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Computational Efficiency of Economic MPC for Power Systems Operation

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Abstract—In this work, we propose an Economic Model Predictive Control (MPC) strategy to operate power systems that consist of independent power units. The controller balances the power supply and demand, minimizing production costs. The control problem is formulated as a linear program that is solved by a computationally efficient implementation of the Dantzig-Wolfe decomposition. To make the controller suitable for real-time applications, we investigate a suboptimal MPC scheme, introducing an early termination strategy to the Dantzig-Wolfe algorithm. Simulations demonstrate that the early termination technique substantially reduces the computation time.

I. INTRODUCTION

For the last two centuries, mankind has depended on fossil fuel. The consequences of the extensive use of these fuels, i.e. global warming and rising costs of fossil fuels, affect our life. Furthermore, the greatest source of CO_2 emissions is the combustion of fossil fuels to generate electricity. A change in energy systems is clearly necessary if we are to become free of dependence on fossil fuels. The introduction of Smart Grids is crucial for future energy systems, as these grids will connect consumers and producers through, for example, renewable energy sources (RES). This innovative scenario requires control actions to ensure that the total energy production can satisfy customer demand.

In this paper we propose an optimization-based controller for dynamic load balancing of a power system consisting of multiple power units that are dynamically decoupled. The control strategy is an Economic MPC strategy that is used to balance power supply and demand in the most economical way. The control problem may be expressed as a linear program tailored for the Dantzig-Wolfe decomposition technique. However, real-time applications require fast computation of the optimal control trajectory; because of this, an early termination strategy is applied to the Dantzig-Wolfe decomposition algorithm. Such early termination significantly reduces the computational time for the MPC at the cost of obtaining only a suboptimal solution.

Economic MPC operates many energy systems, for example refrigeration systems, heat pumps for residential buildings, solar-heated water tanks and batteries in electric vehicles. However, with regard to large-scale systems, a distributed controller to overcome the communication limitation and computational complexity is a better choice [1]. Distributed MPC successfully applies decomposition techniques, i.e. the Dantzig-Wolfe decomposition [2], [3]. Such a decomposition algorithm has been utilized in several applications, including power balancing based on an ℓ_1-penalty function [4] and to operate large-scale power systems [5]. To strengthen the applicability of the controller to real-time applications, this work introduces a suboptimal MPC strategy [6]–[10].

The outline of this paper is as follows: Section II introduces power systems and formulates a linear Economic MPC for linear power systems. Section III describes the Dantzig-Wolfe decomposition algorithm. The early termination strategy is explained in Section IV. Section V-A proposes a model for the power generators included in the power system; Section V-B reports simulation results and, finally, the conclusion and suggestions for future work are presented in Section VI.

II. POWER SYSTEMS AND ECONOMIC MPC

Future power systems will consist of independent power units connected to one common operation center that must control and coordinate such power units, balancing power production and consumption in an economical and realiable way. Operating such a power system means making real-time decisions such as planning the power production in response to customer demand and unpredicted production variations from renewable energy producers, e.g. wind turbines.

A power unit is assumed to be described as a linear stochastic discrete time state space model

$$x_{k+1} = Ax_k + Bu_k + Ed_k \tag{1a}$$
$$y_k = Cx_k + v_k \tag{1b}$$
$$z_k = C_z x_k. \tag{1c}$$

The measurement noise, $v_k \sim N_{iid}(0, R_{vv})$, and $d_k \sim N(\bar{d}_k, R_{dd,k})$ are predicted by external prognosis systems [5]; in many power applications d_k might represent wind speed or sun radiation. While x_k denotes the states, u_k the manipulated variables (MVs), y_k denotes the measurement used for feedback, and z_k is the output variable. The manipulated variable, u_k, is subject to hard constraints

$$u_{min} \leq u_k \leq u_{max} \tag{2a}$$
$$\Delta u_{min} \leq \Delta u_k \leq \Delta u_{max} \tag{2b}$$

The system output z_k must be within an interval $[r_{\min,k}, \ r_{\max,k}]$; such interval may represent electricity demand forecast in advance, or it can define indoor temperature in a building, or temperatures in a refrigeration system or state-of-charge of a battery. However, due to some disturbances or in a specific scenario, it might be impossible to obtain z_k within the defined interval; therefore, the constraints on the output variable include slack variables s_k. The slack variables, s_k, may represent selling or buying power from the short-term market, violation of temperature limits, or violation of state-of-charge limits. Every time s_k is non-zero, a penalty cost, e.g. the cost of buying or selling power on the short-term market must be paid.

