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Optimization of Wind Farm Layout:
A Refinement Method by Random Search

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Abstract

Wind farm layout optimization is to find the optimal positions of wind turbines inside a wind farm, so as to maximize and/or minimize a single objective or multiple objectives, while satisfying certain constraints. Most of the works in the literature divide the wind farm into cells in which turbines can be placed, hence, simplifying the problem from with continuous variables to with discrete variables. In this paper, a refinement method, based on continuous formulation, by using random search is proposed to improve the optimization results based on discrete formulations. Two sets of optimization results of a widely studied test case are refined using the proposed method. One set of the results is from a published work using GA based on discrete formulation, the other set is the improved results using authors' own GA code. Steady improvements are obtained for both sets of results.

1. Introduction

Wind farm is a group of wind turbines located at a site to generate electricity, which is also called as "plant", "cluster", "array" and "park" in the literature. The world's first onshore wind farm was installed in 1980 on the shoulder of Crotched Mountain in southern New Hampshire, USA, with the capacity of 0.6 MW, consisting of 20 wind turbines rated at 30 kW each [1]. In 1991, the world's first offshore wind farm, Vindeby offshore wind farm was erected off the north coast of the Danish Island Lolland, which marked the beginning of offshore wind energy development. It had a total capacity of 4.95 MW and consisted of 11 Bonus 450 kW turbines [2]. Nowadays, the progress of technologies, such as power electronic [3], wind speed forecasting [4], coordinated control [5], together with the increased experience of wind farm construction and operation have enabled the development of modern wind farms, i.e., larger, smarter wind farms, which are typically consisted of hundreds of utility-scale (multi-MW sized) wind turbines and with a total capacity of hundreds MW. In parallel with this trend, the efforts for increasing the percentage of wind power of the total electricity consumption have led to the proliferation of modern wind farms.

Due to the multi-disciplinary nature and the evolution towards larger size, smarter control and more advanced capabilities, the development of wind farm is becoming a highly complex process which pursues multiple and in many cases conflicting objectives under different constraints. It involves different design and engineering tasks, which may come from technical, logistical, environmental, economical, legitimacy and even social considerations [6].

Among all these tasks, the optimization of wind farm layout is a critical one. In the literature, wind farm layout usually refers to the placements of wind turbines inside the wind farm. Therefore, wind farm layout optimization is to determine the positions of turbines inside the wind farm to maximize and/or minimize some objective functions, such as to maximize the energy production and minimize the cost, while meeting various constraints, which may include wind farm boundary, wind turbines proximity, noise emission level, initial investment limit, and so on. In the most general case, i.e., considering the selection of wind turbine number, different wind turbine types, discrete hub heights,

wind farm layout optimization is a multi-objective mixed integer-discrete-continuous nonlinear constrained optimization problem without analytical formulation. It is mathematically complex and can't be solved by using classical analytical optimization techniques.

In the last two decades, this complex problem has received more and more attentions. Different problem formulations have been proposed and various optimization algorithms have been used to tackle this problem. Previous works are based on various simplified formulations, which range from an array of equally spaced turbines [7,8], to an array of unequally spaced turbines [9], to aligned or staggered grid like (row-column) layout [10, 11], to pre-divided discrete grid points for possible turbine location [12], to continuous searching space for possible turbine positions [13,14]; using a range of algorithms, such as Monte Carlo [11,15], genetic algorithm (GA) [12,16], simulated annealing (SA) [14], (PSO) [17]; seeking different kinds of objectives, e.g., maximize the power [9], annual energy production (AEP) [14], profit [8,13], net present value (NPV) [11], minimize the cost of energy (CoE) [12, 15-17], levelized production cost (LPC) [18]. More comprehensive survey of published works can be found in several papers [19-21].

