NON-PARAMETRIC TESTS FOR FIRM EFFICIENCY IN CASE OF ERRORS-IN-VARIABLES: EFFICIENCY DEPTH

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NON-PARAMETRIC TESTS FOR FIRM EFFICIENCY IN CASE OF ERRORS-IN-VARIABLES: EFFICIENCY DEPTH

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ABSTRACT
This paper develops a novel statistic for firm efficiency called *efficiency depth* that allows for statistical inference in case of errors-in-variables. We derive statistical tests that require minimal statistical assumptions; neither the sample distribution nor the noise level is required. An empirical illustration for European banks illustrates that – despite the minimal assumptions- the tests can have substantial discriminating power in practical applications.

KEY WORDS Firm efficiency, nonparametric analysis, errors-in-variables.

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1. INTRODUCTION

The nonparametric approach to analyzing firm efficiency, sometimes dubbed as Data Envelopment Analysis (DEA; Charnes *et al.*, 1978; Banker *et al.*, 1984, Charnes *et al.* 1985), has been credited for its ability to deal with multiple input and output variables while using only minimal assumptions about the relationship between the variables. Perhaps the only serious limitation of the methodology is that the data should be measured by full accuracy. In practice, data are almost always contaminated by errors-in-variables. For example, much empirical research uses accounting data that can give a flawed representation of the underlying economic values, e.g. because of debatable valuation and depreciation schemes. Since efficiency analysis relies on comparison with extreme observations, the results are extremely sensitive to errors; a single outlier can substantially affect the outcomes for the entire sample.

Various approaches have been proposed to account for errors-in-variables, including stochastic programming (e.g. Olesen and Petersen, 1995) and mean-variance analysis (Post, 2001). Unfortunately, the statistical properties of these approaches have not yet been documented, and hence these approaches do not allow for statistical inference. Also, the models invariably rely on a series of possibly restrictive statistical assumptions (including the specification of the noise level), and the robustness with respect to erroneous distribution assumptions remains as an open question.

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This paper develops a novel efficiency statistic called *efficiency depth* that measures the number of observations that is consistent with classifying a firm as efficient. We demonstrate that this statistic allows for statistical inference based on minimal statistical assumptions; neither the sampling distribution nor the noise level is required.

Errors-in-variables for the evaluated firm constitute a major problem. Even in the parametric approach to efficiency analysis, the goodness of the firm efficiency estimates deteriorates rapidly as the noise level increases, and a reliable classification of firms as efficient of inefficient is not possible at high noise levels (see e.g. Waldman, 1984). Inaccuracies for the evaluated firm seriously reduce goodness because the data set contains just a single observation for the evaluated vector (in cross-section studies), and hence errors can not 'cancel out'. Still, the evaluated netput vector can sometimes be measured with great accuracy, e.g. if the analyst is a manager of the evaluated firm. For this reason, we consider both the case with errors-in-variables for the evaluated firm and the case without such errors.

The remainder of this paper is organized as follows. In Section 2 we introduce preliminary concepts and notation related to the traditional case with accurate measurement. Section 3 introduces errors-in-variables, discusses the novel efficiency depth statistic, and presents two Efficiency Depth Theorems to describe the statistical distribution of this statistic. Section 4 shows how the efficiency depth statistic is obtained as the optimal solution to a Mixed Integer Linear Programming problem. Section 5 illustrates the proposed tests by means of an empirical application to 1000 largest European commercial banks. Section 6 concludes by pointing out some interesting avenues for future research.

2. PRELIMINARIES

In theory, efficiency is defined and measured relative to the (super) population of all technically feasible production plans, which can be characterized e.g. by means of a production function or a production possibilities set. In practice, however, efficiency measurement is ultimately based on comparing a discrete set of firms that utilize a common technology. In this paper, we are not primarily concerned of the relationship between population efficiency (efficiency relative to all feasible production plans) and sample efficiency (efficiency relative to the discrete set of observed firms). Rather, we focus on the sample efficiency of n firms indexed by $j \in J \equiv \{1,\cdots,n\}$. Still, it is worth mentioning that sample efficiency is a *necessary* condition for true efficiency, because the firms constitute a subset of the production possibilities. Yet, sample efficiency is not a *sufficient* condition for true efficiency. Therefore, our tests may involve only little discriminating power in small samples (see point 2 in the Conclusions).

