

SIGNAL DETECTION ALGORITHM FOR COGNITIVE RADIO USING SINGULAR VALUE DECOMPOSITION

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ABSTRACT. This paper highlights an algorithm for detecting the presence of wireless signal using the Singular Value Decomposition (SVD) technique. We simulated the algorithm to detect common digital signals in wireless communication to test the performance of the signal detector. The SVD-based signal detector was found to be more efficient in detecting a signal without knowing the properties of the transmitted signal. The performance of the algorithm is better compared to the favorable energy detection. The algorithm is suitable for blind spectrum sensing where the properties of the signal to be detected are unknown. This is also the advantage of the algorithm since any signal would interfere and subsequently affect the quality of service (QoS) of the IEEE 802.22 connection. Furthermore, the algorithm performed better in the low signal-to-noise ratio (SNR) environment.

Keywords: cognitive radio, singular value decomposition (SVD), signal detector.

INTRODUCTION

Cognitive radio (CR) has been a very important function to enable the state of the art technology in revolutionizing spectrum efficient utilization. In responding to the idea of CR coined by Mitola and Maguire (1999), IEEE 802.22 Working Group (WG) was formed in 2004. The WG is expected to develop and incorporate CR functionality in a standard known as Wireless Regional Area Networks (WRAN). The new standard is going to operate in TV bands between 54-862 MHz (IEEE802.22, 2010). The standard is likely to deliver broadband internet access on vacant TV channels at the same time avoiding harmful interference to the licensed users in rural areas within a radius of 17 km to 30 km (Xu *et al.*, 2008a).

In rationalizing the use of CR, a spectrum management function needs to be implemented in cognitive radio module. As stated by Haykin (2005), the spectrum management process consists of four major steps: 1) spectrum sensing, 2) decision making, 3) spectrum sharing and 4) spectrum mobility. The first and second steps are very crucial in enabling the CR technology. CR users are expected to be able to detect primary user (PU) signals and find the spectrum holes or the unused spectrum in order to operate.

Several spectrums sensing algorithms such as classical energy detection (ED), the eigenvalue-based detection, the covariance-based detection and feature-based detection are reported in the literature to detect primary signal. Discussions about these techniques and algorithms as well as their pros and cons are reported by Yucek and Arslan (2009); Chen and Prasad (2009); Akyildiz *et al.* (2006); Akyildiz *et al.* (2008); and Zeng and Liang (2010). According to Kortun *et al.* (2010), the most accurate techniques that can simultaneously

achieve both high probability of detection and low probability of false alarm with very minimal knowledge about the primary user signals and noise spectrum are the eigenvalue-based detection techniques introduced by Zeng and Liang (2007). It is mentioned in the literature that maximum-minimum eigenvalue (MME) method has many advantages over the rest of sensing methods listed above. This is because the decision of signal presence can be done without prior knowledge of the primary signal and noise.

BACKGROUND WORK

Before the introduction of SVD-based detection, researchers discovered that by analyzing the eigenvalues from a received signal matrix, the threshold for detecting primary signal can be calculated. Most of the researchers used the eigenvalue decomposition. This method is known as eigenvalue-based detection. Discussion on the method can be found in (Zeng and Liang, 2007, Zeng and Liang, 2009, Kortun *et al.*, 2010).

The SVD is quite similar to the eigenvalue decomposition technique. However, the SVD is very general in the sense that it can be applied to any $m \times n$ matrix, whereas the eigenvalue decomposition technique can only be applied to certain classes of square matrices. Additionally, the SVD has got several advantages compared to other decomposition methods as listed below (Leach, 2000):

- i. more robust to numerical error;
- ii. exposes the geometric structure of a matrix an important aspect of many matrix calculations; and
- iii. quantify the resulting change between the underlying geometry of those vector spaces.

SVD-based detection was introduced by Xu *et al.* (2008a) and Xu *et al.* (2008b). Both papers used SVD technique to detect wireless microphone signal in a wideband cognitive radio network. In WRAN, the system needs to detect both digital TV and wireless microphone signals since both services are incumbent in the frequency bands. However, in some countries, wireless microphone is not protected by the law as in US where FCC Part 74 protects the service.

In this paper, the SVD technique is used to detect common signals rather than restrict it to the only digital TV and wireless microphone since any other signal would affect the WRAN quality of service.

