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Cross- and Auto-Correlation Effects Arising From Averaging

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Cross- and Auto-Correlation Effects

Arising From Averaging:

the Case of US Interest Rates and Equity Duration

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Abstract

Most of the available monthly interest data series consist of monthly averages of daily observations. It is well-known that this averaging introduces spurious autocorrelation effects in the first differences of the series. It is exactly this differenced series we are interested in when estimating interest rate risk exposures e.g. This paper presents a method to filter this autocorrelation component from the averaged series. In addition we investigate the potential effect of averaging on duration analysis, viz. when estimating the relationship between interest rates and financial market variables like equity or bond prices or exchange rates. In contrast to interest rates the latter price series are readily available in ultimo month form. We find that combining monthly returns on market variables with changes in averaged interest rates leads to serious biases in estimated correlations (R^2), regression coefficients (durations) and their significance (t-statistics). Our theoretical findings are confirmed by an empirical investigation of US interest rates and their relationship with US equities (S&P 500 Index).

Key words : interest rates, duration, averaging, time series properties, spurious autocorrelation

JEL codes : C13, C22, C82, E43, G10

1. Introduction and Summary

Most of the available monthly interest data series consist of monthly averages of daily observations. One example is the extensive *H.15 Statistical Data Release* of the US Federal Reserve Board of Governors, which contains interest rates for selected U.S. Treasury and private money market and capital market instruments.[1] A note to the release states explicitly:

> *Note: Weekly, monthly and annual figures are averages of business days unless otherwise noted.*

Also many of the monthly interest rate series contained in other databases are averages.[2] In no way, of course, we incriminate the Federal Reserve for disseminating averaged data. Averaging smoothes the series which is prefered, for example, when plotting the course of the monthly rate levels over time. However, we warn against the easy use of these series for more sophisticated purposes, such as gauging interest rate risk exposures. As shown by Working [1960] the averaging of daily data within a week or a month introduces spurious autocorrelation effects in the first differences of the series. It is the differenced interest rates we are especially interested in and not in their levels since unexpected *changes* in interest rates constitute interest rate risk. When estimating the interest rate risk exposure of equities or the effective duration of a bond portfolio we relate *changes* in (log) equity or bond prices to *changes* in interest rates.[3] The predictable component of interest rate changes can be removed by applying a time series model; the relationship between changes in market prices and interest rate shocks can be investigated by univariate regressions.

This paper further explores the biases in the differenced series resulting from averaging, both from a theoretical and an empirical point of view. Section 2 introduces notation and summarizes Working s [1960] result: within-month averaging potentially removes almost 20% from the volatility of the differenced series. In addition we derive a filter for removing the induced spurious autocorrelation component from a differenced averaged series. Section 3 investigates the potential effect of averaging on the relationship between interest rates and financial market variables such as equity or bond prices. In contrast to interest rates the latter price series are readily available in ultimo month form. We find that combining monthly returns on market variables with changes in averaged interest rates will lead to serious biases in estimated correlations (R^2s) and hence in regression coefficients (interest rate elasticities) and their significance (t-statistics). Although to a lesser extent this also applies when the induced autocorrelation effect has been filtered from the corrupted interest rate series in an attempt to obtain unexpected interest rate changes. Deriving explicit expressions for the biases we find that interest rate elasticities may be underestimated with more than 20% and proportions explained variance with almost 60%! Section 4 reports our empirical findings. We investigate a broad selection of US interest rates and analyze their relationship with US equities (S&P 500 Index) over the period 1990 to 2000. The empirical results indicate that our theoretical findings provide suitable approximations to the actual biases.

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[1] The release is published weekly at page http://www.federalreserve.gov/releases/H15/data.htm or http://www.bog.frb.fed.us/releases/H15/data.htm. The Federal Reserve Bank of St. Louis (http://www.stls.frb.org/fred/data/irates.html) also publishes the detailed US interest rate data.

[2] For example the *OECD Statistical Compendium*, the *Global Financial Database* (at http://www.globalfindata.com/tbusint.htm), and the *Treasury Management Database* (at http://www.mcs.net/~tryhardz/tmp55.htm).

[3] The possibility of non-stationary disturbances and nonsense regressions when levels are regressed instead of differenced variables provides another argument for first differencing the series; see Granger & Newbold [1974] and Plosser & Schwert [1978].