Different models use different efficiency concepts. It is important to use a concept that correctly reflects the economic objectives of the firms under evaluation. For example, the finding that a particular output can be increased by a substantial amount is not economically meaningful if the price of that output is low or if demand is determined exogenously (as for utilities that operate in a service area that is fixed by regulation). In this paper, we use Nerlove's (1965) concept of profit efficiency, which assumes profit-maximizing behavior. Profit efficiency assesses whether or not, and to what extent, the firms have succeeded in maximizing profit at the input-output prices faced by the firms. To simplify notation, we formulate in terms of netputs (positive elements represent outputs and negative elements represent inputs). The netputs of the observed firms are represented by the matrix $Y \equiv (y_1 ... y_n)$ with $y_j \equiv (y_{1j} ... y_{mj})^T \in T$. In addition, we use prices $p_j \in \Re_+^m$, that are normalized so as to sum to unity, i.e. $p_j^T e = 1$. Using this notation, firm $k \in J$ is efficient if and only if:

(1) $$p_k(p_k, Y) \equiv \max_{j \in J} p_k^T (y_j - y_k) = 0.$$

In many empirical research situations, firms face problems that are more complex than the above problem of unconstrained profit maximization. In practice, we typically need to impose additional restrictions e.g. due to the nondiscretionary nature of exogenously fixed inputs and outputs. Moreover, the firm may face additional cost or revenue constraints; e.g. the cost of some inputs may not exceed a certain a priori budgeted sum. Still, enriching efficiency analysis with those types of additional constraints is relatively straightforward, and operational solutions are well documented elsewhere (e.g. Färe and Primont, 1990; Färe and Grosskopf, 1994; and Grosskopf and Hayes, 1997). Therefore, we choose to focus on the basic case of profit maximization in the following.

Unfortunately, the profit efficiency measure in many cases can not be used directly, as full price information in many cases is not available. Several authors cite this motivation for focussing on technical efficiency (e.g. Charnes and Cooper, 1985). Still, economic theory can guide in the selection of technical efficiency measures that are economically meaningful. For example, Kuosmanen and Post (2001) developed a systematic framework for efficiency measurement with incomplete price information. In this paper, we will use this framework to extend the concept of profit efficiency towards settings with incomplete price information. We assume that the prices belong to the following domain:

(2) $$P(A_k) \equiv \{ r \in \Re_+^m : A_k r \geq 0; r^T e = 1 \},$$

where A_k is a $l \times m$ matrix representing q linear inequalities. In the limiting case of full price information, $P(A_k) = p_k^T$. By contrast, in the limiting case of no price information, A_k is void, and we have $P(A_k) = \{ r \in \Re_+^m : r^T e = 1 \}$. As discussed in Kuosmanen and Post, most research environments involve an intermediate case where some (but not all) information is available.

Using this limited information, we can derive the following necessary condition for efficiency:

(3) $$q_k(A_k, Y) \equiv \min_{r \in P(A_k)} \left[\max_{j \in J} p^T (y_j - y_k) \right] = 0.$$

Interestingly, this necessary test contains both the conventional economic efficiency conditions (i.e. Nerlovian profit efficiency) and the Debreu-Farrell technical efficiency criterion as its limiting special cases. Specifically, in the limiting case of full price information, the embedding minimization problem of (3) becomes redundant, and we are left with the profit efficiency condition (1). By contrast, in the limiting case of no price information, we essentially have the "coefficient of resource utilization" by Debreu (1951) expressed in difference form (rather than ratio form). Recall that this classic efficiency notion underlies the standard Farrell (1957) efficiency measures. Hence, the necessary condition (3) provides a nice generalization of alternative efficiency criteria, and allows for a gradual transition from the most stringent economic efficiency conditions to the weaker technical efficiency criteria, depending on the availability of price information.

3. INCLUDING ERRORS-IN-VARIABLES: EFFICIENCY DEPTH

In contrast to the original methodology, we distinguish between the true values and the observed values for the netputs.[1] We denote *observed* values by

(4) $$\hat{Y} \equiv Y + E \,,$$

with $E \equiv (e_1 \cdots e_n)$ and $e_j \equiv (e_{1j} \cdots e_{mj})^T \in \Re^m$ for errors-in-variables. We assume that the errors are i.i.d. random variables with a symmetric zero-mean distribution characterized by the cumulative distribution function $\Phi_E : \Re \to [0,1]$. Notice that we do not impose a particular distribution function, and that we do not specify the noise level (e.g. in terms of a standard deviation of errors), so as to preserve the nonparametric nature of the original methodology.