$$r_{\min,k} - s_k \leq z_k \leq r_{\max,k} + s_k \qquad (3a)$$
$$s_k \geq 0 \qquad (3b)$$

In order to deal with the stochasticity of the power units, we apply the certainty equivalence principle: in this way, all variables are replaced by their conditional mean values [5]. Furthermore, we implement a Kalman filter to predict $\hat{x}_{k+1+j|k} \ \forall j \in \mathcal{N}$ looking N periods ahead [5]. Consider a power system consisting of P power units (1), the cost of producing power for a power generator is $\phi_{i,k}, \ \forall i \in P$. This economic cost, $\phi_{i,k}$, consists of the cost of operating the power unit, $\hat{c}_{i,k+j|k}$, and the penalties, $\hat{\rho}_{i,k+j+1|k}$, related to the use of slack variables, $\hat{s}_{i,k+j+1|k}$

$$\phi_{i,k} = \sum_{j=0}^{N-1} \hat{c}'_{i,k+j|k}\hat{u}_{i,k+j|k} + \hat{\rho}'_{i,k+j+1|k}\hat{s}_{i,k+j+1|k} \qquad (4)$$

These unit prices are provided by external forecasting systems. The linear constraints and the linear objective functions lead to the formulation of the control problem as a linear program; accordingly, the Linear Economic MPC to operate a power system of P power units is formulated as

$$\min \phi_k = \sum_{i=1}^{P} \phi_{i,k} + \sum_{j=0}^{N-1} \hat{\rho}'_{k+j+1|k}\hat{\hat{s}}_{k+j+1|k} \qquad (5)$$

subject to the local constraints $\forall i \in P$ and $\forall j \in \mathcal{N}$

$$\hat{x}_{i,k+j+1|k} = A_i\hat{x}_{i,k+j|k} + B_i\hat{u}_{i,k+j|k} + E_i\hat{d}_{i,k+j|k} \qquad (6a)$$
$$\hat{z}_{i,k+j+1|k} = C_{z,i}\hat{x}_{i,k+j+1|k} \qquad (6b)$$
$$u_{\min,i} \leq \hat{u}_{i,k+j|k} \leq u_{\max,i} \qquad (6c)$$
$$\Delta u_{\min,i} \leq \Delta \hat{u}_{i,k+j|k} \leq \Delta u_{\max,i} \qquad (6d)$$
$$\hat{z}_{i,k+j+1|k} + \hat{s}_{i,k+j+1|k} \geq \hat{r}_{\min,i,k+j+1|k} \qquad (6e)$$
$$\hat{z}_{i,k+j+1|k} - \hat{s}_{i,k+j+1|k} \leq \hat{r}_{\max,i,k+j+1|k} \qquad (6f)$$
$$\hat{s}_{i,k+j+1|k} \geq 0 \qquad (6g)$$

and subject to the following connecting constraints $\forall j \in \mathcal{N}$ and $\forall i \in P$, where $\hat{\hat{z}}_{k+j+1|k}$ denotes the supply constraints connecting the produced and requested power, $\hat{\hat{r}}_{\min,k}$ and
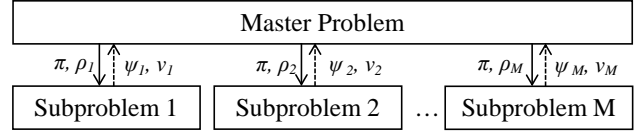

Fig. 1: Dantzig-Wolfe structure. Each subproblem comunicates exclusively with the master problem (MP).

$\hat{\hat{r}}_{\max,k}$ are provided by external forecasts

$$\hat{\hat{z}}_{k+j+1|k} = \sum_{i=1}^{P} \tilde{C}_{z,i}\hat{x}_{i,k+j+1|k} + \tilde{D}_{z,i}\hat{u}_{i,k+j|k} \qquad (7a)$$

$$\hat{\hat{z}}_{k+j+1|k} + \hat{\hat{s}}_{k+j+1|k} \geq \hat{\hat{r}}_{\min,k+j+1|k} \qquad (7b)$$
$$\hat{\hat{z}}_{k+j+1|k} - \hat{\hat{s}}_{k+j+1|k} \leq \hat{\hat{r}}_{\max,k+j+1|k} \qquad (7c)$$
$$\hat{\hat{s}}_{k+j+1|k} \geq 0 \qquad (7d)$$

The optimization control problem (5)-(7) has a block-angular structure tailored for the implementation of Dantzig-Wolfe decomposition to solve efficiently the control linear program.

