Determining Number of Zones in a Pick-and-pack Orderpicking System

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ABSTRACT AND KEYWORDS	
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Free Keywords	Orderpicking, Zone picking, Warehousing, Logistics, Order fulfillment, Distribution
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Determining Number of Zones in a Pick-and-pack Orderpicking System

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Abstract

In this study we consider a pick-to-pack orderpicking system, in which batches of orders are picked simultaneously from different (work) zones by a group of order pickers. After picking, the orders are transported by a conveyor to the next station for packing. Our aim is to determine the optimal number of zones such that the overall (picking and packing) time to finish a batch is minimized. We solve this problem by optimally assigning items to pick routes in each zone. We illustrate the method with data taken from a distribution center of one of the largest online retailers in the Netherlands.

Keywords: Orderpicking; Zone picking; Warehousing; Logistics; Order fulfillment and distribution

1. Introduction and literature review

Within a logistics chain, products need to be physically moved from one location to another, from manufacturers to end users. During this process, they may be buffered or stored at certain places (*warehouses*) for a certain period of time. Many activities are carried out in a warehouse. Among them, orderpicking - *the process of retrieving products from storage (or buffer area) in response to a specific customer request* - is the most critical one. It has long been identified as a very labor intensive operation in manual systems, and a very capital intensive operation in automated systems (Goetschalckx and Ashayeri, 1989). It may consume as much as 60% of all labor activities in the warehouse, and for a typical warehouse, the cost of orderpicking is estimated to be as much as 55% of the total warehouse operating expense (Tompkins et al., 2003). For these reasons, warehousing professionals consider orderpicking as the highest-priority activity for productivity improvements.

Several recent trends both in manufacturing and distribution have made the orderpicking become more and more important and complex. In manufacturing, there is a move to smaller lot-sizes, point-of-use delivery, order and product customization, and cycle time reductions. In distribution logistics, companies tend to accept late orders while providing timely delivery within tight time windows (thus the time available for orderpicking becomes less). Small warehouses are being replaced by larger ones to realize economies of scale. There is also a trend of collecting products, materials, and product carriers from customers back to the warehouse. These new trends make today's orderpicking operations become even more complex.

We can make a distinction between two types of warehouses: conventional (manual picking) and automated warehouses. In this research we focus on the conventional warehouse type. At operational level, there are four issues that have received attention from researchers.

- *Storage assignment.* This is a rule used for assigning stock keeping units (SKUs) to storage locations. The main storage policies mentioned in the literature are randomized, class-based and dedicated storage. The easiest storage method is to randomly allocate incoming products to available storage locations. However, we can reduce the expected travel time of a picking tour by locating high-demand products near the input/ output (I/O) point (depot) of the warehouse. There are two ways of doing that: either on group or on item basis. In practice, class-based storage strategy (see for example, Hausman et al., 1976) is most popular. This strategy divides products and locations into classes, ranks them in decreasing order of ordered frequency, and then assigns in that order to the locations nearest to the I/O point. The dedicated storage strategy (see for example Caron et al., 1998, 2000) rank the items individually and then assigns them in that order to the locations nearest to the I/O point. The cube-per-order index (COI) rule, which is attributed to Heskett (1964), is an example of such a dedicated storage strategy. The COI is the ratio of the space requirement (cube) of a SKU to its turnover rate.

- *Layout problem*. This is the problem of finding a good aisle configuration (i.e. the optimal number and length of aisles) minimizing orderpicking time. Little research has been done in this area. Recently, Roodbergen (2001) proposes a non-linear objective function (i.e. average travel time in terms of number of picks per route and pick aisles) for determining the aisle configuration for random storage warehouses (including single and multiple blocks) that minimizes the average tour length. Also considering minimization of the average tour length as the major objective, Caron et al. (2000) consider 2-block warehouses (i.e., one middle cross aisle) under the COI-based storage assignment. For small (up to 2-block) class-based storage warehouses, Le-Duc and De Koster (2005a,b) propose a travel time model and a local search procedure for determining optimal storage zone boundaries as well as the number of storage aisles.

- *Routing order pickers.* This is the problem of determining the optimal sequence of visits to pick up a number of requested items as quickly as possible. Optimal methods to route (order) pickers depend on the warehouse layout and the location of the depot. Optimal methods for simple warehouse layouts (single or two blocks) are mentioned by Ratliff and Rosenthal (1983), Goetschalckx and Ratliff (1988), De Koster and Van der Poort (1998), Roodbergen and De Koster (2001b) . The disadvantage of the exact algorithm is that it depends on the layout and depot location and that the resulted routes may be too complicated for pickers to follow. For large and more complicated layouts (more than two blocks) several heuristics are documented. The best routing heuristic known so far is probably the combined heuristic (Roodbergen, 2001a). This method combines two basis methods: either traversing a visited aisle from one end to the other or entering and leaving the aisle from the same aisle's end. The choices are made by using a dynamic programming method.

