$F_0(p) = 1 - \frac{(1 - \gamma)(r + c - p)}{2p\gamma},$$

and $\underline{p}_0 = \frac{\frac{1}{2}(1-\gamma)}{\gamma+\frac{1}{2}(1-\gamma)}(r + c)$. Furthermore, $\int_{\underline{p}_0}^{r+c} F_0(p)dp = c$ gives $r = c\frac{1-\kappa}{\kappa}$, with $\kappa = 1 - \frac{\frac{1}{2}(1-\gamma)}{\gamma} \ln \frac{\frac{1}{2}(1-\gamma)+\gamma}{\frac{1}{2}(1-\gamma)}$. This equilibrium holds whenever $\pi_0 > 0$, $\pi_1 < \pi_0$ and $r+c < \theta_L = 1$. Note that the first restriction holds if, and only if, $\pi_0 = \frac{1}{2}(r+c)(1-\gamma) - S > 0$, while the second holds if $\frac{1}{2}(1 - \gamma)r < A$. Substituting the parameter values that are given above shows that these three restrictions do hold for these values. □

The intuition behind the co-existence of multiple equilibria is as follows. For the given parameter values, S is very close to γ. In the equilibrium with high advertised prices described in Section 3.4 the maximum profits from not advertising, realized when the competitor advertises its high price or is not active at all, are $\gamma - S$. When γ is close to S these maximum profits are low and to make not advertising attractive the probability of obtaining these profits should be high. This implies that β has to be low. A low value of β also means that high-valuation consumers have no incentives to search, even when the search costs c are low. The equilibrium with high advertised prices can therefore also exist for low values of c. For such low values of c it is, however, also possible to have equilibria where consumers search (as in the above example). If consumers continue to search, it is not profit maximizing to advertise and a standard consumer search equilibrium emerges as discussed in the example.

We note that in the example above firm profits are strictly positive while in the equilibrium with high advertised prices profits equal zero. The intuition behind this is the difference in search behavior of consumers. In the example all high-valuation consumers search and firms set prices in such a way that these consumers buy at the first firm they find. This implies that firms have monopoly power over the high-valuation consumers while they compete for the low-valuation consumers. Since in our numerical example the fraction of high-valuation consumers, $1 - \gamma$, is relatively high, firms can extract high profits from these consumers. In contrast, in the equilibrium with high advertised prices high-valuation consumers do not search. This means that non-advertising firms only sell to the low-valuation consumers and in the best case realize (low) profits $\gamma - S$, while advertising firms have to pay relative high advertising costs to be able to sell to the high-valuation consumers, and consequently realize low profits as well. Another way to explain the difference in profits is to note that in the equilibrium with high advertised prices, firms are indifferent between being active and not being active. This is needed to ensure that high-valuation consumers do not want to search. Since being inactive leads to zero profits, firms that are active also should earn no profits at all. In the example above high-valuation consumers search and all firms are active, which means their profits can be strictly positive.

In the example above not only are firm profits higher than in the equilibrium with high advertised prices, but consumer welfare is also higher. We note that there are two types of consumers, with low valuations and with high valuations, and that for both types welfare is higher. The reason for this is the same for both consumer types and is twofold. First, in the example the prices are lower. In the numerical example the maximum price is 0.41 while in the equilibrium with high advertised prices the maximum non-advertised price is 1 and the maximum advertised price varies between 1.26 and 2.46, depending on the value of θ_H. The second reason for the difference in consumer welfare is the difference in the level of activity of the firms. In the example above all firms are active and consumers search so in the end all consumers buy. In the equilibrium with high advertised prices firms are active with probability β and therefore there is a probability that no firm is active at all and consumers can not buy. Furthermore, there also is a strictly positive probability that none of the active firms advertises and in that case only the low-valuation consumers buy. We note that in the numerical example β is fairly low[15], and so the difference in consumer welfare can be quite high. To give an example, the low-valuation consumers obtain a welfare of 0.0001 in the equilibrium with high advertised prices while welfare is 0.64 in the example above. The welfare for high-valuation consumers depends on θ_H and, e.g., for $\theta_H = 2$ welfare equals 0.28 and 1.59 respectively.

These observations on firm profits and consumer welfare show that the equilibrium with searching consumers Pareto dominates the equilibrium with high advertised prices. If one uses Pareto dominance as an equilibrium selection criterion the equilibrium with high advertised prices would therefore be rejected in favor of the equilibrium with searching consumers. In the remainder of this section we will show, however, that the co-existence argument crucially depends on the assumed small value of c. The next proposition shows that when θ_H and c are relatively large, the equilibrium of Proposition 3.4.1 is unique.

Proposition 3.5.1 *Fix parameter values $\theta_H^*, c^*, S^*, \gamma^*$ and A^* such that the equilibrium described in Proposition 3.4.1 exists. Then there exists a positive real number $\widehat{x}$ such that for all $x > \widehat{x}$ the equilibrium described in Proposition 3.4.2 exists and is unique for all parameter values $\theta_H^* + x, c^* + x, S^*, \gamma^*$ and A^* .*

The existence part of this proposition has already been shown in Section 3.4. The proof of the uniqueness part can be found in the appendix to this chapter. Here we will

[15] β is between 0.04 and 0.51, depending on the value of θ_H.

provide an intuitive explanation. The equilibrium we defined in Proposition 3.4.1 has high-valuation consumers not searching in case they did not receive an advertisement. Other equilibria where consumers do not search are shown not to overlap with the equilibrium with high advertised prices. This is because in these equilibria the consumers behave in the same way and therefore there is enough continuity in the firms' pricing and advertising decision problem to prevent overlap in the equilibrium regions. Overlap can occur between equilibria with different consumer behavior. For example, as we showed above, the high advertised price equilibrium with no consumer search (partly) overlaps with an equilibrium where high-valuation consumers do search. Therefore it is necessary to rule out the co-existence of equilibria where consumers (partly) search, i.e., equilibria with $\beta > 0$ and $\alpha < 1$.[16] We show in the proof that these equilibria, except for one equilibrium, do not exist for x high enough. The equilibrium that does exist even when x gets infinitely large is shown not to overlap for x high enough.

The two main arguments used to obtain this result are as follows. First, as argued before, consumers who did not receive any advertisements need to compare the search costs c with the possible gains from searching, $\frac{\beta(1-\alpha)}{1-\alpha\beta}(\theta_H - p)$. When a constant x is added to both the search costs and θ_H, the search costs generally increase more than the gains from searching. Therefore, the higher the constant x, the more difficult it is to have an equilibrium where consumers search. Second, when high-valuation consumers search, non-advertising firms may decide to concentrate on high-valuation consumers and since the costs of searching a non-advertising firm are sunk at the moment the consumer has arrived at the shop, these firms can ask a price equal to the maximum advertised price plus the search costs c without loosing any customers. When the search costs increase, this option of not advertising and setting a high price becomes increasingly more attractive. This pricing strategy is, however, detrimental for consumer search since consumers who expect a high price will not search. So, this second argument exploits the fact that firms that advertise are committed to the price they offer before consumers search, whereas non-advertising firms are not committed. The two arguments together rule out other equilibria for the case where θ_H and c are relatively large.

3.6 Conclusion and extensions

The core of the argument developed in this Chapter centers around the uncertainty consumers face concerning the shops that carry the product they are looking for: some firms do have the product, others do not. This uncertainty is important in explaining consumer search behavior, but so far this type of uncertainty has not been considered in the large literature on consumer search. An important role of advertising in such a situation is to inform consumers that the advertising firm indeed sells the product. Advertising therefore can lower consumers' expected search costs. Since visiting an advertising firm comes with lower expected search costs than finding the product in a non-advertising firm, advertising firms have an advantage above non-advertising firms. In this Chapter we show that advertising firms can use this advantage to set higher prices. We have argued that the mechanism we uncover may be important in understanding recent developments in

[16] When $\beta = 0$ no firm is active and searching is not profitable.

emerging markets using multi-media technologies.

We analyzed the case where advertisements contain both information on the availability of the product and on the price the advertising firm asks. As we noted in the introduction, however, in many cases where our argument applies, advertisements may not contain price information. It therefore would be interesting to analyze a model where advertisements only contain information on product availability. The analysis of such a model is not straightforward because of complications arising in consumer search behavior. Without price advertisement, consumers not only have to decide on whether they will search, but also on where to search: an advertising firm or a non-advertising firm. Another complicating factor is that when prices are not advertised, advertising firms may exploit the fact that search costs are sunk at the time the price is revealed to consumers. Still, the argument used in this Chapter suggests that an equilibrium exists with high-valuation consumers visiting and buying at an advertising firm and not searching in case they did not receive an advertisement. Such an equilibrium with advertising firms setting high prices could hold as long as the probability of finding an active firm is low enough. The development of such a model is an interesting area for further research. Future research may also relax some of the restrictive assumptions we have employed, most notably the assumption that advertising costs are linear.

3.A Proofs

3.A.1 Proof of Proposition 3.4.1

The proof consists of two parts. First, we show that the proposed strategy is indeed in the strategy space of the players and second, we show that none of the players has an incentive to deviate from his proposed strategy.

To show that the proposed strategy is indeed in the strategy space of the players we need to show that $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$ and that $F_0(p)$ and $F_1(p)$ are proper cdf's. To show that $0 \leq \alpha \leq 1$, we note that

$$\alpha = \frac{1 - \frac{A+S}{(1-\gamma)(\theta_H - c)}}{\beta}$$

and so when $0 < \beta \leq 1$, $\alpha > 0$ whenever $1 - \frac{A+S}{(1-\gamma)(\theta_H-c)} > 0$, which is the second part of restriction 1. Restriction 2 ensures that $\alpha < 1$. The second part of restriction 1 and restriction 2 together also ensure that $\beta > 0$: restriction 2 gives $\beta > 1 - \frac{A+S}{(1-\gamma)(\theta_H-c)}$ and the second part of restriction 1 gives $1 - \frac{A+S}{(1-\gamma)(\theta_H-c)} > 0$. Restriction 3 finally ensures that $\beta < 1$.

To show that $F_0(p)$ and $F_1(p)$ are proper cdf's note that both $F_0(p)$ and $F_1(p)$ are increasing in p, $F_0(\underline{p}_0)$ and $F_1(\underline{p}_1)$ equal 0, and $F_0(\overline{p}_0)$ and $F_1(\overline{p}_1)$ are equal to 1. We also note that for $\underline{p}_1 > \overline{p}_0 = \theta_L$ to hold, $A + S$ should be larger than $(1-\gamma)\theta_L$. The first part of condition 1 ensures that this is indeed the case.

We now show that under the conditions specified, none of the players has an incentive to deviate. We first consider the search behavior of consumers. It is clear that since low-valuation consumers have no search costs they will search all firms and so know all active firms and their prices. For them it is optimal to buy at the cheapest of these firms, provided the price is not above the valuation for the product. High-valuation consumers who get two advertisements know that both firms are active and also know both prices. Again it is optimal to buy at the cheapest of the two firms.

Next, consider the case where a high-valuation consumer did not receive any advertisements. Suppose such a consumer has searched already once and found an inactive firm so that the consumer has to decide whether to search once more. He will not search for a second time when the utility from searching is smaller than the utility from not searching, which gives

$$-c + \frac{\beta(1-\alpha)}{1-\alpha\beta} \int_{\underline{p}_0}^{\theta_H} (\theta_H - p) f_0(p) dp < 0. \tag{A.1}$$

Integrating by parts and rearranging terms gives that searching a second time is not profitable when $\int_{\underline{p}_0}^{\theta_H} F_0(p)dp < \frac{1-\alpha\beta}{\beta(1-\alpha)}c$. This is the case if

$$\theta_H(1 - \frac{S}{\theta_L \gamma}) - \frac{S}{\gamma}\ln(\theta_L \frac{\gamma}{S}) < \frac{A+S}{(1-\gamma)(\theta_H - c)}c,$$

which is condition 4.

Now suppose the consumer has searched once and found an active firm asking price $p^* \leq \theta_H$. The utility from searching a second firm is given by

$$-c + \frac{\beta(1-\alpha)}{1-\alpha\beta}\left[\int_{\underline{p}_0}^{p^*}(\theta_H - p)f_0(p)dp + (1-F_0(p^*))(\theta_H - p^*)\right] +$$

$$(1 - \frac{\beta(1-\alpha)}{1-\alpha\beta})(\theta_H - p^*).$$

Integrating by parts and rearranging terms gives

$$-c + \theta_H - p^* + \frac{\beta(1-\alpha)}{1-\alpha\beta}\int_{\underline{p}_0}^{p^*} F_0(p)dp.$$

Since the utility from not searching is $\theta_H - p^*$ and $p^* \leq \theta_H$, it is easy to see that under condition 4 searching a second time is also, in this case, not profitable.

We conclude that if a consumer searches one time, he will (under condition 4) certainly not search a second time. However, this implies that the consideration of whether or not to search the first time is exactly identical to the consideration of searching a second time after having found an inactive firm. Therefore, under condition 4 it is indeed not optimal to search at all if no advertisement was received.

If a high-valuation consumer receives a single advertisement with price $p^* \leq \theta_H - c$, his utility from buying at the advertising firm is $\theta_H - p^* - c$ (remember that a consumer who visits an advertising firm incurs search costs c). The utility of searching the non-advertising firm is

$$-c + \frac{\beta(1-\alpha)}{1-\alpha\beta}\left[\int_{\underline{p}_0}^{p^*+c}(\theta_H - p)f_0(p)dp + (1-F_0(p^*+c))(\theta_H - p^* - c)\right] +$$

$$(1 - \frac{\beta(1-\alpha)}{1-\alpha\beta})(\theta_H - p^* - c).$$

Integrating in parts and comparing the two utilities shows that again under condition 4 searching is not profitable. This shows that consumers have no incentives to deviate from the strategy outlined in the proposition.

We next consider the behavior of firms. Let $\pi_0(p)$ denote the profits from not advertising and setting a price p and let $\pi_1(p)$ denote the profits from advertising a price p. Given the consumer behavior specified in the proposition we have for $p \leq \theta_L$,

$$\pi_0(p) = p\gamma(1 - \beta(1-\alpha)F_0(p)) - S.$$

Substituting α, β and $F_0(p)$ gives that $\pi_0(p) = 0$ for all $\underline{p}_0 \leq p \leq \theta_L$. For $p < \underline{p}_0$ we have that $\pi_0(p) = p\gamma - S < \underline{p}_0\gamma - S = 0$ and for $p > \theta_L$ the firm does not sell anything so that $\pi_0(p) = -S < 0$. This shows that for a non-advertising firm it is indeed optimal to choose a price between $\underline{p}_0$ and $\overline{p}_0$.

An advertising firm setting a price between θ_L and $\theta_H - c$ makes a profit of

$$\pi_1(p) = p(1-\gamma)(1-\beta\alpha F_1(p)) - S - A.$$

For prices above $\theta_H - c$, it does not generate any sales so that

$$\pi_1(p) = -S - A$$

and for prices below θ_L we have that

$$\pi_1(p) = p\gamma(1-\beta(1-\alpha)F_0(p)) + p(1-\gamma) - A - S.$$

When we substitute the values for α, β, $F_0(p)$ and $F_1(p)$ we see that for $\underline{p}_1 \leq p \leq \overline{p}_1$, $\pi_1(p) = 0$. Moreover, $\pi_1(p) < 0$ for $p > \overline{p}_1$. For $\theta_L < p \leq \underline{p}_1$, $\pi_1(p) = p(1-\gamma)-S-A \leq \underline{p}_1(1-\gamma)-S-A = 0$. For $\underline{p}_0 < p \leq \theta_L$, $\pi_1(p) = p(1-\gamma)-A \leq \theta_L(1-\gamma)-A < 0$ where in the last inequality we use the first part of restriction 1. And lastly, for $p \leq \underline{p}_0$, $\pi(p) = p - A - S \leq \underline{p}_0 - A - S = \frac{S}{\gamma} - A - S = S(\frac{1}{\gamma} - 1) - A < \theta_L(1-\gamma) - A < 0$, where the second inequality comes from restriction 2 and the last inequality from the first part of restriction 1. We conclude that for an advertising firm deviating from $F_1(p)$ is not profitable.

Since the profits from advertising and the profits from not advertising equal each other there are no gains from deviating from the proposed advertising probability, α, and since both profits equal 0 deviating from the proposed probability of being active, β, also does not lead to more profits. We conclude that firms have no incentives to deviate from the firm strategy outlined in the proposition.

3.A.2 Proof of Proposition 3.4.2

Assume that for A^*, S^*, θ_H^*, c^* and γ^* the equilibrium exists, and so all four restrictions hold. The first three restrictions do not depend on θ_H or c in isolation but instead only depend on $\theta_H - c$. Since this value does not change when x is added to both θ_H^* and c^*, these three restrictions still hold. The fourth restriction depends on θ_H and c in isolation. When x is added to both θ_H^* and c^*, this restriction changes to

$$\theta_H^*(1-\frac{S^*}{\gamma^*}) + x(1-\frac{S^*}{\gamma^*}) - \frac{S^*}{\gamma^*}\ln\frac{\gamma^*}{S^*} < \frac{A^*+S^*}{(1-\gamma^*)(\theta_H^*-c^*)}c^* + \frac{A^*+S^*}{(1-\gamma^*)(\theta_H^*-c^*)}x. \quad \text{(A.2)}$$

Since (restriction 4 of Proposition 3.4.1)

$$\theta_H^*(1-\frac{S^*}{\gamma^*}) - \frac{S^*}{\gamma^*}\ln\frac{\gamma^*}{S^*} < \frac{A^*+S^*}{(1-\gamma^*)(\theta_H^*-c^*)}c^*$$

and (due to restriction 3)

$$x(1-\frac{S^*}{\gamma^*}) < \frac{A^*+S^*}{(1-\gamma^*)(\theta_H^*-c^*)}x,$$

inequality (A.2) holds and so when x is added to θ_H^* and c^*, all restrictions are still satisfied and the equilibrium exists.

3.A.3 Derivation of the reservation price r.

In this appendix we derive the optimal consumer behavior. To ease this derivation we first note that advertised prices never exceed $\theta_H - c$. If they would, sales would be zero and profits would be negative. For the same reason non-advertised prices never exceed θ_H. This implies that for high-valuation consumers buying from an advertising firm always gives a higher utility than not buying at all. Equivalently, if a high-valuation consumer has found an active non-advertising firm, buying from this firm always gives a higher utility than not buying at all.

We define a reservation price r as $\int_{\underline{p}_0}^{r+c} F_0(p)dp = \frac{1-\alpha\beta}{(1-\alpha)\beta}c$. Using this reservation price optimal consumer behavior is as follows.

- Low-valuation consumers buy from the cheapest active firm provided the price is not above θ_L.

  Proof
  Since these consumers have zero search costs they search all non-advertising firms and consequently know all prices. The behavior stated above is therefore optimal. □

- High-valuation consumers who receive two advertisements buy from the cheapest of these firms.

  Proof
  Since there are two firms in the market these consumers know all prices and therefore the stated behavior is optimal. □

- High-valuation consumers who receive one advertisement with price p_1 buy immediately if $p_1 \leq r$. In case $p_1 > r$ it is optimal to search the second firm. If the second firm is non-active consumers will buy from the advertising firm. If the second firm is active consumers will buy from it if the price is at or below $p_1 + c$; else they will buy from the advertising firm.

  Proof
  The second part, on what to do after having searched the second firm is obvious once taking into account that buying from the advertising firm bears costs c while after searching buying from the non-advertising firm is free. To show the first part note that the utility of buying immediately is $\theta_H - c - p_1$ while the utility of searching is

  $$-c + \frac{(1-\alpha)\beta}{1-\alpha\beta}\left(\int_{\underline{p}_0}^{p_1+c} (\theta_H - p)f_0(p)dp + (1 - F_0(p_1+c))(\theta_H - p_1 - c)\right)$$
  $$+(1 - \frac{\beta(1-\alpha)}{1-\beta\alpha})(\theta_H - p_1 - c).$$

  Integrating in parts and rewriting gives

  $$-c + \frac{(1-\alpha)\beta}{1-\alpha\beta}\int_{\underline{p}_0}^{p_1+c} F_0(p)dp + \theta_H - p_1 - c$$

and so search is profitable if and only if

$$\int_{\underline{p}_0}^{p_1+c} F_0(p)dp > \frac{1-\alpha\beta}{(1-\alpha)\beta}c,$$

or $p_1 > r$. □

- When $r+c > \theta_H$ high-valuation consumers who have not received any advertisements do not search. When $r+c < \theta_H$ these consumers will search. If they find an active firm asking a price at or below $r+c$ they stop searching and buy at that firm. If the price found is above $r+c$ or if the firm is inactive consumers will search the second firm as well and buy at the cheapest active firm. When $r+c = \theta_H$ consumers are indifferent between searching and not searching. We propose that a fraction μ of the consumers searches. If the firm is active consumers buy immediately and if it is not active consumers again are indifferent between searching and not searching. We propose that a fraction μ of the searching consumers also searches the second firm.

Proof
The behavior for $r+c > \theta_H$ has already been shown in the proof of Proposition 4.1.

Now look at $r+c < \theta_H$. Suppose the first search gave an active firm asking a price p_0. Buying immediately gives a utility $\theta_H - p_0$ while searching further gives expected utility

$$-c + \frac{(1-\alpha)\beta}{1-\alpha\beta}\left(\int_{\underline{p}_0}^{p_0}(\theta_H - p)f_0(p)dp + (1-F_0(p_0))(\theta_H - p_0)\right)$$

$$+\left(1 - \frac{\beta(1-\alpha)}{1-\beta\alpha}\right)(\theta_H - p_0).$$

Rewriting gives that buying immediately is profitable if

$$\int_{\underline{p}_0}^{p_0} F_0(p)dp \leq \frac{1-\alpha\beta}{\beta(1-\alpha)}c,$$

or $p_0 \leq r+c$. Note that searching the second firm is profitable when $p_0 > r+c$.

If the first search led to an inactive firm the utility of a second search is

$$-c + \frac{(1-\alpha)\beta}{1-\alpha\beta}\int_{\underline{p}_0}^{\theta_H}(\theta_H - p)f_0(p)dp. \tag{A.3}$$

Rewriting gives that this utility is strictly positive for $r+c < \theta_H$ and so a second search is profitable.

This explains the second part of the behavior when $r + c < \theta_H$. For the first part of the behavior we note that in some cases, for instance when the first search leads to an inactive firm, it is profitable to search a second time. This implies that the utility of the first search is higher than the utility of a first search when consumers would not search a second time at all. This utility if consumers would not search a second time is given by (A.3), which is strictly positive for $r+c < \theta_H$. Therefore also the first search is profitable.

Finally, we look at $r+c = \theta_H$. Note that since prices are never above θ_H prices also never are above $r + c$. As in the case $r + c < \theta_H$ it is best to stop search if the price found is at or below $r + c$, implying that if an active firm is found it is optimal to buy there immediately. If no active firm is found the utility of a second search is given by (A.3), which equals zero for $r+c = \theta_H$. Since consumers buy immediately after finding an active firm and are indifferent between searching and not searching after finding an inactive firm the utility of the first search again is given by (A.3), equalling zero. □

3.A.4 Proof of Proposition 3.5.1.

To prove Proposition 3.5.1 we need to check all possible equilibria of the model and show that neither of these overlaps with the equilibrium of Proposition 3.4.1 for x large enough. To this end, we classify the possible equilibria by the probabilities α and β. In the equilibrium of Proposition 3.4.1 these probabilities are strictly between 0 and 1. To save space in this appendix we only look at equilibria for which it is easy to show that they do not overlap with the equilibrium of Proposition 3.4.1 at all. The proof for the other equilibria can be found in the internet appendix.

(i) An equilibrium where $\beta = 0$ implies that profits of being active should be below zero. If a firm deviates and chooses not to advertise and sets a price $\theta_L = 1$, its profits are $\gamma - S$. Hence, an equilibrium where $\beta = 0$ only holds for $\gamma < S$, while condition (2) of Proposition 3.4.1 stipulates that $\gamma > S$.

(ii) In an equilibrium with $\alpha = 1$ and $0 < \beta < 1$ we should have that $\pi_1 = 0$. First suppose $\underline{p}_1 > \theta_L$. If a firm deviates to not advertising a price $\theta_L = 1$, profits are at least equal to $\gamma - S$. As deviating should not be profitable, this equilibrium only holds for $\gamma < S$. Now suppose $\underline{p}_1 \leq \theta_L$. It is easy to show that in this case θ_L is in the support of $F_1(p)$ and so $\pi_1(\theta_L) = 1-\beta+\beta(1-F_1(\theta_L))-A-S = 0$, or $1-\beta+\beta(1-F_1(\theta_L)) = A+S$. Deviating to not advertising a price θ_L gives a profit that is at least as large as $\pi_0(\theta_L) = \gamma(1-\beta+\beta(1-F_1(\theta_L)))-S = \gamma(A + S) - S$. As deviating should not be profitable it follows that $\gamma(A + S) - S$ has to be smaller than 0. This restriction leads to a parameter region that does not overlap with a region defined by $S < \gamma$ and $A > 1 - \gamma$.

(iii) $\alpha = 1$ and $\beta = 1$. In this case an equilibrium does not exist. When every firm is active and advertises, we get Bertrand competition and equilibrium prices equalling 0. This leads to negative profits $-A - S$.

Note that the equilibria that are left to analyze are equilibria with $\alpha < 1$ and $\beta > 0$. For these equilibria we cannot universally show that they do not overlap with our equilibrium and in fact some of these equilibria partially overlap. As we show in the internet appendix these equilibria, however, only overlap for relatively small values of θ_H and c.

Chapter 4

Joining forces to attract consumers: clusters of shops in a consumer search model

4.1 Introduction

Casual observation suggests that many shops are clustered together in main streets and shopping centers. Since these clusters of shops allow for one stop shopping they are popular with consumers, and this popularity of malls in turn makes it attractive for shops to locate in a mall. There is however a drawback of locating together: clustering makes it easy for consumers to shop around to find the best product match. This is not so much a problem when the shops offer heterogeneous goods, but it is when the products are (almost) homogeneous. In that case shopping around by consumers will increase competition and lower prices. Even so, quite some shops in shopping malls sell (almost) homogeneous goods, such as bakeries, butcher's, drugstores, opticians, dry-cleaners and banks.

This paper investigates why shops selling homogeneous[1] goods would locate together and in a sense would choose to compete with each other. To capture the main idea that malls make it easy to shop around, consumers are modeled as in a sequential search model with costly initial search (see Stahl (1989), Janssen et al. (2005)), except that the search costs inside the mall are lower than the search costs outside the mall. In the model a fraction γ of consumers, called shoppers, incur no search or travel costs and therefore know all prices. The other consumers, called non-shoppers, incur costs when entering a shop, independent of whether a shop is located in a mall or not, and non-shoppers incur travel costs when traveling between malls or isolated[2] shops. When a non-shopper

[1] The model used in this paper assumes that the goods sold by shops are perfectly homogeneous and the shops compete in prices. Although examples of really homogeneous goods are hard to find, there are many goods that are almost homogeneous. Modeling the shops as selling perfectly homogeneous goods will clearly distinguish the different effects of locating together and if locating together is profitable under perfectly homogeneous goods it will certainly be profitable under almost homogeneous goods as well.

[2] The term isolated does not necessarily imply that the shop is not located in some shopping center, but is used to signify that the shop does not have direct competitors selling the same good in the same

however stays in the mall where he currently is, no travel costs are incurred.

This model will offer some insight in the effects that play a role in location choice and will show under which conditions homogeneous shops choose to locate together. The main idea is that the increased competition caused by locating together will indeed lead to lower prices, but these lower prices will attract more consumers to the mall. This happens in two distinct ways. First, shops in a mall attract more non-shoppers than isolated shops and, second, lower prices can make more non-shoppers willing to leave their house and start searching.

