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Classifications	

Time Variation in Asset Return Dependence: Strength or Structure?*

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Abstract

The dependence between asset returns varies. Its strength can become stronger or weaker. Also, its structure can change, for example, when asymmetries related to bull and bear markets become more or less pronounced. To analyze these different types of variations, we develop a model that separately accommodates these changes. It combines a mixture of structurally different copulas with time variation. Our model shows both types of changes in the dependence between several equity market returns. Ignoring them leads to biases in risk measures. An underestimation of Value-at-Risk by maximum 15% occurs exactly when most harmful, during crisis periods.

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JEL classification: C32, F3, G15

1 Introduction

The dependence between asset returns is neither linear nor constant. Instead, asset returns exhibit asymmetric dependence, as negative returns show stronger dependence than positive returns. Furthermore, they exhibit tail dependence, because the dependence does not vanish when returns become more extreme. Also, the dependence between returns becomes stronger when volatilities increase or markets become more integrated. These changes can correspond with a strengthening or a weakening of the overall level of the dependence or can alternatively concern particular aspects of it such as tail dependence or asymmetry. We refer to the overall level of the dependence as its strength, and to particular aspects of the dependence as its structure.

Distinguishing changes in the strength of the dependence from changes in its structure is important. Mistaking a change in the structure for a change in the strength (or vice versa) can have dire consequences, in particular for risk management. For example, what if two asset returns change from being tail independent to being tail dependent? If a risk manager uses a model that only accommodates changes in the strength by explicitly specifying the dynamics of the correlation, then the new strength of the dependence is biased upward to compensate for the new structure. As a consequence, measures for the overall risk such as the portfolio's volatility are overestimated; but, because the model does not exhibit tail dependence, measures of tail risk like Value-at-Risk and Expected Shortfall are underestimated. In both cases, risk management is based on erroneous measures.

More generally, an investigation into the nature of the time variation in the dependence between asset returns can contribute to our understanding of the driving forces for this variation. We first have to understand what is changing before we can explain why it is changing. Changes in the strength, for example in correlations, can be hard to relate to other changes in the economy or financial markets when some of these changes are actually changes in the structure. This difficulty holds in particular when changes in the dependence strength happen at different points in time or with a different frequency than changes in the dependence structure.

In this paper, we develop a novel framework that allows for both changes in the structure and in the strength of the dependence. We propose mixture copulas[1] where both the mixture weights and the copula parameters can vary over time. Because copulas separate the dependence from the marginal distributions, they offer full flexibility in constructing a multivariate distribution for asset returns. Copulas that exhibit tail dependence and asymmetry are useful in various applications such as forecasting, risk management, derivative pricing, and asset allocation (see Patton, 2009, 2012, for surveys). Time variation in the copula parameters enables the overall level of the dependence to become stronger or weaker, while the structure stays the same. A time-varying mixture of copulas with different properties for (a)symmetry or tail (in)dependence can include the time variation in the structural aspects of the dependence.

We model the changes in the strength and the structure as switches between regimes. For the strength as well as the structure, one out of a fixed set of regimes prevails at each point in time, and switches between the regimes occur separately for both aspects. The regime processes are latent and follow first-order Markov chains as pioneered by Hamilton (1989, 1990). In this approach, we can formally test whether both types of time variation in the dependence are present. More generally, we can assess the relative importance of the changes in the strength and the structure of the dependence and examine whether the two types of changes occur independently or coincide.

So far, the research on the time variation in the dependence between asset returns has considered either changes in the strength or changes in the structure. The oldest literature on changes in the strength analyzes multivariate extensions of GARCH models, such as the

[1] See Joe (1997) and Nelsen (2006) for general introductions to copula theory and Cherubini et al. (2004) for applications in finance.

BEKK model introduced by Engle and Kroner (1995) and the DCC model proposed by Engle (2002) (see Bauwens et al., 2006; Silvennoinen and Teräsvirta, 2009, for a survey). Pelletier (2006) considers regime switching in correlation parameters. In the literature on stochastic volatility, some models with a separate latent process for correlations have been proposed (see Chib et al., 2009, for an overview). Patton (2006b) introduces conditional copulas where parameters depend on past observations (see also Bartram et al., 2007; Hafner and Manner, 2012). Jondeau and Rockinger (2006) introduce regime switching in copula parameters. Rodriguez (2007), Okimoto (2008) and Chollete et al. (2009) provide evidence of changes in the dependence structure with changes in the functional form of the copula that are governed by a Markov chain. Our approach joins these strands of literature that investigate regime switching by building one framework with both types of changes.

We build our framework on Markov regime-switching models that have become popular in financial modeling over the last decade.[2] In particular, prior research finds these models well suited to model changes in the dependence. Regime-switching models can accommodate changes between a limited set of dependence configurations. Ang and Chen (2002) and Ang and Bekaert (2002) find regime switching in correlations that can be related to cycles of bull and bear markets. Guidolin and Timmermann (2006a,b, 2007, 2008) provide evidence for regime switches in the correlations between bond and stock markets, and between international equity markets. Pelletier (2006) shows that regime switches in correlations can provide a better fit than the autoregressive changes of the DCC model of Engle (2002). Although most of these applications concern recurring changes among a limited set of dependence configurations, Chib (1998) shows that regime-switching models can also be used to accommodate nonrecurring changes. Such changes can typically be related to an increase in financial integration (see Bekaert and Harvey, 1995; Goetzmann et al., 2005; Quinn and Voth, 2010).

[2]See the surveys by Ang and Timmermann (2011) and Guidolin (2011a,b).

We apply our framework to examine the dependence between several different stock markets during the period of 1995-2008. Our mixture copula consists of the Gaussian copula, which is symmetric and tail independent, and the survival Gumbel copula, which exhibits asymmetry and lower tail dependence. Our results provide significant evidence of changes in the strength, i.e. the copula parameters, as well as the structure, i.e. the mixture weights. Periods of weak and strong dependence alternate frequently and can be related to periods of low and high volatility. The changes between the symmetric and asymmetric structures happen less frequently. For some market pairs, the periods with an asymmetric dependence structure occur after crisis periods, while for other pairs we only observe a single switch. The combination of asymmetric and strong dependence can be related to periods of turmoil in financial markets, such as the Asian crisis, the burst of the dotcom bubble, and the credit crunch. The changes in strength do not necessarily coincide with the changes in structure, which stresses the different nature of these changes.

To determine the practical relevance of properly capturing the time variation in the dependence, we take a risk-management perspective and compare Value-at-Risk (VaR) measures under different model specifications. Restricting changes to either the strength or the structure leads to biases in the VaR compared to the unrestricted model with both types of changes. Risk is mostly underestimated when the dependence is strong, asymmetric, and tail dependent, with measures that are up to 15% too low. Overestimation by a maximum of 10% generally occurs when the dependence is weak, symmetric, and tail independent. Taken together these results imply that risk is underestimated during periods of turmoil in financial markets, and overestimated during quiet periods. So, the consequences of the misspecification that arises from ignoring one kind of changes occur when their impact is worst.

Our finding of distinct changes in the strength and the structure contributes to the discussion on the causes of changes in the dependence. With mixed success, researchers have tried to link the time variation in correlations to explanatory variables. Most convincing is the evidence that market liberalizations and increased globalization lead to stronger dependence between equity markets (see Bekaert and Harvey, 1995; Goetzmann et al., 2005; Quinn and Voth, 2010). However, Bekaert et al. (2009) doubt that this explanation applies to the correlation movements between developed markets. Longin and Solnik (1995) and Bracker and Koch (1999) find that the state of the world economy contributes to explaining international correlation movements, though the evidence is weaker than for liberalization and globalization. Baele et al. (2010) find that stock-bond correlations can hardly be related to the state of the economy, though liquidity plays an important role. The difficulty of linking changes in the dependence to fundamental changes in markets and economies might be caused by our limited comprehension of what actually constitutes these changes. By allowing for distinct changes in different aspects of the dependence, our approach can disclose what is changing, which can help to explain why it is changing.

This paper proceeds as follows. In Section 2 we outline the time-varying mixture copula approach. In Section 3 we describe the international stock market returns data and the specific modeling choices for the empirical application. We discuss the estimation results in Section 4 and their implications for risk management in Section 5. We conclude in Section 6.

2 Methodology

In this section we describe the general set-up of the copula framework that comprises time variation for both the strength and the structure of the dependence. For the ease of exposition the model is described for the bivariate case, but the generalization to more than two variables is straightforward. We also show how tests for time variation in the strength and structure of the dependence can be implemented and provide details on the estimation procedure for our model.

2.1 General framework

We consider two random variables X and Y with realizations x and y. In our empirical application, X and Y represent the daily returns on different stock markets. The dependence between X and Y is completely characterized by their joint distribution $F_{\mathrm{XY}}(x, y)$. Sklar (1959)'s theorem states that we can express any joint distribution in terms of the marginal distributions F_{X} and F_{Y} and a copula function C:

$$F_{\mathrm{XY}}(x, y; \boldsymbol{\theta}_{\mathrm{X}}, \boldsymbol{\theta}_{\mathrm{Y}}, \boldsymbol{\theta}_{\mathrm{C}}) = C(F_{\mathrm{X}}(x; \boldsymbol{\theta}_{\mathrm{X}}), F_{\mathrm{Y}}(y; \boldsymbol{\theta}_{\mathrm{Y}}); \boldsymbol{\theta}_{\mathrm{C}}), \tag{1}$$

where $\boldsymbol{\theta}_{\mathrm{X}}$ and $\boldsymbol{\theta}_{\mathrm{Y}}$ denote parameter vectors for the marginals, and $\boldsymbol{\theta}_{\mathrm{C}}$ is a vector of copula parameters. If the marginal distributions F_{X} and F_{Y} are continuous, the copula function C is unique.

The decomposition in Equation (1) immediately shows the attractiveness of the copula approach for flexibly modeling dependence. Because the marginal distributions F_{X} and F_{Y} only contain information on the individual variables, the dependence between X and Y is governed completely by the copula C. Consequently, a wide range of joint distributions can be obtained by combining different marginals with different copulas. We assume that the marginal distributions F_{X} and F_{Y} are continuous and specified parametrically with coefficient vectors $\boldsymbol{\theta}_{\mathrm{X}}$ and $\boldsymbol{\theta}_{\mathrm{Y}}$. In this paper, we concentrate on the possible specifications of the copula function C to accommodate time-varying features in the dependence.

A useful way to characterize a copula is its so-called "quantile dependence" and the limiting case of tail dependence. Quantile dependence $\tau(q)$ is defined as the conditional probability that a realization of one variable lies above or below a given quantile q of its marginal distribution, given that the other realization lies above or below the same quantile,

$$\tau(q) = \begin{cases} C(q,q)/q & \text{for } q \leq 0.5 \\ (1-2q+C(q,q))/(1-q) & \text{for } q > 0.5. \end{cases} \tag{2}$$

The lower and upper tail dependence coefficients are defined as the limits of the quantile dependence measure, $\tau_{\mathrm{L}} = \lim_{q \downarrow 0} \tau(q)$ and $\tau_{\mathrm{U}} = \lim_{q \uparrow 1} \tau(q)$. Different copula specifications (also referred to as copula families) have different quantile and tail dependence characteristics. Elliptical copulas, such as the Gaussian and Student's t copulas, are symmetric with $\tau(q) = \tau(1-q), \forall q \in [0, 0.5]$; whereas other copulas, such as the Clayton and Gumbel copulas, are asymmetric. Correspondingly, copulas can have independence in both tails (e.g., the Gaussian copula); lower and upper tail dependence (e.g., the Student t copula); tail independence in one direction and tail dependence in the other (Clayton and Gumbel copulas).

Recent applications of copulas to asset returns frequently conclude that a single copula is not sufficient to describe the dependence between these series adequately (see Hu, 2006; Rodriguez, 2007; Okimoto, 2008; Chollete et al., 2009). A mixture of copulas yields more flexibility and a wider range of dependence patterns. Two copulas C_{a} and C_{b} can produce a mixture copula:

$$C(u,v) = \omega C_{\mathrm{a}}(u,v;\boldsymbol{\theta}^{\mathrm{a}}) + (1-\omega)C_{\mathrm{b}}(u,v;\boldsymbol{\theta}^{\mathrm{b}}), \tag{3}$$

where $0 \leq \omega \leq 1$ is the weight that determines the relative importance of the two copulas, and $u \equiv F_{\mathrm{X}}(x)$ and $v \equiv F_{\mathrm{Y}}(y)$ denote the marginal probability integral transforms (PITs). The copulas C_{a} and C_{b} can be from the same family, though with different parameters, but they can also be from different families with different properties.

A time-invariant copula such as Equation (3) cannot capture the changes in the dependence between asset returns for which ample empirical evidence exists. Assuming that the functional forms of the copulas C_{a} and C_{b} do not change, we can incorporate time variation in two ways. First, the copula parameters $\boldsymbol{\theta}^{\mathrm{a}}$ and $\boldsymbol{\theta}^{\mathrm{b}}$ can change over time, as considered by Jondeau and Rockinger (2006) and Patton (2006b) among others. Such dynamics lead to a time-varying *strength* of the dependence. Second, the mixture weights ω can vary over time, as in Rodriguez (2007), Okimoto (2008) and Chollete et al. (2009). Assuming that the constituents C_{a} and C_{b} are copulas from different families, such changes result in a time-varying *structure* of the dependence.[3]

Changes in the strength of the dependence can have rather different implications than changes in the structure of the dependence, though it is likely that the two can be mistaken for each other. Hence, it is important to distinguish between the two types of time variation. Obviously, it may also happen that both the strength and the structure change, but at different points in time. Both reasons demand a model that jointly accommodates both types of time variations in the dependence. We propose a time-varying mixture copula for changes in the strength of the dependence via the copula parameters and in the structure of the dependence via the mixture weight:

$$C_t(u,v) = \omega_t C_{\mathrm{a}}(u,v;\boldsymbol{\theta}_t^{\mathrm{a}}) + (1-\omega_t) C_{\mathrm{b}}(u,v;\boldsymbol{\theta}_t^{\mathrm{b}}). \tag{4}$$

To make the flexible mixture copula in Equation (4) work we opt for a regime-switching approach. The parameter vectors $\boldsymbol{\theta}_t^{\mathrm{a}}$ and $\boldsymbol{\theta}_t^{\mathrm{b}}$ can each switch between two different values. The switching between these states is governed by a first-order Markov process S_t^{θ} with transition probabilities $p_{ij}^{\theta} \equiv \Pr[S_t^{\theta} = j | S_{t-1}^{\theta} = j]$. For the dependence structure, we adopt a similar idea. We assume that the mixture weight ω_t can take two different values that depend on the value of a second Markov process S_t^{ω} with transition probabilities $p_{ij}^{\omega} \equiv \Pr[S_t^{\omega} = j | S_{t-1}^{\omega} = i]$.