Most of the published works are using grid based discrete formulation, which simplifies the searching space of the optimization problem from continuous space to discrete space. By removing the grid and formulating the problem in the continuous space, the searching space will be enlarged and the potential to find better solutions will be increased. Considering this, a refinement method is proposed in this paper, which formulates the locations of turbines as continuous variables and tries to improve the optimization results from any other methods. This method is applied to a widely studied ideal test case. This case was first proposed and solved using GA by Mossetti et al. [12]. Later improved results were obtained by Grady et al. [16] also using GA. In order to verify the effectiveness of the method over different results, the same problem is also solved using a GA code developed by the authors, which obtains better results for two wind cases than Grady et al. [16]. These two sets of GA optimization results are then both refined by the proposed method, which both obtains steady improvement.

2. Problem formulation

In their seminal work in 1994, Mossetti et al. [12] proposed an ideal test case. This test case is set to find the optimal number and positions of wind turbines in a 50D×50D square field, where D=80m is the rotor diameter of wind turbine and the minimum distance between two turbines is 5D. The area is subdivided into 100 cells with same size. The center of each cell is a possible location for placing wind turbine, which makes the whole field with 100 possible locations and the searching space with 2^{100} possible layouts. The subdivided wind farm area is shown in Fig. 1, where solid lines represent the boundary of wind farm, dashed lines represent the effective boundary, i.e., the boundary of possible locations for wind turbines, and every cross at the center of each cell means a possible location.
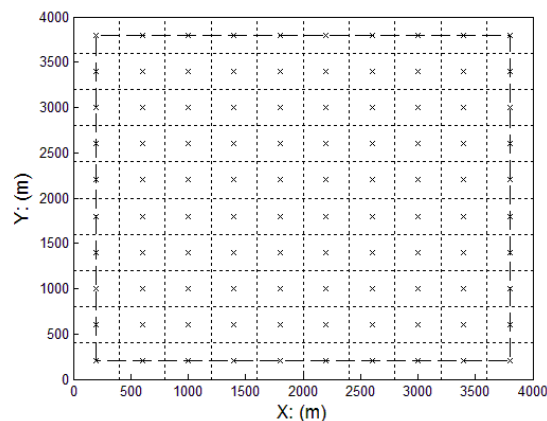
Fig. 1. Subdivided wind farm area.

In order to calculate the wind field in the wind farm, the wake effects between turbines have to been modeled appropriately. In Mossetti study, Jensen wake model [22] is used, which is developed by considering that momentum is conserved within the wake, and that the wake region expands linearly in the direction of wind flow. Fig. 2 shows the schematic of this model.
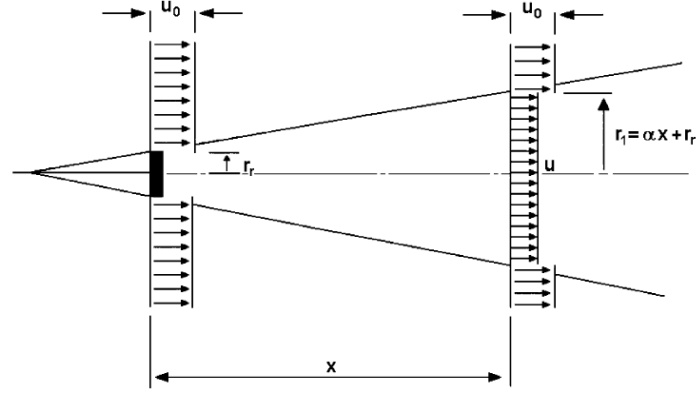
Fig. 2. Schematic of Jensen wake model.

The wind speed downstream of the turbine is governed by the following expression:

$$u = u_0 \left[1 - \frac{2a}{(1 + \alpha(x/r_r))^2} \right], \tag{1}$$

where a is the axial induction factor, x is the distance downstream the turbine, r_r is the downstream rotor radius, which is related to rotor radius r_0 by the following expression:

$$r_r = r_0 \sqrt{\frac{1-a}{1-2a}}, \tag{2}$$

and α is the entrainment constant, also known as the wake decay constant, which is empirically computed as

$$\alpha = \frac{0.5}{\ln(z/z_0)}, \tag{3}$$

where z is the hub height and z_0 is the surface roughness of the terrain. The axial induction factor a can be calculated from the turbine thrust coefficient C_T as

