Example 1

A Health Agency has to decide about opening new ambulatories to serve a set of m users ($i = 1, \dots, m$). A preliminary investigation suggests a set of n potential locations. Each users can be served by an ambulatory on location j ($j = 1, \dots, n$) if the location is not "too far" from the user's home. Thus, we are given a matrix A whose entry a_{ij} has value 1 if user i can be served by an ambulatory open in location j and 0 otherwise. The cost of opening an ambulatory on location j is equal to c_j. Moreover, the ambulatory in location j can serve at most M_j users. Finally, for planning reasons, the Health Agency wants to open at least P ambulatories, serve all users and minimize the overall cost.

Integer Linear Programming Model:

$$x_{ij} := \begin{cases} 1 & \text{if user } i \text{ is served by ambulatory } j \\ 0 & \text{otherwise} \end{cases} \qquad i = 1, \dots, m; j = 1, \dots, n;$$

$$y_j := \begin{cases} 1 & \text{if ambulatory } j \text{ is open} \\ 0 & \text{otherwise} \end{cases} \qquad j = 1, \dots, n;$$

$$\min \sum_{j=1}^{n} c_j y_j \tag{1a}$$

$$\sum_{j=1}^{n} a_{ij} x_{ij} = 1 \qquad \forall i = 1, \dots, m \tag{1b}$$

$$\sum_{i=1}^{m} x_{ij} \leq M_j y_j \qquad \forall j = 1, \dots, n \tag{1c}$$

$$\sum_{j=1}^{n} y_j \geq P \tag{1d}$$

$$y_j \in \{0, 1\} \qquad \forall j = 1, \dots, n \tag{1e}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i = 1, \dots, m; \quad \forall j = 1, \dots, n \tag{1f}$$

Example 2

The Ministry of Home Security must decide concerning the opening of new Fire Stations so as to serve q neighborhoods ($i = 1, \dots, q$) in Bologna. A preliminary investigation led to the consider a set of s potential areas j, $j = 1, \dots, s$ where a Fire Station can be built. Each neighborhood i can be served by a Fire Station on area j if that is not "too far". Then, we are given a binary matrix A whose entry a_{ij} is 1 if neighborhood i can be served by a Fire Station on area j and 0 otherwise. The opening cost for the Fire Station in area j is r_j. For security reasons, each neighborhood must be potentially served by at least 2 (open) Fire Stations, and at least B Fire Stations must be open. The constraints must be satisfied by minimizing the overall opening cost.

Integer Linear Programming Model:

$$y_j := \begin{cases} 1 & \text{if Fire Station } j \text{ is open} \\ 0 & \text{otherwise} \end{cases} \qquad j = 1, \dots, s;$$

$$\min \sum_{j=1}^{s} r_j y_j \tag{2a}$$

$$\sum_{j=1}^{s} a_{ij} y_j \geq 2 \qquad \forall i = 1, \dots, q \tag{2b}$$

$$\sum_{j=1}^{s} y_j \geq B \tag{2c}$$

$$y_j \in \{0, 1\} \qquad \forall j = 1, \dots, s \tag{2d}$$

(2e)

Example 3

A pet shop has n water pools to be used to show a set of m tropical fishes. Each pool j has a cost of c_j and a volume V_j. The shop owner wants to minimize the overall cost of the pools used by considering that, for space reasons, the shop can use at most K pools simultaneously. It is given a graph $G = (V, E)$ whose generic edge a_{ih} defines the incompatibility between fishes i and j, respectively, of being put in the same pool (because of the natural nutrition chain). Moreover, we are given a binary matrix B whose entry b_{ij} is equal to 1 if fish i can be assigned to pool j and 0 otherwise. Finally, each fish i needs a living space of v_i, thus the sum of the living space of the fishes assigned to the same pool j cannot be higher than the volume of the volume V_j.

Integer Linear Programming Model:

$$y_j := \begin{cases} 1 & \text{if pool } j \text{ is used} \\ 0 & \text{otherwise} \end{cases} \qquad j = 1, \dots, n;$$

$$x_{ij} := \begin{cases} 1 & \text{if fish } i \text{ is assigned to pool } j \\ 0 & \text{otherwise} \end{cases} \qquad i = 1, \dots, m; , \quad j = 1, \dots, n;$$

$$\min \sum_{j=1}^{n} c_j y_j \tag{3a}$$

$$\sum_{j=1}^{n} y_j \leq K \tag{3b}$$

$$\sum_{j=1}^{n} b_{ij} x_{ij} = 1 \qquad \forall i = 1, \dots, m \tag{3c}$$

$$\sum_{i=1}^{m} v_i x_{ij} \leq V_j y_j \qquad \forall j = 1, \dots, n \tag{3d}$$

$$x_{ij} + x_{hj} \leq y_j \qquad \forall (i, h) \in A, j = 1, \dots, n \tag{3e}$$

$$x_{ij}, y_j \in \{0, 1\} \qquad \forall i = 1, \dots, m, j = 1, \dots, n \tag{3f}$$

Discuss alternatives and/or possible improvements to the above model.