III. DANTZIG-WOLFE DECOMPOSITION TECHNIQUE

The Dantzig-Wolfe decomposition algorithm is a decomposition technique to solve efficiently linear programs that have a block-angular structure, such as (5)-(7), [2], [3]. The Economic MPC expressed, as a linear program in (5)-(7), can be formulated as

$$\min_{\{w_{i,k}\}_{i=1}^{M}} \varphi = \sum_{i=1}^{M} e'_i w_{i,k} \qquad (8a)$$

$$s.t. \begin{bmatrix} F_1 & F_2 & \dots & F_M \\ G_1 & & & \\ & G_2 & & \\ & & \ddots & \\ & & & G_M \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix} \geq \begin{bmatrix} g \\ h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} \qquad (8b)$$

where $i \in \mathcal{M} = \{1,...,P,P+1\}$ as the slack variables $\hat{\hat{s}}_{k+j+1|k}$ in (5) and (7) are considered as an independent unit.

In this decomposition approach, the linear programming problem can be separated into independent subproblems, which are coordinated by a master problem (MP), as depicted in Figure 1. Within each iteration, the MP sends its Lagrange multipliers to all the subproblems to update their objective function. Then, the subproblems are solved and they send their solutions and objective function values to the MP. The solution to the original problem can be shown to be equivalent to solving the subproblems and the MP through a finite number of iterations [11].

When describing the Dantzig-Wolfe decomposition, it is necessary to introduce the theorem of convex combination, or Dantzig-Wolfe transformation [11], [12].

Theorem 1 (Convex Combination): Consider $\mathcal{W} = \{w \mid Gw \geq h\}$ is a nonempty, bounded and closed set, i.e. a polytope. v^j denotes the extreme point of $\mathcal{W}$ with

$j \in \{1, 2, ..., V\}$.

Then any point w in the polytope $\mathcal{W}$ can be written as a convex combination of its extreme points

$$w = \sum_{j=1}^{V} \lambda_j v^j \qquad (9a)$$

$$s.t \quad \lambda_j \geq 0, \qquad j = 1, 2, ..., V \qquad (9b)$$

$$\sum_{j=1}^{V} \lambda_j = 1 \qquad (9c)$$

Proof: See [11]. ∎

Via the theorem of convex combination (9), the MPC can be formulated as follows, assuming that the feasible regions of subproblems are bounded

$$\min_{\lambda} \quad \varphi = \sum_{i=1}^{M} \sum_{j=1}^{V_i} f_{ij} \lambda_{ij} \qquad (10a)$$

$$s.t \quad \sum_{i=1}^{M} \sum_{j=1}^{V_i} p_{ij} \lambda_{ij} \geq g \qquad (10b)$$

$$\sum_{j=1}^{V_i} \lambda_{ij} = 1, \qquad i = 1, 2, ..., M \qquad (10c)$$

$$\lambda_{ij} \geq 0, \qquad i = 1, 2, ..., M; j = 1, 2, ..., V_i \qquad (10d)$$

where the coefficients are

$$f_{ij} = e_i' v_i^j, \quad p_{ij} = F_i v_i^j \qquad (11)$$

The linear program (10), known as master problem (MP), is equivalent to the block-angular linear problem (8). It is worth noting that (10) has fewer rows in the coefficient matrix than the original problem (8). However, in the MP the number of columns is larger due to an increase in the number of variables with the extreme points of all subproblems.