Unfortunately, $q_k(A_k, \hat{Y}) = 0$ does not provide a necessary condition for efficiency, and it is not clear how to interpret violations of this condition. This provides the direct motivation for considering alternative efficiency statistics.

DEFINITION The *efficiency depth* of firm $k \in J$ is the maximum number of observations that is consistent with the efficiency classification of firm $k \in J$, i.e.:

(5) $$d_k(A_k, \hat{Y}) \equiv \max_{r \in \mathrm{P}(A_k)} \left[\mathrm{card}\left\{ j \in J : r^T(\hat{y}_j - \hat{y}_k) \leq 0 \right\} \right].$$

Alternatively stated, $n - d_k(A_k, \hat{Y})$ is the minimum number of observations that are inconsistent with classifying firm k as efficient, and hence would need to be excluded from the data set for classifying firm k as efficient. If a firm is efficient, no violations occur, i.e. $d_k(A_k, \hat{Y}) = n$, and the lower the efficiency depth, the more empirical evidence the data set contains against efficiency of firm k.

To illustrate the concept of efficiency depth, consider Figure 1. The figure displays 10 firms (A-J) that operate under a common single-input single-output technology. In

[1] The observed values are possible estimators. However, depending on the particular theory and data available, alternative estimators could be used, such as the mean of a sample of multiple observations. Still, for the sake of comparison, we assume here that the standard statistics use the same estimates as our statistics.

case of full price information, with relative prices represented by the iso-profit line *L*, 4 firms are inconsistent with efficiency for firm A: firms G, C, J and E. The remaining 6 firms are consistent with the efficiency hypothesis (efficiency depth = 6). By contrast, in case of no price information, *at least* two firms are inconsistent with efficiency for firm A (efficiency depth = 8); for example, removing firm E and firm G suffices to classify firm A as efficient. A possible solution for the optimal prices (i.e. prices that require only 2 removals to classify firm A as efficient) is represented by the iso-profit line *L'*.

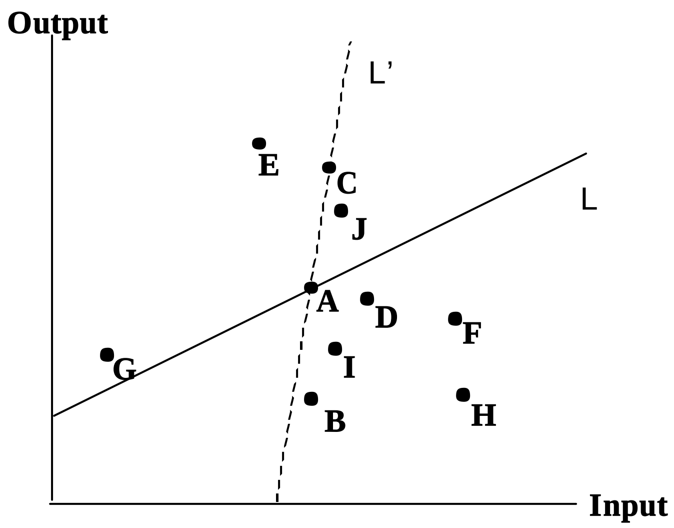

Figure 1 Efficiency depth at different price sets

If the true netputs are known, efficiency can be equivalently tested using either the conventional efficiency measure (3) or the efficiency depth; both $q_k(A_k, Y) = 0$ and $d_k(A_k, Y) = n$ are necessary conditions for efficiency. But in contrast to the conventional statistic, the efficiency depth also allows for statistical inference in case of errors-in variables. The statistical distribution of the efficiency depth generally depends on the statistical distribution of the true production vectors within the production set and the statistical distribution of the errors-in-variables. As discussed above, strong statistical distribution assumptions do not fit well to the nonparametric nature of the original methodology. Nevertheless, the minimal assumptions discussed in above do allow for statistical inference.

