Inventory Control with product returns: the impact of (mis)information

Marisa P. de Brito* Erwin A. van der Laan†

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Abstract

Product returns are often characterized by a dual uncertainty on time and quantity. In the literature on inventory management with product returns, best forecasts have been associated with methods that use the most information regarding product return history.

In practice however, data is often scarce and unreliable. In this paper we investigate the impact of (mis)information on inventory performance. An exact analysis shows that in case of misestimation the most informed method does not necessarily lead to best performance. Further we provide a simulation study to investigate the impact of misinformation w.r.t. inventory costs. The results have relevant implications regarding investments in product return information systems.

keywords: Product returns, forecasting, inventory control, information management

1 Introduction

Products, distribution items and equipments go back in the supply chain for a diversity of reasons (Dekker and De Brito, 2002). For instance, beverage containers are returned by the consumer to the retailer against reimbursement, single-use photo cameras are turned in to be developed, and millions of products purchased through mail-order-companies, e-tailers and other distant sellers, are being returned right now.

There is a lot of money involved with the handling of product returns (Rogers and Tibben-Lembke, 1999). One of the difficulties in handling returns efficiently is that return flows are often characterized by a considerable uncertainty regarding time and quantity. If one could know exactly how much is going to be returned and when, one would certainly benefit from

*Erasmus University Rotterdam, Econometric Institute, Burg. Oudlaan 50, PO Box 1738, 3000 DR Rotterdam, The Netherlands. E-mail: debrito@few.eur.nl

†Erasmus University Rotterdam, Department of Decision and Information Sciences, Burg. Oudlaan 50, PO Box 1738, 3000 DR Rotterdam, The Netherlands. E-mail: elaan@fbk.eur.nl

incorporating this perfect information a priori in the management of production, inventory, and distribution. In many cases this is far from feasible (see Flapper, 1997; Trebilcock, 2002). Nevertheless, one may attempt to forecast the timing and the amount of product returns. To do so, one has to hypothesize about the return flow properties based on historic demand and return data.

There are not many papers that investigate the impact of information on inventory management with product returns. The ones that do so assume known return probabilities or consider specific cases where the most-informed method leads to the best forecast. However, in environments where data is scarce or unreliable, or in environments that are volatile, information may be misleading. In this paper, we elaborate on the impact of such (mis)information on inventory control. We identify situations in which the most informed method does not necessarily lead to the best performance. Furthermore, we investigate for a wide range of parameters the impact on inventory related costs of having inaccurate estimates of the time-to-return distribution. In addition, we show the implications for the practice of inventory management with returns.

The remainder of the paper is structured as follows. The next section reviews the literature on the topic. After that we illustrate that estimates of the return rate can be massively erroneous in practice. Subsequently we elaborate on procedures to estimate the net demand. By means of an exact analysis, in Section 5 we identify situations in which more information does not necessarily lead to better performance. Section 6 then presents a simulation study to quantify the impact of misinformation with respect to inventory related costs. In Section 7, we discuss the managerial implications and and Section 8 we give recommendations for future research.

2 Literature Review

There are few articles that simultaneously consider the forecasting of product returns, inventory management and information issues. Yet, the references below have helped to fill the research gap.

Goh & Varaprasad (1986) developed a methodology to compute life-cycle parameters of returned containers by means of employing data on demand and returns. The authors claim that a careful estimation of these parameters aids to effective inventory. They apply their method to data on soft drinks as Coca-Cola and Fanta from the Malaysia and Singapore markets. The approach requires a time series of aggregated demand and another one of aggregated return data. They used a 50 point time series of monthly data. The precision of the method depends on how accurate is the estimation of the return distribution. This estimation has for basis the Box-Jenkins time series techniques. Goh and Varaprasad call the attention of the reader to the fact that a data set with less than 50 points is too short to employ the methodology. Large time series should also be avoided. Therefore, they recommend a time series of 50 points, or a 4-year period of data coming from a stable marketing environment. Our research contributes for the opposite situations: with imperfect or deceiving data.

Kelle & Silver (1989) proposed four forecasting procedures of net demand during lead time in the case of reusable containers. Every procedure has a different level of information requirement. The least-informed method uses the expected and variance of the net demand together with the probability of return. The most-informed one calls for individual trace and tracking of containers. The authors evaluate the forecasting methods taking the most-informed method as a benchmark. The analysis, however, applies only to the case of perfect information. In our paper we elaborate on the potential impact of (mis)information.

Toktay et al. (2000) consider the real case of new circuit boards for Kodak's single use remanufactured camera. The goal is to have an ordering policy that minimizes the procurements, holding inventory and the lost sales costs. A six-node closed queueing network is employed to represent Kodak's supply chain. Accordingly, the returns of cameras depend on past sales by a return probability and an exponential time lag distribution. Some of the procedures of Kelle & Silver are used to predict the unobservable inventory at the customer-use network node. The authors compare several forecasting methodologies with different levels of information. In our paper we provide an exact and more general analysis of the impact of (mis)information. Furthermore, we numerically compare four methods for a wide range of parameter values with respect to misinformation. Therefore, besides an intra-method comparison our analysis is also an inter-method comparison.

Marx-Gómez et al. (2002) consider a case of photocopiers that may return to the producer after being used. The authors put forward a method to forecast the amount and time of returned photocopiers. Firstly, data is generated according to two scenarios: successful vs. not so successful return incentives. The previous, together with expert knowledge, constitute the basis for developing a set of forecasting rules. Factors as demand, life cycle parameters, and return incentives are taken into account. An extended approach of the model is suggested by allowing a follow-up period to self-learn the rules. This neuro-fuzzy process calls for demand and returns data. Besides this, one should notice that the method of Marx-Gómez et al. depends on the a priori knowledge on predicting product returns acquired by the producer.

Overall we can conclude that the proposed procedures are very demanding with respect to reliable data. In case of reliable data, the most-informed method leads to the best performance. In this paper we take the more realistic perspective of misinformation and analyze its impact.

3 (Mis)information in practice: an illustrative example

Consider the example of a large Western European mail-order-company. Well before the start of the season, the company has to decide on what to offer for sale in the upcoming catalogue. A great part of the total stock to cover all the season has to be ordered many weeks in advance provided that production lead times are lengthy. This is many times the case in Europe where cheap manufacturing comes from Asia or from far East (see Mostard and Teunter, 2002). The order decisions take into account the expected future demand and returns for each line of products. Since there is however no data available on the sales of the aimed season, the forecast is not much more than a rough estimate. In particular, a mail-order-company in the Netherlands uses estimates that on average are more than 20% off the real return rate and sometimes even more than 80%. The above illustrates that large mis-estimation of the return rate does occur in practice. Next, we analyze its impact on inventory control.

