Testing Changing Harmonic Regressors

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Econometric Institute Report 2009-13

Abstract

Econometric models for economic time series may include harmonic regressors to describe cyclical patterns in the data. This paper focuses on the possibility that the cycle periods in these regressors change over time. To this end, a smooth regime-switching harmonic regression is proposed, and a diagnostic test for changing cycle periods is proposed. An application to annual GDP growth in the Netherlands (for 1969-2007) shows that around 1975 the business cycle period shifted from about 3 years to about 11 years.

Key words: Harmonic regressors, smooth regime-switching model

June 26 2009

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1. Introduction and motivation

Many macroeconomic time series variables, like GDP, unemployment and inflation, experience cycles. The literature on macroeconomic fluctuations assumes two views on such cycles. The first is that cycles are governed purely by shocks, and, as a consequence, the best model to capture cycles is an autoregression with at least two complex roots. The second view is that part of the cyclical behaviour is deterministic. In this latter case, econometric models can include the harmonic regressors

$$\cos(\frac{2\pi t}{\kappa}) \text{ and } \sin(\frac{2\pi t}{\kappa}), \text{ where } t = 1,2,\ldots, n,$$

with n is the number of observations and κ is the period of the cycle. A potential beneficial consequence of including harmonic regressors is that long-run forecasts can be meaningful. Indeed, autoregressive time series models give forecasts that in the end converge to the unconditional mean, while models with deterministic cycles generate forecasts with such cycles.

Insert Table 1 about here

Insert Figure 1 about here

In practice it can occur that the key parameter κ is not constant over time. It may well be that outside shocks make the cyclical pattern in economic data to change over time. For example, Figure 1 displays the annual growth in Gross Domestic Product (GDP) in the Netherlands from 1969-2007 (as available in October 2008). The actual data are given in Table 1. Clearly, the data show cyclical patterns, which may be associated with harmonic regressors. What is also clear from Figure 1 is that prior to 1980 the data show a cyclical pattern with a potentially smaller period than that of a cycle after 1980.

In Section 2 I propose a smooth regime-switching harmonic regression model. This model allows for changes in the period of the cycles. The model implies a simple diagnostic tool to see if such changes occur indeed. In Section 3, an illustration to the Netherlands GDP data shows that the business cycle from 1969 to 1975 had a period of 3 years, while from 1975 to 2007 that period became 11 years. Neglecting this change would lead to an estimated cycle period of 12 years. The final section suggests a few areas for further research.

2. A smooth regime-switching harmonic regression

Consider the basic harmonic regression model

$$(1) \qquad y_t = \mu + \alpha_1^* \cos(\frac{2\pi t}{\kappa}) + \alpha_2^* \sin(\frac{2\pi t}{\kappa}) + u_t$$

which can also be written as

$$(2) \qquad y_t = \mu + \alpha_1 \cos(\frac{2\pi t}{\kappa} - \alpha_2) + u_t ,$$

where α_1^* and α_2^* are functions of α_1 and α_2, and u_t is an error term. To save notation, assume that α_2 is 0, which can be done without loss of generality.

To allow for a smooth transition from a cycle with a period κ_1 to that with a period κ_2, a smooth function is introduced. This reads as,

$$(3) \qquad F(t;\tau) = \frac{1}{1 + \exp(-\gamma(t - \tau))}$$

The parameter γ measures the steepness of the transition from 0 to 1, while τ is the threshold value. Figure 2 gives a graphical example of this logistic function, which is frequently used to test for changing parameters, see van Lundbergh et al. (2003).

Insert Figure 2 about here

There are now various ways to include (3) in a version of (2), but only the last one below will lead to a simple testing methodology. A first possibility is to change (2) into

(4) $$y_t = \mu + \alpha_1 \cos(\frac{2\pi t}{\kappa_1 + \kappa_2 F(t;\theta)}) + u_t$$

This model is however not easy to linearize, which is the usual starting point for deriving a test method; see Granger and Teräsvirta (1993). Another option is to consider

(5) $$y_t = \mu + \alpha_1 \cos[(1 - F(t;\tau))\kappa_1^* t + F(t;\tau)\kappa_2^* t] + u_t$$

where κ_1^* is short hand for $\frac{2\pi}{\kappa_1}$ and κ_2^* for $\frac{2\pi}{\kappa_2}$. Here the same problems as for (4) occur. Therefore, a better idea is to consider

(6) $$y_t = \mu + \alpha_1[(1 - F(t;\tau))\cos\kappa_1^* t + F(t;\tau)\cos\kappa_2^* t] + u_t$$

