# INNOVATION, INVESTMENT AND DEVELOPMENT

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# Searching as a localized process

- Recent economic literature has persuasively argued that efforts directed at seeking ways to upgrade technologically the productive apparatus is a bounded search in a space of opportunities
- This search is necessarily local and subject to evolution as skills and competence are acquired and learning from experience takes place.
- The first important observation is that searching is a process cast into agents' bounded rationality.
- The second and by no means less relevant one is that agents can neither fully scan the entire domain of technological opportunities in theory available in the whole economic system nor can they immediately or instantaneously translate actual observations of better or applicable techniques into adoptable plans to upgrade and invest.

# Bounded rationality and local searching

- The second and by no means less relevant one is that agents can neither fully scan the entire domain of technological opportunities in theory available in the whole economic system nor can they immediately or instantaneously translate actual observations of better or applicable techniques into adoptable plans to upgrade and invest.
- Identifying informative sources and collecting information are uncertain activities which depend on acquired capabilities, established technological prowess, consolidated knowledge.
- The purpose of this lecture is to investigate the evolutionary process of imitation and innovation as a process of searching in a given neighbourhood of firms.

# The economy

- In each point in time it is possible, at least in theory, to identify the technological frontier.
- Consider an economy made up by a large number $J$ of sectors.
- It is assumed that in each sector $j$ there is a leader pursuing innovation, and $F_j - 1$ followers trying to imitate it.
- Let us concentrate our attention to the $J$ leaders who invest and search to innovate.

# An historical sequence of technologies

- Consider the following technological sequence:

$$T^j(1) < T^j(2) < \ldots < T^j(N_t^j) \equiv T_{j,t}$$

- This is a sequence of frontier technologies each of which appeared at time $T^j$in sector $j$. They are identified by a progressive number.
- $N_t^j$ is the last technology to appear in sector $j$ and still in existence at time $t$. The sequence is a full 'history' of technology: the $n-th$ technique having been adopted at time $T^j(n)$.

# Information

- It is a well established fact that information is broadcast by leading and innovating firms.
- This information concerns new technological principles and paradigms. Thus, these firms allow their diffusion across sectors and industries.
- When a new technological breakthrough is achieved its content has a positive probability to find applications in other sectors of activity.
- The reason lies with the fact that there exists interfirm connectivity and that firms carry out an active search.

# Broadcasting information

- The capability of understanding and processing information coming from a different sector and a different technological context depends on a common knowledge basis.
- The transmission of such information depends on the strength of this shared knowledge measuring the potential intensity of firms' interaction and the probability of actually passing it on.
- Let this measure be defined, in general, by $\varepsilon^{i,j} \in [0,1]$ f or any two leaders belonging to different sectors $i$ and $j$.
- In what follows, the average of these broadcasting strengths will be used.

## Definition

- Consider the following assumption:
- *The average probability for the economy as a whole that information be passed on is*

$$\hat{\varepsilon} = 1 - \alpha \frac{1}{J}$$

- $J$ has an important role to play: it is the number of sectors or industries that are present in the economy.
- Thus, the higher is this number, that is, the denser is the economy's structure, i.e. the more overlapping are the neighbourhoods within which firms search, the greater is the strength of information transmission.

# Technological search

- Leading firms collect information *packets* from other leading firms within a small cognitive neighbourhood. Let these *packets* be called *bits*, borrowing this concept from computer terminology.
- Collection of information is idiosyncratic, gradual and it is sequential. Accordingly, consider the following assumption:
- *For each innovation, the generic innovator $i$ stands probability $\pi_z$ of locking into a technological search $z$, requiring an informative sequence of $S_z$* bits, $z = 1, 2, ..., Z$.
- Obviously $\sum_{z=1}^{Z} \pi_z = 1$. Without loss of generality, rank these possible technological searches as $S_1 < S_2 < ... < S_Z$

# Information and productivity growth

- Refer to $S_j(n)$ to denote the information content of the n-th innovation in sector $j$. Hence $S_j(N_t^j)$ is the content of the extant technology $N_t^j$
- It is a fairly well established fact that, at least on average, innovations requiring a greater effort, which is here measured by the number of bits collected, yield a greater productivity increase.
- This fact justifies the following assumption. Let us denote by $\lambda_z$ the productivity growth rate resulting from the innovation produced by a search of type $z$, then

$$\lambda_1 < \lambda_2 < ... < \lambda_Z$$

- When a sector is involved in an innovation wave, productivity gains occur at a rate which is specific of the sector and depends on the amount of information gathered.

