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Characterizing QALYs by Risk Neutrality

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Abstract

This paper shows that QALYs can be derived from more elementary conditions than thought hitherto in the literature: it suffices to impose risk neutrality for life years in every health state. This derivation of QALYs is appealing because it does not require knowledge of concepts from utility theory such as utility independence. Therefore our axiomatization greatly facilitates the assessment of the normative (non)validity of QALYs in medical decision making. Moreover, risk neutrality can easily be tested in experimental designs, which makes it straightforward to assess the descriptive (non)validity of QALYs.

Key words: QALYs, value of life, utility independence, risk neutrality

JEL Classification: D81, I10

Quality-adjusted life years (QALYs) are the most common outcome measure in cost utility analyses of health care programs. They offer a straightforward procedure for combining the two most important outcomes of health care programs, quality of life and quantity of life, into one single measure. QALYs have the advantage of being easy to calculate, which is especially important for complex decisions regarding non-constant health profiles and social decisions. Another advantage of QALYs is their direct and intuitively appealing interpretation. A disadvantage is that they require the individual preference relation to satisfy some restrictive conditions. Given the importance of QALYs and the many discussions of their appropriateness, further insights into those restrictive conditions is important.

The aim of this paper is to provide a characterization of QALYs for the case of chronic health states that is more elementary and fundamental than those provided hitherto in the literature. Throughout, we assume expected utility. It has often been criticized for the many empirical violations and also sometimes for its normative validity (Allais, 1953; Machina, 1982). In prescriptive applications, however, it still is the prevailing theory (Edwards, 1992). Pragmatic reasons for its application are that the modern nonexpected utility theories have not yet reached the stage of implementability, whereas the application of expected utility has a long tradition (Keeney & Raiffa, 1976; von Winterfeldt &

Edwards, 1986; Eeckhoudt, 1996). Expected utility is used throughout medical decision making (Weinstein, et al., 1980). Hence it is also the basis of this paper. QALYs for nonexpected utility theory are studied by Bleichrodt & Quiggin (1997).

The conditions commonly used to characterize QALYs are "utility independence," "constant proportional tradeoffs," and "risk neutrality for life years." These conditions were established by Pliskin, Shepard, and Weinstein (1980), and studied also by others (Torrance and Feeny, 1989; Loomes and McKenzie, 1989; Bleichrodt, 1995). The surprising result provided here is that, in the presence of a condition that is unobjectionable in the medical context, risk neutrality for all health states alone already suffices to imply QALYs. That is, in the medical context risk neutrality simply implies the other two conditions.

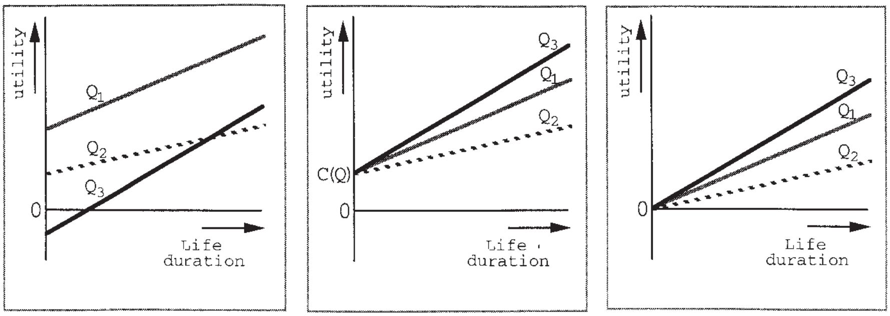
Characterizations aim to identify the preference conditions that underly a particular preference representation. This is important both for normative and for descriptive reasons. Normatively, by examining the preference conditions a decision maker can be persuaded to use a particular model or, alternatively, the preference conditions can be used as an argument for not using a model. Descriptively, identifying the preference conditions allows for the testing of the model in an experimental setting. The attractiveness of a particular characterization depends crucially on the conditions used. Conditions that are easy to understand and intuitively appealing facilitate the tasks of assessing the normative and descriptive properties of a model.

The central condition in our characterization, risk neutrality for life years, is well-known and can easily be explained. It does not require knowledge of utility-theory concepts such as utility independence. Thus, our result is both more elementary and more general than the existing results in the literature. Also, by finding a shorter road to QALYs, we can provide an extremely simple proof that is easily illustrated graphically. The proof is so simple that it is given in the main text.

