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2	A Direct Method for Measuring Discounting and QALYs
3	more Easily and Reliably ¹
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12	Submitted to Medical Decision Making
13	May, 2011
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15	ABSTRACT. Time discounting and quality of life are two important factors in
16	evaluations of medical interventions. The measurement of these two factors is
17	complicated because they interact. Existing methods either simply assume one factor
18	given, based on heuristic assumptions, or invoke complicating extraneous factors,
19	such as risk, that generate extra biases. We introduce a new method for measuring
20	discounting (and then quality of life) that involves no extraneous factors and that
21	avoids all distorting interactions. Our method is considerably simpler and more
22	realistic for subjects than existing methods. It is entirely choice-based and, thus, can
23	be founded on economic rationality requirements. An experiment demonstrates the
24	feasibility of our method, and its advantages over classical methods.
25	Key Words: Utility Measurement, Discounting, QALY, Utility of Life Duration,
26	Time Tradeoff

¹ A first version of this paper circulated as Attema, Bleichrodt, and Wakker (2007), "Measuring the Utility of Life Duration in a Risk-Free versus a Risky Situation." Arthur Attema's and Han Bleichrodt's research was made possible through grants from the Netherlands Organization for Scientific Research (NWO).

Word count 7032

1 INTRODUCTION

2 QALY evaluations integrate two components, quality of life and discounting 3 (utility of life duration). These components interact, and it is hard to measure one 4 without knowing the other (1). Most measurements of one component assumed the 5 other component known, based on heuristic assumptions. Thus in time tradeoff 6 (TTO) measurements of quality of life, utility of life duration is usually assumed to be 7 linear (no discounting). In measurements of discounting, outcomes are usually 8 monetary, with the utility of money assumed to be linear. Whether discounting of 9 health can be equated with discounting of money remains a point of debate (2-5). 10 The few attempts that have tried to avoid the interaction between discounting and 11 quality of life invoked extraneous factors such as risky or interpersonal (utilitarian) 12 aggregations (1,6-10). Then attitudes towards risk and welfare intervene and generate 13 extra biases (11-16). 14 This paper presents a new method to measure discounting within the QALY

15 model that avoids the aforementioned problems: our method (a) needs no extraneous 16 factors; (b) is not affected by the interactions between discounting and quality of life; 17 and (c) uses stimuli that can be simpler and more realistic than those for existing 18 methods. We can measure any general discount function, constant or not. Because 19 we avoid all extraneous factors and interactions, we call our method the direct method 20 (DM). With utility of life duration (discounting) measured, we can also measure 21 quality of life by correcting traditional TTO measurements. We can thus measure the 22 whole QALY model.

Unlike classical methods for measuring the utility of life duration and discounting
(the standard gamble method and the certainty equivalence (CE) method) which are
based on risky decisions, we need not invoke the outcome of immediate death. This

outcome is very aversive and is known to arouse negative and distorting emotions.² 1 2 Especially problematic for classical measurements is that, besides immediate death, they also need scenarios of sure death at some precisely fixed future time point 3 4 already specified and known at present. Subjects have great difficulties imagining such unrealistic scenarios, leading to misunderstandings and distortions. Our method 5 6 for measuring the utility of life duration and discounting avoids such scenarios and leaves the time of death unspecified, as it is in reality.³ This enhances realism and 7 applicability. Further, our method can entirely be based on observable decisions 8 9 (revealed preferences) and does not require introspective data (stated preferences). 10 Hence it is entirely grounded on the rationality requirements of economics used to establish normative gold standards.⁴ 11

12

13 METHODS

14 Existing QALY Measurements and Their Restrictions

15 To prepare for our method, we present a number of classical evaluation models in16 increasing order of generality.

17

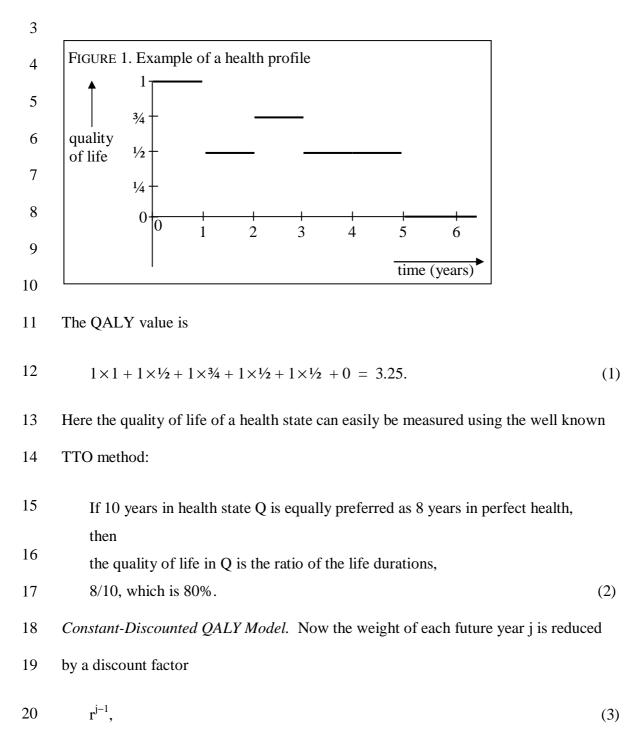
The Linear QALY Model (no Discounting). This is the first refinement of life duration
as outcome measure. Now life years are adjusted for quality of life. Discounting is

 $^{^{2}}$ Sometimes an outcome of death within a week or month is used instead, mitigating the aversiveness at the price of a small inaccuracy in measurement (8,59).

