Physically based finite element strength prediction in notched woven laminates under quasi-static loading

H. Ahmad*[1], A. D. Crocombe[2] and P. A. Smith[2]

The present paper is concerned with modelling damage and fracture in notched woven fabric composites. Previous experimental work has shown that, under tensile loading, damage at a notch in a variety of glass fibre reinforced plastic (GFRP) and carbon fibre reinforced plastic (CFRP) composites based on woven fabric reinforcement comprises matrix damage and fibre tow fracture along the plane of maximum stress. It is these experimental observations that inform the failure modelling developed here, in which a cohesive zone approach is used within a two-dimensional extended finite element method framework. The traction–separation parameters used in the extended finite element method implementation are based on previously reported experimental measurements for the strength and toughness of the woven fabric materials under investigation. The approach is shown to provide predictions of notched strength that are in very good agreement with experimental results from the literature for a range of glass fibre reinforced plastic and carbon fibre reinforced plastic woven fabric systems and also agree well with results obtained from closed form analytical models, which require calibration.

Keywords: Circular hole, Notched plate, Strength prediction, Stress, Woven composite, XFEM

Introduction

Woven fabric based composite materials are an attractive option for a range of engineering applications. Although the mechanical properties are not as good as those of their non-woven counterparts, they still offer reasonable specific stiffness and strength with particularly good impact and energy absorption characteristics. Moreover, they offer some economies and greater flexibility in processing options. A particular problem with woven composites (as with other composites) is their sensitivity to the presence of stress concentrations, such as those provided by an open hole and bolted joint. Unlike many structural metallic materials, composite materials in general lack the capability to deform plastically in the macroscopic sense and this means that failure at a stress raiser in a composite occurs in a relatively brittle fashion. Notwithstanding this, composite materials have the capability to sustain damage before fracture; this means that the notched strength is greater than would be expected simply on the basis of the elastic stress concentration factor. The damage mechanisms in notched woven composite plates differ from those in notched non-woven composite plate and this has implications for the modelling approach.

The failure of composite materials from a stress raiser, particularly an open hole, has been researched extensively over the years. Early closed form models (of which the most well known are perhaps the point and average stress criteria of Whitney and Nuismer[1]) have been followed by finite element (FE) based approaches of varying degrees of complexity. Some of the latter models treat the composite as a homogeneous material for the purpose of the stress analysis and then use laminate theory and two-dimensional in-plane failure models to simulate progressive damage at the ply level, followed by a degradation of the elastic properties to account for the stress redistribution before ultimate failure, e.g. Chang and Chang.[2] Other models use a quasi-two-dimensional approach,[3] or a full three-dimensional analysis, e.g. Hallett *et al.*[4] These approaches represent the observed damage more realistically – in particular, they incorporate through thickness damage (delamination) in addition to in-plane damage. Kortschot *et al.*[3] proposed a model for notched strength prediction which represents the effect of intraply damage (local unidirectional ply splitting) and interply (associated delamination) cracking on the stress distribution near the notch tip in the longitudinal plies of notched cross-ply laminates and uses a statistical Weibull based failure criterion in two-dimensional (2D) model to predict the failure of the 0° plies and hence the notched strength. Hallett *et al.*[4] modelled damage within an FE framework and used the

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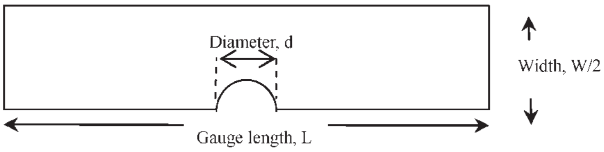
approach to investigate the effect of a number of parameters (hole diameter, ply thickness and laminate thickness) on the strength of notched quasi-isotropic carbon fibre reinforced plastic (CFRP) laminates subjected to tensile stress. Another class of failure model (discussed further below) is based on modelling macroscopic through thickness crack growth from the notch, an approach which is perhaps not consistent with the observed failure mechanism in laminates based on unidirectional layers of reinforcement such as discussed above.

Many of the more widely used approaches to predict notched strength are semiempirical in nature and this limits their general applicability. In particular, the subcritical damage that develops at notches (and which needs to be represented in some way within any model) varies according to the details of the composite under investigation. Hence, it is only through the use of physically based models that there is the possibility of a failure theory being applicable to more than one class of problem without extensive recalibration of the model. In a notched woven fabric glass fibre reinforced plastic (GFRP) composite subjected to tension, it has been shown experimentally using a model (transparent) GFRP system that before failure, an intense zone of damage develops in the region of maximum tensile stress.[5] Damage within that zone comprises matrix cracking and splitting, but also fibre tow fracture. In that sense, the damage resembles stable self-similar crack growth, suggesting that a fracture mechanics based model is a physically reasonable one to use. A similar study was carried by Belmonte *et al.*[6] on woven quasi-isotropic and cross-ply CFRP. Although the opaque nature of the CFRP means that direct observations of damage are less straightforward, there was evidence of tow failure before specimen failure. Other valuable data concerning notched tensile strength of woven composites had been presented previously by Kim *et al.*[7] More recent work by Zahari *et al.*[8] has considered the notched compressive behaviour of woven GFRP.