2. Averaging and spurious autocorrelation

notation

Suppose we have a time series with T monthly observations on the continuous compounded interest rate $\tilde{r}$ p.a. Let $t = 1,...,T$ denote the month index and d the number of trading days in each month (assumed fixed). The interest rate on day i of month t is $\tilde{r}_{(t-1)d+i}$. Hence month $t+1$ contains the rates $\tilde{r}_{td+1},...,\tilde{r}_{td+i},...,\tilde{r}_{(t+1)d}$. Using the backward difference operator Δ we define the difference between the rates on days td and $td+1$ by $\Delta\tilde{r}_{td+1} \equiv \tilde{r}_{td+1} - \tilde{r}_{td}$. The rate on day i of month $t+1$ can be expressed as the sum of the rate ultimo month t and the following daily changes of the rate through day i:

$$(2.1) \qquad \tilde{r}_{td+i} = \tilde{r}_{td} + \sum_{j=1}^{i} \Delta\tilde{r}_{td+j}$$

The difference between the rates on the last day (ultimo) of month $t+1$ and the last day of month t is denoted by $\Delta\tilde{r}_{td,(t+1)d} \equiv \tilde{r}_{(t+1)d} - \tilde{r}_{td}$. Hence the monthly difference between ultimo interest rates is the sum of the differences between the daily rates:

$$(2.2) \qquad \Delta\tilde{r}_{td,(t+1)d} = \sum_{i=1}^{d} \Delta\tilde{r}_{td+i}$$

effects of averaging

We assume that the first differences of the daily interest rates are i.i.d. with constant variance $\mathrm{var}(\Delta\tilde{r})$. Hence the variance of the monthly differenced rates, denoted by $\mathrm{var}(\Delta\tilde{r}_t)$, is:

$$(2.3) \qquad \mathrm{var}(\Delta\tilde{r}_t) = d\ \mathrm{var}(\Delta\tilde{r})$$

where $\mathrm{var}(\cdot)$ is the variance operator.

Now suppose that we first average the daily observations within each month and then take the first differences between these averages. As Working [1960] has shown, this averaging induces a spurious first order serial correlation effect in the differenced series. The average rate in month $t+1$, denoted by $\bar{r}_{td,(t+1)d}$, is:

$$(2.4) \qquad \bar{r}_{td,(t+1)d} = \frac{1}{d}\sum_{i=1}^{d} \tilde{r}_{td+i} = \tilde{r}_{td} + \frac{1}{d}\sum_{i=1}^{d}\sum_{j=1}^{i} \Delta\tilde{r}_{td+j}$$

where the last equality follows from eq.(2.1). The difference between the averaged rates, denoted by $\Delta\bar{r}_{td,(t+1)d}$, is:

$$(2.5) \qquad \Delta\bar{r}_{td,(t+1)d} = \bar{r}_{td,(t+1)d} - \bar{r}_{(t-1)d,td} = \frac{1}{d}\left[\sum_{i=1}^{d}\sum_{j=1}^{i} \Delta\tilde{r}_{td+j} - \sum_{i=1}^{d-1}\sum_{j=1}^{i} \Delta\tilde{r}_{td+1-j}\right]$$

Using Working s [1960] results, the variance of this differenced average is:

$$\text{(2.6)} \qquad \operatorname{var}(\Delta\bar{r}_{td,(t+1)d}) = \frac{2d^2+1}{3d}\operatorname{var}(\Delta\tilde{r}) = \left[\frac{2d^2+1}{3d^2}\right]\operatorname{var}(\Delta\tilde{r}_t)$$

and the first order serial correlation of the differenced average, r_1, is:

$$\text{(2.7)} \qquad r_1 \equiv \operatorname{corr}(\Delta\bar{r}_{td,(t+1)d}, \Delta\bar{r}_{(t-1)d,td}) = \frac{d^2-1}{2(2d^2+1)}$$

where corr(·,·) is the correlation operator. Higher order serial correlation coefficients remain zero.