For simplicity, we first consider the case where the evaluated netput vector is measured accurately, i.e. $e_k = 0$. In our opinion, this is a reasonable assumption in many situations, including for example the following: 1) The analyst is associated with the evaluated firm (which allows for an in-depth analysis to ascertain the quality of the data of the evaluated firm), while the data for the referencing firms come from an external source (e.g. accounting data collected by an industry organization). 2) The data are basically reliable, but there may be errors due to differences in the definition of variables, measurement practices, and/or accounting standards (and hence we may reasonably adopt the standards of the evaluated unit as the basis for the analysis). 3) The analysis is targeted at evaluating feasibility of a (set of) hypothetical fixed netput vector(s), e.g. for planning purposes, or for recovering the production technology (e.g. in terms of a production frontier or production set).

THEOREM 1 ('1ST EFFICIENCY DEPTH THEOREM') If firm $k \in J$ is efficient and the data for $k \in J$ are accurate, i.e. $\boldsymbol{e}_k = 0$, then $P\left[d_k(A_k, \hat{Y}) \leq q\right]$, $q \in \{1, \cdots, n\}$, is bounded from above by the cumulative binomial density function $\Phi_B(q) \equiv \sum_{i=0}^{q-1} \binom{n-1}{i} 0.5^{n-1}$.

PROOF Define $x_k(p_k, \hat{Y}) \equiv \mathrm{card}\{j \in J \setminus k : p_k^T \boldsymbol{e}_j \leq 0\}$. If firm $k \in J$ is efficient, then $p_k^T(\hat{y}_j - \hat{y}_k) \leq p_k^T \boldsymbol{e}_j \quad \forall j \in J \setminus k$.[2] Therefore, using $p_k \in P(A_k)$, we find $d_k(A_k, \hat{Y}) \geq x_k(p_k, \hat{Y}) + 1$, and hence

(i) $\quad P\left[d_k(A_k, \hat{Y}) \leq q\right] \leq P\left[x_k(p_k, \hat{Y}) \leq q - 1\right]$.

Since the errors are i.i.d. random variables with a symmetric zero-mean distribution, $P\left[p_k^T \boldsymbol{e}_j \leq 0\right] = 0.5 \ \forall j \in J \setminus k$, and $P\left[x_k(p_k, \hat{Y}) \leq q - 1\right]$ obeys a binomial distribution. Specifically,

(ii) $\quad P\left[x_k(p_k, \hat{Y}) \leq q\right] = \sum_{i=0}^{q-1} \binom{n-1}{i} 0.5^{n-1}$.

Combining (i) and (ii) gives

(iii) $\quad P\left[d_k(p_k, \hat{Y}) \leq q\right] \leq \sum_{i=0}^{q-1} \binom{n-1}{i} 0.5^{n-1}$. ■

The theorem effectively follows the statistical convention of defining the significance level as the supremum of the rejection probability over all cases satisfying the null hypothesis; the probability of rejecting the null hypothesis (i.e. the evaluated firm is efficient) when it is true is greatest in the limiting case that all firms maximize profit at p_k, i.e. $p_k^T y_j = p_k^T y_k$. Focusing on the least favorable limit may seem to involve unnecessary loss of discriminating power, as data sets generally contain inefficient firms and firms that face different prices than the evaluated firm. However, the results apply for all samples Y that satisfy the null hypothesis. Therefore, this approach circumvents the need to account for the distribution of the firms. In our opinion, this is an attractive feature, because the specification of the sampling distribution is even more problematic than the specification of the error distribution; in contrast to the latter, the former requires detailed assumptions about the economic behavior of the firms (e.g. prices faced by the firms and possible causes of inefficiency). Also, the binomial distribution obtained for the limiting case does not require the specification of the noise level.

The Efficiency Depth Theorem can directly test whether the evaluated firm is efficient. Specifically, the probability that the efficiency depth statistic is less than or equal to $d_k(A_k, \hat{Y})$ if the evaluated firm is efficient (i.e. the probability of exceedance or *p*-value) is bounded from above by $\Phi_B(d_k(A_k, \hat{Y}))$. Hence, we can reject efficiency at all levels of confidence less than or equal to 1-$\Phi_B(d_k(A_k, \hat{Y}))$. Alternatively, we can compute a critical value for the efficiency depth statistic, and compare that critical

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[2] **Seems correct to me, as y=y^ minus epsilon.**

value with the observed value. Table 1 reports some critical values of efficiency depth computed at some selected confidence levels and sample sizes. For example, the null hypothesis can be rejected at 95% confidence in a sample of 500 firms if the efficiency depth is less than or equal to 230.