4 Forecasting methods

Consider a simple single product, single echelon, periodic review inventory system. Each individual demand returns with probability p according to some return distribution. The production lead time is a fixed constant L. Demands that cannot be satisfied immediately are fully backordered. In this setting, a simple base-stock policy is optimal as long as the expectation and variance of the net lead time demand, $ND_L = D_L - R_L$, are perfectly estimated (see Silver et al., 1998). If ND_L follows a normal distribution the optimal base stock level is given as

$$S = \mathrm{E}(ND_L) + k \cdot \sqrt{\mathrm{Var}(ND_L)}, \tag{1}$$

with $E(ND_L)$ and $\mathrm{Var}(ND_L)$ the expectation and variance of the net demand during lead time. The safety factor k is determined according to some desired performance level (see e.g. Silver et al., 1998).

In order to estimate $\mathrm{E}(ND_L)$ and $\mathrm{Var}(ND_L)$ we make use of the four methods first put forward by Kelle and Silver (1989). Apart from the expectation and variance of the demand during lead time ($\mathrm{E}(D_L)$ and $\mathrm{Var}(D_L)$ respectively), each method requires a different level of information for estimating the lead time returns. Below, in increasing order of information need, we list the information used by the four methods, denoted A–D.

Method A - Average behavior
This method requires the following information:

- p, the overall return probability, i.e., the probability that a product is being returned eventually.

This method is an approximation in the sense that all the returns during the lead time are assumed to be perfectly correlated with the demand during that same lead time and independent of previous demands. No historical information is used with respect to demands and returns, so in a static environment the resulting base stock level is constant in time.

Method B - Return distribution
Suppose that we are at the end of period t. This method requires information on previous demand per period and the knowledge of the return distribution as follows:

- u_i, purchased amount during period $i \leq t$.
- p_j, the probability of an item being returned exactly after j periods, $j = 1, ..., n$, with n being the largest j for which the return probability is non-zero.

Method C - Return distribution & return information per period
Suppose that we are at the end of period t. In addition to the requirements of method 2, this method makes use of observed data on aggregated returns:

- u_i, purchased amount during period $i, i \leq t$.
- p_j, the probability of an item being returned exactly after j periods, $j = 1, ..., n$, with n being the largest j for which the return probability is non-zero.
- y_i, the total amount of returned products in each period $i, i \leq t$.

This method is an approximation in the sense that the expression of the covariance was not analytically obtained, but instead based on a multidimensional normal vector, as explained in Kelle and Silver (1989). The authors point out that this is a good approximation when the purchased amount is relatively large and the return probabilities are positive for several periods (in practice $n \geq 4$).

Method D - Return distribution & tracked individual returns
Let t be the last observed period. Besides the requirements of method 2 this method requires to track back in what period each individual return has been sold:

- u_i, purchased amount during period $i, i \leq t$.
- p_j, the probability of an item being returned exactly after j periods, $j = 1, ..., n$, with n being the largest j for which the return probability is non-zero.
- Z_i, the observed product returns from each past purchase $u_i, i < t$.

Under perfect information this method makes optimal use of all relevant information.

Summarizing, all four methods make use of the expectation and variance of demand. Additionally, each method has different requirements with respect to product return information. Method A is the least demanding: only an estimate of the return rate is needed. Apart from the return rate, Method B requires the return distribution. On top of that, Method C also needs a record of the aggregated returns per period. Finally, to employ Method D one needs to invest in a system that allows to scan individual returns and track them back to the period in which they were originally sold.

Under perfect information we expect that the method that uses more information outperforms the methods that use less. In the remainder of the paper we investigate how the various methods perform in presence of imperfect information. First, in Section 5, we identify situations in which Method B may outperform the most informed method, Method D. Then in Section 6 we compare Methods A–D with respect to inventory related costs and confirm our findings of Section 5 by means of a simulation study.

5 Forecasting performance

In this section we analyze the relative performance of the exact Methods B and D under misestimation. Methods A and C contain approximations which makes them less interesting for an exact analysis. Besides that, Method A is a rather naive forecasting method, which we do not expect to perform very well in general (this will be confirmed in Section 6) and the performance of Method C tends to be very close to that of Method D (Kelle & Silver, 1989).

To assess the relative performance of Methods B and D it suffices to look at that part of the lead time net demand for which both methods differ, i.e. the expectation and variance due to returns that come from issues before time t, assuming that we are currently at time t (see Appendix 1). We denote these by $E(<t)$ and $V(<t)$ respectively:

$$E(<t)_B = -\textstyle\sum_{i=t-n+1}^{t-1} u_i R_i(p_j' s) \tag{2}$$

$$E(<t)_D = -\textstyle\sum_{i=t-n+1}^{t-1} (u_i - Z_i) Q_i(p_j' s) \tag{3}$$

$$V(<t)_B = \textstyle\sum_{i=t-n+1}^{t-1} u_i R_i(p_j' s)(1 - R_i(p_j' s)) \tag{4}$$

$$V(<t)_D = \textstyle\sum_{i=t-n+1}^{t-1} (u_i - Z_i) Q_i(p_j' s)(1 - Q_i(p_j' s)) \tag{5}$$

Since *under perfect estimation* Method D always leads to the best performance we use this as a benchmark for our study. Under *imperfect* estimation both Methods B and D will do worse than the benchmark, but it is interesting to know whether there are situations in which Method B outperforms Method D. Therefore, with respect to the expected lead time net demand we would like to analyze

$$\left|\widehat{E}(<t)_B - E(<t)_D\right| - \left|\widehat{E}(<t)_D - E(<t)_D\right|.$$

Here '$\widehat{\ }$' denotes a forecast, based on forecasts $\{\widehat{p}_i\}$ of $\{p_i\}$. However, as can be seen from equations (3) and (5), the above expression depends on the observation Z_i of past product returns coming from demand issue u_i. So, to have a meaningful performance measure we take the conditional expectation with respect to the return process given the history of the demand process, $\mathcal{E}_{R|D}$:

$$F_{\{E\}} = \mathcal{E}_{R|D}\left\{\left|\widehat{E}(<t)_B - E(<t)_D\right| - \left|\widehat{E}(<t)_D - E(<t)_D\right|\right\}. \tag{6}$$

Note that if $F_{\{E\}} < 0$ then, on average, Method B outperforms Method D with respect to the expected net lead time demand and *vice versa* if $F_{\{E\}} > 0$. Similarly, the performance measure with respect to the variance of the lead time net demand is

$$F_{\{V\}} = \mathcal{E}_{R|D}\left\{\left|\widehat{V}(<t)_B - V(<t)_D\right| - \left|\widehat{V}(<t)_D - V(<t)_D\right|\right\}. \tag{7}$$

To make the analysis in this section more readable we define

$$\pi_i = \sum_{j=1}^{t-i} p_j, \qquad \alpha_i = \frac{(1-\pi_i)}{(1-\widehat{\pi}_i)}.$$

and we write '$\sum$' for '$\sum_{i=t-n+1}^{t-1}$', 'R_i' for '$R_i(p_j' s)$', and 'Q_i' for '$Q_i(p_j' s)$'.
In section 5.1 we analyze the expectation of the net demand during lead time and in section 5.2 its variance.