$$a = \frac{1 - \sqrt{1 - C_T}}{2}, \tag{4}$$

And the radius of the downstream wake is increased linearly with the distance as

$$r_1 = \alpha x + r_r, \tag{5}$$

For a wind turbine affected by multiple wakes and/or partial wakes, the effective wind speed it experienced is derived based on the kinetic energy deficit balance assumption, which is obtained as

$$\bar{u}_i = u_0 \left[1 - \sqrt{\sum_{j=1, j \neq i}^{N} \frac{A_{ij}}{A_0} \left(1 - \frac{u_{ij}}{u_0} \right)^2} \right], \tag{6}$$

where A_0 is the rotor area, A_{ij} is the part of the area of the ith turbine's rotor which is affected by the wake generated by the jth turbine (in the case the ith turbine is no effected by the wake of the jth turbine, $A_{ij} = 0$), and u_{ij} is the wind speed of the wake generated by the jth turbine at the position of the ith turbine, which can be determined by Eq. (1).

Using an ideal power curve [16], the total power extracted from the wind by a wind farm consisting of N wind turbines are given by

$$P_{tot} = \sum_{i=1}^{N} 0.3\bar{u}_i^3 \qquad (7)$$

Then the wind farm efficiency can be defined as

$$\eta = \frac{P_{tot}}{N \cdot P_{iso}} \qquad (8)$$

where P_{iso} is the power produced by an isolated wind turbine under the same wind condition.

The wind turbine properties used in this test case are listed in Table 1, and the surface roughness is assumed as $z_0 = 0.3$ m.

Table 1. Wind turbine properties

Hub height (h)	60 m
Rotor radius (r_0)	40 m
Rotor diameter (D)	80 m
Thrust coefficient (C_T)	0.88

The cost of wind farm is modeled by a simple function which only depends on the number of turbines. The total cost per year for the entire wind farm is expressed as follows [12]

$$\text{Cost} = N \left(\frac{2}{3} + \frac{1}{3} e^{-0.00174N^2} \right) \qquad (9)$$

The following objective function, which represents the cost of per unit of energy produced, will be minimized

$$\text{CoE} = \text{Cost}/P_{tot} \qquad (10)$$

Besides, there are constraints about the locations of turbines which are not explicitly stated but satisfied by the 5D width cells setting, which means that the minimal distance between any two turbines is 5D. The minimal distance constraints can be stated as

$$d_{ij} = \sqrt{(x_i^2 - x_j^2) + (y_i^2 - y_j^2)} \geq 5D, \quad \text{for } i,j = 1,2, \cdots N, \text{and } i \neq j \qquad (11)$$

where d_{ij} is the distance between two turbines, x_i and y_i are the coordinates of the ith turbine.

Three wind cases are considered [12]: Case (a): uniform north wind with a speed of 12m/s; Case (b): Equally distributed (36 directions) wind with a speed of 12m/s; Case (c): Ununiformly distributed (36 directions) wind with speeds of 8, 12 and 17 m/s. The distribution of the wind case (c) is shown in Fig. 3.