Note that (6) can be written as

(7) $$y_t = \mu + \alpha_1[\cos\kappa_1^* t - F(t;\tau)\cos\kappa_1^* t + F(t;\tau)\cos\kappa_2^* t] + u_t$$

which means that (7) nests (2). Linearizing the *F* function (up to 3-th order terms and replacing the parameters by their estimates), one gets as the null model (after including the parameter estimates)

(8) $$y_t = \mu + \alpha_1 \cos\hat{\kappa}_1^* t + u_t$$

and as the alternative

(9) $$y_t = \mu + \alpha_1 \cos \hat{\kappa}_1^* t + \beta_1 t \cos \hat{\kappa}_1^* t + \beta_2 t^2 \cos \hat{\kappa}_1^* t + \beta_3 t^3 \cos \hat{\kappa}_1^* t + \gamma_1 t \cos \kappa_2^* t + \gamma_2 t^2 \cos \kappa_2^* t + \gamma_3 t^3 \cos \kappa_2^* t + u_t$$

Of course, the value of κ_2^* is not known, so one can run the regression in (9) for a range of κ_2^* values and examine which gives the maximum fit. In the next section I will illustrate this method for GDP in the Netherlands.

3. Annual GDP growth in the Netherlands

The data in Table 1 are log-transformed and first differenced to arrive at the growth rates thus defined as

(10) $$y_t = \log GDP_t - \log GDP_{t-1}$$

A suitable model for these annual data, running from 1970 to 2007 appears to be

(11) $$y_t = \hat{\mu} + \hat{\alpha}_1 \cos(\frac{2\pi t}{\hat{\kappa}} - \hat{\alpha}_2)$$

with (standard errors in parentheses)

$\hat{\mu} = 0.027\ (0.002)$,

$\hat{\alpha}_1 = -0.010\ (0.004)$,

$\hat{\alpha}_2 = -0.002\ (0.007)$, and

$\hat{\kappa} = 12.124\ (0.689)$.

Diagnostic tests on the estimated residuals do not suggest misspecification. The Wald test value for the null hypothesis $\alpha_1^* = \alpha_2^* = 0$ (hence, when (11) is written in the format of (1)) gets a value of 8.202. Comparing this value with the fractiles in Table 2, this test is significant at 10%.

Insert Table 2 about here

Insert Table 3 about here

The next step is to run the test regression in (9) for a few values of κ_2^*. Table 3 gives the R^2 values for various regressions, and clearly the R^2 in case κ_2 = 5 is highest. Given the visible nature of the data in Figure 1, I only look at cycles with periods smaller than about 12.

The next step involves estimating the parameters in model (6), that is,

$$(12) \qquad y_t = \mu + \alpha_1[(1-F(t;\tau))\cos(\frac{2\pi t}{\kappa_1} - \alpha_2) + F(t;\tau)\cos(\frac{2\pi t}{\kappa_2} - \alpha_3)] + u_t$$

When the starting value of κ_2 is set at 5, and that of κ_1 at 12, the following estimation results are obtained:

$\hat{\mu} = 0.027\ (0.002)$,

$\hat{\alpha}_1 = 0.025\ (0.009)$,

$\hat{\alpha}_2 = 0.005\ (0.029)$,

$\hat{\kappa}_1 = 3.139\ (0.327)$,

$\hat{\alpha}_3 = -0.013\ (0.005)$

$\hat{\kappa}_2 = 10.924\ (0.458)$

The parameters in the *F* function are estimated as

$\hat{\gamma} = 27.273$

$\hat{\tau} = 6.530$

where the standard errors are deleted because they are very large (which means that the transition from one regime to the other is very steep)

Insert Figure 3 about here

The estimated value for τ implies that the point of inflection is around 1975/1976. In Figure 3 the graph of the estimated *F* function is displayed.

Insert Figure 4 about here

The fit of model (12) is displayed in Figure 4. Evidently, there is a cycle with period around 3 years before 1975 and a cycle with period around 11 years after 1976. The Wald test for the null hypothesis $\alpha_1^* = \alpha_2^* = 0$ is 9.836 for the first cycle, while for the second cycle it is 19.017. Comparing these with the fractiles in Table 2 shows strong evidence for these two cycles.

4. Conclusion

In this paper I proposed a nonlinear harmonic regression model which allows the cycle period to change over time. Of the various possible representations of the model, only one representation led to a simple test method that can be easily used in practice. An illustration of the method to GDP data for the Netherlands showed the merits of the method.