5.1 Analysis regarding the expectation of lead time net demand

Define

$$E_{\{BD\}} = \widehat{E}(<t)_B - E(<t)_D = \sum u_i \left(R_i - \widehat{R}_i\right),$$

and

$$E_{\{DD\}} = \widehat{E}(<t)_D - E(<t)_D = \sum (u_i - Z_i) \left(Q_i - \widehat{Q}_i\right)$$

where we have used equations (2) and (3). Obviously, $E_{\{BD\}} = 0 \Rightarrow B \succsim D$ and $E_{\{DD\}} = 0 \Rightarrow D \succsim B$. Conditioning on the sign of $E_{\{BD\}}$ and $E_{\{DD\}}$ we analyze the remaining cases EXP1–EXP4 with respect to the performance measure as defined in (6).

Case EXP1: $E_{\{BD\}} > 0$ and $E_{\{DD\}} > 0$

$$\Longrightarrow \begin{cases} \sum u_i R_i > \max\left\{\sum u_i \widehat{R}_i, \sum (u_i - Z_i)\widehat{Q}_i\right\} \\ F_{\{E\}} = \sum u_i R_i(\widehat{p}_j)\left[\alpha_i - 1\right] \end{cases}$$

where we have used that $\mathcal{E}_{R|D}\{Z_i\} = u_i \pi_i$. It follows immediately that $\forall_i : \alpha_i \leq 1 \Longrightarrow F_{\{E\}} \leq 0$ and thus $B \succsim D$, while $\forall_i : \alpha_i \geq 1$ is not feasible.

Case EXP2: $E_{\{BD\}} < 0$ and $E_{\{DD\}} < 0$

$$\Longrightarrow \begin{cases} \sum u_i R_i < \min\{\sum u_i \widehat{R}_i, \sum (u_i - Z_i)\widehat{Q}_i\} \\ E_3 = \sum u_i \widehat{R}_i \left[1 - \alpha_i\right] \end{cases}$$

It follows immediately that $\forall_i : \alpha_i \geq 1 \Longrightarrow F_{\{E\}} \leq 0$ and thus $B \succsim D$, while $\forall_i : \alpha_i \leq 1$ is not feasible.

Case EXP3: $E_{\{BD\}} < 0$ and $E_{\{DD\}} > 0$

$$\Longrightarrow \begin{cases} \sum u_i \widehat{R}_i < \sum u_i R_i < \sum (u_i - Z_i)\widehat{Q}_i\} \\ F_{\{E\}} = \sum u_i \left[\widehat{R}_i (1 + \alpha_i) - 2R_i\right] \end{cases}$$

If $\forall_i : \alpha_i \geq 1$ then Case EXP3 is not feasible, since $\alpha_i \geq 1 \Rightarrow \widehat{R}_i \geq R_i$.

If $\forall_i : \alpha_i \leq 1$ then $F_{\{E\}} \leq 0$ and thus $B \succsim D$, since $\alpha_i \leq 1 \Rightarrow \widehat{R}_i \leq R_i$ and $1 + \alpha_i \leq 2$.

Case EXP4: $E_{\{BD\}} > 0$ and $E_{\{DD\}} < 0$

$$\Longrightarrow \begin{cases} \sum (u_i - Z_i)\widehat{Q}_i < \sum u_i R_i < \sum u_i \widehat{R}_i\} \\ F_{\{E\}} = \sum u_i \left[2R_i - \widehat{R}_i (1 + \alpha_i)\right] \end{cases}$$

If $\forall_i : \alpha_i \leq 1$ then Case EXP4 is not feasible, since $\alpha_i \leq 1 \Rightarrow R_i \geq \widehat{R}_i$.

If $\forall_i : \alpha_i \geq 1$ then $F_{\{E\}} \leq 0$ and thus $B \succsim D$, since $\alpha_i \geq 1 \Rightarrow \widehat{R}_i \geq R_i$ and $1 + \alpha_i \geq 2$.

We conclude from all cases EXP1–EXP4 that, on average, Method B results in a better estimate of the expected lead time net demand as long as $\forall_i : \alpha_i \leq 1$ or $\forall_i : \alpha_i \geq 1$. A special case that satisfies this condition occurs if all the probabilities are misestimated by the same percentage, i.e. $\forall_j : \widehat{p}_j = ap_j$ for some $a : 0 \leq a \leq 1/p$. This is exactly the case if we under- or overestimate the overall return probability p with factor a, while the shape of the return distribution remains intact. From cases EXP1–EXP4 it is also clear that the difference between the methods increases as the R_i get bigger. In other words, for higher return rates the differences are also higher with respect to the forecasts of the expected lead time net demand.

5.2 Analysis regarding the variance of lead time net demand

The analysis with respect to the variance is similar. Using (4) and (5) we define

$$\begin{aligned} V_{\{BD\}} &= \widehat{V}(<t)_B - V(<t)_D \\ &= \sum \left[u_i \widehat{R}_i(1-\widehat{R}_i) - (u_i - Z_i)Q_i(1-Q_i)\right] \end{aligned}$$

$$\begin{aligned} V_{\{DD\}} &= \widehat{V}(<t)_D - V(<t)_D \\ &= \sum (u_i - Z_i)\left[\widehat{Q}_i(1-\widehat{Q}_i) - Q_i(1-Q_i)\right] \end{aligned}$$

To analyze the performance measure as defined in (7) we consider the following cases.

Case VAR1: $V_{\{BD\}} > 0$ and $V_{\{DD\}} > 0$.

$$\Longrightarrow \begin{cases} \sum (u_i - Z_i)Q_i(1-Q_i) < \min\left\{\sum u_i \widehat{R}_i(1-\widehat{R}_i), \sum (u_i - Z_i)\widehat{Q}_i(1-\widehat{Q}_i)\right\} \\ F_{\{V\}} = \sum u_i \widehat{R}_i \left[(1 - \widehat{R}_i - \alpha_i(1-\widehat{Q}_i)\right] \end{cases}$$

If $\forall_i : \alpha_i \leq 1$ then $F_{\{V\}} \geq 0$ and $D \succsim B$, since $\alpha_i \leq 1 \Rightarrow 1 - \widehat{Q}_i \leq 1 - \widehat{R}_i$.