Other possibilities for the evolution of the copula parameters (and also of the mixture

[3]When C_{a} and C_{b} are copulas from the same family, a time-varying mixture weight ω is observationally equivalent to time variation in the copula parameters $\boldsymbol{\theta}^{\mathrm{a}}$ and $\boldsymbol{\theta}^{\mathrm{b}}$.

weight) are available. Jondeau and Rockinger (2006), and Patton (2006b) have considered autoregressive specifications for the copula parameters $\boldsymbol{\theta}_t^{\mathrm{a}}$ and $\boldsymbol{\theta}_t^{\mathrm{b}}$. These kinds of models often show very strong persistence. The volatility literature argues that strong persistence points to the presence of large infrequent breaks or regime switches (see Diebold and Inoue, 2001; Gouriéroux and Jasiak, 2001; Lamoureux and Lastrapes, 1990, among others). Dias and Embrechts (2009) provide direct evidence in the context of copulas for the presence of infrequent structural changes in the dependence between exchange rates. These results motivate our choice of using Markov processes.

The Markov-switching nature of our model straightforwardly leads to testing that the strength of the dependence is constant while the structure can vary over time, and vice versa. Testing whether strength is constant corresponds with the null hypothesis $\boldsymbol{\theta}_1^{\mathrm{a}} = \boldsymbol{\theta}_2^{\mathrm{a}}$ and $\boldsymbol{\theta}_1^{\mathrm{b}} = \boldsymbol{\theta}_2^{\mathrm{b}}$ in Equation (4). A constant structure of the dependence implies the null hypothesis $\omega_1 = \omega_2$. These (likelihood ratio) tests suffer from the usual complications involved in specification tests in Markov-switching models due to the presence of unidentified nuisance parameters under the null hypothesis. Simulations are needed to obtain the distribution under the null hypothesis and the appropriate critical values (see Hansen, 1992; Garcia, 1998).

Our model can be easily extended in several directions. First, we assume that the Markov processes S_t^{θ} and S_t^{ω} are independent of each other (and independent of F_{X} and F_{Y}), that is, changes in the strength of the dependence and in the structure of the dependence occur independently. This assumption reduces the number of parameters. A more extensive model relaxing this assumption can also be used. Second, increasing the number of states of the Markov processes allows for more than two different values for the copula parameters and the mixture weight. This extension can be particularly useful when the framework of Markov processes is used for modeling nonrecurring structural changes. As demonstrated by Chib (1998) this type of time-variation may be achieved in the Markov-switching approach by restricting the transition probabilities such that the regimes occur in an irreversible sequence (see Pástor and Stambaugh, 2001; Pesaran et al., 2006; Pettenuzzo and Timmermann, 2011, for applications of this approach). Third, a mixture of more than two copulas can be considered.

2.2 Estimation

We use Maximum Likelihood (ML) to estimate the parameters in the time-varying mixture copula of Equation (4) with Markov-Switching specifications for S_t^θ and S_t^ω. ML is the obvious approach, because the marginal distributions F_X and F_Y are specified parametrically up to the unknown parameter vectors $\boldsymbol{\theta}_\mathrm{X}$ and $\boldsymbol{\theta}_\mathrm{Y}$. When the marginal distributions are unknown or to prevent the adverse effects from misspecification of the marginal models, the semiparametric copula-based multivariate dynamic models of Chen and Fan (2006) or other techniques based on quasi-ML can be used. Fermanian and Scaillet (2005) point out that misspecification of the marginal models can severely impact the estimation of the copula parameters.

In typical empirical applications, the total number of parameters quickly becomes large. For example, even though we consider fairly basic specifications for the marginals and the constituent copulas our most general time-varying mixture copula contains 26 parameters. In such cases, numerical optimization of the log likelihood function becomes a daunting task. As an alternative two-stage estimation method we adopt the Inference Function for Margins (IFM) procedure described in Joe (1997). The IFM method uses the decomposition of the complete log likelihood function in the log likelihood functions for the margins and for the copula. Differentiating Equation (1) yields the log likelihood function for the observation at time t:

$$\ell_t(x_t, y_t; \boldsymbol{\theta}_{\mathrm{X}}, \boldsymbol{\theta}_{\mathrm{Y}}, \boldsymbol{\theta}_{\mathrm{C}}) = \log c_t\big(F_{\mathrm{X}}(x_t; \boldsymbol{\theta}_{\mathrm{X}}), F_{\mathrm{Y}}(y_t; \boldsymbol{\theta}_{\mathrm{Y}}); \boldsymbol{\theta}_{\mathrm{C}}\big) + \log f_{\mathrm{X}}(x_t; \boldsymbol{\theta}_{\mathrm{X}}) + \log f_{\mathrm{Y}}(y_t; \boldsymbol{\theta}_{\mathrm{Y}}) = \ell_{\mathrm{c},t}(\boldsymbol{\theta}_{\mathrm{X}}, \boldsymbol{\theta}_{\mathrm{Y}}, \boldsymbol{\theta}_{\mathrm{C}}) + \ell_{\mathrm{X},t}(\boldsymbol{\theta}_{\mathrm{X}}) + \ell_{\mathrm{Y},t}(\boldsymbol{\theta}_{\mathrm{Y}}), \tag{5}$$

where f_{X} and f_{Y} are the densities that correspond to the marginals F_{X} and F_{Y}, and c_t is the density of the copula C_t. The IFM method boils down to the estimation of the parameters $\boldsymbol{\theta}_{\mathrm{X}}$ and $\boldsymbol{\theta}_{\mathrm{Y}}$ in the margins first by univariate ML,

$$\hat{\boldsymbol{\theta}}_{\mathrm{Z}} = \arg\max_{\boldsymbol{\theta}_{\mathrm{Z}}} \ell_{\mathrm{Z}}(\boldsymbol{\theta}_{\mathrm{Z}}), \quad Z = \mathrm{X},\ \mathrm{Y}, \tag{6}$$

where $\ell_Z(\boldsymbol{\theta}_Z) = \sum_{t=1}^{T} \ell_{Z,t}(\boldsymbol{\theta}_Z)$, with T denoting the sample size. In a second step, the parameters in the copula are estimated conditional on the estimated parameters for the margins:

$$\hat{\boldsymbol{\theta}}_{\mathrm{C}} = \arg\max_{\boldsymbol{\theta}_{\mathrm{C}}} \ell_c(\hat{\boldsymbol{\theta}}_X, \hat{\boldsymbol{\theta}}_Y, \boldsymbol{\theta}_{\mathrm{C}}) \tag{7}$$

where $\ell_c(\boldsymbol{\theta}_{\mathrm{X}}, \boldsymbol{\theta}_{\mathrm{Y}}, \boldsymbol{\theta}_{\mathrm{C}}) = \sum_{t=1}^{T} \ell_{c,t}(\boldsymbol{\theta}_{\mathrm{X}}, \boldsymbol{\theta}_{\mathrm{Y}}, \boldsymbol{\theta}_{\mathrm{C}})$. This two-step estimation procedure leads to consistent and asymptotically efficient estimators, (see Joe, 2005; Patton, 2006a). We compute standard errors for $\hat{\theta}_C$ that take into account the additional uncertainty due to the use of the estimated parameters for the margins. For the parameter vector $\hat{\boldsymbol{\theta}} = (\hat{\boldsymbol{\theta}}'_{\mathrm{X}}, \hat{\boldsymbol{\theta}}'_{\mathrm{Y}}, \hat{\boldsymbol{\theta}}'_{\mathrm{C}})'$ it holds that (see Patton, 2006a),

$$\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \overset{\mathrm{a}}{\sim} \mathrm{N}\left(0, \hat{\boldsymbol{H}}^{-1}\hat{G}\hat{\boldsymbol{H}}^{-1}\right), \tag{8}$$

where $\boldsymbol{H}$ and G are the Hessian and the outer product of the gradients.

Because the time-varying mixture copula Equation (4) depends on the latent Markov processes S_t^{θ} and S_t^{ω}, we follow the conventional approach of the EM algorithm as described in Hamilton (1989) to estimate the copula parameters. Applying the IFM approach requires that the parameters of the marginal models can be separated from the copula parameters. In our case, this means that the marginals F_{X} and F_{Y} cannot be subject to the regime-switching induced by S_t^{θ} and S_t^{ω}.

3 Data and model specification

3.1 Data

We examine the dependence between nine major stock markets: the United Kingdom (UK), Germany (GE), France (FR), the United States (US), Canada (CA), Mexico (MX), Japan (JP), Hong Kong (HK) and Korea (KO) over the period of 1995–2008. This set of stock markets contains important developed and emerging markets. Moreover, these markets have recently encountered different periods of tranquility and turmoil. This choice of markets and period ensures that our results are not driven by one particular event in financial markets, such as the credit crisis. The Asian crisis, the burst of the dotcom bubble and the introduction of the Euro are also part of our sample period.

We implement the time-varying mixture copula for six pairs of stock markets: UK-GE, UK-FR, US-CA, US-MX, JP-HK, and JP-KO. We choose these specific combinations because the two markets in each of these pairs do not suffer from asynchronous trading. If we combined Asian, European and American markets, their non-overlapping trading hours would seriously distort the dependence patterns in the daily returns. Although we could deal with the asynchronous trading hours by lowering the data frequency, this lower frequency might lead to estimation difficulties. Accurate estimation of the two Markov processes in combination with mixture copulas requires many observations.

We use the daily market index returns starting on July 3, 1995, when the emerging market data became available, to November 7, 2008. We use MSCI indices for all countries except Mexico and Korea, for which we use the IFC-S&P indices. To avoid any spurious correlations caused by holidays or other non-trading days, we remove days on which at least one of the markets is closed. This procedure results in a sample with $T = 3,250$ observations.

[Table 1 about here.]

Table 1 reports descriptive statistics of the daily returns. Canada and Mexico render the highest annualized returns, while Japan is the only market with a negative mean return. Volatilities are in the range 20–25% on an annualized basis, but Mexico and Korea show substantially higher volatilities, which reflect the higher levels of risk in these emerging markets. The skewness is positive for Japan and Korea, but negative for all of the other countries, which suggests that large negative returns occur more frequently in most markets. The kurtosis estimates range from 8.00 to 18.28 and indicate fat tails. The unconditional correlations between the stock markets pairs UK-GE, UK-FR, US-CA, US-MX, JP-HK, and JP-KO are 0.73, 0.81, 0.64, 0.60, 0.41, and 0.35 respectively. The European markets have the highest degree of co-movement in terms of correlation, followed by the American and Asian markets.

To get a first indication of the dependence structure, Figure 1 displays the exceedance correlations as used in Longin and Solnik (2001), Ang and Chen (2002), and Patton (2006b), among others. We compute the correlations given that both returns lie above or below a given quantile q of their marginal distributions.[4] For most country pairs, correlations in the left tail are higher than correlations in the right tail of the return distributions. The difference is most pronounced for the Asian countries, which have the lowest unconditional correlation. The UK and France have a higher correlation conditional on a positive return in both markets than conditional on two negative returns. They also

[4]We discuss the specification of the marginal distributions in more detail in Section 3.2.

have the highest unconditional correlation. The kink at zero indicates that for all country pairs the correlations conditional on negative returns are higher than the correlations conditional on positive returns. This kink shows the importance of allowing for asymmetry when modeling the dependence.

[Figure 1 about here.]

3.2 The marginal distributions

For the marginal distributions of the daily stock index returns X_t we use an AR(1)-Threshold GARCH(1,1) [TGARCH] model with a (standardized) skewed Student's t distribution for the innovations (cf. Jondeau and Rockinger, 2006; Chollete et al., 2009, among others). The GARCH part of the model captures the heteroscedasticity in the asset returns, while the skewed Student's t distribution can accommodate the remaining skewness and kurtosis. The model reads

$$X_t = \phi_0 + \phi_1 X_{t-1} + \varepsilon_t \tag{9}$$

$$\varepsilon_t = \sigma_t z_t \tag{10}$$

$$\sigma_t^2 = \psi + \alpha^+(\varepsilon_{t-1}^+)^2 + \alpha^-(\varepsilon_{t-1}^-)^2 + \beta\sigma_{t-1}^2 \tag{11}$$

$$z_t \sim \mathrm{st}(\nu, \lambda), \tag{12}$$

where $\varepsilon_t^+ = \max(\varepsilon_t, 0)$, and $\varepsilon_t^- = \min(\varepsilon_t, 0)$. The skewed Student's t density is given by

$$f_{\mathrm{st}}(z; \nu, \lambda) = \begin{cases} bc\left(1 + \frac{1}{\nu-2}\left(\frac{bz+a}{1-\lambda}\right)^2\right)^{-(\nu+1)/2} & \text{if } z < -a/b \\ bc\left(1 + \frac{1}{\nu-2}\left(\frac{bz+a}{1+\lambda}\right)^2\right)^{-(\nu+1)/2} & \text{if } z \geq -a/b, \end{cases} \tag{13}$$

with

$$a = 4\lambda c \frac{\nu - 2}{\nu - 1}, \quad b^2 = 1 + 3\lambda^2 - a^2, \quad \text{and} \quad c = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi(\nu-2)}\Gamma\left(\frac{\nu}{2}\right)}.$$

The skewness and kurtosis of X_t are nonlinear functions of the parameters ν and λ. A negative value of the parameter λ corresponds with a left-skewed density. To ensure positivity and stationarity of the conditional variance σ_t^2 we impose the restrictions $\psi > 0$, $\alpha^+, \alpha^-, \beta \geq 0$, and $(\alpha^+ + \alpha^-)/2 + \beta \leq 1$ in Equation (11).

We do not allow for regime-switching in the marginal distribution (cf. Jondeau and Rockinger, 2006; Chollete et al., 2009). Of course the possibility exists to include regime switching in, for example, the conditional volatility σ_t as in Okimoto (2008) (either induced by S_t^θ and S_t^ω or by a separate Markov process). However, this approach would preclude the use of the IFM method for parameter estimation. Instead we would have to resort to a one-step ML estimation of all parameters in the margins and the copulas. Another possible extension of the model for the marginal distribution is time variation in the parameters ν and λ as considered by Jondeau and Rockinger (2006, 2009).

3.3 The constituent copulas

The time-varying mixture copula should accommodate a variety of different dependence structures. Stock returns exhibit tranquil periods with symmetric dependence, but also periods of turmoil with asymmetric dependence and (lower) tail-dependence. Therefore, we select the Gaussian copula and the survival Gumbel copula to constitute the mixture copula. The Gaussian copula is the standard copula for tranquil periods. Okimoto (2008) shows that the survival Gumbel copula performs better in modeling asymmetry and tail dependence compared to other copulas such as the Joe and Clayton copulas. Hu (2006) also uses these copulas to examine the dependence structure of returns in developed equity markets (albeit in a mixture copula with constant weights).

The Gaussian copula has cdf

$$C_{\text{Gau}}(u, v; \rho) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v); \rho), \tag{14}$$

where $u \equiv F_{\text{X}}(x)$ and $v \equiv F_{\text{Y}}(y)$ are defined as before, Φ_ρ is the bivariate normal cdf with correlation ρ, and Φ^{-1} is the inverse of the univariate standard normal cdf. The corresponding density is given by

$$c_{\text{Gau}}(u, v; \rho) = \frac{1}{\sqrt{1-\rho^2}} \exp\left(\frac{-(r^2 - 2\rho rs + s^2)}{2(1-\rho^2)} + \frac{r^2+s^2}{2}\right), \tag{15}$$

where $r = \Phi^{-1}(u)$ and $s = \Phi^{-1}(v)$. The normal copula exhibits independence in both the lower and upper tails unless $|\rho| = 1$ (see Embrechts et al., 2003).