- *Batching* and *zoning.* Batching determines which orders are released together (in batch picking, multiple orders are picked together in one pick tour and need later sorted by order). Batching is designed to reduce the average travel time per order by sharing a pick tour with other orders. There are basically two criteria for batching: the proximity of pick locations and the time window. Proximity batching, the clustering of orders based on retrieval locations, is studied in, for example, Hwang et al. (1988), Gibson and Sharp (1992), Elsayed et al. (1993), Rosenwein (1994), Elsayed and Lee (1996), De Koster et al. (1999), Gademann et al. (2001) and Gademann and van de Velde (2005). With time-window batching the number of orders per batch can be fixed or variable. Variable time-window batching groups all orders that arrive during the same time interval or window. With fixed-number-of-orders time-window batching, a time window is the variable length until a batch has a predetermined number of orders (Le-Duc and De Koster, 2003a,b). Zoning is closely related to batching; it divides the pick area into sub-divisions (or zones), each will be dedicated to, normally, one or few pickers. The major advantages of zoning are: reduction of the travel time (because of the smaller traversed area and also the familiarity of the picker with the zone) and of the traffic congestion. Depending on the process sequence, zoning can be further classified as progressive zoning or synchronized zoning. With progressive zoning, orders are picked zone by zone sequentially (this system is also called pick-and-pass); a batch is finished until all (order) lines of the orders in the batch are picked. In contrast, in synchronized (or parallel) zoning, pickers in all zones can be working on the same batch at the same time. Choe and Sharp (1991) notice that zoning has received little attention in the literature despite its important impact on the performance of OPS. Mellema and Smith (1988) examine the effects of aisle configuration, stocking policy and batching and zoning rules by using a simulation model. They suggest that a combination of batching and zoning can significantly increase the productivity (pieces per man-hour). Choe et al. (1993) study the effects of three strategies for OP in an aisle-based system: single-order-pick, sort-while-pick, and pick-and-sort. They propose analytical tools for planner to quickly evaluate various alternatives without using simulation. Petersen (2002) uses simulation to show that the zone size, the number of items on the pick list, and the storage policy have a significant effect on the zoning configuration (i.e. the aisle’s length is a variable).

A critical problem associated with zoning is to define the zone storage capacity (or zone borders). More specifically, for a given layout, operational policies (routing, batching method) and a storage assignment policy, it is the problem of how to divide the picking area into zones such that a certain objective is maximized or minimized. Example objectives include the system throughput time (Petersen, 2002) and the load balance between zones (Jane and Laih, 2005). If we assume that all aisles are identical and all zones are of the same size (an equal number of identical aisles), then the zone partitioning problem becomes the problem of determining the optimal number of aisles constituting a zone. It should be mentioned here that this problem has not been studied in the literature. The most related publication is Petersen (2002), where the effects on the travel distance in a zone of the number of aisles in the zone, of storage assignment methods, and of the number of items in the pick list are investigated. However, the zone storage capacity is fixed (i.e. aisle length is a decision variable). Therefore, the problem essentially differs with the problem of determining the number of zone (or zone storage capacity). In this study, we will consider the problem of finding the optimal number of zones to minimize the system throughput time for an orderpicking system, where orders are picked in groups and after picked they are sorted and packed per order (synchronized zone picking). This system type is commonly observed in many mail-order companies (e.g., Amazon - Germany, Wehkamp - the Netherlands). The paper is organized as follows. In the next section, we describe the orderpicking operation. Then, we present a mathematical model for the problem of optimally assign items-to-routes in each zone in Sections 3. Next, we apply the model to find the optimal number of zones for a mail-order company in Section 4. Finally, we conclude and propose outlooks for further research in Section 5.

2. Orderpicking system

The schematic layout of the OP system that we consider is sketched in Figure 1. Basically, we have two functional areas: one area for picking and one for packing. Items are stored in rectangular bin-shelving storage racks. Batched orders are picked simultaneously from different zones in the picking area by a group of order pickers. After an order picker has completed a pick tour, the picked items are deposited on a conveyor and transported to the buffer area. When all items of an order have been picked, they are sorted and picked.

- *Batch generation*: orders (requests from a customer consist of one or several *items*[1]) arriving within a predetermined interval are grouped together in one *batch* for joint release to the order pickers. Within a batch, orders are spread over the zones based on the storage locations of their items. They are consolidated later at the packing area.

[1] 'Item' here means stock keeping unit (SKU), in the literature it is also called 'order-line'

- *Picking operation*: all batched items from the same zone are picked by one order picker or a group of order pickers designated to the zone. Each order picker can only be assigned to at most one zone (zone picking). As each order picker can only pick a limited number of items (e.g. due to the capacity limitation of the picking cart) in one pick route, the batched items from a zone may require *t pick shifts* to be completed, where $1 \le t \le \tau$ with $\tau = \max_{zones} \{t\}$. (In the case of a single order picker per zone, the number of pick shifts required is the number of pick routes.) The order picker starts a batch by obtaining a picking cart and pick lists (each is a list of items to be picked in one pick route) from a central location. The order picker then goes to the left-most aisle in the zone to start a pick route. After picking all requested items, the order pickers place them on the transportation conveyor, and go back to the left-most aisle to start a new pick route. The transportation conveyor runs continuously to move all picked items to the buffer area. For each batch of orders, it is assumed that the order picker receives all pick instructions at the beginning of the batch. For the ease of discussion later on, we divide the throughput time of a batch into periods from *1* to τ *+1*, where periods are defined as follows. Period *1* is the time lapse between the starting time (to pick the batch) and the moment when all the order pickers (from all zones) have finished the first pick route. Period *2* starts from the end of the period *1* and ends when all the second pick routes in all zones have been completed, and so on. The last period starts from the moment when all last pick routes have been completed, and ends when all items are sorted (no picking operation is carried out, only the packing).
- *Packing operation*: a conveyor runs continuously in the buffer area for buffering incomplete orders (an order is called incomplete if not all of its items are picked). Orders only enter the sorter when they are complete. It means that newly-picked items enter the sorter if and only if all the items in the orders they belong to either have been picked or were previously picked (waiting in the buffer area). The complete orders are sorted to sorter exits (see Figure 1)

according to destinations (e.g. each shipping lane is assigned to a group of proximity destination postcodes). A group of packers manually pack the orders. After packing, orders are transported to the shipping docks for delivery to the customers.

