Extreme Linkages in Financial Markets: Macro Shocks and Systemic Risk

Charnchai Leuwattanachotinan* Earamus University Rotterdam

Casper G. de Vries Chapman University

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Abstract

In this paper we measure extreme loss linkages in financial markets during severe macroeconomic conditions. Specifically, we employ a count estimator, which is a non-parametric univariate approach, to compute probabilities of the extreme linkage between daily S&P500 and German DAX index returns conditioned on extreme levels of macroeconomic factors (i.e. inflation, industrial production, unemployment and money supply). According to the results, we conclude that, the factor related to real economy, i.e. industrial production, has most impact on the extreme loss linkage between US and German equity markets comparing to the other factors which are more related monetary policies. Additionally, the same procedure is also implemented to the equity returns by sector of both markets and we find that industrial production is still the most dominant macro factor. Health care and utilities sectors are the two sectors least affected by the severe macroeconomic circumstances.

1 Introduction

There are numerous works and research dedicated to studying rare events and their co-movement in the financial markets. Nevertheless, with best of our knowledge, few researchers attempt to link such extreme linkages with macro-related situations. Here, we post two main questions. First, what are extreme linkages of asset returns during severe macroeconomic conditions? Second, do such linkages differ from those in an ordinary period?

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One excellent work that investigates the extreme linkage in financial market was carried out by Hartmann, Straetmans, and de Vries (2004). They studied asset market linkages between and across equity and bond markets in crisis periods but unconditional on macro factors. Our work here is mainly based on their work with extension to the situations where the number of macroeconomic factors are stressed.

The remainder of the paper is organised as follows. The next section introduces the concept of heavy tails and the measure of extreme dependency. Section 3 theoretically investigates the fat-tailed distribution of macro factors using a standard closed economy macro model. The equity return and macroeconomic data are described in Section 4. In Section 5, empirical results of estimating the extreme loss linkage probabilities are discussed. Section 6 concludes.

2 Heavy Tails and Extreme Dependence Measure

This section is devoted to review some theory of heavy tails and present an estimator to measure the extreme dependence.

2.1 Fundamentals of Heavy Tails

Suppose that $F(x)$ is a distribution function of a random variable x. $F(x)$ exhibits heavy tails if its tails vary regularly at infinity. Specifically, for the upper tail, we have

$$\lim_{t\to\infty} = \frac{1-F(tx)}{1-F(t)} = x^{-\alpha}, x > 0, \alpha > 0, \tag{1}$$

where α is a tail index.

Furthermore, random variables whose tails are regularly varying also have an additivity property, i.e. Feller's Convolution Theorem (1971, VIII.8). That is, assume that

$$P\{X > x\} = 1 - F(x) = Ax^{-\alpha} + o(x^{-\alpha}), \text{ as } x \to \infty. \tag{2}$$

Then, if X_1 and X_2 are i.i.d. with c.d.f. $F(x)$ in (2),

$$P\{X_1 + X_2 > s\} \sim 2As^{-\alpha}, \text{ as } s \to \infty. \tag{3}$$

If X and Y are two random variables, where $P\{X > x\} \sim A_1 x^{-\alpha}, P\{Y > x\} \sim A_2 x^{-\gamma}$, and $\alpha > \gamma$, it can be shown that

$$P\{X + Y > s\} \sim As^{-\alpha}, \text{ as } s \to \infty. \tag{4}$$

In other words, the convolution is dominated by the heavier tail.

2.1.1 Tail Index Estimator

To estimate the tail index α, one may use the Hill (1975) estimator. Suppose that $X_1 \leq X_2 \leq \ldots \leq X_n$ is the ascending order statistics. Thus, the Hill estimator is the inverse tail index

$$\widehat{\gamma} = \widehat{\frac{1}{\alpha}} = \frac{1}{k}\sum_{i=1}^{k} \log \frac{X_{n+1-i}}{X_{n-k}}, \tag{5}$$

where X_{n-k} is an appropriate threshold which is frequently selected from the sample data. A suitable threshold or cutoff point can be chosen by using the eye-balling technique, introduced by Embrechts et al. (1997). More precisely, the cutoff point is selected where the plot of the estimated tail indices against threshold values is first relatively stable. Alternatively, the Dekkers-Einmahl-de Haan (1989) (DEdH) estimator may also be used for estimating the tail index such that

$$\widehat{\gamma} = \widehat{\frac{1}{\alpha}} = 1 + H + \frac{1}{2}\frac{K/H}{H - K/H}, \tag{6}$$

where

$$H = \frac{1}{k}\sum_{i=1}^{k} \log \frac{X_{n+1-i}}{X_{n-k}}, \qquad K = \frac{1}{k}\sum_{i=1}^{k} (\log \frac{X_{n+1-i}}{X_{n-k}})^2.$$

It is noted that H is the Hill estimator and if the distribution varies regularly at infinity, $K/2H$ is an alternative to the Hill estimator.

2.2 Extreme Dependence Measure

To measure the extreme dependence of a bivariate data, we discuss the count measure which is the core methodology used in the next section. Then, some data transformation method for the raw data will also be presented.

2.2.1 Count Measure

At first, one may think of using correlation to measure dependence. Nevertheless, it is known that correlation concept much depends on the multivariate normal distribution in which it might not a reliable dependence measure for data at the tails (Ang and Chen, 2002). Moreover, the amount of correlation is also quite not meaningful in the sense that it tells us nothing about the probability. More detail regarding the pitfalls of using correlation measure can be found in Embrechts et al. (1999).

Therefore, we directly quantify the extreme dependence in terms of probability by employing a count measure which does not require any distributional assumption for the data. Suppose that X and Y are two random variables whose failure regions are defined by $X > s$ and $Y > s$, where s is a constant. Then, it can be shown that *the expected number of market crashes given that at least one market crashes* is

$$\begin{aligned} \mathbb{E}[k|k \geq 1] &= 1 + \frac{P(X > s, Y > s)}{1 - P(X \leq s, Y \leq s)} \\ &= \frac{P(X > s) + P(Y > s)}{1 - P(X \leq s, Y \leq s)} \\ &= 1 + \frac{P(\min[X, Y] > s)}{P(\max[X, Y] > s)} \\ &\approx 1 + \frac{\#\min[X, Y] > s}{\#\max[X, Y] > s}, \end{aligned} \tag{7}$$

where $k = \{1, 2\}$ is the number of market crashes.

From (7), we can notice that the conditional expected number of market crashes is in fact equal to the summation of marginal probabilities of each random variable divided by a joint probability. Typically, there are several methodologies that can be used to estimate the joint distribution such as Copula (a parametric approach) and tail dependence function (a non-parametric multivariate approach). However, the count measure will here be used since it is a simpler method that can turn the two-dimensional problem to a univariate problem of counting the number of times that $\min[X, Y] > s$ and $\max[X, Y] > s$; see Hartmann et al. (2010). In other words, the expected number of market crashes given that at least one market crashes can be approximated just by counting the number of minimums and maximums of X and Y over a threshold.

2.2.2 Conditional Count Measure

The count measure can be applied to measure the extreme linkages during severe macroeconomic conditions. It can simply be done by conditioning the probabilities of X and Y more on thresholds of macroeconomic factors. For example, the expected number of market crashes given that at least one market crashes when macroeconomic factor π are simultaneously stressed in both countries can be defined by

$$\mathbb{E}[k|k \geq 1, \pi_x > a, \pi_y > b] = \frac{P(X > s|\pi_x > a) + P(Y > s|\pi_y > b)}{1 - P(X \leq s, Y \leq s|\pi_x > a, \pi_y > b)}, \tag{8}$$

where a and b are constant thresholds, and π_x and π_y are the macroeconomic factor π of the countries of returns X and Y respectively.

Empirically, we first define thresholds for macroeconomic factors in order to specify periods of the severe conditions. Consequently, the data of those particular periods will be extracted for computing conditional extreme linkage probability.

2.2.3 Eliminating the Effect of Marginal Distributions

To investigate the dependence structure of random variables, it is conventional to eliminate the effect of their marginal distributions. One way to deal with this is to transform the raw data to common unit Pareto marginals (Hartmann et al., 2006). More precisely, suppose that we have the random variables X_i for $i = 1, \ldots, M$. Then, X_i can be transformed to $\tilde{X}_i$ such that

$$\tilde{X}_i = \frac{1}{1 - F_X(X_i)}, \text{ for } i = 1, \cdots, M,$$

where $F_{X_i}(X_i)$ denotes the marginal cumulative distribution function for X_i. After transformation, each $\tilde{X}_i$ will obtain the common marginal distribution in which the dependence structure still remains same as that of X_i. Nevertheless, since the marginal distributions are unknown, it is suggested to use their empirical counterparts instead. Hence, we eventually achieve

$$\tilde{X}_i = \frac{n+1}{n+1-R_{X_i}},$$

where $R_{X_i} = \text{rank}(x_{ik}, k = 1, \cdots, n)$. In addition, one may use other transformation methods such as unit Frétchet marginals (see Poon et al., 2004).

3 Macroeconomics with Shocks

In this section, we use a standard closed economy macro model to study how shocks propel through the macro economy in equilibrium. Both supply and demand shocks are studied. It is shown how standard assumptions on the shock distribution lead to a power law distribution explaining the bouts of severe changes and asymptotic dependency between the various macro factors.

3.1 Demand Side

Current macro models typically entertain a two sector model. One sector is competitive and the other sector produces differentiated products. The pricing power in the latter sector determines price setting behavior.

The macro literature has focussed almost exclusively on the Dixit and Stiglitz (1977) specification for the differentiated goods demand, see e.g. Walsh (2010, ch.8). The familiar Dixit-Stiglitz (DS) specification with endogenous labor supply is derived from the following utility function.

$$U = Z^{1-\theta} \left[\frac{1}{n} \sum_{i=1}^{n} Q_i^{\rho} \right]^{\theta/\rho} - \frac{1}{1+\gamma} L^{1+\gamma}, \tag{9}$$

where Z is the composite good, the Q_i are the differentiated goods and L is labor. To guarantee concavity and allow for zero demand, the parameter ρ is constrained to $\rho\epsilon(0,1)$. Macro literature, see e.g. Walsh (2010, ch.8), mostly uses a continuum of differentiated goods, here we use a specification with a discrete number n, to facilitate the computation of the distribution of the various macro factors in equilibrium. We envision the Z good to be a staple good like agricultural produce, while the Q_i goods capture the production of other goods and services.