If $\forall_i : \alpha_i \geq 1$ then we distinguish two cases:
(i) If $\sum (u_i - Z_i)Q_i(1-Q_i) < \sum u_i \widehat{R}_i(1-\widehat{R}_i) < \sum (u_i - Z_i)\widehat{Q}_i(1-\widehat{Q}_i)$ then $V3 < 0$ and $B \succ D$.
(ii) If $\sum (u_i - Z_i)Q_i(1-Q_i) < \sum (u_i - Z_i)\widehat{Q}_i(1-\widehat{Q}_i) < \sum u_i \widehat{R}_i(1-\widehat{R}_i)$ then $V3 > 0$ and $D \succ B$.

Case VAR2: $V_{\{BD\}} < 0$ and $V_{\{DD\}} < 0$.

$$\Longrightarrow \begin{cases} \sum (u_i - Z_i)Q_i(1-Q_i) > \max\left\{\sum u_i \widehat{R}_i(1-\widehat{R}_i), \sum (u_i - Z_i)\widehat{Q}_i(1-\widehat{Q}_i)\right\} \\ F_{\{V\}} = \sum u_i \widehat{R}_i \left[\alpha_i(1-\widehat{Q}_i) - (1-\widehat{R}_i)\right] \end{cases}$$

If $\forall_i : \alpha_i \leq 1$ then $F_{\{V\}} \leq 0$ and $B \succsim D$, since $\alpha_i \leq 1 \Rightarrow 1 - \widehat{Q}_i \leq 1 - \widehat{R}_i$.

If $\forall_i : \alpha_i \geq 1$ then we distinguish two cases:
(i) If $\sum (u_i - Z_i)Q_i(1-Q_i) > \sum (u_i - Z_i)\widehat{Q}_i(1-\widehat{Q}_i) < \sum u_i \widehat{R}_i(1-\widehat{R}_i)$ then $V3 < 0$ and $B \succ D$.
(ii) If $\sum (u_i - Z_i)Q_i(1-Q_i) > \sum u_i \widehat{R}_i(1-\widehat{R}_i) > \sum (u_i - Z_i)\widehat{Q}_i(1-\widehat{Q}_i)$ then $V3 > 0$ and $D \succ B$.

Case VAR3: $V_{\{BD\}} < 0$ and $V_{\{DD\}} > 0$.

$$\Longrightarrow \begin{cases} \sum u_i \widehat{R}_i(1-\widehat{R}_i) < \sum (u_i - Z_i)Q_i(1-Q_i) < \sum (u_i - Z_i)\widehat{Q}_i(1-\widehat{Q}_i) \\ F_{\{V\}} = \sum u_i \left[2R_i(1-Q_i) - \widehat{\alpha}_i \widehat{R}_i(1-\widehat{Q}_i) - \widehat{R}_i(1-\widehat{R}_i)\right] \end{cases}$$

Case VAR4: $V_{\{BD\}} > 0$ and $V_{\{DD\}} < 0$.

$$\Longrightarrow \begin{cases} \sum(u_i - Z_i)\widehat{Q}_i(1-\widehat{Q}_i) < \sum(u_i - Z_i)Q_i(1-Q_i) < \sum u_i\widehat{R}_i(1-\widehat{R}_i) \\ F_{\{V\}} = \sum u_i \left[\alpha_i\widehat{R}_i(1-\widehat{Q}_i) + \widehat{R}_i(1-\widehat{R}_i) - 2R_i(1-Q_i)\right] \end{cases}$$

From the above analysis we conclude that with respect to the variance there are situations for which Method B outperforms D, but also situations for which D outperforms B. We can enhance the analysis by looking again at the special case that we misestimate the overall return probability, i.e. $\widehat{p} = ap$ for some $a : 0 \leq a \leq 1/p$, while the shape of the return distribution remains intact. Then we can write $\widehat{R}_i = aR_i$ and $\widehat{Q}_i = aR_i/(1-a\pi_i)$. Still it is difficult to analyze $F_{\{V\}}$ as a whole, because of the summations. So, instead we analyze the individual *coefficients* of u_i in $V_{\{BD\}}$, $V_{\{DD\}}$,and $F_{\{V\}}$, denoted $\nu_{\{BD\},i}$, $\nu_{\{DD\},i}$, and ϕ_i, for a specific value of Z_i, say $Z_i = \beta_i u_i$, $0 \leq \beta \leq 1$:

$$\nu_{\{BD\},i} = [a(1-aR_i) - (1-R_i/(1-\pi_i))(1-\beta_i)/(1-\pi_i)]\, R_i$$

$$\nu_{\{DD\},i} = [a(1-aR_i/(1-a\pi_i))/(1-a\pi_i) - (1-R_i/(1-\pi_i))/(1-\pi_i)]\, R_i(1-\beta_i)$$

$$\phi_i = \begin{cases} \phi_{1,i} = & aR_i\,[(1-aR_i) - (1-aR_i/(1-a\pi_i))(1-\pi_i)/(1-a\pi_i)] & \text{(VAR1)} \\ -\phi_{1,i} = & aR_i\,[(1-aR_i/(1-a\pi_i))(1-\pi_i)/(1-a\pi_i) - (1-aR_i)] & \text{(VAR2)} \\ \phi_{2,i} = & R_i\,[2(1-R_i/(1-\pi_i)) - a(1-aR_i/(1-a\pi_i))(1-\pi_i)/(1-a\pi_i) - a(1-aR_i)] & \text{(VAR3)} \\ -\phi_{2,i} = & R_i\,[a(1-aR_i/(1-a\pi_i))(1-\pi_i)/(1-a\pi_i) + a(1-aR_i) - 2(1-R_i/(1-\pi_i))] & \text{(VAR4)} \end{cases}$$

Figure 1 gives an example of how the preference regions are constructed by $\nu_{\{BD\},i}$, $\nu_{\{DD\},i}$, and ϕ_i in terms of a and R_i. Note that the boundary between the preference regions of Method B and D only depends on ϕ_i, which does not depend on β_i. Thus, the analysis is independent of a particular realization of observed returns.