[Insert Figure 1 here]

With a given work force level (the number of workers at both picking and packing stages), the objective of our study is to minimize the total time to complete a batch of orders (*throughput time*). There are two decision problems that may impact the overall time to complete an order batch.

- At the operational level, the problem is how to assign items to different routes in each zone (recall that completion of a batch in one zone may require more than one pick route to be completed). The item assignment and sequence in which we pick routes in each zone has an important impact on the latter stage when the items are consolidated. Let us consider a simple example. We have two picking zones A and B, each with one order picker, with pick capacity of one unit per pick route. In a batch, we have to complete two orders: order1=A1+B1, order2=A2+B2. For this situation, we have four possible pick sequences: (A1→A2, B1→B2), (A1→A2, B2→B1), (A2→A1, B1→B2) and (A2→A1, B2→B1). It is clear that the second and third sequence result in the longest throughput time, as there is no order to pack after the first pick shift. In the general case, when we have a set of orders, a given layout (number of zones, the size of zone), and a work force level at both the picking and packing area, we can formulate this problem as a mixed integer-linear program. We will discuss this in the next section.

- At the tactical level, we have to decide the number of zones into which the overall picking area should be divided (or in other words, how large the zone size should be). When the zone size increases, the route time (to pick a given number of items) also increases. And consequently, the throughput time may also increase. However, on the other hand, large zones reduce the consolidation problem, as orders are spread over fewer zones. This makes it simpler to arrange the pick sequence (item-to-route assignment in each zone) in such a way that the number of complete orders arriving at the packing area (per time unit) increases. And thus, the throughput time may be shorter. The best zoning scheme is the one that brings the best compromise between these two opposite effects.

In practice, the number of aisles in a warehouse is limited. Therefore, when we assume that zones are identical, we can choose from only a limited number of possible zone sizes (number of aisles per zone). For example, if we have 20 aisles then we have the following zone-size possibilities: 1, 2, 4, 5, 10 and 20 aisles (with 20, 10, 5, 4, 2 and 1 zones respectively). Because of that, our solution strategy is as follows. For each zoning scheme, we first solve the item-to-route assignment problem. In a next step, we vary the zone sizes and choose the zone size that provides the shortest overall throughput time. In the next section we will step by step formulate a mathematical model for the item-to-route assignment problem and discuss a solution approach.

3. Mathematical model for item-to-route assignment problem

In the model, the following assumptions are made:

- (Storage) aisles are identical.
- A zone is a set of adjacent entire aisles (i.e. one aisle can not belong to more than one zone). All zones have the same number of aisles; this assumption is made to keep the workforce balanced among zones.
- The picking capacity per pick route is determined by the number of items to be picked in one pick route.
- Order pickers always start from the left-most aisle (of the assigned zones). Within a zone, the average route length depends only on the number of items per route, the zone size, the storage assignment and the routing method used.

- The travel time between from one side of the aisle to the other is negligible. It means that an order picker can pick items from both sizes of the aisles in a single pass. No additional time is needed to reach the higher-level storage locations in an aisle.
- Multiple order pickers can work in one zone at the same time (i.e. traffic congestion is negligible).
- The item transportation time (μ) between the picking and packing area is a constant.
- Routes between order pickers in different zones are synchronized. Synchronized zoning usually gives a shorter response time than progressive zoning (at the expense of order integrity).
- Only complete orders can enter the sorter, incomplete orders are buffered. The buffer capacity is sufficiently large to buffer all order needed.

Data

q	the maximum number of items that an order picker can pick in a pick route. We assume that this is identical for all order pickers as the pick capacity of an order picker mainly depends on the picking vehicle or cart.
a	number of aisles per zone
L	length (in travel time unit) of a storage aisle
w_b	centre-to-centre distance (in travel time unit) between two consecutive storage aisles
t_s	set-up time of a pick route
μ	transportation (conveyor) time
r_{pi}	picking rate (number of units per time unit that an order picker can pick). It is assumed to be identical for all order pickers.
r_{pa}	overall packing rate (number of orders per time unit). This rate depends on the average order size (number of items per order) and the average packing time per unit.

N_k number of order pickers in zone k

t, i, o, k indices of period, item, orders and zones

K set of zones

O set of all orders

I_o set of all items in order o

I_k set of all items in zone k

I set of all items, $I = \bigcup_{o \in O} I_o = \bigcup_{k \in K} I_k$

Intermediate variables

τ the maximum number of required pick shifts in the zones, $\tau = \max_{k \in K} \left\{ \left\lceil \frac{|I_k|}{qN_k} \right\rceil \right\}$.