The budget constraint reads

$$wL + \Pi(Q) = qZ + \frac{1}{n}\sum_{i=1}^{n} p_i Q_i, \tag{10}$$

where w is the wage rate and q, p_i are the goods prices, while $\Pi(Q)$ are the profits received from the differentiated goods sector.

The first order conditions for optimality conditions entail

$$(1-\theta)\, Z^{-\theta} n^{-\theta/\rho} \left[\sum_{i=1}^{n} Q_i^{\rho}\right]^{\theta/\rho} - \lambda q = 0,$$

$$\theta \left(\frac{Z}{\left[\sum_{i=1}^{n} Q_i^{\rho}\right]^{1/\rho}}\right)^{1-\theta} n^{-\theta/\rho} \left[\sum_{i=1}^{n} Q_i^{\rho}\right]^{\frac{1}{\rho}-1} Q_i^{\rho-1} - \lambda \frac{1}{n} p_i = 0,$$

$$-L^{\gamma} + \lambda w = 0$$

and

$$wL + \Pi(Q) = qZ + \frac{1}{n}\sum_{i=1}^{n} p_i Q_i.$$

The first order conditions imply the familiar price and wage ratios

$$\frac{p_i}{p_j} = \frac{Q_i^{\rho-1}}{Q_j^{\rho-1}},$$

$$\frac{p_i}{q} = \frac{\theta}{1-\theta}\frac{Z}{Q_j}\frac{p_j^{\rho/(\rho-1)}}{\frac{1}{n}\sum_{i=1}^{n} p_i^{\rho/(\rho-1)}},$$

and

$$\frac{w}{q} = \left(q^{-1}P\right)^{\theta} \frac{L^{\gamma}}{(1-\theta)^{1-\theta}\,\theta^{\theta}},$$

where the price index for differentiated goods is defined as

$$P = \left(\frac{1}{n}\sum_{i=1}^{n} p_i^{\rho/(\rho-1)}\right)^{\frac{\rho-1}{\rho}}.$$

Then the labor supply can be written as

$$L = \left((1-\theta)^{1-\theta} \theta^{\theta} \frac{w}{q^{1-\theta} P^{\theta}} \right)^{1/\gamma}. \tag{11}$$

The goods demanded can be expressed as

$$Z = (1-\theta) \frac{wL + \Pi(Q)}{q} \tag{12}$$

the goods demanded and

$$Q_i = \theta \frac{wL + \Pi(Q)}{p_i} \left(\frac{p_i}{P} \right)^{\rho/(\rho-1)}. \tag{13}$$

3.2 Supply Side

Assume Ricardian technologies for all the goods, where

$$Z = BN$$

and

$$Q_i = AN_i.$$

Here A and B are the productivity coefficients while N and N_i are the respective labor inputs. Both A and B are random variables. These TFP shocks are the familiar supply side total factor productivity shocks due to innovation and nature.

Suppose that the market for Z is perfectly competitive

$$\Pi(Z) = qZ - wN = \left(q - \frac{w}{B} \right) Z = 0,$$

so that

$$q = w/B. \tag{14}$$

3.2.1 DS Differentiated Goods

In the DS specification the differentiated goods profit function reads

$$\begin{aligned} \Pi(Q_i) &= p_i Q_i - wN_i = \left(p_i - \frac{w}{A} \right) Q_i \\ &= \left(p_i - \frac{w}{A} \right) \theta \frac{wL + \Pi(Q)}{p_i} \left(\frac{p_i}{P} \right)^{\rho/(\rho-1)}. \end{aligned}$$

The producer exploits his pricing power, but ignores his pricing effect on the price index P of the differentiated goods and the consumer income $wL + \Pi(Q)$.[1] Differentiation

[1] One can easily incorporate this effect as well if desired, see Heijdra and Yang (1993). But for two reasons we do not follow this route. One may doubt that producers take this macro effect of their pricing behavior into account. Moreover, it adds little to the insights derived form specifying the differentiated goods sector.

gives

$$\frac{\partial \Pi(Q_i)}{\partial p_i} = \frac{1}{\rho - 1} Q_i \left\{ \rho - \frac{1}{A} \frac{w}{p_i} \right\}.$$

Exploiting the pricing power therefore implies setting prices

$$p_i = \frac{w}{\rho A}. \tag{15}$$

Hence, $P = w/\rho A$ as all prices are identical. Total profits in the differentiated goods sector are

$$\begin{aligned} \Pi(Q) &= \frac{1}{n} \sum_{i=1}^{n} \Pi(Q_i) = \frac{1}{n} \sum_{i=1}^{n} \left(1 - \frac{w/p_i}{A}\right) \theta \left[wL + \Pi(Q)\right] \left(\frac{p_i}{P}\right)^{\rho/(\rho-1)} \\ &= (1-\rho)\,\theta \left[wL + \Pi(Q)\right]. \end{aligned}$$

Solve for the total sectorial profits as

$$\Pi(Q) = \frac{(1-\rho)\,\theta}{1 - (1-\rho)\,\theta} wL. \tag{16}$$

3.3 Macro Equilibrium

It follows that in equilibrium after substituting the price levels into the labor supply equation (11)

$$L = \left(\theta^{\theta} (1-\theta)^{1-\theta} A^{\theta} B^{1-\theta}\right)^{1/\gamma} \rho^{\theta/\gamma} = \varphi \rho^{\theta/\gamma}, \tag{17}$$

say, and where

$$\varphi = \left(\theta^{\theta} (1-\theta)^{1-\theta} A^{\theta} B^{1-\theta}\right)^{1/\gamma}.$$

Furthermore, from (12), (16) and (17)

$$Z = (1-\theta) \frac{B}{1 - (1-\rho)\,\theta} \varphi \rho^{\theta/\gamma}. \tag{18}$$

Similarly, using (13), (16) and (17)

$$Q_j = \theta \frac{A}{1 - (1-\rho)\,\theta} \rho \varphi \rho^{\theta/\gamma}.$$

Hence

$$\frac{1}{n} \sum_{j=1}^{n} Q_j = \theta \frac{A}{1 - (1-\rho)\,\theta} \varphi \rho^{\theta/\gamma + 1}. \tag{19}$$

To determine the price level, we also assume a simple quantity type relation for the money supply process

$$M = wL. \tag{20}$$

3.4 Equilibrium Price Distribution

With the above preparations, we can now derive the implications for the equilibrium prices, quantities and macro factors. Most macro models consider supply shocks originating from total factor productivity A and B and demand shocks originating from the money supply process M or from the markup elasticity ρ, see Smets and Wouters (2003). We will first look at the implication of such shocks for the prices p_i of industrial production.

From the above (14) combined with (20) and (17), we get that

$$q = \frac{w}{B} = \frac{M}{B}\frac{1}{L} = M\frac{1/\rho^{\theta/\gamma}}{B\left(\theta^{\theta}\left(1-\theta\right)^{1-\theta}A^{\theta}B^{1-\theta}\right)^{1/\gamma}}.$$

Similarly, using (15) combined with (20) and (17) yields

$$p_i = p = \frac{w}{\rho A} = \frac{M}{\rho A}\frac{1}{L} = M\frac{1/\rho^{\theta/\gamma+1}}{A\left(\theta^{\theta}\left(1-\theta\right)^{1-\theta}A^{\theta}B^{1-\theta}\right)^{1/\gamma}}.$$

Consider a supply shock A such that A follows a beta distribution:

$$\Pr\left\{A \le t\right\} = t^{\beta}$$

on $[0,1]$ and $\beta > 0$. Consider the implication for the distribution of the differentiated goods Q_i. Some calculation reveals

$$\begin{aligned}
\Pr\left\{p_i \le s\right\} &= \Pr\left\{M\frac{1/\rho^{\theta/\gamma+1}}{A\left(\theta^{\theta}\left(1-\theta\right)^{1-\theta}A^{\theta}B^{1-\theta}\right)^{1/\gamma}} \le s\right\} \\
&= \Pr\left\{\frac{M/\rho^{\theta/\gamma+1}}{\left(\theta^{\theta}\left(1-\theta\right)^{1-\theta}B^{1-\theta}\right)^{1/\gamma}}A^{-(1+\theta/\gamma)} \le s\right\} \\
&= \Pr\left\{cA^{-(1+\theta/\gamma)} \le s\right\}
\end{aligned}$$

say, where $c = M/\left[\rho^{\theta/\gamma+1}\left(\theta^{\theta}\left(1-\theta\right)^{1-\theta}B^{1-\theta}\right)^{1/\gamma}\right]$. So that

$$\begin{aligned}
\Pr\left\{p_i \le s\right\} &= \Pr\left\{cA^{-(1+\theta/\gamma)} \le s\right\} \\
&= \Pr\left\{c/s \le A^{(1+\theta/\gamma)}\right\} \\
&= \Pr\left\{(c/s)^{1/(1+\theta/\gamma)} \le A\right\} \\
&= 1 - \Pr\left\{A \le (c/s)^{1/(1+\theta/\gamma)}\right\} \\
&= 1 - c^{\beta/(1+\theta/\gamma)}s^{-\beta/(1+\theta/\gamma)}
\end{aligned}$$

with support on $[c, \infty)$. The distribution of equilibrium prices is heavy tailed.