The second effect, more non-shoppers willing to search, only holds for relatively high values of the search costs. When the search costs are low all non-shoppers are active, independent of whether there is a mall or not. When the search costs are high enough, not all non-shoppers are active and the presence of a mall makes more non-shoppers willing to leave their house. The analysis in this paper shows that this effect is very strong. When either all shops are isolated or all shops are located in the same mall consumers necessarily divide evenly over all shops and so the only effect of the mall is that more non-shoppers are active. For high enough search costs this effect is strong enough to make the profits when all shops are in the same mall higher than the profits when all shops are isolated.

To analyze the uneven distribution of non-shoppers over mall and isolated shops and to analyze the location choice of shops an intermediate case needs to be studied where some shops are in a mall and some other shops are isolated. Although several results, like the uneven distribution of consumers over shops, can be obtained analytically, the intermediate case is too complex to completely analyze in an analytic way. Simulations are therefore used to obtain an insight in the magnitude of the uneven distribution of consumers and in the outcome of the location choice game.

The location choice game is analyzed under the assumption that only one mall can be formed. One can think of a town that has one large out-of-town shopping center with ample space for new shops and several much smaller in-town mini malls that have no space to expand and accommodate new shops. Starting from some initial situation with $k \geq 1$ shops in the mall and $n-k$ isolated shops, a randomly chosen shop gets the opportunity to relocate. After a possible relocation profits are realized and another randomly chosen shop gets the opportunity to relocate. An equilibrium is reached when none of the shops has an incentive to relocate anymore and the final outcome of this game depends on the initial situation and on the model parameters. The simulations show that there exist parameter values for which a situation with only isolated shops is an equilibrium. There also are parameter values for which in equilibrium a mall with at least two shops exists. The simulations suggest that an equilibrium with only isolated shops can only be obtained for relatively low search costs and for an initial situation with either no mall or a mall with two shops. Other initial situations or relatively high search costs in the simulations always lead to an equilibrium with a mall.

This can be explained from the two effects on sales mentioned before. Recall that the increase in active non-shoppers only holds for relatively high values of the search

shopping center.

costs. This paper also proves that, for a given mall size, mall shops always attract more non-shoppers than isolated shops and that this uneven distribution of non-shoppers over shops holds for all values of the search costs. For low search costs the only effect on sales therefore is the uneven distribution of consumers over shops and, as the simulations show, this increase in sales is not always large enough to make a mall an equilibrium. For high search costs both effects on sales hold and the simulations suggest that the combined effect is strong enough to always make a mall an equilibrium outcome of the location choice game.

The role of the initial situation can be explained from the uneven distribution of consumers over mall and isolated shops. When the initial situation has only isolated shops the consumers necessarily divide evenly over the n shops. The increase in sales from forming a mall therefore is an increase from a fraction $\frac{1}{n}$ of non-shoppers to some fraction above $\frac{1}{n}$. When there already exists a mall, an isolated shop who decides to join the mall increases its sales from a fraction *below* $\frac{1}{n}$ to a fraction above $\frac{1}{n}$. This is a larger increase in sales than when a mall is formed starting from an initial situation of only isolated shops and apparently this increase in sales is large enough to make joining the mall a profitable strategy.

In the general case of a mall with $2 \leq k \leq n-1$ shops finding expressions for all the possible equilibrium price distributions is very difficult. There are however some interesting properties that can be analytically derived and that hold for all equilibria. Consumers behave as in a standard sequential search model with costly initial search (see Janssen et al. (2005)), except that in this paper travel costs are added. These travel costs are not incurred when a non-shopper stays in the mall and it is therefore not surprising that the price distribution of mall shops resembles the price distribution obtained in Janssen et al. (2005). Mall shops compete for the shoppers and have some market power over the non-shoppers. They balance these two effects by randomizing over prices. The price distribution has a maximum price equal to the average mall price plus the entering costs, such that a non-shopper who finds the maximum price has no incentive to continue searching in the mall. Interestingly, the price distribution of isolated shops has a different form. Consider a non-shopper who visited an isolated shop and who has to decide on whether to continue searching in the mall. To do this he has to travel to the mall, which is costly. Moreover, for every visit to a shop that has not previously been visited entering costs are incurred. The total costs of searching thus are the travel costs plus the entering costs and searching is only profitable when the average mall price plus the entering costs plus the travel costs is below the price found in the isolated shop. Since the maximum mall price equals the average mall price plus the entering costs a non-shopper who currently is in an isolated shop will only move on to the mall when the price found in the isolated shop is above the maximum mall price plus the traveling costs. Therefore, the maximum price p that an isolated shop can ask without making non-shoppers willing to search is the maximum mall price plus the travel costs. Note that p is relatively high and therefore attractive to ask. If an isolated shop would however only ask price p a rational consumer expecting this price would never search an isolated shop in the first place. Moreover, asking a low price can be profitable as well since it attracts the shoppers. It turns out that isolated shops randomize over setting price p or setting a price far below the maximum mall price.

This can be interpreted as isolated shops usually being more expensive than mall shops, but sometimes offering a large discount. An isolated shop will attract non-shoppers who hope to be lucky enough to find a discount. But even if the non-shopper does not find a discount he will stay at the isolated shop since to search further the non-shopper has to incur travel and entering costs.

One of the first papers considering the location of shops is the well-known paper by Hotelling (1929). The Hotelling model differs in important aspects from the model in this paper. For instance, the model by Hotelling assumes a spatial structure where consumers and shops are located along a line, consumers incur transportation costs that are linear in the distance between consumer and shop and consumers incur no entering costs. These assumptions ensure that along a part of the Hotelling line the price and sales effect work in the same direction: locating closer to a competitor leads to higher prices *and* more sales. It is therefore no surprise that for this part of the line shops have an incentive to locate closer to each other.

Of the more recent research combining location choice and consumer search Dudey (1990, 1993) are the only papers on homogeneous products. The work by Dudey differs in two important ways from the research in this paper. First, consumers in Dudey's model are assumed to search only one shop. Although in the model in the current paper in equilibrium consumers search only once as well, in principle consumers can search several times and this threat of continued search increases competition between shops. Second, in Dudey's model shops do not compete in price, but in quantity. Dudey finds that a situation where all shops locate together is an equilibrium. The main idea behind this equilibrium is that if a shop would deviate it would set the monopoly quantity and price and consequently no consumer would visit the deviating shop. This argument resembles the uneven distribution of consumers over mall shops and isolated shops as found in this paper. Dudey however shows that in his model an uneven clustering of shops cannot be an equilibrium since the largest mall (or largest malls if there are two or more malls with the same size) will attract all the consumers. In particular, this implies that in Dudey's model isolated shops cannot exist. In my model the existence of isolated shops is possible since, as mentioned above, isolated shops randomize over prices and attract some consumers hoping to find a low price.

There are several papers combining consumer search and location choice in a heterogeneous products setting, see e.g. Stahl (1982a, 1982b), Wolinsky (1983), Fischer and Harrington (1996), Gehrig (1998) and Konishi (2005). In these models searching in a mall is attractive, not only because mall prices are lower but also because in a mall the probability of finding a product that closely matches the taste of the consumer is higher. In general, the product match and pricing effect together increase the sales so much that it is profitable to locate together. The current paper contributes by showing that even without the product match effect locating together can be profitable. There are more important differences between this paper and the papers mentioned above. For instance, Wolinsky (1983) only investigates whether a situation with all shops in the same mall can be an equilibrium. In contrast, Stahl (1982a) only investigates under which conditions a situation with only isolated shops cannot be an equilibrium and Stahl (1982b) limits the analysis to only two shops. Both papers by Stahl also assume that consumers only search one location (either a mall or an isolated shop). In Fischer and Harrington (1996) and Gehrig (1998) the search costs are assumed to be so low that in equilibrium all consumers are always active. This implies that the formation of a mall will never lead to more active consumers and the only effect on sales is that larger malls attract more consumers. Finally, Konishi (2005) mentions both effects on sales, but analyzes this under the assumption that consumers can visit only one location. Moreover, Konishi analyzes the location choice game only for some very specific cases, where e.g. there are only two possible locations.

The next section presents the model that will be used. Section 4.3 analyzes the two opposite cases where in one case all shops are isolated and in the other case all shops are in the same mall. Section 4.4 concentrates on the intermediate case of $2 \leq k \leq n-1$ mall shops. In Section 4.4.1 some general features of the equilibria are derived and Section 4.4.2 derives one specific equilibrium. Section 4.5 analyzes the location choice of shops and Section 4.6 concludes. The proofs are in the appendix to this chapter.

4.2 The model

The model has $n > 2$ shops in the market that sell a homogeneous good. Production costs are linear and without loss of generality they are taken to be zero. The shops compete in prices and the strategy of a firm i can therefore be denoted by a price distribution $F_i(p)$, where $F_i(p)$ is the cdf, the maximum price is denoted by $\overline{p}_i$ and the minimum price by $\underline{p}_i$. Note that if shop i chooses a pure price strategy with price p_i the price distribution is given by $F_i(p) = 0$ for $p < p_i$ and $F_i(p) = 1$ for $p \geq p_i$. In the next sections it will however become clear that there is no symmetric pure strategy equilibrium.

The shops play a location choice game. In this game there is one possible mall with k shops and the remaining $n-k$ shops are all isolated. At the start of game one of the shops has the option to either join the mall at no relocation costs or to leave the mall to be an isolated shop at no relocation costs. Fixed relocation costs can be introduced easily without changing much of the analysis. After the shop has decided on whether or not to relocate the pricing game is played and profits are realized. After that again a randomly chosen shop has the option to relocate, profits are realized, again a randomly chosen shop can relocate, profits are realized, and so on. Shops do not take future actions of competitors into account when deciding on their own strategy. This is quite realistic since in reality possibilities to relocate occur only occasionally. A model where shops discount future expected profits and the time span between realizing profits and relocation is long enough would approximately have the same outcome as the model where shops are not forward-looking at all. In equilibrium, none of the shops has an incentive to relocate.

The model has a unit mass of consumers, all having unit demand and a valuation θ for the product. The consumers are aware of all the locations of the shops, but they do not know the prices in the shops. They however form rational price expectations and base their decisions on these expectations.

There are two different types of consumers. A fraction γ of consumers consists of shoppers who have zero search costs. As a consequence shoppers know all the prices and buy at the cheapest shop.[3] A fraction $1-\gamma$ of consumers consists of consumers that incur strictly positive search costs. These consumers are referred to as non-shoppers. Non-shoppers incur costs c_e when entering a shop. These costs are incurred whenever a not previously visited shop is entered and do not depend on whether a shop is in a mall with several shops or is an isolated shop. The entering costs are equivalent to the continuation costs in a standard consumer search model. These costs reflect the time spent in the shop, finding the product on the shelf, finding the price of the product, waiting for a shop assistant to help you, etc. Note that positive entering costs are essential in the model. Without entering costs non-shoppers could without additional costs search all the shops in the mall. This would drive the mall prices to zero, and no shop would ever locate in a mall. In addition to the entering costs, non-shoppers incur travel costs c_t whenever they travel from their house to a cluster of shops or travel between clusters of shops, where the cluster can be either a shopping mall with several shops selling the product or an isolated shop.[4] The travel costs can be interpreted as the costs of, say, a bus ticket or petrol costs. The travel costs ensure that searching h shops in the same mall comes at less costs than searching h shops spread over different clusters. Finally, the analysis is restricted to values of c_e and c_t for which $c_t + c_e \leq \theta$.

Non-shoppers search sequentially. This means that non-shoppers first decide on whether to stay at home, visit a mall shop or visit an isolated shop. Let μ denote the fraction of non-shoppers who decide to visit a shop and let $1-\mu$ denote the fraction of non-shoppers who decide to stay at home. The fraction μ of non-shoppers will be referred to as active non-shoppers. Based on the price found in the first shop, an active non-shopper decides on whether to search a second shop and whether this second search will be in the same cluster as the first search (if possible) or in an other cluster. Then, based on the outcome of the second search, active non-shoppers decide on whether or not to search a third time and where the third search will be, etc.

In the analysis below I will focus on symmetric equilibria in the sense that all shops in the same cluster choose identical price distributions and clusters of the same size have identical price distributions as well. Note that identical price distributions does not necessarily imply identical prices since realized prices could differ from each other.

[3]One could think of shoppers as consumers who obtain a strictly positive utility from the shopping experience, even if travel expenses are taken into account. For the results it is not strictly necessary that there are consumers with zero search costs who know all prices. The less restrictive assumption that some fraction γ of consumers gets to know the prices of two or more random shops without incurring search costs would be sufficient to obtain the results in this paper. For simplicity I however assume the presence of a fraction of consumers with zero search costs.

[4]The travel costs are incurred every time a non-shopper travels between clusters of shops, and therefore are also incurred when returning to a previously visited shop that is in a different cluster than the cluster where the non-shopper currently is. These return costs are necessary to prevent arbitrage. Imagine a situation of one mall with shops 1 and 2 and two isolated shops, 3 and 4. Now suppose a non-shopper's first search was in shop 1 and the second search was in shop 3. If there are no return costs and the non-shopper would like to visit shop 2 in his third search he could go there immediately at cost $c_t + c_e$, but he could also return to shop 1 at no costs and then visit shop 2 at cost c_e. To prevent such a situation return costs of at least c_t are necessary when returning to a shop in a different cluster.

Since price distributions are symmetric where possible and since I only consider situations where there is at most one shopping mall I drop the shop index i in $F_i(p)$ and in $\underline{p}_i$ and $\overline{p_i}$ but instead use an index k denoting the number of firms in the shopping mall. Where necessary I add an index m for mall or i for an isolated shop. So $F_1(p)$ is the price distribution used by all shops when they are all isolated shops and $F_n(p)$ is the price distribution when all shops are in the same shopping mall. $F_k^i(p)$ and $F_k^m(p)$ are the price distributions of an isolated shop while there also is a cluster of k shops and of a shop located in a mall with k shops, respectively. The index k will also be added to μ, such that μ_k denotes the fraction of active non-shoppers when there is a mall with k shops.

Because shops choose symmetric pricing strategies, non-shoppers a priori have no preferences over shops that are located in the same mall. Moreover, non-shoppers a priori have no preferences over the isolated shops. Once a non-shopper has chosen to visit the mall he will therefore choose a random shop from this mall. In the same vein, once a non-shopper has decided to visit an isolated shop he will choose such a shop at random.

4.3 Two opposite cases: only isolated shops and only mall shops

4.3.1 Only isolated shops

In the model where all shops are isolated each visit to a shop comes at cost $c_t + c_e$, and each return visit to a previously visited shop comes at cost c_t. This model is equivalent to the model in Janssen, Moraga-Gonzalez and Wildenbeest (2005) (henceforth JMW) except for the return costs, which are absent in the JMW model. It is relatively easy to show that the equilibrium derived in JMW also holds in a model with return costs and in this section I will focus on this equilibrium.[5]

The equilibrium derived in JMW splits the parameter region in two non-overlapping parts. When the search costs c_t+c_e are small enough, all non-shoppers are willing to search at least one shop ($\mu_1 = 1$). This is called a full search equilibrium. When the search costs $c_t + c_e$ get too high non-shoppers are indifferent between searching and staying home and only a fraction $0 < \mu_1 < 1$ of the non-shoppers is active. This is called a partial search equilibrium. Recall that in the model $c_t + c_e \leq \theta$. This ensures that in equilibrium $\mu_1 > 0$. If no non-shopper would be active ($\mu_1 = 0$) shops would only sell to the shoppers and prices would be zero. But with prices equal to zero the utility of searching once is $\theta - c_e - c_t$, which is positive, a contradiction.

The equilibrium specification of the full search equilibrium in JMW uses a reservation price r_1. This reservation price is implicitly defined by

$$\int_{\underline{p}_1}^{r_1} (r_1 - p) dF_1(p) = c_t + c_e.$$

[5]Return costs complicate a full analysis considerably and could potentially lead to multiple equilibria. See Janssen and Parakhonyak (2008) for an analysis of consumer behavior under the assumption of return costs. Janssen and Parakhonyak currently work on shop behavior under return costs.

When $r_1 < \theta$ in JMW the full search equilibrium holds and the maximum price in the price distribution is r_1. When $r_1 = \theta$ a partial search equilibrium holds with a maximum price of θ. Propositions 4.3.1 and 4.3.2 will show that also when return costs are added to the model a full search equilibrium with $\overline{p}_1 = r_1$ holds for $r_1 < \theta$ and a partial search equilibrium with $\overline{p}_1 = \theta$ holds for $r_1 = \theta$. The proofs also show that these equilibria are the only ones with $\overline{p}_1 \leq r_1$.

The role of the reservation price is fairly easy to explain if one considers a situation where a non-shopper has only one opportunity left to search. Denote by p^* the price found in the current shop and denote by p^{min} the minimum price found in previous searches (if the current shop is the first one searched let p^{min} be infinitely large). If a non-shopper decides to stop searching he will obtain utility $\theta - \min(p^*, p^{min} + c_t)$. (Note that when all shops are isolated, if the non-shopper decides to return to a previously visited shop he incurs return costs c_t and therefore the total expenses in that case would be $p^{min} + c_t$.) Now suppose that $r_1 \leq \theta$ and $\min(p^*, p^{min} + c_t) = r_1$. If a non-shopper decides to perform one more search the expected utility gain of this search is given by the lhs of (4.3.1) and the costs of this search are given by the rhs. Thus, for $r_1 \leq \theta$, r_1 is defined in such a way that if $\min(p^*, p^{min} + c_t) = r_1$ a non-shopper is indifferent between continuing search and stopping search. If $\min(p^*, p^{min} + c_t) < r_1$ a non-shopper will stop searching and if $\min(p^*, p^{min} + c_t) > r_1$ the non-shopper will continue his search. The proofs of Propositions 4.3.1 and 4.3.2 will also consider the general case where a non-shoppers has $h > 1$ opportunities left to search. The proofs show that also in the general case the reservation price has exactly the same interpretation.

In addition, the reservation price in combination with the valuation of the product θ also affects the decision whether to search or stay at home. The definition of the reservation price given by (4.3.1) gives $r_1 - Ep = c_t + c_e$, with Ep the expected price. Searching once gives (expected) utility $\theta - Ep - c_t - c_e$ and plugging in Ep gives expected utility $\theta - r_1$. This already shows that if $r_1 < \theta$ all non-shoppers want to search. The proof of Proposition 4.3.2 also considers the general case where a non-shopper can search more than once and shows that if $r_1 = \theta$ non-shoppers are indifferent between searching and staying at home. For $r_1 < \theta$ the following is an equilibrium.

Proposition 4.3.1 *(Full search equilibrium)*

If $c_t + c_e < \theta(1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma}ny^{n-1}}dy)$ then all non-shoppers are active and non-shoppers will stop searching as soon as $\min(p^*, p^{min} + c_t) \leq r_1$, with r_1 defined as

$$r_1 = \frac{c_t + c_e}{1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma}ny^{n-1}}dy}.$$

Shops randomize over prices according to the price distribution

$$F_1(p) = 1 - (\frac{1-\gamma}{\gamma n}\frac{r_1 - p}{p})^{\frac{1}{n-1}}.$$

The maximum price asked by a shop is r_1, the minimum price equals $\frac{1-\gamma}{1+\gamma(n-1)}r_1$ and expected profits are given by $\pi_1 = r_1\frac{1-\gamma}{n}$.

In a partial search equilibrium it must be that $r_1 = \theta$ to make non-shoppers indifferent between staying home and searching. Only a fraction μ_1 of the non-shoppers is active and this fraction is determined by the condition $r_1 = \theta$, or $Ep = \theta - c_t - c_e$. Note here that μ_1 determines Ep since an increase in the fraction of active non-shoppers increases the prices.

Proposition 4.3.2 *(Partial search equilibrium)*
If $c_t + c_e > \theta(1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma} n y^{n-1}} dy)$ a fraction $0 < \mu_1 < 1$ of the non-shoppers is active, while the remaining fraction $1 - \mu_1$ of non-shoppers stays at home and does not buy at all. The fraction μ_1 is implicitly defined by

$$h(\mu_1) \equiv \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1-\gamma)\mu_1} y^{n-1}} dy = \frac{\theta - c_t - c_e}{\theta}.$$

Active non-shoppers stop searching as soon as $\min(p^*, p^{min} + c_t) \leq \theta$. Shops randomize over prices according to the price distribution

$$F_1(p) = 1 - (\frac{(1-\gamma)\mu_1}{\gamma n} \frac{\theta - p}{p})^{\frac{1}{n-1}}.$$

The maximum price asked by a shop is θ, the minimum price equals $\frac{(1-\gamma)\mu_1}{\gamma n + (1-\gamma)\mu_1}\theta$ and expected profits are given by $\pi_1 = \theta \mu_1 \frac{1-\gamma}{n}$.

In equilibrium, the pricing strategy is a mixed strategy. Because of the presence of a fraction of shoppers who know all prices a pure pricing strategy or even an atom in the price distribution is not possible. If shops would use a pure pricing strategy or a price distribution with an atom, it would always be profitable to undercut the pure price or atom slightly and attract all shoppers. Undercutting is not possible when the pure pricing strategy or the atom is at price 0, but in that case asking a price $c_t + c_e$ keeps the non-shoppers at the shop and leads to higher profits than a price of 0.

The mixed strategy balances two different effects: the shops have monopoly power on their share of non-shoppers but compete for the shoppers. Asking a high price would extract a high profit from the non-shoppers, but the probability of selling to the shoppers is low. For a low price this is the other way around: the profits from the non-shoppers are low, but there is a large probability of attracting the shoppers. Shops balance these effects by randomizing over prices.

In the equilibria stated in the Propositions non-shoppers will search at most once. In the first shop they visit they will find a price at or below r_1 (θ in case of partial consumer search) and will stop searching. In a model without return costs it is easy to see that in equilibrium no shop would ever want to price so high that non-shoppers search on. If the maximum price in an atomless price distribution would be so high that non-shoppers search on, a shop asking this maximum price would make no profits, while asking a lower price would lead to strictly positive profits. This simple idea does not hold anymore in a model with return costs since a shop that asks the maximum price and is the last (nth)

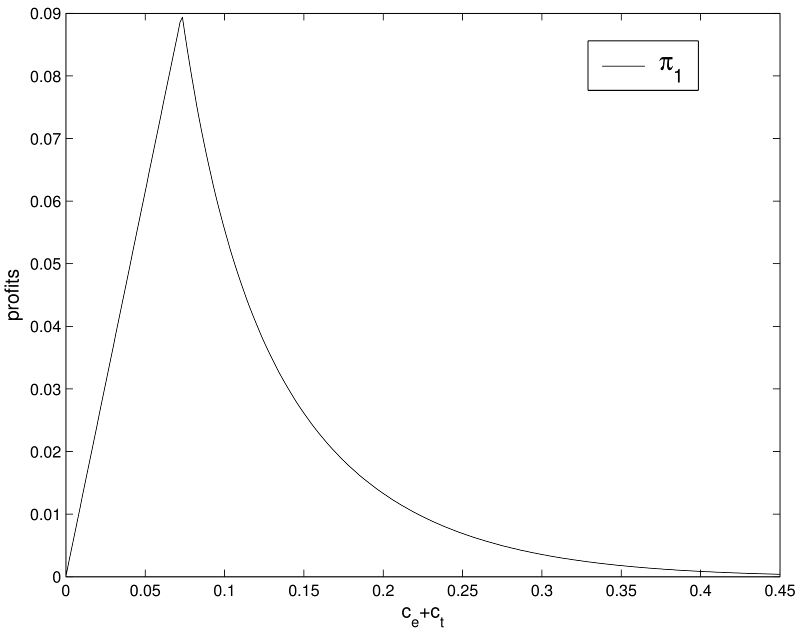


Figure 4.1: Expected profits as a function of the search costs when all shops are isolated.

shop to be searched will sell to non-shoppers as long as its competitors ask a price above $\overline{p} - c_t$. Still, in a model with return costs, as long as all competitors make sure that non-shoppers do not search a shop will never want to let non-shoppers search since this leads to zero profits.

Figure 4.1 shows the expected profits as a function of the search costs $c_t + c_e$. In this figure, the number of shops n equals 10, $\gamma = 0.1$ and $\theta = 1$. With these parameter values the full search equilibrium holds for $c_t + c_e < 0.073$ and the partial search equilibrium holds for $0.073 < c_t + c_e < 1$. At first sight the full search equilibrium seems to hold only for very small search costs but note that a search cost value of 0.073 implies that the search costs are still 7.3% of the valuation of the product. The expected profits are plotted for $c_t + c_e < 0.45$; for higher values of $c_t + c_e$ the profits are decreasing and when $c_t + c_e$ approaches 1 the profits approach 0. In the full search equilibrium the maximum price r_1 increases linearly with $c_t + c_e$ and the profits are linearly increasing in $c_t + c_e$ as well. As soon as the search costs $c_t + c_e$ are above 0.073 profits however decrease in search costs. In this case the partial search equilibrium obtains, the maximum price equals θ and the profits depend on the fraction of consumers who search, μ_1. This fraction decreases in $c_t + c_e$, leading to decreasing profits.

4.3.2 Only mall shops

When all the shops are in the same mall non-shoppers incur costs $c_e + c_t$ for the first search, they incur costs c_e for every next search and have no return costs. In such a setup it is possible to derive a unique equilibrium, again with a parameter region where all non-shoppers search and a parameter region where only a fraction of non-shoppers searches.

The analysis of this case follows the same lines as the analysis of the model where all shops are isolated. More specifically, define a reservation price r_n by

$$\int_{\underline{p}_n}^{r_n} (r_n - p) dF_n(p) = c_e.$$

As in section 4.3.1, in a full search equilibrium the reservation price combined with the prices found thus far determines whether a non-shopper wants to continue search or wants to stop searching. Note that here the definition of the reservation price uses only c_e instead of $c_e + c_t$. This is because in the model where all shops are in the same mall continuing search comes at cost c_e, while in section 4.3.1 continuing search comes at cost $c_e + c_t$.

The consideration whether to start searching or stay at home (which determines whether a full search equilibrium can exist) is however different from section 4.3.1. When all the shops are isolated the first search is as costly as all the other searches. But when all shops are in the same mall the first search comes at cost $c_t + c_e$ while continuing search comes at cost c_e. The reservation price is defined using only this continuation costs, such that $r_n - Ep = c_e$. A consumer will only travel to the mall when the benefits, θ, are higher than the costs, $Ep + c_t + c_e$. This can be rewritten as $r_n < \theta - c_t$ and so when $r_n < \theta - c_t$ a full search equilibrium holds. In a partial search equilibrium $r_n = \theta - c_t$. This also has implications for the maximum prices that will be asked in equilibrium. In a full search equilibrium the maximum price is, as before, the reservation price. This ensures that no non-shoppers are searching more than once and, since in a model where all the shops are in a mall there are no return costs, this also is the unique equilibrium. In a partial search equilibrium the maximum price however is $\theta - c_t$. In a partial search equilibrium the prices will be chosen in such a way that $\theta - Ep - c_t - c_e = 0$, since this makes non-shoppers indifferent between searching and not searching. But because the continuation costs are only c_e the utility of continuing search is at least $\theta - Ep - c_e$, which equals c_t. Buying at a price above $\theta - c_t$ gives a utility below c_t and for prices above $\theta - c_t$ continuing search is therefore profitable. If the maximum price would be above $\theta - c_t$ a shop setting this price would not sell anything, which cannot be an equilibrium.

The full search equilibrium has the following form.

Proposition 4.3.3 *(Full search equilibrium)*

If $c_e < (1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma} n y^{n-1}} dy)(\theta - c_t)$ all non-shoppers are active and non-shoppers will stop searching as soon as they find a price at or below r_n, with r_n defined as

$$r_n = \frac{c_e}{1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma} n y^{n-1}} dy}.$$

Shops randomize over prices according to the price distribution

$$F_n(p) = 1 - \left(\frac{(r_n - p)(1-\gamma)}{n\gamma p}\right)^{\frac{1}{n-1}}.$$

The maximum price asked by a shop is r_n, the minimum price equals $\frac{1-\gamma}{(1-\gamma)+\gamma n} r_n$ and expected profits are given by $\pi_n = r_n \frac{1-\gamma}{n}$.