$\Re(q,a)$ time needed to finish a pick route of q items (or picks) in a zone containing a aisles and return to the left-most aisle of the assigned zone. It consists of four components: travel time, setup time, picking time and correction time. (It has to note that the number of items in the last pick route (in each zone) can be less than the route's capacity.) If the random storage assignment and the S-shape routing method are used, then it can be calculated by (see details in Appendix A):

$$\Re(q,a) = La\left[1 - \left(1 - \frac{1}{a}\right)^q\right] + 2w_b \sum_{i=1}^{a} (i-1)\left[\left(\frac{i}{a}\right)^q - \left(\frac{i-1}{a}\right)^q\right] + CR(q,a) + t_s + \frac{q}{r_{pi}} \qquad \textbf{(1)}$$

Decision variables

$$x_{ti} = \begin{cases} 1 & \text{if item } i \text{ is picked in period } t(t = 1..\tau) \\ 0 & \text{otherwise} \end{cases}$$

$$y_{to} = \begin{cases} 1 & \text{if order } o \text{ has been completely picked in or before period } t(t = 1..\tau) \\ 0 & \text{otherwise} \end{cases}$$

TL_{to} total number of items of order o completely picked at the end of period t

NCO_t number of newly complete orders in period $t\,(t = 1..\tau)$

UCO_t number of complete (but unpacked) orders transferred from period $t\,(t = 1..\tau)$ to period $t+1$.

This is because in a period of length $\Re(q,a)+\mu$, we can only pack a limited number of complete orders: $P = \lfloor [\Re(q,a)+\mu].r_{pa} \rfloor$.

$$z_t = \begin{cases} 1 & \text{if } UCO_t = 0 \\ 0 & \text{otherwise} \end{cases} \quad \forall(t = 1..\tau)$$

The whole batch is completed only when all orders have been packed. Therefore, the throughput time, the overall time (ψ) to complete a batch, is the summation of time required to pick all items (the total picking time), the transportation (for all pick shifts) and the time needed to pack all remaining unpacked orders after the last pick shift. The throughput time can be calculated by:

$$\psi = (\tau-1)\{\Re(q,a)+\mu\} + \{\Re(q_M,a)+\mu\} + \{NCO_\tau + UCO_{\tau-1}\}/r_{pa} \qquad \textbf{(2)}$$

where $\Re(q_M,a)$ is the longest pick-route time in period τ; q_M is the maximum number of items which need to be picked from some zone in period τ. Having mentioned all assumptions and variables, we now can formulate the item-to-route assignment problem as follows.

MODEL

Objective Min ψ

Such that

$$\sum_{t=1}^{\tau} x_{ti} = 1 \qquad \forall(k \in K, i \in I_k) \qquad \textbf{(3)}$$

$$\sum_{i \in I_k} x_{ti} \le qN_k \qquad \forall(k \in K, t = 1..\tau) \qquad \textbf{(4)}$$

$$TL_{to} = \sum_{j=1}^{t} \sum_{i \in I_o} x_{ji} \qquad \forall(o \in O, t = 1..\tau) \qquad \textbf{(5)}$$

$$[1-(|I_o| - TL_{to})](1-y_{to}) \le 0 \qquad \forall(o \in O, t = 1..\tau) \qquad \textbf{(6)}$$

$$-|I_o| + TL_{to} \le y_{to} - 1 \qquad \forall (o \in O, t = 1..\tau) \tag{7}$$

$$NCO_t = \sum_{o \in O} \sum_{j=1}^{t} y_{jo} - \sum_{j=1}^{t-1} NCO_j \qquad \forall (t = 1..\tau) \tag{8}$$

$$(NCO_t + UCO_{t-1} - P)\left[UCO_t - (NCO_t + UCO_{t-1} - P)\right] \le 0 \qquad \forall (t = 1..\tau) \tag{9}$$

$$UCO_t\left[1 - (NCO_t + UCO_{t-1} - P)\right] \le 0 \qquad \forall (t = 1..\tau) \tag{10}$$

$$UCO_t \ge (NCO_t + UCO_{t-1} - P) \qquad \forall (t = 1..\tau) \tag{11}$$

$$NCO_0 = 0 \tag{12}$$

$$UCO_0 = 0 \tag{13}$$

$$UCO_t \ge 0 \ \forall (t = 1..\tau) \tag{14}$$

$$x_{ti}, y_{to}, z_t \in \{0,1\} \qquad \forall (o \in O, t = 1..\tau, i \in I_k, k \in K) \tag{15}$$

In the objective function, we minimize the throughput time to finish a batch of q orders. Constraint (3) ensures that each item is assigned to exactly one pick route. Constraint (4) is the capacity constraint. It indicates that the maximum number of items that can be picked from zone k by N_k order pickers in one period cannot exceed the total capacity of the N_k order pickers. Constraints (5)-(7) indicates that $y_{to} = 1$ if order o is completed by the end of period t (meaning that all items belong to order o are picked in pick shift t), and $y_{to} = 0$ otherwise. Constraint (6) can be linearized by using the big-M method: $|I_o| - TL_{to} \le M_1(1 - y_{to}) \ \forall (o \in O, t = 1..\tau)$, where M_1 is the smallest possible constant, it would equal to $\max_{o \in O}\{|I_o|\}$. Constraints (8)-(14) imply that $UCO_t = (NCO_t + UCO_{t-1} - P)$ $\forall (t = 1..\tau)$ if $NCO_t + UCO_{t-1} - P > 0$, and $UCO_t = 0$ otherwise. In words, it means the number of complete orders left over period *t+1* equals the number of newly complete order during period *t* plus the number of complete orders left over from period *t-1* minus the number of orders that have been packed in period *t*. Constraints (9) and (11) can also be linearized by the big-M method:

$UCO_t - (NCO_t + UCO_{t-1} - P) \le M_2(1 - z_t) \quad \forall(t = 1..\tau)$ and $(NCO_t + UCO_{t-1} - P) < M_2 z_t \ \forall(t = 1..\tau)$, respectively. M_2 equals the number of orders $|O|$. The last constraint defines the nonnegative and binary property of variables x_{ti}, y_{to} and z_t.