Not all the combinations of R_i and a in Figure 1 are feasible, since $R_i \leq p - \pi_i$ and $a \leq 1/p$. Figures 2a-f depict the preference regions for the feasible area only and for various values of p and π_i. From these figures it appears that Method B performs better as R_i gets smaller, π_i gets larger or p gets smaller, particularly if $a < 1$. Note that the R_i tend to be small if the base of the time-to-return distribution, n, is large compared to the lead time L. At this time we would like to stress that the analysis of this section does not depend on time t, since equations (2)–(7) do not depend on t, nor did we make any assumptions on realizations of the demand process $\{u_i\}$ and observed returns $\{Z_i\}$.

Insert Figure 2 about here

Summarizing, we conclude that it is not at all obvious that Method D, which is the most informed method, performs better than Method B. In fact, we have identified situations in which Method B performs better, on average, with respect to the expectation and variance of lead time net demand. In particular when all return probabilities are underestimated or all return probabilities are overestimated Method B has opportunities to outperform Method D. In the next section we will quantify the impact of misestimation with respect to costs.

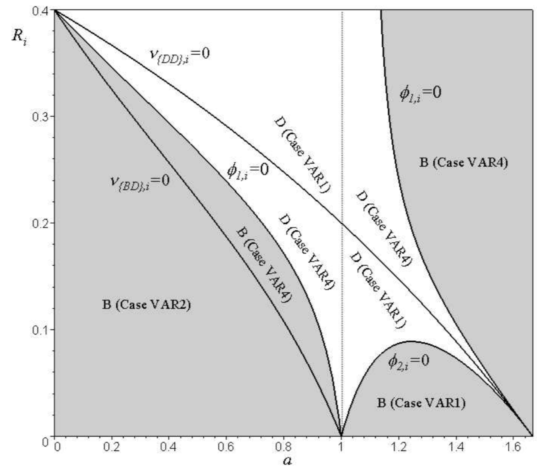

Figure 1: Preference regions in terms of a and R_i; $\pi_i = \beta_i = 0.6$, $p = 0.7$.

6 Cost performance

6.1 Experimental Design

In order to quantify the impact of misinformation on inventory cost performance we conducted a simulation study. The experiments are based on the inventory system that was introduced in Section 4 and are conducted in the following manner. Each period t we draw the cumulative demand D_t from a normal distribution with mean μ_D and variance σ_D^2 (values are rounded to integers; negative numbers are treated as zero). For each individual item of this cumulative demand we determine the time to return based on the pre-specified return probabilities, $\{p_j\}$. Each period, estimates of the expectation and variance of the net demand during lead time are computed according to one of methods A–D. These estimates are subsequently used to compute the base stock level, S. At the end of each period, overstocks are charged with a holding cost \$ h per item, per period, whereas stockouts are penalized with \$ b per occurrence. At the end of each simulation experiment we calculate the total average cost per period as the total average holding plus backorder costs per period. Note that all methods use *estimates,* $\{\widehat{p}_j\}$, of the real return probabilities, $\{p_j\}$, since the latter are not known. The same holds for the overall return probability, p, which is estimated as $\widehat{p}$.

Each simulation experiment consists of at least ten simulation runs of 5.000 periods, preceded by a warm-up run of the same length. The simulation stops as soon as the relative error in the total average costs is less than 1%. In order to make a better comparison among simulation experiments we make use of common random numbers.

Based on the estimates $E(ND_L)$ and $V(ND_L)$ of the mean and variance of net lead time demand the base stock level S is computed as in (1). Assuming that the net demand during lead time is normally distributed, the value of the safety factor, k, can be chosen such that total costs are optimized:

$$G(k^*) = 1 - \frac{h}{b},$$

where $G(.)$ is the standard normal distribution.

As a base case scenario for our simulation experiments we use the parameter set as given in Table 1. We assume that *if* an item returns, the time-to-return follows a geometric distribution with conditional expected return time 1/q. So, the return probabilities are given as

$$p_j = pq(1-q)^{j-1}, \quad j = 1, 2, ..\infty.$$

All parameters are assumed to be constant over time.

μ_D	σ_D^2	L	p	q	h	b
30	36	4	0.5	0.6	1	50

Table 1 Base case scenario

6.2 Misinformation

In case of perfect information, i.e. $\hat{p} = p$ and $p_j = p_j$, method D will outperform all other methods since it is using all of the available information in a correct way. In order to investigate the effect of misinformation, we consider two types of errors in the parameter estimates. The first is a misestimation of the overall return probability, p, while the shape of the real distribution is preserved. The second is a misestimation of the conditional expected time-to-return, $1/q$. This affects the shape of the time-to-return distribution, but the estimated overall return probability is kept equal to the real return probability. For example, suppose that the real time to return distribution is given by $\{p_1, p_2, p_3\} = \{0.2, 0.4, 0.1\}$. Then an estimate of $\{\widehat{p}_1, \widehat{p}_2, \widehat{p}_3\} = \{0.1, 0.2, 0.05\}$ would have the same shape, but a 50% lower estimated return probability. An estimate of $\{\widehat{p}_1, \widehat{p}_2, \widehat{p}_3\} = \{0.3, 0.2, 0.2\}$ would have the same overall return rate (80%), but a different *shape* of the time to return distribution.

6.3 Numerical results

Comparing the four methods in case of perfect information (Table 2) we observe here that method D is indeed superior to the other methods, although the differences with respect to methods B and C are rather small for $p \leq 0.8$ (less than 1 percent).

The performance of Method A is extremely poor. It uses the assumption that all lead time returns are correlated with the lead time demands. This causes a systematic underestimation of the variance in the lead time net demand, especially for high return rates and large lead times.

	A		**B**		**C**		**D**
	abs.	rel.	abs.	rel.	abs.	rel.	abs.
base case:							
$p = 0.5, b = 50, L = 4$	32.57	24.9	26.11	0.2	26.06	-0.0	26.07
$\mathbf{p = 0.8}, b = 50, L = 4$	48.39	111.8	23.06	0.9	22.95	0.4	22.85
$\mathbf{p = 0.9}, b = 50, L = 4$	48.71	129.3	21.90	3.1	21.37	0.6	21.24
$p = 0.5, \mathbf{b = 10}, L = 4$	21.30	11.0	19.22	0.1	19.20	0.2	19.18
$p = 0.5, \mathbf{b = 100}, L = 4$	39.21	36.7	28.72	0.1	28.62	-0.2	28.69
$p = 0.5, b = 50, \mathbf{L = 8}$	36.41	10.6	33.00	0.2	32.96	0.1	32.94
$p = 0.5, b = 50, \mathbf{L = 16}$	45.14	4.1	43.42	0.1	43.37	0.0	43.37

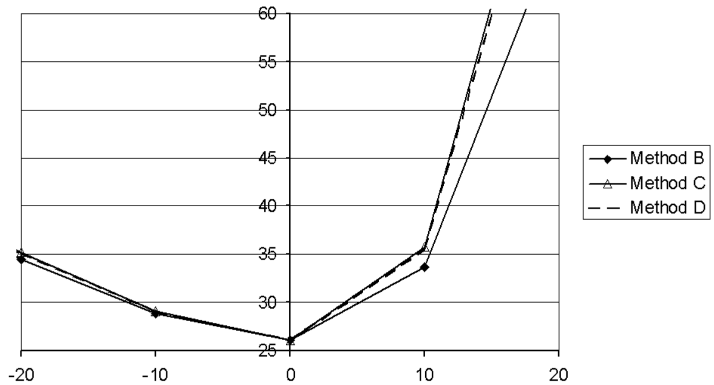

Figure 3 Misestimation (-30%,+30%) of the return probability for $p = 0.8$.