The partial search equilibrium is as follows.

Proposition 4.3.4 *(Partial search equilibrium)*
If $c_e > (1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma}ny^{n-1}}dy)(\theta - c_t)$ *a fraction* $0 < \mu_n < 1$ *of non-shoppers is active and a fraction* $1 - \mu_n$ *of non-shoppers does not search at all, where* μ_n *is defined by*

$$h(\mu_n) \equiv \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1-\gamma)\mu_n} y^{n-1}} dy = \frac{\theta - c_t - c_e}{\theta - c_t}.$$

Active non-shoppers will stop searching as soon as they find a price at or below $\theta - c_t$*. Shops randomize over prices according to price distribution*

$$F_n(p) = 1 - (\frac{(\theta - c_t - p)(1-\gamma)\mu_n}{n\gamma p})^{\frac{1}{n-1}}.$$

The maximum price asked by a shop is $\theta - c_t$*, the minimum price equals* $(\theta - c_t)\frac{\mu_n(1-\gamma)}{\gamma n + \mu_n(1-\gamma)}$ *and expected profits are given by* $\pi_n = (\theta - c_t)\frac{\mu_n(1-\gamma)}{n}$*.*

Figure 4.2 shows the expected profits as a function of the search costs $c_t + c_e$. Recall that the reservation value r_n depends only on the continuation costs of search, c_e. Moreover, the decision whether or not to search depends on c_t. Therefore, in contrast to the model where all shops are isolated, the expected profits do not depend on total costs $c_e + c_t$, but on c_t and c_e in isolation. To be able to make a plot of the expected profits as a function of the total costs $c_e + c_t$ I assume that c_e and c_t are related to each other in a fixed proportion, that is, $c_t = \beta(c_t + c_e)$ and $c_e = (1 - \beta)(c_t + c_e)$, or consequently $c_t = \frac{\beta}{1-\beta}c_e$. In the figure, $\beta = 0.8$. As before, the number of firms n equals 10, $\gamma = 0.1$ and $\theta = 1$. The expected profits are plotted for $c_t + c_e < 0.8$. For higher values of $c_t + c_e$ the expected profits decrease to 0. The figure shows the same pattern as in the case where all the shops are isolated. When the search costs are low enough (for the current parameter values $c_t + c_e$ should be below 0.28) the full search equilibrium holds. In this case the expected profits increase in the search costs since the maximum price that can be asked, r_n, increases in the search costs. When the search costs are high enough ($c_t + c_e$ above 0.28) the partial search equilibrium holds and the expected profits decrease in the search costs. As before, the fraction of searching consumers, μ_n, decreases in the search costs and moreover the maximum price $\theta - c_t$ decreases in the search costs as well.

4.3.3 Comparing the two opposite cases

Figure 4.3 combines figures 4.1 and 4.2 by showing the expected profits as a function of the search costs $c_t + c_e$ in the case where all shops are isolated and in the case where all shops are located in the same shopping mall. Again, the number of firms n equals 10, $\gamma = 0.1$ and $\theta = 1$. In the figure $c_t = 0.8(c_t + c_e)$ and $c_e = 0.2(c_t + c_e)$. The expected profits are plotted for $c_t + c_e < 0.5$.

The figure can be split in different parts. First, when the search costs $c_t + c_e$ are small enough (for the current parameter values $c_t + c_e < 0.073$) the full search equilibrium holds in both cases and $\pi_1 > \pi_n$. The intuition for this result is straightforward. By

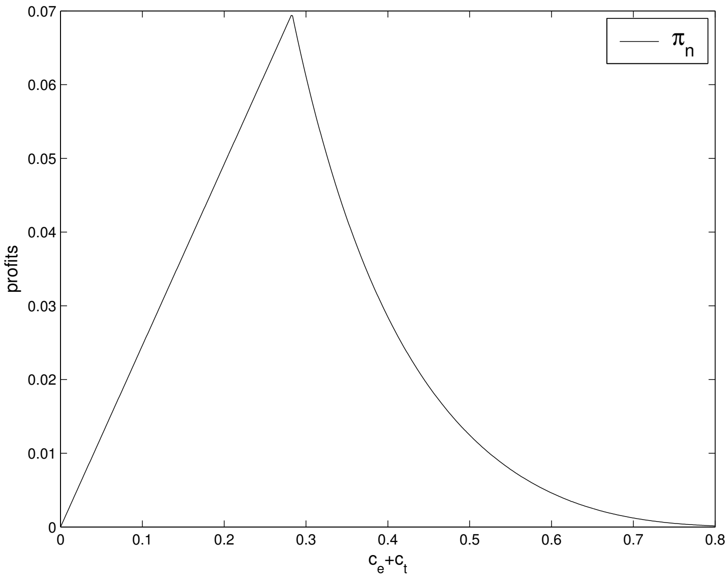


Figure 4.2: Expected profits as a function of the search costs when all shops are located in the same shopping mall.

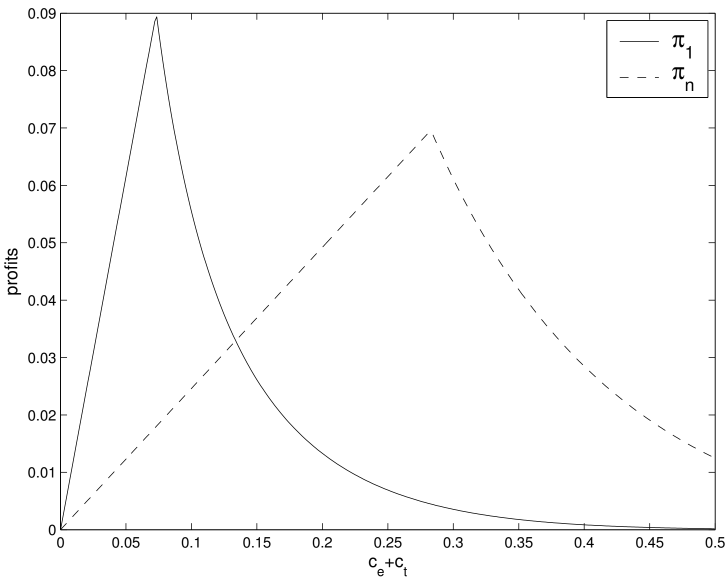


Figure 4.3: Expected profits as a function of the search costs when all shops are isolated and when all shops are located in the same shopping mall.

locating together in a single shopping mall shops decrease the costs to continue search from $c_e + c_t$ to c_e, leading to stronger competition and lower prices and profits. Second, when the search costs $c_t + c_e$ have an intermediate value (for the current parameter values $0.073 < c_t + c_e < 0.28$) the full search equilibrium holds in the case where all shops are located together and the partial search equilibrium holds in the case where all shops are located separately. The intuition for this is as before: when all shops are located together consumers expect lower prices and therefore consumers are more willing to search. This implies that when all shops are located together all non-shoppers are active and the expected profits increase in the search costs $c_t + c_e$. When all shops are located separately however only a fraction of the non-shoppers is active and expected profits decrease in the search costs $c_t + c_e$. When the search costs $c_t + c_e$ are high enough the expected profits when locating together are higher than the expected profits when locating separately. Finally, when the search costs are high enough (for the current parameter values $c_t + c_e > 0.28$) the partial search equilibrium holds in both cases. The fraction of active consumers is however higher when all firms are located together and this leads to higher expected profits when all firms are located together.

The pattern shown in Figure 3 does not depend on the specific parameter values chosen. Name the value of $c_t + c_e$ where the full search equilibrium changes into a partial search equilibrium the inflection value. A close look at Propositions 4.3.1 and 4.3.3 shows that the inflection value is always higher when all shops are located together. It is also easy to see that when in both cases a full search equilibrium holds, that is, when the search costs $c_t + c_e$ are below the inflection value for the case when all shops are separate, $\pi_1 > \pi_n$. With somewhat more effort it can be shown that for $c_t + c_e$ at or above the inflection value when all shops are located together $\pi_n > \pi_1$. This gives Proposition 4.3.5.

Proposition 4.3.5 *Let $c_t = \beta(c_t + c_e)$ with $0 < \beta < 1$. Then there exists a number c with*

$$\theta(1 - \int_0^1 \frac{1}{1 + \frac{\gamma}{1-\gamma} n y^{n-1}} dy) < c < \theta \frac{1 - \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma} n y^{n-1}} dy}{1 - \beta \int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma} n y^{n-1}} dy}$$

such that for $c_t + c_e < c$ $\pi_1 > \pi_n$ and for $c_t + c_e > c$ $\pi_1 < \pi_n$

Note that the total welfare (profits + consumer surplus) is given by $\gamma\theta + \mu_k(1-\gamma)(\theta - c_t - c_e)$, where $\mu_k = 1$ in a full search equilibrium. The shoppers always buy without incurring search costs and therefore add θ to the total welfare. Of the non-shoppers only a fraction μ_k searches, and since the searching non-shoppers search only once in equilibrium they add $\theta - c_t - c_e$ to the total welfare. When comparing the total welfare when all shops are in the same mall with the total welfare when all shops are isolated the only parameter that is important is μ_k. A close look at Propositions 4.3.1 - 4.3.4 shows that for all parameter values $\mu_n \geq \mu_1$ and therefore total welfare is higher when all shops are located together[6]. Intuitively, locating together lowers prices and the lower prices increase consumer participation.

[6]For $c_t + c_e$ below the inflection value of the case where all shops are separate $\mu_1 = \mu_n = 1$. For $c_t + c_e$ between the inflection value of the case where all shops are separate and the inflection value of the case where all shops are in the same mall $\mu_n = 1$ and $\mu_1 < 1$. For $c_t + c_e$ above the inflection value of the case

4.4 The intermediate case

The previous section has shown that under some conditions it could be profitable for shops to locate together instead of locating separately. In this section I will investigate the situation where $2 \leq k \leq n-2$ shops are located together in a shopping mall and the remaining $n-k$ shops are located outside the shopping mall and separately from each other. The analysis is restricted to a single shopping mall because a more general case with several shopping malls complicates the consumer behavior considerably. Note that under the restriction $k \leq n-2$ there are at least two shops outside the shopping mall. The pricing behavior of shops partly depends on this restriction. For instance, when there are at least two shops outside the shopping mall an isolated shop has at least one isolated competitor. This implies that if all active non-shoppers visit the mall the isolated shops compete for the shoppers, driving isolated prices down to zero. If there would be only one isolated shop there is no competition with other isolated shops and the isolated shop could make a profit even when all active non-shoppers visit the mall. The case $k = n-1$ is interesting because it shows whether all shops in the same mall can be an equilibrium. The analysis however turns out to be prohibitively difficult, if not impossible, and is therefore not contained in this paper.

Recall that $F_k^m(p)$ is the price distribution used by the shops that are in a shopping mall with k shops. Denote by π_k^m the expected profits of such a shop and define r_k^m as

$$\int_{\underline{p}_k^m}^{r_k^m} (r_k^m - p) dF_k^m(p) = c_e.$$

The same can be done for the isolated shops: $F_k^i(p)$ is the price distribution used by them, π_k^i denotes the expected profits and r_k^i is defined as

$$\int_{\underline{p}_k^i}^{r_k^i} (r_k^i - p) dF_k^i(p) = c_e + c_t.$$

Note that the definition of r_k^m uses c_e while the definition of r_k^i uses $c_t + c_e$. The reason for this is that a non-shopper who is in an isolated shop and wants to continue search has to incur a search cost $c_e + c_t$, while a non-shopper who is in a mall can continue searching in the mall at cost c_e. As before, the reservation prices determine whether a consumer wants to continue search and moreover determine whether a full search or a partial search equilibrium holds. As in the previous section, I will concentrate on equilibria where $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$.

In the previous section there was only one full search equilibrium with $\overline{p}_k \leq r_k$ and there was only one partial search equilibrium with $\overline{p}_k \leq r_k$. If $2 < k < n-1$ it cannot be ruled out that there are several full search equilibria with $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$ and there are several partial search equilibria with $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$. Moreover, some of these equilibria cannot be analytically solved anymore. It is however possible to analytically show some interesting features that all these equilibria have in common. In this section

where all shops are in the same mall μ_1 and μ_n are defined by $h(\mu_1) = \frac{\theta - c_t - c_e}{\theta}$ and $h(\mu_n) = \frac{\theta - c_t - c_e}{\theta - c_t}$. Since $h(\mu)$ is increasing in μ $\mu_n > \mu_1$.

I will first derive these common features. To compare profits in different cases and ultimately analyze the location decision of shops it is needed to derive specific equilibrium price distributions. In the second part of this section I will concentrate on one equilibrium that exists for a large range of parameter values. Even though the ultimate analysis is restricted to one equilibrium which is not unique, it will show that a shopping mall can indeed exist in equilibrium. Moreover, the analysis will give some insight in the conditions under which shopping malls are an equilibrium outcome.

4.4.1 Common features of all equilibria

Before some common features of all equilibria can be derived, the optimal consumer behavior should be specified. This consumer behavior is also needed in the second part of this section where one specific equilibrium is derived. The optimal consumer behavior is quite complex because of the wealth of options for consumers. After one or more searches they can decide to buy at the current shop, possibly return to a previously visited shop (incurring return costs), continue search in the mall or continue search in an isolated shop. The complete specification of optimal consumer behavior is only used in the formal proofs of the propositions in this section and to save space the complete specification of consumer behavior is therefore placed in the appendix to this chapter.

A first general result that can be derived is on the relation between r_k^i and r_k^m. When $\overline{p}_k^m \le r_k^m$ and $\overline{p}_k^i \le r_k^i$ (4.4) and (4.4) can be rewritten as $r_k^m = Ep_k^m + c_e$ and $r_k^i = Ep_k^i + c_t + c_e$, where Ep_k^m is the expected mall price, and Ep_k^i is the expected price in an isolated shop. If $Ep_k^i > Ep_k^m$ all active non-shoppers prefer to search in the mall and isolated shops only attract shoppers. Since the number of isolated shops, $n-k$, is at or above 2, this drives the prices in the isolated shops down to zero and $Ep_k^i > Ep_k^m$ cannot hold. The reverse argument holds when $Ep_k^m > Ep_k^i$ and so in equilibrium $Ep_k^m = Ep_k^i$. Using (4.4) and (4.4) this implies that $r_k^i = r_k^m + c_t$.

Proposition 4.4.1 *In any equilibrium with $\overline{p}_k^m \le r_k^m$ and $\overline{p}_k^i \le r_k^i$, $r_k^i = r_k^m + c_t$.*

A consequence of Proposition 4.4.1 is that non-shoppers who have to choose where to search first are indifferent between searching in the mall and in isolated shops. In equilibrium they will therefore spread randomly over mall shops and isolated shops. But I will show later that they do not spread evenly: mall shops attract a larger share of non-shoppers than isolated shops.

Applying Proposition 4.4.1 to the optimal consumer behavior as defined in the appendix to this chapter shows that when $\overline{p}_k^m \le r_k^m$ and $\overline{p}_k^i \le r_k^i$ non-shoppers will stop searching after their first search. When a mall shop deviates to a price above r_k^m every non-shopper who finds this price will continue to search in the mall. Similarly, when an isolated shop deviates to a price above r_k^i every non-shopper who finds this price will continue to search in some other isolated shop. Using this optimal consumer behavior it is easy to see that indeed an equilibrium with $\overline{p}_k^m \le r_k^m$ and $\overline{p}_k^i \le r_k^i$ can exist. If a shop would deviate to a higher price it would not sell anything and profits would be zero. Non-deviating shops obtain strictly positive profits since they at least sell to some non-shoppers. This shows that deviating is not profitable.

A natural equilibrium would be an equilibrium where $F^i_k(p) = F^m_k(p) = F_k(p)$. This is however not possible. Note that $\overline{p}_k = \overline{p}^m_k \leq r^m_k = r^i_k - c_t$ and that because of the shoppers $F_k(p)$ should be atomless. An isolated shop setting price $\overline{p}_k$ would sell only to those non-shoppers who visit the isolated shop on their first search. But raising price to r^i_k would not deter any of these non-shoppers from buying and profits would be higher. Note that non-shoppers are willing to buy at price r^i_k since to continue search they not only incur continuation costs c_e but also travel costs c_t. In equilibrium therefore $F^m_k(p) \neq F^i_k(p)$.

Let x_k be the fraction of active non-shoppers who decide to first visit a shop in the shopping mall and let $1 - x_k$ be the fraction of active non-shoppers who first visit an isolated shop, with $0 < x_k < 1$. Note that $x_k = \frac{k}{n}$ means an equal division of non-shoppers over all the shops.

Using the fact that non-shoppers stop searching after their first search the profit functions are as follows. For $p \leq r^m_k$,

$$\pi^m_k(p) = \gamma p(1 - F^m_k(p))^{k-1}(1 - F^i_k(p))^{n-k} + (1-\gamma)\mu_k \frac{x_k}{k} p.$$

For $p \leq r^i_k$

$$\pi^i_k(p) = \gamma p(1 - F^m_k(p))^{k}(1 - F^i_k(p))^{n-k-1} + (1-\gamma)\mu_k \frac{1 - x_k}{n-k} p.$$

A standard undercutting argument shows that atoms in $F^m_k(p)$ are only possible for those prices p^* at which $F^i_k(p^*) = 1$. Similarly, atoms in $F^i_k(p)$ are only possible for those prices p^* at which $F^m_k(p^*) = 1$. The profit functions also show that in equilibrium $\overline{p}^m_k = r^m_k$, since for a lower maximum price it would be profitable to deviate to r^m_k. Similarly, in equilibrium $\overline{p}^i_k = r^i_k$. Equilibrium expected profits are $\pi^m_k = r^m_k \frac{x_k}{k}\mu_k(1-\gamma)$ and $\pi^i_k = r^i_k \frac{1-x_k}{n-k}\mu_k(1-\gamma)$.

Note that for $p \geq r^m_k$

$$\pi^i_k(p) = (1-\gamma)\mu_k \frac{1 - x_k}{n-k} p.$$

This shows that isolated shops will never set a price between r^m_k and r^i_k and that there will be an atom at r^i_k. $F^i_k(p)$ should also have some probability mass below r^m_k since else the definition of r^i_k as given by (4.4) cannot hold. This probability mass is atomless, as well as $F^m_k(p)$. Let z_k denote the probability that an isolated shop sets a price r^i_k. Proposition 4.4.2 summarizes.

Proposition 4.4.2 *In any equilibrium, $0 < F^i_k(r^m_k) < 1$ and for $p < r^m_k$ $F^i_k(p)$ is atomless. $F^i_k(p)$ is constant for $r^m_k \leq p < r^i_k$ and has an atom at $p = r^i_k$. Moreover, in equilibrium $F^m_k(p)$ is atomless and $\overline{p}^m_k = r^m_k$.*

The intuition behind $F_k^m(p)$ is fairly standard: shops in a mall randomize over prices to balance the effects of the shoppers and the non-shoppers. $F_k^i(p)$ has a non-standard shape, with an atom at $\overline{p}_k^i$ and a gap below $\overline{p}_k^i$. Moreover, $\overline{p}_k^i > \overline{p}_k^m$ and the difference between the two maximum prices is exactly c_t. The maximum prices differ because of the search costs. Once a non-shopper is in the mall he can search at a relatively low cost c_e while if a non-shopper is in an isolated shop, continuing search will cost $c_e + c_t$. Consequently, an isolated shop has more power over the non-shoppers, which leads to a maximum price that is higher by exactly the difference in search costs. Despite the higher maximum price, non-shoppers are willing to search in isolated shops. This is because isolated shops randomize over the high maximum price and much lower prices. In this way isolated shops balance the effects of shoppers and non-shoppers. Note that the difference in maximum price and the other prices is at least c_t. Balancing the effects of shoppers and non-shoppers therefore only is possible when the fraction of non-shoppers who decide to search in an isolated shop, $1 - x_k$, is relatively low, making the fraction of shoppers more important for the shop. The next Proposition addresses this.

Proposition 4.4.3 *In any equilibrium,* $\pi_k^i < \pi_k^m$ *and* $\frac{1-x_k}{n-k} < \frac{x_k}{k}$ *or, equivalently,* $x_k > \frac{k}{n}$.

The main intuition behind this result is as argued above. When the isolated shops have expected profits at or above the expected profits of shops in the mall then the fraction of non-shoppers that isolated shops attract, $1 - x_k$, necessarily is relatively high[7]. The proof of Proposition 4.4.3 shows that when $1-x_k$ is that high an isolated shop makes more profit from setting a price r_k^i than from setting a lower price. A situation where isolated shops ask a price r_k^i for sure however cannot be an equilibrium situation, since in that case consumers prefer to search in the mall and x_k would be 1.

The same line of thought can be used when thinking about the minimum prices. When an isolated shop sets a price $\underline{p}_k^i$ it competes with the mall for the shoppers. To make sure that setting this minimum price is as profitable as setting price r_k^i $1 - x_k$ should be relatively small, but also the probability of attracting the shoppers when setting price $\underline{p}_k^i$ should be high. To give an extreme example, if $\underline{p}_k^i = \overline{p}_k^m$, the probability of selling to the shoppers would be zero and profits from a price $\underline{p}_k^i$ are below the profits from a price r_k^i. To make sure that the probability of attracting the shoppers is high enough, $\underline{p}_k^i$ cannot be much higher than $\underline{p}_k^m$. In fact, it can be proven that in equilibrium $\underline{p}_k^i \leq \underline{p}_k^m$.

Proposition 4.4.4 *In any equilibrium,* $\underline{p}_k^i \leq \underline{p}_k^m$.

4.4.2 One specific equilibrium

In the remainder of this section I will derive one specific equilibrium. Expressions (4.4.1) and (4.4.1) show that in general both $\pi_k^i(p)$ and $\pi_k^m(p)$ depend on both $F_k^i(p)$ and $F_k^m(p)$ and this makes a general analysis very difficult. When however the isolated shops either set a price below $\underline{p}_k^m$ or set a price at r_k^i, $\pi_k^i(p)$ only depends on $F_k^i(p)$ and $\pi_k^m(p)$ only

[7]Note that it need not be the case that $\frac{1-x_k}{n-k} \geq \frac{x_k}{k}$, but x_k should be such that $r_k^i \frac{1-x_k}{n-k} \geq r_k^m \frac{x_k}{k}$.

depends on $F_k^m(p)$. In this case closed form expressions for $F_k^i(p)$ and $F_k^m(p)$ can be derived and therefore this section focusses on an equilibrium where the isolated shops either set a price below $\underline{p}_k^m$ or set a price at r_k^i.[8] Note that this is also consistent with Propositions 4.4.2 and 4.4.4.

As before, there is a full search equilibrium with $\mu_k = 1$ and $r_k^i < \theta$ and there is a partial search equilibrium with $0 < \mu_k < 1$ and $r_k^i = \theta$. In the full search equilibrium $\pi_k^m = r_k^m(1-\gamma)\frac{x_k}{k}$ and equating $\pi_k^m(p)$ with π_k^m gives

$$F_k^m(p) = 1 - [\frac{(r_k^m - p)(1-\gamma)\frac{x_k}{k}}{\gamma p (z_k)^{n-k}}]^{\frac{1}{k-1}}$$

with

$$\underline{p}_k^m = r_k^m \frac{(1-\gamma)\frac{x_k}{k}}{\gamma (z_k)^{n-k} + (1-\gamma)\frac{x_k}{k}}.$$

Isolated shops expect profits equal to $\pi_k^i = r_k^i(1-\gamma)\frac{1-x_k}{n-k}$ and equating this with $\pi_k^i(p)$ gives that for $p \leq \underline{p}_k^m$

$$F_k^i(p) = 1 - [\frac{(r_k^i - p)(1-\gamma)\frac{1-x_k}{n-k}}{\gamma p}]^{\frac{1}{n-k-1}}$$

with

$$\underline{p}_k^i = r_k^i \frac{(1-\gamma)\frac{1-x_k}{n-k}}{\gamma + (1-\gamma)\frac{1-x_k}{n-k}}.$$

The maximum prices r_k^m and r_k^i are defined by

$$\int_{\underline{p}_k^m}^{r_k^m} (r_k^m - p) dF_k^m(p) = c_e$$

and

$$\int_{\underline{p}_k^i}^{r_k^i} (r_k^i - p) dF_k^i(p) = c_e + c_t.$$

Plugging in the expressions for $F_k^m(p)$ and $F_k^i(p)$ and rewriting (details are in the appendix to this chapter) gives

$$r_k^m = \frac{c_e}{\int_0^1 1 - \frac{1}{1 + \frac{\gamma}{1-\gamma}(z_k)^{n-k}\frac{k}{x_k}y^{k-1}} dy}$$

[8] Note that such an equilibrium cannot hold when $k = n-1$. Since in that case the isolated shop has no isolated competitors it will never set a price below $\underline{p}_k^m$. Moreover, an equilibrium where the isolated shop randomizes between a price of $\underline{p}_k^m$ and a price of r_k^i is not possible since mall shops then have an incentive to deviate to a price slightly below $\underline{p}_k^m$. More in general, it can be shown that when $k = n-1$ the supports of $F_k^i(p)$ and $F_k^m(p)$ overlap, $\pi_k^i(p)$ depends on $F_k^m(p)$ and $\pi_k^m(p)$ depends on both $F_k^m(p)$ and $F_k^i(p)$, which makes an analysis very difficult.

and

$$r_k^i = \frac{c_e + c_t}{\int_{z_k}^1 1 - \frac{1}{1+\frac{\gamma}{1-\gamma}\frac{n-k}{1-x_k}y^{n-k-1}}dy}.$$

The probability that an isolated shop sets a price equal to r_k^i, z_k, is implicitly defined by $z_k = 1 - F_k^i(\underline{p}_k^m)$. Moreover, as Proposition 4.4.1 shows, $r_k^i = r_k^m + c_t$. These two equalities together define x_k and z_k. Unfortunately, it is impossible to solve explicitly for x_k and z_k, and in the next section I will use computer simulations to obtain numerical values.

The full search equilibrium can only hold when $r_k^i < \theta$ (see the optimal consumer behavior) and no shop has an incentive to deviate. Using the expressions for $F_k^m(p)$ and $F_k^i(p)$ given above it can be shown that an isolated shop never has an incentive to deviate from $F_k^i(p)$. For a shop in the mall it is clear that deviation to a price above r_k^m is never profitable. Deviating to a price below $\underline{p}_k^i$ is also not profitable, but deviating to a price between $\underline{p}_k^i$ and $\underline{p}_k^m$ could be profitable. In that case deviating gives profits $\pi_k^m(p) = \gamma p(1 - F_k^i(p))^{n-k} + (1-\gamma)\frac{x_k}{k}p$. Plugging in $F_k^i(p)$ and twice differentiating shows that the second derivative is positive. This implies that the maximum value of $\pi_k^m(p)$ is obtained either at $p = \underline{p}_k^i$ or at $p = \underline{p}_k^m$. Deviating is not profitable if and only if $\pi_k^m(\underline{p}_k^m) \geq \pi_k^m(\underline{p}_k^i)$, or $r_k^m \frac{x_k}{k}(\gamma + (1-\gamma)\frac{1-x_k}{n-k}) \geq r_k^i \frac{1-x_k}{n-k}(\gamma + (1-\gamma)\frac{x_k}{k})$.