If the MP is solved via the Simplex method, then only the basic set is needed and it has the same number of basic variables as the number of rows. Hence, not all the extreme points are necessary to be known. This yields to the reduced master problem (RMP), dynamically constructed at a fixed size by incorporating column generation techniques

$$\min_{\lambda} \quad \varphi = \sum_{i=1}^{M} \sum_{j=1}^{l} f_{ij} \lambda_{ij} \qquad (12a)$$

$$s.t \quad \sum_{i=1}^{M} \sum_{j=1}^{l} p_{ij} \lambda_{ij} \geq g \qquad (12b)$$

$$\sum_{j=1}^{l} \lambda_{ij} = 1, \qquad i = 1, 2, ..., M \qquad (12c)$$

$$\lambda_{ij} \geq 0, \qquad i = 1, 2, ..., M; j = 1, 2, ..., l \qquad (12d)$$

where $l \leq V_i$ for all $i \in \{1, 2, ..., M\}$. Solving the RMP provides the Lagrange multipliers π associated with the inequality constraint (12b), the Lagrange multipliers ρ, associated with equalities (12c), and the Lagrange multipliers κ for the positivity constraints (12d). As depicted in Figure 1, the MP sends the Lagrange multipliers to each subproblem.

The Lagrangian associated to the MP (10) yields to the following necessary and sufficient optimality conditions, for $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, V_i$

$$\nabla_{\lambda_{ij}} \mathcal{L} = f_{ij} - p_{ij}' \pi - \rho_i - \kappa_{ij} = 0 \qquad (13a)$$

$$\sum_{i=1}^{M} \sum_{j=1}^{V_i} p_{ij} \lambda_{ij} - g \geq 0 \quad \perp \quad \pi \geq 0 \qquad (13b)$$

$$\sum_{j=1}^{V_i} \lambda_{ij} - 1 = 0 \qquad (13c)$$

$$\lambda_{ij} \geq 0 \quad \perp \quad \kappa_{ij} \geq 0 \qquad (13d)$$

The conditions (13a) and (13d) yield to

$$\kappa_{ij} = f_{ij} - p_{ij}' \pi - \rho_i = [e_i - F_i' \pi]' v_i^j - \rho_i \geq 0 \qquad (14)$$

The Karush-Kuhn-Tucker conditions (KKT-conditions) for (10) are for $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, V_i$

$$\sum_{i=1}^{M} \sum_{j=1}^{V_i} p_{ij} \lambda_{ij} - g \geq 0 \quad \perp \quad \pi \geq 0 \qquad (15a)$$

$$\sum_{j=1}^{V_i} \lambda_{ij} - 1 = 0 \qquad (15b)$$

$$\lambda_{ij} \geq 0 \quad \perp \quad \kappa_{ij} = [e_i - F_i' \pi]' v_i^j - \rho_i \geq 0 \qquad (15c)$$

Initially, a feasible extreme point to the MP is necessary; we adopt an initialization technique that uses the previous optimal solution and the output constraints (3)-(7) to compute the initial vertex [5]. This initial point is then used to form the RMP (12) considering $l = 1$. Assuming λ_{ij}^{RMP} to be a solution of RMP, so that a feasible solution to MP (10) is

$$\lambda_{ij} = \lambda_{ij}^{RMP} \qquad i = 1, 2, \ldots, M; j = 1, 2, \ldots, l \qquad (16a)$$

$$\lambda_{ij} = 0 \qquad i = 1, 2, \ldots, M; j = l+1, l+2, \ldots, V_i \qquad (16b)$$

λ_{ij}^{RMP} satisfies the KKT-conditions for $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, l$; however, the optimal solution needs to satisfy these conditions for all $i = 1, 2, \ldots, M$ and $j = l+1, l+2, \ldots, V_i$. We only know the extreme points, v_i^j for $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, l$. Because of this the KKT-conditions are satisfied for $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, V_i$ if $\min_i \quad \psi_i - \rho_i \geq 0$ where

$$\psi_i = \min_{v_i^j} [e_i - F_i' \pi]' v_i^j \qquad (17)$$

v_i^j is an extreme point of the polytope $\mathcal{W}_i$. Then, we form the following linear program to solve (17)

$$\psi_i = \min_{w_i} \varphi = [e_i - F_i' \pi]' w_i \qquad (18a)$$

$$s.t \qquad G_i w_i \geq h_i \qquad (18b)$$

These linear programs are called subproblems and can be solved via either parallel or sequential computation. This possible parallel computation of the subproblems represents one of the advantages of the Dantzig-Wolfe decomposition algorithm. Let (ψ_i, w_i) be the optimal value-minimizer pair for the linear problem (18); an optimal solution is reached if the following condition is satisfied

$$\psi_i - \rho_i \geq 0 \qquad i \in \{1, 2, ..., M\} \tag{19}$$

Therefore the solution of the original control problem (8) is given by

$$w_i^* = \sum_{j=1}^{l} v_i^j \lambda_{ij} \qquad i \in \{1, 2, ..., M\} \tag{20}$$

When condition (19) is not satisfied, the number of extreme points considered, l, is not enough to satisfy the KKT-conditions and a new vertex v_i^{l+1} needs to be included.