³ If we also measure quality of life, and want to include (scalings relative to) death, then we obviously cannot avoid using this health state. See the comment in the elaborated example presented later.

1 not yet incorporated. Figure 1 illustrates this evaluation for a health profile of five

2 years at various levels of quality of life, followed by death.



⁴ For an illuminating discussion of the fundamental difference between hypothetical revealed preference, as used in our study, and introspection, see Savage (1954, p. 28) (15). Savage's revealed-preference based gold standard for rational behavior is the most famous one in economics.

with 1-r the discount rate. In standard cost-effectiveness studies, discounting as in
 Eq. 3 is commonly used. The discount rate 1-r is often assumed to be 0.03.
 Then the resulting QALY in Figure 1 is

4 $1 \times 1 + 0.97 \times \frac{1}{2} + 0.97^2 \times \frac{3}{4} + 0.97^3 \times \frac{1}{2} + 0.97^4 \times \frac{1}{2} = 3.09.$ (4)

The *utility of a period of life* is the QALY value of this period when spent in perfect health. That is, it is the discounted number of years in question. Periods are denoted by their beginning and their end, as in [5,10] for years 6 to 10. Here 5 refers to the time point after five years, which is the start of year 6, and the difference 10-5= 5 is the duration of the period. We chose this notation to be consistent with interval notation for continuous time. Writing U[5,10] for the utility of this period, we have, with r=0.97:

12
$$U[5,10] = \sum_{j=6}^{10} r^{j-1} = \sum_{j=6}^{10} 0.97^{j-1} = 4.04.$$

13 The utility of the first 10 years to come, U[0,10], then is

14
$$\sum_{j=1}^{10} r^{j-1} = \sum_{j=1}^{10} 0.97^{j-1} = 8.75.$$

15 Conversely, if the utility U of life duration is given then the discount factor for 16 year j can be obtained as $U[j-1,j] (= r^{j-1})$, being the incremental utility of prolonging 17 j–1 life years to j life years. Utility of life duration and discounting are two different 18 but equivalent ways of expressing time preference. We often suppress the beginning 19 of a period if it is 0, writing U(10) as shorthand for U[0,10].

21 The (General) QALY Model (Non-Constant Discounting). In general, discounting

22 need not be constant, and year j may have general utility U[j-1,j] that is not as in Eq.

23 3. Then the (general) QALY value in Figure 1 is

1	$U[0,1] \times 1 + U[1,2] \times \frac{1}{2} + U[2,3] \times \frac{3}{4} + U[3,4] \times \frac{1}{2} + U[4,5] \times \frac{1}{2}.$ (5)
2	We still have the following relations between utility of life years and discounting:
3	The discount factor for a year j is its utility $U[j-1,j]$;
4	the utility U of a period is equal to the sum of the discounted life years in that period. (6)
5	In the continuous case, the discount factor is the derivative of the utility of life
6	duration and, vice versa, the utility of life duration is the integral of the discount
7	factor.
8	Not only discounting (i.e., U), but also quality of life is unknown beforehand, and
9	has to be measured. The TTO observation of Eq. 2 now implies a quality of life of
10	U(8)/U(10) (7)
10 11	U(8)/U(10) (7) With U unknown we cannot easily know what the quality of life of health states is.
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needing any additional assumption., This has been demonstrated in follow-up studies
 (18,19).⁵

3

More general models. More general models can be considered, with interactions between different time periods or with nonmultiplicative interactions between discounting and quality of life. Such general models are not commonly used in health because it is not clear how they can be measured or implemented, and we will not consider them either. The degree to which violations of the general QALY model lead to violations of our method, relative to violations of other methods, is a topic for future research.

11

12 The Direct Method for Measuring Utility of Life Duration

We assume the general QALY model throughout. Suppose that an improvement in quality of life from $\frac{1}{2}$ to $\frac{3}{4}$ is possible in Figure 1, and that it is possible either in period [1,2] (year 2), or in period [3,5] (years 4 and 5). Write X for the quality-of-life difference $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$. Assume further that these two improvements are equally preferred, implying that their QALY gains are the same: U[1,2]X = U[3,5]X. (8)

- 19 Dropping the common factor X gives
- 20 U[1,2] = U[3,5]. (9)

⁵ These papers used the sample and utility measurements of this paper to correct TTO measurements and to provide better insights into biases such as loss aversion and procedural invariance.

1	A convenient feature of the measurement just described is that we need not know
2	quality of life X because it drops out anyhow. To see this point in general, imagine
3	that some health profile yields a health state β (β abbreviates bad) in two different
4	periods $P = [P_1, P_2]$ and $Q = [Q_1, Q_2]$. Imagine that improving β into a health state γ (γ
5	abbreviates good) is equally preferred for period P as for period Q. Then we have the
6	following equality for the total QALY gains, where X denotes the quality-of-life
7	difference between health states β and γ :

8
$$U[P_1,P_2]X = U[Q_1,Q_2]X.$$
 (10)

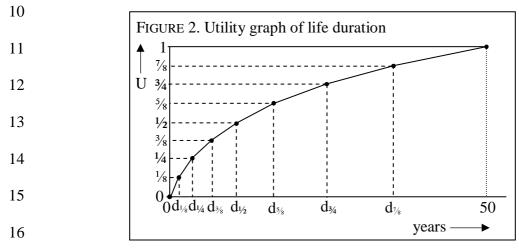
9 This implies

10
$$U[P_1,P_2] = U[Q_1,Q_2].$$
 (11)

11 because we can drop X irrespective of what it is (as long as it is not zero).