Table 2 Comparison of Methods A–D in case of perfect information.

6.3.1 Misestimation of the overall return probability

Figure 3 shows that a misestimation of 10% leads to a cost increase of 10–30%. A 20% misestimation may even lead to a cost increase of more than 200 % relative to the perfect information case. Clearly, an overestimation of the return probability is far worse than an underestimation, since overestimation leads to costly stockouts.

Table 3 shows that Method B structurally outperforms the other methods in case of misestimation of 10% or more, whereas Method C always performs worse. The differences between the methods become bigger as the return rate goes up or the lead time goes down. The analysis of Section 5 showed that in case of misestimation of the return rate Method B is better than Method D with respect to the expected lead time net demand, while the difference between the methods increases as the return rate increases. With respect to the variance the analysis was less straightforward. Sometimes Method B outperforms Method D and sometimes the other way around. From the numerical results we conclude that the effect of the expected lead time net demand dominates the effect of the variance.

		B	**C**	**D**			**B**	**C**	**D**
	error	rel.	rel.	abs.		error	rel.	rel.	abs.
base case:	−20%	−1.9	0.3	35.15	$p = 0.5$,	−20%	−1.7	0.3	38.11
$p = 0.5$,	−10%	−0.9	0.2	29.07	$\mathbf{b = 100}$,	−10%	−0.8	0.2	31.79
$b = 50$,	+10%	−5.2	0.7	35.54	$L = 4$	+10%	−5.7	0.8	40.11
$L = 4$	+20%	−17.9	2.4	84.33		+20%	−19.8	2.9	105.85
$\mathbf{p = 0.8}$,	−20%	−5.3	1.2	42.45	$p = 0.5$,	−20%	−0.8	0.1	87.60
$b = 50$,	−10%	−3.9	1.3	31.18	$b = 50$,	−10%	−0.7	0.1	61.95
$L = 4$	+10%	−43.1	8.5	92.26	$\mathbf{L = 16}$	+10%	−5.1	0.5	170.99
	+20%	−67.1	6.6	692.04		+20%	−6.9	0.6	902.68

Table 3 Comparison of Methods B–D in case of misestimation of the return probability.

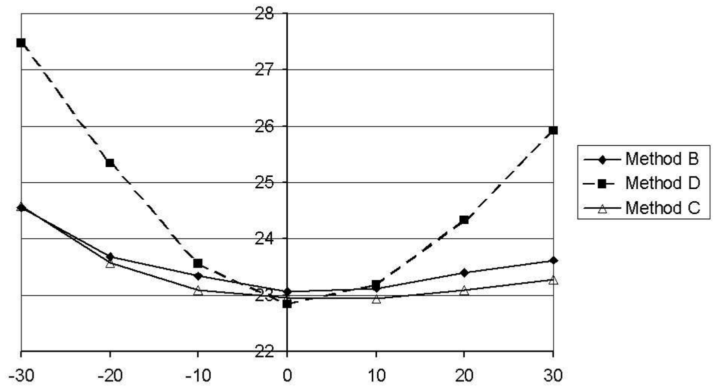

Figure 4 Misestimation (-30%,+30%) of the shape of the time-to-return distribution for $p = 0.8$.

6.3.2 Misestimation of the conditional expected time-to-return

According to Table 4, misestimation of the conditional expected time-to-return has little effect if return rates are small. For $p = 0.8$ though (Figure 4), Methods B and C are far more robust with respect to misspecification than the benchmark, Method D. Again the difference in performance with respect to the benchmark is positively correlated with the return rate and negatively correlated with the lead time.

	error	B rel.	C rel.	D abs.		error	B rel.	C rel.	D abs.
base case:	-20%	-0.4	-0.6	26.46	$p = 0.5$,	-20%	-0.6	-0.8	29.13
$p = 0.5$,	-10%	-0.1	-0.3	26.20	$\mathbf{b = 100}$,	-10%	0.0	-0.3	28.76
$b = 50$,	$+10\%$	0.1	0.0	26.05	$L = 4$	$+10\%$	0.1	-0.1	28.70
$L = 4$	$+20\%$	-0.5	-0.2	26.13		$+20\%$	0.2	0.1	28.71
$\mathbf{p = 0.8}$,	-20%	-6.5	-7.0	25.34	$p = 0.5$,	-20%	-0.2	-0.2	43.64
$b = 50$,	-10%	-0.9	-2.0	23.56	$b = 50$,	-10%	0.0	-0.1	43.46
$L = 4$	$+10\%$	-0.2	-1.1	24.33	$\mathbf{L = 16}$	$+10\%$	0.1	0.0	43.35
	$+20\%$	-3.8	-5.1	24.33		$+20\%$	-0.1	-0.2	43.43

Table 4 Comparison of Methods B–D in case of misestimation of the expected time-to-return.

7 Discussion and managerial implications

Under perfect information, Method A performs in general very poorly and is not recommended for practical implementation. Thus, to *only* have knowledge on average behavior does not seem to be cost effective. Including information on the return distribution, however, does seem to provide a sufficient level of sophistication, as the performance of Method B shows. With respect to inventory related costs, the differences between Methods B, C and D are not such that they justify investments in recording detailed return data.

Both for misinformation on the return rate and on the return distribution, the differences between the methods become smaller with the decrease of the return probability and the increase of the lead time. Obviously, if there are few returns forecasting of the lead time returns is not really an issue. To understand the latter one should note that when L is large, most of the items that return during the lead time were also purchased during the lead time. The forecast of this type of returns is not based on historical data, so all methods B–D give exactly the same forecast.

With respect to misestimating the return rate, in general it is better to underestimate the return rate than to overestimate, since stockouts are usually much more costly than overstocks. Therefore, if an interval estimate of the return rate is available, one may opt to use a value that is closer to the lowerbound rather than the upperbound.

