DO WE NEED ALL EURO DENOMINATIONS?

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Abstract

We show that there are theoretical arguments, which indicate that leaving out the 1-euro cent and 2-euro cent leads to more efficient cash payment behavior. Next, we show that having no access to 100-euro notes and 10-euro notes is less harmful than having no 50-euro notes. Finally, we argue that the transition from the Dutch guilder to the euro should not have led to different payment behavior, at least in theory.

Key words: Cash payment; euro transition

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1. Introduction

January 1, 2002 marked the launch of the euro in 12 European countries. In the Netherlands the transition from the guilder to the euro involved a transition to a different denominational structure. The rather unique 1-2½-5 series was replaced by the more common 1-2-5 series, which is generally accepted as the optimal denominational series of banknotes and coins. Indeed, Boeschoten and Fase (1989) already concluded that the Dutch guilder range was efficient to some extent, but that a transition to the more common 1-2-5 range would give benefits. This suggests that the structure of the euro range could decrease the number of notes and coins needed in everyday cash payments as compared to the less common 1-2½-5 range that was applied in the guilder era.

The guilder banknotes 1000, 250, 100, 50, 25 and 10 were replaced by the euro notes 500, 200, 100, 50, 20, 10 and 5 which in fact are rather similar in value (EUR 1 = NLG 2.20371), except for the 200-euro note. This banknote is new, as the guilder range did not include any banknote with a comparable value. Furthermore, the new euro coins consist of denominations 2, 1, 0.50, 0.20, 0.10, 0.05, 0.02 and 0.01, and this range involves two more coins than the guilder range used to have, which were 5, 2.50, 1, 0.25, 0.10 and 0.05 guilders. The 20-eurocent coin is new as it amounts to about 50 guilder cents, as well as the 1-eurocent coin, which is about 2 cents in guilders. It may be that the re-introduction of a 1-cent coin, though its value in euros is higher than a 1-cent guilder, could cause an increase in the use of coins in cash payments as all amounts to be paid are no longer rounded off to 5 cent amounts.

Given the changes in value and in the amount of coins and notes, it is of interest to see what the implications are of the transition from guilders to euros. In this paper we address this question by examining if the new euro range is more efficient than the old guilder range, where we take into account all kinds of different aspects of the two denominations. For this purpose we use the theoretical concept of efficient payments introduced in Cramer (1983).

In Section 2 of this paper, we discuss the concept of efficient payments. Next, in Sections 3 and 4, we see what happens if coins are or a single note is deleted from the denominational range. Finally, we conclude with a review of the main theoretical results.

2. Efficient payments in euros and guilders

The theoretical model of individual payment behavior in Cramer (1983) is based on the 'principle of least effort'. If individuals would behave according to this principle, each amount would be paid such that the number of notes and coins exchanged is minimized. Such payment schemes are the efficient payments.

Each payment amount has one or more efficient payment schemes. An illustration is the amount of € 11.30 that can be efficiently paid along three different ways, that is (i) 10 + 1 + 0.20 + 0.10, (ii) 10 + 1 + 0.50 and 0.20 returned and (iii) 10 + 2 and 0.50 + 0.20 returned.

In practice, it is unlikely that all actual payments are efficient because individuals might not behave according to the 'principle of least effort', nor will they all have the necessary denominations in their wallet. However, the Cramer model does provide a simple way to illustrate basic differences between denominational ranges, which is due to the fact that it can be applied to any denominational range. Cramer (1983) also gives an algorithm to generate all efficient payment schemes for a given range of amounts. This algorithm is described in full detail in the appendix of Kippers et al (2003). In this paper we use it to generate efficient payment schemes for all amounts between NLG 0.05 and NLG 220.35 for the guilder range. For the euro range, efficient payment schemes are generated using the same amounts, converted to euro, with EUR 99.99 being the highest. This will enable us to compare the difference between the two ranges.

In comparing the efficient payment schemes of two denominational ranges, one can distinguish between two aspects of efficiency. First, the smaller number of tokens that is exchanged on average, the more efficient is the range. Second, the more efficient payment schemes there exist for an amount, the more opportunities for individuals there are to make an efficient payment. Otherwise stated, the higher the probability that an efficient payment is made, the more efficient is the range. If we look at all efficient payment schemes and at the number of tokens used in each payment, we combine the two aspects of efficiency. That is, the more there exist efficient payment schemes with a small amount of tokens, the higher the efficiency of the range.

Table 1 shows the relevant statistics for all payment schemes for the guilder and euro range for NLG 0.05 up to and including NLG 100. Clearly, the differences between the guilder and euro range are small. The euro range does provide more efficient payment

schemes, but in contrast, the difference between the averages of tokens exchanged per payment scheme is negligible. This small difference between the two ranges is also shown by the equality of the median, minimum and maximum. Hence, we note that the payment process seems to have improved nor deteriorated in the Netherlands with the introduction of the euro.

3. Leaving out the 1-eurocent and 2-eurocent coin

Now, what happens if we get rid of 1- and 2-eurocent coins? In Finland, all amounts to be paid in cash are rounded to the nearest multiple of EUR 0.05. Although the 1-euro and 2-eurocent are still legal tender in Finland, their need is abolished due to rounding. We can investigate the theoretical effect of the Finnish example by applying Cramer´s algorithm to all amounts between EUR 0.01 and EUR 100, where the amounts are multiples of EUR 0.01 in one case and multiples of EUR 0.05 in another, thus starting with the amount EUR 0.05. The second exercise then shows the effects of abolishing the need for 1-euro and 2-eurocent coins.