The above problem formulation defines a complete ideal test case for wind farm layout optimization, which is especially suitable for algorithm study. It was first developed and solved with GA by Mossetti et al. [12], then Grady et al. [16] tackled this case with their improved GA and obtained better solutions, other algorithms, such as Monte Carlo [15], PSO [17] have also been applied to study this test case.
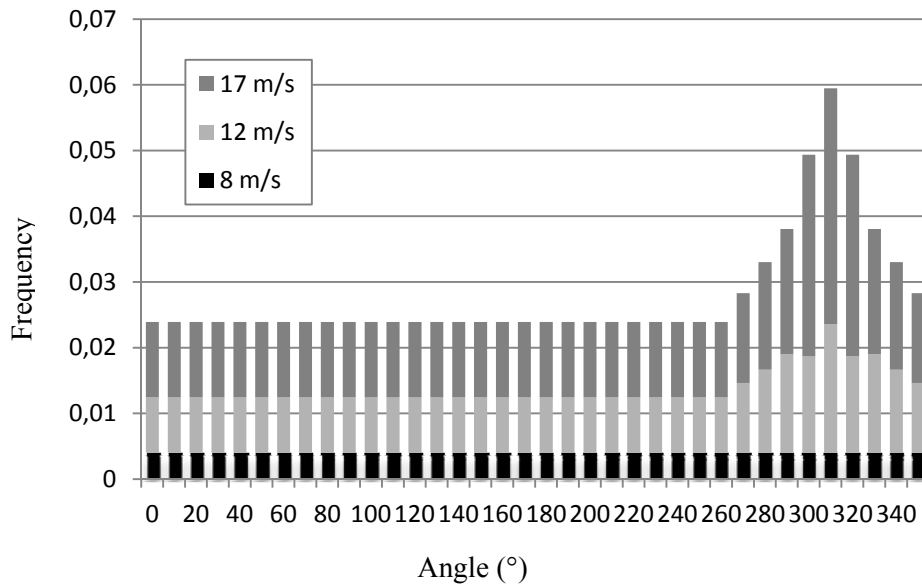
Fig. 3. Distribution of wind case (c): ununiformly distributed wind [16]

3. Refinement algorithm

Because of the discrete nature of the formulation stated in Section 2, it is very convenient to apply all kinds of discrete type meta-heuristics [23], such as the binary coded GA [12, 16]. In this test case, one solution, i.e., one possible layout can be represented by a 100 bit string consisting of 0 and 1, where 1 means that a turbine is located in the relative cell, and 0 means no turbine.

For a general wind farm layout optimization problem, if the number of turbines is not specified, the searching space for optimal solution is mixed integer-continuous type, where the number of turbines is represented by an integer and the locations of turbines are represented by continuous variables. By assuming that all turbines can only be placed in the center of the pre-divided cells, the searching space is simplified into a discrete type, by which solutions can be easily formulated as binary strings. Although this simplification is very important for the application of discrete type meta-heuristics, there are no physical constraints that the turbines have to be placed in the center of cells, and there are no real cells or grids in the actual wind farm field. Therefore by removing this grids and cells setting, the possible searching space for layout is largely increased, from discrete space to continuous space. This increase of searching space may be utilized by certain algorithms to find better solutions, i.e., to refine the optimization results based on the discrete searching space.

In this work, a simple method is proposed to find better solution, using the optimization results obtained from any discrete type of optimization methods. The algorithm is shown as follows:

Algorithm 1: Random search (RS) algorithm for optimization refinement of wind farm layout

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Initialize:
*Select initial solution **s0** from the optimization result of an existing method*
*Evaluate fitness value: **f0=f(s0)***

***While** stop condition is not true:*
    1. ***Random Move***
        Select a turbine randomly, move its position in a random direction with a random step:
        s=s0+Δs

It should be noticed that there is a *feasibility check* step in the algorithm, which deals with the constraints of the problem. As stated in Section 2, the minimal distance constraints, i.e., the requirements of locations of turbines governed by equation (11), also the effective boundary constraints (as shown in Fig. 1), are automatically satisfied by the cells setting of the problem formulation. Now, when the grids and cells are removed and the turbines are allowed to randomly move, these constraints have to be treated explicitly. So for every new solution introduced by random move, its feasibility has to be checked. Only when all the constraints are satisfied, the solution is considered to be feasible and the algorithm moves to next step.

The other feature needs to be pointed out is that the number of turbines is fixed in this algorithm, which stays the same as in the initial solution from any other optimization method.

This algorithm is simple, intuitive and easy to implement. It can be used as a last stage refinement tool to improve the results obtained by any other algorithms, especially for those based on discrete formulation.

4. Results and Comparisons

In this section the proposed refinement algorithm is used to improve the results obtained by Grady et al. [16], which are improved GA optimization results comparing to the original results presented in Mossetti study [12]. In order to verify the effectiveness of this algorithm, the test case in Section 2 is also solved by using a GA developed by the authors. Comparing to Grady study [16], our GA treats the same test case with the same problem formulation, but uses different crossover, mutation and selection methods, also adds an elitist strategy, which successes in finding the same optimal solution for wind case (a), and better solutions for wind cases (b) and (c).