The most robust method under misestimation of the return rate is Method B. Method B systematically outperforms Methods C and D if the return rate is misestimated by merely 10%,or more. The cost differences are particularly high if return rates are overestimated. With respect to misestimating the conditional time to return distribution, Method C and again Method B are much more robust than Method D.

The above results strongly suggest that Method B has a sufficient level of sophistication both in case of perfect estimation and imperfect estimation. In the latter case Method B is far more robust than the most informed method, Method D. For our mail order company of Section 3 this means that orders only have to be based on the return distribution and realized demand per period, but there is no need to track individual returns.

8 Summary of conclusions and further research

In this paper we have investigated the impact of (mis)information on forecasting performance and performance with respect to inventory costs by analyzing four forecasting methods as proposed by Kelle and Silver (1989). All methods make use of the expectation and variance of the demand, but different levels of information with respect to returns. The least demanding method, Method A only uses the return rate. Method B also requires the return distribution. Method C additionally uses a periodic record of returns. Finally, Method D needs to track back the period in which each individual product return was sold.

Under perfect information, forecasting performance increases as the level of information increases, Method D naturally being the best method. Yet, from the analysis we have concluded that Method B presents a reasonable level of sophistication under perfect information and is exceptionally robust under misinformation in comparison with the other methods. Method D does not appear to be very robust under misestimation. This leads to the conclusion that companies are not likely to recover investments on advanced return data with inventory savings, especially in volatile environments. Naturally, it is worth to further investigate the value of return information, with respect to multiple criteria, like for instance production scheduling and human resource management.

The huge gap in performance between Methods A and B suggest that refinements of Method A are possible. We believe that a promising research direction is the search for simple methods with low information requirements but a reasonable performance.

In the analysis of misinformation we considered a static situation where a systematic error was introduced. This is natural for small errors, since it is difficult in general to reason whether observations indicate trend changes or mere stochastic behavior. However, large variations could be dealt with by an adaptive method. This is a topic for further research.

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Appendix 1: Forecasting methods

For more details and demonstrations we refer to Kelle and Silver (1989). Methods A and B are respectively Methods 1 and 2 in Kelle and Silver (1989), while Method C and D are respectively methods 4 and 3. Please consider the notation as defined in Appendix 2.

Method A - Average behavior

The expected lead time net demand, forecasted according to method A is

$$E_A(ND) = \quad (1-p)E(D_L) \tag{8}$$

and the respective forecasted variance is

$$V_A(ND) = \quad (1-p)V(D_L) + p(1-p)E(D_L) \tag{9}$$

Method B - Return distribution
Suppose that we are at the end of the period t.
The expected lead time net demand, forecasted according to method B is

$$E_B(ND) = \quad \sum_{i=t+1}^{t+L} \mu_i - E_B(R) \tag{10}$$

with $E_B(R)$, the total amount returned during the lead time, defined by

$$E_B(R) = \quad \sum_{i=t-n+1}^{t} u_i R_i(p_{j's}) + \sum_{i=t+1}^{t+L-1} \mu_i R_i(p_{j's}) \tag{11}$$

with μ_i standing for the average demand in period i.
The forecasted variance according to method B is

$$V_B(R) = \quad \sigma^2_{t+L} + V_B(R_{prev}) \tag{12}$$

with σ_i^2 standing for the variance of the demand in period i, and $V_B(R_{prev})$, the total amount returned from previous issues during the lead time, defined by

$$V_B(R_{prev}) = \sum_{i=i_m}^{t} u_i R_i(p_{j's}) \left[1 - R_i(p_{j's})\right] \tag{13}$$

with

$$i_m = \max\{1, t-n+1\} \tag{14}$$

$R_i(p_{j's})$ is the success probability of having units from issue $u_i$returning during any of the the lead time periods $i = t+1, ..., t+L$, and it is defined as follows.

$$\begin{cases} R_i(p_{j's}) = 0, & \text{for } i < t-n+1 \\ R_i(p_{j's}) = \sum_{j=1}^{j_m} p_{t-i+j}, & \text{for } i_m \leq i \leq t \\ R_i(p_{j's}) = \sum_{j=1}^{j_n} p_j, & \text{for } t < i < t+L \end{cases}$$

where

$$\begin{aligned} j_m &= \min\{L, n+i-t\} \\ j_n &= \min\{n, t-i+L\}. \end{aligned}$$

Method C - Return distribution & return information per period

Suppose that we are at the end of the period t. Let W_i be the amount returned during the lead time period $i = t+1, ..., t-L$ and $\underline{c}$ a vector of covariances defined for $j = 1, ..., n-2$ by,

$$\begin{aligned} c_j &= Cov\left(\sum_{i=i_m}^{t-1} W_i y_{t-j+1}\right) \\ &= -\sum_{i=i_m}^{t-j} u_{t-n+i} p_{n-j+1-i} \sum_{m=1}^{j_m} p_{n-i+m} \end{aligned} \tag{15}$$

with $i_m = \max\{1, t-n+1\}$ and $j_m = \min\{i, L\}$

The expected lead time net demand, forecasted according to method C is

$$\begin{aligned} E_C(ND) &= \sum_{i=t+1}^{t+L} \mu_i - E_C(R) \\ &= E_B(ND) - \underline{c}T^{-1}(\underline{y} - E(\underline{y})) \end{aligned} \tag{16}$$

with $\underline{y} = (y_t, y_{t-1}, ..., y_{t-n+2})$ being the vector of recent aggregated returns, T the covariance matrix of vector $\underline{y}$ and $T^{-1}$the respective inverse matrix.The elements of matrix, $T_{j,k} = cov(y_{t-j+1}, y_{t-k+1})$ are defined as follows:

$$\begin{cases} T_{j,k} = -\sum_{j=i_j}^{t-k} p_{t-j+1-i}\ p_{t-k+1-i}, & \text{for } j = 1, ..., n-2,\ j \leq k \leq n-2 \\ T_{k,j} = T_{j,k}, & \text{for } k < j \\ T_{j,j} = \sum_{j=i_j}^{t-k} p_{t-j+1-i}\ (1 - p_{t-j+1-i})\ , & \text{for } j = 1, ..., n-2 \end{cases}$$

with $i_j = \max\{1, t-j+1-n\}$.
The forecasted variance in conformance with Method C is

$$V_C(ND) = V_B(R) - \underline{c}T^{-1}\underline{c}^{'} \tag{17}$$

Method D - Return distribution & tracked individual returns

The expected lead time net demand, forecasted according to method D is

$$E_D(ND) = \sum_{i=t+1}^{t+L} \mu_i - \left[\sum_{i=i_m}^{t-1} (u_i - V_i) Q_i(p_{j's}) + u_t R_t(p_{j's}) + \sum_{i=t+1}^{t+L-1} \mu_i R_i(p_{j's}) \right] \tag{18}$$

with $Q_i(p_{j's})$ being the success probability associated to the Binomial conditional random variable W_i given the observation of V_i, and defined as follows.