Using the same method as before one can derive that in a partial search equilibrium for $p \leq r_k^m$

$$F_k^m(p) = 1 - [\frac{(r_k^m - p)(1-\gamma)\mu_k \frac{x_k}{k}}{\gamma p(z_k)^{n-k}}]^{\frac{1}{k-1}}$$

with

$$\underline{p}_k^m = r_k^m \frac{(1-\gamma)\frac{x_k}{k}\mu_k}{\gamma(z_k)^{n-k} + (1-\gamma)\frac{x_k}{k}\mu_k}$$

and that for $p \leq \underline{p}_k^m$

$$F_k^i(p) = 1 - [\frac{(r_k^i - p)(1-\gamma)\frac{1-x_k}{n-k}\mu_k}{\gamma p}]^{\frac{1}{n-k-1}}$$

with

$$\underline{p}_k^i = r_k^i \frac{(1-\gamma)\frac{1-x_k}{n-k}\mu_k}{\gamma + (1-\gamma)\frac{1-x_k}{n-k}\mu_k}.$$

Since this is a partial search equilibrium, $r_k^i = \theta$ and $r_k^m = \theta - c_t$. The three parameters x_k, z_k and μ_k are jointly defined by $z_k = 1 - F_k^i(\underline{p}_k^m)$, $r_k^m = \int_{\underline{p}_k^m}^{\theta - c_t} pdF_k^m(p) + c_e = \theta - c_t$ and $r_k^i = \int_{\underline{p}_k^i}^{\theta} pdF_k^i(p) + c_t + c_e = \theta$. Again, solving explicitly for x_k, z_k and μ_k is not possible and I will resort to simulations.

An analysis similar to the one for the full search equilibrium shows that deviating is never profitable for isolated shops and that mall shops will not deviate if and only if $r_k^m \frac{x_k}{k}(\gamma + (1-\gamma)\frac{1-x_k}{n-k}\mu_k) \geq r_k^i \frac{1-x_k}{n-k}(\gamma + (1-\gamma)\frac{x_k}{k}\mu_k)$.

4.5 Location choice

In this section I will use computer simulations to analyze the equilibrium that has been derived in Section 4.4.2. With these simulations also the location choice game will be analyzed. Let β be such that $c_t = \beta(c_t + c_e)$, or $c_t = \frac{\beta}{1-\beta}c_e$. The system of equations that defines x_k and z_k in the full search equilibrium of Section 4.4.2 only depends on β, and not on c_t and c_e. This implies that for a given β in the full search equilibrium of Section 4.4.2 x_k and z_k are constant and r_k^m, r_k^i, π_k^m and π_k^i are linear in $c_e + c_t$. Note that the full search equilibrium only holds when $r_k^i < \theta$, or $c_t + c_e$ small enough. The value of r_k^i depends on k, but if $c_t + c_e$ is chosen small enough the full search equilibrium holds for all k with $2 \leq k \leq n-2$. Moreover, when $c_t + c_e$ is chosen small enough the full search equilibrium also holds in the model where all shops are located in the same mall and in the model where all shops are located separately. Therefore, when $c_t + c_e$ is small enough, the profits in the different models are all linear in $c_t + c_e$ and can be easily compared. This is what I will do in the first part of this section. Recall that in Section 4.3 μ_1, μ_n and the value of $c_t + c_e$ where the full search equilibrium changes in a partial search equilibrium (the inflection value) played an important role in the comparison of the profits in the two extreme cases. In the comparison of profits in the first part of this section μ_k and the inflection value do not play any role. The fraction of non-shoppers visiting the mall, $\frac{x_k}{k}$, and the reservation prices r_k^m and r_k^i are the only determinants of the relative profits. The effects of μ_k and the inflection value will be discussed later in this section, when I also consider the partial search equilibrium of Section 4.4.2

Tables 4.1, 4.2 and 4.3 give the expected profits for several values of γ, β, n and k. For ease of notation, the profits are given relative to $c_t + c_e$. Note that the profits of mall shops and isolated shops can only be given for $k \leq n-2$ since for $k = n-1$ the analysis of Section 4.4 does not hold. Moreover, the equilibrium that has been derived in the previous section does not always hold. When the equilibrium does not hold this is denoted by 'na'. The tables show that for several parameter values $\pi_1 > \pi_2^m$. This implies that, starting from a situation with only isolated shops, if a shop would get the option to join a competitor the shop would choose not to do so, and the situation without a mall is an equilibrium. Starting from a situation where there is a mall with two shops two things can happen. First, if one of the mall shops gets the option to relocate it would do so, since $\pi_1 > \pi_2^m$. This would lead to an equilibrium situation with only isolated shops. Second, if one of the isolated shops gets the option to join the mall it would do so, since $\pi_3^m > \pi_2^i$. Note that for $k \geq 2$ $\pi_{k+1}^m > \pi_k^i$ and therefore in subsequent periods mall shops would never want to leave the mall. Isolated shops however would prefer to join the mall and this would result in an equilibrium where the mall attracts all the shops, at least until the equilibrium from Section 4.4.2 no longer holds. When the initial situation has a mall

Table 4.1: Expected profits in the full search equilibrium of Section 4.4.2 for different values of n, β and k, with $\gamma = 0.05$ and $\theta = 1$.

	n=5		n=7		n=10		
	$\beta = 0.8$	$\beta = 0.9$	$\beta = 0.6$	$\beta = 0.9$	$\beta = 0.7$	$\beta = 0.8$	$\beta = 0.9$
π_1	4.12	4.12	3.07	3.07	2.27	2.27	2.27
π_2^m	2.91	2.54	2.75	3.03	2.44	2.92	3.36
π_2^i	2.07	1.37	2.39	0.97	1.70	1.38	0.87
π_3^m	na	1.70	na	1.95	2.03	2.13	2.21
π_3^i	na	1.07	na	0.91	1.59	1.30	0.83
π_4^m			na	1.38	1.71	1.55	1.62
π_4^i			na	0.84	1.51	1.28	0.78
π_5^m			na	na	na	1.36	1.36
π_5^i			na	na	na	1.16	0.67
π_6^m					na	na	1.03
π_6^i					na	na	0.65
π_7^m					na	na	0.76
π_7^i					na	na	0.62
π_8^m					na	na	na
π_8^i					na	na	na

with three or more shops it is clear from the analysis above that again the mall attracts all the shops, at least until the equilibrium from Section 4.4.2 no longer holds. Thus, when $\pi_1 > \pi_2^m$ the simulations suggest that there are two equilibria possible, depending on the initial situation and on which type of shop gets the option to relocate. As the tables show, there are also parameter values for which $\pi_1 < \pi_2^m$. In this case, starting from a situation where there are only isolated shops, a shop that gets the option to join a competitor would do so, and they would form a mall with 2 shops. Note that again for $k \geq 2$ $\pi_{k+1}^m > \pi_k^i$ and therefore in subsequent periods mall shops will stay in the mall and isolated shops will join the mall, at least until the equilibrium from Section 4.4.2 no longer holds. It is clear that the same thing happens when the initial situation has a mall with two or more shops. Thus, when $\pi_1 < \pi_2^m$ the simulations suggest that only one equilibrium is possible, where the mall attracts all the shops, at least until the equilibrium from Section 4.4.2 no longer holds. Note that this final equilibrium gives lower profits than the situation where there is no mall. What drives the result is that for a small mall the mall shops have higher profits than when there is no mall at all. Once there is a mall, joining is profitable for isolated shops since shops in the mall attract more non-shoppers than isolated shops and this effect is stronger than the decrease in prices. This increase in mall size is however decreasing the profits of the existing mall shops. Every time an isolated shop joins the mall the mall prices decrease. In addition, as more extensive simulations show, every time an isolated shop joins the mall the fraction of non-shoppers per mall shop, $\frac{x_k}{k}$ decreases.

The analysis above suggests that there are two possible outcomes in the location choice game. There either is no mall at all or there is a mall with at least k^* shops[9], where k^*

[9] If $\pi_{k^*}^i > \pi_{k^*+1}^m$ then a mall with k^* shops is an equilibrium. Now suppose $\pi_{k^*}^i < \pi_{k^*+1}^m$. If $\pi_{k^*+1}^i >$

Table 4.2: Expected profits in the full search equilibrium of Section 4.4.2 for different values of n, β and k, with $\gamma = 0.1$ and $\theta = 1$.

	n=5	n=7		n=10		n=15	
	$\beta = 0.8$	$\beta = 0.6$	$\beta = 0.9$	$\beta = 0.7$	$\beta = 0.9$	$\beta = 0.6$	$\beta = 0.8$
π_1	2.09	1.60	1.60	1.23	1.23	0.93	0.93
π_2^m	1.45	1.40	1.73	1.37	1.80	1.09	1.23
π_2^i	1.11	1.27	0.52	0.92	0.52	0.81	0.70
π_3^m	na	na	1.03	1.08	1.20	0.92	1.00
π_3^i	na	na	0.52	0.88	0.49	0.79	0.66
π_4^m		na	0.71	0.90	0.88	0.82	0.85
π_4^i		na	0.49	0.84	0.46	0.77	0.63
π_5^m		na	na	na	0.66	na	0.75
π_5^i		na	na	na	0.43	na	0.59
π_6^m				na	0.53	na	0.65
π_6^i				na	0.39	na	0.57
π_7^m				na	na	na	0.60
π_7^i				na	na	na	0.52
π_8^m				na	na	na	0.53
π_8^i				na	na	na	0.50
π_9^m						na	na
π_9^i						na	na

Table 4.3: Expected profits in the full search equilibrium of section 4.4.2 for different values of n, β and k, with $\gamma = 0.25$ and $\theta = 1$.

	n=5		n=7		n=10		
	$\beta = 0.8$	$\beta = 0.9$	$\beta = 0.7$	$\beta = 0.9$	$\beta = 0.6$	$\beta = 0.7$	$\beta = 0.9$
π_1	0.81	0.81	0.66	0.66	0.53	0.53	0.53
π_2^m	0.72	0.52	0.55	0.62	0.51	0.54	0.73
π_2^i	0.42	0.30	0.49	0.29	0.46	0.42	0.27
π_3^m	na	na	na	0.39	na	0.44	0.50
π_3^i	na	na	na	0.27	na	0.40	0.25
π_4^m			na	na	na	na	0.36
π_4^i			na	na	na	na	0.24
π_5^m			na	na	na	na	0.29
π_5^i			na	na	na	na	0.21
π_6^m					na	na	na
π_6^i					na	na	na

is the maximum number of mall shops for which the equilibrium of Section 4.4.2 holds. Which of the two possible outcomes is obtained depends on the initial situation and on whether $\pi_1 > \pi_2^m$ or $\pi_1 < \pi_2^m$. When comparing π_1 and π_2^m there are two forces at work. First, when two shops are located in the mall the expected prices will be lower, which lowers the expected profits. But at the same time the shops in the mall will attract more searching non-shoppers: $\frac{x_k}{k}$ instead of $\frac{1}{n}$. The relative strength of these two effects depends on the values of β, n and γ. The tables suggest that for small values of n the pricing effect is larger while for large values of n the sales effect is larger. Similarly, for small values of β the pricing effect outweighs the sales effect, while for large values of β the sales effect is larger. The mechanism behind the influence of n is difficult to investigate since both π_1 and π_2^m depend on n. The mechanism behind the influence of β is however quite intuitive. Recall that π_1 only depends on $c_t + c_e$ and not on c_t or c_e. Therefore, π_1 is constant in β and β only influences π_2^m. When β increases and $c_t + c_e$ is kept constant c_t increases relative to c_e. A relatively low value of c_e increases competition inside the mall and leads to lower prices. At the same time, the difference between mall prices and isolated prices, which is c_t, increases. This makes the mall more attractive to visit than an isolated shop, and the fraction of non-shoppers that a mall shop attracts, $\frac{x_k}{k}$, increases. Thus, an increase in β leads to a decrease in prices and an increase in sales for mall shops. As the table shows, sales increase faster than prices decrease, and π_2^m increases in β.[10]

Another observation from the tables and from more extensive simulations is that the equilibrium of Section 4.4.2 does not exist for γ high, β low or k high. Recall from the previous section that mall shops can have an incentive to deviate to a price $\underline{p}_k^i < \underline{p}_k^m$ since in that way they are sure to capture all the shoppers. This deviation is only profitable when a relatively large part of the sales of the shops in the mall will come from the shoppers. This happens when γ is high, explaining why the equilibrium can not hold for high values of γ. As observed before, when β is low the sales to non-shoppers, as determined by $\frac{x_k}{k}$, are relatively low and again a relatively large part of the sales of the shops in the mall will come from the shoppers. Therefore the equilibrium does not exist for low values of β. More extensive simulations than those reported in the tables show that $\frac{x_k}{k}$ decreases in k. Therefore a high value of k again increases the relative share of the shoppers and leads to deviations.

Thus far, I have looked at the full search equilibrium of Section 4.4.2. In the remainder of this section I will also look at the partial search equilibrium of Section 4.4.2. This equilibrium type is more complicated to analyze, since the fraction of active non-shoppers visiting the mall, x_k, and the probability that an isolated shop sets a price r_k^i, z_k, now not only depend on β, but also on c_t and c_e. Therefore, instead of tables, I will give several plots of expected profits as a function of $c_t + c_e$. Simulations show that the plots

$\pi_{k^*+2}^m$ then a mall with k^*+1 shops is an equilibrium. If $\pi_{k^*+1}^i < \pi_{k^*+2}^m$ and $\pi_{k^*+2}^i > \pi_{k^*+3}^m$ then a mall with k^*+2 shops is an equilibrium. This can be continued until a mall with k^*+h shops is found to be an equilibrium or $\pi_{n-1}^i < \pi_n^m$, in which case a mall with n shops is an equilibrium.

[10]Exactly the same mechanism explains the effect of β on π_k^m for $k > 2$. Note however from the table that for $k > 2$ it could be that π_k^m *decreases* in β. In that case prices decrease faster in β than sales increase in β.

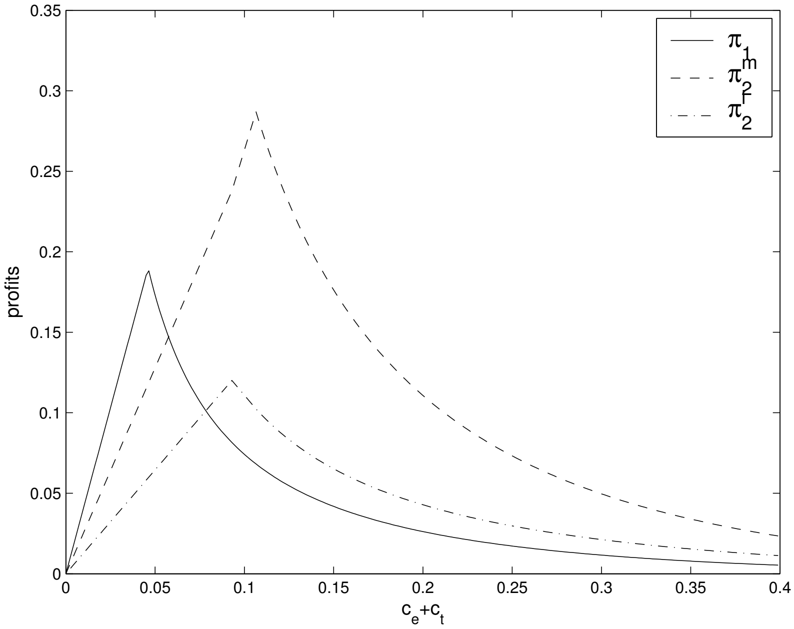


Figure 4.4: Expected profits as a function of the search costs when all shops are located separately, when all shops are located in the same shopping mall and when there is a mall of two shops.

of the combined full and partial search equilibrium with $2 \leq k \leq n-2$ have the same pattern as in the extreme cases analyzed in Section 4.3. Again, there is a value of $c_t + c_e$, called the inflection value, such that for $c_t + c_e$ below this value the full search equilibrium holds and above this value the partial search equilibrium holds. The fraction of searching non-shoppers, μ_k, is decreasing in $c_t + c_e$ and as a consequence the profits in the partial search equilibrium decrease in $c_t + c_e$.

Recall that the equilibrium derived in Section 4.4.2 does not always hold, since for some parameter values the mall shops have an incentive to deviate to a price $\underline{p}_k^i < \underline{p}_k^m$. This incentive is larger when the fraction of shoppers is more important. The simulations suggest that if for some parameter values the full search equilibrium does not hold, also the partial search equilibrium does not hold. But as the plots in this section show, the existence of the full search equilibrium of Section 4.4.2 is however no guarantee that the partial search equilibrium of Section 4.4.2 holds for all values of $c_t + c_e$ above the inflection value. This is caused by μ_k. In a partial search equilibrium μ_k decreases in $c_t + c_e$, decreasing the fraction of active non-shoppers and increasing the importance of the shoppers. When $c_t + c_e$ is large enough, and consequently μ_k is low enough, the shoppers are important enough to make deviation to $\underline{p}_k^i$ profitable for the mall shops, even when the full search equilibrium (with $\mu_k = 1$) does hold.

Figures 4.4, 4.5 and 4.6 show the expected profits for several values of k. In these figures γ is set at 0.05, $n = 5$, $\beta = 0.9$ and $\theta = 1$. Figure 4.4 depicts π_1, π_2^m and π_2^i and figure 4.5 depicts π_2^i, π_3^m and π_3^i. A first observation is that the inflection point shifts to the right when k increases. This is intuitively clear. Expected prices will be lower when

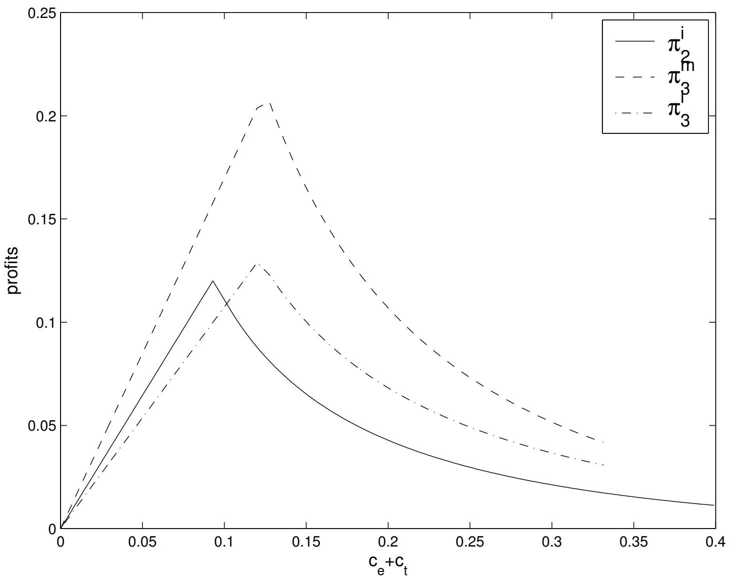


Figure 4.5: Expected profits as a function of the search costs when there is a mall of two shops and when there is a mall of three shops.

more shops are located in the mall.[11] Therefore, when more shops are located in the mall more non-shoppers will be tempted to search, shifting the inflection point to the right. A consequence of this is that for $k \geq 2$ $\mu_k \geq \mu_{k-1}$. Comparing π_1 and π_2^m there are three regions of interest. As in Section 4.3, there is a region of low search costs where there is a full search equilibrium for both $k = 1$ and $k = 2$, there is a region of high search costs where there is a partial search equilibrium for both $k = 1$ and $k = 2$ and there is a region of intermediate search costs where for $k = 1$ there is a partial search equilibrium while for $k = 2$ there is a full search equilibrium. The region with low search costs has been analyzed in the first part of this section. In the region with intermediate search costs π_1 is decreasing in the search costs while π_2^m is increasing in search costs. Note that the decrease in π_1 is caused by the decrease in μ_1. When $k = 2$ there is a full search equilibrium and the increase in π_2^m is caused by an increase in prices. For search costs equal to the inflection value for $k = 2$, that is, for the maximum value of search costs such that for $k = 2$ a full search equilibrium exists, $\pi_2^m > \pi_1$. In the region with high search costs $\pi_2^m > \pi_1$. There are three effects that together explain why $\pi_2^m > \pi_1$. Two effects also played a role when comparing the full search equilibria: when $k = 2$ the prices are lower but the fraction of active non-shoppers visiting the mall ($\frac{x_k}{k}$) is above $\frac{1}{n}$. When comparing the full search equilibria the pricing effect is stronger than the sales effect. But when comparing the partial search equilibria the fraction of active non-shoppers, μ_k, also plays a role. Since $\mu_2 > \mu_1$ the effect of this fraction of active non-shoppers enforces the positive effect of the fraction of active non-shoppers visiting the mall and the combined positive effect on sales can outweigh the negative effect of lower prices. When this happens, a situation with only isolated shops cannot be an equilibrium. If such a situation occurs and one of the isolated shops would get the opportunity to join a competitor the isolated shop would do

[11] Simulations not reported here show that also the inflection point for $k = 3$ is to the left of the inflection point for $k = 5$

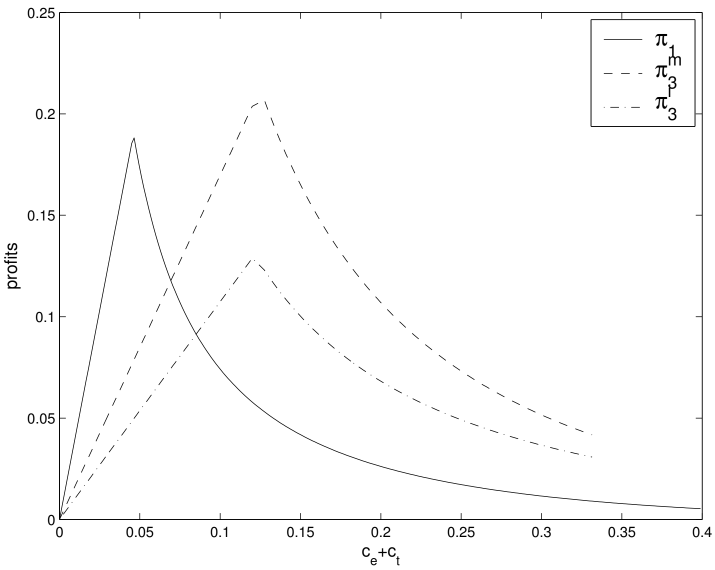


Figure 4.6: Expected profits as a function of the search costs when there is a mall of three shops, when all shops are isolated and when all shops are in the same mall.

so. Note that also the remaining isolated shops can profit from the formation of a mall. The formation of a mall leads to lower prices and a loss of active non-shoppers visiting an isolated shop ($\frac{1-x_k}{n-k} < \frac{1}{n}$) but also to an increase in active non-shoppers and this can offset the previous two effects.

To save space only the plots for $\gamma = 0.05$, $n = 5$, $\beta = 0.9$ and $\theta = 1$ are included in this paper. Plots for other parameter values show the same pattern. When the search costs $c_t + c_e$ are high enough, $\pi_1 < \pi_2^m$ and a situation with only isolated shops cannot be an equilibrium. Recall from the tables in the first part of this section that for some parameter values $\pi_1 < \pi_2^m$ in the full search equilibrium. In the plots for these parameter values $\pi_1 < \pi_2^m$ for all values of $c_t + c_e$ and a situation with only isolated shops can never be an equilibrium.

To complete the analysis, Figure 4.5 depicts π_2^i, π_3^m and π_3^i. The figure shows that for all values of $c_t + c_e$ π_3^m is above π_2^i and therefore shops have an incentive to join the mall. For an isolated shop joining the mall means lower prices but a larger fraction of active non-shoppers visiting the shop ($\frac{x_{k+1}}{k}$ instead of $\frac{1-x_k}{n-k}$) and for high enough search costs it also means a higher fraction of active non-shoppers ($\mu_{k+1} > \mu_k$). Note that for low search costs the remaining isolated shops lose profits when one isolated shop joins the mall because of the lower prices. For high search costs the remaining isolated shops however again profit from the increase in μ_k. Plots for other parameter values show exactly the same pattern and if relevant also show that $\pi_k^i < \pi_{k+1}^m$ for $k > 2$. This suggests that the location choice game has exactly the same outcome as in the first part of this section. An equilibrium either has only isolated shops or has a mall with at least k^* shops, where k^* is the maximum number of mall shops for which the equilibrium of Section 4.4.2 holds. Which equilibrium holds depends on whether $\pi_1 < \pi_2^m$ or $\pi_1 > \pi_2^m$ and on the initial situation. Note that in the simulations for high enough search costs $\pi_1 < \pi_2^m$ and therefore for high enough search costs the only possible equilibrium is an equilibrium with a mall with at least k^* shops. Figure 4.6 compares the profits when $k = 3$ with the profits when all shops are isolated. Recall that in the analysis in the first part of the paper an equilibrium with a mall with at least k^* shops in the end makes all shops worse off than when all shops would be isolated. This is not necessarily the case when search costs are high enough. As the figure shows, for high enough search costs $\pi_3^m > \pi_1$ and even $\pi_3^i > \pi_1$. This is again caused by the fraction of searching non-shoppers, μ_k. Apparently the difference between μ_3 and μ_1 is high enough to offset the negative pricing effect of a mall and even the negative effect of the fraction of active non-shoppers who visit an isolated shop, $\frac{1-x_k}{n-k}$.

4.6 Conclusion

This paper analyzed the incentives of a shop to locate together with similar shops in a shopping mall. As in a standard sequential consumer search model, the consumers incur costs when entering a shop, independent of where the shop is located. Consumers also incur travel costs when traveling between shops that are not in the same mall, a novel feature in a sequential search setting. The addition of travel costs implies that searching in a shopping mall is more attractive than searching isolated shops. This has several implications for the profitability of shops. First, the lower search costs in a shopping mall reduce the prices. Second, because mall prices are lower, the shopping mall attracts more consumers than the isolated shops. And, third, because the existence of a shopping mall leads to lower prices, the fraction of active consumers is increasing in the size of the shopping mall.

Simulations suggest that because of the positive second and third effect a shopping mall always is an equilibrium outcome of the location choice game analyzed in this paper. If a shop would leave the shopping mall the drop in sales would be so large that profits would decrease. This shopping mall equilibrium is, however, not necessarily the unique equilibrium. When the search costs are low enough simulations show that an equilibrium with only isolated shops can exist. For low enough search costs the third effect is absent since for low enough search costs all consumers are active, independent of the size of the shopping mall. The second effect alone is not always strong enough to counter the negative pricing effect and in this case an equilibrium exists with only isolated shops. Even in this case however an equilibrium with a shopping mall of at least three shops exists. The reason for this is that the second effect is stronger when a mall already exists than when an isolated shop joins another isolated shop to form a mall with two shops. When all shops are isolated the consumers divide evenly over the shops and the increase in sales when forming a mall with two shops is not as high as when an isolated shop with less than an even share of consumers joins an existing mall.

An interesting feature of the equilibria in this model is that the profits of mall shops are always above the profits of isolated shops. This has two causes. First, as mentioned before, mall shops attract more consumers than isolated shops. Second, even though isolated shops can set prices above the mall prices, the increase in prices is limited since if isolated shops would set a price that is too high all consumers would go to the mall. Consumers in an isolated shop are willing to buy at a slightly higher price since to continue search they not only incur the entering costs from the standard model but also the travel costs. When the price difference between mall shops and isolated shops is however above the travel costs consumers in an isolated shop continue their search in the mall. An interesting simulation result is that isolated shops not necessarily loose profits when a mall is formed. Even though isolated shops loose customers who go to the mall, the fraction of active consumers increases and the isolated shops get some share of these consumers. When the search costs are high enough the increase in active consumers is high enough to offset the decrease in sales caused by consumers visiting the mall instead of an isolated shop.

4.A Proofs of Section 4.3

4.A.1 Proof of Propositions 4.3.1 and 4.3.2

To prove Propositions 4.3.1 and 4.3.2 first note that in an equilibrium where some non-shoppers search a shop will never ask a price above θ. If a shop would ask a price above θ it would not make any sales and profits would be 0. Asking a price $c_t + c_e$ however prevents non-shoppers from searching further and guarantees a strictly positive profit. This implies that if a non-shopper is in a shop he can always obtain a non-negative utility by buying from this shop.