12 Another desirable feature of the measurement just proposed is that we need not 13 know or specify the health states outside the two periods considered, as long as these 14 are kept constant. In the above calculation based on QALY gains we simply did not 15 need such information. Whatever the other periods contribute to the total QALY 16 evaluations is immaterial for the QALY gains considered. In Figure 1, with the 17 improvement in year 2 equally preferred as the improvement in years 4 and 5, the 18 health states in years 1 and 3 are immaterial for the conclusion of our QALY analysis. 19 Importantly, after 5 years no immediate death has to follow, but any realistic health 20 profile may be assumed. All of this does not affect our inference of U[1,2] = U[3,5].

1 The observations just made allow measuring the utility of life duration to any desired degree of precision.⁶ If [0,D] is the total period of interest, then we can 2 normalize U(D) = 1. Obviously, U(0) = 0. We first find $d^{1/2}$, with 1/2 a superscript 3 whose role will become clear later, such that the period $[0,d^{\frac{1}{2}}]$ has the same utility as 4 $[d^{\frac{1}{2}}, D]$. Then $U(d^{\frac{1}{2}}) = \frac{1}{2}$. We next find $d^{\frac{1}{4}}$ and $d^{\frac{3}{4}}$ such that $U[0, d^{\frac{1}{4}}] = U[d^{\frac{1}{4}}, d^{\frac{1}{2}}]$ and 5 $U[d^{\frac{1}{2}}, d^{\frac{3}{4}}] = U[d^{\frac{3}{4}}, D]$. Then $U(d^{\frac{1}{4}}) = \frac{1}{4}$ and $U(d^{\frac{3}{4}}) = \frac{3}{4}$. We can continue this bisection 6 7 procedure to any desired degree of precision, and obtain the entire graph of U this 8 way. Figure 2 depicts such a graph of U. The method just described is called the 9 direct method (DM). As explained by Eq. 6, the graph also captures discounting.

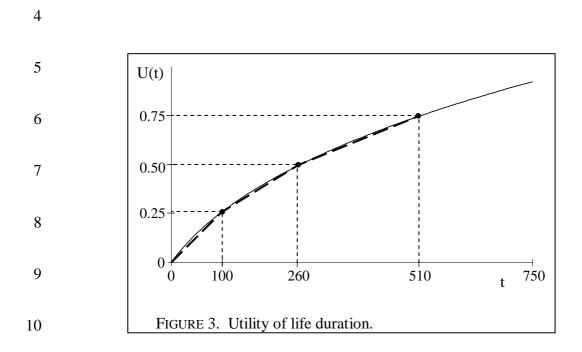


17 ELABORATED EXAMPLE

This section presents a numerical example to demonstrate the simplicity and generality of the DM, and the way it can also be used to measure not only discounting but also quality of life. Assume health states P (poor health), M (mediocre health), and F (fair health). ([i,j]: Q) denotes health state Q in period [i,j] (in days), good health for the rest of the coming two years, and a regular health profile thereafter (which need not be specified). Assume the indifferences $([0,100]: P) \sim ([100,260]: P) \sim ([260,510]: P).$

⁶ In what follows, we will use bisection techniques from the psychological literature (60) within a revealed-preference setup.

Then U[0,100] = U[100,260] = U[260,510]. Scaling these values to be 0.25, linear
 interpolation gives the bold dashed line in Figure 3.



11 Assume that we further observe

12
$$([0,100]: P) \sim ([0,260]: M) \sim ([0,510]: F).$$

13 Because the utility of period [0,260], i.e. the total discounted value of the timepoints

14 in this period, is twice the utility of period [0,100], as we have just measured, we infer

15 that the loss of quality of life due to P is twice that due to M. Similarly, it is three

16 times the loss of quality of life due to F. Thus curing P is worth three times more than

17 curing F when it is over the same period.

18

19 REMARK. If we want to scale quality of life on the 0-1 death-life scale, then we

20 obviously have to include the death outcome. An indifference

21 ([0,100]; perfect health; death after) ~ ([0,260]; M; death after)
 22

then reveals
$$U(M) = 1/2$$
, implying $U(P) = 1/4$ and $U(F) = 3/4$.

It is natural to combine the death health state in some period with death following for ever after. Then we cannot assume the sequel of life-as-usual after, losing one advantage of our method. To retain that advantage we can use, instead of death, a 1 health state equivalent to death that can be combined with life-as-usual after, such as

- 2 being unconscious (20,21). \Box
- 3

4 The measurements and derivations in this example were all elementary, fully

5 preference-based under the normative principles of economics, and valid under all

6 QALY models described before. They completely identify utility of life duration (i.e.,

time discounting) and quality of life, where these components have been completelydisentangled.

9

10 **EXPERIMENT**

We implement the DM in an experiment to demonstrate its feasibility, and to
compare it with the CE method, the classical method for measuring the utility of life
duration in health research.

14

15 Subjects

N=70 students (30 female) from different departments of the Erasmus University
in Rotterdam participated. They were recruited using e-mail, poster advertisements,
and flyers distributed at the university campus.