The model is a mixed integer-linear program. The most difficult constraints are (3) and (4). Constraints (5)-(16) are used just for keeping track of the ‘inventory’ level after each period. If we have only one zone, then our problem can be interpreted as a vehicle routing problem (VRP) with minimizing the ‘inventory’ level after the last period as the objective function. When we have k zones, our problem is a type of multiple VRP. The VRP is an NP-hard problem. Therefore, our problem also belongs to this class.

Our computational experience with this model was that the running time of the model mainly depends on four factors: the total number of items, the order size (average number of items per order), and the number of periods and zones. For a problem size of 6 zones, 1000 items to be picked in 4 periods, 10 items per order on average, the time required to run the model to optimality was about 15 seconds (using LINGO release 8.0, 2.4 GHz CPU). However, for larger instances the running time went up very rapidly; it increased to more than 41 hours when the number of periods increased to 7. For real-life warehouses, the number of periods (per batch) in each zone can be rather few. However, the number of items per order and the number of zones can be large. A heuristic approach is needed for solving large instances.

4. Case study and numerical experiments

In this section, we first introduce the case we have investigated. Then we discuss the results obtained by using the model that proposed in Section 3.

4.1 Introduction

The case we consider is based on the distribution center of Wehkamp, one of the largest online retailers in the Netherlands. Its mission is "being an innovative home-shopping organization with a wide assortment of consumer products against competitive prices and recognizable better service". The company uses a pick and pack system (which was simplified and sketched in Figure 1). About 15000 orders have to be picked per day, each containing 1.6 items (in total 2.3 units per order) on average. Since the picking and packing department have a limited capacity, orders received from customers are processed several times (in batches) a day; each batch contains about 1000 items in total. The picking process is described in Section 2. The order picker starts a batch by picking up a picking cart and obtaining pick lists from the central location. Pick routes always start from the left-most aisle in the zone. The picked items are dropped on the transportation conveyor, which conveys them to the packaging area. At the packaging area, complete orders are sorted by packing destination station (automatically) and then per order (manually), while incomplete orders (i.e. items) are buffered until they are complete (see Figure 1). In this case, all the buffering takes place at the packing station. When an order at packing station is complete, a light indicator turns on to signal the packers that packaging can start.

As previously discussed, the zone size may strongly influence the system throughput time. Therefore, it is a crucial decision for the manager to decide how large zones should be, or, equivalently, the number of zones the pick area should be divided into, such that the system throughput of the system is minimized. In the next section, we will use the model of Section 3 to answer this question for the case.

4.2 Numerical experiments and results

Table 1 shows the current operational data as well as the size of the picking area. The company has 36 storage aisles and uses 18 order pickers. Therefore, there are 6 possible zoning schemes (see Table

2). The packing rates depend on the average order size (average number of items per order); they are 8, 3, 1 and 0.5 order(s) per minute for order sizes of 1.6, 5, 10, and 20 items respectively.

[Insert Table 1 here]

In order to determine the optimal number of zones, we carried out a number of experiments. We considered four pick-list sizes (10, 20, 30 and 40 items per pick route), and four order sizes (1.6, 5, 10, and 20 items per order on average). Combining this with 6 zoning schemes, we have 96 scenarios in total, including the current situations (1.6 items per order, maximum 40 items per pick route). An order batch was generated as follows. We fixed the number of items per batch. For each item, a storage location (in one of the 36 aisles) and an order (to which the item belongs, from 1 to κ) were randomly drawn from a uniform distribution (implying that random storage assignment is used). The average order size was controlled by adjusting κ: $\#\text{orders} = \kappa\left(1 - 1/\kappa\right)^{\#\text{items}}$. For each scenario we generated 5 order batches, and after solving the item-to-route assignment problem mentioned in Section 3, we calculated the average throughput time value. The average travel time per pick route can be calculated, based on the zone size, the number of items per route, and the routing method used. In our case, the S-shape method is used and the route time is calculated by using formulation (1). The route times for the different pick-list and zone sizes are tabulated in Appendix B.

[Insert Table 2 here]

We used LINGO (version 8.0) to solve the item-to-route assignment problem (discussed in Section 3). It turns out that we can find the optimal solution for 78 among 96 scenarios within 10 seconds (2.4 MHz Pentium CPU). For the other scenarios, we could not find the optimal solution within 1 hour. However, we observed that the gap between the feasible solution and the best lower bound provided by LINGO after about 5 minutes of running is very small (less than 5% for small, medium and large order sizes, and less than 10% for very large order sizes). For this reason, we decided to use the truncated solution for the problem. The results of the experiments are presented in Table 3, where the truncated solutions are printed in bold and italic.