These two sets of optimization results are both refined with the same refinement algorithm (RS), the numerical results presented in Table 2 show steady improvements for all wind cases.

It can be seen from Table 2 that the proposed refinement algorithm with RS can obtain steady improvement for both sets of GA optimization results. The greatest improvements are achieved for the wind case (a), in which the power and wind farm efficiency increases by more than 6%, and the cost of energy decreases by more than 5%. For the wind cases (b) and (c), improvements are also achieved for Grady GA results and present GA results. Comparing these two sets of GA results, it can be concluded that our own GA is able to find better layouts than the results in Grady study [16]. Based on the improved initial solutions, better refined results are also obtained for these two wind cases. The fact that the greatest improvements are for wind case (a) may be explained by the smallest number of turbines of this case, which means the wind farm field is less densely filled, thus the space for

improvement is also larger. The results in Table 2 suggest that the quality of initial solutions, which are the starting point of random evolution towards better results, is very important for the success of the proposed refinement method.

Table 2. Comparison of layout performance of GA optimization results and refined results

|	Case (a)				Case (b)				Case (c)			
	N	CoE	P(kW)	η(%)	N	CoE	P(kW)	η(%)	N	CoE	P(kW)	η(%)
Grady GA Published	30	0.001544	14310	92.0	39	0.001567	17220	85.2	39	0.000803	32038	86.6
Grady GA Re-eval.	30	0.001545	14294	91.9	39	0.001578	17056	84.4	39	0.000859	31336	84.2
Discrepan--cy (%)	/	0.06	0.11	0.11	/	0.70	0.95	0.94	/	6.97	2.19	2.85
RS result	30	0.001454	15190	97.7	39	0.001558	17279	85.5	39	0.000840	32044	86.5
Exe. Time	36045 s				39479 s				48341 s			
Improve--ment (%)	/	-5.90	6.27	6.27	/	-1.29	1.31	1.31	/	-2.21	2.26	1.31
Present GA					41	0.001511	18572	87.4	39	0.000855	31478	85.0
Exe. Time	The same as for Grady GA				86420 s				345871 s			
RS Result					41	0.001462	19195	90.3	39	0.000837	32178	86.9
Exe. Time					39741 s				50403 s			
Improve--ment (%)					/	3.25	3.35	3.35	/	-2.18	2.22	2.22

Note: 1. The 'Grady GA Re-eval.' are re-evaluated results of the published Grady study, using our code, the discrepancy is introduced by the difference in detailed wake modeling, coding, computation, and especially the exact data of wind case (c) (this paper uses data extracted from the figure in [16]);
2. Improvement percentage is calculated based on re-evaluated results of Grady GA;
3. The improve percentage of CoE is negative, which means the CoE is minimized further;
4. 'Exe. Time' is the execution time using a personal computer with 2.33GHz CPU and 1.96GB RAM.