Further research can cover the detection of changing cycle periods in case an economic time series experiences more than one deterministic cycle. Second, it would be interesting to know what happens if changes in cycle periods are ignored and harmonic regressors with a constant cycle period are included.

Table 1:
Annual real GDP in the Netherlands 1967-2007 (as available in October 2008)
(in millions of euros, price level 2000)

Year	GDP	Year	GDP
1969	170054	1988	281312
1970	180568	1989	293746
1971	188388	1990	306034
1972	193110	1991	313499
1973	204260	1992	318847
1974	212974	1993	322857
1975	213236	1994	332417
1976	223445	1995	342776
1977	227738	1996	354452
1978	233046	1997	369617
1979	237758	1998	384119
1980	242984	1999	402113
1981	241931	2000	417960
1982	238996	2001	426009
1983	243782	2002	426334
1984	252216	2003	427765
1985	257936	2004	437332
1986	266496	2005	446282
1987	271953	2006	461349
		2007	477315

Source: Statistics Netherlands (Statline)

Table 2: Critical values for the Wald test α_1=α_2=0 in the auxiliary regression

$$y_t = \mu + \alpha_1 \cos(\frac{2\pi t}{\kappa}) + \alpha_2 \sin(\frac{2\pi t}{\kappa}) + u_t .$$

Size	Critical value
1%	12.17
2.5%	10.27
5%	8.90
10%	7.29
20%	5.62

Source: Table 4 in Franses, De Groot and Legerstee (2009).

Table 3: R^2 of regression (9)