[Insert Table 3 here]

The results in Table 3 show that for the current situation (1.6 items per order on average and 40 items per route) the 18-zone configuration gives the shortest throughput time for the system. Given the pick-list size varies between 10 and 40, when the order size increases, this configuration is still the best for not very large orders (i.e., less than 20). It means that for not very large order sizes, the configuration that minimizes the picking time (i.e. the 18-zone configuration) also minimizes the system throughput time. It is because, when orders are small, the reduction in picking time is dominant the increase in packing time. Furthermore, the 18-zone option would be more favorable if we take aisle congestion into account (in the 18-zone configuration, each zone has only one order picker, thus it is free from the travel congestion). For very large orders, it appears that the 6-zone configuration outperforms the other zoning schemes. It shows the effect of spreading orders over zones: large zone may reduce the picking time, but may increase the consolidation time. This effect seems to be clear for large order sizes.

For the current situation, a pick-list of 40 items per route is not optimal. A pick-list size of 20 appears to be optimal in most of the cases. When the pick-list size changes from 10 to 40, the throughput time decreases and then goes up. We can explain this behavior as follows. If we increase the pick-list size, the overall travel time to complete a batch (and also the total set-up time) will decrease. Therefore, the overall picking time of a batch will be reduced. However, the accumulative number of complete orders which have to be packed in the last period, when the picking is completed, will grow (potentially). That increases the overall packing time, thus the system throughput time. Clearly, there exists a trade-off between picking time and packing time when increasing the pick-list size.

5. Concluding remarks

In this paper, we have elaborated the problem of choosing the right number of zones at a manual pick-and-pack OP system. At the first phase, we formulated the problem of assigning items to pick routes in each zone such that the throughput time is minimized as a mixed integer-linear program. At the second phase, we used this problem as a tool for evaluating different zone-size options to find the optimal one. We also illustrated the method with a pick-and-pack OP system which is used in a distribution center of the one of the largest online retailers in the Netherlands.

In the case we considered above, random storage assignment and the S-shape routing method are used. However, our model can be applied for other operational policies (like the return routing, class-based or COI-based assignments), as long as we can estimate the travel time of a pick route.

There are still several limitations to this study. First, we do not take into account the congestion in the aisles that may result from having more than one order picker per zone. Second, though we found the optimal solutions for most scenarios investigated, it does not guarantee that we can find a 'good' solution for the item-to-route assignment problem when the problem size increases (i.e. more aisles, periods, items). We propose to use a 2-opt heuristic procedure to cope with large problems, like the one given in Appendix C. Indeed, some efforts are needed to testify its efficiency. Certainly, it is our future research direction.

Appendix A Picking time estimation

Travel time consists of three components: travel time in the cross-aisles aisles, travel time within the storage aisles, and travel time back to the left-most aisle of the zone (e.g. to start a new pick route). As we assume that the order picker always starts a pick route from the left-most aisle of the zone, the last component equals to the cross-aisle travel time. With the S-shape routing method and random storage, the average travel time within storage aisles can be estimated by $L\left[a-a\left(1-\frac{1}{a}\right)^{q}\right]+$ $CR(q,a)$, where the term in brackets is the expected number of visited aisles. $CR(q,a)$ is the correction time, its takes into account the fact that from the last pick position (in the last visited aisle) the order picker has to return to the drop-off point (the transportation conveyor). Such a turn has to be made if and only if the number of visited aisles is odd. $CR(q,a)$ can be estimated by:

$$CR(q,a)=\sum\nolimits_{g\in G:odd}\left[\binom{a}{g}\left(\frac{g}{a}\right)^{q}X(g)\left(2L\frac{\frac{q}{g}}{\frac{q}{g}+1}-L\right)\right],$$

where $X(g)$ is 1 minus the probability that all q items are in less than g aisles ($g\in\{G\,|\,1\le g\le a,\ g\text{ is odd}\}$), conditional on the fact that all q items in at most g specific aisles:

$$X(g)=1-\sum\nolimits_{j=1}^{g-1}(-1)^{j+1}\binom{g}{g-j}\left(\frac{g-j}{g}\right)^{q}.$$

If we number the aisles of a zone from 1 to a (from the left to the right), the cross-aisle travel time can be estimated by $w_b\sum_{i=1}^{a}(i-1)\left[\left(\frac{i}{a}\right)^{q}-\left(\frac{i-1}{a}\right)^{q}\right]$. Where $\left(\frac{i}{a}\right)^{q}-\left(\frac{i-1}{a}\right)^{q}$ is the probability that q picks fall in aisles $1,\ldots,i$ minus the probability that q picks fall in aisles $1,\ldots,i-1$, and w_b is the travel time between two consecutive storage aisles (see Figure 1).