Also note that one can then easily obtain that the price changes are also fat tailed distributed as ratios of random variables that have fat tails are also fat tailed distributed. Interestingly, if we look at the implication for nominal output of the sector or profits, we do not get the fat tail implication since

$$\begin{aligned} p_i Q_i &= M \frac{1/\rho^{\theta/\gamma+1}}{A\varphi} * \theta \frac{A}{1-(1-\rho)\theta} \rho\varphi\rho^{\theta/\gamma} \\ &= M \frac{\theta}{1-(1-\rho)\theta} \end{aligned}$$

and

$$\Pi(Q) = \frac{(1-\rho)\theta}{1-(1-\rho)\theta} wL = \frac{(1-\rho)\theta}{1-(1-\rho)\theta} M.$$

But if we have demand shocks of the sorts discussed in Smets and Wouters (2003) regarding ρ, assuming that (recalling that by assumption $\rho\epsilon(0,1)$)

$$\Pr\{1-\rho \le t\} = t^{\beta}$$

then

$$\begin{aligned} \Pr\{\Pi(Q) \le s\} &= \Pr\left\{\frac{(1-\rho)\theta}{1-(1-\rho)\theta} M \le s\right\} \\ &= \Pr\left\{(1-\rho)\theta \le \frac{s}{M} - \frac{s}{M}(1-\rho)\theta\right\} \\ &= \Pr\left\{\left[1+\frac{s}{M}\right](1-\rho)\theta \le \frac{s}{M}\right\} \\ &= \Pr\left\{(1-\rho) \le \frac{s}{\theta}\frac{1}{M+s}\right\} \\ &= \left(\frac{s}{\theta}\frac{1}{M+s}\right)^{\beta} \\ &= \frac{1}{\theta^{\beta}}\left(1-\frac{M}{M+s}\right)^{\beta} \end{aligned}$$

with support $[0, \frac{\theta M}{1-\theta}]$. If $\theta = 1$, only differentiated goods, we have again a heavy upper tail. The ratio of profits in the change of profits, though, is certainly heavy tailed. To see this, note that

$$\begin{aligned} \Pr\left\{\frac{1}{\Pi(Q)} \le x\right\} &= \Pr\left\{\frac{1}{x} \le \Pi(Q)\right\} = 1 - \Pr\left\{\Pi(Q) \le \frac{1}{x}\right\} \\ &= 1 - \frac{1}{\theta^{\beta}}\left(1-\frac{M}{M+1/x}\right)^{\beta} \\ &= 1 - \frac{1}{\theta^{\beta}}\left(\frac{1}{Mx+1}\right)^{\beta} \end{aligned}$$

on $[\frac{1-\theta}{\theta M},\infty)$. The inverses has tail index β.

Looking at nominal GDP, we get

$$\begin{aligned} qZ+\frac{1}{n}\sum_{i=1}^{n}p_iQ_i &= wL+\Pi(Q) \\ &= \left[1+\frac{(1-\rho)\,\theta}{1-(1-\rho)\,\theta}\right]wL \\ &= \frac{1}{1-(1-\rho)\,\theta}M. \end{aligned}$$

So if we assume again that $\Pr\{1-\rho\leq t\}=t^{\beta}$, then due to θ in the denominator $1-(1-\rho)\,\theta\epsilon[1-\ \theta,1]$ and there are no fat tails. But if we assume that for example that M is exponentially distributed, then the ratio of the money supply and time t divided by the time t-1 supply is fat tailed, since if

$$\Pr\{M\leq t\}=1-e^{-t},$$

then

$$\Pr\{\frac{1}{M}\ \leq\ s\}=\Pr\{\frac{1}{s}\leq M\}=\exp(-1/s),$$

which is a Frechet extreme value distribution with a tail index of one. Note that we can obtain the distribution of the change as follows (use the last result in the third step and the exponential distribution for the numerator in the fourth step)

$$\begin{aligned} \Pr\left\{\frac{M(t)}{M(t-1)}-1\leq x\right\} &= E_{M(t)}\left[\Pr\left\{\frac{m(t)}{M(t-1)}-1\leq x\right| M(t)=m(t)\right\}\right] \\ &= E_{M(t)}\left[\Pr\left\{\frac{1}{M(t-1)}\leq\frac{x+1}{m(t)}\right| M(t)=m(t)\right\}\right] \\ &= E_{M(t)}\left[e^{-\frac{m(t)}{1+x}}\right] \\ &= \int_0^{\infty}e^{-\frac{m}{1+x}}e^{-m}dm \\ &= \int_0^{\infty}e^{-\frac{2+x}{1+x}m}dm \\ &= \frac{1+x}{2+x} \\ &= 1-\frac{1}{2+x} \end{aligned}$$

which is a Burr distribution with tail index 1 and support $[-1,\infty)$.

3.5 Implication for Systemic Risk

As for a start, consider the asymptotic dependence between the GDP measure

$$qZ + \frac{1}{n}\sum_{i=1}^{n} p_i Q_i = wL + \Pi(Q) = \frac{1}{1-(1-\rho)\,\theta} M$$

and industrial output in nominal terms

$$p_i Q_i = M \frac{\theta}{1-(1-\rho)\,\theta,}$$

where M itself follows a Pareto law

$$\Pr\{M \leq t\} = 1 - t^{-\alpha}.$$

This example may be less interesting as M is directly assumed to be fat tailed but we can use the above ideas to derive the fat tail property endogenously. Given the assumption on M, we get immediately that

$$\Pr\{\frac{1}{1-(1-\rho)\,\theta} M > t\} = \Pr\{M > [1-(1-\rho)\,\theta]\,t\} = [1-(1-\rho)\,\theta]^{-\alpha}\, t^{-\alpha}$$

and

$$\Pr\{\frac{\theta}{1-(1-\rho)\,\theta} M > t\} = \theta^{\alpha}\,[1-(1-\rho)\,\theta]^{-\alpha}\, t^{-\alpha}.$$

Since $\theta\epsilon(0,1)$ we find that

$$\begin{aligned}
\Pr\{\frac{1}{1-(1-\rho)\,\theta} M > t, \frac{\theta}{1-(1-\rho)\,\theta} M > t\} &= \Pr\{\frac{\theta}{1-(1-\rho)\,\theta} M > t\} \\
&= \theta^{\alpha}\,[1-(1-\rho)\,\theta]^{-\alpha}\, t^{-\alpha},
\end{aligned}$$

while

$$\begin{aligned}
1 - \Pr\{\frac{1}{1-(1-\rho)\,\theta} M \leq t, \frac{\theta}{1-(1-\rho)\,\theta} M \leq t\} &= \Pr\{\frac{1}{1-(1-\rho)\,\theta} M > t\} \\
&= [1-(1-\rho)\,\theta]^{-\alpha}\, t^{-\alpha}.
\end{aligned}$$

Hence, the measure for asymptotic dependence gives

$$1 + \frac{\Pr\{\frac{1}{1-(1-\rho)\theta} M > t, \frac{\theta}{1-(1-\rho)\theta} M > t\}}{1 - \Pr\{\frac{1}{1-(1-\rho)\theta} M \leq t, \frac{\theta}{1-(1-\rho)\theta} M \leq t\}} = 1 + \theta^{\alpha} > 1.$$

4 Data

The equity return and macroeconomic data are described in this section. Furthermore, the stressed levels for macro factors are also discussed.

4.1 Equity Returns

Figure 1 illustrates the scatter plot of daily S&P500 and DAX index returns from January 1973 to June 2012 (10,304 days). From the plot, it can be seen that there are several extreme returns which are related to some main events. For instance, the pair of most extreme returns in the left below quadrant represents the well-known Black Monday co-crash in October 1987. During the credit crisis in 2008, both markets dramatically slumped on the same day and after a while a large rebound occurred. Moreover, there is also the event that the German market realised the biggest loss in the past 40 years, while the US market slightly reacted. That event is pertaining to the German unification. For the results in next sections, we will particularly consider the linkage probability of the extreme large loss.

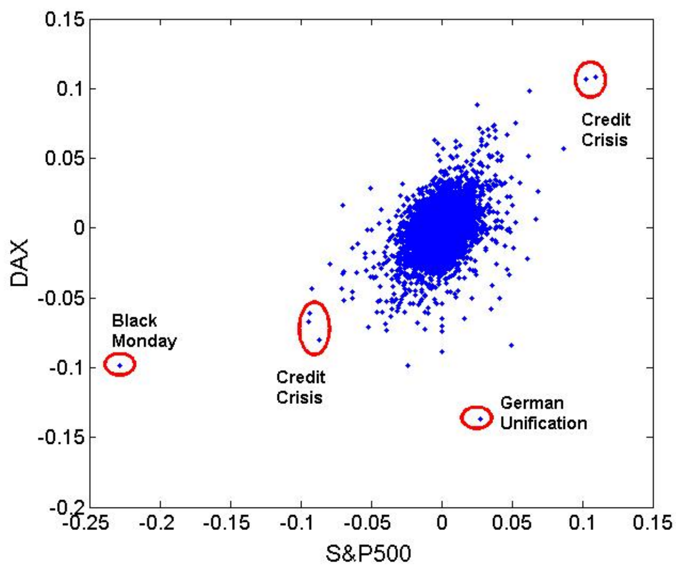

Figure 1: Scatter plot of daily S&P500 and DAX index returns from January 1977 to June 2012 (10,304 days).

4.2 Macroeconomic Factors

For macroeconomic factors, we consider the following four main indicators.

1. Inflation (Consumer Price Index; CPI)

2. Industrial production output

3. Unemployment rate
4. Money supply (M2)

The data are obtained from DataStream inc. at a monthly frequency and all are seasonally adjusted.[1] Table 1 presents the correlations of each factor between two countries in which we can see that it can be divided into two categories: strong positive and mild negative correlations.

S&P500 and DAX Index Returns	0.41
US and German Inflation	0.73
US and German Industrial Production Output	0.63
US and German Unemployment Rate	-0.29
US and German Money Supply (M2)	-0.23

Table 1: Correlations of index returns and macroeconomic factors between US and Germany.

To begin with, we look at US and German (seasonally adjusted) inflations from January 1970 to December 2012 as illustrated in Figure 2. At first glance, we can observe that US inflation is relatively higher for the first 15 years while the German CPI has diminished. In order to condition the return data on inflation of these two countries, an immediate question arises how can we determine a stressed level for these inflation data?