The primary focus is on monthly series. Assuming that there are on average $d = 21$ trading days in a month, the term between square brackets in eq.(2.6) approaches its limit value of $2/3$. This means that within-month averaging reduces the volatility of the differenced series with almost 20%. The reduction of volatility is summarized in Table 1 for various values of d. Between the two extremes (the lowest value $d = 2$ and the limit value $d \to \infty$) we find the weekly ($d = 5$) and monthly ($d = 21$) averages. The decrease in volatility from averaging may imply a substantial loss of information contained in the original (ultimo) series. This issue is further investigated in section 3.

a correction for spurious autocorrelation

Since second and higher order serial correlation coefficients are zero we infer that the differenced averaged series $\Delta\bar{r}_{td,(t+1)d}$ follows a MA(1)-process:

$$\text{(2.8)} \qquad \Delta\bar{r}_{td,(t+1)d} = \tilde{e}_{td,(t+1)d} + a\tilde{e}_{(t-1)d,td}$$

where $\tilde{e}_{td,(t+1)d}$ is a zero mean, serially uncorrelated and constant variance disturbance term and a is the moving average parameter. Inverting the MA(1)-process into an AR(∞)-process we have:[4]

$$\text{(2.9)} \qquad \tilde{e}_{td,(t+1)d} = \frac{\Delta\bar{r}_{td,(t+1)d}}{1+aL} = \Delta\bar{r}_{td,(t+1)d}\left[1 - aL + a^2L^2 - \ldots\right]$$

where L is the lag operator. The variate $\tilde{e}_{td,(t+1)d}$ now represents the monthly interest rate change, corrected for the spurious first order serial correlation from the averaging:

$$\text{(2.10)} \qquad \tilde{e}_{td,(t+1)d} = \Delta\bar{r}^*_{td,(t+1)d}$$

In order to perform this correction we first determine the theoretical expected value of the spurious first order serial correlation. For the monthly case $d = 21$ we have from eq. (2.7) $r_1 = 0.2492$. As the first order serial correlation for the MA(1)-process (2.8) is $r_1 = a \cdot (1+a^2)^{-1}$, this implies that $a = 0.2670$. Truncating eq. (2.9) after, say, four lags we can compute the corrected series $\Delta\bar{r}^*_{td,(t+1)d}$ by:

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[4] The first order serial correlation for the MA(1)-process (2.8) is $r_1 = a/(1+a^2)$. As d has a lower bound of 2, it follows from (2.7) that $1/6 \le r_1 < 1/4$. This in turn implies that $-0.1623 \le a < 0.2679$. As $|a| < 1$, invertibility of the MA(1)-process is guaranteed.

$$(2.11) \quad \Delta\bar{r}^*_{td,(t+1)d} = \Delta\bar{r}_{td,(t+1)d} - 0.2670 \cdot \Delta\bar{r}_{(t-1)d,td} + 0.0713 \cdot \Delta\bar{r}_{(t-2)d,(t-1)d} + \\ - 0.0190 \cdot \Delta\bar{r}_{(t-3)d,(t-2)d} + 0.0051 \cdot \Delta\bar{r}_{(t-4)d,(t-3)d}$$

Moving forward, a simple updating scheme is $\Delta\bar{r}^*_{(t+1)d,(t+2)d} = \Delta\bar{r}_{(t+1)d,(t+2)d} - a\Delta\bar{r}^*_{td,(t+1)d}$. After applying this filter, the actual first order serial correlation of $\Delta\bar{r}^*_{td,(t+1)d}$ is zero. It directly follows from the MA(1)-process that the variance of the filtered averaged interest rate $\Delta\bar{r}^*_{td,(t+1)d}$ is:

$$(2.12) \quad \mathrm{var}(\Delta\bar{r}^*_{td,(t+1)d}) = \frac{\mathrm{var}(\Delta\bar{r}_{td,(t+1)d})}{1+a^2} = \frac{2d^2+1}{3d^2(1+a^2)} \mathrm{var}(\Delta\tilde{r}_t)$$

It also directly follows that the correlation between the filtered and unfiltered changes in the averaged rate is:

$$(2.13) \quad \mathrm{corr}(\Delta\bar{r}_{td,(t+1)d}, \Delta\bar{r}^*_{td,(t+1)d}) = \left(1+a^2\right)^{-1/2}$$

So the volatility reduction of filtering a differenced averaged series is small compared to the reduction in volatility that results from within-month averaging. Table 1 again summarizes this effect for various values of d. As we will see in section 3, however, the effect of filtering on cross correlations is substantial.

It is unlikely that interest rate changes exhibit constant variance and that the interest rate follows a random walk. We therefore do not expect the results to hold exactly. However, even when the interest rate process shows weak mean-reversion and heteroskedasticity effects we hope that the MA(1)-process in (2.8) holds as a suitable approximation and that the filter (2.11) will remove the spurious component from the autocorrelation.