To prove the optimality of the consumer behavior stated in the Propositions an induction argument will be used. Consider a non-shopper who expects the shops to price according to some price distribution $F_1(p)$ with $\overline{p} \leq \min(\theta, r_1)$, where r_1 is defined by

$$\int_{\underline{p}}^{r_1} (r_1 - p) dF_1(p) = c_e + c_t.$$

Denote by p^* the price the non-shopper found in his last search and denote by p^{min} the minimum price he found in previous searches, with p^{min} infinite when there are no previous searches. Let q denote $\min(p^* + c_t, p^{min} + c_t)$. If the non-shopper has already searched $n-1$ shops the utility from buying is $\theta - \min(p^*, p^{min} + c_t)$. If the non-shopper decides to search the nth shop as well and he finds a price below q he will buy in the nth shop. Else he will return to a previously visited shop. The expected utility from searching is given by

$$U(search) = -c_t - c_e + \int_{\underline{p}_1}^{q} (\theta - p) dF_1(p) + (1 - F_1(q))(\theta - q).$$

Note that the utility above holds even when $q > \theta$. For $p > \theta$ $F_1(p) = 1$ and therefore when $q > \theta$ the utility above reduces to $-c_t - c_e + \int_{\underline{p}_1}^{\theta} (\theta - p) dF_1(p)$, which is exactly the expected utility of search in case $q > \theta$. The utility from searching can be rewritten as

$$U(search) = -c_t - c_e + \theta - q + \int_{\underline{p}_1}^{q} (q - p) dF_1(p).$$

If $\min(p^*, p^{min} + c_t) > r_1$ it must be that $q > r_1$. Using that $\overline{p} \leq r_1$, $\int_{\underline{p}_1}^{q} (q-p) dF_1(p) = \int_{\underline{p}_1}^{r_1} (q-p) dF_1(p) = q - r_1 + \int_{\underline{p}_1}^{r_1} (r_1 - p) dF_1(p) = q - r_1 + c_t + c_e$. This gives that the utility of searching equals $\theta - r_1$ and since the utility of buying immediately is $\theta - \min(p^*, p^{min} + c_t)$ searching is profitable for $\min(p^*, p^{min} + c_t) > r_1$.

If $\min(p^*, p^{min} + c_t) \leq r_1$ both $q > r_1$ and $q \leq r_1$ are possible. For $q > r_1$ the utility of search equals $\theta - r_1$ (see previous paragraph) and $U(buy) \geq \theta - r_1$. Search therefore is not profitable. For $q \leq r_1$, $\int_{\underline{p}_1}^{q} (q-p) dF_1(p) < c_t + c_e$ and $U(search) < \theta - q$. Since $U(buy) \geq \theta - r_1 \geq \theta - q$ search is not profitable. So for $\min(p^*, p^{min} + c_t) \leq r_1$ the non-shopper will stop searching while for $\min(p^*, p^{min} + c_t) > r_1$ he will continue to search.

This shows that the consumer behavior stated in the Propositions is indeed optimal when a consumer has searched $n-1$ shops. Now suppose that he has searched $h \geq 1$ shops and that the stated consumer behavior holds whenever he has searched $h+1$ or more shops. Since the consumer expects $\overline{p}$ to be at or below r_1 the optimal consumer behavior tells him to stop searching after searching the $h+1$th shop. Therefore, after searching the hth shop, the consumer expects to search only one more shop and the utilities of continuing search and of stopping search are the same as before. After searching the hth shop the non-shopper will therefore continue his search if and only if $\min(p^*, p^{min} + c_t) > r_1$ and the stated consumer behavior also holds in the case $h \geq 1$ shops have been searched.

This leaves the case where no shops have been searched yet. Again, given the optimal consumer behavior for $h \geq 1$, the non-shopper expects to search only once and the utility of search equals

$$U(search) = -c_t - c_e + \int_{\underline{p}}^{\min(\theta, r_1)} (\theta - p) dF_1(p).$$

When $r_1 < \theta$ this reduces to $-c_t - c_e + \int_{\underline{p}}^{r_1} (\theta - p) dF_1(p) = -c_t - c_e + \theta - r_1 + \int_{\underline{p}}^{\min(\theta,r_1)} (r_1 - p) dF_1(p) = \theta - r_1 > 0$, so for $r_1 < \theta$ all non-shoppers will search. When $r_1 = \theta$ the expression above can be rewritten as $-c_t - c_e + \int_{\underline{p}}^{r_1} (r_1 - p) dF_1(p) = 0$ and so non-shoppers are indifferent between searching and staying home.

Before deriving an explicit expression for r_1, consider the pricing behavior of shops. First look at the full search case with $r_1 < \theta$. A standard undercutting argument shows that the price distribution has no atoms. For $p \leq r_1$ profits are given by

$$\pi_1(p) = p\gamma(1 - F_1(p))^{n-1} + p(1 - \gamma)\frac{1}{n}$$

Under the assumption $\overline{p} \leq r_1$ it must be that $\overline{p} = r_1$. If $\overline{p} < r_1$ deviation to a price r_1 would be profitable. This gives that in equilibrium profits equal $\pi_1(r_1) = r_1(1 - \gamma)\frac{1}{n}$ and equating this with $\pi_1(p)$ gives

$$F_1(p) = 1 - (\frac{1 - \gamma}{\gamma n} \frac{r_1 - p}{p})^{\frac{1}{n-1}}.$$

Finally, the minimum price is the price $\underline{p}$ such that $F_1(\underline{p}) = 0$. This gives $\underline{p} = r_1 \frac{1-\gamma}{\gamma n + 1 - \gamma}$. Note that deviation to a price below $\underline{p}$ is not profitable and that deviation to a price above r_1 gives zero profits and therefore is not profitable as well.

Given $F_1(p)$ the reservation price r_1 can be derived. Rewriting the definition of r_1 gives $r_1 - \int_{\underline{p}}^{r_1} p dF_1(p) = c_t + c_e$. Rewriting $F_1(p)$ gives

$$p = \frac{r_1}{1 + \frac{\gamma n}{1-\gamma}(1 - F_1(p))^{n-1}}$$

and therefore

$$\int_{\underline{p}}^{r_1} p dF_1(p) = \int_0^1 \frac{r_1}{1 + \frac{\gamma n}{1-\gamma}(1 - y)^{n-1}} dy.$$

This can be rewritten as

$$\int_{\underline{p}}^{r_1} p dF_1(p) = \int_0^1 \frac{r_1}{1 + \frac{\gamma n}{1-\gamma} y^{n-1}} dy.$$

The definition of r_1 then finally gives

$$r_1 = \frac{c_t + c_e}{1 - \int_0^1 \frac{1}{1+\frac{\gamma n}{1-\gamma} y^{n-1}} dy}.$$

Now look at the partial search case with $r_1 = \theta$. A standard undercutting argument shows that the price distribution has no atoms. For $p \leq r_1$ profits are given by

$$\pi_1(p) = p\gamma(1 - F_1(p))^{n-1} + p(1-\gamma)\frac{\mu_1}{n}$$

It must be that $\overline{p} = r_1 = \theta$. If $\overline{p} < \theta$ deviation to a price θ would be profitable. This gives that in equilibrium profits equal $\pi_1(\theta) = \theta(1-\gamma)\frac{\mu_1}{n}$ and equating this with $\pi_1(p)$ gives

$$F_1(p) = 1 - (\frac{(1-\gamma)\mu_1}{\gamma n} \frac{r_1 - p}{p})^{\frac{1}{n-1}}.$$

Finally, the minimum price is the price $\underline{p}$ such that $F_1(\underline{p}) = 0$. This gives $\underline{p} = r_1 \frac{(1-\gamma)\mu_1}{\gamma n + (1-\gamma)\mu_1}$. Note that deviation to a price below $\underline{p}$ is not profitable.

The condition $r_1 = \theta$ defines μ_1 and the condition $0 < \mu_1 < 1$ defines the parameter region for which the equilibrium holds. The definition of r_1 and $r_1 = \theta$ gives $\theta - \int_{\underline{p}}^{\theta} p dF_1(p) = c_t + c_e$. Using the same method as before, this can be rewritten as

$$\theta(1 - \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1-\gamma)\mu_1} y^{n-1}} dy) = c_t + c_e$$

or,

$$h(\mu_1) = \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1-\gamma)\mu_1} y^{n-1}} dy = \frac{\theta - c_t - c_e}{\theta},$$

defining μ_1. Note that $h(\mu_1)$ is increasing in μ_1, with $h(0) = 0$ and $h(1) = \int_0^1 \frac{1}{1+\frac{\gamma n}{(1-\gamma)} y^{n-1}} dy$. The condition $0 < \mu_1 < 1$ therefore gives

$$0 < \frac{\theta - c_t - c_e}{\theta} < \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1-\gamma)} y^{n-1}} dy.$$

Recall that by assumption $\theta - c_t - c_e > 0$ and so the only relevant part is $\frac{\theta - c_t - c_e}{\theta} < \int_0^1 \frac{1}{1+\frac{\gamma n}{(1-\gamma)} y^{n-1}} dy$, or $c_t + c_e > \theta(1 - \int_0^1 \frac{1}{1+\frac{\gamma n}{(1-\gamma)} y^{n-1}} dy)$.

4.A.2 Proof of Propositions 4.3.3 and 4.3.4

Once a non-shopper has searched one shop he is in the situation described by Stahl (1989) with search costs c_e and so he will stop searching as soon as he finds a price at or below r_n, with r_n defined by

$$\int_{\underline{p}}^{r_n} (r_n - p) dF_n(p) = c_e.$$

Stahl (1989) also shows that the maximum price is at or below r_n. This implies that non-shoppers search at most once. The expected utility of the first search therefore is

$$-c_t - c_e + \int_{\underline{p}}^{r_n} (\theta - p) dF_n(p).$$

For $r_n \leq \theta$ this can be rewritten as

$$-c_t - c_e + \theta - r_n + \int_{\underline{p}}^{r_n} (r_n - p) dF_n(p),$$

which equals $\theta - r_n - c_t$. Therefore, for $r_n < \theta - c_t$ all non-shoppers will search and for $r_n = \theta - c_t$ non-shoppers are indifferent between searching and staying home. For $\theta - c_t < r_n \leq \theta$ searching clearly is not profitable and for $r_n > \theta$ $\int_{\underline{p}}^{r_n} (\theta - p) dF_n(p) < c_e$ and so the utility of searching is strictly negative as well.

In a full search equilibrium $r_n < \theta - c_t$ and the profits for $p \leq r_n$ are given by

$$\pi_n(p) = p\frac{1-\gamma}{n} + p\gamma(1 - F_n(p))^{n-1}).$$

This expression shows that $\overline{p} = r_n$ since else deviation to r_n would be profitable. Equilibrium profits are therefore $\pi_n(r_n) = r_n \frac{1-\gamma}{n}$ and equating $\pi_n(p)$ and $\pi_n(r_n)$ gives

$$F_n(p) = 1 - (\frac{(r_n - p)(1-\gamma)}{n\gamma p})^{\frac{1}{n-1}}$$

with $\underline{p}_n = r_n \frac{1-\gamma}{\gamma n + (1-\gamma)}$. It is clear that deviation to a price below $\underline{p}_n$ is not profitable. The same argument as in the proof of Propositions 4.3.1 and 4.3.2 finally shows that

$$r_n = \frac{c_e}{1 - \int_0^1 \frac{1}{1 + \frac{\gamma}{1-\gamma} n y^{n-1}} dy}.$$

In a partial search equilibrium $r_n = \theta - c_t$ and a fraction μ_n of the non-shoppers searches. For $p \leq r_n$ the profits are

$$\pi_n(p) = p\frac{\mu_n(1-\gamma)}{n} + p\gamma(1 - F_n(p))^{n-1}).$$

This expression shows that $\overline{p}_n = r_n$ and equilibrium profits are $\pi_n(r_n) = r_n \frac{\mu_n(1-\gamma)}{n}$. Equating $\pi_n(p)$ with $\pi_n(r_n)$ gives

$$F_n(p) = 1 - (\frac{(r_n - p)(1-\gamma)\mu_n}{n\gamma p})^{\frac{1}{n-1}}$$

with $\underline{p}_n = r_n \frac{\mu_n(1-\gamma)}{\gamma n + \mu_n(1-\gamma)}$. It is clear that deviating to a price below $\underline{p}_n$ is not profitable. The fraction of searching non-shoppers, μ_n, is defined by the condition $r_n = \theta - c_t$. The same procedure as in the proof of Propositions 4.3.1 and 4.3.2 gives

$$h(\mu_n) \equiv \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1-\gamma)\mu_n} y^{n-1}} dy = \frac{\theta - c_t - c_e}{\theta - c_t}.$$

Finally, because $h(\mu_n)$ is increasing in μ_n the condition $0 < \mu_n < 1$ gives

$$0 < \frac{\theta - c_t - c_e}{\theta - c_t} < \int_0^1 \frac{1}{1 + \frac{\gamma}{1-\gamma} n y^{n-1}} dy$$

where the first part, $\frac{\theta-c_t-c_e}{\theta-c_t}>0$, is automatically satisfied because of the assumption $\theta-c_t-c_e>0$.

4.A.3 Proof of Proposition 4.3.5

For ease of notation, let q denote $\int_0^1 \frac{1}{1+\frac{\gamma}{1-\gamma}ny^{n-1}}dy$.

For $c_t+c_e<\theta(1-q)$ the full search equilibrium holds both when all shops are located together and when all shops are isolated. Therefore $\pi_1=\frac{1-\gamma}{n}\frac{c_t+c_e}{1-q}>\frac{1-\gamma}{n}\frac{c_e}{1-q}=\pi_n$.

Next, I show that for $c_t+c_e>\theta\frac{1-q}{1-\beta q}$ $\pi_1<\pi_n$. In this case the partial search equilibrium holds both when all shops are located together and when all shops are isolated. Define the function $g(\mu)\equiv\int_0^1\frac{1}{1+\frac{\gamma}{1-\gamma}\frac{n}{\mu}y^{n-1}}dy$ and note that μ_1 is defined by $g(\mu_1)=\frac{\theta-c_t-c_e}{\theta}$ and that μ_n is defined by $g(\mu_n)=\frac{\theta-c_t-c_e}{\theta-c_t}$. Expected profits are given by $\pi_1=\theta\mu_1\frac{1-\gamma}{n}$ and $\pi_n=(\theta-c_t)\mu_n\frac{1-\gamma}{n}$ and therefore $\pi_1<\pi_n$ holds if and only if $\mu_n>\frac{\theta}{\theta-c_t}\mu_1$. Using that $g(\mu)$ is strictly increasing in μ this can be rewritten as $g(\mu_n)>g(\frac{\theta}{\theta-c_t}\mu_1)$ or

$$\frac{\theta-c_t-c_e}{\theta-c_t}>\int_0^1\frac{1}{1+\frac{\gamma}{1-\gamma}\frac{n}{\mu_1}\frac{\theta-c_t}{\theta}y^{n-1}}dy.$$

Since $\int_0^1\frac{1}{1+\frac{\gamma}{1-\gamma}\frac{n}{\mu_1}\frac{\theta-c_t}{\theta}y^{n-1}}dy=\frac{\theta}{\theta-c_t}\int_0^1\frac{1}{\frac{\theta}{\theta-c_t}+\frac{\gamma}{1-\gamma}\frac{n}{\mu_1}y^{n-1}}dy$, $\pi_1<\pi_n$ if and only if

$$\int_0^1\frac{1}{\frac{\theta}{\theta-c_t}+\frac{\gamma}{1-\gamma}\frac{n}{\mu_1}y^{n-1}}dy<\frac{\theta-c_t-c_e}{\theta}.$$

The definition of μ_1 gives $\frac{\theta-c_t-c_e}{\theta}=\int_0^1\frac{1}{1+\frac{\gamma}{1-\gamma}\frac{n}{\mu_1}y^{n-1}}dy$ and so $\pi_1<\pi_n$ if and only if

$$\int_0^1\frac{1}{\frac{\theta}{\theta-c_t}+\frac{\gamma}{1-\gamma}\frac{n}{\mu_1}y^{n-1}}dy<\int_0^1\frac{1}{1+\frac{\gamma}{1-\gamma}\frac{n}{\mu_1}y^{n-1}}dy$$

and this holds always since $\frac{\theta}{\theta-c_t}>1$.

For $\theta(1-q)<c_t+c_e<\theta\frac{1-q}{1-\beta q}$ a full search equilibrium holds when all shops are located together. This implies that $\pi_n=\frac{(1-\beta)(c_t+c_e)}{1-q}\frac{1-\gamma}{n}$ is linearly increasing in c_t+c_e. When all shops are located separately a partial search equilibrium holds with $\pi_1=\theta\mu_1\frac{1-\gamma}{n}$ and $g(\mu_1)=\frac{\theta-c_t-c_e}{\theta}$. Since $g(\mu)$ is strictly increasing in μ and $\frac{\theta-c_t-c_e}{\theta}$ decreases in c_t+c_e, μ_1 decreases in c_t+c_e and therefore π_1 decreases in c_t+c_e. Since for $c_t+c_e<\theta(1-q)$ $\pi_1>\pi_n$ and for $c_t+c_e>\theta\frac{1-q}{1-\beta q}$ $\pi_1<\pi_n$ this implies that there exists a unique value c with $\theta(1-q)<c<\theta\frac{1-q}{1-\beta q}$ where $\pi_1=\pi_n$, with $\pi_1>\pi_n$ for $c_t+c_e<c$ and $\pi_1<\pi_n$ for $c_t+c_e>c$.

4.B Optimal consumer behavior

A first useful result is the following.

Proposition 4.B.1 *In equilibrium $\pi_k^m>0$ and $\pi_k^i>0$. Consequently, $\overline{p}_k^m\leq\theta$ and $\overline{p}_k^i\leq\theta$.*

Proof

Suppose to the contrary that $\pi_k^m = 0$ and $\pi_k^i > 0$. This implies that $\underline{p}_k^i > 0$ since for $\underline{p}_k^i = 0$ $\pi_k^i = \pi_k^i(\underline{p}_k^i) = 0$. If some of the non-shoppers visit the shopping mall in their first search then for a shop in the mall setting a price c_e will prevent the non-shoppers from continuing search, leading to positive profits, a contradiction. If none of the non-shoppers search in the shopping mall shops in the mall compete for the shoppers, leading to a maximum price of 0. But then non-shoppers would prefer to search in the shopping mall, a contradiction.

The case $\pi_k^m > 0$ and $\pi_k^i = 0$ is the same as above, reversing the roles of the shops inside and outside the mall.

This leaves the case $\pi_k^m = 0$ and $\pi_k^i = 0$. If non-shoppers would search then the shops attracting some non-shoppers could set a price c_e and make a strictly positive profit. If non-shoppers do not search the firms compete for the shoppers and all set a price 0. But in that case non-shoppers would find it optimal to search, a contradiction.

Since a price above θ would not lead to any sales, the profits of setting such a price are 0, which contradicts the fact that in equilibrium profits are strictly positive.

□

Using Proposition 4.B.1 and assuming that $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$ the behavior of non-shoppers can be derived. This behavior depends on the prices found in previous searches and on whether the consumer currently is in a shop inside the shopping mall or whether he currently is in a shop outside the shopping mall. First consider the case where the consumer currently is in the shopping mall and let $\tilde{p}_k^m$ denote the lowest price found on current and previous searches in the shopping mall. Let $\tilde{p}_k^i$ denote the lowest price found on previous searches outside the shopping mall, with $\tilde{p}_k^i = \infty$ when on previous searches no shops outside the mall have been visited. Note that if the non-shopper decides to stop searching and $\tilde{p}_k^m \leq \tilde{p}_k^i + c_t$ then he will buy from the cheapest shop in the shopping mall, at price $\tilde{p}_k^m$ (note that $\tilde{p}_k^m \leq \theta$ and therefore buying at $\tilde{p}_k^m$ is a better strategy than not buying at all). If the non-shopper decides to stop searching and $\tilde{p}_k^m > \tilde{p}_k^i + c_t$ then he will buy from the cheapest shop outside the shopping mall, incurring return costs c_t and buying at price $\tilde{p}_k^i$. In the proposition that follows I will use the term 'buy from the cheapest option' to denote this behavior.

Proposition 4.B.2 *Consider a non-shopper who expects $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$ and who currently is in a shop in the shopping mall.*

When $r_k^m \leq r_k^i$ ($r_k^m > r_k^i$) his optimal behavior is as follows. When $\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) \leq r_k^m$ ($\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) \leq r_k^i$) stop search and buy from the cheapest option. When $\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) > r_k^m$ ($\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) > r_k^i$) search further in (outside) the shopping mall, if possible. If there are no shops left to search in (outside) the shopping mall and $\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) \leq r_k^i$ ($\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) \leq r_k^m$) buy from the cheapest option. If there are no shops left to search in (outside) the shopping mall and $\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) > r_k^i$ ($\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) > r_k^m$) search further outside (in) the shopping mall, if possible. If there also are no shops left to search outside (in) the shopping mall buy from the cheapest option.

Now consider the case where the consumer currently is outside the shopping mall and let $\breve{p}_k^i$ denote the price found in the current shop. Let $\tilde{p}_k^i$ denote the lowest price found on previous searches outside the shopping mall, with $\tilde{p}_k^i = \infty$ when on previous searches no shops outside the mall have been visited. Let $\tilde{p}_k^m$ denote the lowest price found on previous searches inside the shopping mall, with $\tilde{p}_k^m = \infty$ when on previous searches no shops inside the mall have been visited. Note that if the non-shopper decides to stop searching and to buy and $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) = \breve{p}_k^i$ then he will buy from the shop he currently is, at price $\breve{p}_k^i$. If $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) = \tilde{p}_k^i + c_t$ then he will buy from the cheapest shop outside the shopping mall visited before, incurring return costs c_t and buying at price $\tilde{p}_k^i$. If $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) = \tilde{p}_k^m + c_t$ then he will buy from the cheapest shop inside the shopping mall, incurring return costs c_t and buying at price $\tilde{p}_k^m$. In the proposition that follows I will use the term 'buy from the cheapest option' to denote this behavior.

Proposition 4.B.3 *Consider a non-shopper who expects $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$ and who currently is in a shop outside the shopping mall.*

When $r_k^i \leq r_k^m + c_t$ ($r_k^i > r_k^m + c_t$) his optimal behavior is as follows. When $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) \leq r_k^i$ ($\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) \leq r_k^m + c_t$) stop search and buy from the cheapest option. When $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) > r_k^i$ ($\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) > r_k^m + c_t$) search further outside (in) the shopping mall, if possible. If there are no shops left to search outside (in) the shopping mall and $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) \leq r_k^m + c_t$ ($\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) \leq r_k^i$) buy from the cheapest option. If there are no shops left to search outside (in) the shopping mall and $\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) > r_k^m + c_t$ ($\min(\breve{p}_k^i, \tilde{p}_k^i + c_t, \tilde{p}_k^m + c_t) > r_k^i$) search further in (outside) the shopping mall, if possible. If there also are no shops left to search in (outside) the shopping mall buy from the cheapest option.

Proof

A complete proof of Propositions 4.B.2 and 4.B.3 is available on request. Here I only give a short sketch of the proof.

Let h denote the number of shops that have not yet been searched, let h^m denote the number of shops in the mall that have not yet been searched and let h^i denote the number of isolated shops that have not yet been searched, with $h = h^m + h^i$. The proof uses several induction arguments. First, it is easy to see that both propositions hold when $h = 0$. A second step is to prove that both propositions hold when $h^m = 1$ and $h^i = 0$. Using a standard induction argument it can then be shown that both propositions also hold for $h^i = 0$ and $h^m > 1$. A third step is to prove that both propositions also hold when $h^m = 0$ and $h^i = 1$. Again using a standard induction argument it can then be shown that both propositions also hold for $h^m = 0$ and $h^i > 1$.

These three steps together prove Propositions 4.B.2 and 4.B.3 for some corner cases. These cases together form the basis of one final induction step. This final step shows the following. If the propositions hold for $h = x - 2$ and for $h = x - 1$ then the propositions also hold for $h = x$, with $h^m \geq 1$ and $h^i \geq 1$. Since the propositions hold for $h = 0$ and $h = 1$, they will also hold for $h > 1$, $h^m \geq 1$ and $h^i \geq 1$. Note that steps two and three have already shown that the propositions hold for $h^m = 0$, $h^i > 1$ and for $h^i = 0$, $h^m > 1$.

□

Propositions 4.B.2 and 4.B.3 specify the optimal behavior of non-shoppers when they have searched at least one shop under the conditions $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$. Proposition 4.B.4 specifies the optimal behavior of non-shoppers when they have not yet searched any shop.

Proposition 4.B.4 *Let shops price according to $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k^i \leq r_k^i$.*

If $r_k^i < r_k^m + c_t$ ($r_k^i > r_k^m + c_t$) non-shoppers prefer to first search an isolated shop (shop in the mall) above first searching a shop in the mall (an isolated shop). If $r_k^i < \theta$ ($r_k^m + c_t < \theta$) all *non-shoppers will search, if* $r_k^i = \theta$ $(r_k^m + c_t = \theta)$ *non-shoppers are indifferent between staying at home and searching an isolated shop (shop in the mall) and if* $r_k^i > \theta$ $(r_k^m + c_t > \theta)$ *all non-shoppers prefer to stay at home.*

If $r_k^i = r_k^m + c_t$ *non-shoppers are indifferent between searching in an isolated shop or in a shop in the mall. When* $r_k^i < \theta$ *all non-shoppers will search, when* $r_k^i = \theta$ *non-shoppers are indifferent between searching and staying at home and when* $r_k^i > \theta$ *all non-shoppers prefer to stay at home.*

Proof

First look at the case $r_k^i = r_k^m + c_t$. If a non-shopper would start his search in a shop in the mall he expects to find a price at or below r_k^m and as Proposition 4.B.2 shows the non-shopper thus expects to stop searching after the first search. Expected utility of searching in the mall is

$$U(\text{mall}) = -c_t - c_e + \int_{\underline{p}_k^m}^{r_k^m} (\theta - p) dF_k^m(p),$$

which can be rewritten as

$$U(\text{mall}) = -c_t - c_e + \theta - r_k^m + \int_{\underline{p}_k^m}^{r_k^m} (r_k^m - p) dF_k^m(p) = \theta - r_k^m - c_t.$$

A non-shopper who starts his search in an isolated shop expects to find a price at or below r_k^i and as Proposition 4.B.3 shows he expects to stop searching after the first search. Expected utility is

$$U(\text{isolated}) = -c_t - c_e + \int_{\underline{p}_k^i}^{r_k^i} (\theta - p) dF_k^i(p),$$

which can be rewritten as

$$U(\text{isolated}) = -c_t - c_e + \theta - r_k^i + \int_{\underline{p}_k^i}^{r_k^i} (r_k^i - p) dF_k^i(p) = \theta - r_k^i.$$

Since $r_k^i = r_k^m + c_t$, $U(\text{mall}) = U(\text{isolated})$ and non-shoppers are indifferent between searching in an isolated shop or in a shop in the mall. When $r_k^i < \theta$ $U(\text{isolated}) > 0$ and all non-shoppers will search. When $r_k^i = \theta$ $U(\text{isolated}) = 0$ and non-shoppers are indifferent between searching and staying at home. When $r_k^i > \theta$ $U(\text{isolated}) < 0$ and all non-shoppers prefer to stay at home.

The proof for the cases $r_k^i > r_k^m + c_t$ and $r_k^i < r_k^m + c_t$ follows the same arguments, but is mathematically slightly more complicated since non-shoppers sometimes expect to search twice instead of once. Details are available on request.