SYSTEM MODEL

In detecting a signal, two hypotheses are involved: \mathcal{H}_0 , signal is not present; and \mathcal{H}_1 , signal is present. The received signal samples under two hypotheses are given respectively as follows (Cabric *et al.*, 2006, Chen *et al.*, 2007, Sahai and Cabric, 2005, and Urkowitz, 1969):

$$\begin{aligned}\mathcal{H}_0: y_i(n) &= \eta_i(n) \\ \mathcal{H}_1: y_i(n) &= x_i(n) + \eta_i(n)\end{aligned}\tag{4}$$

where $x_i(n)$ is the transmitted signal samples, (N), through a wireless channel consisting of path loss, multipath fading and time dispersion effects, and $\eta_i(n)$ is the white noise which is independent and identically distributed (iid) with zero mean and σ_η^2 variance. Note that $x_i(n)$ can be the superposition of the received signals from multiple primary users, hence, no synchronization is needed here.

There are two probabilities involved for signal detector: probability of detection, P_d , which defines the alternative hypothesis \mathcal{H}_1 , the probability of the detecting algorithm having detected the presence of the primary signal; and probability of false alarm, P_{fa} , which defines

a null hypothesis \mathcal{H}_0 , the probability of the detecting algorithm claiming the presence of the primary signal.

In this paper we consider three types of signals: rectangular pulse, raised cosine and root-raised cosine since these are common signals in today's communication system.

SVD-BASED SIGNAL DETECTOR

SVD plays an important role in signal processing and statistics, particularly in the area of a linear system. $y(n)$ should be defined as time series for the received signal with $n = 1, 2, \dots, N$, commonly, we can construct a Henkel matrix with $M = N - L + 1$ rows and L columns as follows:

$$\mathbf{R} = \begin{bmatrix} y(1) & y(2) & \dots & y(L) \\ y(2) & y(3) & \dots & y(L+1) \\ \vdots & \vdots & \vdots & \vdots \\ y(N-L+1) & y(N-L+2) & \dots & y(N) \end{bmatrix} \quad (2)$$

then \mathbf{R} is an $M \times L$ matrix. Its elements can be found by substituting $y(n)$

$$\mathbf{R}_{ml} = y(m+l-1), \quad m = 1, 2, \dots, M \text{ and } l = 1, 2, \dots, L. \quad (3)$$

Using SVD, \mathbf{R} can be factorized as

$$\mathbf{R} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \quad (4)$$

where \mathbf{U} and \mathbf{V} are an $M \times M$ and an $L \times L$ unitary matrix, respectively. The columns of \mathbf{U} and \mathbf{V} are called left and right singular vectors, respectively. The $\mathbf{\Sigma} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_m)$ is a diagonal matrix whose nonnegative entries are the square roots of the positive eigenvalues of $\mathbf{R}^H\mathbf{R}$ or $\mathbf{R}\mathbf{R}^H$. These nonnegative entries are called the singular values of \mathbf{R} and they are arranged in a decreasing manner with the largest number in the upper left-hand corner. The $[\]^H$ denotes the Hermitian transpose of a matrix.

Whenever no primary signal or other signal is present, the received signal $y(n)$ includes only AWGN contribution such that its singular values are similar and close to zero. Whenever other signals whose power is higher than a threshold value are active, there will exist several dominant singular values to represent these signals. As a result, the signals can be detected by examining the presence of dominant singular values.

THRESHOLD DEFINITION

The decision threshold and probability of false alarm are derived based on limiting distribution of eigenvalues based on random matrix theory. The decision statistic for the maximum-minimum eigenvalue (MME) detection is defined as the ratio of maximum to minimum eigenvalues of the received signal as follows:

$$T_y = \frac{\lambda_{max}}{\lambda_{min}} \quad (5)$$

Based on the decision statistic in (5), the detection threshold, γ , must be estimated for a required probability of false alarm. To define the threshold in terms of P_{fa} or vice versa, the density of the test statistic, T_y , is required. The density can be found asymptotically i.e. both the threshold values and the probabilities of detection and false alarm are derived based on asymptotical distributions of eigenvalues that is mathematically tractable and less complicated (Kortun *et al.*, 2010).

An asymptotic formula of signal detection threshold in term of desired probability of false alarm for MME has been proposed by Zeng and Liang (2007). The detection threshold in

terms of desired probability of false alarm is calculated by using the results of the theorem stated by Johnstone (2001) and Zeng and Liang (2007), as follows (in our case, $M=1$):

$$\gamma = \frac{(\sqrt{N_s} + \sqrt{L})}{(\sqrt{N_s} - \sqrt{L})} \cdot \left(1 + \frac{(\sqrt{N_s} + \sqrt{L})^{-\frac{2}{3}}}{(N_s L)^{\frac{1}{6}}} \cdot F_1^{-1}(1 - P_{fa}) \right) \quad (6)$$

where F_1^{-1} denotes the inverse of cumulative distribution function (CDF) of the Tracy-Widom distribution of order 1 (Tracy and Widom, 2000).