Table 1: Autocorrelation effects from averaging.

r_1 is the first order spurious autocorrelation and α is the MA(1) parameter.
Standard deviation σ of differenced averages compared to differenced ultimos. The table entries are the proportionality factors of indicated averaged case with respect to ultimo case.

	$d=2$	$d=5$	$d=21$	$d \to \infty$
r_1	0.167	0.235	0.249	0.250
α	0.172	0.250	0.267	0.268
wrt $s(\Delta\tilde{r}_t)$:				
$s(\Delta\bar{r}_t)$	0.764	0.808	0.816	0.816
$s(\Delta\bar{r}^*_t)$	0.753	0.784	0.788	0.788

3. Averaging and the effect on cross correlations

Suppose we want to investigate the relationship between some factor $\tilde{y}$ and the interest rate $\tilde{r}$. $\tilde{y}$ could for example be the logarithm of an equity index, a bond index, or an exchange rate. Considering efficient market variables we may assume that $\tilde{y}$ follows a random walk:

$$\tilde{y}_{td+1} = \tilde{y}_{td} + \Delta\tilde{y}_{td+1} \tag{3.1}$$

where the increments $\Delta\tilde{y}_{td+i}$ (the daily continuous compounded returns) are i.i.d. with constant variance $\mathrm{var}(\Delta\tilde{y})$. Hence the variance of the monthly factor return, $\mathrm{var}(\Delta\tilde{y}_t)$, is:

$$\mathrm{var}(\Delta\tilde{y}_t) = d\ \mathrm{var}(\Delta\tilde{y}) \tag{3.2}$$

Using the same notation as before, the monthly factor return can be expressed as the sum of the daily returns:

$$\Delta\tilde{y}_{td,(t+1)d} = \sum_{i=1}^{d} \Delta\tilde{y}_{td+i} \tag{3.3}$$

effect on correlation

We denote the contemporaneous covariance between daily factor returns and interest rate changes as $\mathrm{cov}(\Delta\tilde{y}, \Delta\tilde{r})$. We assume that cross correlations across time are absent. From (2.2) and (3.3) it follows that the covariance between monthly differences, $\mathrm{cov}(\Delta\tilde{r}_t, \Delta\tilde{y}_t)$, is:

$$\mathrm{cov}(\Delta\tilde{y}_t, \Delta\tilde{r}_t) = d \cdot \mathrm{cov}(\Delta\tilde{y}, \Delta\tilde{r}) \tag{3.4}$$

What will now be the effect of substituting the differenced average interest rates for the differenced ultimo rates? From (2.5) and (3.3) we obtain:

$$\begin{aligned} \mathrm{cov}(\Delta\tilde{y}_{td,(t+1)d}, \Delta\bar{r}_{td,(t+1)d}) &= \mathrm{cov}\left(\sum_{i=1}^{d} \Delta\tilde{y}_{td+i} , \frac{1}{d} \sum_{i=1}^{d} \sum_{j=1}^{i} \Delta\tilde{r}_{td+j} \right) \\ &= \frac{1}{d} \sum_{i=1}^{d} \mathrm{cov}\left(\Delta\tilde{y}_{td+i} , (d+1-i)\, \Delta\tilde{r}_{td+i}\right) \\ &= \frac{1}{d} \sum_{i=1}^{d} i \cdot \mathrm{cov}\left(\Delta\tilde{y} , \Delta\tilde{r}\right) \\ &= \tfrac{1}{2}(d+1)\,\mathrm{cov}\left(\Delta\tilde{y} , \Delta\tilde{r}\right) = \frac{d+1}{2d}\mathrm{cov}\left(\Delta\tilde{y}_t , \Delta\tilde{r}_t\right) \end{aligned} \tag{3.5}$$

where the first equality follows from the independent increments assumption and the last equality from (3.4). Even for relatively small values of d averaging the interest rate series almost halves the covariance with the economic factor return. Since averaging also affects variance we determine the correlation between the factor return and the change in the averaged interest rate:

$$(3.6)\qquad \operatorname{corr}(\Delta\tilde{y}_{td,(t+1)d}, \Delta\bar{r}_{td,(t+1)d}) = \frac{\operatorname{cov}(\Delta\tilde{y}_{td,(t+1)d}, \Delta\bar{r}_{td,(t+1)d})}{\left[\operatorname{var}(\Delta\tilde{y}_{td,(t+1)d})\operatorname{var}(\Delta\bar{r}_{td,(t+1)d})\right]^{1/2}}$$

$$= \left[\tfrac{1}{2}(d+1)\sqrt{\frac{3}{2d^2+1}}\right]\operatorname{corr}\left(\Delta\tilde{y}_{td,(t+1)d}, \Delta\tilde{r}_{td,(t+1)d}\right)$$