Table 1: Some examples of critical efficiency depth values when the data of the evaluated firm may include errors

	Sample size (n) →		
↓Confidence	50	100	500
99%	15	37	223
95%	18	40	230
90%	19	42	234

It is worth to note that in large samples, the binomial cumulative density $\Phi_B(d_k(A_k,\hat{Y}))$ can be approximated using the cumulative normal density

(9) $$\Phi_N\left(\frac{2q-n-2}{\sqrt{n-1}}\right),$$

where $\Phi_N(\cdot)$ denotes the cumulative standard normal density function.

In Theorem 1, we assumed that the data of the evaluated firm are measured with full accuracy. Although this seems to be a reasonable assumption in many situations, there are other circumstances where this condition appears overly restrictive. For these cases, the following theorem applies:

THEOREM 2 ('2ND EFFICIENCY DEPTH THEOREM') If firm $k \in J$ is efficient and the data for $k \in J$ are may be inaccurate, then $P[d_k(A_k,\hat{Y}) \le q]$, $q \in \{1,\cdots,n\}$, is bounded from above by is bounded from above by the cumulative uniform density $\Phi_U(q) \equiv \frac{q}{n}$.

PROOF Define $x_k(p_k,\hat{Y}) \equiv \text{card}\{j \in J \setminus k : p_k^T e_j \le p_k^T e_k\}$ and $z_k(p_k,\hat{Y},z) \equiv \text{card}\{j \in J \setminus k : p_k^T e_j \le z\}$. By analogy to the proof for Theorem 1,

(i) $$P[d_k(A_k,\hat{Y}) \le q] \le P[x_k(p_k,\hat{Y}) \le q-1] = \sum_{s=0}^{q-1} P[x_k(p_k,\hat{Y}) = s].$$

Since the errors are i.i.d. random variables, $P[p_k^T e_j \le z] = \Phi_E^*(z) \equiv \int_{e:p_k^T e \le z} \partial\Phi_E(e)$ $\forall j \in J \setminus k$. Hence, $P[z_k(p_k,\hat{Y},z) = s]$ obeys a binomial distribution. Specifically,

(ii) $$P[z_k(p_k,\hat{Y},z) = s] = \binom{n-1}{s} \Phi_E^*(z)^i (1-\Phi_E^*(z))^{n-1-i}.$$

Therefore,

(iii) $$P\left[x_k(p_k,\hat{Y})=s\right]=\int_0^1 P\left[z_k(p_k,\hat{Y},z)=s\right]\partial\Phi_E^*$$

$$=\binom{n-1}{s}\int_0^1 \Phi_E^*(z)^i\left(1-\Phi_E^*(z)\right)^{n-1-i}\partial\Phi_E^*(z)$$

$$=\binom{n-1}{s}\sum_{t=0}^{n-1-i}\left(\binom{n-1-i}{t}(-1)^{2+t}\int_0^1\Phi_E^*(z)^{i+t}\partial\Phi_E^*(z)\right)$$

$$=\binom{n-1}{s}\sum_{t=0}^{n-1-i}\left(\binom{n-1-i}{t}\frac{(-1)^{2+t}}{i+t+1}\right).$$

The third equality is obtained from $(1-\Phi_E^*(z))^{n-1-i}=\sum_{t=0}^{n-1-i}\left(\binom{n-1-i}{t}(-1)^{2+t}\Phi_E^*(z)^t\right)$,

and the fourth equality from $\int_0^1\Phi_E^*(z)^{i+t}\partial\Phi_E^*(z)=\frac{1}{i+t+1}$. The right-hand side series solves $\frac{1}{n}$. Substituting $P\left[x_k(p_k,\hat{Y})=s\right]=\frac{1}{n}$ in (i) gives:

(iv) $$P\left[d_k(A_k,\hat{Y})\le q-1\right]\le\frac{q}{n}.\blacksquare$$

Like above in case of Theorem 1, the 2nd Efficiency Depth Theorem can be used for testing efficiency. The *p*-value associated with the observed efficiency depth is bounded from above by $\Phi_U\left(d_k(A_k,\hat{Y})\right)$. Hence, we can reject efficiency at all levels of confidence less than or equal to 1- $\Phi_U\left(d_k(A_k,\hat{Y})\right)$. Conversely, to reject the efficiency hypothesis at the confidence level of a , the efficiency depth should be small enough to satisfy $d_k(A_k,\hat{Y})\le(1-a)n$. Table 2 reports the critical values for efficiency depth at the selected confidence levels and sample sizes used in Table 1.