κ_2	R^2
5	0.545
6	0.471
7	0.457
8	0.514
9	0.524
10	0.468

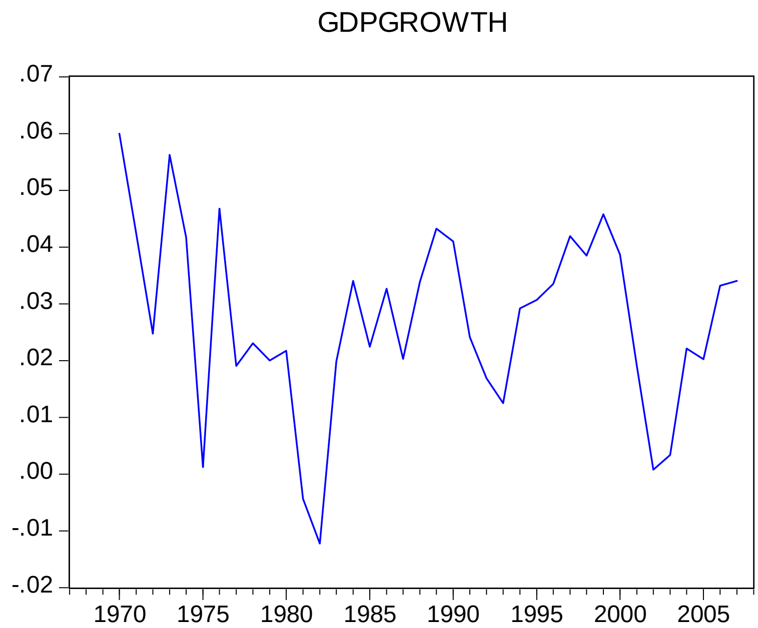

Figure 1: Annual growth in real GDP in the Netherlands

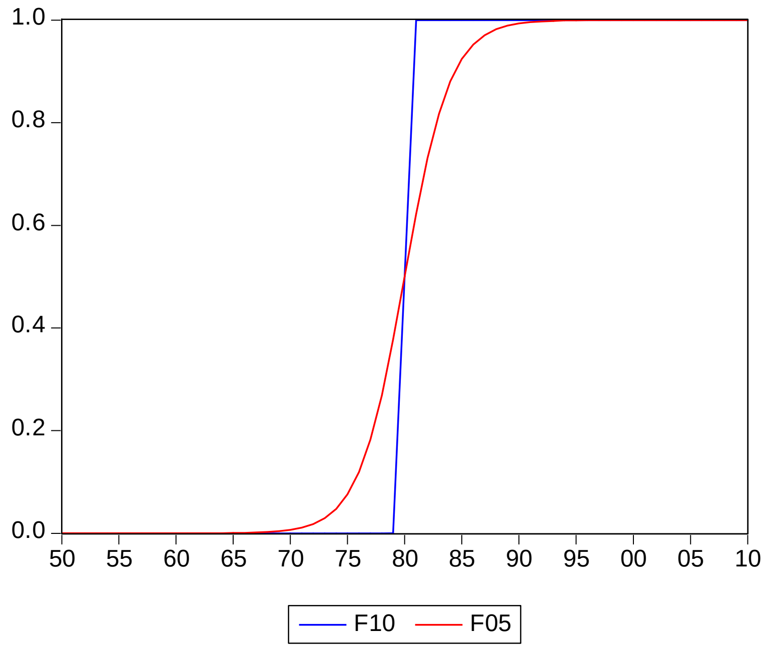

Figure 2: Switching function (3) with $\gamma = 10$ and with $\gamma = 0.5$, while $\tau = 30$, where the horizontal axis marks annual data ranging from 1950 to 2010.

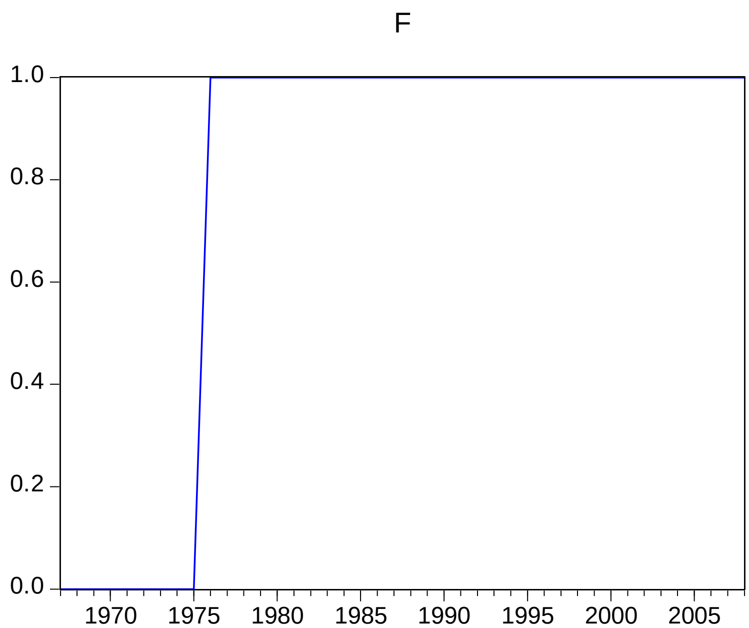

Figure 3: Switching function

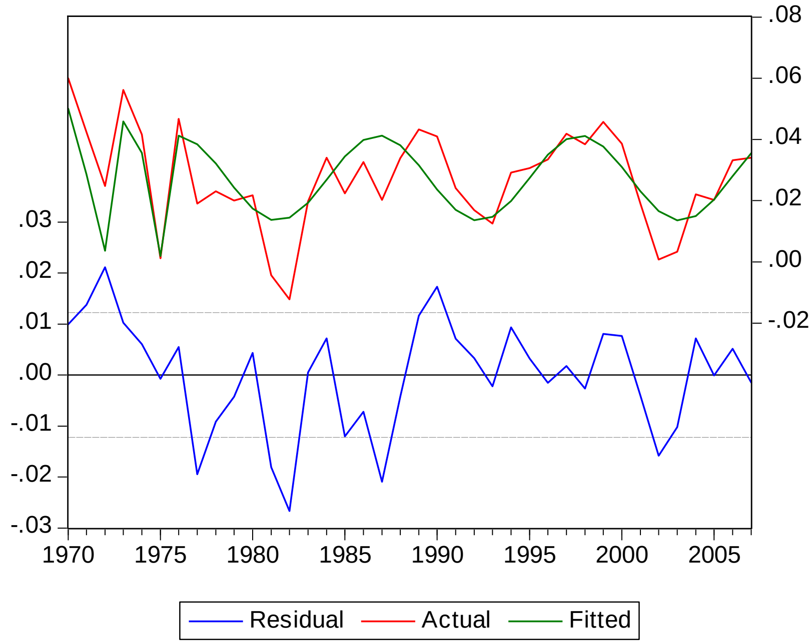

Figure 4: Model fit

References

Franses, P.H., E.A. de Groot and R. Legerstee (2009), Testing for harmonic regressors, *Journal of Applied Statistics*, 36, 339-346.

Granger, C.W.J. and T. Teräsvirta (1993), *Modeling Nonlinear Economic Relationships*, Oxford: Oxford University Press.

Lundbergh, S., T. Teräsvirta and D. van Dijk (2003), Time-varying smooth transition autoregressive models, *Journal of Business & Economic Statistics*, 21, 104-121.