The results shown in Table 2 are quite striking. The average number of tokens exchanged per payment scheme decreases from 5.83 to 4.93. Also, the maximum number of required tokens decreases from 8 to 7. This exercise tells us that payments can be done considerably more efficiently without the 1-eurocent and 2-eurocent coins.

4. Leaving out a single note

The current euro range contains one banknote denomination more than the previous guilder range. We now use the concept of efficient cash payments to analyze the effects of removing a single banknote denomination from the euro range.

We are interested in this issue for several reasons. First, it is generally accepted that a banknote range should not contain too many different denominations. This might cause confusion and perhaps inefficient payment behavior. If empirical analysis would show this is indeed the case for euro cash payments, it might be worthwhile to investigate the possibility of withdrawing a banknote denomination from circulation. Second, in cases of emergency it might be necessary to temporarily put one banknote denomination out of use (e.g. due to a

counterfeiting attack, strikes, or delivery failure). With the concept of efficient payments, we can understand the theoretical effects of removing one denomination from the current euro banknote range.

The starting point for our calculations is the complete euro banknote range complemented by a 1-euro coin. The resulting (virtual) range consists of the following denominations: 500, 200, 100, 50, 20, 10, 5 and 1 euro. Our focus is on banknote denominations, and by limiting the range to the smallest denomination of 1 euro we reduce computational efforts. We apply Cramer´s (1983) algorithm again to compute efficient payment schemes for all amounts between 1 and 1000 euro, for six different denominational ranges. The first is our basic range, which includes all denominations listed above. We subsequently remove 200, 100, 50, 20 and 10 euro from the denominational range. Therefore, these five ranges have one denomination less than the basic range.

Table 3 shows some characteristics of the resulting efficient payment schemes for each of the denominational ranges. The first row of the table shows that the number of efficient payment schemes decreases rapidly if a single banknote denomination is removed from the range. For example, in the full range, amounts can be paid efficiently with up to a maximum of 18 different payment schemes, while this maximum decreases to 10 or less if a single denomination is left out of the range. This means that individuals have lesser opportunities to make an efficient payment. As can be expected, and see the last row of Table 3, the average number of tokens exchanged, in an arbitrary amount that is paid efficiently, increases when the denominational range becomes smaller. This effect is largest when the 200- or 20-euro note is removed from the range, with an average of 4.8 tokens required to pay an amount efficiently, in contrast to the 4.5 tokens with the full range available.

If we consider the tokens exchanged per payment scheme, in which both aspects of efficiency are combined, we find the differences to be small. In all cases the maximum number of tokens used in a payment scheme is 8. The average number of tokens exchanged in a payment scheme even decreases if the 100- or 10-euro note is removed from the full range. This is explained by the fact that the reduction in efficient payment schemes, when the 100- or 10-euro note is removed from the range, mainly concerns those efficient payment schemes that involve many tokens (7 or 8).

In sum, we can conclude from this theoretical analysis that the removal of one banknote denomination does have a negative effect on the payment system, but the effect is not as

dramatic as one might have expected. If we compare across the different denominational ranges, we can conclude that the withdrawal of the 100- or 10-euro banknotes has the smallest negative effects. It will reduce the number of efficient payment schemes by 25%, and the average number of tokens exchanged per amount by only 2-2.5%. The 50-euro banknote, on the other hand, seems to be more important. Removing this banknote from the denominational range will increase the average number of tokens exchanged by 7%.

5. Conclusions

The transition to euro notes and coins does not seem to make a difference for the Dutch paying public, although the new 1-2-5 range of the denominations is perhaps a little more efficient than the old 1-2½-5 range of guilder notes and coins.

Our theoretical calculations show, that the payment system would benefit from a removal of the 1-cent and 2-cent coins, which would make the euro range more efficient. Finally, our computations suggest that the removal of a 10-euro or 100-euro banknote causes the range to be less efficient, but this loss in efficiency is small.

A natural subsequent study would involve observing payment behavior in practice with a full range of denominations as well as with limited ranges with a single denomination removed.

Table 1 Statistics on all payment schemes for amounts between NLG 0.05 and NLG 100

Efficient payment schemes	**guilder range**	**euro range**[1)]
number	14,928	16,151
average number of tokens exchanged	5.68	5.84
median	6	6
minimum	1	1
maximum	8	8

1) Amounts in NLG converted to amounts in EUR (1 EUR = NLG 2.20371)

Table 2 Statistics on all payment schemes for amounts between EUR 0.01 and EUR 100

Efficient payment schemes	**all euro denominations[1)]**	**without 1 and 2 euro cent coins[2)]**
number	36,591	5,957
average number of tokens exchanged	5.83	4.93
median	6	5
minimum	1	1
maximum	8	7

1) Amounts are multiples of EUR 0.01
2) Amounts are multiples of EUR 0.05

Table 3 Statistics on efficient payment schemes for all ranges. The amounts are in between 1 and 1000 euro with multiples of 1 euro

	all	leaving out a single note				
	denominations	200	100	50	20	10
payment schemes						
number	2553	1766	1873	2034	1862	1964
maximum	18	6	9	8	8	10
tokens used for each payment scheme						
average	5.18	5.26	5.10	5.34	5.27	5.12
Median	5	5	5	5	5	5
Minimum	1	1	1	1	1	1
Maximum	8	8	8	8	8	8
tokens used for each amount						
average	4.52	4.83	4.62	4.84	4.82	4.65

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