Table 2: Some examples of critical efficiency depth values when the data of the evaluated firm may include errors

	Sample size (n) →		
↓Confidence	50	100	500
99%	0	1	5
95%	2	5	25
90%	5	10	50

These critical values are substantially smaller than those reported in Table 1; the probability of exceedance $\Phi_U(q)$ falls below the conventional levels of significance only for extremely values of efficiency depth. Efficiency tests building on Theorem 2 therefore generally involve less power than tests based on Theorem 1. This illustrates the key importance of ensuring that the evaluated netput vector is accurately measured.

Also, the upper bounds derived in Theorems 1 and 2 do not use any information on the price domain $P(A_k)$. Hence, the same *p*-values also apply in all price situation, regardless of whether we measure more stringent profit efficiency or weaker technical efficiency. Still, the efficiency depth statistic generally decreases as the price domain is restricted. Thus, the power of the efficiency test can be improved by incorporating additional price restrictions. This implies high priority of collecting reliable information on relative prices.

4. COMPUTATIONAL CONSIDERATIONS

The original efficiency measure (2) by straightforward Linear Programming. By contrast, computing the efficiency depth is more complicated, because it involves the selection of the prices that minimize the number of violations of the efficiency condition. Still, standard mathematical programming techniques can solve this problem.

We can reformulate the efficiency depth statistic in the following way. Our purpose is to select a price vector p from the domain P to maximize the number of firms with a profit difference $p^T(\hat{y}_j - \hat{y}_k)$ is non-positive. This gives us an alternative expression of the efficiency depth statistic as:

(6) $$d_k = \max_{p,k}\left\{\vec{1}k \,\middle|\, p^T(\hat{y}_j - \hat{y}_k) \le c_k(\vec{1} - k); k_j \in \{0,1\} \;\; \forall j \in J; Ap \ge 0; \vec{1}p = 1\right\},$$

where c_k should be a sufficiently large number for which $p^T(\hat{y}_j - \hat{y}_k) \le c_k \;\; \forall j \in J \setminus k$. To guarantee feasibility of problem (6), one can e.g. use the optimal solution to the LP problem:

(7) $$\max_{p,c}\left\{c \,\middle|\, p^T(\hat{y}_j - \hat{y}_k) \le c \;\; \forall j \in J \setminus k; Ap \ge 0; \vec{1}p = 1\right\}$$

Problem (6) can be solved using Mixed Integer Linear Programming (MILP) techniques. MILP is computationally more complex than linear programming. However, with modern-day solvers and computation power, problem (6) should not involve substantial computational burden, even for large-scale problems with numerous firms and numerous netput variables. For example, using the CPLEX Mixed Integer Optimizer, the computations for the example below (involving 1000 firms and 4 netput variables) required only minimal effort using an ordinary PC desktop.

5. EXAMPLE APPLICATION

The nonparametric approach to efficiency analysis has seen extensive application for studying the financial industry. For example, Berger and Humphrey (1997) found that 69 out of 122 frontier efficiency studies for financial institutions use the nonparametric approach. To illustrate our approach to errors-in-variables, we performed an empirical application in this area. Specifically, we used a data set with 1997 financial statement data of the 1000 largest banks in the European Union[3].

[3] We use BankScope data provided by Bureau van Dijk Nederland.

For convenience, we use a simplified representation of the bank technology that involves a single input: total cost, and three outputs: (1) total loans, (2) total other earning assets (OEA) and (3) off-balance-sheet (OBS) items. Table 3 lists some descriptive statistics for the data set.

Table 3: Descriptive statistics data set

	Cost	Loans	OEA	OBS
Minimum	9.02	0.00	0.00	1.20
Maximum	29141.99	294484.64	238366.42	269003.01
Mean	987.82	8592.61	4310.23	7976.46
St. dev.	2591.46	23120.81	17246.90	21598.58
Skewness	6.21	6.69	8.20	5.95
Kurtosis	47.80	58.57	83.76	45.82

Unfortunately, reliable information on the prices of the netput variables was not available. Hence, we resorted to testing for technical efficiency. Table 4 gives descriptive statistics for the efficiency depth and the associated upper bounds for the *p*-value (see Theorem 1 and 2).