□

4.C Proofs of Section 4.4

4.C.1 Proof of Proposition 4.4.1

Recall that in the model $c_t + c_e < \theta$ and therefore at least some non-shoppers will search. First assume that $r_k^i < r_k^m + c_t$. Proposition 4.B.4 shows that under this assumption all searching non-shoppers will first search in an isolated shop. Using Proposition 4.B.3 and using that $\overline{p}_k^i \leq r_k^i$ it is easy to see that the searching non-shoppers will stop searching after their first search and will buy from the isolated shop they visited. Consequently, shops in the mall will compete for the shoppers and mall prices will be zero. The definition of r_k^m in that case gives $r_k^m = c_e$, a contradiction of the initial assumption that $r_k^i < r_k^m + c_t$. For $r_k^i > r_k^m + c_t$ all searching non-shoppers will first search in the mall, and because $\overline{p}_k^m \leq r_k^m$ they will stop searching after their first search and buy from the mall shop they visited. But in that case, using that there are $n - k \geq 2$ isolated shops, the isolated shops would compete for the shoppers, isolated prices would be zero and $r_k^i = c_t + c_e$, a contradiction of $r_k^i > r_k^m + c_t$. The only possibility is that in equilibrium $r_k^i = r_k^m + c_t$ and as Proposition 4.B.4 shows non-shoppers are indifferent between first searching in the mall or first searching in an isolated shop.

4.C.2 Proof of Proposition 4.4.3

First note that

$$\pi_k^i = \pi_k^i(\underline{p}_k^i) = \gamma \underline{p}_k^i (1 - F_k^m(\underline{p}_k^i))^k + (1-\gamma)\frac{1-x_k}{n-k}\underline{p}_k^i \mu_k$$

and

$$\pi_k^m \geq \pi_k^m(\underline{p}_k^i) = \gamma \underline{p}_k^i (1 - F_k^m(\underline{p}_k^i))^{k-1} + (1-\gamma)\frac{x_k}{k}\underline{p}_k^i \mu_k.$$

Suppose contrary to the proposition that $\pi_k^m = \pi_k^i$, implying that $r_k^m \frac{x_k}{k} = r_k^i \frac{1-x_k}{n-k}$, or, using Proposition 4.4.1, $\frac{x_k}{k} > \frac{1-x_k}{n-k}$. This gives $\pi_k^i = \pi_k^i(\underline{p}_k^i) < \pi_k^m(\underline{p}_k^i) \leq \pi_k^m$, a contradiction to the assumption $\pi_k^m = \pi_k^i$.

Now suppose contrary to the proposition that $\pi_k^m < \pi_k^i$. Note that $\pi_k^m < \pi_k^i$ implies $\pi_k^i(\underline{p}_k^i) > \pi_k^m(\underline{p}_k^i)$ and therefore $\frac{1-x_k}{n-k} > \frac{x_k}{k}$. Moreover, $\pi_k^m = \pi_k^m(r_k^m) \geq \pi_k^m(\underline{p}_k^i)$ gives $(r_k^m - \underline{p}_k^i)(1-\gamma)\frac{x_k}{k}\mu_k \geq \gamma \underline{p}_k^i(1 - F_k^m(\underline{p}_k^i))^{k-1}$. Combining these two inequalities gives

$$\begin{aligned}
\pi_k^i(\underline{p}_k^i) &\leq \gamma \underline{p}_k^i (1 - F_k^m(\underline{p}_k^i))^{k-1} + (1-\gamma)\frac{1-x_k}{n-k}\underline{p}_k^i \mu_k \\
&\leq (r_k^m - \underline{p}_k^i)(1-\gamma)\frac{x_k}{k}\mu_k + (1-\gamma)\frac{1-x_k}{n-k}\underline{p}_k^i \mu_k \\
&< r_k^m (1-\gamma)\frac{1-x_k}{n-k}\mu_k \\
&< r_k^i (1-\gamma)\frac{1-x_k}{n-k}\mu_k = \pi_k^i(r_k^i),
\end{aligned}$$

a contradiction.

Since both $\pi_k^m = \pi_k^i$ and $\pi_k^m < \pi_k^i$ are not feasible, it should be that $\pi_k^m > \pi_k^i$, or $r_k^m \frac{x_k}{k} > r_k^i \frac{1-x_k}{n-k}$. Proposition 4.4.1 then gives that $\frac{x_k}{k} > \frac{1-x_k}{n-k}$.

4.C.3 Proof of Proposition 4.4.4

Suppose to the contrary that $\underline{p}_k^m < \underline{p}_k^i$. I will show that in that case isolated shops have an incentive to deviate to a price $\underline{p}_k^m$.

For $p \leq \underline{p}_k^i$

$$\pi_k^m(p) = \gamma p(1 - F_k^m(p))^{k-1} + (1-\gamma)\frac{x_k}{k}p.$$

Proposition 4.4.2 shows that for $p \leq \underline{p}_k^i$ there will not be any atoms in $F_k^m(p)$. Moreover, for $p \leq \underline{p}_k^i$ a gap in $F_k^m(p)$ is not possible. To prove this, suppose $F_k^m(p)$ would be constant for all prices in (p_1^*, p_2^*), with $p_2^* \leq \underline{p}_k^i$. Then $\pi_k^m(p_1^*) < \pi_k^m(p_2^*)$, a contradiction. If $F_k^m(p)$ would be constant for all prices in (p_1^*, p_2^*), with $p_2^* > \underline{p}_k^i$ then $\pi_k^m(p_1^*) < \pi_k^m(\underline{p}_k^i)$, again a contradiction.

Given that there are no atoms or gaps for $p \leq \underline{p}_k^i$, in equilibrium $\pi_k^m(\underline{p}_k^m) = \pi_k^m(\underline{p}_k^i)$. This gives

$$1 - F_k^m(\underline{p}_k^i) = \left[\frac{\gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}(\underline{p}_k^m - \underline{p}_k^i)}{\gamma \underline{p}_k^i}\right]^{\frac{1}{k-1}}.$$

Note that $\pi_k^i(\underline{p}_k^i) = \gamma \underline{p}_k^i (1 - F_k^m(\underline{p}_k^i))^k + (1-\gamma)\frac{1-x_k}{n-k}\underline{p}_k^i$. Plugging in the expression given above gives

$$\pi_k^i(\underline{p}_k^i) = (1 - F_k^m(\underline{p}_k^i))(\gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}(\underline{p}_k^m - \underline{p}_k^i)) + (1-\gamma)\frac{1-x_k}{n-k}\underline{p}_k^i.$$

Note that $\pi_k^i(\underline{p}_k^m) = \gamma \underline{p}_k^m + (1-\gamma)\frac{1-x_k}{n-k}\underline{p}_k^m$ and that deviation to $\underline{p}_k^m$ is profitable when $\pi_k^i(\underline{p}_k^m) > \pi_k^i(\underline{p}_k^i)$. This can be rewritten as

$$\gamma \underline{p}_k^m + (1-\gamma)\frac{1-x_k}{n-k}(\underline{p}_k^m - \underline{p}_k^i) > (1 - F_k^m(\underline{p}_k^i))(\gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}(\underline{p}_k^m - \underline{p}_k^i)).$$

I will show that this inequality always holds. First note that $1 - F_k^m(\underline{p}_k^i) \geq 0$ implies that $\gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}(\underline{p}_k^m - \underline{p}_k^i) \geq 0$.

Now if $0 < 1 - F_k^m(\underline{p}_k^i) \leq 1$, $(1 - F_k^m(\underline{p}_k^i))(\gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}(\underline{p}_k^m - \underline{p}_k^i)) \leq \gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}(\underline{p}_k^m - \underline{p}_k^i)$. Since $\frac{x_k}{k} > \frac{1-x_k}{n-k}$ (Proposition 4.4.3) and $\underline{p}_k^m - \underline{p}_k^i < 0$, $\gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}(\underline{p}_k^m - \underline{p}_k^i) < \gamma \underline{p}_k^m + (1-\gamma)\frac{1-x_k}{n-k}(\underline{p}_k^m - \underline{p}_k^i)$. Combining gives $(1 - F_k^m(\underline{p}_k^i))(\gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}(\underline{p}_k^m - \underline{p}_k^i)) < \gamma \underline{p}_k^m + (1-\gamma)\frac{1-x_k}{n-k}(\underline{p}_k^m - \underline{p}_k^i)$t.

If $1 - F_k^m(\underline{p}_k^i) = 0$, the inequality reduces to $\gamma \underline{p}_k^m + (1-\gamma)\frac{1-x_k}{n-k}(\underline{p}_k^m - \underline{p}_k^i) > 0$. Since $\gamma \underline{p}_k^m + (1-\gamma)\frac{1-x_k}{n-k}(\underline{p}_k^m - \underline{p}_k^i) > \gamma \underline{p}_k^m + (1-\gamma)\frac{x_k}{k}(\underline{p}_k^m - \underline{p}_k^i) \geq 0$ this inequality always holds.

4.C.4 Details on the derivation of the equilibrium of Section 4.4.2

In the full search equilibrium r_k^m is defined by

$$\int_{\underline{p}_k^m}^{r_k^m} (r_k^m - p)dF_k^m(p) = c_e.$$

Rewriting gives

$$r_k^m - \int_{\underline{p}_k^m}^{r_k^m} pdF_k^m(p) = c_e.$$

Note that

$$p = \frac{r_k^m}{1 + \frac{\gamma}{1-\gamma}(z_k)^{n-k}\frac{k}{x_k}(1 - F_k^m(p))^{k-1}}.$$

Therefore,

$$\int_{\underline{p}_k^m}^{r_k^m} pdF_k^m(p) = \int_0^1 \frac{r_k^m}{1 + \frac{\gamma}{1-\gamma}(z_k)^{n-k}\frac{k}{x_k}y^{k-1}}dy$$

and so the definition of r_k^m gives

$$r_k^m - \int_0^1 \frac{r_k^m}{1 + \frac{\gamma}{1-\gamma}(z_k)^{n-k}\frac{k}{x_k}y^{k-1}}dy = c_e,$$

or

$$r_k^m(\int_0^1 1 - \frac{1}{1 + \frac{\gamma}{1-\gamma}(z_k)^{n-k}\frac{k}{x_k}y^{k-1}}dy) = c_e.$$

In the full search equilibrium r_k^i is defined by

$$\int_{\underline{p}_k^i}^{r_k^i} (r_k^i - p)dF_k^i(p) = c_e + c_t.$$

Since $F_k^i(p)$ is constant for $r_k^m \leq p < r_k^i$ and since for $p = r_k^i$ $r_k^i - p = 0$ the definition can be rewritten as

$$\int_{\underline{p}_k^i}^{r_k^m} (r_k^i - p)dF_k^i(p) = c_e + c_t.$$

Rewriting gives

$$r_k^i F_k^i(r_k^m) - \int_{\underline{p}_k^i}^{r_k^m} pdF_k^i(p) = c_e + c_t,$$

or, using that $F_k^i(r_k^m) = 1 - z_k$ by definition of z_k,

$$r_k^i(1 - z_k) - \int_{\underline{p}_k^i}^{r_k^m} pdF_k^i(p) = c_e + c_t.$$

Note that

$$p = \frac{r_k^i}{1 + \frac{\gamma}{1-\gamma}\frac{n-k}{1-x_k}(1 - F_k^i(p))^{n-k-1}}.$$

Therefore,

$$\int_{\underline{p}_k^i}^{r_k^m} p dF_k^i(p) = \int_0^{1-z_k} \frac{r_k^i}{1 + \frac{\gamma}{1-\gamma}\frac{n-k}{1-x_k}(1-y)^{n-k-1}} dy.$$

A change of variables gives

$$\int_{\underline{p}_k^i}^{r_k^m} p dF_k^i(p) = \int_{z_k}^{1} \frac{r_k^i}{1 + \frac{\gamma}{1-\gamma}\frac{n-k}{1-x_k} y^{n-k-1}} dy.$$

The definition of r_k^m now gives

$$r_k^i(1-z_k) - \int_{z_k}^{1} \frac{r_k^i}{1 + \frac{\gamma}{1-\gamma}\frac{n-k}{1-x_k} y^{n-k-1}} dy = c_e + c_t,$$

or

$$r_k^i(\int_{z_k}^{1} 1 - \frac{1}{1 + \frac{\gamma}{1-\gamma}\frac{n-k}{1-x_k} y^{n-k-1}} dy) = c_e.$$

Part II

Director ties

Chapter 5

Interlocking boards and firm performance: evidence from a new panel database

5.1 Introduction

A director can hold several directorships in different firms. Such a director constitutes a link between the firms he serves. Firms that are linked in this way are said to be interlocked. There is much research on interlocks, ranging from a description of what the network of interlocked firms looks like to studies on the influence of interlocks on firm strategy and performance. We address this last topic by analyzing a new large and unique panel data set concerning firms in The Netherlands.

There are several views on the origin and effect of interlocks, see Mizruchi (1996) for an extensive review. To mention a few views, Dooley (1969) and Mizruchi and Stearns (1994) have argued that interlocks are a way for firms to coopt and/or monitor each other. For example, an interlock between a financial institution and a nonfinancial firm enables the financial institution to monitor the nonfinancial firm and it reduces the risk for the financial firm to provide capital to the nonfinancial firm. In this view, interlocks would have a positive effect on firm performance. Another favorable view on interlocks states that interlocks provide firms with information on business practice (see e.g. Davis, 1991). For example, a director who encountered a new practice in one of the firms where he serves could bring this practice to the other firms where he serves as well.

Apart from favorable views on interlocks, there also are some hypotheses on possible negative effects of interlocks. One of the most well-known views is the busyness hypothesis of Ferris et al. (2003). This hypothesis states that multiple directorships place an excessive burden on directors, resulting in bad governance and diminishing firm performance (see also Fich and Shivdasani, 2006). Another explanation of why having many interlocks could diminish firm performance is that interlocks reflect upper-class cohesion (Useem, 1984). If this is indeed true, the board members of a firm with many interlocks per director form a cohesive upper-class group. It has been shown (see Janis, 1982, and Mullen et al., 1994) that cohesive groups perform worse in decision making, as they strive for unanimity and often suffer from a reduction in independent critical thinking. Moreover, the members of a cohesive upper class mostly have the same background and hence such a board is less diverse, while it is precisely such diversity that has been shown to improve firm performance (see Carter et al., 2003).

Given these different views, it is not surprising that much empirical research on the effect of interlocks on performance has been carried out. Results of these studies are mixed, see for example Bunting (1976), Pennings (1980), Burt (1983), Fligstein and Brantley (1992), and Phan et al. (2003). Most research is based on US data. We are aware of only two studies which concern The Netherlands, and these are Meeusen and Cuyvers (1985) and Van Ees et al. (2003). The first study documents a positive relation between interlocks within financial firms and their performance. The second study mentions a negative effect of the percentage share of outsiders on firm performance, where outsiders are defined as directors who have at least two directorships.

In this chapter we again take up the issue of interlocks and performance by presenting empirical results based on a newly created large panel database for The Netherlands. In contrast to previous studies, the database we use constitutes a panel for many years, instead of the commonly used cross section. Hence, we can also examine the dynamic effects of interlocks, which, as we will document, are quite prominent. Besides this, a panel allows us to correct for both firm and time effects, which can be important in explaining firm performance.

The outline of the chapter is as follows. First, in Section 5.2 we will put forward three main hypotheses on the effect of interlocks on firm performance in The Netherlands. Next, in Section 5.3, we give a description of the data. Then in Section 5.4 we will describe the method we will use to test the hypotheses. The hypothesis testing results are presented in Section 5.5 and in Section 5.6 we summarize our findings.

5.2 Hypotheses

In the previous section we mentioned some views on the relation between interlocks and firm performance. In this section we will argue which views are most relevant for The Netherlands and based on this we will outline our hypotheses.

Practically all of the views that predict a positive effect of interlocks are based on an information transmission or a monitoring argument. However, as Haunschild and Beckman (1998) argue, interlocks are not the only way of information transmission. They show that alternative information sources, like CEO membership in the Business Roundtable, reduce the impact of interlocks. In The Netherlands we expect these alternative information sources to play an important role. The number of large firms in The Netherlands is limited and therefore the number of people that serve on the boards of these firms is limited as well. Furthermore, the boards of Dutch firms currently predominantly consist of Dutch citizens. These directors seem to form a cohesive group (see Stokman et al., 1988, and Van Hezewijk, 1986, 1988) and meet each other regularly outside the boardroom in selective informal meetings.

We propose that not only the effect of interlocks on information transmission is limited but also that interlocks do not play an important role in monitoring other firms in

The Netherlands. The reason for this is as mentioned before. Even without interlocks the Dutch directors form a dense network of friendship ties. We acknowledge that interlocks could formalize these relations and make monitoring easier, but in the absence of interlocks, monitoring could still take place via informal networks.

The views that predict a negative effect of interlocks thus seem to have more relevance for The Netherlands. The busyness hypothesis seems to have a universal scope. Around the world articles appear in the popular press about directors attracting many directorships and being 'too busy to mind the business' and The Netherlands is no exception. In 2004 the code Tabaksblat became effective. This is a Dutch corporate governance code that, among other things, advised to limit the number of directorships to five, with a chair position counting as double. This code Tabaksblat partly was a reaction to societal concerns about directors amassing directorships. The upper class cohesion view also seems to be relevant for The Netherlands. As argued before, the Dutch directors seem to form a cohesive group, which could hamper decision making.

We are now ready to state our first hypothesis.

Hypothesis 1a (H1a) The immediate effect of interlocks between Dutch firms on firm performance is negative.

Moreover, as we use a panel data set it is possible to look at the lagged effects of interlocks. One of the important tasks of the board of directors is to make strategic decisions that have a long-term impact on firm performance. We therefore expect that there is a lagged effect of interlocks as well. For the same reasons why we expect the immediate effect of interlocks to be negative we expect the lagged effect to be negative as well. Furthermore, as the main task of the board of directors is to make long-term strategic decisions we expect that the effect of interlocks shows off more strongly when looking at future realized results instead of immediate results. We thus state the following hypothesis.

Hypothesis 1b (H1b) The lagged effect of interlocks between Dutch firms on firm performance is negative and it is stronger than the immediate effect.

Hypotheses 1a and 1b concern the general effect of interlocks. Theory leads us to suppose that this effect, if existent, will be partly caused by the busyness of directors. We would like to test this and therefore propose the following four hypotheses.

Hypothesis 2a (H2a) The fraction of busy directors in the board has a negative *immediate* effect on performance.

Hypothesis 2b (H2b) When the fraction of busy directors is taken into account the *immediate* effect of interlocks diminishes.

Hypothesis 2c (H2c) The fraction of busy directors in the board has a negative *lagged* effect on performance, and this effect is stronger than the immediate effect.

Hypothesis 2d (H2d) When the fraction of busy directors is taken into account the *lagged* effect of interlocks diminishes.

Further, we expect upper class cohesion to play a role in the effect of interlocks in The Netherlands. We will look at the fraction of upper class directors. The effect of this fraction of upper class directors however is not likely to be linear. Having one or two upper class directors in the board increases the diversity of information and expertise in the board while at the same time the risk of group think does not play a role as long as the fraction of upper class directors in the board is small. Therefore, for small fractions of upper class directors we expect their effect to be positive. On the other hand, when the majority of the board already consists of upper class directors adding an extra upper class director would not provide much more information. And, at the same time, the risk of group think increases and diversity will be lacking. So, for high fractions of upper class directors we expect their effect to be negative. We therefore hypothesize the following effects.

Hypothesis 3a (H3a) The fraction of upper class directors in the board has a quadratic *immediate* effect on performance and the quadratic effect takes the form of an inverse u.

Hypothesis 3b (H3b) When the fraction of upper class directors is taken into account the *immediate* effect of interlocks diminishes.

Hypothesis 3c (H3c) The fraction of upper class directors in the board has a quadratic *lagged* effect on performance and the quadratic effect takes the form of an inverse u. This effect is stronger than the immediate effect.

Hypothesis 3d (H3d) When the fraction of upper class directors is taken into account the *lagged* effect of interlocks diminishes.

5.3 A new panel database

In The Netherlands virtually all firms have a two-tier board structure. There is an executive board, which consists of a Chief Executive Officer (CEO), a Chief Financial officer (CFO) and of other executive directors. Additional to this executive board, there also is a supervisory board. The main task of this board is to monitor the executive board itself and to monitor the major business decisions taken by the executive board. The supervisory board largely consists of retired executive directors. Almost all of the interlocks of a firm are formed by members of the supervisory board who also serve as supervisory director for other firms. It is not uncommon for a high-profile director to have four or five supervisory directorships. As interlocks are mainly formed by the supervisory board, we focus on these directors.

5.3.1 Defining interlocks

We gathered data on supervisory boards of 101 large, listed, Dutch firms in the period 1994 to 2004, using annual reports and the REACH database. The annual reports give us, for each firm, the directors in the supervisory board by the end of July in each year. Using this information, we count the number of interlocks of a firm with other firms in the database. For example, suppose firm A has two directors, X and Y. X also sits on the supervisory boards of firms B and C, and Y also sits on the board of firm D. As such, firm A then has three interlocks. Multiple interlocks are counted as one. Suppose Y is on the boards of A, D and also B, like X. Then, firm A still has only three interlocks, as the multiple interlock with firm B is counted as one interlock.

We divide the number of interlocks by the number of directors to correct for the size of the board[1]. In our database there are a few firms with very large boards (15 or more members) as well as firms with smaller (3 or less members) boards. Clearly, large boards can have much more interlocks than small boards.

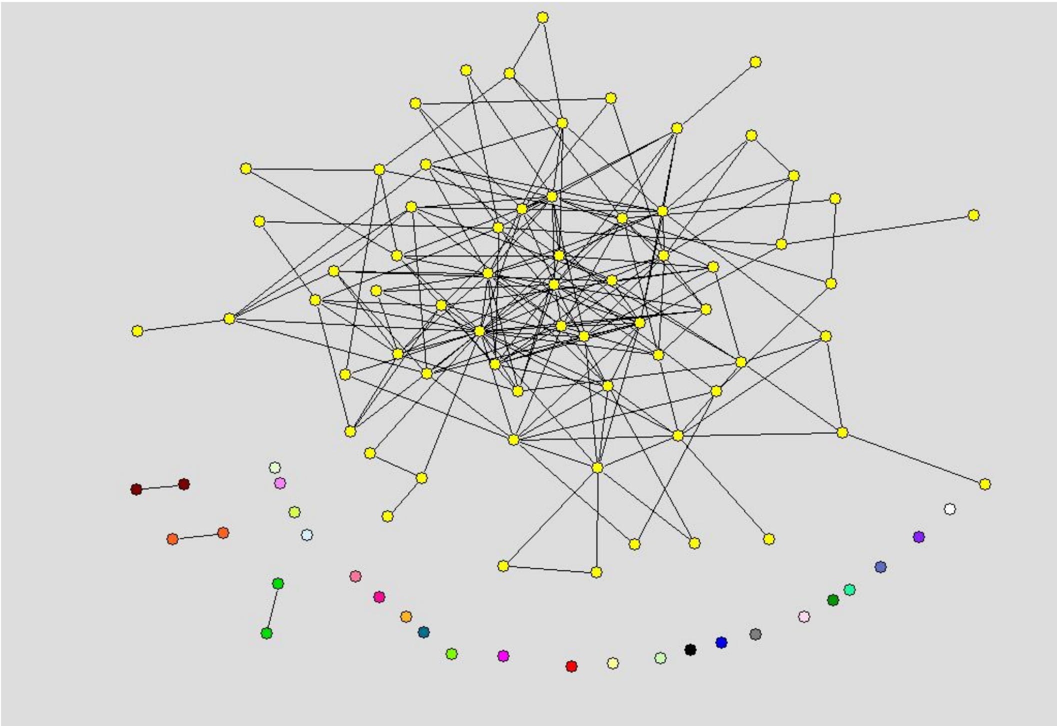
Since we expect that the effect of interlocks in The Netherlands is driven by busyness and upper class cohesion the number of interlocks corrected by board size is a more relevant measure than the mere number of interlocks. It is clear that a board with 10 interlocks and 3 directors is more busy than a board with 10 interlocks and 7 directors. In the same vein, the previous board will employ more upper class directors than the second board. We acknowledge that the number of interlocks without correction for board size is the best variable to measure the amount of information the firm gets about its environment. Therefore using the correction for board size could bias our results towards H1a. To exclude this possibility we did the same analysis as reported below using the number of interlocks instead of the number of interlocks divided by the size of the board, and the results are qualitatively the same. In what follows we will therefore use the term 'interlocks' for the number of interlocks divided by the size of the board.

Figure 5.1 shows the network of interlocks for the year 1998. For other years the network looks roughly the same (although of course various little changes occur over time). We see one giant component of firms that are linked to each other and several fringe firms that have no links or are only linked to another fringe firm.

Next to a measure of interlocks we need to have a measure of board busyness. For each director in our database we count his or her number of directorships. In line with the code Tabaksblat, we define a director as being busy when he has more than four directorships. The reason not to use five as a threshold is that the number of directors with more than five directorships is very limited. In addition, we have no information on chair positions and therefore cannot count these positions twice. Another reason to use four directorships as a threshold instead of five is that we capture only the 101 largest firms in The Netherlands. There are of course other firms either in The Netherlands or abroad that a director could serve, and we do not count these directorships. Furthermore, some directors are also active in government organizations, like the Dutch central bank. This information is however not available, and so we can only use the 101 firms in our dataset. For each firm and year we count the number of supervisory directors in the board,

[1]In the rare case a firm has no supervisory directors (in one or two cases it happened that the entire supervisory board steps down by the end of July) we deleted the observation.

Figure 5.1: Interlock network in 1998. Dots denote firms and lines denote interlocks between firms.



and the number of these directors who are busy (at least, according to our definition). Division of these numbers gives the fraction of busy directors in the board.

Concerning upper class membership, we define a director as belonging to the upper class when he has more than two directorships in the 101 firms in the dataset. The threshold of two directorships is not based on previous evidence. Therefore, we also estimated the models using a threshold of one directorship and using a threshold three directorships, but it turns out that the model based on a threshold of two directorships gives easily interpretable and partly significant results. Hence, we stick to the threshold of two directorships.

5.3.2 Performance measures

Additional to information on the supervisory boards, we also gathered data on the performance of the firms during 1994 to 2004 using the REACH database. We gathered data on stock returns, the price-earnings ratio and the price-to-book ratio, the return on assets and the return on equity. Furthermore, for each firm we store the BIK codes[2] (four-digit level), the turnover and the growth of the turnover.

Some of the dependent variables (stock returns, the price-earnings ratio and the price-to-book ratio) show serious skewness and excess kurtosis. To accommodate this, we transform the stock return by taking the log of (1 + return/100). For the price-earnings ratio we delete all observations on firms that make losses, as the price-earnings ratio is not

[2]BIK codes are the Dutch equivalent of SIC codes.

Table 5.1: Description of BIK codes.

category	description	number of firms
C	oil and mineral mining	1
D	manufacturers	43
F	construction	6
G	trade	12
I	transport and communication	8
J	financials	12
K	provision of services and renting	19

Table 5.2: Statistics per year.

year	stock	p/e ratio	p/b ratio	roa	roe	turnover	board size	interlock
1994	2.67	11.11	1.53	6.05	7.99	NA	6.18	0.69
1995	7.26	14.14	2.83	11.32	12.18	3.06	5.98	0.70
1996	45.38	17.54	3.61	9.90	22.42	3.34	5.77	0.66
1997	27.56	22.03	4.62	9.83	28.83	3.85	5.64	0.67
1998	1.02	25.82	5.56	10.27	26.09	3.57	5.63	0.79
1999	16.66	29.15	4.52	8.19	21.91	4.18	5.46	0.77
2000	1.65	25.65	3.50	5.91	18.59	5.52	5.59	0.73
2001	-13.97	20.86	2.40	3.73	6.05	5.61	5.60	0.72
2002	-27.72	12.23	1.69	1.12	-3.38	5.66	5.60	0.72
2003	36.32	19.32	2.00	2.84	8.97	5.42	5.49	0.69
2004	21.95	28.54	2.16	5.87	9.26	5.82	5.36	0.60

defined for these firms. We then transform the resulting price-earnings ratios by taking natural logs. Finally, the price-to-book ratio is also taken in natural logs.

We have the BIK codes of the firms in the sample at the four-digit level. Most firms have several BIK codes as our database concerns large firms active in several related areas. For each firm we take the main sector in which it is active and reduce the corresponding BIK code to the one-digit level. This way, firms are divided in seven different groups, like finance, transport and communication, industry, and construction. In Table 5.1 these categories are summarized.