19

20 Procedure

We tested our design in several pilot sessions. The experiment was computerrun and was administered in sessions of at most two persons. An experimenter was present during each session. All subjects finished the experimental session within 45 minutes. They were paid a fixed amount of €12.50 for participating. To avoid order effects, the order of the direct and the CE method was randomized across sessions. The two methods were administered successively. Both methods were preceded by
 two practice questions.

All indifferences were elicited using sequences of at most five binary choices rather than using direct matching. Choice-based elicitation is more time consuming but causes fewer inconsistencies (22). The indifferences were elicited iteratively. After each choice the subject was asked to confirm it. At the end of the iteration process, the first choice of the process was repeated. If the respondent changed this choice the iteration process recommenced. To further check for consistency, the elicitation of the first indifference value was repeated at the end of each method.

10

11 Stimuli of the DM

12 For the bad health state we took regular back pain (β = bad back) because it is 13 well known. We described this health state using the EQ-5D questionnaire that has 14 been widely used and validated (23). For the good health state γ we took full health. 15 It was explained that this health state meant being able to function perfectly well on 16 all five EuroQol dimensions, irrespective of age. The descriptions of β and γ were 17 printed on cards and handed to the subjects. The descriptions that we used are 18 provided in Appendix A.

19 We investigated the utility function over the next 50 years, i.e. over the time 20 interval [0,50]. (In the notation used before, D=50.) We normalized U(50)=1. The 21 reference health profile was β during all 50 years. We told the subjects that after 50 22 years all options gave the same health profile without further specifying it. Subjects 23 could choose between periods during which β would be improved to γ . In the first 24 question we determined d^{1/2} such that U(d^{1/2}) = 1/2*U(50) = 1/2. That is, improving β to γ

1	during the period $[0,d^{\frac{1}{2}}]$ is equally preferred as doing so during the period $[d^{\frac{1}{2}},50]$
2	We further elicited $d^{\frac{1}{8}}$, $d^{\frac{1}{4}}$, $d^{\frac{3}{4}}$, and $d^{\frac{7}{8}}$, with utilities $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{7}{8}$.
3	
4	

.

5 The CE Method: Stimuli (Risk Involved)

In the CE part of the experiment, we assumed full health throughout and
considered outcomes in terms of years, with for instance b denoting b years in full
health followed by immediate death. In general, subjects had to determine a risk-free
option b for sure that was equivalent to a risky option denoted a¹/₂c (probability ¹/₂ at a
years in full health and probability ¹/₂ at c years in full health).⁷ We assume a > b > c.
We thus successively obtained the following right-hand sides c^{superscript} from
indifferences:

13	501/20	equally preferred as	$c^{\frac{1}{2}};$
----	--------	----------------------	--------------------

equally preferred as	$c^{\frac{1}{4}}$
C	0 equally preferred as

15	$50_{\frac{1}{2}}c^{\frac{1}{2}}$	equally preferred as	$c^{3/4}$.
15	J01/2C	equally preferred as	ι,

16 0	$2^{\frac{1}{4}}$	equally 1	preferred as	c ^{1/8} ;
------	-------------------	-----------	--------------	--------------------

17 $50_{1/2}c^{3/4}$ equally preferred as $c^{7/8}$.

18 The (Classical) CE Method: Utility Analysis Using Expected Utility

19 We next turn to the classical method of analysis, based on expected utility. The

- 20 utility function will be denoted by the same symbol U as in the QALY model,
- assuming that they are the same. The assumption of one unifying concept of utility
- 22 that can be applied to different decision contexts has been questioned in the economic

⁷ The CE method resembles the standard gamble method except that in the standard gamble the riskfree option is held constant and indifference is achieved by varying the probabilities in the risky option.

2 evaluations such as in the QALY model is a nonlinear transformation of utility for 3 risk. Transferability of utility, based on one concept of utility, has been commonly 4 assumed in QALY analyses, however. There, for example, utilities measured under risk are used to analyze intertemporal decisions. Avoiding reliance on this 5 6 controversial assumption is one of the reasons why we introduce the DM. We will 7 first, however, present the traditional analyses that assume the same U. 8 For an observation that b is equally preferred as $a_{1/2}c$, expected utility implies that 9 U(b) is the midpoint of U(a) and U(c), i.e. (25) 10 $U(b) = \frac{U(a) + U(c)}{2}.$ (12)11 Under the normalization U(50) = 1 and U(0) = 0, superscripts give utility levels, i.e. $U(c^{\frac{1}{2}}) = \frac{1}{2}$, $U(c^{\frac{1}{4}}) = \frac{1}{4}$, and so on. This, classical, way to derive utilities from CE data 12 13 is called the *CEE method*, where the last letter E refers to expected utility. 14 15 The CEP Method: CE Analyzed Using Prospect Theory 16 Wakker and Stiggelbout (26) showed how to analyze CE data using the 17 empirically more realistic prospect theory (27,28) instead of expected utility. With 18 immediate death as reference point, b being equally preferred as $a_{\frac{1}{2}c}$ (with a > b > c) 19 now implies: 20 $U(b) = w(\frac{1}{2}) \times U(a) + (1 - w(\frac{1}{2})) \times U(c).$ (13)21 Here $w(\frac{1}{2})$ is the weight of probability $\frac{1}{2}$. Tversky and Kahneman's (28) estimate 22 $w(\frac{1}{2}) = 0.42$ has been found to perform well at the aggregate level. It implies that the

23 worst outcome c is relatively overweighted. This overweighting captures part of the

risk aversion that is empirically observed but that cannot be properly captured by the

literature (24). It has been assumed, for instance, that utility in intertemporal