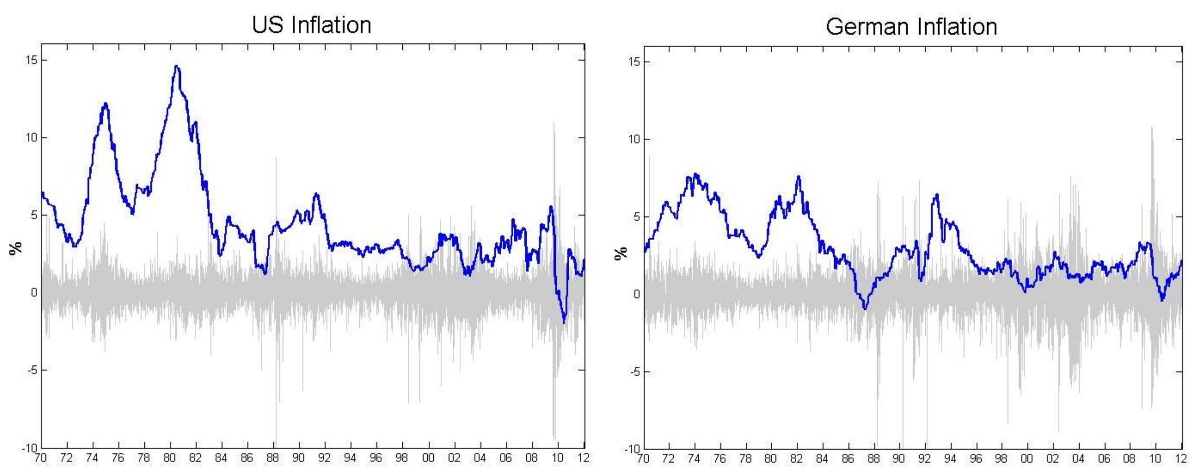

Figure 2: US inflation with daily S&P500 index returns (left) and German inflation with daily DAX index returns (right) from January 1970 to December 2011. The data are seasonally adjusted.

[1]the data of all four macro factors except unemployment rate are year-on-year percentage change.

4.2.1 Constant versus Time-Varying Stressed Thresholds

It is natural to think of the following two alternatives for specifying threshold levels for a macro factor.

- First and the simplest way is to define thresholds by computing constant lower and upper (i.e. 5% and 95%) quantiles for the whole period of data. The plots of US and German inflations with constant 5% and 95% quantile thresholds are demonstrated in Figure 3 (top row). It can be seen from the figure that if we define the stressed levels of inflation using these upper constant thresholds, the data of recent 25 years will be excluded, meaning that several extreme returns (especially, during the credit crisis) will not be taken into account for investigating the conditional extreme linkage probability. Most importantly, it is known that monetary policy related to inflation typically changes over time and those extremely high levels of inflation in 1970s are unlikely to happen again as the central banks of both countries has recently targeted more on inflation. Accordingly, using the constant thresholds is therefore rather not suitable and hence it may be more sensible if we allow thresholds to vary over time.
- Second alternative is to employ N-year moving average 5% and 95% quantile thresholds. In Figure 3 (bottom row), the US and German inflations with 10-year moving average thresholds are presented. Clearly, we can see that making use of moving average allows the thresholds to change over time in which it seems more reasonable than using the constant thresholds.
- Figures 4, 5 and 6 plot the remaining three macroeconomic factors (industrial production output, unemployment rate and money supply (M2) respectively) with constant and 10-year moving average 5% and 95% quantile thresholds.

To this end, two observations are made as below.

- Specifying a window for computing the thresholds is rather subjective. Considering 5-, 10- and 15-year moving average 5% and 95% quantile thresholds (not shown in the figure), it can be found that the narrower the window is, the quicker the thresholds moves.
- The public announcement date of macroeconomic indicators might not have a significant effect to our study since we are not investigating the impact of an announcement date to extreme events but merely employing thresholds to specify the stressed periods of macro factors. Note that, in the US and Germany, the official macro figures are generally released around two weeks after the end of every month and the date is not precisely predetermined.

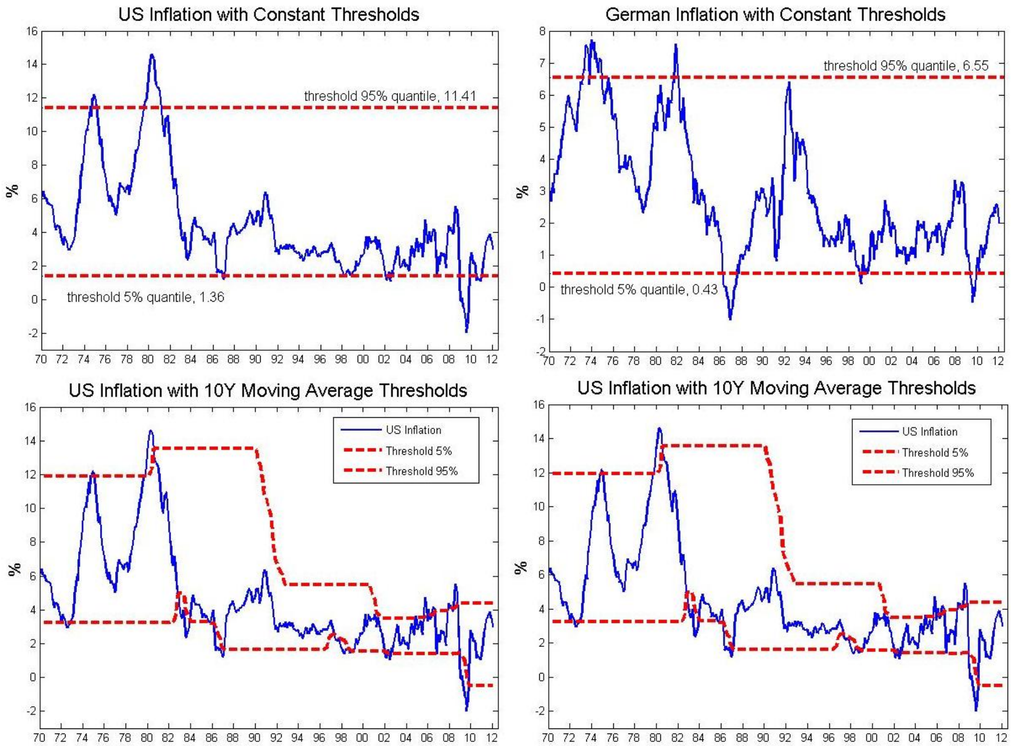

Figure 3: US (left) and German (right) seasonally adjusted CPI percentage changes (year-on-year) from January 1970 to December 2011 with constant (top) and 10-year moving average (bottom) 5% and 95% quantile thresholds.

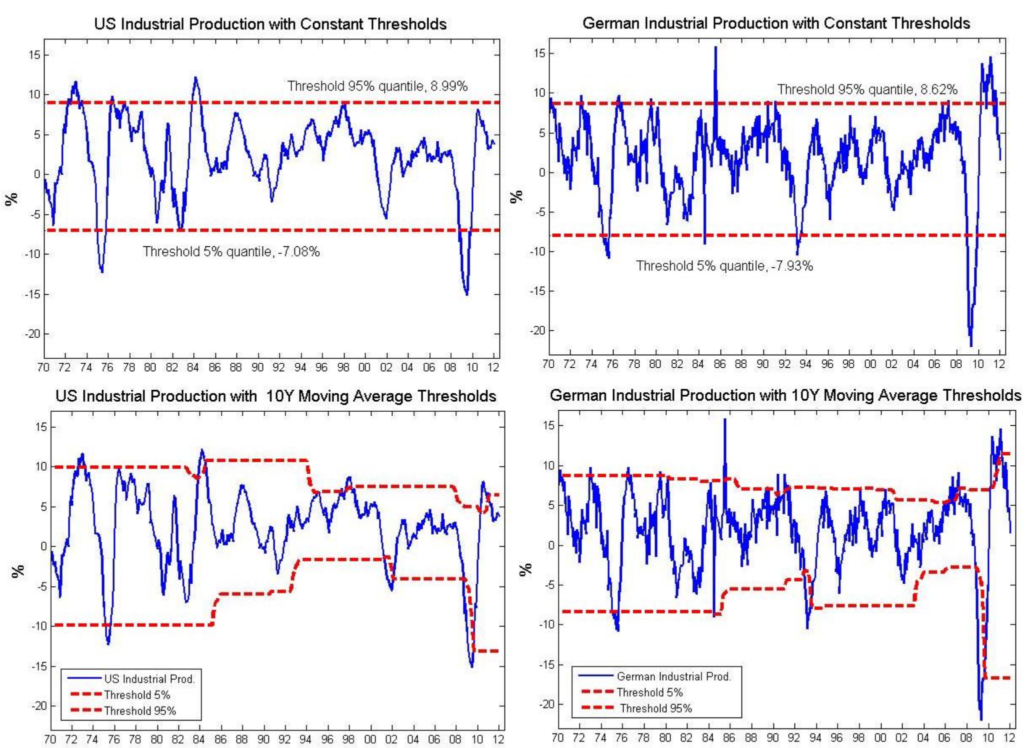

Figure 4: US (left) and German (right) monthly seasonally adjusted industrial production index percentage changes (year-on-year) from January 1970 to December 2011 with constant (top) and 10-year moving average (bottom) 5% and 95% quantile thresholds.