The survival Gumbel copula has cdf

$$C_{\text{Gum}}(u, v; \delta) = u + v - 1 + \exp\left(-[(-\ln(1-u))^\delta + (-\ln(1-v))^\delta]^{\frac{1}{\delta}}\right) \tag{16}$$

and the corresponding density

$$c_{\text{Gum}}(u, v; \delta) = \frac{(\ln(1-u)\ln(1-v))^{\delta-1} C^{Gum}(1-u, 1-v; \delta)}{(1-u)(1-v)((-\ln(1-u))^\delta + (-\ln(1-v))^\delta)^{2-\frac{1}{\delta}}} \times (\delta - 1 - \ln C_{\text{Gum}}(1-u, 1-v; \delta)), \tag{17}$$

where the parameter $\delta \in [1, \infty)$. The strength of the dependence increases with δ, and $\delta = 1$ ($\delta \to \infty$) corresponds to independence (perfect dependence). The survival Gumbel copula exhibits independence for the upper tails, but tail dependence for the lower tails with coefficient $\tau_L = 2 - 2^{\frac{1}{\delta}}$.

When using the Gaussian and survival Gumbel copulas in Equation (4), the parameters ρ and δ vary according to the value of S_t^θ. For the identification of the regimes with different strengths of dependence, we impose the restriction $\rho_1 < \rho_2$. No restrictions are put on the parameters of the Gumbel copula, but it turns out that for all country pairs the estimates are such that $\delta_1 < \delta_2$. Hence, we can characterize the regime $S_t^\theta = 1$ ($S_t^\theta = 2$) by weak (strong) dependence. Similarly, for identification purposes we impose the restriction $\omega_2 < \omega_1$, so that the weight on the Gaussian copula in the first regime $S_t^\omega = 1$ is larger than in the second regime $S_t^\omega = 2$. For this reason we label these regimes as symmetric and asymmetric dependence.

In addition to the general time-varying mixture copula Equation (4), we estimate several nested, restricted versions of the model. By examining the loss in the likelihood due to the imposed restrictions, we can assess which characteristics are the most important in modeling the dependence of the equity returns. As discussed in the previous section, we conduct a likelihood ratio test of the null hypothesis $\omega_1 = \omega_2$ to test a constant dependence structure. The restricted model with switching copula parameters but a constant mixture weight is given by

$$C(u, v; \boldsymbol{\theta}_{\mathrm{C}}) = \omega C_{\mathrm{Gau}}(u, v; \rho_t) + (1 - \omega) C_{\mathrm{Gum}}(u, v; \delta_t). \tag{18}$$

Similarly, we conduct a likelihood ratio test of the null hypotheses $\rho_1 = \rho_2$ and $\delta_1 = \delta_2$ to test for constant strength in the dependence. These hypotheses correspond with a model where the copula parameters are constant, but the mixture weight can switch according to the value of S_t^ω,

$$C(u, v; \boldsymbol{\theta}_{\mathrm{C}}) = \omega_t C_{\mathrm{Gau}}(u, v; \rho) + (1 - \omega_t) C_{\mathrm{Gum}}(u, v; \delta). \tag{19}$$

We examine two other previously considered restricted copula specifications. We implement the model proposed by Okimoto (2008), which assumes constant copula parameters, while the mixture weights in the two regimes are set equal to $\omega_1 = 1$ and $\omega_2 = 0$, such that the dependence structure switches between a Gaussian copula and a survival Gumbel copula,

$$C(u, v; \boldsymbol{\theta}_{\mathrm{C}}) = \begin{cases} C_{\mathrm{Gau}}(u, v; \rho) & \text{if } S_t^{\omega} = 1, \\ C_{\mathrm{Gum}}(u, v; \delta) & \text{if } S_t^{\omega} = 2. \end{cases} \tag{20}$$

We also consider the specification by Hu (2006) with constant copula parameters and a constant mixture weight,

$$C(u, v; \boldsymbol{\theta}_{\mathrm{C}}) = \omega C_{\mathrm{Gau}}(u, v; \rho) + (1 - \omega) C_{\mathrm{Gum}}(u, v; \delta). \tag{21}$$

4 Estimation Results

We estimate the marginal models and mixture copulas for the selected six combinations of stock markets. In this section, we provide detailed results for the dependence between the stock markets of the UK and of Germany. We analyze the patterns in the regime processes for the variation in the strength and in the structure of the dependence, and test for their significance. We finish with a brief summary of the results for the other combinations. We provide detailed results in the appendix.

4.1 Estimation results for the UK and Germany

The results from the marginal models for the UK and Germany confirm the stylized facts for the time-series of asset returns. Shocks to volatility are highly persistent, but negative shocks have a stronger impact than positive ones. The shocks show evidence of negative skewness and mildly fat tails, which supports our choice for a skewed Student's t distribution. Table A.1 shows the parameter estimates and standard errors.

For the IFM method, it is of crucial importance that the marginal models are correctly specified, because otherwise the estimates of the copula parameters in the second step can be biased (see Fermanian and Scaillet, 2005). We apply the Kolmogorov-Smirnov, Cramer-Von Mises and Anderson-Darling tests to examine the goodness-of-fit. The p-values for the UK exceed 0.90 for all tests, and for Germany they are all above 0.71, indicating we can safely proceed with the second step of the IFM procedure.[5]

In Table 2, we report the results for the unrestricted model with changes in the strength as well as the structure of dependence, and for restricted versions of the model discussed in the previous section. The estimates for the unrestricted model in column 1 show different regimes for both the strength and the structure of the dependence. First, regimes with relatively weak and strong dependence are well defined. For identification purposes, we imposed the restriction $\rho_1 < \rho_2$ for the Gaussian copula. We find in addition that $\delta_1 < \delta_2$ for the survival Gumbel copula. Both copulas thus have weaker dependence in the regime $S_t^\theta = 1$. Second, the weights on the Gaussian copula take the values $\omega_1 = 1$ and $\omega_2 = 0.41$, such that the dependence in the regime $S_t^\omega = 1$ is symmetric and tail independent, while it is asymmetric with lower tail dependence in the regime $S_t^\omega = 2$. The standard errors of the copula parameters are fairly small, which indicates precise estimates.[6]

[Table 2 about here.]

The transition probabilities for the S_t^θ process, $p_{11}^\theta = 0.998$ and $p_{22}^\theta = 0.998$, indicate high persistence of the regimes for strength. The unconditional probabilities for the weak and strong dependence regimes are 0.60 and 0.40.[7] So the process S_t^θ spends slightly more time in the low strength regime, irrespective of the structure of the dependence. For the dependence structure the transition probabilities $p_{11}^\omega = 0.996$ and $p_{22}^\omega = 0.996$ imply that the symmetric and the asymmetric regimes are also highly persistent and that both occur

[5]The actual p-values for the UK (Germany) are Kolmogorov-Smirnov 0.943 (0.831), Cramer-Von Mises 0.904 (0.846), and Anderson-Darling 0.902 (0.710).

[6]Whenever a parameter estimate reaches a boundary, we impose this value and only compute standard errors for the remaining parameters.

[7]The unconditional probabilities are given by $P(S_t^\theta = 1) = (1 - p_{22}^\theta)/(2 - p_{11}^\theta - p_{22}^\theta)$ and similar for $P(S_t^\theta = 2)$.

about half of the time. The unconditional probability for the symmetric (asymmetric) regime is equal to 0.47 (0.53).

Figure 2 shows the smoothed inference probabilities for the weak dependence regime ($S_t^\theta = 1$) and for the symmetric dependence regime ($S_t^\omega = 1$). The regime with asymmetry and tail dependence seems to occur mostly when the UK and German markets are bearish or suffer from periods of turmoil with high volatility. Switches from symmetric to asymmetric dependence frequently coincide with the occurrence of a financial crisis, e.g. the Asian crisis (July 1997), the Russian crisis (August 1998), the burst of the dot-com bubble (March 2001), and the start of the credit crisis (January 2007). The dynamics in the strength of the dependence are less easily related to macroeconomic or financial circumstances, although the strong dependence regime occurred during the burst of the dot-com bubble. Moreover, the strength of the dependence has increased over time, and has been in the strong dependence regime since the beginning of 2004. This might be an indication of higher financial integration, globally or on the European level.

[Figure 2 about here.]

We investigate the restricted models in Equations (18) to (21) (labeled 2–5 in Table 2) to determine the importance of the different sources of the time variation. Imposing a constant dependence structure (column 2) yields one regime with low strength, slight asymmetry, and tail dependence, and a second regime in which these aspects are much more pronounced. The mixture copula puts a weight of 0.707 on the Gaussian copula and 0.293 on the Gumbel copula. This estimate corresponds roughly with the average of ω_1 and ω_2 in the unrestricted model. For the weak dependence regime, the Gaussian correlation parameter increases, but the Gumbel parameter decreases, compared to the unrestricted model. Parameters for the strong dependence regime hardly change. The likelihood ratio (LR) statistic of this model against the unrestricted model equals 16.73.

Due to the presence of the unidentified nuisance parameters p_{11}^{ω} and p_{22}^{ω} under the null hypothesis, the LR-statistic does not follow a $\chi^2(1)$-distribution. A simulation of this distribution as in Hansen (1992) leads to a p-value of 0.032, which indicates the relevance of the changes in the dependence structure. The smoothed inference probabilities for the restricted model in Figure 3a coincide to a large extent with the corresponding smoothed inference probabilities of the unrestricted model (black line). This result indicates that the restricted model only captures the changes in the strength and does not capture the changes in the structure in any way.

[Figure 3 about here.]

In the model that only accommodates changes in the structure of the dependence (column 3 in Table 2), the Gaussian copula dominates one regime with $\omega_1 = 0.962$ and a large correlation parameter of 0.855. This regime exhibits strong and symmetric dependence. In the other regime, the Gumbel copula dominates ($\omega_2 = 0.191$), but its parameter of 1.439 implies only weak asymmetric dependence. The strong asymmetry and the tail dependence of the unrestricted model do not show up in this restricted specification. The LR-statistic of 69.5 strongly rejects this restriction. The smoothed inference probabilities for the symmetric regime of the constant-strength model in Figure 3b do not resemble the pattern for the symmetric regime of the unrestricted model at all. The resemblance with the strong dependence regime of the unrestricted model is clearer. It seems that the restricted model tries to capture changes in strength by changes in structure. As a consequence, the restricted model fails to accommodate strong asymmetry and tail dependence.

The model proposed by Okimoto (2008) in column 4 further restricts the structure-only regime in column 3, because the copula must be either fully Gaussian or fully survival Gumbel. The parameter estimates are close to those for the structure-only model, but lead to a further deterioration of the likelihood function. Even though this model includes the survival Gumbel copula, it misses out on asymmetry and tail dependence. The LR-statistic points at a strong rejection of this model in favor of the more flexible unrestricted model with changing parameters and freely changing mixture weights.

Hu (2006) proposes a constant mixture of Gaussian and survival Gumbel copulas with constant parameters. The estimates in column 5 of Table 2 do not support his unconditional mixture model, because the survival Gumbel copula does not receive any weight. Imposing constant dependence leads to a very large difference in likelihood.

Overall, these results show strong evidence for time variation in the dependence. Moreover, modeling changes only in strength or in structure is not sufficient, because the data provides significant evidence for changes in both.

4.2 Estimation results for other pairs of equity markets

Our analysis for the other country pairs confirms our conclusions based on the UK and German equity markets. We summarize the main results here, and provide a detailed discussion in Appendix A.2.

All country pairs show two distinct regimes for the strength of the dependence. Table 3 shows that both the Gaussian correlation parameter ρ and the Gumbel parameter δ are substantially higher in the strong dependence regime $S_t^\theta = 2$ than in the weak dependence regime $S_t^\theta = 1$. The correlation parameters of the Gaussian copulas indicate positive but mild dependence in the weak regime (estimates vary between 0 (JP-KO) and 0.58 (US-MX)), and strong dependence in the other regime (estimates range from 0.47 (JP-KO) to 0.89 (JP-HK)). The markets of the UK and France are always highly correlated, with parameters $\rho_1 = 0.823$ and $\rho_2 = 0.931$. The parameters for the Gumbel copulas switch from 1.1–1.4 for the weak regime to 1.8–3.5 for the strong regime. So, both copulas contribute to the increase in the dependence when the process switches to the strong dependence regime.

[Table 3 about here.]

For all of the market pairs, the Gaussian and Gumbel copulas are necessary to model dependence. Table 3 shows that the Gaussian copula dominates the structure regime $S_t^\omega = 1$ with weights ω_1 varying between 0.864 (US-MX) and 1 (US-CA). The Gumbel copula contributes the most to the other regime with weights for the Gaussian copula between 0 (US-CA) and 0.453 (JP-KO). So, for all of the pairs, the first regime is mostly symmetric with some asymmetric influence, but the second regime can be characterized as predominantly asymmetric. The US-CA pair has a fully symmetric Gaussian regime as $\omega_1 = 1$, and a fully asymmetric Gumbel regime as $\omega_2 = 0$. For the other pairs, we reject the restriction $\omega_1 = 1$ and $\omega_2 = 0$. We conclude that most market pairs need the mixture of symmetric and asymmetric copulas in both regimes.

The transition probabilities are high (the lowest estimate is $p_{22}^\theta = 0.871$ for the JP-KO pair), which implies that all regimes are highly persistent. In particular, Figure 4 shows only occasional switches in the structure of dependence. For the pairs UK-FR, US-ME and JP-KO, only one switch in the dependence structure is present with the symmetric regime dominating in the second half of our sample period (roughly after July 2001). The strength regimes switch more often. The low strength regime prevails mostly during quiet periods when markets increase. The Japanese and Korean markets switch to a regime with strong dependence only once, after July 2001. Though all market pairs show pronounced switches around July 2001, the smoothed inference probabilities do not indicate that the switches are highly synchronized.

[Figure 4 about here.]

All market pairs show switches in both the structure and the strength of the dependence. Models in which changes are limited to strength are rejected with p-values of 0.022 and lower. The evidence for changes in strength are even stronger, as models with only changes in structure are rejected with p-values below 0.001 for all market pairs. The models of Okimoto (2008) and Hu (2006) are also strongly rejected in favor of the unrestricted model. We conclude that accounting for either the time variation in the structure of the dependence, as in Rodriguez (2007), Okimoto (2008) and Chollete et al. (2009), or the time variation in the strength of dependence, as in Jondeau and Rockinger (2006), Bartram et al. (2007) and Hafner and Manner (2012), does not suffice to accurately model the dependence between international stock markets.