1	U function. From Eq. 13 we can calculate all utilities $U(c^{superscript})$. They will be	
2	lower than the utilities under expected utility. We call this method the CEP method	od,
3	where the last letter P refers to prospect theory.	
4		
5	Convenience of the Methods	
6	At the end of the experiment, the subjects were asked to rate both the DM and	l the
7	CE on a scale from 1 (worst) to 7 (best), in terms of understandability and cognitiv	/e
8	burden.	
9		
10	Statistical Analyses of DM versus CEE (Assuming the Same Utility Function U)	
11	We first test the null hypothesis H_0 that expected utility holds for the risky CE	2
12	questions and that utility U from expected utility is the same as the utility to be use	ed
13	in QALY evaluations. Under H_0 , $c^{superscript} = d^{superscript}$ should hold for every	
14	superscript. Because both the c-values and the d-values are chained, they are not	
15	independent and direct tests of the above equalities would not be independent.	
16	Miyamoto and Eraker (29) devised a solution of this problem, by proposing to test	
17	proportional matches. We follow their proposal and test the following five equality	ies
18	predicted by H ₀ , using Wilcoxon signed ranks tests:	
19	$c^{1/2} = d^{1/2}$:	(14a)
17		(14b)
20	$(c^{3/4}-c^{1/2})/(50-c^{1/2}) = (d^{3/4}-d^{1/2})/(50-d^{1/2});$	(14c)
	$c^{1/8}/c^{1/4} = d^{1/8}/d^{1/4};$	(14d)
21	$(c^{\frac{1}{3}}-c^{\frac{3}{4}})/(50-c^{\frac{3}{4}}) = (d^{\frac{1}{3}}-d^{\frac{3}{4}})/(50-d^{\frac{3}{4}}).$	(14e)

For each subject and for each method we determined the shape of the utility
 function for life duration. The degree of concavity was measured by computing the
 area under the normalized utility function that results from linear interpolation:

4
$$50 - 50 \times (\frac{1}{8}d^{\frac{1}{8}} + 3d^{\frac{1}{4}}/16 + \frac{1}{4}d^{\frac{1}{2}} + 3d^{\frac{3}{4}}/16 + \frac{1}{8}d^{\frac{7}{8}})$$

5 To smooth response errors, we also fitted exponential utility

$$U(x) = (1-e^{-rx})/(1-e^{-r})$$
 (with $U(x) = x$ for $r = 0$)

for both methods, minimizing nonlinear squared distances. We did this both for each individual and for the median data. Exponential utility is widely used and generally gives a good fit (30). An additional advantage is that the estimated exponential coefficient equals the discount rate. Utility is concave if r>0, convex if r<0, and linear if r=0. All tests reported below are nonparametric and two-sided.

12

6

13 Statistical Analyses of DM versus CEP (Assuming the Same Utility Function U)

Under prospect theory there is no easy way to compare the c-values of the CE method directly to the d-values of the DM method. Hence, we did not carry out an analog of the test of Eqs. 14 for prospect theory. That is, we did not test the null hypothesis H₀ that prospect theory holds for the risky CE questions, nor that the utility function U from prospect theory can be used in QALY evaluations. The tests of curvature through area under the utility curve and through fitted exponential could easily be adapted to prospect theory and were carried out accordingly.

21

22 **RESULTS**

We excluded the data of three subjects who did not understand the task or who werenot willing to make risky choices about life duration for religious reasons. This left

67 subjects. The consistency tests revealed satisfactory test-retest reliability. The
 correlations between original and repeated indifference values were high and
 significantly different from 0: 0.75 for CE and 0.74 for DM (p<0.05 in both cases).

5 Utility Curvature

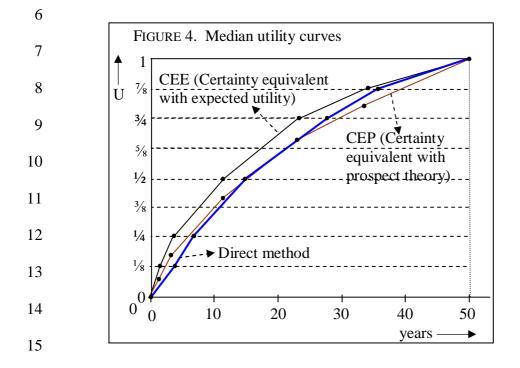


Figure 4 shows the utility functions based on the (point-wise) median data. All functions were clearly concave, the CEE curve most so. Table 1 shows the implied discount rates based on the median data. The implied rates for the DM and the CEP were lower than for the CEE. Discount rates are commonly found to decline over time (31,32). This is indeed what we observed for the CEE, but not for the DM. The discount rates implied by the DM were approximately constant and close to the 3% that is widely used in CEAs.