After the presentation of the main theorem of this paper, we provide a detailed analysis of the relations between our conditions and the ones customary in the literature. This will further clarify the points where we generalize existing results.

1. Structural assumptions

We restrict attention to *chronic health states* in this paper, that is, we assume that quality of life is constant until death. Thus a pair (Q,T) designates the outcome where a person lives for T years in health state Q and then dies. We adopt the structural assumptions commonly used in the study of multiattribute utility and medical decision making (Keeny and Raiffa, 1976; Pliskin et al., 1980). That is, we study an individual preference relation on lotteries over chronic health states. By $[p_1,(Q_1,T_1);\ldots;p_n,(Q_n,T_n)]$ we denote a lottery yielding outcome (Q_i,T_i) with probability p_i. The preference relation satisfies the von Neumann-Morgenstern axioms (1947). Hence there exists a utility function U, assigning to each chronic health state (Q,T) the utility $U(Q,T)$, so that the expectation of U, $(p_1U(Q_1,T_1) + \ldots + p_nU(Q_n,T_n)$ for the above lottery), governs the choices between lotteries over chronic health states.



1a. Risk neutrality *1b. Zero-condition* *1c. Rescaling*

Figure 1. Illustration of a derivation of QALYs. The figure shows U(Q,T) for three qualities of life Q.

2. The main result

The individual preference relation satisfies *risk neutrality for life years* if, with quality of life held fixed, the individual is indifferent between a lottery over life years and the expected life duration of that lottery. Risk neutrality means that, for any particular health state Q, the individual is indifferent between:

(i) a probability p of T years in Q and a probability $(1 - p)$ of S years in Q;
(ii) $pT + (1 - p)S$ years in Q for certain.

If we draw $U(Q,T)$, holding quality of life Q constant, then risk neutrality for life years implies that the graph of $U(Q,T)$ is linear. Risk neutrality is illustrated in figure 1a where the utility function over life years has been drawn with quality of life held fixed at three different levels. Let us emphasize that linearity of utility is necessary and sufficient for risk neutrality, and does not require that the slope of utility be one or that the intercept be zero. By risk neutrality, the von Neumann Morgenstern utility function $U(Q,T)$ is of the form $C(Q) + V(Q)T$, where $C(Q)$ is a constant that depends on Q, but is independent of T and $V(Q)$ is a positive constant that depends on Q, but is independent of T. In figure 1a, $C(Q)$ is the intercept of $U(Q,T)$ and $V(Q)$ the slope.

It is obvious that under the QALY model, where $U(Q,T) = V(Q)T$ for a function V, $U(Q,0)$ must be the same for all health states. That condition is not implied by the assumptions made so far. In particular, it is not satisfied in figure 1a. Therefore, the QALY model fails in figure 1a. Let us display the condition.

Zero-condition. For a duration of zero life years, all quality of life levels are equivalent.

The zero-condition is entirely self-evident in the medical context. Figure 1b illustrates the effect of imposing the zero-condition in addition to risk neutrality for life years. By the zero-condition, $U(Q,0)$ is constant for all health states. Thus $U(Q,0) = C(Q) + V(Q)0 = C(Q)$ is constant. For a duration of zero life years the zero-condition implies that al linear utility lines pass through the same point $C(Q)$, i.e. they must have the same intercept.

It is well-known in the von Neumann-Morgenstern utility theory that one can add, at one's will, a constant to a von Neumann-Morgenstern utility function. Therefore we can add minus the intercept $C(Q)$ in figure 1b. That is, we may assume figure 1c in which $U(Q,0) = 0$ for all Q. We can now write

$$U(Q,T) = V(Q)T.$$

The above equation has established the QALY model. Let us summarize: If expected utility, risk neutrality for each fixed health state, and the zero-condition hold, then the QALY model holds. It is obvious that the conditions are also necessary for the QALY model. Therefore we have established:

Theorem 1. Under expected utility, the following two statements are equivalent for a preference relation on lotteries over chronic health states:

(i) The QALY model holds:

$$U(Q,T) = V(Q)T.$$

(ii) The zero-condition holds and, for each health state, risk neutrality holds for life years. *Q.E.D.*

Because the zero-condition is unobjectionable in the medical context, the above theorem has demonstrated that risk neutrality for all health states is the essence of the QALY model. The QALY model can be justified normatively if and only if risk neutrality can be, and it can be criticized normatively if and only if risk neutrality can be. Similarly, the QALY model can be verified descriptively if and only if risk neutrality can be, and it can be falsified if and only if risk neutrality can be. The general finding, both normatively and descriptively, is that risk neutrality does not hold (McNeil et al., 1978; Stiggelbout et al., 1994). Therefore QALYs can at best be used as an approximation, in contexts where the violations of risk neutrality are not too extreme.