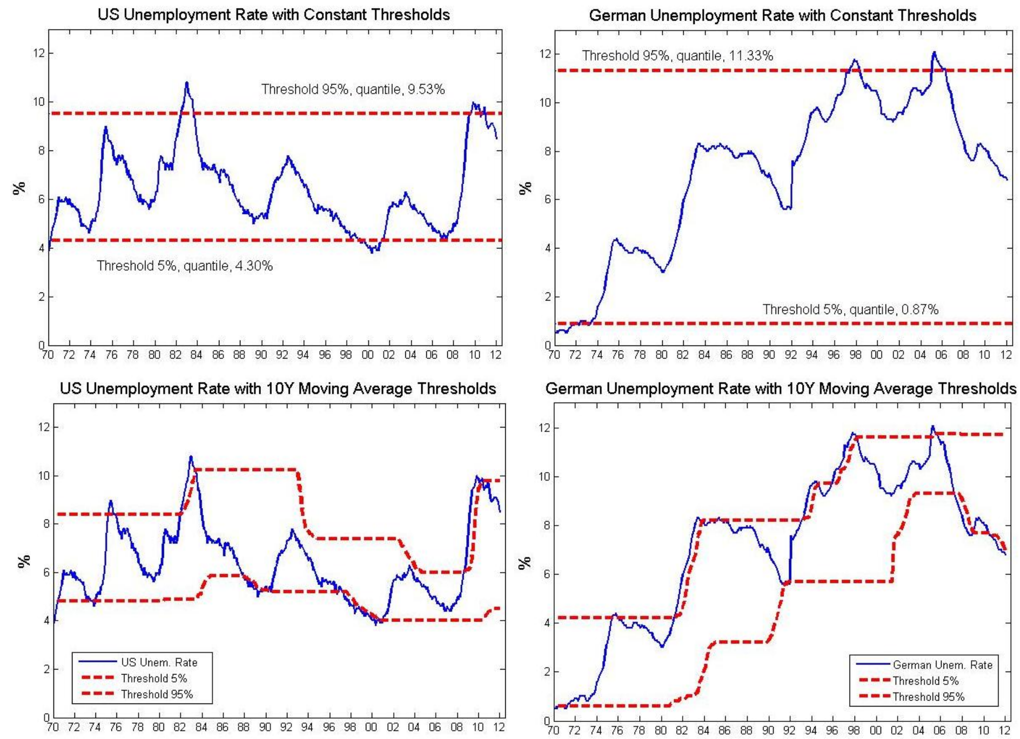

Figure 5: US (left) and German (right) monthly seasonally adjusted unemployment rate from January 1970 to December 2011 with constant (top) and 10-year moving average (bottom) 5% and 95% quantile thresholds

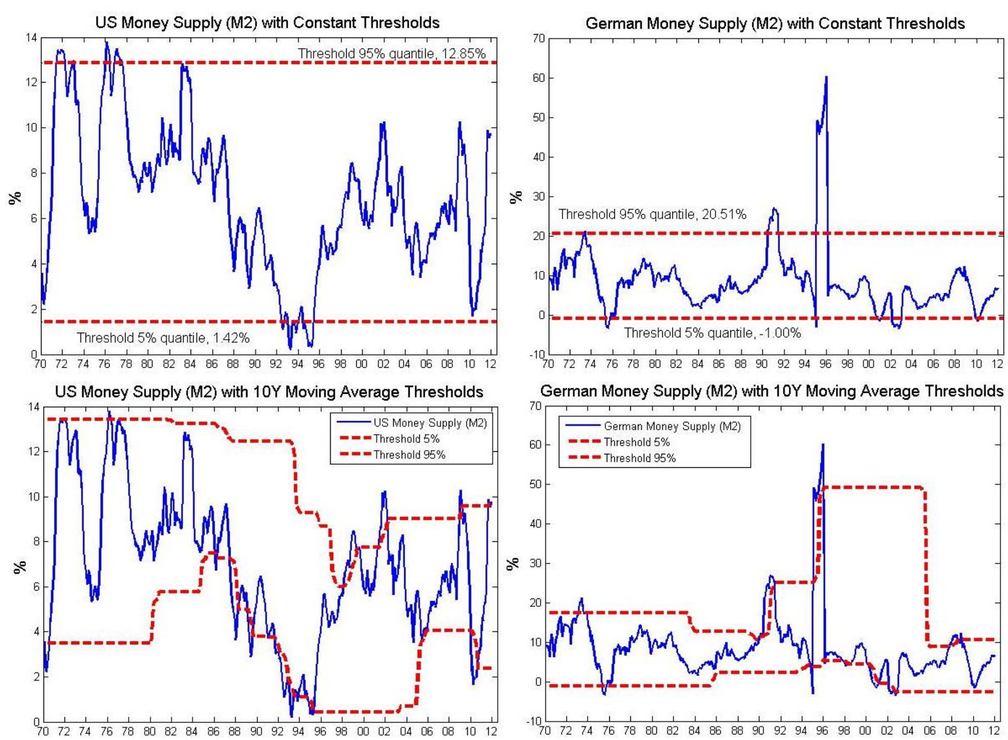

Figure 6: US (left) and German (right) monthly seasonally adjusted money supply (M2) percentage changes (year-on-year) from January 1970 to December 2011 with constant (top) and 10-year moving average (bottom) 5% and 95% quantile thresholds.

5 Empirical Results

In this section, the estimation results of the tail indices of macro factors are first presented. Then, we investigate the extreme loss linkages between S&P500 and DAX returns conditional on stressed macroeconomic factors for both index and sector levels.

5.1 Tail Indices of Macro Factors

We use the DEdH estimator to estimate the tail indices $\widehat{\alpha}$ for monthly US (Figure 7) and German (Figure 8) macro data (inflation, industrial production, unemployment rate and money supply (M2)) from January 1973 to June 2012.

For the German data, it is evident that inflation ($\widehat{\alpha} \sim 2.0$) obtains a heavier tail than unemployment rate and money supply ($\widehat{\alpha} \sim 5.0$). Interestingly, industrial production exhibits non-fat tail since the tail index cannot be estimated. The results by the US data are also similar to those by the German data except money supply that the DEdH plot looks rather strange due to the extremely sharp money supply change in around 1996 of the raw data.

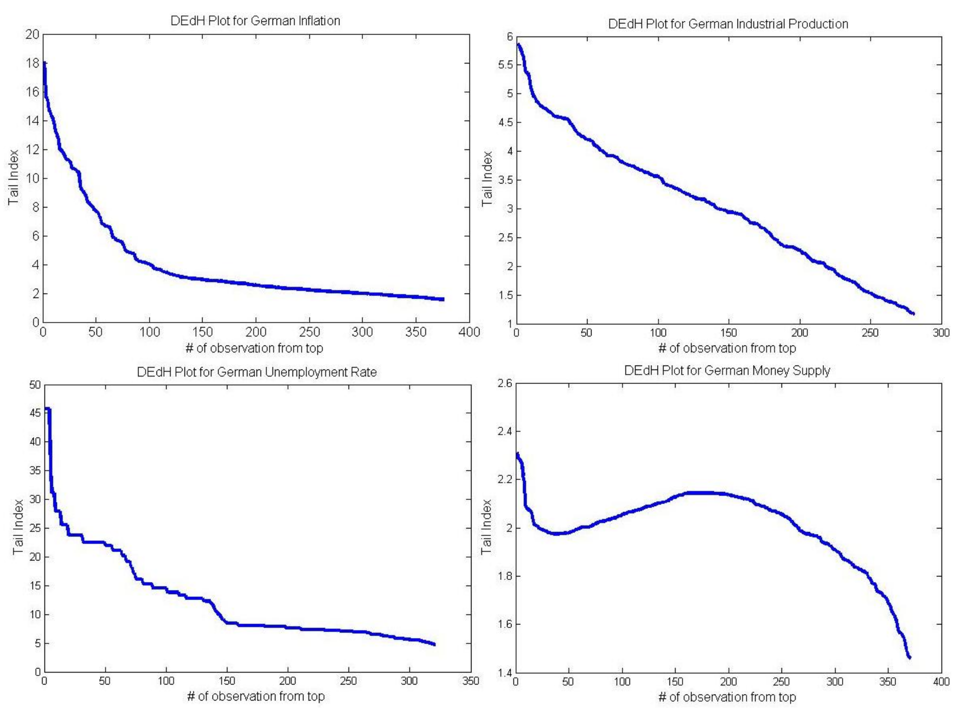

Figure 7: DEdH plots for four US macro factors: inflation, industrial production, unemployment rate and money supply (M2). The monthly data ranges from January 1973 to June 2012 (474 months).

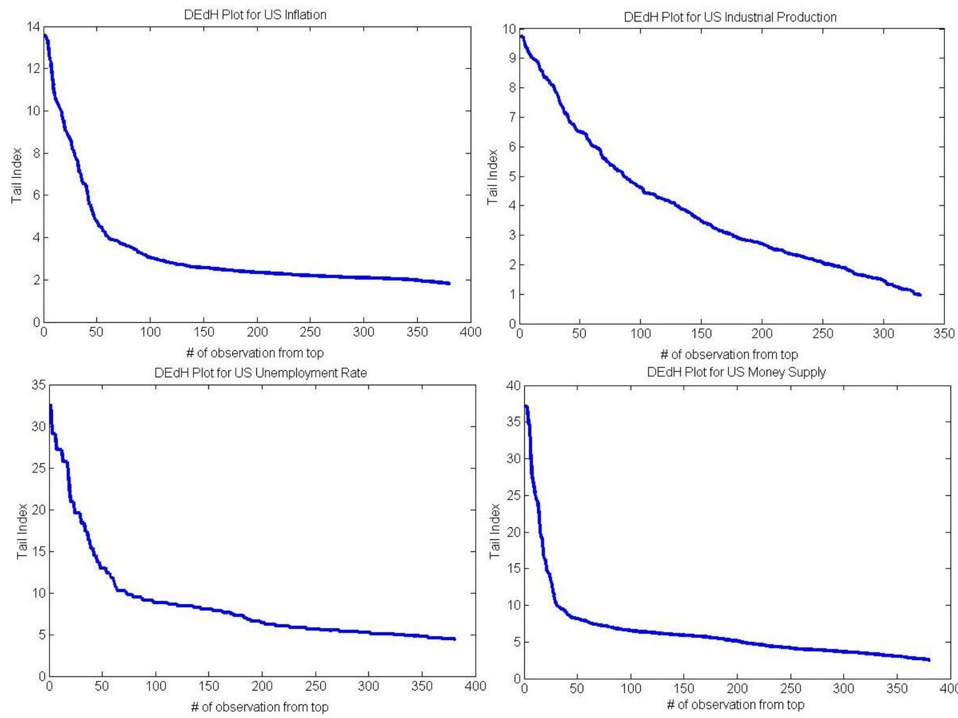

Figure 8: DEdH plots for four German macro factors: inflation, industrial production, unemployment rate and money supply (M2). The monthly data ranges from January 1973 to June 2012 (474 months).

5.2 Extreme Linkages: Equity Index Returns

We now measure the unconditional and conditional extreme loss linkage probabilities using the count measure for daily S&P500 and DAX index returns from January 1973 to June 2012 (10,304 days). The raw data are transformed to unit Pareto marginals in order to eliminate the influence of the marginal distributions.

5.2.1 Unconditional Extreme Linkage

The estimation result of the unconditional (i.e. without conditioning on macroeconomic factors) extreme loss linkage between the US and German daily index returns is presented in Figure 9. To choose a suitable threshold, we use the eye-balling technique in which the cutoff point is selected where the plot is first relatively stable. From the figure, we achieve the linkage probability at around 0.2 which can be interpreted as "approximately once per 5 market crashes, there will be one co-crash".