The optimal layouts for all wind cases, from the original and refined results of both the Grady and our own GA, are shown in Figs. 4-6.
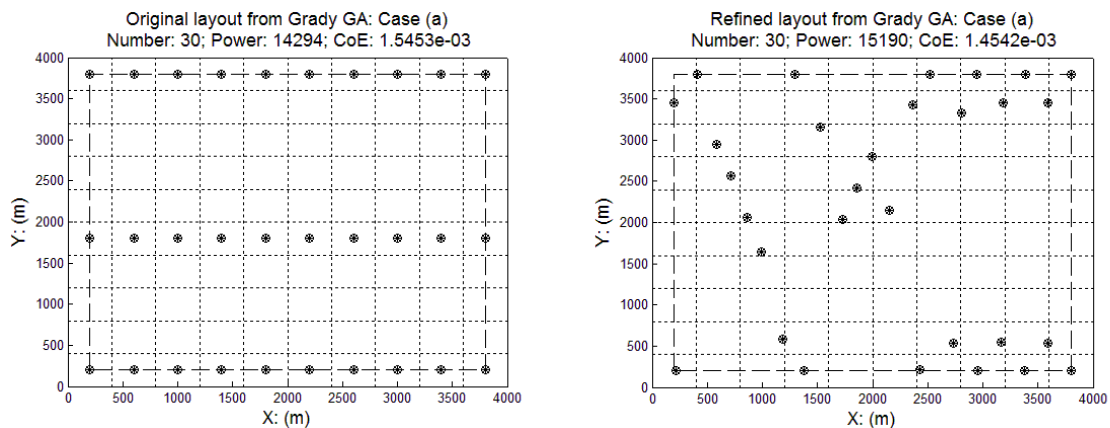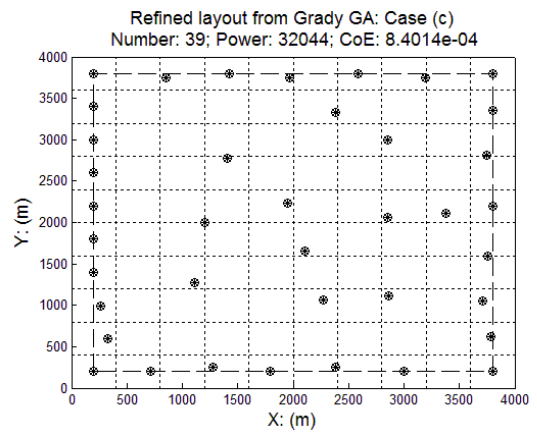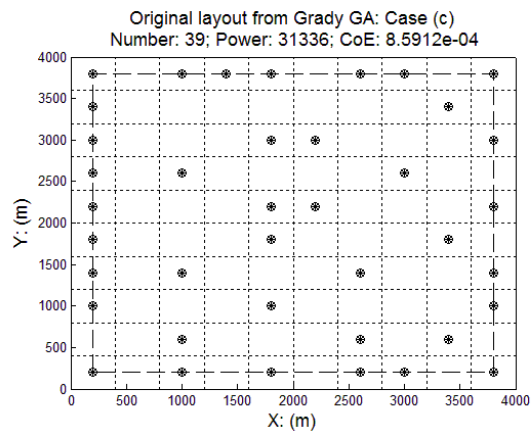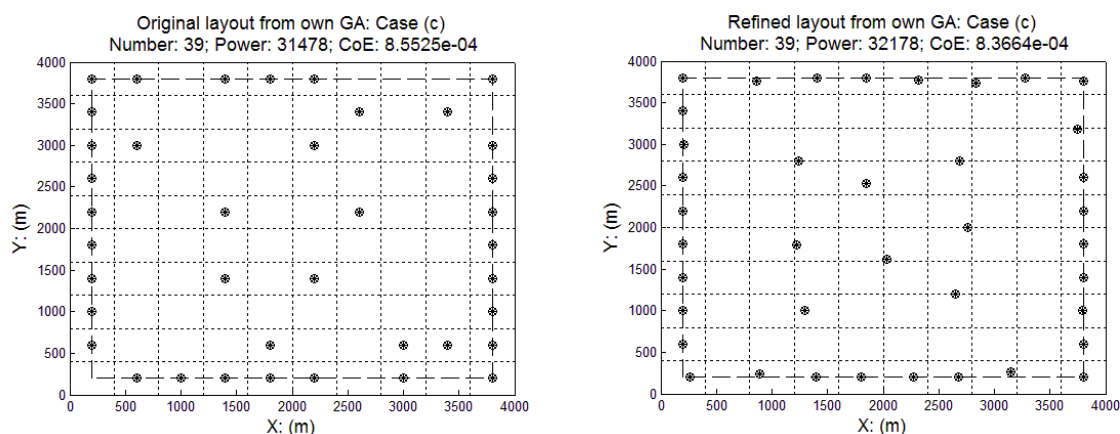

Fig. 4. Comparison of optimal layout for case (a)
(In this case, our own GA and Grady GA get the same solution.)