Finally, $\Re(n,m)$ is estimated as:

$$\Re(q,a) = La\left[1-\left(1-\frac{1}{a}\right)^{q}\right] + 2w_b\sum_{i=1}^{a}(i-1)\left[\left(\frac{i}{a}\right)^{q} - \left(\frac{i-1}{a}\right)^{q}\right] + CR(q,a) + t_s + \frac{q}{r_{pi}} \quad \blacklozenge$$

Appendix B   Average route time (in minute) with different zone and pick-list sizes

Pick-list size	1 zone	2 zones	3 zones	6 zones	9 zones	18 zones
1	7.29	5.79	5.29	4.79	4.63	4.46
2	9.65	7.63	6.94	6.22	5.94	5.54
3	11.46	9.13	8.30	7.33	6.88	6.15
4	13.07	10.53	9.59	8.40	7.76	6.61
5	14.52	11.78	10.72	9.30	8.53	7.02
6	15.93	13.00	11.81	10.10	9.18	7.41
7	17.25	14.14	12.82	10.84	9.74	7.79
8	18.54	15.23	13.78	11.53	10.24	8.16
9	19.78	16.28	14.68	12.17	10.70	8.54
10	20.99	17.29	15.54	12.76	11.13	8.92
11	22.16	18.25	16.36	13.32	11.54	9.29
12	23.32	19.19	17.13	13.83	11.94	9.67
13	24.44	20.09	17.88	14.32	12.33	10.04
14	25.54	20.96	18.59	14.78	12.72	10.42
15	26.62	21.80	19.28	15.22	13.10	10.79
16	27.67	22.61	19.94	15.65	13.49	11.17
17	28.71	23.40	20.58	16.06	13.86	11.54
18	29.72	24.17	21.19	16.47	14.24	11.92
19	30.72	24.92	21.79	16.87	14.62	12.29
20	31.69	25.64	22.36	17.26	15.00	12.67
21	32.65	26.35	22.92	17.65	15.37	13.04
22	33.60	27.03	23.47	18.04	15.75	13.42
23	34.52	27.70	24.00	18.42	16.12	13.79
24	35.43	28.35	24.52	18.80	16.50	14.17
25	36.33	28.99	25.03	19.18	16.87	14.54
26	37.21	29.62	25.52	19.56	17.25	14.92
27	38.08	30.22	26.01	19.94	17.62	15.29
28	38.93	30.82	26.48	20.32	18.00	15.67
29	39.77	31.41	26.95	20.70	18.37	16.04
30	40.59	31.98	27.41	21.08	18.75	16.42
31	41.41	32.54	27.85	21.45	19.12	16.79
32	42.21	33.09	28.29	21.83	19.50	17.17
33	43.00	33.63	28.73	22.20	19.87	17.54
34	43.77	34.17	29.16	22.58	20.25	17.92
35	44.54	34.69	29.58	22.96	20.62	18.29
36	45.29	35.21	30.00	23.33	21.00	18.67
37	46.04	35.71	30.41	23.71	21.37	19.04
38	46.77	36.22	30.82	24.08	21.75	19.42
39	47.50	36.71	31.22	24.46	22.12	19.79
40	48.21	37.20	31.62	24.83	22.50	20.17

Appendix C   A heuristic procedure for large-size problems

We suggest the following heuristic procedure for coping with large problem instances. Our heuristic (depicted in Figure 2) starts from a feasible solution (generated by using LINDO, for example). From this feasible solution, we use a 2-opt procedure to switch items picked in a periods t to the items picked in a period j, given that after the period t, the number of complete orders is greater than the packing capacity, and after the period j, the number of complete orders is less than the packing capacity. The 2-opt procedure is repeated until no further improvement on the objective function can be made.

[Insert Figure 3 here]

The idea behind it is that it may be possible to improve the objective function if we can 'shift' some unpacked orders of a later shift to an earlier shift, which has excess packing capacity (i.e. there is some idle time). Figure 3 shows an example of a feasible solution with 4 periods. If we can increase number of complete order in periods 2 and 3 then we can reduce the number of unpack orders in period 4, and thus the throughput time. A feasible way of doing this is as follows. We define the items arrived in period 4 which are in the same orders with the items left over from period 3. Then we consider whether these items can be picked in the earlier periods where excess capacity occurs (period 2 or 3) or not (by switching the item pick sequence). If not, we move one step forward until no further improvement on the objective function can be achieved.

[Insert Figure 3 here]

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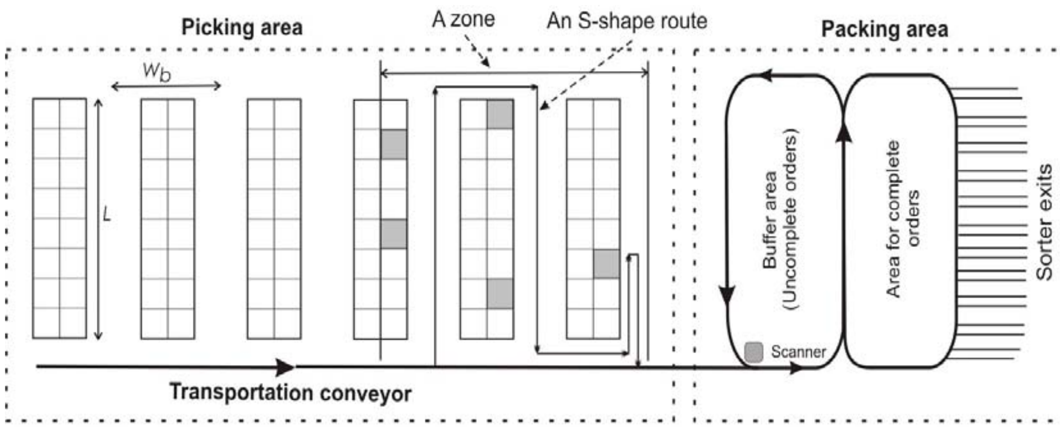