4.3 Robustness checks

As a robustness check we consider two alternatives for the marginal distributions. First, we adopt the semi-parametric approach of Chen and Fan (2006). We estimate the AR(1)-TGARCH(1,1) model with quasi-ML with a normal distribution for the innovations z_t. Then we use the empirical CDF of the standardized residuals $\hat{z}_t$ to obtain the required input for the copula estimation. Second, we use the empirical CDF's of the returns themselves as marginals. The first alternative gives results that are almost identical to those obtained with the fully parametric marginal specification in Equations (9) to (12). For the second alternative the general patterns in the copula estimates remain similar, but for some specific parameters the differences with the fully parametric model are somewhat larger.[8]

An important assumption in our time-varying mixture copula is that changes in the strength and structure of the dependence occur independently. We test the validity of this assumption by estimating the same copula specification, but with the switches among the four regimes driven by a single Markov process with unrestricted transition probabilities. A likelihood ratio test can then be straightforwardly conducted to test the null hypothesis that the transition probabilities can be restricted in accordance with the independence of the Markov processes S_t^θ and S_t^ω. The test statistic gives p-values of 0.27 (UK-GE), 0.69

[8]Details are available upon request.

(UK-FR), 0.77 (US-CA), 0.10 (US-MX), 0.31 (JP-HK), and 0.08 (JP-KO). Therefore, we cannot reject the independence of changes in the strength and structure of the dependence for any of the market pairs.

5 Implications for risk management

The estimation results provide statistical evidence that a dependence model should accommodate time variation in both the strength and the structure of the dependence. In this section we assess the economic implications of our findings. We focus on risk management, because failing to account for particular changes in the dependence can lead to biased risk measures. A misspecified model with a symmetric and tail-independent copula can still produce a reliable estimate of the overall dependence. However, it fails to properly indicate the dependence between extreme returns and leads to (downside) risk measures that are sometimes too low or too high. We first investigate how the dependence between negative extreme returns varies over the different regimes. Next we compare Values-at-Risk to determine the impact of the misspecification of the dependence.

5.1 The dependence between extreme returns

The Gaussian and Gumbel copulas that constitute the mixture copula have different implications for the dependence between returns. These differences pertain in particular to extreme returns. Because the regimes differ in the weights attributed to both copulas and in the parameters that determine the strength of the dependence, the exact impact for the dependence in the different regimes is not obvious. Therefore, we examine tail dependence and exceedance probabilities in each regime.

The lower tail-dependence coefficients in Table 4a show a substantial variation in the dependence between extreme returns. In the asymmetric regimes, coefficients range from

0.11 (US-MX) to 0.36 (UK-GE) when the dependence is weak, and from 0.31 (JP-KO) to 0.78 (US-CA) when the dependence is strong. This strong tail dependence reflects the high weight that the Gumbel copulas receives in the asymmetric regime. When the asymmetric regime prevails, the diversification opportunities decrease or completely vanish. To the contrary, tail dependence is small or absent when a symmetric regime prevails, no matter whether the prevailing strength regime is weak or strong. When the Gaussian copula receives all of the weight in the symmetric regime (UK-GE and US-CA), the coefficient of the lower tail-dependence equals zero, because the Gaussian copula implies tail independence (unless $\rho = 1$). For all other market pairs, the Gumbel copula receives a small weight, which leads to small non-zero coefficients of tail dependence. For US-MX, for example, the weight on the Gumbel copula in the symmetric regime is 0.14, so the lower tail-dependence coefficient is 0.02 (0.07) in the weak (strong) dependence regime.

[Table 4 about here.]

Although these lower tail-dependence coefficients show large differences in the dependence between extreme returns across the different regimes, they do not tell a complete story. The tail dependence coefficients are limit concepts and apply to very extreme returns. In Table 4b, we investigate the exceedance probabilities $\tau(q)$ defined in Equation (2) for the less extreme case of $q = 0.05$. The differences are the largest between the weak and strong dependence regimes. When the dependence is weak, the probability that the return in one market falls below the 0.05 quantile, conditional on a realization in the other market below this quantile ranges from 0.05 (JP-KO) to 0.52 (UK-FR). When the dependence is strong, this probability varies from 0.23 (JP-KO) to 0.79 (US-CA). Under this regime, losses in one market are very likely to be accompanied by losses in the other market. The effect of the changes in the dependence structure are mixed. For the pairs UK-GE and JP-KO, the exceedance probabilities increase substantially by about 0.20 when a switch from the symmetric to the asymmetric regime takes place. For the US-CA combination, the exceedance probability in the weak dependence regime does not change when a switch to asymmetry takes place, but in the strong dependence regime the probability doubles from 0.40 to 0.79. For the pairs US-MX and JP-KO the switches in the structure of the dependence hardly influence the exceedance probabilities. For UK-FR, a switch to the asymmetric regime leads to small decreases.

5.2 Value at Risk

To determine the importance of accurate models for the dynamics of the dependence, we examine VaRs. The estimation results and the differences in the regimes with regard to the dependence between extreme returns indicate that both the structure and the strength of the dependence should be modeled accurately. However, these outcomes do not indicate what the practical consequences of an incorrect specification are. Therefore, we compare the VaR that results from the unrestricted model with both changes in the strength and in the structure to the VaR that results when only one type of change is allowed. We calculate these VaR measures for a portfolio that is equally invested in both markets.

For a confidence level q, VaR_q corresponds with the q-th quantile of the portfolio loss distribution Z,

$$\mathrm{VaR}_q = \arg\max\{z : \Pr(Z \geq z) \geq 1 - q\}.$$

We perform a simulation study to compare the VaR obtained from the unrestricted time-varying mixture copula with the models with time variation in either only the strength or only the structure of the dependence. We vary the probability of the weak dependence regime and the symmetric dependence regime between 0 and 1 with steps of 0.05. This results in $21^2 = 441$ combinations of regime probabilities in the strength-structure plane. For each combination, we generate one million drawings from the copulas. Following

Chollete et al. (2009), we then use the inverse cdf of the normal distribution to transform the copula drawings in the $[0, 1]$-domain to returns. We then form equally-weighted portfolios from the simulated returns and compute the VaR_q by taking the q-th quantile.[9] We follow the same procedure for the restricted models where one of the two regime probabilities can obviously be ignored.

Following Okimoto (2008) and Chollete et al. (2009), we compute the ratio of the VaR measures for the unrestricted model, $\mathrm{VaR}_q^{\mathrm{U}}$ to that of the restricted model, $\mathrm{VaR}_q^{\mathrm{R}}$. We assume that the unrestricted model with changes in both the strength and the structure of the dependence is the true model, because of the strong rejections of the restricted models. Consequently, a ratio of VaRs above (below) one indicates that the restricted model underestimates (overestimates) risk.

We differ in one important aspect from the design of the analysis in Okimoto (2008) and Chollete et al. (2009). We calculate conditional VaR measures, because we fix the regime probabilities at specific values. To the contrary, Okimoto (2008) and Chollete et al. (2009) compute unconditional VaR measures, because they average across the regimes with different structures of the dependence. Although their conclusions are valid in an unconditional sense, they need not necessarily hold at each point in time. Instead, our conditional approach can show in what situation a restricted model leads to the over- or underestimation of risk.

Figure 5 shows the VaR_q ratios for $q = 0.99$. In the left panels, the restricted models accommodate only changes in the strength and impose a constant structure of the dependence. In the right panels, the restricted models accommodate only changes in the structure and impose a constant strength of the dependence. Overall, both restrictions lead to both under- and overestimation of VaR. The underestimation of risk is more often prevalent than the overestimation, and the degree of underestimation is also larger. When

[9] Because we model the returns of the portfolio, our approach actually produces VaR relative to the initial portfolio value.

a model ignores changes in the structure, the VaR ratios vary between 0.92 and 1.10. When changes in the strength are excluded, the ratios lie between 0.90 and 1.15. The size of these effects puts our results in line with those of Okimoto (2008), because he reports ratios between 0.96 and 1.15. The results reported by Chollete et al. (2009) tend to be a bit smaller. In line with the statistical evidence, the changes in the strength are more important than the changes in the structure. When VaR measures are used to determine capital reserves, the model misspecification leads directly to misalignments of capital.

[Figure 5 about here.]

[Figure 5 (continued) about here.]

Next, we consider each market pair separately. For the UK-GE pair (Figure 5a), ignoring changes in the structure of the dependence leads to an overestimation of the $\text{VaR}_{0.99}$ by at most 4%. This overestimation occurs when the low strength and the asymmetric structure regimes prevail. The largest underestimation of 3% occurs for the low strength and the symmetric structure regimes. When a high strength regime applies, the VaR ratios are closer to one. Figure 5b shows that the restriction of constant strength leads to larger deviations in the VaR. The largest underestimation of 5% shows up for the combination of high strength with an asymmetric structure. In the opposite combination of the low strength and the symmetric structure regimes, an overestimation of 10% arises. For both restrictions, the underestimation occurs exactly when the dependence is asymmetric, which happens during crisis periods, like July 2001–July 2004 and from July 2007 onwards (see Figure 2).

For the pair UK-FR in Figure 5c, the absence of changes in structure leads to a maximum underestimation of the VaR of 4%, when the combination of low strength and a symmetric structure applies. Overestimation does not really occur. To the contrary, ignoring the changes in the strength (Figure 5d) leads to an underestimation of a maximum of 4% when the dependence is strong and asymmetric. When the dependence is weak, we observe a small underestimation of the VaR of about 1% independent of the probability for the symmetric regime. When we combine these results with the smoothed inference probabilities in Figure 4a, we see that the changes in the structure matters after March 2001, while the changes in the strength are important around July 1998.

Figures 5e and 5f for the US-CA pair show more severe underestimation. Ignoring changes in the structure of the dependence leads mostly to underestimation with a maximum of 6%. For changes in strength, we observe a maximum underestimation of 10%, and a maximum overestimation of 6%. In both cases, the underestimation is largest when the strong and asymmetric dependence regimes prevail, which happens during the crisis periods around October 1997 (Asian crisis), July 2001 (burst dotcom bubble), and July 2007 (credit crunch).

For the US-MX pair, imposing a constant copula structure leads again mostly to an underestimation of the risk in Figure 5g. This underestimation is largest (3%) when the symmetric and low strength regimes occur. When a constant strength of the dependence is imposed (Figure 5h), the underestimation is more severe with a maximum of 10%. As with the US-CA pair, the largest underestimation results from the combination of the asymmetric structure and the high strength. This combination prevails during the Asian crisis.

The pair JP-HK shows a large contrast in Figure 5i and Figure 5j. Ignoring changes in structure seems almost inconsequential with both over- and underestimations limited to 1–1.5%. To the contrary, ignoring changes in strength can lead to an underestimation of up to 15%. This underestimation is present for most combinations of regime probabilities, and increases when the probability of strong dependence rises. Periods of strong dependence happen frequently between 1997–2001 and particularly after July 2004.

The final pair JP-KO shows large variation in under- and overestimations of the VaR

(Figures 5k and 5l). When the restrictions concern changes in the structure, the VaR-ratios vary between 93% and 110%. The maximum results from the combination of the asymmetric structure and the weak dependence, and the minimum results from the symmetric structure and the weak strength. If the strength becomes higher, the effect of imposing a constant structure becomes less consequential. When changes in strength are ignored, differences are even larger. The maximum underestimation of 15% corresponds with a strong symmetric dependence, which occurs from July 2001 to July 2006. The maximum overestimation of 8% coincides with weak asymmetric dependence, which occurs from July 1995 to March 1998.

This detailed analysis shows that ignoring changes in strength means that risk is underestimated when the actual strength regime is strong. Periods of strong dependence occur frequently, mostly during the bear markets after the Asian crisis, the burst of the dotcom bubble, and the credit crisis. Ignoring changes in the structure is also most costly during these periods. It is more difficult to link the effect directly to the regime that actually applies (i.e., symmetric or asymmetric), because the prevailing structure regimes are not strongly synchronized. Some countries actually show only a single switch.

One might argue that restricting the changes in the strength or in the structure only matters in extreme situations and for risks far out in the tails. In Figure 6 we therefore consider ratios for Value-at-Risk with a 95% confidence level. The graphs in the figure are similar to those in Figure 5, and we also see that the magnitude of the ratios has not diminished. So for less extreme $\text{VaR}_{0.95}$ measurements also, we conclude that accurately modeling the changes in the strength and in the structure of the dependence matter.

[Figure 6 about here.]

[Figure 6 (continued) about here.]

6 Conclusion

Both the strength and the structure of the dependence between asset returns can vary over time. Changes in the strength affect the overall level of the dependence, whereas changes in the structure relate to (a)symmetry and tail (in)dependence. We propose a new approach to model the dynamics of the dependence that combines mixture copulas with Markov regime-switching models. Our analysis of six pairs of stock markets provides evidence for the presence of distinct sets of regimes for both the strength and the structure of the dependence. Switches between a weak or mild dependence regime and a strong one happen regularly. The strong dependence regime often coincides with periods of high volatility following financial crises. Changes from a symmetric, tail-independent structure to an asymmetric, tail-dependent one happen less frequently. For some countries we observe only a single switch, while for others we observe a few. While switches in strength show some commonality, switches in the dependence structure do not seem synchronized. The combination of a high strength regime and an asymmetric, tail-independent regime shows up most around crisis periods such as the Asian crisis, the burst of the dotcom bubble, and the credit crunch.

We find that both types of changes are important for an accurate model of dependence. Models in which either the strength or the structure of the dependence can change can be regarded as misspecified. Likelihood ratio tests reject these restricted models in favor of a model with both types of changes. Applying a misspecified model with only one type of changes in the dependence leads to biases in risk measures. A risk manager that calculates the Value-at-Risk for an equally weighted investment in a pair of stock markets sometimes underestimates the VaR by a maximum of 15%, while maximally overestimating it by 10% at other points in time. The underestimation occurs exactly during crisis periods.

While our findings can lead to improvements in risk management, they can also contribute to our understanding of the determinants of the changes in dependence. Many studies that try to relate the changes in the dependence between the returns of financial assets to other economic changes, focus on the correlations between these returns. Changes in correlations might offer only a limited perspective on what is actually changing in the dependence. They are a poor measure to pick up changes in asymmetry or tail dependence. This narrow focus might explain why relating the variation in correlations to other economic variations is so hard. Our approach that accommodates distinct changes in the strength and the structure of the dependence can show better what is actually changing, which might make it easier to explain why it is changing.