5.2.2 Conditional Extreme Linkage

We here estimate the extreme loss linkage probabilities of the US and German daily index returns conditional on stressed levels of each of four macro factors (described in the previous section). Thresholds for a macro factor are computed by using 10-year moving average 5% and 95% quantiles. Furthermore, we consider to condition the return data on a macro factor in four cases:

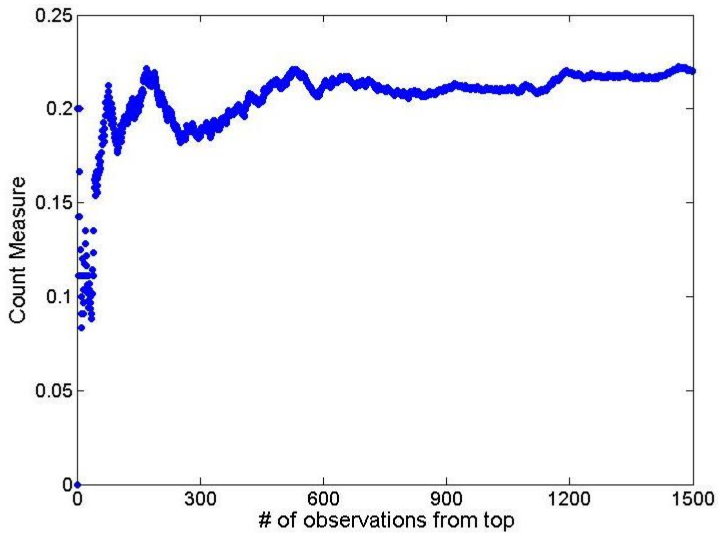

Figure 9: Unconditional count measure for extreme loss of daily S&P500 and DAX index returns from January 1973 to June 2012.

1. Above 95% quantile threshold in both countries.
2. Above 95% quantile threshold at least in one country.
3. Below 5% quantile threshold in both countries.
4. Below 5% quantile threshold at least in one country.

In other words, we are investigating the extreme linkage between two markets during the periods of high and low levels of a macro factor. The stressed periods can also be categorised as either when it occurs in two countries simultaneously or at least one country is facing the severe macroeconomic condition. In addition, due to the nature of macro variables, the daily bivariate returns will be conditioned on 1-month lag macro data. For example, if the value of a macro factor of this month exceeds a specified threshold value, the daily returns of the next whole month will be taken into account for estimating the extreme linkage probability.

Figures 10 demonstrates the scatter plots of daily S&P500 and DAX returns conditional on four cases of stressed inflations. The corresponding estimates for the extreme linkage probabilities are also presented in Figure 11. From the figures, we first can observe that quite a few data are obtained when the returns are conditioned on high inflation in both countries simultaneously. According to the results, it can be found that in almost all cases the extreme linkages between US and German equity markets conditional on stressed inflations are rather close to the unconditional one. The linkage probability when conditioned on low inflation below 5% quantile thresholds in both countries turns to be slightly higher than other cases.

Figures 12 to 17 show the resulting estimates of conditioning the returns on each of the remaining three macro factors: industrial production, unemployment and money supply. Clearly, conditioning on industrial production results in much higher extreme linkage probabilities than the unconditional one (except in case of above 95% quantile at least in one country). Considering the returns conditioned on unemployment, the most impact on the extreme linkage is the case where unemployment is above the 95% quantile threshold at least in one country. Finally, for money supply, we notice that when conditioning the returns on high money supply above 95% quantile in both countries, we achieve a very low extreme linkage probability. In addition, we can see from all the figures that there are some cases that no conditional data at all, for example, when conditioning the returns on low unemployment in both countries and on high money supply in both countries.

Table 2 summarises the impact of stressed macroeconomic factors in four scenarios on the extreme loss linkage probabilities of the daily S&P500 and DAX returns. It can be seen from the table that industrial production clearly has the most impact on the extreme linkages in almost all cases except only when it is higher than the 95% quantile threshold at least in one country. For least impact factors, the results are mixed up among inflation, unemployment and money supply.

To this end, it is worth mentioning that if we use constant (instead of moving average) thresholds for macro factors, the estimated extreme linkage probability can be different. One should bear in mind that a view on the stressed level of a macro factors might affect the estimation results.

<table>
<tr><th colspan="2">10-year Moving Average Thresholds</th><th>Ranking of Macro Factors (when stressed) by the Impact on Extreme Loss Linkages</th></tr>
<tr><td rowspan="2">Above 95% Quantile Threshold</td><td>In Both Countries</td><td>1. Industrial Production
2. Inflation, Unconditional
3. Unemployment</td></tr>
<tr><td>At Least in One Country</td><td>1. Unemployment, Money Supply
2. Industrial Production
3. Inflation, Unconditional</td></tr>
<tr><td rowspan="2">Below 5% Quantile Threshold</td><td>In Both Countries</td><td>1. Industrial Production
2. Inflation
3. Unconditional
4. Money Supply</td></tr>
<tr><td>At Least in One Country</td><td>1. Industrial Production
2. Unemployment, Money Supply
3. Inflation, Unconditional</td></tr>
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Table 2: Ranking of macro factors (when stressed) by the impact on extreme loss linkages between the US and German equity index returns from from January 1973 to June 2012.

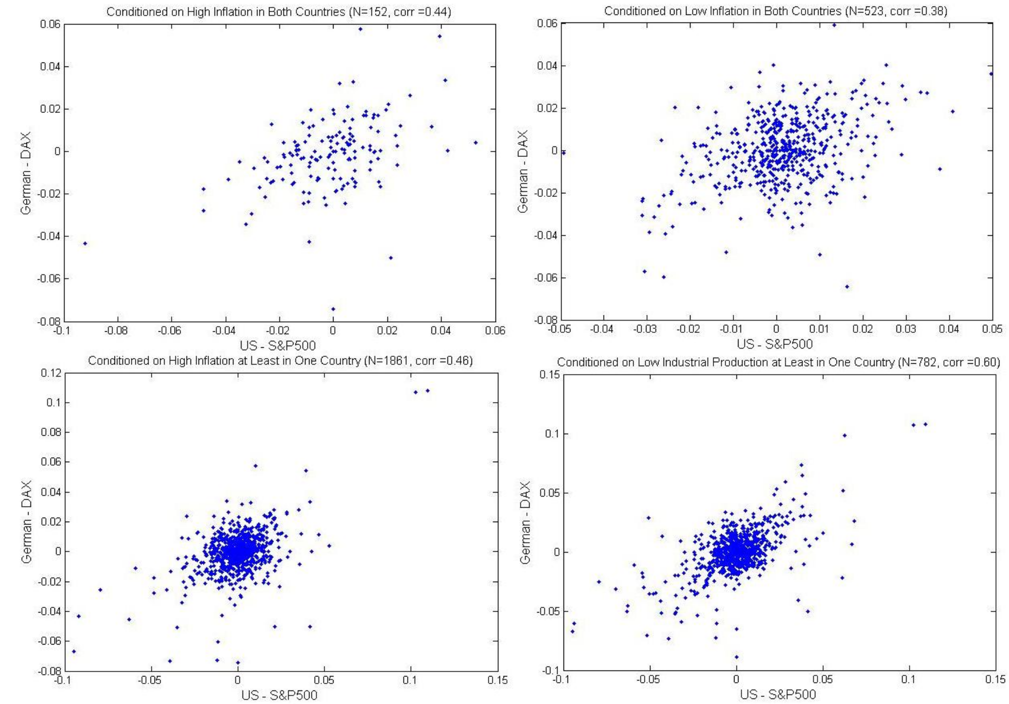

Figure 10: Daily S&P500 and DAX index returns from January 1973 to June 2012 conditional on a) high inflation above 10-year moving average 95% quantile threshold in both countries (top, left) and at least in one country (bottom, left), b) low inflation below 10-year moving average 5% quantile threshold in both countries (top, right) and at least in one country (bottom, right).

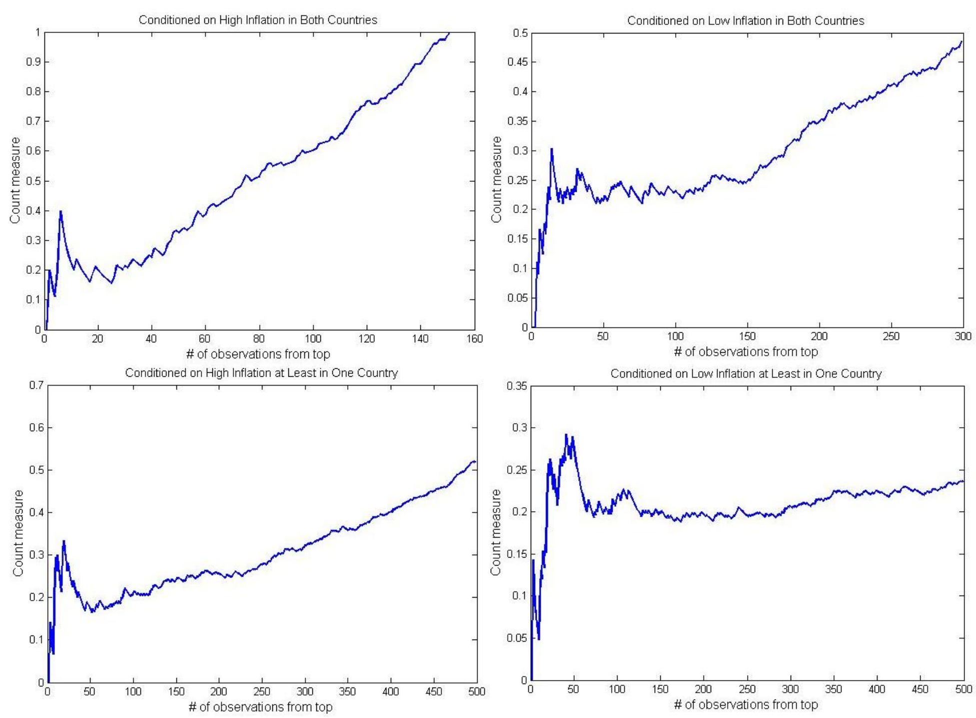

Figure 11: Count measure for daily S&P500 and DAX index returns from January 1973 to June 2012 conditional on a) high inflation above 10-year moving average 95% quantile in both countries (top, left) and at least in one country (bottom, left), b) low inflations below 10-year moving average 5% quantile in both countries (top, right) and at least in one country (bottom, right).