Figure 1 A pick and pack OP system

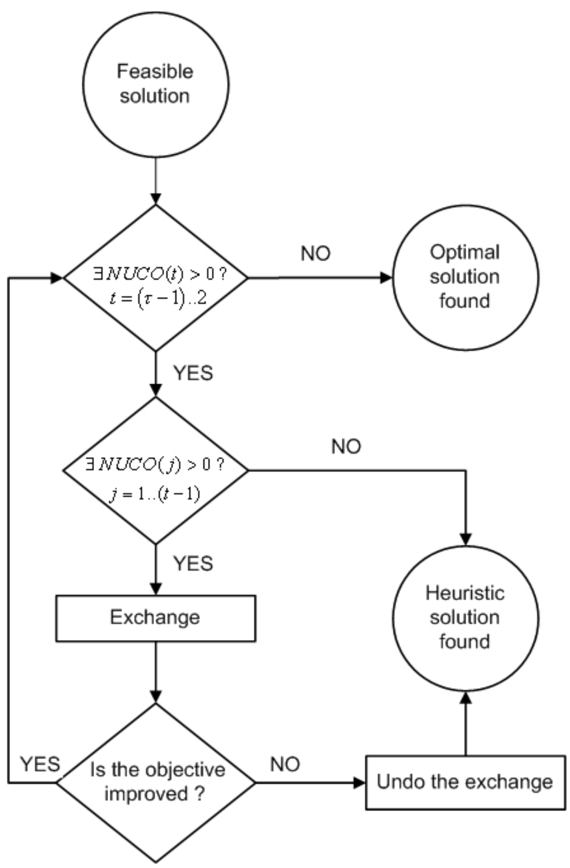

Figure 2 A 2-opt procedure for large size instances

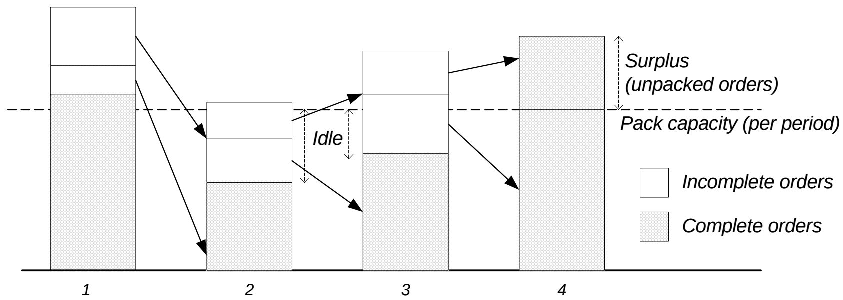

Figure 3 An illustration example of a feasible solution with four pick-shifts

Table 1 Operational data

Operational data		System parameters	
Average number of items per batch	1000	Number of storage aisles	36
Average number of items per order	1.6		
Max. number of items per route (capacity or pick-list size)	40	Aisle length (L) in seconds	60
Number of order pickers	18	Distance between two consecutive aisles (w_b) in seconds	5
Set-up time (t_s) in seconds	180		
Picking time per item ($1/r_{pi}$) in seconds	5		
Packing rate (r_{pa}): 8, 3, 1 and 0.5 order(s) per 60 seconds for 1.6, 5, 10, and 20 items order size respectively			

Table 2 Possible zoning schemes

Number of zones	Number of storage aisles per zone	Number of order pickers per zone
1	36	18
2	18	9
3	12	6
6	6	3
9	4	2
18	2	1

Table 3 Average throughput time (in minutes)

Order size (items)	Pick-list size	1 zone	2 zones	3 zones	6 zones	9 zones	18 zones	Mean
Small (1.6)	10	152.72	133.81	124.84	109.48	112.61	98.79	122.04
	20	110.25	105.61	104.13	100.71	99.61	97.42	102.96
	30	111.34	110.37	108.66	103.89	103.25	101.13	106.44
	40	116.53	114.92	115.12	107.66	106.88	104.83	110.99
	Mean	122.71	116.18	113.19	105.44	105.59	100.54	110.61
Medium (5)	10	***152.81***	***133.90***	***125.47***	***110.32***	***102.78***	***111.66***	122.82
	20	109.91	97.91	90.34	87.58	86.66	84.67	92.85
	30	101.88	96.95	95.07	90.26	89.46	87.58	93.53
	40	114.45	101.42	98.70	94.12	93.29	87.96	98.32
	Mean	119.76	107.55	102.40	95.57	93.05	92.97	101.88
Large (10)	10	***160.47***	***143.56***	***132.47***	***125.98***	***123.02***	***120.41***	134.32
	20	127.38	127.24	125.01	121.58	122.23	119.67	123.85
	30	132.21	131.62	129.41	124.26	124.50	122.25	127.38
	40	136.90	135.42	133.03	128.79	127.25	120.08	130.25
	Mean	139.24	134.46	129.98	125.15	124.25	120.60	128.95
Very large (20)	10	***154.32***	***144.50***	***133.80***	***125.63***	***128.62***	***134.47***	136.89
	20	127.38	127.24	127.01	121.85	122.67	125.61	125.29
	30	133.21	131.62	129.41	122.84	124.25	126.87	128.03
	40	137.09	135.42	133.03	124.79	125.58	128.63	130.76
	Mean	138.00	134.70	130.81	123.78	125.28	128.89	130.24

* *Bold and italic values mean the truncated solutions*

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