IV. SUBOPTIMAL MPC STRATEGY

Real-time applications may restrict the applicability of the Economic MPC, especially because of the limits on the storage space and the computation time. Real-time MPC applications often involve warm-start, explicit MPC and early termination techniques to speed up the online computation [7], [8]; these approaches compute suboptimal solutions. The warm-start technique is used to initialize the Dantzig-Wolfe decomposition algorithm, as described in Section III and [5]. The novel step of this paper is the introduction of the early termination strategy to reduce computation time in the solution of the optimal control trajectory. Section III introduces the RMP (12): the algorithm adds a vertex of the polytope (9) to the RMP as long as the stopping criteria (19) is not satisfied. In a real-time scenario, the Dantzig-Wolfe algorithm tries to compute the optimal solution within a given sampling time. However, if too many vertices of the polytope (9) are necessary to get from the solution of the previous control problem to that of the current one, then the algorithm can be stopped. With regard to the early termination strategy, the CPU time can be limited [9]; for this purpose, in this work we use as heuristic a limit on the number of vertices of the polytope to include in the Dantzig-Wolfe algorithm. It is worth noting that, in the Dantzig-Wolfe algorithm, the suproblem (18) always has a feasible solution if the original linear program (8) does [13]; moreover, each polytope is assumed to be nonempty (9). As a result, the suboptimal solution obtained by early termination is feasible and the resulting MPC is both feasible and, therefore, stable [7], [8].

V. APPLICATION TO A POWER SYSTEM

In this section we apply the Economic MPC controller to a power system consisting of power plants, and the Dantzig-Wolfe decomposition computes the optimal control trajectory. In addition, we implement the early termination strategy in order to reduce computational times.
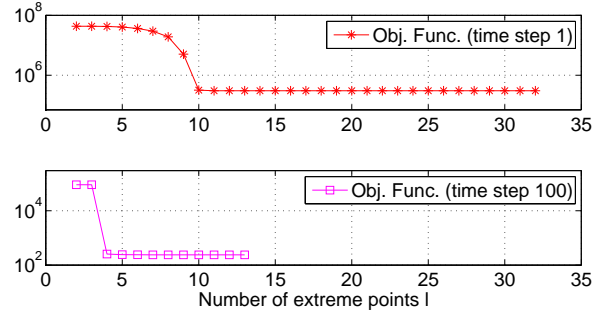

Fig. 2: Master problem (10) objective function and number of extreme points of the polytope (9) at each time step.

As a case study we consider a power system consisting of four power units, described below. Simulations are carried out for 120 time steps with time horizon $T = 70$.

A. Power Units

As a case study we consider a power system that consists of power generators. The individual power units are independent systems, and they can be modeled separately, as the actions in one of them do not directly affect the other units. They are coupled through the objective to follow the overall power system reference and activate secondary resources. Power units are modeled as in (21) [14]; in this way we address three kinds of power units: central thermal power plants, diesel generators and gas turbines. The first kind of power generator has a slow dynamic, while the remaining two show fast dynamics.

$$Z_i(s) = G_i(s)U_i(s) \qquad G_i(s) = \frac{1}{(\tau_i s + 1)} \tag{21}$$

where $z_i(t)$ is the produced power at unit i, while $u_i(t)$ is the corresponding reference signal.