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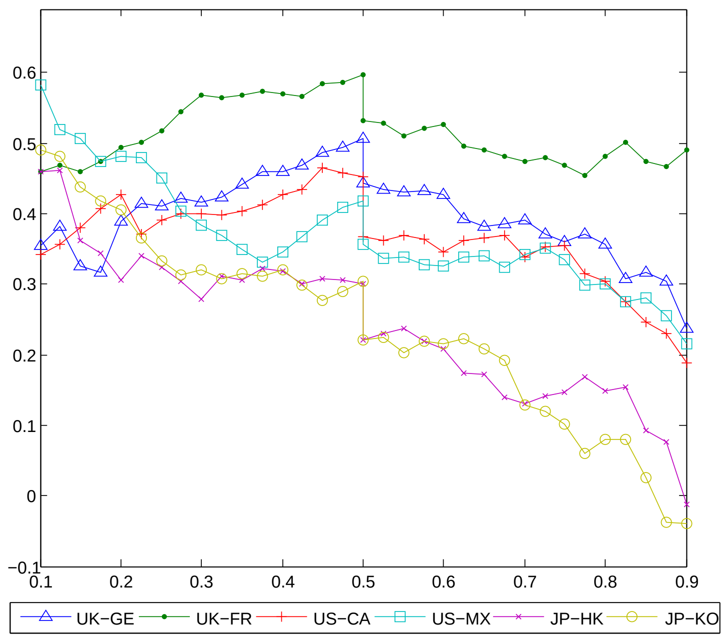
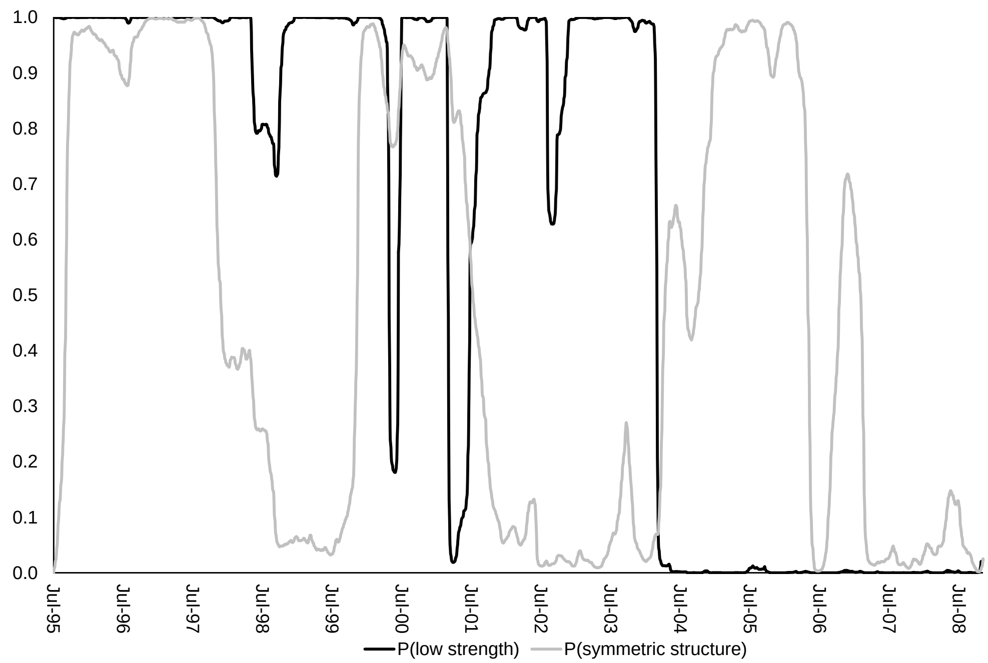
Figure 1: Exceedance correlations



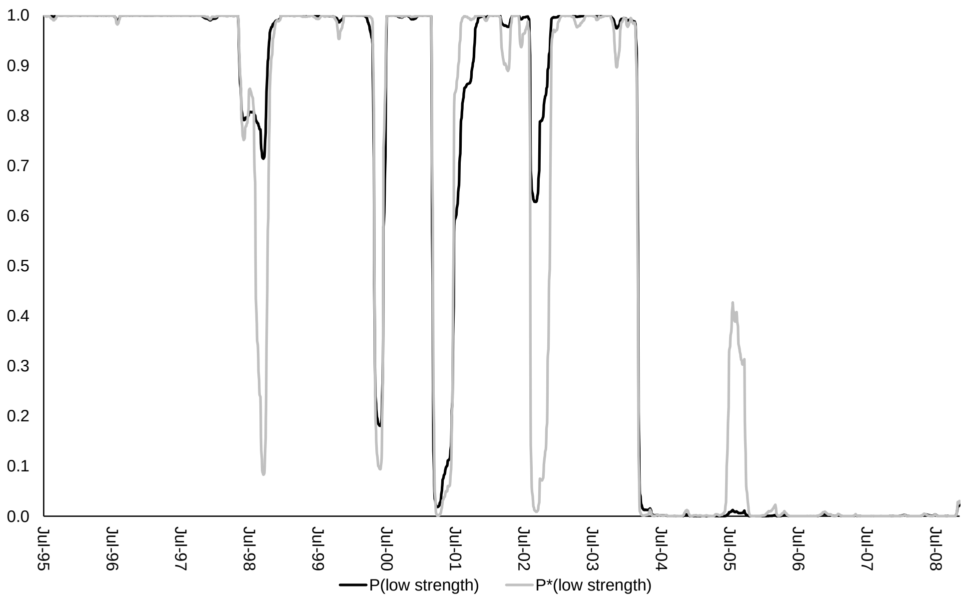
The graph shows the exceedance correlations of Ang and Chen (2002) for the different pairs of stock indices. The sample covers the period from July 3, 1995 to November 7, 2008. The horizontal axis shows the quantiles for which the exceedance correlation is computed. We model the marginal distributions as in Section 3.2

Figure 2: Smoothed inference probabilities for UK-GE

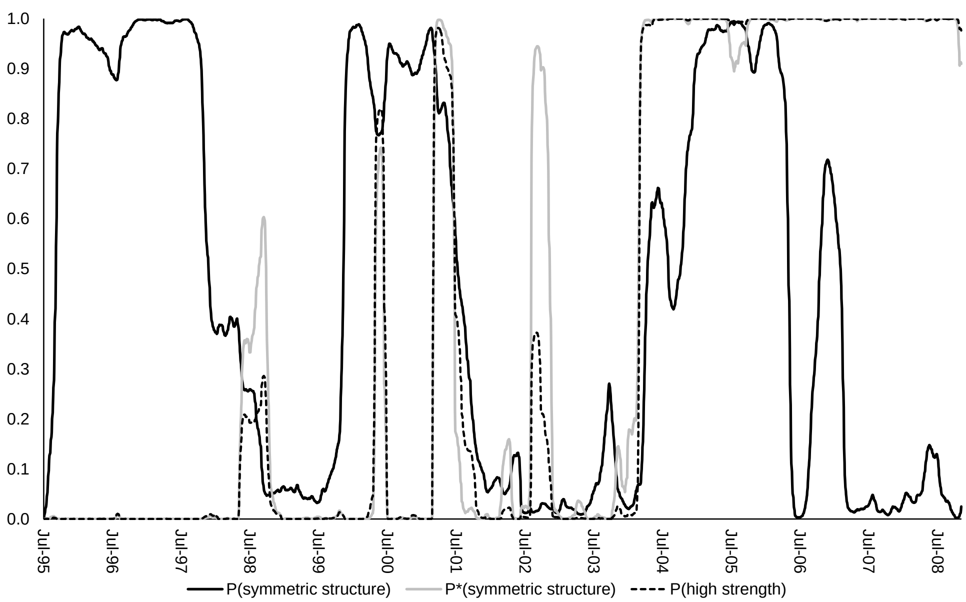


The graph shows the smoothed inference probabilities for the weak dependence regime ($S_t^{\theta} = 1$) and the symmetric dependence regime ($S_t^{\omega} = 1$) in model Equation (4) for the UK and Germany. They correspond with the estimation results for Model 1 in Table 2.

Figure 3: Smoothed inference probabilities for UK-GE – restricted models



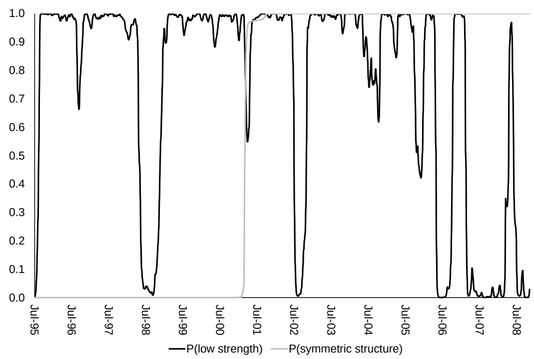
(a) Time-varying strength, constant structure



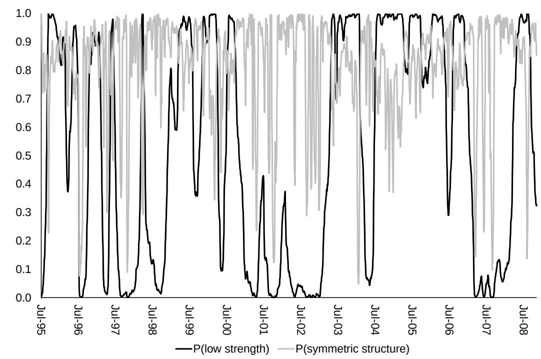
(b) Constant strength, time-varying structure

This figure shows smoothed inference probabilities for the restricted models 2 and 3 and the unrestricted model 1 in Table 2. Panel (a) shows the probabilities for the weak dependence regime in model 2 (time-varying strength, constant structure) with a gray line, and for the weak dependence regime in the unrestricted model 1 with a black line. Panel (b) shows the probabilities for the symmetric regime in model 3 (constant strength, time-varying structure) with a gray line, for the symmetric regime in the unrestricted model with a black solid line and for the strong dependence regime in the unrestricted model 1 with a dashed black line.

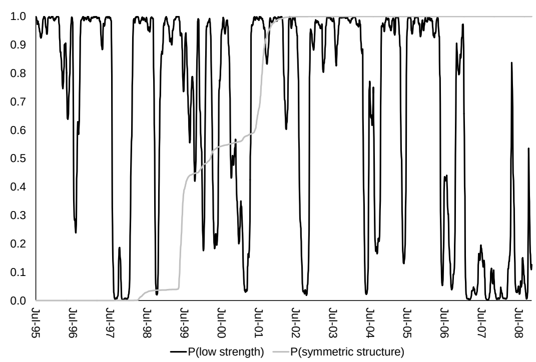
Figure 4: Smoothed probabilities other country pairs



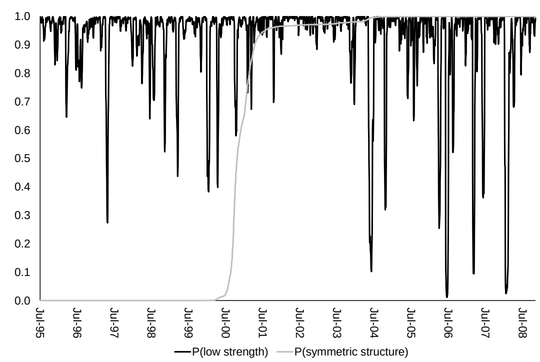
(a) UK-FR



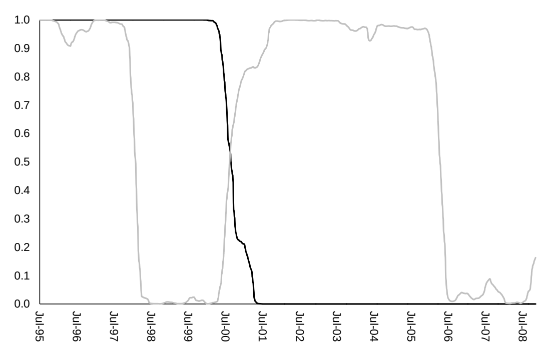
(b) US-CA



(c) US-MX



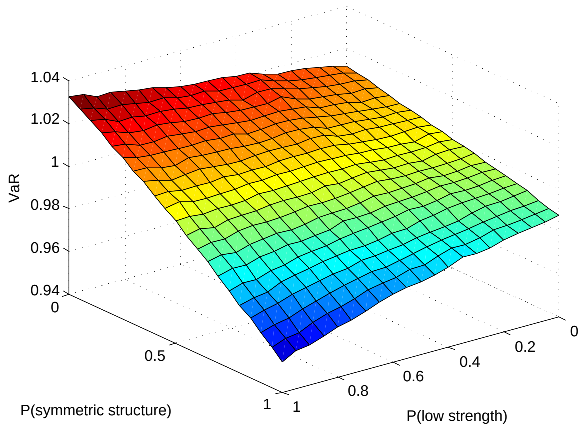
(d) JP-HK



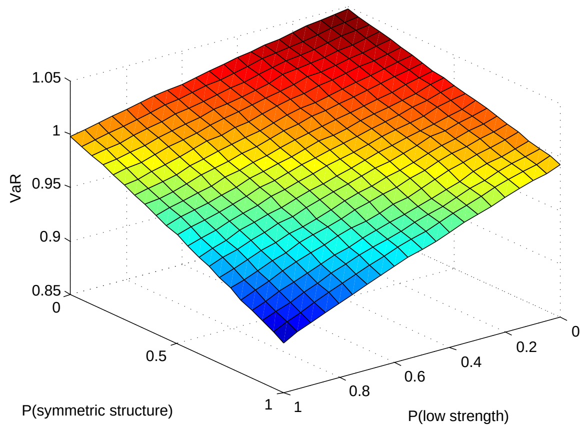
(e) JP-KO

The graph shows the smoothed probabilities of being in the regime with relatively weak dependence ($S_t^\theta = 1$) and the smoothed probabilities of being in the regime with relatively symmetric dependence ($S_t^\omega = 1$) in the model in Equation (4). They correspond with the estimation results in Table 3.