The estimated exponential coefficients based on the median data (best fitting the whole utility curve) were 0.056 for CEE, 0.036 for the DM, and 0.036 for CEP. The corresponding discount rates are 5.6%, 3.6%, and 3.6%. They differed significantly

- 1 between DM and CEE (Z-test, p<0.001) and between CEE and CEP (Z-test, p <
- 2 0.001), but not between DM and CEP.
- 3
- 4

Table 1: implied discount rates

	U ⁻¹ (1/8)	$U^{-1}(1/4)$	$U^{-1}(1/2)$	$U^{-1}(3/4)$	U ⁻¹ (7 / 8)
CEE	8.5%	7.8%	5.3%	5.0%	4.5%
DM	2.8%	3.5%	3.5%	3.5%	3.9%
СЕР	4.5%	5.0%	3.8%	3.4%	2.7%

5 Each entry gives the constant discount rate on $[0, U^{-1}(x)]$ predicting $U^{-1}(x)$.

6

7 The area under the normalized utility function was highest for the CEE (mean 8 area = 37.03). This area was significantly lower for the DM (mean area = 35.03, Z = 9 -2.296, p=0.02). For CEP, the area was not significantly different from the DM 10 (mean area = 35.91, Z = -1.399, p = 0.16), but it obviously was lower than the CEE (Z 11 =-7.115, p < 0.001). All areas differed significantly from 25, the case corresponding 12 to linear utility (p < 0.001 in all tests). The median individual discount rates were 3.5% for DM, 6.2% for CEE, and 13 14 3.6% for CEP. All estimates differed significantly from zero discounting ($p \le 0.001$).

15 Figure 5 shows the distribution of the individual discount rates. The CEE estimate

16 differed significantly from the DM estimate (t=2.58, p=0.01) and from the CEP

17 estimate (t = 3.81, p < 0.001). The CEP and DM estimates did not differ significantly

18 (t=0.63, p=0.53). Thus CEP and DM may measure the same utility, but CEE

19 measures something different.

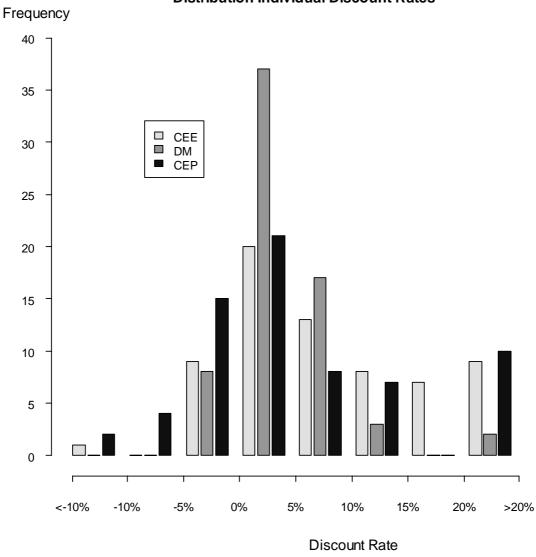
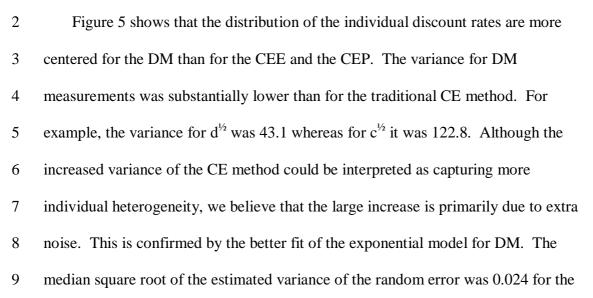
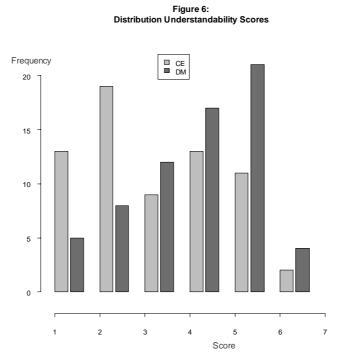


Figure 5: Distribution Individual Discount Rates



DM, and was significantly lower than the 0.056 for the CEE (t=4.87, p<0.001), and the 0.061 for the CEP (t=5.25, p<0.001).⁸ The fit of the CEE and of the CEP did not differ significantly (t=1.11, p=0.27). The relatively poor fit of the CEP indicates that individual prospect theory parameters vary substantially, implying that the method is only reasonable at the group level and does not fit well at the individual level.



6

10 understandability scale were 4.76 for the DM and 3.88 for the CE method. The mode

11 was 6 for the DM and 3 for the CE. The scores differed significantly (p < 0.001).

12

```
13 Directly Testing DM versus CEE (Eqs. 14a-14e)
```

14 Eqs. 14d (proportional match of $c^{\frac{1}{8}}$ versus $d^{\frac{1}{8}}$) and 14e (proportional match of $c^{\frac{7}{8}}$

15 versus $d^{\frac{7}{8}}$) were rejected, with always the c-values smaller than the d-values. The

<sup>Our subjects also considered the DM to be easier than the CE method. Figure 6
shows that the distribution of individual scores for the DM was clearly to the right of
the distribution for the CE. The mean scores on our 1 (worst)-7 (best)</sup>

⁸ This also held true when we used power utility instead of exponential utility.

other three Eqs. 14a, 14b, and 14c were not rejected (14a: Z=0.534, p=0.59; 14b: Z=-1.511, p=0.13; 14c: Z=-1.631, p=0.10; 14d: Z=-2.361, p=0.02; 14e: Z=-2.780, p=0.01). We also compared the two methods over the whole domain by taking the differences between the proportional matches for each question and performing a Friedman test. This yielded a significant difference, indicating more concavity for CEE than for DM ($\chi^2 = 11.570$; p = 0.04).