Table 4: Test results technical efficiency

	Efficiency depth	p-value (1)	p-value (2)
Minimum	12	0.000	0.012
Maximum	1000	1.000	1.000
Mean	641.03	0.703	0.641
St. Dev.	248.66	0.447	0.249
Skewness	-0.425	-0.885	-0.425
Kurtosis	-0.816	-1.182	-0.816

At a level of significance of 95 percent, 271 banks are classified as technically inefficient, using Theorem 2. However, using Theorem 1, only 5 banks are classified as inefficient.

We have thus far focussed on technical efficiency. Profit efficiency aggregates all individual netputs into a single economically meaningful measure, i.e. profit. Aggregation can substantially improve discriminating power. Unfortunately, reliable information on the relative prices is not available. Still, we can illustrate the potential effect of aggregation for hypothetical prices. Specifically, for all banks, we assumed a uniform price of 0.06 for all three outputs, and a price of 1 for the input variable.

Table 5 gives descriptive statistics for the aggregated data (i.e. profit at the hypothetical prices) and the test results.

Table 5: Test results economic efficiency

	Profit	Efficiency depth	p-value (1)	p-value (2)
Minimum	-2276.170	1	0.000	0.001
Maximum	14395.990	1000	1.000	1.000
Mean	255.830	500.50	0.499	0.501

St. Dev.	1162.080	288.82	0.491	0.289
Skewness	7.270	0.000	0.002	0.000
Kurtosis	68.880	-1.200	-1.978	-1.200

At a level of significance of 95 percent, 475 banks are classified as economically inefficient, using Theorem 2. Using Theorem 1, 50 banks are classified as inefficient.

6. CONCLUSIONS AND SUGGESTIONS

The novel efficiency depth statistic allows for statistical inference based on minimal assumptions about the sampling distribution and the statistical distribution of errors. The empirical illustration for European banks suggests that –despite the minimal assumptions- the tests can have substantial discriminating power in practical applications, especially if reliable information is available on the netput vector and the prices of the evaluated firm.

For these reasons, we believe efficiency depth is a useful complement or substitute to traditional efficiency measures. Still, we see the following routes for future research:

1. A rigorous analysis of the power of our tests. We think detailed statistical distribution assumptions are not consistent with our nonparametric orientation. Still, there is value added in analysis the power of our tests for a wide range of different distribution structures, e.g. using computer simulations.

2. As in all nonparametric analysis, using minimal assumptions can introduce small sample error. Small samples generally do not contain sufficient observations to fully represent the production possibilities. This can cause small sample error, i.e. inefficient firms can be wrongly classified as efficient, or the degree of population inefficiency can be substantially underestimated (see e.g. Banker, 1993, Kneip *et al.*, 1998, and Gijbels *et al.*, 1999). Recently, it has been suggested to analytically derive the sampling distribution (e.g. and Gijbels *et al.*, 1999) or to approximate it using bootstrapping techniques (e.g. Simar and Wilson, 1998); knowledge of the sampling distribution can correct for small sample bias and construct confidence intervals. Future research could try to integrate our tests with these approaches.

3. Inclusion of heteroskedasticity and interdependency. Despite our nonparametric orientation, we have assumed homoskedasticity and independence across firms and independence across netputs. If reliable information about the variance-covariance structure is available, that information could be used to improve the analysis. We have not considered this issue in this paper, but future research could focus on this subject. One possibility is to apply the 'standard' econometric approach of using data transformations to transform heteroskedatisic and interdependent observations into homoskedastic and independent observations.

4. We have focused on testing whether or not firms are efficient. Efficiency measurement is one application of the nonparametric approach to production analysis; another application is recovering the production technology (characterized e.g. by means of a production function or a production possibilities set) faced by the firms (e.g. for studying scale and scope properties). Future research could focus on extending our approach towards this application. In a

follow-up paper (Cherchye *et al.*, 2000), we have already developed some preliminary ideas for such an extension.

REFERENCES

Afriat, S. (1972): Efficiency Estimation of Production Functions, *International Economic Review* 13(3), 568-598.