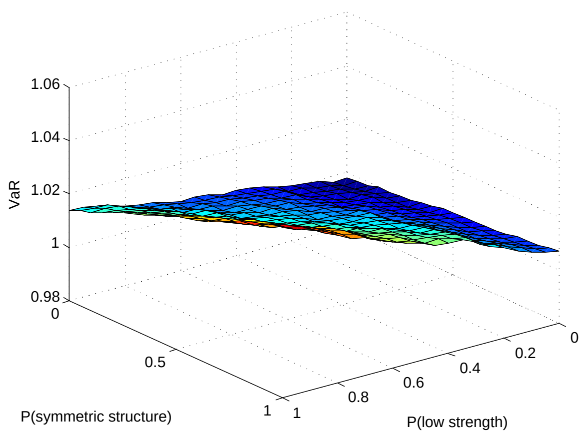
Figure 5: 99% Value-at-Risk Ratios



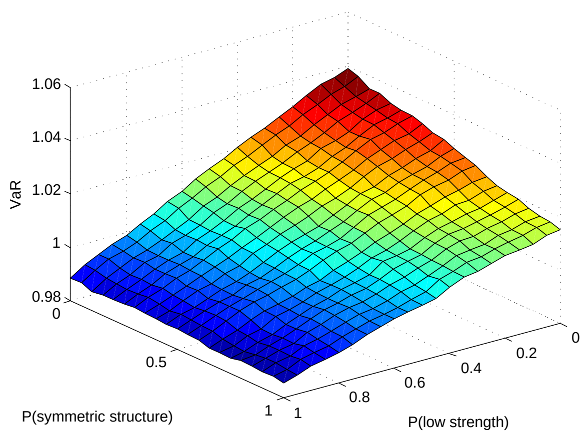
(a) UK-GE – Strength regimes only



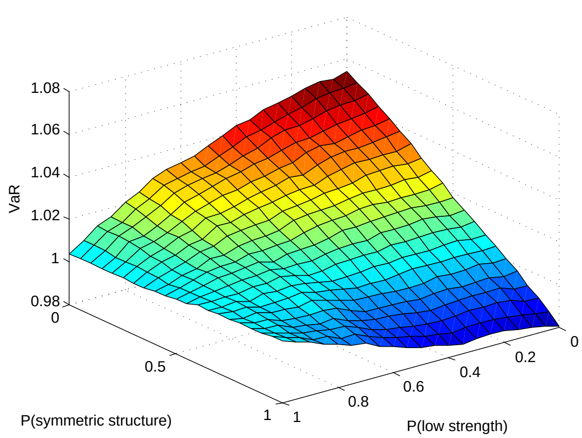
(b) UK-GE – Structure regimes only



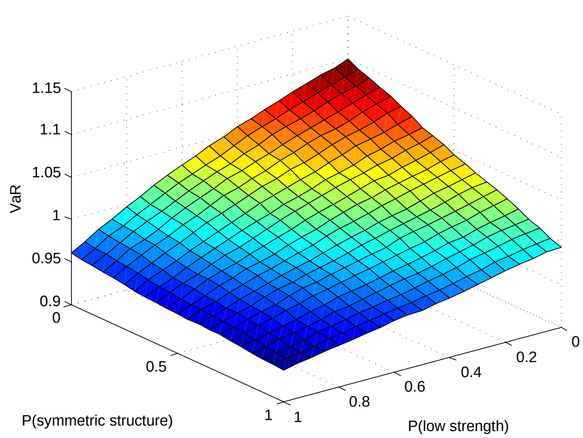
(c) UK-FR – Strength regimes only



(d) UK-FR – Structure regimes only



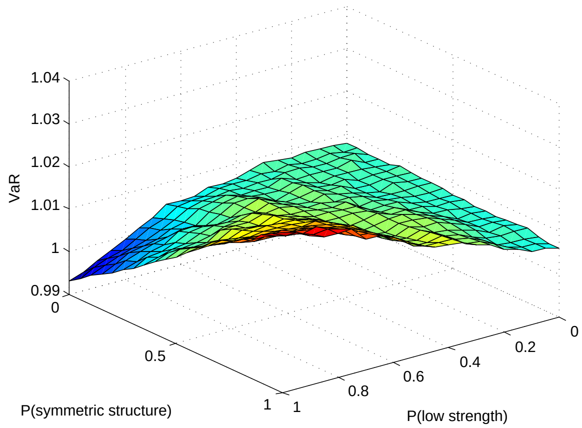
(e) US-CA – Strength regimes only



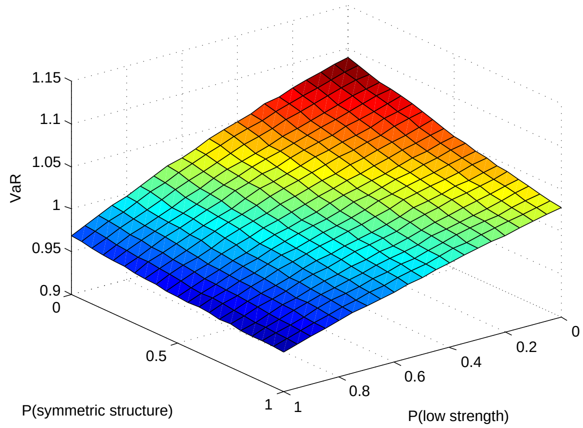
(f) US-CA – Structure regimes only

This figure shows the ratio of VaR_q that results from the unrestricted time-varying mixture copula to the Var_q that results from a restricted specifications as a function of the probability for the regime with a symmetric dependence structure and of the probability for the regime with a low dependence strength. We choose $q = 0.99$. In the left-hand (right-hand) figures the restricted model imposes a constant dependence structure (strength), while allowing for time-variation in the strength (structure) of the dependence. – *Note continues on next page.*

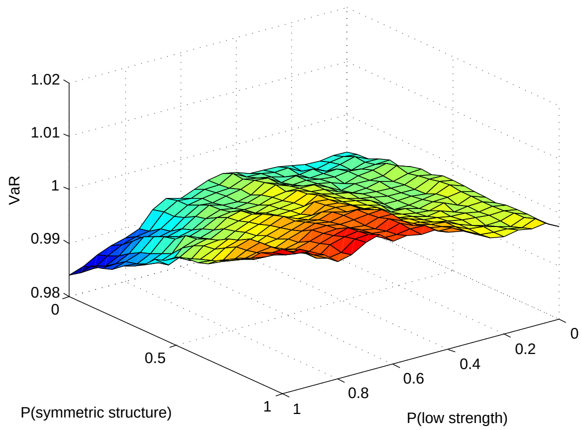
Figure 5: 99% Value-at-Risk Ratios – *continued*



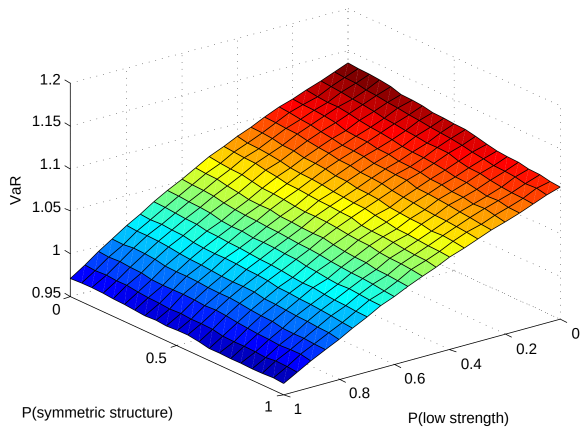
(g) US-MX – Strength regimes only



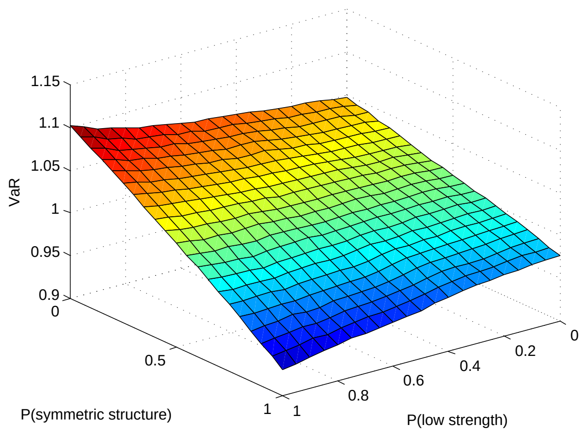
(h) US-MX – Structure regimes only



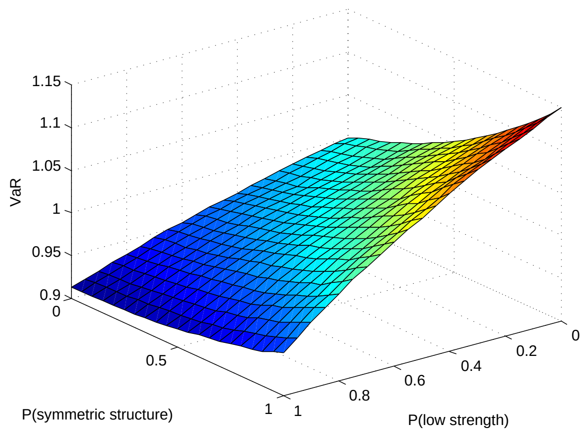
(i) JP-HK – Strength regimes only



(j) JP-HK – Structure regimes only



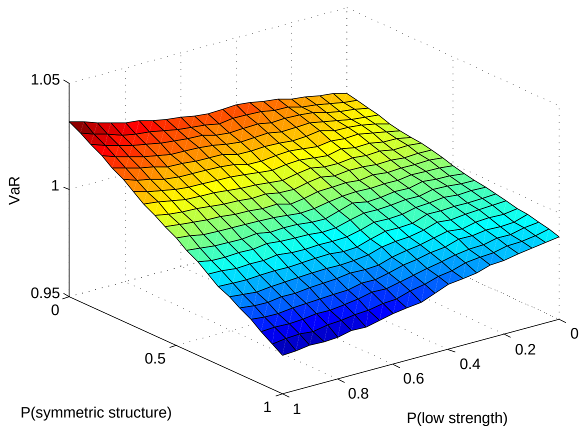
(k) JP-KO – Strength regimes only



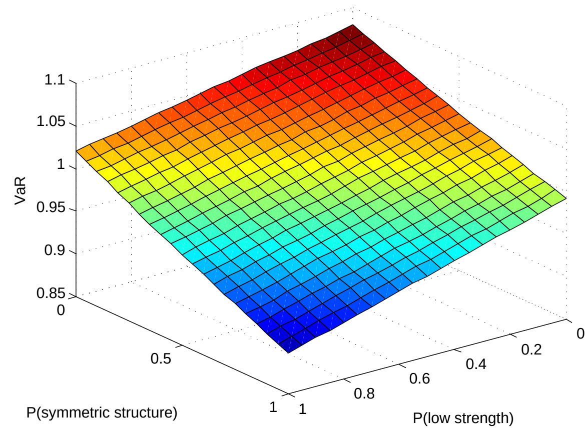
(l) JP-KO – Structure regimes only

Note continued from previous page. – The ratios are evaluated for all probability combinations in the two dimensional grid with a distance of 0.05 between the points. We construct pseudo returns to calculate VaR_q for a portfolio that is equally invested in both stock markets. We approximate the distribution of the mixture copula that the probability combinations imply by one million draws. We transform these draws to pseudo returns using the inverse of the cdf of the standard normal distribution. For the restricted model either the probability for the strength regime or for the structure regime can be ignored.

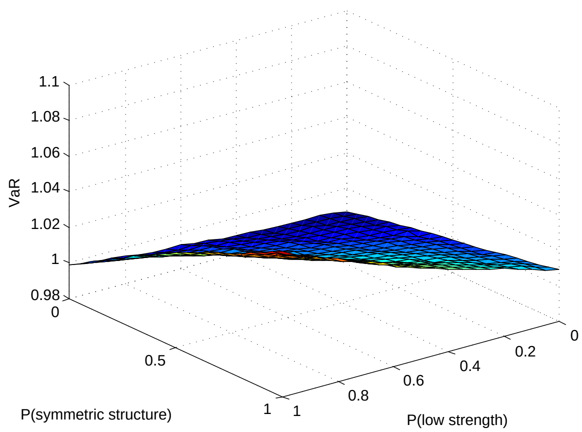
Figure 6: 95% Value-at-Risk Ratios



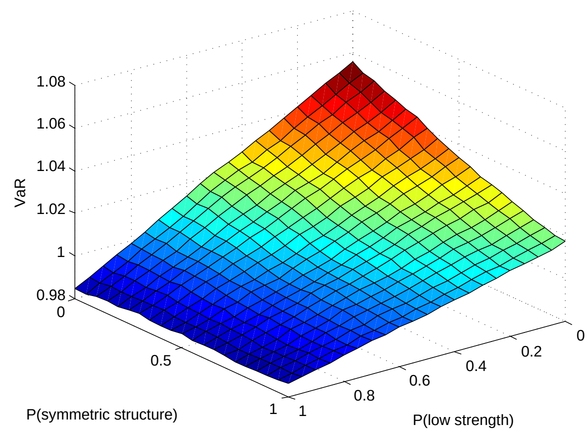
(a) UK-GE – Strength regimes only



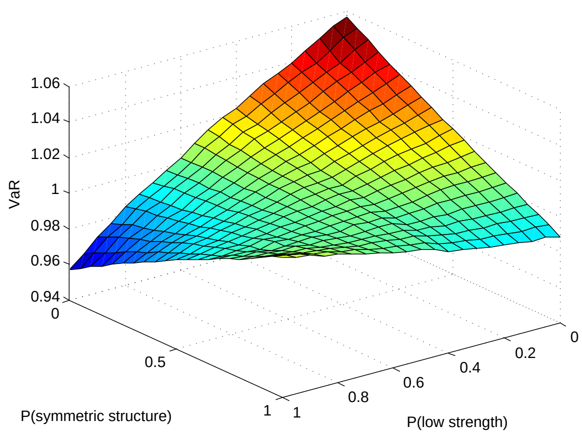
(b) UK-GE – Structure regimes only



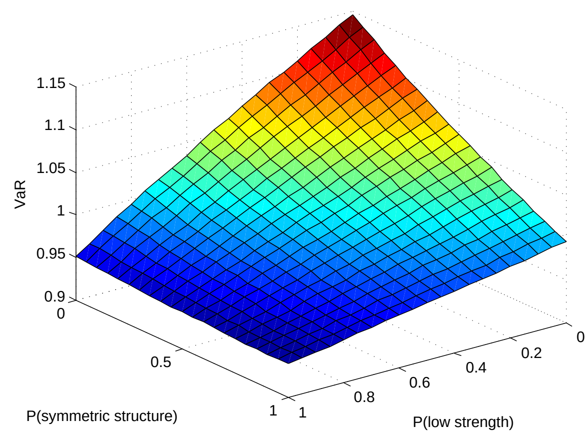
(c) UK-FR – Strength regimes only



(d) UK-FR – Structure regimes only



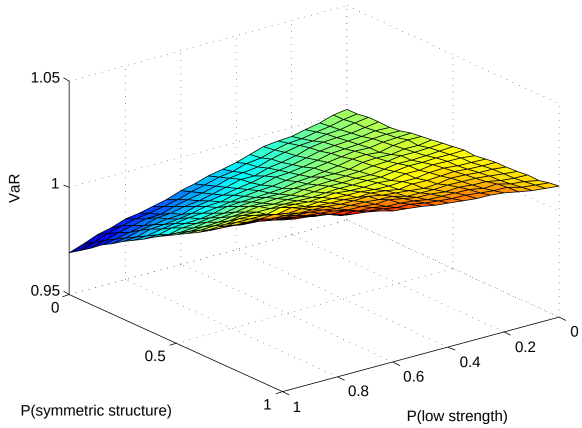
(e) US-CA – Strength regimes only



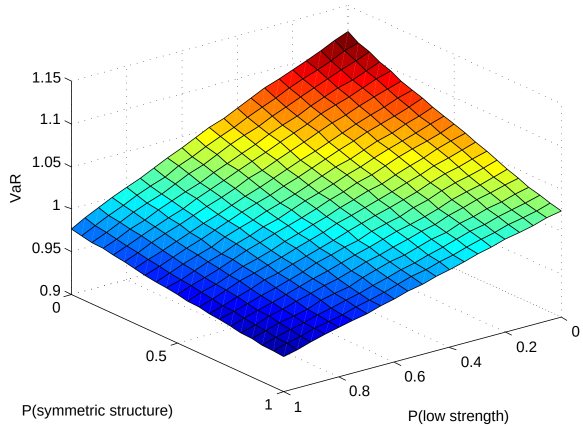
(f) US-CA – Structure regimes only

This figure shows the ratio of VaR_q that results from the unrestricted time-varying mixture copula to the Var_q that results from a restricted specifications as a function of the probability for the regime with a symmetric dependence structure and of the probability for the regime with a low dependence strength. We choose $q = 0.95$. For further explanation, see Figure 5.