For the lowest value d can take, $d = 2$, the term between square brackets in eq.(3.6) equals 0.866 and for $d \to \infty$ this proportionality factor becomes $\sqrt{3/8} = 0.612$. For $d = 21$ the term is very close to this limit value: 0.641. Since the square of this term equals 0.411, about 60% of the factor variance explained by interest rate changes vanishes by averaging the interest rate! These effects on R^2 are summarized in Table 2.

From (2.8) it follows that the covariance between the monthly factor return and the change in the averaged interest rate does not change when the filter (2.11) is applied. The correlation between the monthly factor return and the filtered averaged interest rate then becomes:

$$(3.7)\qquad \operatorname{corr}(\Delta\tilde{y}_{td,(t+1)d}, \Delta\bar{r}^{*}_{td,(t+1)d}) = \sqrt{1+a^2}\,\operatorname{corr}(\Delta\tilde{y}_{td,(t+1)d}, \Delta\bar{r}_{td,(t+1)d})$$

$$= \left[\tfrac{1}{2}(d+1)\sqrt{\frac{3(1+a^2)}{2d^2+1}}\right]\operatorname{corr}\left(\Delta\tilde{y}_{td,(t+1)d}, \Delta\tilde{r}_{td,(t+1)d}\right)$$

Filtering raises the proportion explained variance with a factor a^2. For $d = 2$, we have $r_1 = 1/6$ and $a = 0.1716$, so the term between square brackets in (3.7) equals 0.879. For $d \to \infty$ the proportionality factor becomes 0.634. For $d = 21$ the term is again very close to this limit value: 0.663. Since the square of this term equals 0.440, about $a^2 = 7\%$ more factor variance can be explained by filtering monthly averaged interest rate changes; see Table 2.

Table 2: Cross-correlation effects from averaging.

α is the MA(1) parameter of the differenced average series. Determination coefficients R^2 and regression coefficients *b* of differenced averages compared to differenced ultimos. The table entries are the proportionality factors of indicated averaged case with respect to ultimo case.

	$d=2$	$d=5$	$d=21$	$d\to\infty$
α	0.172	0.250	0.267	0.268
wrt $R^2(\Delta\tilde{y}_t, \Delta\tilde{r}_t)$:				
$R^2(\Delta\tilde{y}_t, \Delta\bar{r}_t)$	0.750	0.529	0.411	0.375
$R^2(\Delta\tilde{y}_t, \Delta\bar{r}_t^{*})$	0.772	0.562	0.440	0.402
wrt $b(\Delta\tilde{y}_t, \Delta\tilde{r}_t)$:				
$b(\Delta\tilde{y}_t, \Delta\bar{r}_t)$	1	0.882	0.785	0.750
$b(\Delta\tilde{y}_t, \Delta\bar{r}_t^{*})$	1.030	0.937	0.841	0.804

The case considered above in which the interest rate is averaged but the market factor not is in our opinion most prevailing. For the sake of completeness, however, we investigate the effect of averaging both series. Defining the monthly first difference of the market factor $\tilde{y}_t$ analogous to eq.(2.5) we have:

$$(3.8)\qquad \operatorname{cov}(\Delta\bar{y}_{td,(t+1)d},\Delta\bar{r}_{td,(t+1)d}) = \frac{1}{d^2}\operatorname{cov}\left(\left[\sum_{i=1}^{d}\sum_{j=1}^{i}\Delta\tilde{y}_{td+j} - \sum_{i=1}^{d-1}\sum_{j=1}^{i}\Delta\tilde{y}_{td+1-j}\right], \left[\sum_{i=1}^{d}\sum_{j=1}^{i}\Delta\tilde{r}_{td+j} - \sum_{i=1}^{d-1}\sum_{j=1}^{i}\Delta\tilde{r}_{td+1-j}\right]\right)$$

$$= \frac{1}{d^2}\left[\sum_{i=1}^{d}(d+1-i)^2 + \sum_{i=1}^{d-1}(d-i)^2\right]\operatorname{cov}(\Delta\tilde{y},\Delta\tilde{r})$$