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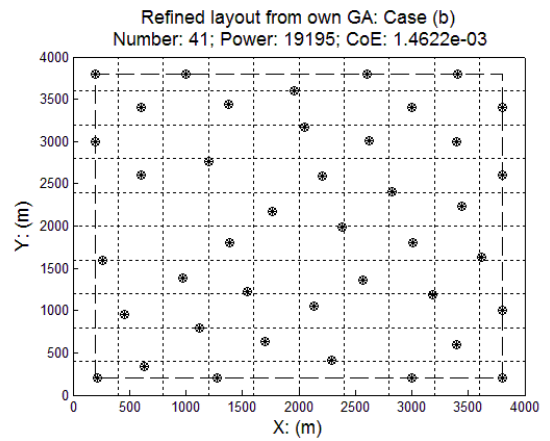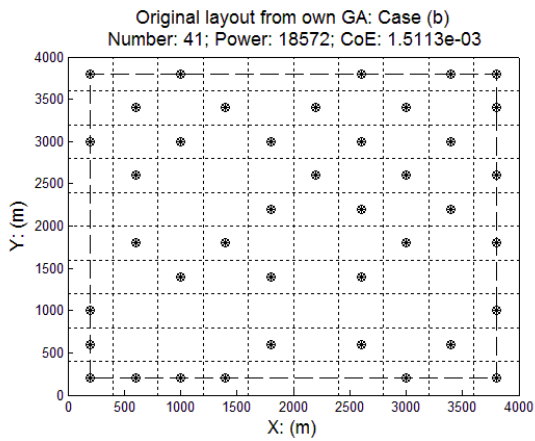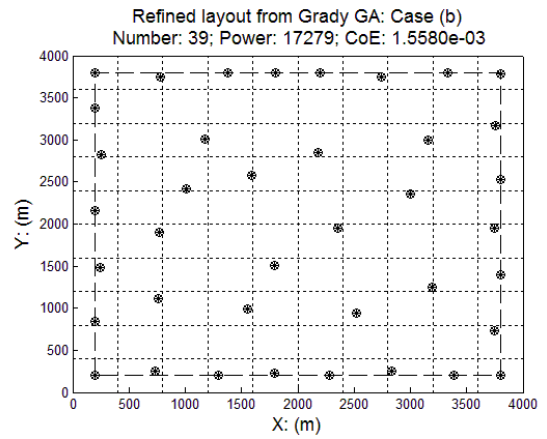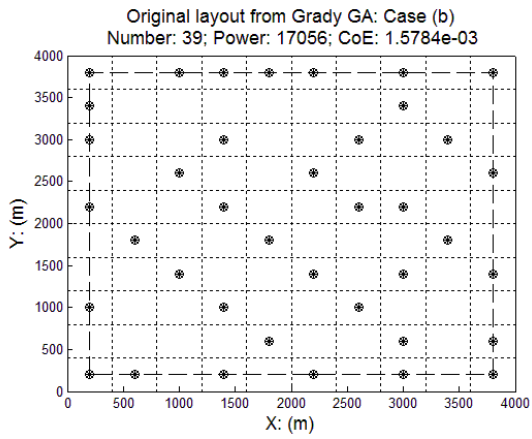
Fig. 5. Comparison of optimal layout for case (b)

Fig. 6. Comparison of optimal layout for case (c)

5. Conclusions

In this paper, a refinement method by random search is developed. The algorithm is simple, intuitive and easy to implement, it has the ability of treating the constrained wind farm layout optimization in the continuous formulation. The application of this method to improve the optimization results from GA of a widely studied test case demonstrates its effectiveness. The performance of the algorithm is dependent on the quality of initial solutions, which may be obtained by other optimization methods. The results shown in this paper indicate that this algorithm can serve as a last stage refinement tool, by improving existing optimization results and obtaining better solutions. This algorithm could be further investigated and combined with other optimization methods, to tackle more realistic wind farm layout optimization in future study.

Acknowledgment

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