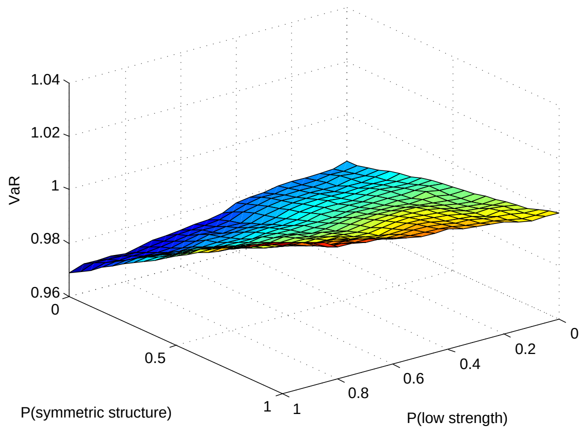
Figure 6: 95% Value-at-Risk Ratios – *continued*



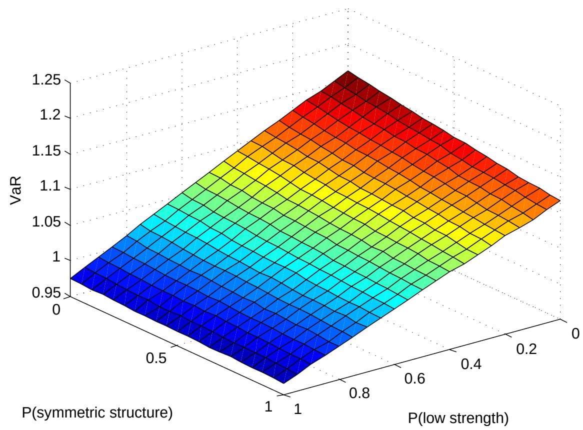
(g) US-MX – Strength regimes only



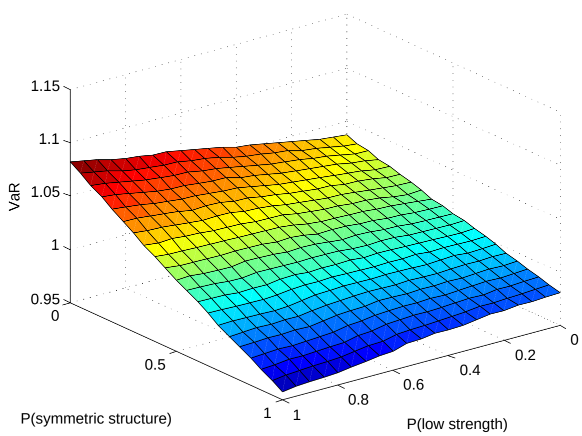
(h) US-MX – Structure regimes only



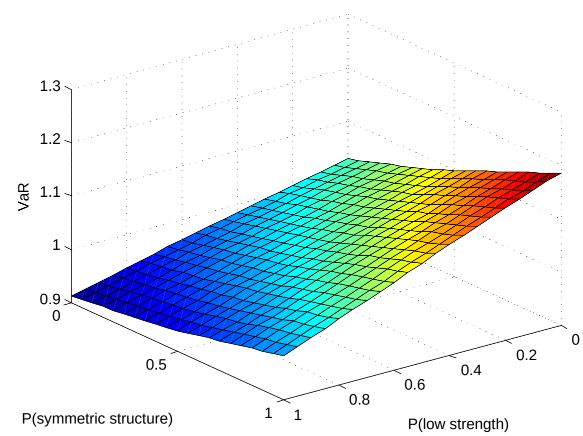
(i) JP-HK – Strength regimes only



(j) JP-HK – Structure regimes only



(k) JP-KO – Strength regimes only



(l) JP-KO – Structure regimes only

Figure note on previous page.

Table 1: Descriptive statistics

	Mean (%)	Vol (%)	Skewness	Kurtosis
US	4.31	19.84	−0.16	11.19
Canada (CA)	8.68	22.01	−0.72	12.12
Mexico (MX)	9.68	30.06	0.09	13.85
UK	1.99	20.49	−0.23	13.83
Germany (GE)	3.80	24.92	−0.14	8.00
France (FR)	4.86	23.24	−0.22	9.83
Japan (JP)	−3.38	24.74	0.17	8.09
Hong Kong (HK)	0.59	27.23	−0.19	11.07
Korea (KO)	−0.27	43.59	0.26	18.28

The table reports the annualized mean (in %), annualized volatility (in %), skewness and kurtosis coefficients of the daily stock market returns over the period of July 3, 1995, to November 7, 2008 ($T = 3,250$. observations)

Table 2: Estimation results for UK-GE

Model	1	2	3	4	5
Strength	varying	varying	constant	constant	constant
Structure	varying	constant	varying	varying	constant
ρ_1	0.480	0.638	0.855	0.844	0.677
	(0.042)	(0.033)	(0.076)	(0.008)	(0.006)
ρ_2	0.818	0.823			
	(0.028)	(0.023)			
δ_1	2.120	1.314	1.439	1.541	ND
	(0.209)	(0.108)	(0.254)	(0.035)	
δ_2	3.761	3.953			
	(0.381)	(0.740)			
ω_1	1	0.707	0.962	1	1
	(–)	(0.091)	(0.175)		(–)
ω_2	0.405		0.191	0	
	(0.232)		(0.289)		
p^{θ}_{11}	0.998	0.997			
	(0.002)	(0.002)			
p^{θ}_{22}	0.998	0.996			
	(0.002)	(0.003)			
p^{ω}_{11}	0.996		0.997	0.995	
	(0.006)		(0.003)	(0.003)	
p^{ω}_{22}	0.996		0.997	0.996	
	(0.007)		(0.003)	(0.002)	
ℓ_C	1196.1	1187.7	1161.3	1153.9	991.3
LR test		16.7	69.5	84.3	409.5
p-value		0.032	< 0.001	< 0.001	0.001

The table reports the estimation results for the mixture copula specifications for the daily stock index returns in the UK and Germany over the period of July 3, 1995, to November 7, 2008. Model 1 is the unrestricted time-varying mixture copula in Equation (4) that allows for changes in both the strength and the structure of the dependence. Model 2 corresponds with Equation (18) and accommodates switches in the copula parameters (time-varying strength) but has no switches in the mixture weight (constant structure). Model 3 corresponds with Equation (19) and has no switches in the copulas parameters (constant strength) but accommodates switches in the mixture weights (time-varying structure). Model 4 corresponds with Equation (20) and has no switches in the copulas parameters but accommodates switches in the mixture weight that are restricted to $\omega_1 = 1$ and $\omega_2 = 0$. Model 5 corresponds with Equation (20) and does not accommodate any regime switches at all. The marginal distributions are modeled by the AR(1)-TGARCH(1,1) specification for all of the copula models. We report asymptotic standard errors in parentheses. For the restricted models the parameters that are assumed constant across regimes are reported in the 'regime 1'-row. When a boundary is reached during the estimation the boundary value is imposed for this parameter. In those cases, a (–) appears instead of a standard error. Unidentified parameters are indicated with ND. The last three rows report the values for the log likelihood of the copula models (ℓ_C), the likelihood ratio statistic of the restricted Models 2–5 against Model 1, and the corresponding p-values based on 2,500 simulations.

Table 3: Estimation results for other country pairs

	UK-FR	US-CA	US-MX	JP-HK	JP-KO
ρ_1	0.823	0.519	0.578	0.435	0.003
	(0.014)	(0.073)	(0.048)	(0.057)	(0.103)
ρ_2	0.931	0.703	0.837	0.893	0.473
	(0.007)	(0.016)	(0.029)	(0.032)	(0.035)
δ_1	1.334	1.215	1.148	1.128	1.425
	(0.061)	(0.087)	(0.048)	(0.050)	(0.112)
δ_2	1.986	3.533	1.543	1.899	1.959
	(0.194)	(0.299)	(0.534)	(0.608)	(0.239)
ω_1	0.958	1	0.864	0.868	0.992
	(0.018)	–	(0.138)	(0.184)	(0.115)
ω_2	0.348	0	0.353	0.273	0.453
	(0.071)	–	(0.317)	(0.253)	(0.172)
p_{11}^{θ}	0.996	0.99	0.99	0.989	0.999
	(0.003)	(0.006)	(0.006)	(0.005)	(0.001)
p_{22}^{θ}	0.988	0.988	0.98	0.871	1
	(0.006)	(0.004)	(0.012)	(0.030)	–
p_{11}^{ω}	1	0.971	1	1	0.999
	–	(0.012)	–	–	(0.001)
p_{22}^{ω}	0.999	0.877	0.999	0.999	0.999
	(0.001)	(0.030)	(0.002)	(0.001)	(0.002)
LR structure	79.7	21.9	21.5	15.6	18.4
	[< 0.001]	[0.014]	[0.011]	[0.022]	[0.011]
LR strength	96.6	36.4	26.5	24.7	77.2
	[< 0.001]	[< 0.001]	[< 0.001]	[< 0.001]	[< 0.001]
LR Okimoto	219.2	36.9	55.4	31.3	80.7
	[< 0.001]	[< 0.001]	[< 0.001]	[< 0.001]	[< 0.001]
LR Hu	524.4	151.1	238.8	76.0	193.9
	[< 0.001]	[< 0.001]	[< 0.001]	[< 0.001]	[< 0.001]

The table reports the estimation results for the mixture copula specifications for the daily stock index returns for the pairs UK and France (UK-FR), US and Canada (US-CA), US and Mexico (US-MX), Japan and Hong Kong (JP-HK), and Japan and Korea (JP-KO) over the period of July 3, 1995 to November 7, 2008. The estimates correspond with the time-varying mixture copula in Equation (4) that allows for changes in both the strength and the structure of the dependence. The marginal distributions are modeled by the AR(1)-TGARCH(1,1) specification for all of the copula models. We report asymptotic standard errors in parentheses. When a boundary is reached during the estimation the boundary value is imposed for this parameter. In those cases, a (–) appears instead of a standard error. We report four likelihood ratio statistics with p-values in brackets below based on 2,500 simulations. The statistic "LR structure" tests the restriction that the dependence structure is constant as in Equation (18). The statistic "LR strength" tests the restriction that the strength of the dependence is constant as in Equation (19). The statistics "LR Okimoto" and "LR Hu" test the restricted models of Okimoto (2008) in Equation (20) and Hu (2006) in Equation (21) versus the unrestricted model.

Table 4: The dependence between the lower tails

s_θ	s_ω	UK-GE	UK-FR	US-CA	US-MX	JP-HK	JP-KO
Weak	symmetric	0	0.01	0	0.02	0.02	0.00
Weak	asymmetric	0.36	0.21	0.23	0.11	0.12	0.21
Strong	symmetric	0	0.02	0	0.07	0.07	0.00
Strong	asymmetric	0.47	0.38	0.78	0.35	0.43	0.31

(a) Lower tail dependence coefficients

s_θ	s_ω	UK-GE	UK-FR	US-CA	US-MX	JP-HK	JP-KO
Weak	symmetric	0.23	0.52	0.26	0.28	0.2	0.05
Weak	asymmetric	0.47	0.41	0.27	0.23	0.2	0.24
Strong	symmetric	0.52	0.69	0.4	0.53	0.62	0.23
Strong	asymmetric	0.69	0.63	0.79	0.55	0.59	0.43

(b) Exceedance probabilities

Panel A reports the tail dependence for the different combinations of the regimes of the strength and structure of the dependence, implied by the estimation results of the dependence model. The tail dependence is calculated as $\tau_L = (1 - \omega_{s^\omega})(2 - 2^{1/\delta_{s_\theta}})$. Panel B reports the probability of observing one return in the lower 5%-quantile given that the other return belongs to this quantile. This probability is computed as $C(0.05, 0.05)/0.05 = \omega_{s_\omega} C_{\mathrm{Gau}}(0.05, 0.05; \rho_{s_\theta})/0.05 + (1 - \omega_{s_\omega}) C_{\mathrm{Gum}}(0.05, 0.05; \delta_{s_\theta})/0.05$.

A Detailed results

A.1 Results for the margins

Table A.1 reports the estimation results for the marginal AR(1)-TGARCH(1,1) models with skewed Student's t innovations. The parameter estimates largely reflect the well-known stylized facts of univariate daily stock return distributions. First, for all countries the volatility is highly persistent as $(\alpha^+ + \alpha^-)/2 + \beta$ is estimated to be close to one. This persistence indicates prolonged periods of relatively high and low volatilities. Second, for all of the countries, the estimate of α^- substantially exceeds the estimate of α^+, reflecting the property that negative return shocks have the larger impact on the conditional volatility. Third, the degrees of freedom ν that range from 6.6 (KO) to 18.5 (UK) indicate fat tails. Fourth, the skewness parameter λ is negative for all of the countries and is significant at the 1% level for all of the countries but Japan and Mexico. Fifth, the AR(1) parameter ϕ_1 is small and significant for only half of the countries, which corresponds with the small first-order autocorrelation in the daily stock returns.

[Table A.1 about here.]

The IFM method requires a correct specification of the marginal distributions to prevent biases in the second step. The results for the Kolmogorov-Smirnov, Cramer-Von Mises and Anderson-Darling tests for the goodness-of-fit indicate that this requirement is generally met. For all markets except Hong Kong, the test results in the final three columns of Table A.1 fail to reject the null hypothesis of correct specification. For Hong Kong, we re-estimate the AR(1)-TGARCH(1,1) model with a generalized error distribution for the innovations. Because this distribution cannot be rejected, we use the generalized error distribution for the Hong Kong stock return innovations.

A.2 Results for other country pairs

The results for the UK and France in Table A.2 indicate a regime with an almost symmetric dependence structure (mixture weight for the Gaussian copula $\omega_1 = 0.958$) and a regime with a more asymmetric dependence structure (mixture weight for the Gaussian copula

$\omega_2 = 0.348$). Figure 4a shows one switch between these regimes at the burst of the dot-com bubble in April 2001. Because there is only one switch, the transition probability p_{11}^{ω} is set equal to one. The difference in the strength of the dependence is mainly concentrated in the survival Gumbel copula, with $\delta_2 = 1.99 > \delta_1 = 1.33$. The correlation parameter of the Gaussian copula is high in both regimes ($\rho_1 = 0.82$ and $\rho_2 = 0.93$). The strong dependence regime prevails during crisis periods and the subsequent bear markets. In Figure 4a the crisis periods of July 1997, April 2001, September 2002, and the recent financial crisis are clearly visible.

[Table A.2 about here.]

The results for the restricted Models 2–5 indicate that these models capture dynamics in the dependence less accurately than the unrestricted model. All restricted models are rejected with p-values lower than 0.001. For all of the restricted models, the LR-statistics exceed the values reported in Table 2, which indicates that properly modeling the time variation in the dependence is even more important for the combination of the UK and French stock markets than for the UK and Germany. The parameter estimates of the restricted models can again be seen as a convex combination of the unrestricted parameter estimates.