$$= \frac{2d^2+1}{3d}\operatorname{cov}(\Delta\tilde{y},\Delta\tilde{r}) = \frac{2d^2+1}{3d^2}\operatorname{cov}(\Delta\tilde{y}_t,\Delta\tilde{r}_t)$$

where first step follows from the independence of the increments over time and the last equality follows from (3.4). Since the variances change in the same proportion as the covariance, the correlation between the differenced averaged series is identical to the correlation between the differenced ultimo series:

$$(3.9)\qquad \operatorname{corr}(\Delta\bar{y}_{td,(t+1)d},\Delta\bar{r}_{td,(t+1)d}) = \operatorname{corr}(\Delta\tilde{y}_t,\Delta\tilde{r}_t)$$

effect on regression coefficients

Suppose we investigate the relationship between the factor and the interest rate by univariate regression. Such a regression can be used to measure the interest rate risk exposure of equities or to estimate the effective duration of a bond portfolio. This is an important issue in the context of interest rate risk.[5] Since we already have determined the variances of the various differenced series and their correlations, the regression slopes from regressing the factor return on the change in (ultimo, averaged and corrected) interest rates readily follow.[6] Trivially, the regression coefficient is invariant under the length of the differencing interval of the original data:

$$(3.10)\qquad b(\Delta\tilde{y},\Delta\tilde{r}) = \frac{\operatorname{cov}(\Delta\tilde{y},\Delta\tilde{r})}{\operatorname{var}(\Delta\tilde{r})} = b(\Delta\tilde{y}_t,\Delta\tilde{r}_t)$$

From (2.6) and (3.5) the slope for the changes in the averaged interest rate is:

$$(3.11)\qquad b(\Delta\tilde{y}_{td,(t+1)d},\Delta\bar{r}_{td,(t+1)d}) = \left[\frac{3d(d+1)}{2(2d^2+1)}\right] b(\Delta\tilde{y}_t,\Delta\tilde{r}_t)$$

[5] See for example Leibowitz [1986] and Hallerbach [1994].

[6] The regression equations include a constant term. Note that the regression slope *b(y,x)* from regressing *y* on *x* and the slope *b(x,y)* from regressing *x* on *y* are related by $R^2 = b(y,x) \times b(x,y)$.

For $d \to \infty$ the proportionality factor between square brackets becomes ¾. For $d = 21$ the term is very close to this limit value, 0.785, so using monthly averaged interest rates would underestimate the actual slope by more than 20%!

Next we consider the slope on the filtered averaged series.[7] From (2.12) and (3.7) we find:

$$(3.12)\quad b(\Delta\tilde{y}_{td,(t+1)d}, \Delta\bar{r}^{*}_{td,(t+1)d}) = (1+a^2)\, b(\Delta\tilde{y}_{td,(t+1)d}, \Delta\bar{r}_{td,(t+1)d})$$

$$= \left[(1+a^2) \frac{3d(d+1)}{2(2d^2+1)} \right] b(\Delta\tilde{y}_t, \Delta\tilde{r}_t)$$

For $d \to \infty$ the proportionality factor between square brackets becomes 0.804. For $d = 21$ the term is again close to this limit value: 0.841. Filtering thus would reduce the downward bias to just over 15%. Table 2 summarizes the effect of averaging on various parameters.

Finally, since averaging changes the variances and covariances of differences with the same proportionality factor $(2d^2+1)/3d^2$ we have:

$$(3.13)\quad b(\Delta\bar{y}_{td,(t+1)d}, \Delta\bar{r}_{td,(t+1)d}) = b(\Delta\tilde{y}_t, \Delta\tilde{r}_t)$$

effect on significance levels

We consider univariate regressions of $\tilde{y}_t$ on interest rate variables using T observations. The regression s *F*-statistic with 1 and T-2 degrees of freedom equals the square of the *t*-statistic with T-2 degrees of freedom. Hence we can use the simple relationship between the *t*-statistic and the (unadjusted) determination coefficient R^2:

$$(3.14)\quad |t| = \sqrt{F} = \sqrt{\frac{T-2}{R^{-2}-1}}$$

Using the correlation coefficients derived above and given the total number of observations T, the *t*-statistics of the various regressions can easily be determined. For the relevant averaging periods and for a choice of R^2 s, Table 3 gives the ratios of the *t*-statistics relative to the case in which differenced ultimos are regressed on differenced ultimos. The longer the averaging interval the greater the decrease in significance. Even for a moderate averaging interval of one week the *t*-statistic drops substantially. Also the stronger the relationship between differenced ultimos the larger the loss in significance by averaging. Filtering the differenced average only marginally improves significance. Note from (3.9) and (3.13) that averaging both factor and interest rate series does not affect slope, correlation nor significance.