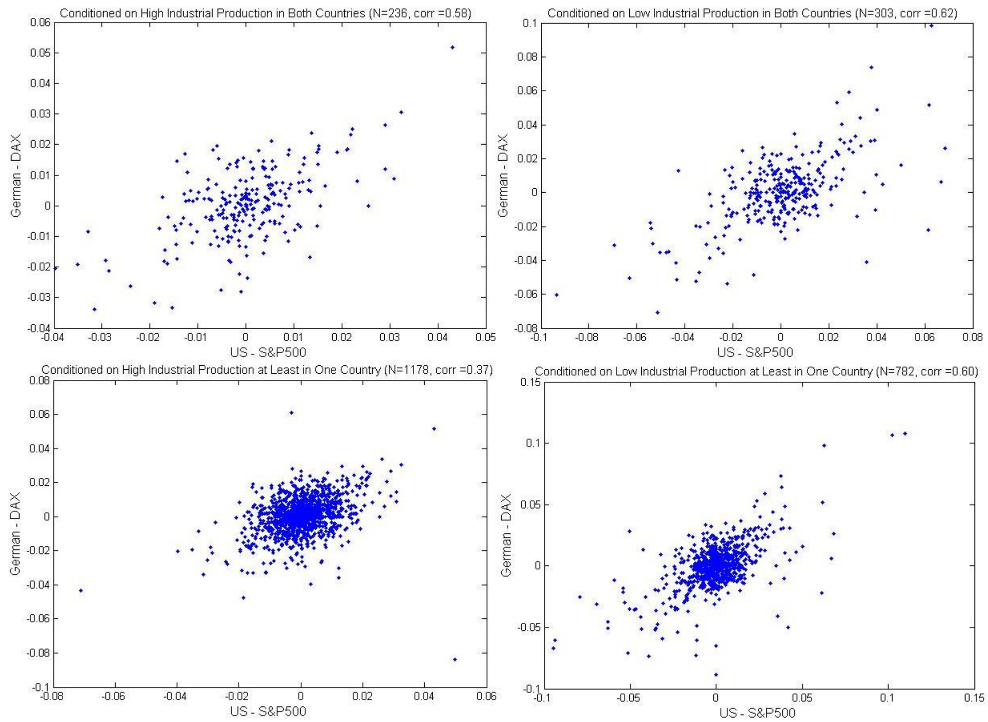

Figure 12: Daily S&P500 and DAX index returns from January 1973 to June 2012 conditional on a) high industrial production above 10-year moving average 95% quantile threshold in both countries (top, left) and at least in one country (bottom, left), b) low industrial production below 10-year moving average 5% quantile threshold in both countries (top, right) and at least in one country (bottom, right).

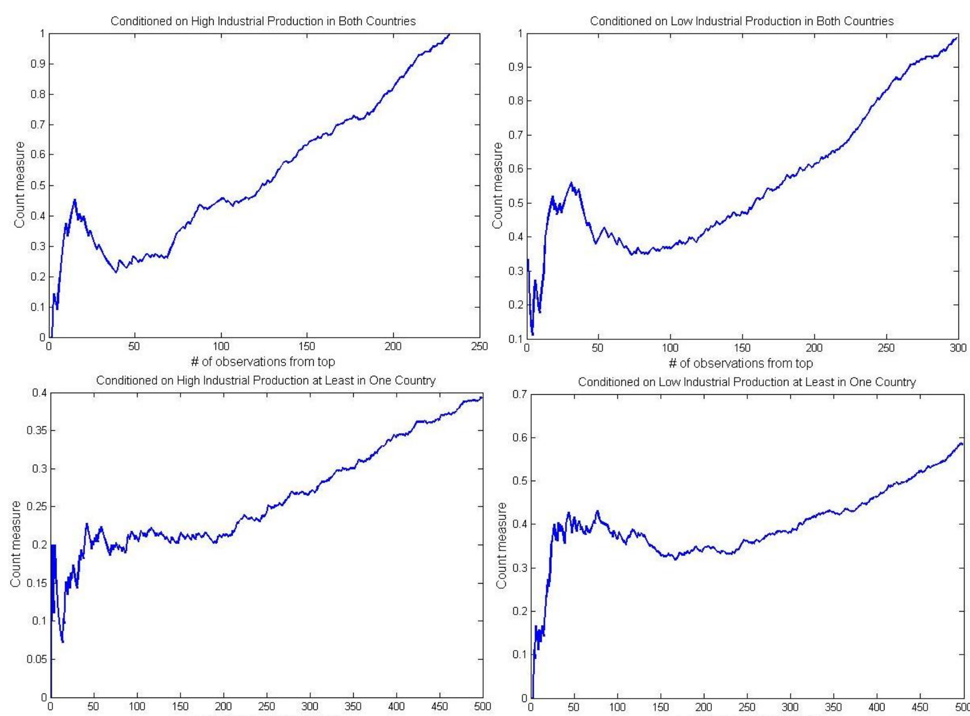

Figure 13: Count measure for daily S&P500 and DAX index returns from January 1973 to June 2012 conditional on a) high industrial production above 10-year moving average 95% quantile in both countries (top, left) and at least in one country (bottom, left), b) low industrial production below 10-year moving average 5% quantile in both countries (top, right) and at least in one country (bottom, right).

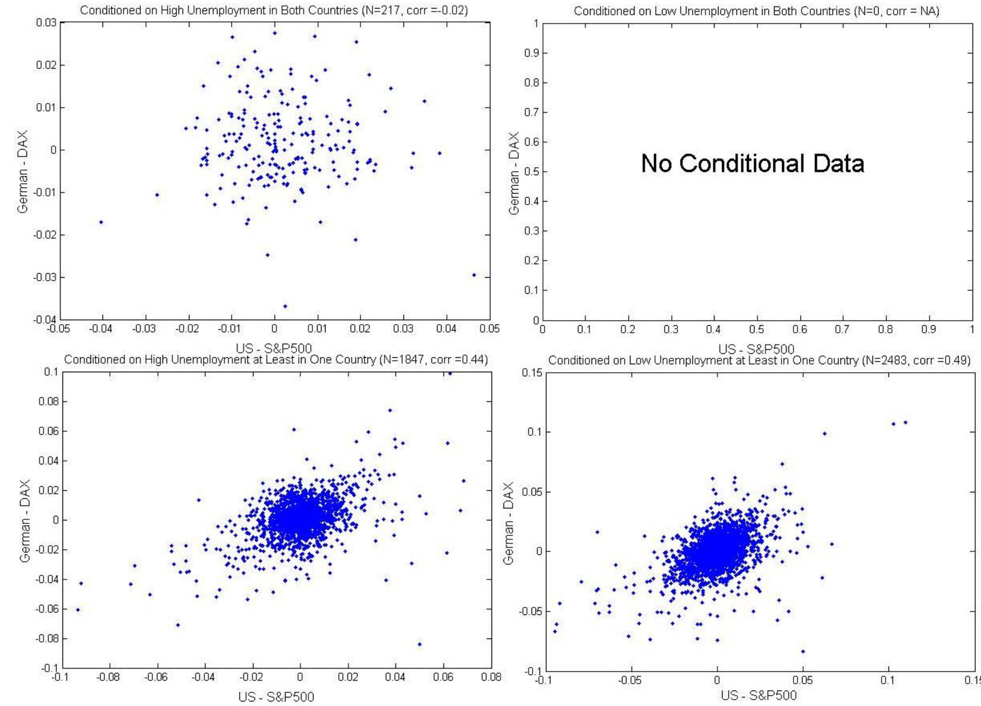

Figure 14: Daily S&P500 and DAX index returns from January 1973 to June 2012 conditional on a) high unemployment rate above 10-year moving average 95% quantile threshold in both countries (top, left) and at least in one country (bottom, left), b) low unemployment rate below 10-year moving average 5% quantile threshold in both countries (top, right) and at least in one country (bottom, right).

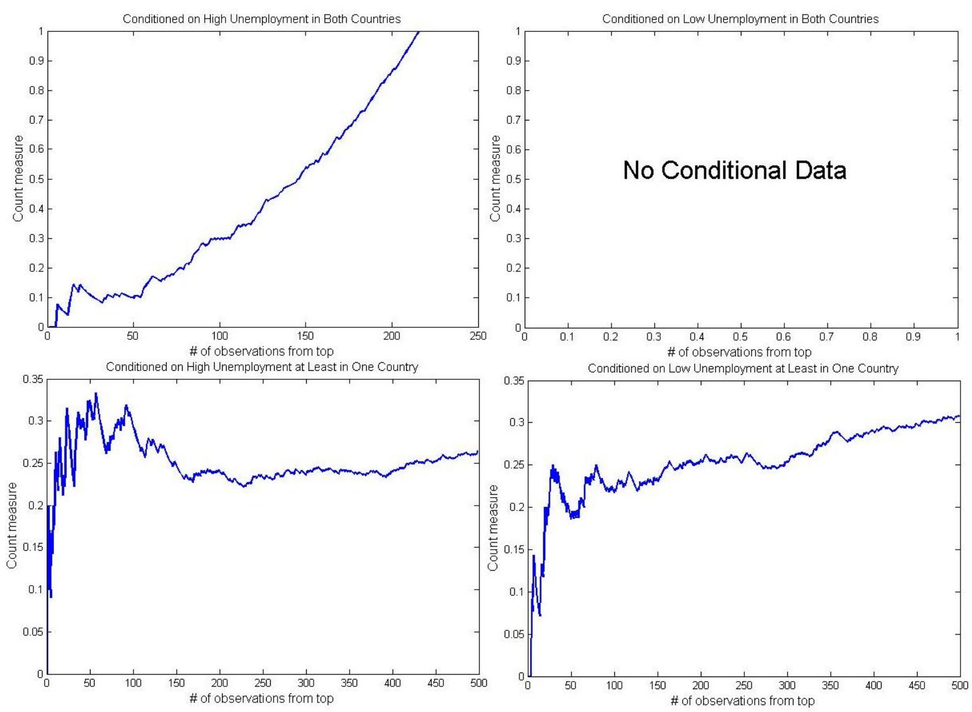

Figure 15: Count measure for daily S&P500 and DAX index returns from January 1973 to June 2012 conditional on a) high unemployment rate above 10-year moving average 95% quantile in both countries (top, left) and at least in one country (bottom, left), b) low unemployment rate below 10-year moving average 5% quantile in both countries (top, right) and at least in one country (bottom, right).