# Neighbours

- Information is retrieved within a neighbourhood of firms.
- It is assumed that the longer are the informative sequences, the larger is the number of neighbours $S^*$, which each leader contacts in his search.
- The average number of neighbours is equal to the mean value of the random variable length of the informative sequence:

$$S^* = \sum_{z=1}^{Z} \pi_z S_z$$

# Autonomous innovation activity

- Firms are also assumed to autonomously obtain information through an independent research and development (R.&D.) activity.
- There clearly cannot be any spreading of innovative principles if there is no autonomous, self-generation.
- The search process, therefore, requires that autonomous searching be also conducted since if there were no primary innovative input, no informative interaction could take place.
- Accordingly consider an entirely exogenous innovative event, i.e. of obtaining an information bit, as occurring with a Poisson arrival rate equal to $h$.
- Hence, there are two overlapping processes: an exogenous one and an interactive one.

# Definitions

- The process of gleaning information is gradual. At any point in time there are firms that have gone through the whole or part of an information sequence.
- Consider, for simplicity's sake, an economy with just one information sequence. Assume also that there are $S$ states in such sequence:

$$0, 1, 2, .......k, , k+1......., c, a$$

- 0 indicates that no information has been collected yet; $k$ that state $k$ has been reached and that $k$ bits have been collected; $c$ indicates that a state has been attained where only one bit is required to complete the sequence, it is therefore a critical state;
- $a$ signals the completion of the sequence and the firm that happens to be there becomes active and innovates after which it starts the process back from 0.

# Firms and states of information

- Always in the spirit of simplification it is assumed that the number $S$ of bits required by the informational sequence coincides with the average number of neighbours $S^*$.
- Denote by $\rho_a$ the share of firms that have reached a complete set of bits and consequently innovate.
- These firms then spread information to others that are still in the process of collecting the required ones and begin again their searching and learning process.
- $\rho_c$ is the share of firms that are in the critical stage, that require just an additional bit to complete the sequence; thus, the number of firms that will eventually reach the active stage $a$ depends on this critical number.
- $\rho_k$ denotes the share of firms that are still in stage k. Note that $\rho_0 = \rho_a$ and that $\sum_{k=1}^{a} \rho_k = 1$

# The probability of gaining information

- Since all firms carry an independent search, they can gain an information bit with probability $h$.
- Firms stand a positive probability of obtaining informative bits through interaction from firms that having innovated broadcast information.
- This information comes from the share $\rho_a$ of firms that are innovating having completed the information chain.
- A firm obtains it from a neighbourhood $S^*$ of which, on average, $S^*\rho_a$ are innovators that broadcast information, each of which does so with probability $S^*\rho_a\hat{\varepsilon}$.
- Hence the probability of gaining an informative bit is $h + S^*\rho_a\hat{\varepsilon}$.

# The equations

- The following system of equations describes the gradual process of gleaning information.

$$\dot{\rho}_a = -\rho_a + (h + S^* \hat{\varepsilon} \rho_a)\rho_c$$

$$\dot{\rho}_c = -(h + S^* \hat{\varepsilon} \rho_a)\rho_c + (h + S^* \hat{\varepsilon} \rho_a)\rho_{c-1}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\dot{\rho}_k = -(h + S^* \hat{\varepsilon} \rho_a)\rho_k + (h + S^* \hat{\varepsilon} \rho_a)\rho_{k-1}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\dot{\rho}_0 = -(h + S_a^* \hat{\varepsilon} \rho_a)\rho_0 + \rho_a$$