8 **DISCUSSION**

7

9 DM versus CE utility. Traditional measurements result in more concave (higher) 10 utility than the DM. This discrepancy leaves open the question which of the utilities 11 is more valid. One argument against traditional CEEs is that they are based on 12 expected utility and there is much empirical evidence against this theory (27,33-36). 13 When the CE data are analyzed using the empirically more realistic prospect theory 14 (CEP), they are adjusted downwards and become statistically the same as DM 15 utilities. This suggests that the traditionally analyzed CE measurements (CEE) 16 overestimate utility and, hence, lead to discount rates that are too high, and that the 17 DM measurements are more valid.

When choosing between DM and CEP utility, we prefer the former. Although the empirical violations of expected utility have been corrected for by CEP for group averages, individual variations in those violations still generate errors at the individual level, which was reflected in the better fit of the DM for the individual level data.
Further, CEP retains the other drawbacks of risky choice.
The absence of significant differences between CEP and DM group average utilities supports the transferability of utility across different domains (risk versus 1 intertemporal). It corroborates similar transfers found between risky utility and other 2 forms of utility if risky utility is analyzed using prospect theory (37).

3

4 Extraneous Devices Other than Risk. Alternative extraneous devices have been used to measure discounting. The person trade-off method replaces probabilities by 5 6 proportions of affected people (38-40). Then equity considerations generate 7 distortions much as risk aversion does for risk (41,42). The Rawls-Harsanyi veil of 8 ignorance (43) demonstrates the close relationship between the person-tradeoff 9 method and the risk approach. 10 Another approach to measuring discount rates without invoking risky choice is 11 based on inconsistencies in traditional TTO measurements (6,44). Such 12 inconsistencies can result if different durations are used and linear utility is 13 erroneously assumed. The resulting inconsistencies have been used to estimate a 14 discount factor (45-47). These approaches assume one-parameter discounting, usually 15 constant discounting, and do not provide general discount functions. Cairns (3) is 16 closest to us. He did not use extraneous devices and he used similar stimuli. He, 17 however, used parametric fitting to measure QALYs. 18 19 Violations of the OALY Model. The DM was developed for the general QALY model, 20

and is valid only to the extent that the QALY model is valid. The central condition

21 underlying the QALY model is an independence condition (Miyamoto et al. (48), who

22 generalize Pliskin et al. (49)), which ensures that the utility of life duration can be

23 measured independently of health quality. Empirical evidence on this condition is

24 limited and mixed, with some violations documented (50-52), but also some support

25 (29,53,54). Other objections against the QALY model have been raised as well, for instance that it may lead to discrimination of people who have a limited capacity to
benefit from health care, such as the disabled. However, no tractable alternative
model is available yet. Kahneman et al. (55) showed that QALY-evaluations can be
restored if quality of life (or its analog called instant utility) is sufficiently
comprehensive in the sense of incorporating pleasures and pains now felt because of
past and future events.

7

8 Applications

9 Our method is not only applicable at the individual level, but also in societal cost-10 effectiveness analyses (CEA). For the latter, several authors have argued that the 11 social rate of time preference should be based on the diminishing marginal utility for 12 life-years (56,57). Our method readily provides this information.

Existing algorithms, used to measure health quality in CEAs, are usually based on the TTO (EQ-5D) and the standard gamble (SF6D, HUI). These methods are known to be systematically biased, because of discounting, violations of expected utility, and other distortions (58). Our method avoids these biases and will, we hope, be further investigated as a possible alternative.

18

Origin of Our Method. It appears natural, when measuring the utility of life duration in the QALY model, to proceed analogously to the measurement of utility of outcomes in the expected utility model (15), and this is what all methods have done so far. In a mathematical sense our method is, however, analogous to the measurement of *subjective probability* under expected utility, and not of utility. To see this point, assume indifference between:

25 (a) A wealth improvement of \$100 conditional on the event of no rain tomorrow;

(b) The same wealth improvement conditional on the event of rain tomorrow.⁹
Then the two weather events are apparently considered equally likely. They must
then have the same subjective probability ¹/₂. Our method is mathematically
equivalent by substituting period for event and health improvement for wealth
improvement.

6

7 CONCLUSION

8 To evaluate medical interventions we have to correct the number of resulting life 9 years for two factors: (1) quality of life; (2) discounting (utility of life duration). It 10 has traditionally been thought that measuring either of these factors is difficult given 11 that the other factor is also unknown. Complex and distorting extraneous devices 12 have been invoked such as risk or welfare, or assumptions were made heuristically. 13 The direct method (DM) resolves these problems. It is surprisingly simple both for 14 subjects and for data analyses. It directly measures the utility of life duration 15 irrespective of quality of life (which may be unknown). Then, with discounting and 16 utility of life duration available, quality of life can readily be measured in a second 17 stage (Eq. 7). The whole QALY model can thus be measured in full generality, 18 without using any extraneous device. 19 An experiment has implemented the DM, confirming prior expectations: The DM

is considerably easier to administer and to understand for subjects than classical
methods. It avoids the aversive and implausible scenarios needed in traditional
measurements. DM utilities agree with risky utilities (if the latter are analyzed using
prospect theory, a proper descriptive risk theory) on average, but they fit better at the

⁹ The same conclusion obviously holds if we consider an improvement in health (from β to γ) rather

individual level. We conclude that the DM measures the utility of life duration in a
 more tractable, reliable, and valid manner than methods used before.

3

4 ACKNOWLEDGMENT

Bas Karreman, Tom Van Ourti, Kirsten Rohde and Stefan Trautmann made useful
suggestions for the experiment. Guido Attema helped developing the computer
program.