The threshold definition in (6) is formulated based on deterministic asymptotic values of the minimum and maximum eigenvalues of the Hankel matrix, \mathbf{R} , when the number of samples, N_s is very large. As shown in the equation, it is defined only in terms of number of samples, N_s , level of Hankel matrix, L and the desired probability of false alarm, P_{fa} .

SIGNAL DETECTION ALGORITHM

In implementing the SVD-based signal detector, we adopt method by Zeng and Liang (2007). The algorithm to detect the presence of a signal is as follows:

- Step 1:* Select number of column of a Hankel matrix, L such that $k < L < N - k$ (Tufts and Kumaresan, 1982), where N is the number of sampling points and k is the number of dominant singular values. In this paper, $k = 2$ and $L = 16$.
- Step 2:* Factorize the Hankel matrix to form the equation as in (4).
- Step 3:* Obtain the maximum and minimum eigenvalues of the Hankel matrix which are λ_{max} and λ_{min} .
- Step 4:* Compute threshold value, γ , as discussed in the previous section.
- Step 5:* Compare the ratio with the threshold. If $\frac{\lambda_{max}}{\lambda_{min}} > \gamma$, the signal is present, otherwise, the signal is not present.

SIMULATION

Simulation parameters

It is assumed that the channel is not changing during the period of samples. The level of the Hankel matrix, i.e. the column of the matrix is $L = 16$. The results are averaged over 10^5 tests using Monte-Carlo Simulations written in Matlab. Simulation results are taken using QPSK modulated random primary signal and independent and identically distributed (i.i.d.) noise samples with Gaussian distribution are used. Three types of signal namely rectangular pulse, raised cosine and root-raised cosine were tested and compared. To find the threshold, we require the probability of false alarm is $P_{fa} \leq 0.1$ and probability of detection is $P_d > 0.9$ as required by IEEE 802.22 standard.

Simulation results

Figure 1 shows simulation results of the P_d when the SVD-based method and energy detector (ED) are used for comparison when SNR is from -18dB to -4dB. It can be noticed from the graphs that the performance of the ED drops dramatically for SNR values below -8 dB. Even though ED at certain points better than the SVD based detection, but the overall performance of the detector is better than the ED. It is also shown in the graphs that the SVD-based method works better in detecting the rectangular pulse signal, raised cosine the second and root-raised cosine the third. The performance of ED for all three signals is quite similar. From these figures, it can be concluded that the SVD-based detection can overcome the flaws in ED when dealing with low SNR signals.

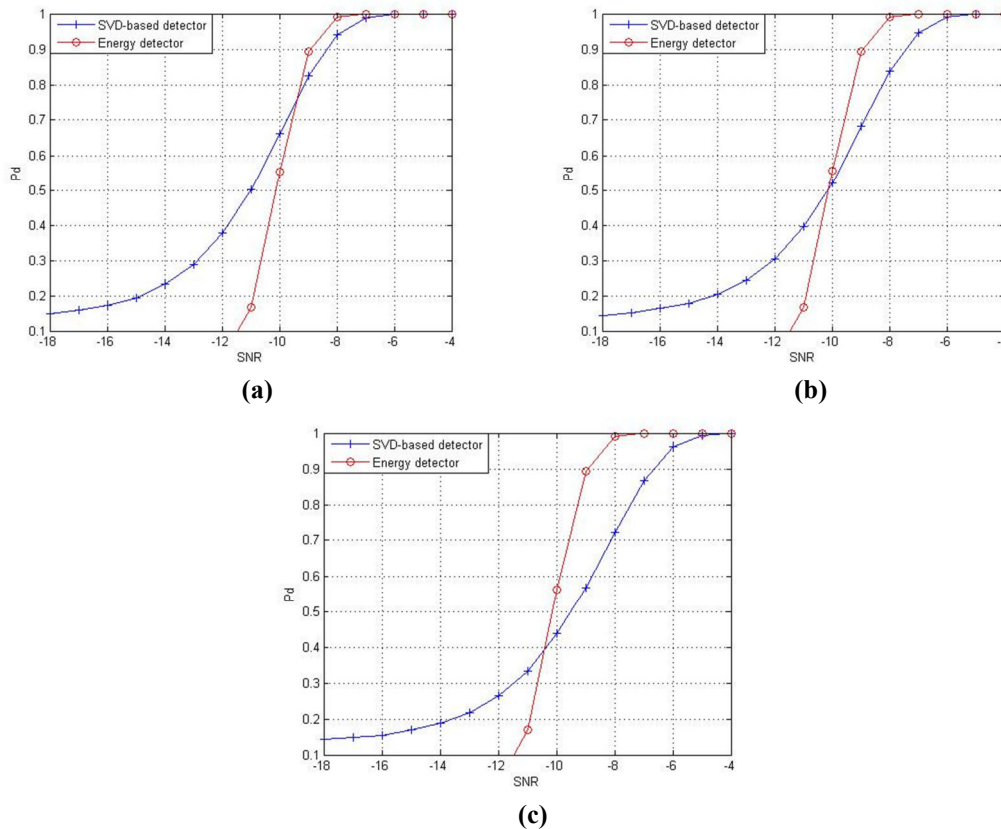


Figure 22. Comparison of P_d between the SVD-based detector and energy detector for a) Rectangular pulse, b) Raised cosine and c) Root-raised cosine.