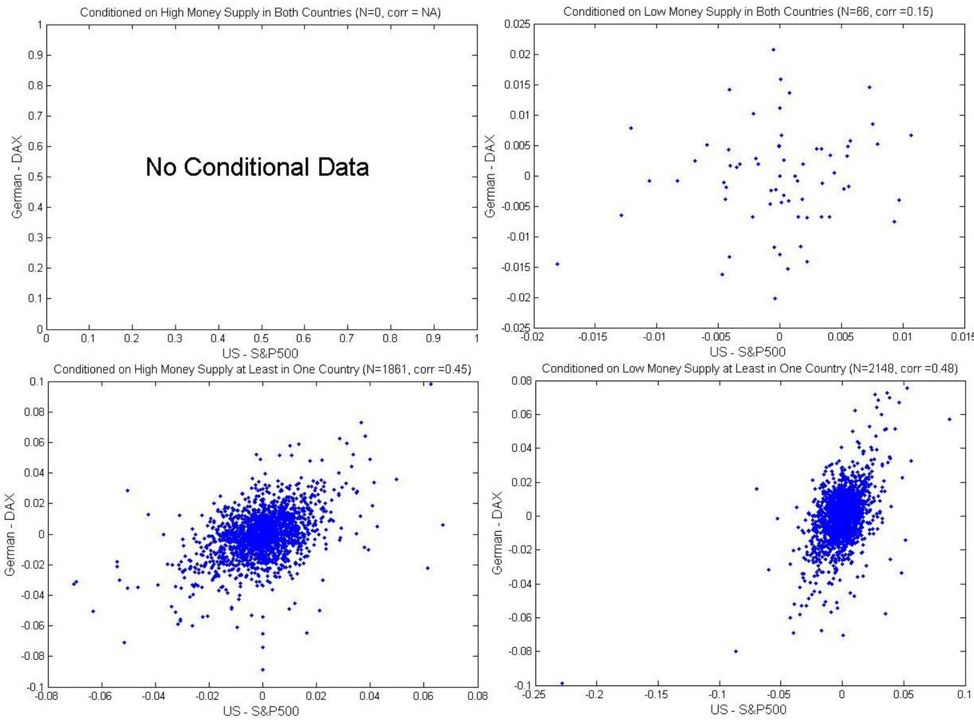

Figure 16: Daily S&P500 and DAX index returns from January 1973 to June 2012 conditional on a) high money supply change above 10-year moving average 95% quantile threshold in both countries (top, left) and at least in one country (bottom, left), b) low money supply change below 10-year moving average 5% quantile threshold in both countries (top, right) and at least in one country (bottom, right).

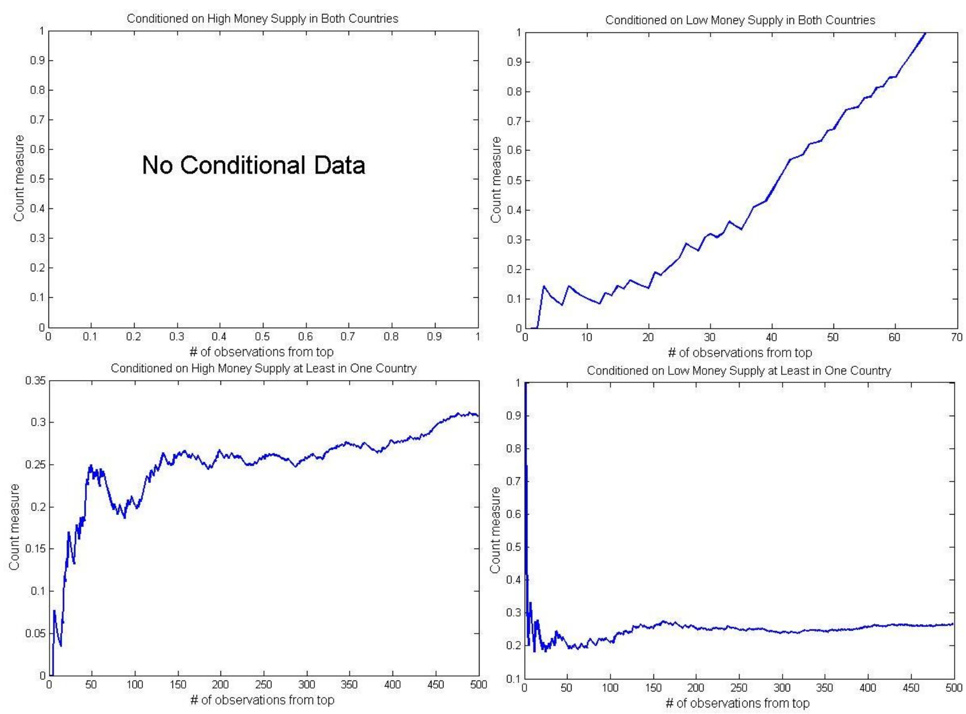

Figure 17: Count measure for daily S&P500 and DAX index returns from January 1973 to June 2012 conditional on a) high money supply change above 10-year moving average 95% quantile in both countries (top, left) and at least in one country (bottom, left), b) low money supply change below 10-year moving average 5% quantile in both countries (top, right) and at least in one country (bottom, right).

5.3 Extreme Linkages: Equity Returns by Sector

We continue on investigating the extreme loss linkage of US and German equity markets but now in a sector level. The eight sectors considered are financials, industrials, materials, consumer goods, consumer services, utilities, health care and telecom. The results are summarised and analysed as follows.

- Comparing the unconditional extreme linkages among all sectors, we are generally able to categorise the sectors as three main groups as shown in Table 3. From the table, we can observe that sectors in the first group achieve the extreme linkage probability at around 0.20 which is close to that of the index returns. The second group gets the linkage probability lower, while the last group obtains the lowest linkage probability.

	Sector	Extreme Linkage Probability
1.	Financials, Industrials, Materials	≈ 0.20
2.	Consumer Services, Consumer Goods, Telecom	≈ 0.15
3.	Utility, Health Care	≈ 0.12

Table 3: The unconditional extreme loss linkage probabilities for the US and German equity returns by sector from January 1973 to June 2012.

- For each sector[2], it turns out that industrial production is still the most influential macro factor on the extreme linkage, except for health care sector where the impact by all four macro indicators are rather close. Furthermore, we can also notice that inflation (during high stressed level) has more impact in materials, industrials, telecom and health care sectors than those of the index returns.
- Table 4 demonstrates which sectors are affected most and least for each stressed macro factor. In general, we can see that three sectors (industrials, financials and materials), which are in the first group in Table 3, are most affected when each of four macro factors is stressed. For the least affected sectors, the results are mixed up but all are in the second and third groups.

6 Conclusion

In this paper, we attempt to associate multivariate extreme value theory with macroeconomic circumstances. The contribution is therefore the investigation of the extreme loss linkage in the financial markets conditional on the stressed levels of macro factors, where the unconditional linkage probability is also considered for comparison. The methodology used is the count estimator which is easy to implement for dealing

[2]To save the space, the figures are not shown here but available upon request.

Macro Factors	Above 95% Quantile Thresholds		Below 5% Quantile Thresholds	
	Most Impact	Least Impact	Most Impact	Least Impact
Inflation	Industrials Materials	Consumer Services Consumer Goods	Industrials Materials	Health Care Telecom Consumer Services
Industrial Production	Industrials Financials Materials	Telecom Health Care Consumer Services	Industrials Financials Materials	Telecom Health Care Utilities
Unemployment	Industrials Materials Financials	Health Care Telecom Consumer Services	Industrials Financials Materials	Consumer Services Telecom Utilities
Money Supply	Materials Industrials Financials	Telecom Utilities Health Care	Industrials Materials Consumer Services	Utilities Health Care Consumer Services

Table 4: The impact on the extreme loss linkage for the US and German equity returns by sector (from January 1973 to June 2012) for each macro factors.

with the estimation of MEVT for bivariate data. More precisely, the count estimator allows us to measure the extreme linkage by counting the number of minimum and maximum values of two return series over a threshold. According to the empirical results, it turned out that when conditioning the daily S&P500 and DAX index returns on each of four macro indicators (i.e. inflation, industrial production, unemployment and money supply), where the stressed levels are computed by using 10-year moving average, industrial production distinctly has more impact on the linkage probabilities than other macro indicators (i.e. inflation, unemployment and money supply), which are more related to monetary policies. It should be cautioned that when investigating the extreme linkage conditional on a macro factor, choosing a stressed level for macro factors does affect the extreme linkage probability. Further, when determining the extreme linkages in sector level, we also found that industrial production is still the most influential macro factor. Industrials, materials and financials sectors are the three sectors that typically obtain a higher extreme linkage probability than the other sectors. The extreme linkage of the returns of health care and utilities sectors are likely to be less affected by stressed macro factors. Nevertheless, inflation tends to have more impact for some particular sectors than the index, given a high stressed level.

Additionally, macro variables are also theoretically investigated for the heavy tails using a standard closed economy macro model (Dixit and Stiglitz specification). The result shows that the nominal output has no fat tail implication, in line with the empirical results in which its tail indices cannot be estimated. This is interesting since we already observed that industrial production output has most impact on the extreme linkage of the US and German equity returns. Further investigation may be carried out.

Clearly, there is still a plenty of room for further research. The methodology may be improved, or otherwise it is interesting to try other methods for the estimation. Also, more macro factors may be considered and the analysis is not limited only to the US and German equity markets. Other pairs of markets such as emerging and developed markets would also be very interesting. Finally, this piece of work can be a useful tool for both policymakers and investors, who much concern about extreme large loss of their asset portfolios during harsh macroeconomic environment.

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