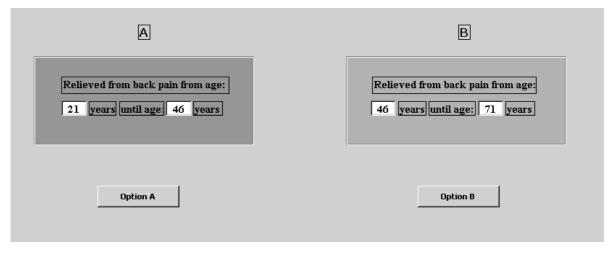
8 APPENDIX A: HEALTH STATE DESCRIPTIONS (Translated 9 from Dutch)

10 11 Card 1 – Regular Back Pain 12 You have regular back pain. This has the following consequences for your functioning in daily life: 13 14 You have no problems in walking about. 15 16 17 You have no problems in washing or dressing yourself. 18 You have some problems with your usual activities. 19 20 21 You have *moderate* pain or other discomfort. 22 23 You are not anxious or depressed. 24 25 26 *Card* 2 – *Full Health* 27 You have no complaints and are in perfect health. This has the following 28 consequences for your functioning in daily life: 29 30 You have no problems in walking about.

1 2 You have no problems in washing or dressing yourself. 3 4 You have no problems with your usual activities. 5 You have no pain or other discomfort. 6 7 You are not anxious or depressed. 8 9 10 **APPENDIX B: EXAMPLE OF UTILITY ELICITATION** 11 12 This appendix illustrates the DM by showing the complete elicitation for a typical 13 14 subject (of age 21) in our experiment. The DM always starts with the elicitation of $d^{1/2}$. The first question takes a starting value of $d^{1/2}$ equal to the midpoint of the 15 16 period considered. This period was [0,50] in our experiment. That is, the subject first 17 had to compare U[0,25] and U[26,50], as shown by the screen shot of the first

- 18 question for this subject (Figure B1).
- 19

20 FIGURE B1



- 21 22
- 23

24 We subsequently adjusted this starting value of $d^{1/2}$ (25 in this example) upwards or

25 downwards depending on the option chosen. Because this subject chose A in this

26 question, implying U[0,25] > U[26,50], we made option A less attractive in the

27 second question and, therefore, adjusted $d^{1/2}$ downwards to a value of 13 (all numbers

28 were rounded to integers). This generated the following screen shot:

1 FIGURE B2

A	В
Relieved from back pain from age: 21 years until age: 34 years	Relieved from back pain from age: 34 years until age: 71 years
Option A	Option B

2 3

4 Suppose that the subject now switched preference and preferred option B. The third question therefore had to make A somewhat more attractive again, by increasing $d^{1/2}$. 5 This was accomplished by using a change half the size of the change in the previous 6 7 question, i.e., by taking the midpoint between 13 and 25 (=19). We continued this 8 way until the fifth question, after which we ended with a small interval containing the 9 indifference value, for which we finally took the midpoint of the small interval. 10 Having elicited an indifference value for $d^{1/2}$, we could next find $d^{1/4}$ and $d^{3/4}$ such that 11 $U[0,d^{1/4}] = U[d^{1/4},d^{1/2}]$ and $U[d^{1/2},d^{3/4}] = U[d^{3/4},50]$. The order of these two was 12 randomized. Suppose that we continued with elicitating $d^{1/4}$ for this subject. This 13 required replacing D=50 by $d^{1/2}=20$. The remainder of the procedure was similar to 14

15 the elicitation of $d^{1/2}$. Hence, the first choice was represented (Figure B3).

16

17 FIGURE B3



1 Table B1 shows all choices and answers of this subject.

- 3 TABLE B1
- $d^{1/2}$

Period with back pain relief in option	Period with back pain relief in	Option
A $(0,d^{1/2})$	option B (d ^{1/2} ,50)	chosen
21-46	46-71	А
21-34	34-71	В
21-40	40-71	В
21-43	43-71	A
21-41	41-71	A
Indifference value	d ^{1/2} =19.5 (=40.5-21)	

d^{1/4}

Period with back pain relief in option	Period with back pain relief in	Option
A $(0,d^{1/4})$	option B $(d^{1/4}, d^{1/2})$	chosen
21-31	31-41	А
21-26	26-41	В
21-28	28-41	В
21-30	30-41	А
21-29	29-41	В
Indifference value	d ^{1/4} =8.5 (=29.5-21)	

d^{3/4}

Period with back pain relief in option	Period with back pain relief in	Option
A $(d^{1/2}, d^{3/4})$	option B (d ^{3/4} ,50)	chosen
41-56	56-71	A
41-48	48-71	В
41-52	52-71	А
41-50	50-71	В
41-51	51-71	В
Indifference value	d ^{3/4} =30.5 (=51.5-21)	

1	
2	d ^{1/8}

Period with back pain relief in option	Period with back pain relief in	Option
A $(0, d^{1/8})$	option B (d ^{1/8} ,d ^{1/4})	chosen
21-25	25-29	А
21-23	23-29	В
21-24	24-29	А
Indifference value	d ^{1/8} =2.5 (=23.5-21)	

3 4 $d^{7/8}$

Period with back pain relief in option	Period with back pain relief in	Option
A $(d^{3/4}, d7^{/8})$	option B (d ^{7/8} ,50)	chosen
51-61	61-71	А
51-56	56-71	В
51-59	59-71	А
51-58	58-71	А
51-57	57-71	A
Indifference value	d ^{7/8} =35.5 (=56.5-21)	

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