Banker, R. D. (1993): 'Maximum likelihood, consistency and Data Envelopment analysis: a statistical foundation', *Management Science* 39 (10), 1265-1273.

Banker, R.D., Charnes, A. and Cooper W. W. (1984): Some models for estimating technical and scale inefficiencies in data envelopment analysis, *Management Sci*ence, 30(9), 1078-1092.

Berger, A.N. and D.B. Humphrey (1997): Efficiency of Financial Institutions: International Survey and Directions for Future Research, *European Journal of Operational Research* 98, 175-212.

Charnes, A. and W. W. Cooper (1985): 'Preface to topics in data envelopment analysis', *Annals of Operations Research* 2, 59-94.

Charnes, A., W. Cooper, B. Golany, L. Seiford, J. Stutz (1985): 'Foundations of Data Envelopment Analysis for Pareto-Koopmans Efficient Empirical Production Functions', *Journal of Econometrics* 30 (1): 91-107.

Charnes, A., Cooper, W. W. and Rhodes, E. (1978): 'Measuring the efficiency of decision making units', *European Journal of Operational Research* 2 (6): 429-444.

Charnes, A., and Neralic (1990): 'Sensitivity analysis of the additive model in data envelopment analysis', *European Journal of Operational Research* 48: 332-341.

Cherchye, L., T. Kuosmanen and T. Post (2000), New Tools for Dealing with Errors-in-Variables in DEA, Discussion Paper D/2000/2020/6, Centre for Economic Studies, Catholic University Leuven.

Debreu, G. (1951): 'The Coefficient of the Resource Utilization', *Econometrica* 19 (3): 273 - 292.

Diewert, W.E., and C. Parkan (1983): *Linear programming Tests of Regularity Conditions for Production Frontiers*, in W. Eichhorn, R. Henn, K. Neumann and R.W. Shephard (Eds.): *Quantitative Studies on Production and Prices*, Physica-Verlag, Würzburg.

Färe, R., and S. Grosskopf (1994): *Cost and Revenue Constrained Production*, Bilkent University Lecture Series, vol. 4, New York; Heidelberg; London and Tokyo: Springer.

Färe, R., and D. Primont (1990): A Distance Function Approach to Multioutput Technologies, *Southern Economic Journal* 56, 879-891.

Farrell, M.J. (1957), The Measurement of Productive Efficiency, *Journal of the Royal Statistical Society* Series A 120, 253-281.

Gijbels, I., E. Mammen, B.U. Park and L. Simar (1999): "On Estimation of Monotone and Concave Frontier Functions", *Journal of the American Statistical Association* 94 (445): 220-228.

Grosskopf, S., and K.J. Hayes (1997): Budget-constrained frontier measures of fiscal equality and efficiency in schooling, *Review of Economics & Statistics*, 79(1), 116-124.

Kneip, A., B. U. Park and L. Simar (1998): 'A note on the convergence of nonparametric DEA efficiency measures', *Econometric Theory* 14: 783-793.

Kuosmanen, T. and G.T. Post (1999b): 'Measuring Economic Efficiency under Imperfect Price Information: With an Application to European Commercial Banks', Rotterdam Institute for Business Economics Studies (RIBES) working paper 9934, forthcoming in *European Journal of Operational Research*.

Nerlove, M. (1965): *Estimation and Identification of Cobb-Douglas Production Functions*, Rand McNally, Chicago, Illinois.

Olesen, O. B. and N. C. Petersen (1995), 'Chance Constrained Efficiency Evaluation', *Management Science* 41, 442-457.

Post, G.T. (2001), 'Performance benchmarking in stochastic environments using Mean-Variance Data Envelopment Analysis', *Operations Research* March-April.

Simar, L. and P. Wilson (1998): 'Sensitivity Analysis of Efficiency Scores: How to Bootstrap in Nonparametric Frontier Models', *Management Science* 44 (1): 49-61.

Varian, H (1984): 'The nonparametric approach to production analysis', *Econometrica* 52: 579-597.

Varian, H (1985): 'Nonparametric Analysis of Optimising Behaviour with Measurement Error', *Journal of Econometrics* 30 (1/2): 445-458.

Waldman, D. M. (1984): 'Properties of technical efficiency estimators in the stochastic frontier model', *Journal of Econometrics* 25, 353-364.

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