In terms of performance of the detector, the receiver operating characteristics (ROC) curves are shown in Figure 2. Both methods were simulated at -8 dB SNR and tested for three types of signal. We plot the P_d under \mathcal{H}_1 against P_{fa} under \mathcal{H}_0 when P_{fa} changes from 0.01 to the desired value of 0.1. It is clearly shown that the ROC curves of the SVD-based detection are much higher than the ED's which proves the good performance of the detector.

CONCLUSIONS

In this paper, we highlight an SVD-based approach to detect common signals in today's digital communication system. The rationale of detecting common signals is that, in order for a CR system to operate with an acceptable QoS, the CR need to avoid interference not only to/from primary users but any signals which could affect the delivery of information to the system. The simulation results show that SVD of the data matrix is very useful in finding the dominant singular values in which the presence of other signals can be detected. The method is more robust to numerical errors and very fast. These qualities are desirable in IEEE 802.22 standard since it is easily suits the need to shorten the period of sensing and hence rendering the system to be more precise and reliable.

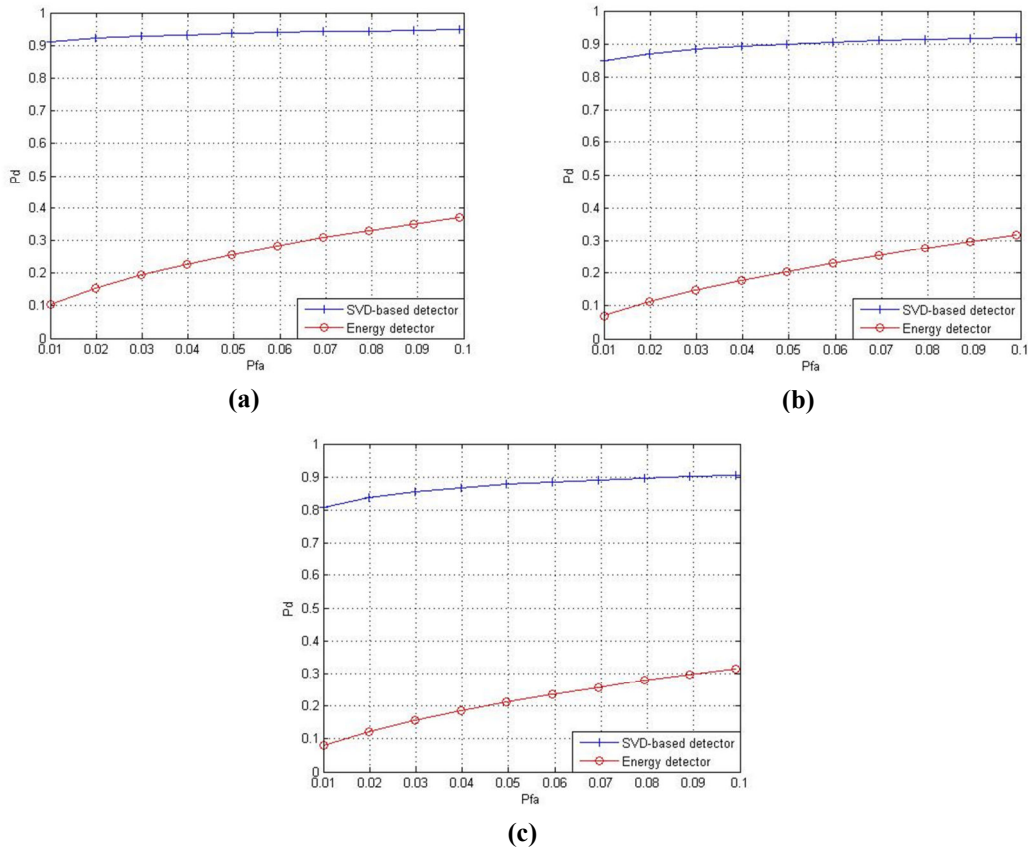


Figure 23. ROC curves of the SVD method and energy detector for a) Rectangular pulse, b) Raised cosine and c) Root-raised cosine.

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