B. Simulations Results

The controller developed in this work implements an Economic MPC policy, where the Dantzig-Wolfe decomposition technique computes the optimal control sequence. Section IV introduces the early termination strategy that leads to a suboptimal MPC. Simulation reveals that ten vertices of the feasible polytope (9) provide the optimal value of the objective function, as shown in Figure 2: the algorithm reduces the objective function until it reaches the minimum. However, the Dantzig-Wolfe algorithm takes 32 extreme points to satisfy the stopping criteria (19). As a result, the controller can terminate iteration before the stopping criterion is satisfied and the solution obtained is, therefore, suboptimal. Furthermore, at each time step, the Dantzig-Wolfe algorithm implements the warm-start described in Section III; because of this, the algorithm has a better initial point and the optimal value of the objective function decreases at each time step, see Figure 2. In order to demonstrate the early termination effectiveness, we simulate four different scenarios: the first is the exact Dantzig-Wolfe algorithm, while the remaining three include limits on the number of vertices of the polytope, respectively 15, 10 and
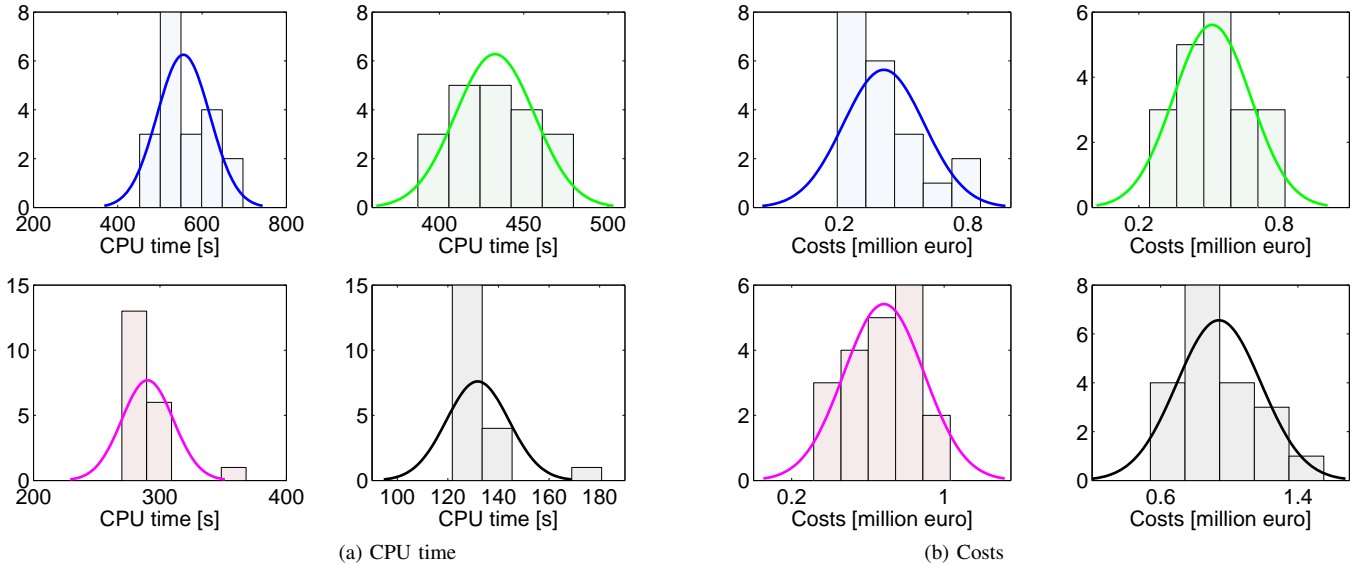
Fig. 3: CPU time and cost distributions based on 20 stochastic simulations. Blue distribution is for Economic MPC solved via exact Dantzig-Wolfe algorithm. Green distribution is for an early termination strategy with 15 vertices. Magenta distribution is for an early termination strategy with 10 vertices. Black distribution is for an early termination strategy with five vertices.

5. Figure 3a reports the CPU time for closed-loop simulations: the early termination technique substantially reduces the computational time by comparison with the implementation of the exact Dantzig-Wolfe algorithm. It should, however, be noted that the decrease in computational time is linked to an increase in the production costs, see Figure 3b.

VI. Conclusion

In this work we have proposed a subotpimal Economic MPC to operate power systems. We have introduced power systems and their independent power units; consequently, we have defined the control problem as a linear program tailored for the Dantzig-Wolfe decomposition technique. After the description of the Dantzig-Wolfe algorithm, we have introduced the early termination strategy that provides a suboptimal solution of the control problem. Closed loop simulations have demonstrated that the algorithm developed noticeably decreases computational times. On the other hand, such reductions cause unavoidable extra costs. This finding should be explored in future work.

Acknowledgment

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