Empirical research has shown that individuals do not attach equal weight to different years of life (Johannesson and Johansson, 1996). Individuals apply a discounted utility model rather than a linear utility model (Viscusi and Moore, 1989; Moore and Viscusi, 1990). The realism of the QALY model can be increased if the numbers T do not designate life years, but discounted life years. Obviously, such a different interpretation of the number T in our theorem does not change its mathematical correctness. Thus risk neutrality with respect to discounted life years holds if and only if the QALY model holds with discounted instead of absolute life years. Note that risk neutrality with respect to discounted life years implies risk aversion in terms of (undiscounted) life years, in agreement with the general empirical finding.

3. A comparison with the result of Pliskin et al.

The characterization of QALYs that is commonly invoked in the literature has been established by Pliskin et al. (1980). Instead of the zero-condition, Pliskin et al. impose mutual utility independence and constant proportional tradeoffs. One reason for Pliskin et al. to consider these conditions is that they can serve to characterize models that are more general than the risk neutral QALY model studied in this paper. The importance of risk neutrality for QALY characterizations was already suggested by Johannesson (1995). That paper, however, did not provide a complete characterization and derivation. In more general contexts, without risk neutrality, the zero-condition appeared in Miyamoto and Eraker (1988) and Peters (1992). We now turn to a discussion of constant proportional tradeoffs.

The *constant proportional tradeoffs* assumption holds if, for all health states Q_1 and Q_2, there exists a positive number q such that $U(Q_1,T) = U(Q_2,qT)$ for all life durations T. In other words, constant proportional tradeoffs hold if the proportion of life years the individual is willing to give up for a given quality of life improvement is invariant with respect to life duration. Pliskin et al. imposed constant proportional tradeoffs only for a best and worst state of health, and then proved that, in the presence of mutual utility independence, constant proportional tradeoffs for all states of health is implied. That, in turn, immediately implies our zero-condition, simply by substituting $T = 0$ in the above definition. This implication also demonstrates how the QALY axiomatization of Pliskin et al. can be derived from ours: One derives the zero-condition as just indicated, and then by risk neutrality and Theorem 1 the QALY model follows. The zero-condition can be viewed as a weakened version of constant proportional tradeoffs. Risk neutrality and the zero-condition imply constant proportional tradeoffs as follows immediately from the representation $U = V(Q)T$ in Theorem 1 (define $q = V(Q_1)/V(Q_2)$ in the above definition). Next we discuss some notions of utility independence.

- Quality of life is *utility independent* from quantity of life if preferences over lotteries for quality of life with quantity of life held fixed at level T are invariant with respect to the particular level T.
- Quantity of life is *utility independent* from quality of life if preferences over lotteries for quantity of life with quality of life held fixed at level Q are invariant with respect to the particular level Q.
- If both conditions hold, we say that quality of life and quantity of life are *mutually utility independent.*

If quality of life is utility independent from quantity of life, then $[p,(Q_1,T); 1 - p,(Q_2,T)]$ is preferred to (Q,T) if and only if, for any life duration T' different than T, $[p,(Q_1,T'); 1 - p,(Q_2,T')]$ is preferred to (Q,T'). If quantity of life is utility independent from quality of life, then $[p,(Q,T_1); 1 - p,(Q,T_2)]$ is preferred to (Q,T) if and only if, for any health state Q' different than Q, $[p,(Q',T_1); 1 - p,(Q',T_2)]$ is preferred to (Q',T).

Obviously, if risk neutrality holds irrespectively of the quality of life, then for a fixed health state the preferences are governed by expected life duration, irrespective of the health state, and quantity of life is utility independent from quality of life. Conversely, if risk neutrality holds for perfect health and quantity of life is utility independent from quality of life, then risk neutrality holds for all qualities of life. This follows from the fact that by utility independence all utility functions over life years are strategically equivalent regardless at which level quality of life is held fixed; thus, if risk neutrality holds for life years in full health, risk neutrality for life years must, by utility independence, hold for all health states. Therefore the following observation is not surprising.