## The stationary state

- The dynamics of this system can be very complex. It is, however, interesting to observe it in the stationary state, i.e. when all $\dot{\rho}_k = 0$.
- This is a way to observe the system as it were in equilibrium: all firms moving from one state of information to the next without altering the share of each state.
- In this case, solutions for the shares of firms in the various stages of information collection are:

$$\rho_c = \ldots\ldots = \rho_k = \rho_{k-1} = \ldots\ldots. = \rho_0$$

- Since it is $\sum_{k=1}^{a} \rho_k = 1$, a sum of $S^*$elements, $\rho_c = \frac{1}{S^*}$ and from the first of equations $\rho_a = \frac{h}{S^*(1-\hat{\varepsilon})}$.

# Measuring the avalanche

- The question to ask is the following: assuming the system in equilibrium, what happens if there is an increase in the exogenous probability that a bit of information enters it?
- In other words, what is the expected number of firms capable of innovating (completing there informative chain)?
- Since it is wished that this sudden increase occurs once and for all, i. e. the system be perturbed only marginally around $h = 0$, the answer is:

$$E(V_T) = \frac{\partial}{\partial h} \rho_a |_{h=0} = \frac{1}{S^*(1 - \hat{\varepsilon})}$$

# The expected number of innovating firms

- Taking into account the components of $\hat{\varepsilon}$

$$E(V_T) = \frac{J}{S^* \alpha}$$

- This can be described as an innovation wave or avalanche. It is clear that this is a wave of an average, expected dimension: waves of all sizes can actually occur.
- It is interesting to note that the higher is $J$, the overall number of sectors, the higher is the number of innovators. This is due to the fact that in this case the higher is the probability of transmission.
- The higher is the shadow cost of gaining informative bits, $S^*$, the lower is such a number which is also lower the higher is $\alpha$.

# The waiting time between avalanches

- It is also straightforward to compute the average waiting time between two avalanches.
- Consider that an exogenous shock that may initiate a new process has a probability of $h$ in the time unit.
- Hence, $\frac{1}{h}$ is the period that must be awaited on average for such a shock to materialize.
- Since there are $S^*$to be completed to arrive at innovation, it follows that the waiting time $\omega = E(T(N_t+1) - T(N_t)$ is

$$\omega = E(T(N_t+1) - T(N_t) \backsim \frac{S^*}{h}$$

# The expected number of innovations at time t

- Taking into account the average waiting time, we can ask what is the expected number of innovations at any point in time in a given sector $j$ .
- Since the expected average avalanche arriving in sector $j$ is $\frac{E(V_T)}{J} = \frac{1}{S^*\alpha}$, then divided by $\omega$: $\frac{h}{S^{*2}\alpha}$
- and over a period $t$ :

$$E(N_t^j) \sim \frac{h}{S^{*2}\alpha} t$$

- as an average over a long period of time.

# Productivity growth and the economy's growth rate

- Let it be assumed that productivity lowers by $\lambda_j$ real costs of production, equally the cost of labour $a_{j,N_t^j}$ and the cost of capital inputs $a_{k_j,N_t^j}$ ,in sector $j$ and specific to technology $N_t^j$. Thus:

$$a_{j,N_t^j} = a_{j,N_t^j-1} e^{-\lambda_j E(N_t^j)}$$

$$a_{k_j,N_t^j} = a_{k_j,N_t^j-1} e^{-\lambda_j E(N_t^j)}$$

- Consider the average long-term growth of such an economy. It is simply the number of expected innovations per sector over a period $t$ , times the average productivity growth rate $\lambda^*$

$$\gamma_I = \frac{h}{\alpha S^2} \lambda^* t$$

# Final comments

- This is a model that intends to explain an economy's growth rate in terms of the innovative capability that it is able to afford.
- This capability is achieved through a gradual, localized process of searching due to firms' interaction and also due to an independent research and development activity.
- Firms exploit the informative potential of a neighbourhood of firms that together makes up a full network.
- The process unfolds over a sequence which is a trajectory out of many possible ones.
- This activity self-organizes to generate innovations waves.

# References


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