[7] Since the corrected series is in fact a residual from an auxiliary estimated time series process, this could induce biases in the estimated regression coefficient. However, as Pagan [1984] has shown, the OLS estimators for the auxiliary generated residuals in the main regression are both consistent and efficient.

Table 3: Averaging and the significance of the estimated relationship.

Significance of regression coefficients b (or of determination coefficients R^2) of estimated relationships between factor return $\Delta\tilde{y}_t$ and various interest rate changes. The table entries are the proportionality factors of the $|t|$ values of the indicated averaged case with respect to ultimo case, for various levels of the ultimo case R^2.

	$d = 2$	$d = 5$	$d = 21$	$d \to \infty$
$\Delta\tilde{y}_t$ and $\Delta\bar{r}_t$:				
$R^2 = 0.90$	0.481	0.318	0.255	0.239
0.75	0.655	0.468	0.386	0.361
0.50	0.775	0.600	0.509	0.481
0.25	0.832	0.676	0.587	0.557
0.10	0.854	0.709	0.621	0.592
0.05	0.860	0.718	0.632	0.602
$\Delta\tilde{y}_t$ and $\Delta\bar{r}_t^*$:				
$R^2 = 0.90$	0.503	0.338	0.270	0.251
0.75	0.677	0.493	0.405	0.379
0.50	0.793	0.625	0.531	0.502
0.25	0.847	0.700	0.609	0.579
0.10	0.868	0.732	0.643	0.614
0.05	0.873	0.741	0.653	0.624

4. An application to US interest rates and equities: gauging equity durations

In order to put our theoretical results in a practical perspective we investigate a selection of US interest rates over the period January 2, 1990 through December 31, 2000. The monthly interest rate series available from the Federal Reserve Board s *H.15 Statistical Data Release* consist of monthly averages of daily observations. We selected the complete Treasury Constant Maturity Series which offers yield to maturities on the most actively traded marketable Treasuries, interpolated for ten remaining maturities ranging from three months up to 30 years. We used the available daily data series to construct the ultimo month series.[8] In order to study the cross-correlation effects we take the S&P 500 Total Return Index as the other financial market variable. The ultimo month index series is readily available; we used daily index data to construct the within-month averaged index data series. Finally we have log-transformed all series so that the first differences represent log returns.[9]

The derivations in section 2 and 3 assume that the daily series exhibit constant variance and no autocorrelation. In practice we would expect some mean reversion effects

[8] Data on the 20-year Constant Maturity Treasury are only available starting from October 1, 1993.
[9] This is common practice. Repeating all estimations for the original data yielded comparable results.

Table 4: Actual cross- and autocorrelation effects of averaging.

Using 120 monthly observations over the period 1990:1 through 1999:12. Actual standard deviations σ (in %) of differenced ultimo month interest rates, and regression coefficients b, determination coefficients R^2 and t-statistics of regression of S&P500 returns on these interest rate shocks. The entries in the wrt columns show the proportionality factors of the indicated differenced average case with respect to differenced ultimo case. For the fractional t-statistics in the last two columns we used T=120 and the actual R^2s (see eq.(3.14)).