$$Q_i(p_{j's}) = \frac{R_i(p_{j's})}{1 - \sum_{j=1}^{t-i} p_j} \tag{19}$$

The forecasted variance according to method D is

$$\begin{aligned} V_D(ND) = \quad & \sigma_{t+L}^2 + \sum_{i=i_m}^{t-1} (u_i - V_i) Q_i(p_{j's}) \left[1 - Q_i(p_{j's})\right] \\ & + u_t R_t(p_{j's}) \left[1 - R_t(p_{j's})\right] \\ & + \sum_{i=t+1}^{t+L-1} \left\{ \sigma_i^2 \left[1 - R_t(p_{j's})\right] + \mu_i R_i(p_{j's}) \left[1 - R_i(p_{j's})\right] \right\} \end{aligned} \tag{20}$$

Appendix 2: Basic notation

b,	penalty per time unit and per item backordered.
D_L,	random variable representing (r.v.r.) the demand during the lead time L.
$E(.)$,	expected value of a random variable.
$E_m(.)$,	expected value, $E(.)$, estimated according to forecasting method $m = A, B, C, D$.
h,	holding cost per item and per time unit.
k,	safety factor, which is determined such that total average costs are minimised.
L,	fixed amount of time, between an order takes place and it is put in inventory.
ND_L,	r.v.r. the net demand during the lead time, equal to$D_L - R_L$.
p,	the probability of an item ever being returned, i.e. $\sum_{j=1}^{n} p_j$.
p_j,	the probability of an item being returned after exactly j periods, $j = 1, ..., n$ with n being the largest j for which the return probability is larger than zero.
R_L,	random variable representing the return during the lead time L.
s,	re-order point associated with a base-stock policy.
t,	the current period.
u_i,	purchased amount during period $i \in N$ and $i \leq t$.
$V(.)$,	variance of a random variable.
$V_m(.)$,	forecast of a variance, $V(.)$, according to forecasting method $m =\ A, B, C, D$.
W_i,	r.v.r. the returned amount during the lead time period $i = t + 1, ..., t + L$.
y_k,	the total amount returned in each previous period $k \leq t$.
μ_i,	demand expected value for period $i \in IN$
σ_i^2,	demand variance for period $i \in IN$
Z_i,	the returned amount from each past $u_{i,}$until period t (inclusive).

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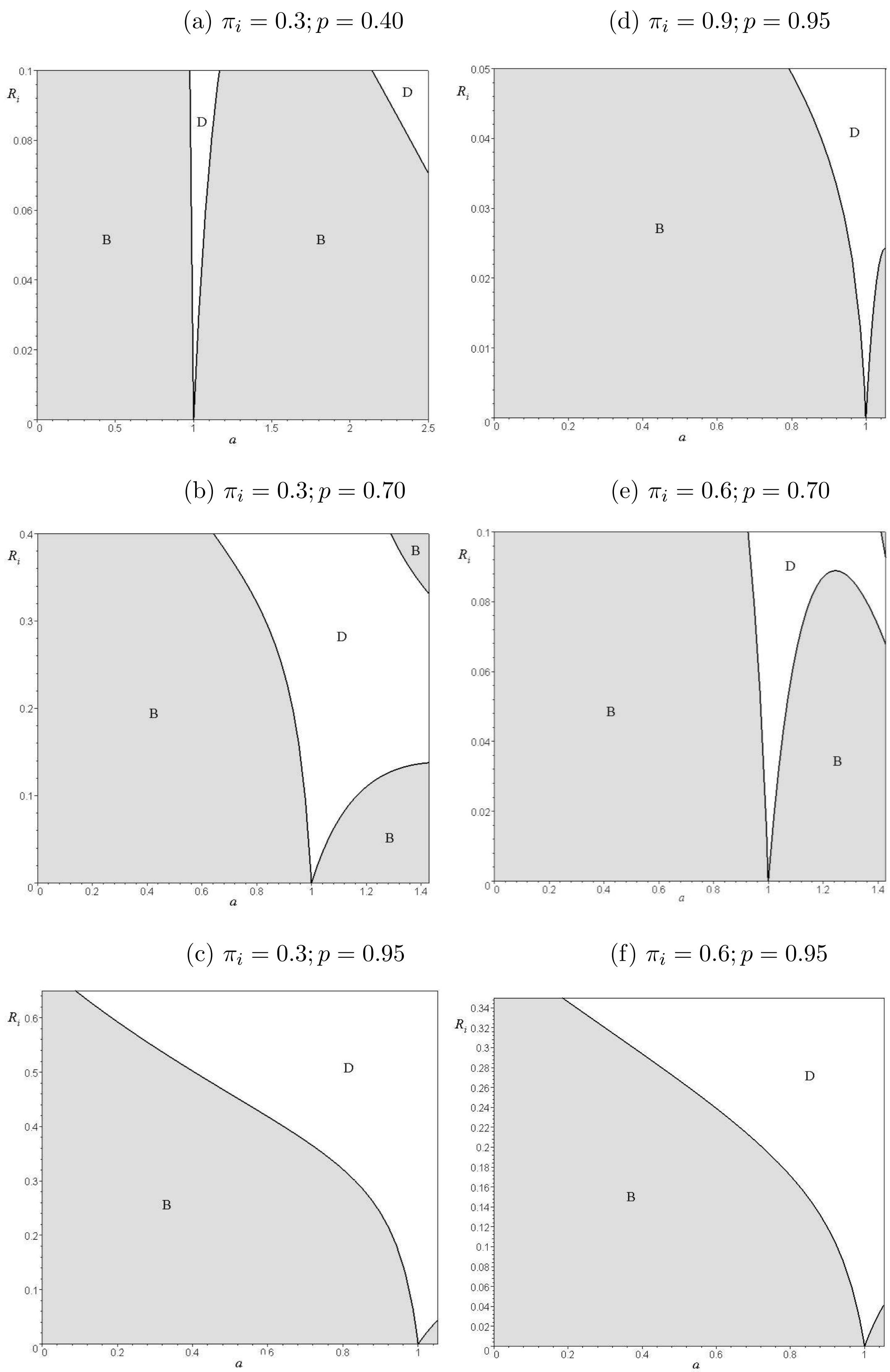

Figure 2: Preference regions in terms of a and R_i for various values of π_i and p.