The estimation results of the US and Canada in Table A.3 show that the dependence is either fully asymmetric or fully symmetric because $\omega_1 = 1$ and $\omega_2 = 0$. Because the estimates for the mixture parameters reach their boundaries, the dependence structure is described by a Gaussian copula in the symmetric regime and a Gumbel copula in the asymmetric regime. This implies that for this country pair the joint Markov process switches between four different and distinct copulas. The parameters in the low strength regime differ substantially from the parameters in the high strength regime, with $\rho_1 = 0.52 < \rho_2 = 0.70$, and $\delta_1 = 1.22 < \delta_2 = 3.53$. The smoothed inference probabilities in Figure 4b for the dependence structure switches relatively infrequently, mostly being in the asymmetric regime during financial crises, and in the symmetric regime during tranquil periods. It is harder to link the state of the dependence structure process to the financial market conditions for the US-CA, although the process seems to be more often in the high strength state during turmoil periods.

[Table A.3 about here.]

Also for the combination of the US and Canadian stock markets, we reject the restricted Models 2–5. Model 2 with no changes in the structure of the dependence is rejected with a p-value of 0.014. For all other models, the p-values are below 0.001. When a model only accommodates changes in strength as for Model 2, the dependence is asymmetric in both regimes. When a model only accommodates changes in structure, one regime remains fully Gaussian, while the other regime puts most but not all weight on the Gumbel copula. The Gaussian regime shows strong dependence, while the Gumbel-dominated regime shows weak dependence. The estimates for Model 3 are actually quite close to those for Model 4 as proposed by Okimoto (2008).

For the US and Mexico, we find a clear difference between the weak and strong dependence regimes as both $\rho_1 < \rho_2$ and $\delta_1 < \delta_2$, although the difference between the Gumbel parameters is not as large as for the US and Canada. The Gaussian copula is the most important in the symmetric regime ($\omega_1 = 0.864$), although the Gumbel copula still receives a non-negligible weight of 0.136. In the asymmetric regime, the copula is a weighted average of the Gaussian and Gumbel copulas with weights of 0.353 and 0.647. The smoothed inference probabilities in Figure 4c show that the dependence structure switches only once during the summer of 2001, from the asymmetric regime to the more symmetric regime. Here the Markov-switching framework accommodates a nonrecurring change in dependence, and consequently $p^{\omega}_{11} = 1$. The strength regimes are persistent with probabilities of 0.990 and 0.980. The strong dependence regime occurs during most of the financial crises and since 2006.

[Table A.4 about here.]

Comparing the restricted Models 2–5 with the unrestricted models shows again that both changes in strength and structure are important. Restricting changes to only the strength of the dependence is rejected with a p-value of 0.011; for all of the other restrictions the p-values are below 0.001. Restricting the changes in the structure of the dependence leads to mild asymmetry in both strength regimes (Model 2). Restricting the changes in the strength of the dependence produces a symmetric regime with strong dependence and an asymmetric regime with weak dependence.

Japan and Hong Kong also experience a single change from a symmetric dependence structure to an asymmetric structure ($\omega_1 = 0.868$ versus $\omega_2 = 0.273$), which occurred around the burst of the dotcom bubble. The Gumbel parameter estimates in Table A.5 are fairly small, and suggest that the lower tail dependence between these markets is not particularly strong. The transition probability $p^{\theta}_{22} = 0.87$ is relatively low, showing that most of the time the dependence strength is in the weak regime. Figure 4d confirms that the strength is mostly low, although we observe that the strong strength regime starts to occur more and more often after March 2004.

[Table A.5 about here.]

The evidence for changes in the structure of the dependence for the combination of the Japanese and Hong Kong equity markets is a bit less than for the other combinations, but a p-value of 0.022 is still clearly below the conventional confidence level of 0.05. When the structure of the dependence cannot change, we see that the strong dependence regime becomes more asymmetric. When changes in the strength are restricted, we observe a Gaussian-dominated regime and a Gumbel-dominated regime. The parameter for the Gaussian copula is in between the two parameters in the unrestricted model, but the Gumbel parameter remains low.

Japan and Korea constitute the only pair that shows independence in the weak and symmetric regime ($\omega_1 = 0.992, \rho_1 = 0.003$), as shown in Table A.6. The regime with an asymmetric structure puts approximately equal weights on the Gaussian and Gumbel copulas. The parameters for the Gumbel copula do not indicate independence, as they are both well above one. The smoothed inference probabilities in Figure 4e show that all of the regimes are highly persistent. The strength is weak before June 2001, and remains strong thereafter (and hence $p^{\theta}_{2}2 = 1$). The structure also switches only occasionally with more asymmetry during the dotcom bubble (June 1999–July 2001) and the credit crisis (after July 2007). The regime combination that implies (near) independence prevails from the start of our sample until July 1999.

[Table A.6 about here.]

When we restrict the changes in the dependence structure, the estimates indicate a mostly Gaussian dependence structure with limited influence for the Gumbel copula ($\omega_1 = 0.792$). This specification is actually the only one where the low value for the correlation parameter in the Gaussian copula is combined with the high value for the Gumbel copula, that is, $\rho_1 < \rho_2$ but $\delta_1 > \delta_2$. The restrictions for this specification are rejected with a p-value of 0.011. When the changes in the strength are rejected, the estimates indicate a regime with a mixture of a near-independent Gaussian and a mildly dependent Gumbel copula, and a regime that is fully determined by the Gumbel copula. This regime is strongly rejected with a p-value below 0.001. The models with further restrictions are also rejected.

Table A.1: The estimation results for the margins

	ϕ_1	$\psi \cdot 10^4$	α^+	α^-	β	ν	λ	$\ell \cdot 10^{-3}$	K-S	C-M	A-D
UK	−0.024	0.021	0.011	0.128	0.915	18.487	−0.094	10.28	0.943	0.904	0.902
	(0.018)	(0.004)	(0.007)	(0.015)	(0.011)	(5.106)	(0.026)				
GE	−0.006	0.025	0.028	0.135	0.909	12.638	−0.097	9.58	0.831	0.846	0.710
	(0.018)	(0.006)	(0.010)	(0.016)	(0.010)	(2.463)	(0.025)				
FR	0.002	0.026	0.018	0.123	0.916	11.792	−0.088	9.78	0.981	0.892	0.749
	(0.018)	(0.006)	(0.009)	(0.015)	(0.010)	(2.177)	(0.025)				
US	−0.047	0.013	0.000	0.139	0.922	9.478	−0.123	10.38	0.639	0.711	0.058
	(0.018)	(0.003)	(0.015)	(0.017)	(0.012)	(1.448)	(0.023)				
CA	0.074	0.025	0.032	0.120	0.910	7.549	−0.115	10.01	0.776	0.812	0.649
	(0.018)	(0.007)	(0.010)	(0.018)	(0.012)	(0.905)	(0.024)				
MX	0.103	0.098	0.014	0.179	0.874	7.115	−0.030	8.95	0.697	0.781	0.846
	(0.018)	(0.020)	(0.009)	(0.024)	(0.016)	(0.814)	(0.021)				
JP	−0.021	0.032	0.040	0.099	0.918	10.415	−0.021	9.31	0.715	0.591	0.450
	(0.018)	(0.010)	(0.010)	(0.014)	(0.012)	(1.662)	(0.027)				
HK	0.016	0.024	0.038	0.081	0.915		1.204	9.42	0.394	0.386	0.115
	(0.018)	(0.006)	(0.012)	(0.015)	(0.011)		0.035				
KO	0.097	0.047	0.042	0.115	0.916	6.632	−0.061	8.07	0.167	0.252	0.215
	(0.018)	(0.013)	(0.010)	(0.017)	(0.011)	(0.777)	(0.022)				

This table reports the estimation results of the AR(1)-TGARCH(1,1,1) model with skewed Student's t innovations as in Equations (9) to (13). For the Hong Kong stock market a generalized error distribution (GED) is used for the innovations. The parameter of the GED distribution is given in the column headed λ. The sample period runs from July 3, 1995, to November 7, 2008 ($T = 3,250$ observations). The table reports the estimates for the autoregressive component (column 2), the parameters in the TGARCH specification (columns 3–6) and the degrees of freedom and skewness parameters (columns 7–8), with the standard errors given in parentheses. The table also reports the log likelihood values ℓ (column 9) and p-values for the Kolmogorov-Smirnov (K-S), Cramer-Von Mises (C-M) and Anderson-Darling (A-D) tests for the uniformity of the standardized residuals in columns 10–12.

Table A.2: Estimation results for UK-FR

Model	1	2	3	4	5
Strength	varying	varying	constant	constant	constant
Structure	varying	constant	varying	varying	constant
ρ_1	0.823	0.722	0.873	0.577	ND
	(0.014)	(0.078)	(0.010)	(0.021)	
ρ_2	0.931	0.890			
	(0.007)	(0.021)			
δ_1	1.334	1.188	1.443	2.829	2.090
	(0.061)	(0.125)	(0.067)	(0.084)	(0.041)
δ_2	1.986	2.545			
	(0.194)	(1.472)			
ω_1	0.958	0.723	0.969	1	0
	(0.018)	(0.079)	(0.027)		(–)
ω_2	0.348		0.250	0	
	(0.071)		(0.070)		
p^{θ}_{11}	0.996	0.994			
	(0.003)	(0.004)			
p^{θ}_{22}	0.988	0.995			
	(0.006)	(0.008)			
p^{ω}_{11}	1.000		0.997	0.993	
	(–)		(0.003)	(0.003)	
p^{ω}_{22}	0.999		0.996	0.994	
	(0.001)		(0.002)	(0.002)	
ℓ_C	1542.942	1503.111	1494.663	1433.328	1280.737
LR test		79.662	96.558	219.227	524.410
p-value		< 0.001	< 0.001	< 0.001	< 0.001

The estimation results for the mixture copula specifications for UK-FR. For further details see Table 2.

Table A.3: Estimation results for US-CA

Model	1	2	3	4	5
Strength	varying	varying	constant	constant	constant
Structure	varying	constant	varying	varying	constant
ρ_1	0.519	0.473	0.724	0.723	0.596
	(0.073)	(0.054)	(0.017)	(0.013)	(0.011)
ρ_2	0.703	0.731			
	(0.016)	(0.019)			
δ_1	1.215	1.297	1.320	1.346	ND
	(0.087)	(0.115)	(0.060)	(0.038)	
δ_2	3.533	2.375			
	(0.299)	(0.567)			
ω_1	1	0.833	1	1	1
	(–)	(0.087)	(–)		(–)
ω_2	0		0.094	0	
	(–)		(0.148)		
p_{11}^{θ}	0.990	0.987			
	(0.006)	(0.005)			
p_{22}^{θ}	0.988	0.987			
	(0.004)	(0.004)			
p_{11}^{ω}	0.971		0.988	0.987	
	(0.012)		(0.004)	(0.003)	
p_{22}^{ω}	0.877		0.984	0.982	
	(0.030)		(0.007)	(0.006)	
ℓ_C	786.574	775.611	768.387	768.104	711.034
LR test		21.927	36.375	36.940	151.081
p-value		0.014	< 0.001	< 0.001	< 0.001

The estimation results for the mixture copula specifications for US-CA. For further details see Table 2.

Table A.4: Estimation results for US-MX

Model Strength Structure	1 varying varying	2 varying constant	3 constant varying	4 constant varying	5 constant constant
ρ_1	0.578 (0.048)	0.512 (0.051)	0.758 (0.073)	0.729 (0.017)	ND
ρ_2	0.825 (0.029)	0.837 (0.033)			
δ_1	1.140 (0.048)	1.148 (0.094)	1.238 (0.082)	1.262 (0.038)	1.559 (0.092)
δ_2	1.832 (0.534)	1.543 (0.189)			
ω_1	0.864 (0.138)	0.744 (0.121)	0.929 (0.056)	1	0.000 (–)
ω_2	0.353 (0.317)		0.126 (0.130)	0	
p^{θ}_{11}	0.990 (0.006)	0.992 (0.005)			
p^{θ}_{22}	0.980 (0.012)	0.987 (0.008)			
p^{ω}_{11}	1.000 (–)		0.987 (0.006)	0.983 (0.004)	
p^{ω}_{22}	0.999 (0.002)		0.986 (0.014)	0.977 (0.007)	
ℓ_C	712.815	702.044	688.809	685.101	593.412
LR test		21.541	26.470	55.428	238.805
p-value		0.011	< 0.001	< 0.001	< 0.001

The estimation results for the mixture copula specifications for US-MX. For further details see Table 2.

Table A.5: **Estimation results for JP-HK**

Model	1	2	3	4	5
Strength	varying	varying	constant	constant	constant
Structure	varying	constant	varying	varying	constant
ρ_1	0.435	0.387	0.585	0.337	0.379
	(0.057)	(0.039)	(0.062)	(0.019)	(0.005)
ρ_2	0.893	0.883			
	(0.032)	(0.043)			
δ_1	1.128	1.192	1.164	2.591	ND
	(0.050)	(0.052)	(0.039)	(0.276)	
δ_2	1.899	2.559			
	(0.608)	(0.957)			
ω_1	0.868	0.645	0.749	1	1.000
	(0.184)	(0.145)	(0.130)		(–)
ω_2	0.273		0.228	0	
	(0.253)		(0.139)		
p^{θ}_{11}	0.989	0.990			
	(0.005)	(0.005)			
p^{θ}_{22}	0.871	0.872			
	(0.030)	(0.035)			
p^{ω}_{11}	1.000		1.000	0.988	
	(–)		(–)	(0.007)	
p^{ω}_{22}	0.999		0.999	0.875	
	(0.001)		(0.001)	(0.029)	
ℓ_C	289.007	281.209	276.660	273.339	251.001
LR test		15.595	24.694	31.337	76.011
p-value		0.022	< 0.001	< 0.001	< 0.001

The estimation results for the mixture copula specifications for JP-HK. For further details see Table 2.

Table A.6: Estimation results for JP-KO

Model	1	2	3	4	5
Strength	varying	varying	constant	constant	constant
Structure	varying	constant	varying	varying	constant
ρ_1	0.003	−0.045	−0.079	0.010	0.371
	(0.103)	(0.063)	(0.140)	(0.049)	(0.008)
ρ_2	0.473	0.552			
	(0.035)	(0.041)			
δ_1	1.425	1.730	1.420	1.411	ND
	(0.112)	(0.335)	(0.075)	(0.031)	
δ_2	1.959	1.260			
	(0.239)	(0.153)			
ω_1	0.992	0.792	0.770	1	1.000
	(0.115)	(0.082)	(0.192)		(–)
ω_2	0.453		0.000	0	
	(0.172)		(–)		
p^{θ}_{11}	0.999	0.998			
	(0.001)	(0.002)			
p^{θ}_{22}	1.000	0.999			
	(–)	(0.001)			
p^{ω}_{11}	0.999		0.997	0.997	
	(0.001)		(0.003)	(0.002)	
p^{ω}_{22}	0.999		0.999	0.999	
	(0.002)		(0.001)	(0.001)	
ℓ_C	336.522	327.309	297.917	296.163	239.572
LR test		18.427	77.209	80.719	193.900
p-value		0.011	< 0.001	< 0.001	< 0.001

The estimation results for the mixture copula specifications for JP-KO. For further details see Table 2.