In Table 5.10 in the Appendix we give some statistics for each of the firms in our panel database. We report the BIK code of the firm and the average values (over the years) of the untransformed performance measures, the turnover in billions of euros, the board size and the number of interlocks per director. From this table it is clear that the firms differ widely on these features.

In Table 5.2 we give statistics per year, now averaged over all 101 firms. The first column gives the year. Columns 2 to 6 give the average untransformed performance

Table 5.3: Sample sizes for the five performance measures.

stock return	734
price-earnings ratio	616
price-to-book ratio	722
return on assets	716
return on equity	734

Table 5.4: Summary statistics for each of the performance measure samples.

		stock return	p/e ratio	p/b ratio	roa	roe
	mean	-0.018	2.667	0.731	6.047	14.379
dependent	s.d.	0.438	0.794	0.859	13.361	31.166
	median	0.001	2.550	0.642	7.225	15.735
	mean	0.638	0.668	0.638	0.638	0.638
interlock	s.d.	0.552	0.560	0.554	0.555	0.551
	median	0.500	0.571	0.536	0.500	0.571
	mean	5.347	5.665	5.295	5.420	5.400
turnover	s.d.	17.140	18.189	17.192	17.336	17.143
	median	0.849	0.917	0.821	0.903	0.900
	mean	10.527	11.927	10.925	10.427	10.614
growth turnover	s.d.	33.980	28.160	33.899	34.267	33.709
	median	5.950	8.060	6.210	5.850	5.940

measures in each year. 'Stock' denotes the percentage growth of the stock price (stock return), 'p/e ratio' is the price-earnings ratio (stock price divided by earnings per share), 'p/b ratio' is the price-to-book ratio (stock price divided by equity per share), 'roa' denotes the return on assets (in percentages) and 'roe' denotes return on equity (in percentages). The seventh column gives the average turnover in billions of euros. The last two columns give board characteristics: the size of the board (average number of board members) and the average number of interlocks per director. For the number of interlocks per director we exclude all firms that started only after 1994. These firms have somewhat less interlocks per director and, consequently, when these firms are included the number of interlocks per director decreases over time. As one can see from the last column of Table 5.2, the number of interlocks per director does not have a clear upward or downward trend. Interestingly, the size of the board declines over time, also when the firms that start after 1994 are excluded. As expected, we see that the average firm performance is lower during 2001 and 2002 and we also note that the turnover increases over time.

For Tables 5.10 and 5.2 we used all available observations for the computations. However, for the regressions below we need to delete all firm-year observations that have either a missing value on one of the performance measures or on turnover or growth of turnover. We also removed some outliers. In Table 5.3 we report the available sample sizes for the five performance measures. The sample size clearly differs between performance measures.

In Table 5.4 we give some statistics of the (transformed) data, using the samples of Table 5.3. Each column in Table 5.4 corresponds to the sample of one of the (transformed) performance measures: the log of (1+ stock return/100), the logs of the price-earnings and price-to-book ratios, the return on assets (in percentages, no transformation applied) and the return on equity (in percentages, no transformation applied). The first panel in Table 5.4 gives the mean, the standard deviation (s.d.) and the median of the (transformed) performance measures itself. The second panel gives the same statistics on the number of interlocks per director. Panel three gives the same statistics on the turnover (in billions of euros) and the last panel gives the same statistics on the percentage growth of the turnover. Note that the columns of the panels concern different sample sizes. From Table 5.4 one can see that the statistics in panels two, three and four differ only slightly between the different performance measures.

5.4 Methodology

We have five different financial performance measures and we analyze each of these separately. For each performance measure we first choose the panel data model that best fits the data. The (panel data) models we choose from are a fixed effects (FE) model, a random effects (RE) model and a model based on the BIK codes. We will denote this last model as the BIK model. We use the AIC for model selection as, in contrast to the BIC, this criterion yields plausible final models. This AIC-based selected model is used to test the hypotheses. We will now first give a description of the three panel data models we choose from.

5.4.1 Models

The FE model is given by

$$y_{it} = \alpha_i + x_{it}\beta + \gamma_t + \varepsilon_{it}, \tag{5.1}$$

with ε_{it} iid and normally distributed, y_{it} the dependent variable, where x_{it} collects the independent variables and γ_t measures developments over time[3]. Note that the constant α_i depends on the firm i. The estimator of β is the within-estimator, as it is based on differences over time within a firm, and not on differences between firms.

The RE model is written as

$$y_{it} = \alpha + x_{it}\beta + \gamma_t + \alpha_i + \varepsilon_{it}, \tag{5.2}$$

with both ε_{it} and α_i iid and normally distributed. Again, y_{it} is the dependent variable, x_{it} summarizes the independent variables and γ_t concerns time. Note that here α does not depend on the firm i. Instead, correlation over time in the data of the same firm is

[3]Note that γ_t contains parameters that need to be estimated. Thus we allow for a flexible pattern over time. This is especially important for stock returns, as our database contains both years of boom and years of decline of the stock market.

captured by a random variable α_i, which is the same over time for one firm, but potentially differs across firms. The error term $\zeta_{it} = \alpha_i + \varepsilon_{it}$ is not iid and normally distributed as ζ_{it} and $\zeta_{i,t-1}$ are correlated. The resulting estimator of β is based on differences within a firm over time as well as on differences between firms.

For the BIK model, let BIK_i denote the BIK code of firm i. The BIK model is given by

$$y_{it} = \alpha_{BIK_i t} + x_{it}\beta + \varepsilon_{it}, \tag{5.3}$$

with ε_{it} iid and normally distributed. The idea of including $\alpha_{BIK_i t}$ is that firms in the same sector might have the same performance over time. For example, over time the patterns in stock returns might be the same for firms in the same sector. Note that with the BIK model we allow for different patterns over time. For instance, the model allows the performance in sector 1 to increase while at the same time the performance in sector 2 decreases. Note that this is not possible with the time dummies in the FE and RE models.

5.4.2 Variables

As control variables we include in every regression turnover, squared turnover, turnover one year lagged, squared turnover one year lagged, growth of turnover (in short: growth), squared growth, growth one year lagged and squared growth one year lagged. With the squared variables we allow for a potential nonlinear effect of the control variables on performance. Note that turnover serves as a measure of the size of the firm.

To test the hypotheses we include several other independent variables. First, to test H1a we include the number of interlocks corrected for board size. We estimate three different specifications, one with interlock included linearly, one with interlock included quadratically and one with only the square root of interlock included. We do this as the effect of interlock could be nonlinear. For instance, suppose the busyness hypothesis holds true. In that case one or two interlocks per director will not have that much of a negative effect since supervisory directors should be able to handle two or three positions. More interlocks per director however would have a larger negative impact. If H1a holds true the estimate on interlock would be negative and significant.

To test H1b we estimate the same equations as for H1a, except that interlock is now included with a one year lag. If H1b holds true the estimate on interlock again would be negative and significant. Furthermore when the estimates for H1a and H1b are compared the model fit as measured by AIC should be better for H1b, the estimates should be larger in size and and the p-values should be lower.

To test H2a - H2d we not only include the number of interlocks corrected for board size in the equation but also include the fraction of busy directors in the board. We include this variable in a linear fashion while for comparison we still include interlock in the three different specifications mentioned before. If H2a holds true the estimate on the fraction of busy directors is negative and significant. When H2b holds true the estimate on interlock decreases in size when the fraction of busy directors is included and the p-value for interlock increases. Furthermore, when the effect of interlocks is indeed caused by the busyness of directors the model fit as measured by AIC should improve when the fraction of busy directors is included.[4] After all, the fraction of busy directors is a better measure of busyness than the number of interlocks, which is just an indirect measure of busyness. The procedure to test H2c and H2d is similar to the procedure used for H2a and H2b, except that the variables interlock and fraction of busy directors are now lagged.

To test H3a-H3d we include the number of interlocks corrected for board size in the equation and also include the fraction of upper class directors in the board. As posed by H3a and H3c we will include this variable in the model in a quadratic way. For comparison we again include interlock in the three different specifications mentioned before. If H3a holds true the estimates on the fraction of upper class directors and the squared fraction of upper class directors are jointly significant. Furthermore the estimate on the squared fraction of upper class directors will be negative so that the estimated parabola has an inverse u form. Furthermore, to test H3b we follow the same procedure as for H2b and to test H3c and H3d we follow the same procedure as the one to test H3a and H3b except that the variables interlock and fraction of upper class directors (squared) are now lagged.

Previous research on interlocks and firm performance has mentioned the possibility of reverse causality. Not only will the number of interlocks influence performance, but good performing firms could attract more interlocking directors. In the database we use, the number of interlocks is based on the directorships halfway the year, while performance is measured over the complete year, and is quite volatile over years. Hence, reverse causality is unlikely to happen. Moreover, note that all hypotheses concerning lagged effects completely circumvent the possibility of reverse causality.

5.5 Test results

We start our analysis by choosing for each performance measure the panel data model (FE, RE or BIK) that best suits the data. To do this we estimate for each performance measure all equations concerning H1a and H1b using the FE, the RE and the BIK model. Table 5.5 gives for each combination of performance measure and panel data model the lowest and highest obtained AIC's of the six different equations. From the table it becomes clear that for the stock returns the BIK model provides the best fit, while for all other performance measures the FE model is preferred. It is not surprising that the BIK model is favored for stock returns. In the sample there is both a boom (before 2000) and a decline (after 2000) of stock prices. It is well known that stock prices in some sectors lead booms and declines, while others sectors follow. Allowing for different patterns over time for different sectors seems to be reasonable here, and hence the favorable AIC values for the BIK model. In what follows we only report the estimation results based on the best fitting panel data model.

In Table 5.6 we report the estimation results concerning H1a and H1b. Each panel of the table has a different performance measure as the dependent variable in the panel model. The p-value are reported in parentheses. The column 'model' denotes which model is used (FE, RE or BIK model), which transformation of interlock is used (linear,

[4]The AIC corrects for the number of variables included in the regression and so is a proper measure to compare two different model specifications

Table 5.5: AIC's of the different models.

model		lowest AIC	highest AIC
growth of stock price	FE	1.041	1.052
	RE	1.177	1.206
	BIK	0.895	0.898
price-earnings ratio	FE	2.057	2.068
	RE	2.123	2.135
	BIK	2.402	2.403
price to book ratio	FE	1.265	1.295
	RE	1.515	1.542
	BIK	2.299	2.310
return on assets	FE	7.474	7.478
	RE	7.532	7.537
	BIK	7.531	7.547
return on equity	FE	9.428	9.440
	RE	9.458	9.470
	BIK	9.639	9.650

quadratic or square root) and whether interlock is lagged. The column 'F-test' gives the p-value of the F-test of joint significance of interlock and interlock-squared. In each model we also included the control variables mentioned in the previous section but estimation results are not reported to save space.

When looking at the results, we note that the effect of the number of interlocks on stock returns and the return on assets is not significant, neither for current interlocks nor for lagged interlocks. For the price-earnings ratio only the non-lagged quadratic specification gives significant results. The estimates give a parabola of inverse u form with its top at 0.66 interlocks per director. On the other hand, the effect on the price-to-book ratio and the return on equity is (partly) significant. For the price-to-book ratio, all parameters for current interlocks are significant at the 5% or 10% level and the parameters for lagged interlocks are significant at the 1% level. For return on equity only the lagged variables are relevant. The significant estimates in the linear and square root specifications are all negative. The estimated quadratic specification has a U-shape with minimum at approximately 3.9 interlocks per director for the price-to-book ratio and an inverted U-shape with maximum at -0.4 interlocks per director for return on equity (lagged specification). As the number of interlocks per director is a positive variable with a mean of approximately 0.65 and a standard deviation around 0.55, the significant quadratic specifications also suggest a negative effect. We note that for the price-to-book ratio and the return on equity the lagged specification gives a better AIC, while for stock returns, the price-earnings ratio and the return on assets the evidence is mixed.

Concerning the hypotheses we draw the following conclusions. First, we find limited support for H1a. This hypothesis is supported by the price-to-book ratio but not by the other four performance measures. Second, we find somewhat more support for the first

Table 5.6: Estimation results concerning H1a and H1b.

measure	model	interlock	interlock^2	F-test	AIC
stock	BIK, linear	-0.004 (0.888)	-	-	0.896
	BIK, quadr.	-0.007 (0.930)	0.002 (0.968)	0.989	0.898
	BIK, sq.root	0.000 (0.999)	-	-	0.896
	BIK, linear, lag	-0.009 (0.757)	-	-	0.896
	BIK, quadr., lag	-0.089 (0.268)	0.048 (0.285)	0.538	0.897
	BIK, sq.root, lag	-0.016 (0.653)	-	-	0.895
p/e ratio	FE, linear	-0.134 (0.200)	-	-	2.064
	FE, quadr.	0.353 (0.137)	-0.268 (0.023)	0.033	2.057
	FE, sq. root	-0.028 (0.839)	-	-	2.067
	FE, linear, lag	0.038 (0.719)	-	-	2.067
	FE, quadr., lag	0.285 (0.227)	-0.134 (0.241)	0.471	2.068
	FE, sq.root, lag	0.156 (0.257)	-	-	2.065
p/b ratio	FE, linear	-0.158 (0.016)	-	-	1.292
	FE, quadr.	-0.200 (0.169)	0.025 (0.743)	0.053	1.295
	FE, sq. root	-0.201 (0.014)	-	-	1.292
	FE, linear, lag	-0.311 (0.000)	-	-	1.265
	FE, quadr., lag	-0.401 (0.004)	0.053 (0.466)	0.000	1.267
	FE, sq.root, lag	-0.351 (0.000)	-	-	1.270
roa	FE, linear	-1.035 (0.465)	-	-	7.476
	FE, quadr.	-0.403 (0.899)	-0.365 (0.824)	0.747	7.478
	FE, sq. root	-1.410 (0.428)	-	-	7.475
	FE, linear, lag	-1.033 (0.465)	-	-	7.476
	FE, quadr., lag	3.450 (0.265)	-2.572 (0.104)	0.203	7.474
	FE, sq.root, lag	0.544 (0.755)	-	-	7.476
roe	FE, linear	-4.193 (0.268)	-	-	9.439
	FE, quadr.	2.797 (0.741)	-4.033 (0.356)	0.353	9.440
	FE, sq. root	-2.294 (0.629)	-	-	9.440
	FE, linear, lag	-10.483 (0.005)	-	-	9.428
	FE, quadr., lag	-3.286 (0.689)	-4.145 (0.325)	0.013	9.429
	FE, sq.root, lag	-9.071 (0.051)	-	-	9.435

part of H1b. The lagged effect of interlocks is negative for both the price-to-book ratio and the return on equity, while it is not significant for the other performance measures. The second part of H1b is supported as well by the price-to-book ratio and return on equity. For both performance measures the lagged specification gives a better AIC, lower p-values and overall stronger effects.

We now turn to the estimation results on H2a-H2d, which are reported in Table 5.7. We report parameter estimates and p-values on interlock (possibly lagged) and the ratio of busy directors (possibly lagged), as well as the AIC and for the quadratic models the p-value of an F-test testing the joint significance of the linear and quadratic term.

The first outcome we note from the table is that almost all estimates on the ratio of busy directors have the expected negative sign. The estimates are significant in the non-lagged specification concerning the price-earnings and price-to-book ratios while for all other specifications and performance measures the estimates are non-significant. H2a is therefore supported by two out of five performance measures. H2b also is supported by the price-earnings and price-to-book ratios. When the ratio of busy directors is included in the model the AIC improves, the estimates on interlock that were significant now turn insignificant and also the size of the estimates on interlock decreases. We also note that the size of the estimated effect of busyness on the price-earnings and price-to-book ratios is economically quite significant. Finally, H2c is not supported by the data. Although the estimated effect is negative it is not significant for any of the performance measures. As a consequence, H2d is also not supported.

Tables 5.8 and 5.9 give the estimation results on H3a-H3d. Again we report parameter estimates and p-values, this time on interlock (possibly lagged) and the ratio of upper class directors (possibly lagged), as well as the AIC and for the quadratic models the p-value of an F-test testing the joint significance of the linear and quadratic term. Note that the first column 'F-test' gives the p-value of the F-test of joint significance of 'interlock' and 'interlock-squared' and that the second column 'F-test' does the same for the ratio non-upper class. Table 5.8 contains the estimation results on the performance measures log(stock return/100 + 1), log(price-earnings ratio) and log(price-to-book ratio) and Table 5.9 shows the estimation results on the performance measures return on assets and return on equity.

From the tables we see that H3a is supported by the return on assets and the return on equity, but not by the other performance measures. For the return on assets and return on equity the fraction of upper class directors and the squared fraction of upper class directors jointly are significant and the effect takes the inverse u form we hypothesized. For the other three performance measures the estimates are nonsignificant and sometimes take the wrong form as well. H3b is not completely supported by the data. Although for the return on assets and the return on equity the AIC improves, the estimates on interlock generally increase in size and in one case even turn significant. The return on assets and return on equity for a large part also support H3c. The estimates on the ratio of upper class directors are significant and the effect is of the hypothesized form. For the return on equity the lagged effect is indeed stronger than the immediate effect, but for the return on assets the lagged effect is less strong than the immediate effect. Also for H3d the results on the return on assets are mixed while the return on equity indeed has

Table 5.7: Estimation results concerning H2a-H2d.

measure	model	interlock	interlock^2	F-test	ratio busy	AIC
stock	BIK, linear	-0.011 (0.760)	-	-	-0.192 (0.488)	0.898
	BIK, quadr.	-0.021 (0.794)	0.023 (0.662)	0.868	-0.261 (0.414)	0.900
	BIK, sq.root	0.013 (0.742)	-	-	-0.179 (0.473)	0.898
	BIK, linear, lag	-0.017 (0.641)	-	-	0.096 (0.718)	0.898
	BIK, quadr., lag	-0.091 (0.263)	0.052 (0.308)	0.533	-0.051 (0.866)	0.899
	BIK, sq.root, lag	-0.023 (0.577)	-	-	0.082 (0.731)	0.898
p/e ratio	FE, linear	-0.011 (0.923)	-	-	-1.594 (0.030)	2.058
	FE, quadr.	0.334 (0.159)	-0.208 (0.094)	0.244	-1.185 (0.124)	2.056
	FE, sq.root	0.101 (0.486)	-	-	-1.790 (0.009)	2.057
	FE, linear, lag	0.094 (0.427)	-	-	-0.747 (0.294)	2.068
	FE, quadr., lag	0.270 (0.253)	0.105 (0.390)	0.504	-0.520 (0.494)	2.070
	FE, sq.root, lag	0.208 (0.149)	-	-	-0.789 (0.237)	2.065
p/b ratio	FE, linear	-0.100 (0.178)	-	-	-0.770 (0.107)	1.291
	FE, quadr.	-0.210 (0.149)	0.069 (0.380)	0.275	-0.909 (0.071)	1.292
	FE, sq.root	-0.147 (0.089)	-	-	-0.827 (0.063)	1.289
	FE, linear, lag	-0.283 (0.000)	-	-	-0.380 (0.403)	1.266
	FE, quadr., lag	-0.412 (0.004)	0.083 (0.281)	0.000	-0.557 (0.249)	1.267
	FE, sq.root, lag	-0.309 (0.000)	-	-	-0.691 (0.105)	1.269
roa	FE, linear	-0.494 (0.763)	-	-	-6.850 (0.508)	7.478
	FE, quadr.	-0.472 (0.882)	-0.014 (0.994)	0.956	-6.821 (0.534)	7.480
	FE, sq.root	-0.947 (0.617)	-	-	-6.760 (0.479)	7.477
	FE, linear, lag	-0.214 (0.895)	-	-	-10.340 (0.301)	7.477
	FE, quadr., lag	3.354 (0.278)	-2.279 (0.176)	0.397	-5.362 (0.614)	7.476
	FE, sq.root, lag	1.450 (0.434)	-	-	-13.426 (0.147)	7.476
roe	FE, linear	-1.963 (0.652)	-	-	-28.150 (0.303)	9.440
	FE, quadr.	2.565 (0.762)	-2.879 (0.533)	0.744	-22.330 (0.439)	9.442
	FE, sq.root	0.062 (0.990)	-	-	-34.356 (0.174)	9.440
	FE, linear, lag	-9.069 (0.035)	-	-	-17.785 (0.498)	9.430
	FE, quadr., lag	-3.460 (0.674)	-3.585 (0.425)	0.079	-10.036 (0.720)	9.432
	FE, sq.root, lag	-6.825 (0.166)	-	-	-33.391 (0.171)	9.434

Table 5.8: Estimation results concerning H3a-H3d.

measure	model	interlock	interlock^2	F-test	ratio upper class	ratio^2	F-test	AIC
stock	BIK, linear	0.039 (0.503)	-	-	0.061 (0.818)	0.390 (0.299)	0.370	0.898
	BIK, quadr.	-0.068 (0.590)	0.059 (0.337)	0.504	0.263 (0.438)	-0.729 (0.157)	0.233	0.899
	BIK, sq.root	0.030 (0.652)	-	-	0.084 (0.768)	-0.349 (0.381)	0.414	0.899
	BIK, linear, lag	0.003 (0.953)	-	-	0.083 (0.759)	-0.244 (0.535)	0.784	0.900
	BIK, quadr., lag	-0.215 (0.086)	0.119 (0.050)	0.146	0.497 (0.147)	-0.935 (0.076)	0.202	0.897
	BIK, sq.root, lag	-0.028 (0.668)	-	-	0.176 (0.546)	-0.310 (0.459)	0.755	0.900
p/e ratio	FE, linear	-0.146 (0.394)	-	-	0.734 (0.276)	-1.217 (0.171)	0.390	2.067
	FE, quadr.	0.393 (0.250)	-0.292 (0.068)	0.131	-0.142 (0.864)	0.267 (0.824)	0.975	2.063
	FE, sq.root	0.056 (0.775)	-	-	0.317 (0.650)	-1.130 (0.221)	0.242	2.068
	FE, linear, lag	0.057 (0.740)	-	-	-0.139 (0.837)	0.134 (0.887)	0.979	2.074
	FE, quadr., lag	0.526 (0.106)	-0.259 (0.090)	0.225	-0.933 (0.256)	1.538 (0.219)	0.462	2.071
	FE, sq.root, lag	0.266 (0.157)	-	-	-0.563 (0.425)	0.468 (0.627)	0.674	2.070
p/b ratio	FE, linear	-0.211 (0.049)	-	-	0.234 (0.585)	-0.113 (0.844)	0.807	1.297
	FE, quadr.	-0.317 (0.125)	0.061 (0.548)	0.120	0.409 (0.430)	-0.424 (0.584)	0.712	1.299
	FE, sq.root	-0.235 (0.041)	-	-	0.300 (0.497)	-0.418 (0.486)	0.775	1.296
	FE, linear, lag	-0.304 (0.004)	-	-	-0.399 (0.347)	0.676 (0.255)	0.521	1.268
	FE, quadr., lag	-0.302 (0.119)	-0.001 (0.989)	0.015	-0.403 (0.429)	0.683 (0.383)	0.680	1.271
	FE, sq.root, lag	-0.256 (0.019)	-	-	-0.439 (0.320)	0.280 (0.650)	0.441	1.273

Table 5.9: Estimation results concerning H3a-H3d - continued.

measure	model	interlock	interlock^2	F-test	ratio upper class	ratio^2	F-test	AIC
roa	FE, linear	-3.122 (0.179)	-	-	20.587 (0.027)	-24.907 (0.045)	0.078	7.473
	FE, quadr.	-8.914 (0.048)	3.318 (0.133)	0.131	30.225 (0.008)	-41.948 (0.013)	0.026	7.472
	FE, sq.root	-4.437 (0.076)	-	-	23.790 (0.014)	-31.015 (0.017)	0.042	7.470
	FE, linear, lag	0.854 (0.708)	-	-	10.670 (0.253)	-30.007 (0.021)	0.042	7.471
	FE, quadr., lag	1.388 (0.745)	-0.309 (0.882)	0.922	9.744 (0.386)	-28.343 (0.099)	0.155	7.473
	FE, sq.root, lag	2.132 (0.375)	-	-	7.634 (0.432)	-26.773 (0.046)	0.024	7.469
roe	FE, linear	4.681 (0.449)	-	-	-0.942 (0.970)	-49.529 (0.135)	0.057	9.435
	FE, quadr.	3.131 (0.794)	0.888 (0.880)	0.742	1.638 (0.957)	-54.088 (0.229)	0.086	9.438
	FE, sq.root	5.136 (0.440)	-	-	-2.378 (0.926)	-42.665 (0.217)	0.034	9.435
	FE, linear, lag	-2.819 (0.642)	-	-	16.362 (0.509)	-74.769 (0.031)	0.029	9.422
	FE, quadr., lag	-7.977 (0.484)	2.973 (0.593)	0.778	25.371 (0.397)	-90.876 (0.048)	0.041	9.424
	FE, sq.root, lag	-2.602 (0.683)	-	-	16.524 (0.523)	-78.915 (0.028)	0.004	9.422

that the estimates on interlock turn insignificant and are smaller in size.

From the estimates on the return on equity and return on assets one can infer an optimal ratio of upper class directors. For the return on assets the estimated parabola peaks at approximately a ratio of 0.4 (current) and 0.2 (lagged) upper class directors, which implies that it is optimal to have a board which consists of 20% to 40% of upper class directors. For the return on equity the peak is at approximately a ratio of 0 (non-lagged) and 0.1 (lagged), and so the estimated effect of upper class directors is negative almost everywhere. We again note that the estimated effects are quite sizable.

5.6 Conclusion

In this study we used a new and detailed panel database to investigate the effect of interlocks in the Netherlands during 1994 to 2004. Our hypothesis that interlocks have a negative effect is partly supported by the data and we find that interlocks indeed have a stronger effect in the medium run than in the short run. There are two theories that can explain the negative effect of interlocks in The Netherlands. The first is the busyness hypothesis of Ferris et al. (2003). We find support for the hypothesis that busyness causes the immediate negative effect of interlocks, but in the medium run busyness does not play a significant role. The other theory that can explain the negative effect of interlocks is the upper class hypothesis. Again, we find support for the hypothesis that upper class effects cause the immediate negative effect of interlocks and this time we also find weak support that upper class effects are stronger in the medium run than in the short run.

For none of our hypotheses the support is overwhelming. One research question we were not able to answer and which thus deserves further attention is why some performance measures give significant results only on the busyness hypothesis while other performance measures give significant results only on the upper class hypothesis. Another puzzling result is that busyness has an effect in the short run but not in the medium run. Nevertheless, we think that our findings do have managerial implications. Our results on the busyness hypothesis further strengthen the results of Fich and Shivdasani (2006) and extend these results to firms outside the US. The results in both this chapter and the paper of Fich and Shivdasani advise not to appoint directors that are already busy serving other firms. Also, it would be good to be cautious in appointing high profile (upper class) directors since a too large fraction of these directors in the board could harm firm performance.