Observation 2. Risk neutrality holds for all qualities of life if and only if quantity of life is utility independent from quality of life and risk neutrality holds for perfect health. *Q.E.D.*

A remarkable implication of Theorem 1 is that risk neutrality, in the presence of the zero-condition, implies utility independence of quality of life from quantity of life. This is easily seen for the utility function $U(Q,T) = V(Q)T$ in Theorem 1, because the expectation of $V(Q)$ governs preferences over qualities of life for a fixed level of T, independent of what that level of T is.[1]

Risk neutrality in isolation does not imply utility independence of quality of life from quantity of life. This can be seen as follows. Risk neutrality does not exclude $U(Q_3,T) < U(Q_2,T)$ for small T and $U(Q_3,T) > U(Q_2,T)$ for large T (Figure 1a). Here $U(Q_3,.)$ has a larger intercept, but a smaller slope, than $U(Q_1,.)$ and the lines intersect at some T. Then the preference order of Q_2 and Q_3 depends on T and quality of life is not utility independent from quantity of life.

We summarize the above discussion in the following corollary of Theorem 1.

Corollary 3.

(i) Risk neutrality and the zero-condition imply mutual utility independence and constant proportional tradeoffs.

(ii) In the characterization of the QALY model by means of risk neutrality, mutual utility independence, and constant proportional tradeoffs, the following generalizations are possible:

constant proportional tradeoffs can be weakened to the zero-condition

and either

mutual utility independence can be dropped

or

risk neutrality and mutual utility independence can be weakened to risk neutrality for perfect health and utility independence of life years from health states. *Q.E.D.*

Corollary 3 demonstrates that, for empirical investigations of the QALY model, tests of utility independence and constant proportional tradeoffs are tests of *implications* of risk neutrality.

4. Discussion

In this paper we have shown that QALYs can be derived from an individual preference relation that satisfies the von Neumann-Morgenstern axioms by imposing risk neutrality for life years and a very weak condition, that for a duration of zero years all health states are equivalent (the zero-condition). Given that the zero-condition is self-evident in the medical context, the crucial condition in our characterization is risk neutrality for life years. Risk neutrality for life years is a condition that is both easy to understand and straightforward to test in an experimental design. Empirical research generally indicates that risk neutrality for life years is violated to a certain degree.

Mutual utility independence, constant proportional tradeoffs, and risk neutrality for life years are commonly imposed for characterizing the QALY model. Corollary 3 shows that each of these conditions can be relaxed considerably. If the zero-condition, self-evident in the medical context, is accepted, then Theorem 1 shows that two of the three common conditions, mutual utility independence and constant proportional tradeoffs can simply be dropped.

The advantage of high tractability due to risk neutrality becomes more important when outcome evaluation is extended to nonchronic health profiles and/or social evaluations. In the extension to nonchronic health profiles, the assumption of "additive separability" over disjoint time periods is usually added, for one reason because it allows for the application of Markov models. Then every life year is adjusted by a quality correction factor, and the resulting numbers are added up. Here, due to the increased complexity of stimuli, tractability of evaluation is more important, which explains the common assumption of risk neutrality for life years (or discounted life years; see above). A similar observation holds for social evaluations. Here the aggregation over individuals increases complexity and therefore risk neutrality for (discounted) life years, implying that every (discounted) life year counts equally, is also the common assumption.

To avoid misunderstanding, let us emphasize that our characterization of QALYs in terms of preference conditions need not mean a justification or a defense. Just as well can it serve to criticize QALYs. Our result has shown that, for a criticism of QALYs, risk neutrality should be the target. An attempt to criticize QALYs by criticizing constant proportional tradeoffs or mutual utility independence, while accepting risk neutrality, will not work. A test of constant proportional tradeoffs or mutual utility independence always entails an indirect test of risk neutrality. The general purpose of axiomatizations of a quantitative model in terms of preference conditions is to relate theoretical concepts such as QALYs to conditions, such as risk neutrality, that have a direct empirical meaning. Thus a characterization shows the empirical meaning of a theoretical model. It facilitates both defenses and criticisms of the model.

Notes

1. A minor modification should be made that is implicitly assumed throughout this paper: Utility independence is restricted to the domain where the life duration 0 is excluded, and requires that all health states be positive. These points have sometimes been overlooked in the literature.

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