		wrt $s(\Delta\tilde{r}_t)$			wrt $b(\Delta\tilde{y}_t,\Delta\tilde{r}_t)$			wrt $R^2(\Delta\tilde{y}_t,\Delta\tilde{r}_t)$			wrt $t(\Delta\tilde{y}_t,\Delta\tilde{r}_t)$	
	$s(\Delta\tilde{r}_t)$	$s(\Delta\bar{r}_t)$	$s(\Delta\bar{r}_t^*)$	$b(\Delta\tilde{y}_t,\Delta\tilde{r}_t)$	$b(\Delta\tilde{y}_t,\Delta\bar{r}_t)$	$b(\Delta\tilde{y}_t,\Delta\bar{r}_t^*)$	$R^2(\Delta\tilde{y}_t,\Delta\tilde{r}_t)$	$R^2(\Delta\tilde{y}_t,\Delta\bar{r}_t)$	$R^2(\Delta\tilde{y}_t,\Delta\bar{r}_t^*)$	$t(\Delta\tilde{y}_t,\Delta\tilde{r}_t)$	$t(\Delta\tilde{y}_t,\Delta\bar{r}_t)$	$t(\Delta\tilde{y}_t,\Delta\bar{r}_t^*)$
3 month	0.193	0.940	0.895	-2.698	1.084	0.931	0.018	1.040	0.694	1.474	1.020	0.831
6 month	0.207	0.967	0.913	-3.686	0.942	0.905	0.039	0.830	0.682	2.178	0.908	0.820
1 year	0.239	0.940	0.867	-3.936	0.867	0.904	0.059	0.664	0.614	2.708	0.807	0.775
2 year	0.273	0.918	0.846	-3.569	0.858	0.924	0.063	0.621	0.611	2.814	0.778	0.772
3 year	0.282	0.899	0.827	-3.833	0.844	0.923	0.078	0.576	0.582	3.153	0.746	0.750
5 year	0.277	0.879	0.809	-4.131	0.832	0.911	0.087	0.535	0.544	3.346	0.716	0.722
7 year	0.263	0.863	0.801	-5.004	0.818	0.879	0.115	0.498	0.496	3.920	0.684	0.682
10 year	0.253	0.844	0.785	-5.533	0.847	0.911	0.130	0.510	0.511	4.201	0.690	0.690
20 year	0.218	0.871	0.825	-4.988	0.834	0.750	0.077	0.528	0.373	3.128	0.713	0.596
30 year	0.211	0.862	0.812	-7.121	0.817	0.868	0.150	0.496	0.497	4.572	0.675	0.675

for the (short) interest rates and (autoregressive) heteroskedasticity for all daily series.[10] Therefore our results will not hold exactly but hopefully they will provide a good approximation.

Of the ultimo month series the 3- and 6-month rates reveal a weak mean reversion at the 95% confidence level; this effect disappears after applying appropriate heteroskedasticity and autocorrelation consistent standard errors (Newey & West [1987]). For all other series the autocorrelation functions of the first differenced ultimos indicate that the total change in the interest rate may be considered as unexpected.

As expected the differenced monthly averages show a strong first order autocorrelation. After applying the filter eq.(2.11) no significant degree of autocorrelation is left. Since we only want to correct for the serial correlation induced by averaging, we used the theoretical value of r_1 to estimate the moving average parameter α.

Table 4 reports the results. The first column gives the standard deviation of the changes in the ultimo month rates (in percentages). The second column shows the volatility of the changes in the averaged rates, expressed as a fraction of the number in the first column. The third column shows the standard deviation of the filtered rate changes, again expressed as a proportion of the first column entry. We see that although the decrease in volatility from the averaging is not as drastic as one would expect (the theoretical value in column two is .82) the proportion volatility remaining after applying the filter is very close to the predicted value of .78. The shorter maturity range shows the largest discrepancies with the theoretical results. Given mean reversion and heteroskedasticity effects this comes as no surprise. Note that 10 year rate complies in an exemplary fashion with the theoretical results. This is important since given the liquidity of the market segment this rate is often considered as the benchmark long interest rate.

Next we turn to cross-correlation effects. All slopes coefficients from regressing equity returns on interest rate changes are significant at the 95% level, except for the 3-month rate. Since we use log-changes the regression slope represents the interest rate elasticity of the S&P 500 Index. For example, the slope of 5.5 for the 10-year rate indicates that the equity duration for the 10-year rate is 5.5: we may expect a drop of 5.5% of the S&P 500 Index when one plus the 10-year rate increases with 1%, ceteris paribus. Notably for the longer rates the empirical results conform well to the theoretical findings. Using the monthly averaged series would lead to a 15-20% underestimation of the equity duration. Filtering the data in order to obtain interest rate shocks only marginally mitigates this problem. The loss in explained variance when using monthly averages is not as high as the nearly 60% predicted but still a staggering 50%! As a result also the drop in significance is quite close to the 35-40% predicted (see the last two columns of Table 4).

Summarizing we conclude that the theoretical findings provide a good indication for the observed empirical biases. Using available monthly averaged interest rate series seriously distorts empirical estimates ranging from interest rate volatilities to equity durations and their significance. We therefore advice a close inspection of the monthly data before use.

[10] For short rates Chan et al. [1992] find only very weak evidence of mean reversion but a predominant heteroskedasticity effect related to the interest rate level.

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