5.A Statistics per firm

Table 5.10: Statistics per firm.

firm	BIK	stock	p/e ratio	p/b ratio	roa	roe	turn-over	board size	inter-lock
Aalberts	D	24.06	15.38	4.47	10.55	31.88	0.49	3.91	0.72
ABN Amro	J	13.65	11.99	2.42	0.77	20.74	14.90	13.36	1.41
Acomo	G	12.08	9.16	1.96	13.49	25.72	0.19	3.73	0.00
Aegon	J	20.84	22.51	3.14	1.06	14.36	22.51	10.91	1.12
Ahold	G	5.51	0.27	7.34	3.71	16.58	37.88	6.73	1.39
Akzo Nobel	D	6.07	20.04	4.34	10.03	25.83	12.63	9.73	1.02
Alanheri	G	-5.00	53.02	0.75	0.31	1.33	0.13	3.00	0.15
AM	F	7.07	11.05	1.79	9.45	16.64	1.56	5.36	0.43
Arcadis	K	11.22	9.94	1.80	10.03	19.18	0.68	6.55	0.37
ASM International	D	49.82	79.22	6.12	6.05	-3.66	0.50	4.36	0.92
ASML	D	42.11	27.50	8.39	11.51	11.57	1.38	6.30	0.46
Athlon	K	20.37	11.80	2.16	2.95	20.53	0.80	3.91	0.55
Ballast Nedam	F	3.93	7.89	1.08	0.88	2.49	1.87	6.18	0.70
BAM	F	16.72	5.31	1.52	5.26	19.46	2.74	6.55	0.55
Batenburg beheer	F	4.38	8.69	1.54	13.85	18.54	0.11	2.91	0.24
Beterbed	G	12.15	13.61	10.34	18.96	52.31	0.20	3.25	0.44
Boskalis	F	8.78	10.77	1.76	8.54	19.06	0.86	5.45	1.26
Brunel	K	-0.76	32.10	2.63	14.69	16.04	0.23	3.25	0.31
Buhrmann	G	0.62	58.29	1.12	2.29	4.57	7.56	7.55	1.56
Ten Cate	D	7.34	5.32	1.07	6.00	7.53	0.62	5.55	0.84
LogicaCMG	K	60.93	15.92	14.82	1.03	0.43	1.75	4.22	0.24
Coberco	D	0.20	7.61	NA	6.50	12.94	4.32	11.43	0.21
Corio	J	8.43	NA	1.03	5.19	9.76	0.19	5.91	0.73
Corus	D	-16.66	2.39	0.69	5.61	8.10	9.09	8.64	1.27
Crown van Gelder	D	2.29	2.25	0.59	9.41	7.49	0.13	4.00	0.05
Crucell	D	24.42	-7.59	2.08	-29.37	-39.54	0.01	6.00	0.00
CSM	D	4.98	14.15	4.59	13.06	41.37	2.57	6.36	1.19
CTAC	K	-11.12	22.64	5.91	17.00	33.80	0.02	2.71	0.43
Delft Instruments	D	5.89	11.83	1.43	4.68	4.61	0.20	5.27	0.53
Dico international	D	-11.11	1.65	0.58	-13.70	-25.55	0.03	2.82	0.33
DOCdata	D	-9.25	11.60	1.35	4.39	5.35	0.09	3.13	0.44
Draka	D	10.82	18.27	2.22	6.71	17.65	1.20	5.55	1.08
DSM	D	9.87	10.34	1.09	7.68	17.17	6.49	7.82	0.57
Econosto	D	-10.28	21.83	2.50	1.60	-4.14	0.29	4.00	0.11
Elsevier	D	3.76	-98.96	5.21	13.62	11.35	5.90	7.55	0.99
Eriks group	G	6.63	24.15	1.57	12.52	17.34	0.30	3.00	0.00
Eurocommercial properties	J	8.19	15.38	1.00	3.57	6.35	0.07	4.00	0.00
EVC International	D	5.52	-2.09	0.49	-2.32	-18.10	1.06	4.10	0.13
Exact	K	-4.23	25.57	8.70	25.19	35.42	0.18	4.29	0.12
Exendis	D	-5.12	9.19	2.25	5.41	2.27	0.02	3.36	0.30

Table 5.10 – continued from previous page

Firm	BIK	stock	p/e ratio	p/b ratio	roa	roe	turn-over	board size	inter-lock
de Porceleyne Fles	D	8.18	10.00	1.73	5.85	13.02	0.00	3.18	0.24
Fornix	D	37.76	6.72	5.19	16.98	24.65	0.04	3.18	0.00
Fortis	J	13.95	-147.55	2.81	9.09	11.55	1.79	10.80	0.16
Frans Maas	I	2.11	9.38	1.67	5.74	12.00	0.82	4.55	0.50
Fugro	K	27.57	13.24	3.60	10.68	32.41	0.67	5.73	0.61
Gamma Holding	D	0.78	9.85	1.36	8.73	15.33	0.82	6.09	1.49
Getronics	K	32.28	28.15	10.20	5.24	13.67	2.52	4.82	0.67
Grolsch	D	0.30	16.69	2.96	14.87	17.61	0.24	6.00	1.07
Grontmij	K	10.20	13.13	1.23	6.04	13.56	0.39	6.18	0.38
Hagemeijer	G	-9.37	16.12	3.43	4.90	11.60	6.17	5.91	0.85
Heijmans	F	19.30	9.90	1.93	9.32	21.85	1.61	4.82	0.79
Heineken	D	7.18	23.70	4.83	12.38	22.02	6.85	7.36	1.59
Hunter Douglas	D	9.87	10.27	1.93	12.59	19.38	1.44	7.00	0.20
Imtech	K	14.87	9.23	2.39	9.40	34.26	2.10	6.18	1.41
ING	J	16.31	11.88	1.82	NA	18.37	9.29	11.45	1.06
Kas bank	J	15.01	10.36	1.17	NA	14.17	0.10	7.09	0.06
Kendrion	D	-11.25	6.98	3.42	2.72	-3.68	0.73	3.64	0.17
KLM	I	1.17	70.17	0.74	0.32	6.70	6.10	9.36	0.95
KPN	I	7.27	14.54	2.68	1.73	-3.37	10.73	7.55	0.92
Van Lansschot	J	4.81	13.43	2.27	NA	18.21	0.35	8.43	0.74
Macintosh Retail	G	6.55	16.26	1.39	7.74	13.48	0.69	5.00	0.62
Magnus	K	-22.05	35.62	5.39	5.35	4.89	0.03	2.33	0.00
Van der Molen	J	22.55	6.63	4.11	10.09	62.13	0.20	4.64	0.18
Nedap	D	20.58	16.38	3.36	16.63	21.36	0.10	3.91	0.26
Nedschroef	D	16.11	5.62	0.94	7.32	10.77	0.25	5.00	0.18
Neways	D	6.77	7.63	1.79	3.46	-2.15	0.15	2.64	0.06
Numico	D	25.23	20.88	7.44	1.05	48.25	2.52	6.18	1.05
Nutreco	D	9.21	12.31	2.09	4.55	11.78	2.95	3.73	0.46
Nyloplast	D	-6.27	23.39	1.44	8.70	10.61	0.02	3.09	0.11
OCE	D	8.52	27.58	1.93	5.42	11.50	2.64	5.82	1.10
Opg groep	D	9.97	9.85	1.94	10.57	24.73	1.65	7.00	0.23
Ordina	K	31.21	22.45	18.20	29.11	83.72	0.27	3.27	0.52
P&O Nedlloyd	I	19.26	18.34	0.56	-0.26	6.78	2.37	5.73	1.44
Philips	D	22.11	3.74	2.05	4.40	14.58	32.29	7.82	0.68
Randstad	K	27.69	25.28	8.85	13.44	38.44	4.63	5.73	0.99
Reesink	G	-0.27	10.38	1.13	9.23	10.51	0.12	4.00	0.41
De Vries Robbe	K	-12.16	9.02	0.97	-30.97	-57.34	0.01	2.43	0.07
Rodamco Europe	J	8.32	12.69	0.96	5.20	13.12	0.47	6.40	0.28
SBM	D	14.85	24.17	3.50	6.09	17.06	1.16	5.55	1.65
Schuitema	G	16.30	16.76	4.55	11.54	28.01	2.25	4.45	0.27
Shell	C	9.73	51.07	2.81	14.45	15.36	130.77	7.27	1.85
Simac	K	31.67	30.42	5.12	-3.90	-33.07	0.19	3.82	0.08
Sligro food group	G	24.79	15.27	4.54	13.80	32.92	0.88	4.00	0.05
Smit International	I	17.82	8.64	1.09	5.22	18.31	0.32	5.64	0.87

Table 5.10 – continued from previous page

Firm	BIK	stock	p/e ratio	p/b ratio	roa	roe	turn-over	board size	inter-lock
Nieuwe Steen investments	J	3.58	11.27	1.18	5.22	11.36	0.06	4.29	0.00
Stork	D	14.83	3.66	1.29	5.36	10.84	2.19	7.00	1.02
Telegraaf	D	6.07	-8.25	2.23	8.35	6.53	0.68	6.27	0.32
TNT	I	-3.36	23.99	4.57	10.91	22.86	10.48	8.57	0.82
Twentsche kabel	D	14.53	9.70	1.44	8.78	10.45	0.45	5.91	0.73
Unilever	D	10.66	26.93	6.15	10.81	35.58	43.06	10.00	0.43
Unit 4 Agresso	K	17.65	23.19	12.61	25.57	69.78	0.16	3.43	0.00
Univar	I	126.57	11.30	0.72	3.38	5.87	3.93	4.33	1.15
United Services	K	33.15	15.87	6.78	20.21	55.63	0.81	4.29	0.93
Vedior	K	9.43	-8.21	2.09	-2.60	-2.54	5.44	3.88	0.33
Vendex	G	-0.83	31.72	3.32	9.21	28.89	4.52	6.00	0.82
Versatel	I	57.13	-13.96	1.37	-34.02	-30.34	0.31	5.67	0.00
VHS onroerend goed	K	16.23	8.32	1.65	4.54	19.88	0.04	3.00	0.00
VNU	D	15.65	28.05	4.98	8.40	23.53	3.01	6.18	1.46
Wereldhave	J	7.08	11.81	0.99	5.39	9.35	0.14	4.45	0.46
Wessanen	D	-2.21	53.32	1.77	6.75	16.21	2.78	5.18	0.89
Wolters-Kluwer	D	4.39	22.24	7.88	9.70	24.28	3.01	6.36	1.35

Chapter 6

Summary and directions for further research

6.1 Summary

In this thesis I consider the provision of information in two very different settings. In the first part of this thesis, consisting of chapters 2, 3 and 4, micro-economic models are used to analyze the search behavior of consumers. The second part of this thesis, chapter 5, empirically analyzes the effect of director interlocks on firm performance. In this section I will give a short summary of each chapter.

In *Chapter 2* a duopoly model is considered where *ex ante* consumers are uninformed about prices. Price information can come from two different sides. First, firms can decide to advertise and, second, consumers can (sequentially) search for information. Both mechanisms are costly, and the relative costs determine which side takes the burden of information provision. When the search costs are relatively low, or the advertising costs are relatively high, firms do not advertise, but consumers do search. On the other hand, when the search costs are high consumers do not search, but firms do advertise. For intermediate costs, both advertising and search are used and it is shown that advertising and search are 'substitutes' in the sense that if advertising decreases (e.g. because of increasing advertising costs) consumers will search more. Likewise, when consumer search decreases (e.g. because of increasing search costs) firms will advertise more. Despite this substitution result, consumers in general do not know all market prices, and as a result prices are above marginal costs.

One of the novelties in Chapter 2 is that we assume that search costs are only incurred when visiting a non-advertising firm. Visiting a firm that has advertised comes at no costs. This assumption has an interesting effect on the pricing behavior of firms: for some parameter values advertised prices are higher than non-advertised prices. Intuitively, advertising firms have an advantage over non-advertising firms. A consumer incurs search costs when visiting a non-advertising firm and he may be willing to pay a higher price in order to avoid these search costs.

In *Chapter 3* we further examine this mechanism. Instead of assuming that visiting a non-advertising firm comes at no cost, we endogenize the difference between the costs of visiting an advertising and a non-advertising firm by adding an initial phase to the model. In this phase, firms decide on whether or not they want to shelve the product. The costs of visiting an advertising firm are assumed to be equal to the costs of visiting a non-advertising firm, but consumers are sure that an advertising firm has shelved the product. When visiting a non-advertising firm, there is a probability this firm has not shelved the product at all and the consumer would have to continue his search. This makes buying the product in an advertising shop less costly than buying the product in a non-advertising shop, and consequently advertising shops have an advantage. The size of this advantage depends on the probability that a firm decides to shelve the product. If this probability is small, an advertising shop has a large advantage, while a large shelving probability will only lead to a small advantage for the advertising firms. In Chapter 3 it is shown that the model parameters can be chosen in such a way that the advantage for advertising firms is large enough to make advertising a high price an equilibrium strategy.

Chapter 4 presents a model combining sequential consumer search and the location choice of shops. The search costs in this model consist of two elements. Consumers incur costs when traveling between malls and they incur costs when entering a shop. This search costs structure ensures that visiting two or more shops in the same mall comes at less costs than visiting the same number of shops dispersed over different malls. It is assumed that the shops sell homogenous products, allowing to focus on the role of prices. Three roles of prices are found. First, the existence of a mall containing two or more competing shops lowers the prices. Since a mall containing two or more competing shops lowers the consumer search costs, this result is quite intuitive. Second, these lower prices make more consumers active, that is, they make more consumers willing to leave their home and start searching. And, third, malls that contain two or more competing shops attract more consumers per shop than malls where a shop has no direct competitors. Simulations show that the second and third effect can outweigh the first effect, making locating together a profitable strategy.

The topic of *Chapter 5* is director interlocks. A director interlock between two firms occurs when the firms share one or more directors in their boards. In Chapter 5 we use a unique panel dataset on Dutch directors to investigate the effect director interlocks have on firm performance. Although interlocks could provide firms with information and interlocking directors generally are more experienced, we find weak evidence of a negative relation between interlocks and performance. Further investigations suggest that this negative effect has two causes. First, interlocking directors tend to be too busy to give full attention to all of their directorships and second, in The Netherlands interlocking directors form a homogenous, cohesive group, potentially suffering from a lack of critical thinking. We conclude that the common practice of interlocking in The Netherlands has no foundation in increased performance and that firms should be careful in appointing suitable board members.

6.2 Directions for further research

There are several possibilities to extend the research contained in this thesis. In this last part of the thesis I will outline some directions for further research in the areas of search and advertising, search and location choice and director interlocks.

The models in this thesis concerning search and advertising are of a theoretical nature. A first direction for further research would be to estimate the parameters of these models. There currently is an emerging literature trying to estimate consumer search costs and using those estimates for a welfare analysis, see e.g. Moraga-Gonzalez and Wildenbeest (forthcoming). The models used in this literature are pure search models, without any advertising. Including advertising in the empirical analysis would enhance the realism of those models. Estimating the models in Chapters 2 and 3, however, will not be easy. The main complication is the multiplicity of equilibria in the models containing advertising. Models with only consumer search generally have a unique (symmetric) equilibrium, and data on prices is sufficient to identify the model parameters. Intuitively, there is a one-to-one correspondence between the model parameters and equilibrium pricing behavior and therefore an observed price distribution is sufficient to estimate the model parameters. When there are multiple equilibria, it is likely that there is no one-to-one correspondence between model parameters and equilibrium pricing behavior. In every equilibrium type the price distribution takes a different form and it is very well possible that two different sets of parameter values, entered into their respective equilibrium price distributions, give the same final price distribution. If in a search and advertising model with multiple equilibria there still is a one-to-one correspondence between the model parameters and the equilibrium pricing behavior, then price data would be sufficient to identify the parameters of such a model and it would be relatively easy to estimate the parameters by using the same methods as outlined in Moraga-Gonzalez and Wildenbeest (forthcoming). If there is no one-to-one correspondence between the model parameters and equilibrium pricing behavior, price data is not sufficient for identification. The only solution then is to gather data on either advertising costs or on advertising intensity. Data on pricing and advertising together should be sufficient to identify the model parameters, but this data is much more difficult to obtain than data on only prices. Once again, the methods outlined in Moraga-Gonzalez and Wildenbeest (forthcoming) could be used, although the actual estimation would be more complex.

As a theoretical extension of the models one could think of a model with different consumer segments and targeted advertising, where the consumer segments differ in search costs. In the models in this thesis there are two consumer segments, with one segment having zero search costs and the other segment having strictly positive search costs. If targeted advertising would have been possible, the outcome most likely would be that all firms ignore the zero search cost segment. But if there are two or more consumer segments with strictly positive search costs, the analysis gets more complicated. An intuitive outcome would be that the firms concentrate more on the high search costs segment, but whether this result really can be obtained is an interesting topic for future research.

Future research should also complete the analysis of the model in Chapter 4. The analysis of the model in this chapter consists of two parts. First, the pricing behavior is determined while the location of shops is fixed. Once this has been done, the location choice of shops is considered. In the first part of the analysis one ideally considers all location possibilities, but several location possibilities, especially the ones with a relatively large mall, turned out to be quite complex to analyze and are missing in the analysis. This hampers the analysis in the second part. The results obtained thus far in the first part are sufficient to show that a mall can be formed and indicate what factors play a role in this, but it would be very interesting to make a complete analysis of the game. One important location possibility that has not yet been analyzed is the case where there is one isolated shop and one mall containing all the other shops. Once this case has been analyzed, it is possible to show whether one large mall is a Nash equilibrium or not. To show whether one large mall is a *unique* Nash equilibrium, one would also need to analyze the other location possibilities. The main problem with such an analysis is that the pricing behavior can be quite complex. One way to work around this problem would be to focus on profits instead of pricing behavior, since profits are all that is needed in the second part of the analysis. Probably computer simulations could also help to find the equilibrium profits.

The model in Chapter 4 assumes that there is only one mall that has enough space for several competitors. Although in some situations this is realistic, it would be interesting to extend the model by assuming that there is more than one mall with enough space for several competitors. A question that would arise in such a setting is whether shops prefer to spread evenly over several malls, or whether they prefer to locate all together in one mall.

Concerning the analysis in Chapter 5, one avenue for further research would be to gather data on 2005 - 2008. A panel data set like we already have on 1994-2004 can be used to estimate lagged effects. This is important in the case of interlocks because of a causality problem: it could be that interlocks increase performance, but it could as well be that good performing firms attract more interlocks. Estimating the lagged effect of interlocks circumvents this problem. The current dataset allows a one-year lag, but there is not enough data to estimate longer lags. Data on more years will solve this and allow for two- and three-year lags. One possible complication here is the code Tabaksblat, that became effective at the end of 2004. This code Tabaksblat is a Dutch corporate governance code of good conduct and, among other things, it limited the number of board positions a director might have. Even though the code Tabaksblat is not compulsory, the popular press suggests that virtually all directors indeed comply to it. This implies that after 2004 almost no director in the dataset will be busy and that estimating a fixed effects model on busyness is impossible. The models estimating the effect of interlocks on firm performance and the effect of a homogenous group can still be estimated, but the effect of interlocks on firm performance can change when busyness plays no role anymore. A model with a structural break is necessary to investigate this, complicating the analysis.

One of the conclusions of Chapter 5 was that the negative effect of interlocks is partly caused by a cohesive group effect. It would be interesting to investigate this further by comparing different countries. In The Netherlands there is a relatively small group of directors, who often meet each other in an informal setting. Not all countries have such a cohesive group of directors, and this could be visible in the estimation results. For instance, we find that in The Netherlands the effect of the percentage of high profile directors in the board takes the form of an inverse u. In countries where the directors form a less cohesive group, the top of the inverse u shaped curve could lie at a higher percentage of high profile directors, or the effect could be completely linear (with a positive slope).

As a final remark, the data on directors used in Chapter 5 could also be coupled with other data than firm performance. One interesting avenue would be to investigate whether the interlock network has any effect on the compensation of executives. It is regularly suggested in the popular press that interlocking directors earn higher salaries because their network shields them. An investigation on this topic would only require additional data on compensation, which is relatively easy to gather.

Nederlandse samenvatting (Summary in Dutch)

Inleiding

Dit proefschrift bestaat uit twee delen, die beide een ander aspect van de rol van informatie in de economie belichten. Het eerste deel van dit proefschrift, bestaande uit de hoofdstukken 2, 3 en 4, analyseert met behulp van theoretische micro-economische modellen de informatiestromen tussen winkels en consumenten. In dit deel van het proefschrift wordt er van uitgegaan dat consumenten niet (volledig) geïnformeerd zijn over producten en/of prijzen. Om geïnformeerd te raken kunnen consumenten zoeken naar informatie, maar iedere zoekactie brengt kosten met zich mee, hetzij in termen van geld, hetzij in termen van tijd. Winkels kunnen deze kosten beïnvloeden, onder andere door te adverteren of zich bij elkaar in de buurt te vestigen. In het eerste deel van dit proefschrift wordt geanalyseerd of het voor winkels aantrekkelijk is om de zoekkosten van consumenten te beïnvloeden en wat dit voor invloed heeft op de prijzen en op de hoeveelheid informatie die consumenten uiteindelijk hebben.

Het tweede deel van dit proefschrift heeft informatiestromen tussen bedrijven als onderwerp. Veel leden van de raad van commissarissen hebben meerdere commissariaten. Deze dubbelcommissarissen vormen een brug tussen de bedrijven waarin ze commissaris zijn, en zijn een manier om informatie uit te wisselen tussen bedrijven. Dubbelcommissariaten hebben echter ook schaduwzijden. Een deel van de dubbelcommissarissen heeft het zo druk dat de kwaliteit van hun toezicht en advies eronder lijdt. In Hoofdstuk 5 van dit proefschrift wordt het uiteindelijke effect dat dubbelcommissarissen hebben geschat met behulp van data over Nederlandse beursgenoteerde bedrijven.

Deel I. Zoekgedrag van consumenten

De hoofdstukken 2 en 3 van dit proefschrift hebben het zoekgedrag van consumenten en het advertentiegedrag van winkels als onderwerp. In Hoofdstuk 2 wordt aangenomen dat consumenten niet op de hoogte zijn van de prijzen in de verschillende winkels. Winkels kunnen consumenten informeren door te adverteren, maar consumenten kunnen ook zelf zoeken naar informatie. De belangrijkste conclusies van dit hoofdstuk zijn als volgt. Ten eerste blijken adverteren en zoeken substituten te zijn. Wanneer winkels meer adverteren gaan consumenten minder zoeken en wanneer consumenten meer zoeken gaan winkels minder adverteren. Ten tweede blijkt er geen eenduidig verband te zijn tussen zoekkosten en prijzen. Gewoonlijk leiden hogere zoekkosten tot hogere prijzen. Als de zoekkosten hoog zijn, zoeken consumenten minder. Hierdoor zijn consumenten minder goed geïnformeerd en is de concurrentie tussen winkels kleiner. Maar als winkels de mogelijkheid hebben om te adverteren, leiden hoge zoekkosten (en minder zoekactiviteit van consumenten) ook tot meer advertenties, wat weer tot meer concurrentie en lagere prijzen leidt. De analyse in Hoofdstuk 2 laat zien dat het uiteindelijke effect op de prijs van een stijging in de zoekkosten beide kanten op kan vallen. De laatste interessante conclusie is dat geadverteerde prijzen hoger kunnen zijn dan niet geadverteerde prijzen. Een van de aannames in Hoofdstuk 2 is dat een consument geen zoekkosten hoeft te maken om in een adverterende winkel te kopen. Om in een niet adverterende winkel te kopen moeten wel zoekkosten worden gemaakt. Dit zorgt ervoor dat consumenten bereid zijn meer te betalen in adverterende winkels.

Hoofdstuk 3 van dit proefschrift werkt de laatste conclusie van Hoofdstuk 2 verder uit. In plaats van eenvoudigweg aan te nemen dat een consument geen zoekkosten hoeft te maken om in een adverterende winkel te kopen, wordt in Hoofdstuk 3 aangenomen dat consumenten niet weten waar een bepaald product te koop is. Als een consument een advertentie ontvangt, weet hij dat de adverterende winkel het product verkoopt. De consument moet dan nog steeds zoekkosten maken om het product te kopen in de adverterende winkel, maar als hij (voor dezelfde zoekkosten) een niet adverterende winkel bezoekt, loopt hij het risico dat die winkel het product niet verkoopt en dat hij verder moet zoeken. Zolang niet elke winkel het product verkoopt is een bezoek aan een adverterende winkel gemiddeld genomen dus goedkoper. In Hoofdstuk 3 wordt aangetoond dat in bepaalde gevallen inderdaad niet in alle winkels het product verkocht wordt. Verder wordt aangetoond dat in deze gevallen adverterende winkels een hogere prijs kunnen vragen dan niet adverterende winkels.

In Hoofdstuk 4 van dit proefschrift wordt een model behandeld waarin winkels de zoekkosten van consumenten kunnen beïnvloeden door hun locatiekeuze. De zoekkosten van consumenten vallen in dit model uiteen in twee onderdelen. Consumenten maken kosten om naar een winkelcentrum te reizen, en als ze eenmaal in een winkelcentrum zijn, maken ze kosten om een individuele winkel te bezoeken. Winkels kunnen ervoor kiezen om zich te vestigen in een winkelcentrum waar nog geen concurrenten zijn (waar concurrent gedefinieerd is als een winkel die dezelfde producten verkoopt), maar ze kunnen zich ook vestigen in een winkelcentrum waar al wel concurrenten zijn. Als twee of meer concurrenten zich in hetzelfde winkelcentrum vestigen reduceert dit de reiskosten van consumenten. Het gevolg is meer concurrentie en lagere prijzen dan wanneer een winkel geen directe concurrenten heeft. Tegelijkertijd trekken de lagere prijzen meer consumenten naar het winkelcentrum met twee of meer concurrerende winkels. De analyse in Hoofdstuk 4 toont aan dat dit laatste effect vaak sterker is dan de reductie in prijzen: uiteindelijk maken winkels meer winst als ze zich in hetzelfde winkelcentrum vestigen.

Een ander interessant resultaat betreft de prijzen in winkels die geen directe concurrent hebben. Deze winkels kiezen vaak voor een prijs die hoger ligt dan de maximum prijs van concurrerende winkels, maar kiezen er soms ook voor om een prijs te vragen die onder de minimum prijs van concurrerende winkels ligt. Een interpretatie hiervan is dat geïsoleerde winkels gewoonlijk een hoge prijs vragen, maar daarnaast ook regelmatig aanbiedingen tegen zeer lage prijzen hebben.

Deel II. Dubbelcommissariaten

Hoofdstuk 5 van dit proefschrift betreft dubbelcommissariaten in Nederland. Iemand die in meerdere bedrijven de functie van commissaris vervult heeft een potentieel positief effect op het bedrijfsresultaat: een dubbelcommissaris heeft meer informatie en ervaring tot zijn beschikking dan iemand die slechts één commissarispositie bekleedt. Daar staat tegenover dat een dubbelcommissaris het drukker heeft. Bovendien is er een 'homogene groep effect' mogelijk. Dubbelcommissarissen horen vaak bij het 'old boys netwerk', een groep commissarissen die vrijwel allemaal dezelfde achtergrond hebben en elkaar regelmatig bij informele gelegenheden tegenkomen. Een raad van commissarissen die voornamelijk bestaat uit 'old boys' loopt een verhoogd risico op groepsdenken.

In Hoofdstuk 5 wordt met behulp van een unieke dataset onderzocht wat de effecten van dubbelcommissariaten zijn op de uiteindelijke prestatie van bedrijven. De dataset bestaat uit gegevens van 101 Nederlandse beursgenoteerde bedrijven, over de periode van 1994 tot en met 2004. Een eerste analyse betreft het effect van het aantal verbindingen dat een bedrijf via zijn commissarissen met andere bedrijven heeft (in het vervolg het aantal interlocks genoemd). Uit de analyse blijkt dat het aantal interlocks een zwak negatief effect heeft op de prestaties van een bedrijf. Verdere analyse betreft het effect van het tekort aan tijd waarmee dubbelcommissarissen te maken hebben en het 'homogene groep effect'. Het effect van drukke commissarissen is zoals verwacht negatief: meer drukke commissarissen in de raad van commissarissen leidt tot slechtere prestaties. Het 'homogene groep effect' is iets gecompliceerder. Een beperkt aantal 'old boys' in de raad van commissarissen heeft een positief effect op bedrijfsprestaties. Als 'old boys' echter de meerderheid gaan vormen in de raad van commissarissen verslechteren de resultaten.

Concluderend kan gesteld worden dat dubbelcommissarissen een licht negatief effect op de bedrijfsprestaties hebben. Dit wordt met name veroorzaakt door drukke commissarissen en een 'homogene groep effect'. Deze twee effecten zijn sterker dan het positieve informatie effect dat dubbelcommissariaten met zich meebrengen.

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