The previous paragraph suggests that an appropriate failure model for notched woven fabric composites under tensile loading would consider the damage growth from the notch explicitly. There are a number of essentially similar techniques that treat the problem in this way. A particular stress–displacement relation is assumed within the damage zone and the energy absorbed in separating the crack faces corresponds to the material toughness G_c. One of the first models to adopt this method was that of Eriksson and Aronsson[9] who assumed a constant cohesive stress–displacement relationship within the damage zone. An approximate analytical version of this approach, a 'critical damage growth' (CDG) model, was presented by Hitchen *et al.*,[10] building on earlier work by Soutis and Fleck,[11] and this was applied subsequently to woven composites.[5,6] Afaghi-Khatibi *et al.*[12] presented an 'equivalent crack growth model' (ECGM) which assumed a linear decrease in cohesive stress with displacement in the damage zone, which they applied successfully to the data of Kim *et al.*[7]

The present work explores further the applicability of this type of approach, which in the present paper is incorporated within an extended finite element model (XFEM) formulation. A limitation of the previous studies is that the material parameters within the models have generally been calibrated against experimental data rather than determined independently. Here we use independent measurements of unnotched strength and toughness, where available, to develop numerical models for the experimental studies on woven fabric systems conducted by Kim *et al.* using GFRP[7] and Belmonte *et al.* using GFRP[5] and CFRP.[6] In the next section, details of the numerical procedure are provided. Extended finite element model predictions are compared with experiment and other closed form solutions in subsequent sections.



1 Geometry of notched composite plate model, used for all experimental cases

Finite element modelling

Notched plate geometry and material properties

The three experimental studies from the literature were modelled within a two-dimensional FE framework using ABAQUS CAE Version 6.9.1.[13] For all the models, the plate symmetry means that only half of the coupon needed to be modelled (Fig. 1), reducing computational cost and time during the processing stage. The first data set is taken from the extensive experimental work conducted by Kim *et al.*[7] using three types of woven fabric composite systems with various combinations of notch sizes, laminate width and volume fraction. All plates have a length L of 200 mm with varying widths W and a circular hole of diameter d of 5 mm. Table 1 shows the geometry and material properties for all the woven fabric systems investigated experimentally by Kim *et al.*[7] that are being modelled in current paper. The elastic properties are required for the stress analysis while to implement the damage model, the unnotched strength σ_o and the fracture energy G_c are also required. Independently measured values for the latter were not reported by Kim *et al.*[7] Instead, they derived their values from an analysis of the notched strength experimental data and this is the origin of the notch size dependent data (strength and fracture energy) seen in Table 1, apparent in the glass–epoxy in particular. In the absence of independently measured properties, these fracture energy values are used in the current work as it enables comparison not only with the experimental data from Ref. 7 but also with the ECGM approach, which has been applied to the same data set.[12]

The second and third data sets were taken from the work of Belmonte *et al.* for woven GFRP laminates[5] and woven CFRP laminates[6] respectively. The plates had 130 mm gauge length (GFRP) or 150 mm gauge length (CFRP) and a constant width of 25 mm with notch sizes (hole diameters) d of 2·5, 5·0 and 10·0 mm giving d/W ratios of 0·1, 0·2 and 0·4 respectively. Belmonte also included an additional series of larger CFRP specimens from the thickest plates that had gauge length 380 mm, width 120 mm and a 20 mm diameter hole corresponding to $d/W=0\cdot17$. Belmonte *et al.*[5,6] carried out experiments on two different types of woven composite systems, i.e. GFRP and CFRP. The material properties are shown in Tables 2 and 3. The GFRP systems comprised two stacking sequences of a four-layer quasi-isotropic composite (Table 2). The woven CFRP systems investigated had two types (a plain weave and a five harness satin weave), two different lay-ups (cross-ply and quasi-isotropic) and three different plate thickness for each giving 12 systems in total (Table 3). In contrast with Kim *et al.*,[7] Belmonte *et al.*[5,6] reported values of fracture energy that were measured independently for each lay-up using single edge notch fracture mechanics specimens in accordance with ASTM E399-90.

Implementation of constitutive law in FE modelling

A typical plane stress mesh used in the FE modelling is shown in Fig. 2. The meshes are refined in the vicinity of notch edge, while away from the notch, the mesh can be made coarser. Mesh refinement was investigated and is discussed in a later section on 'Woven composite systems tested by Kim *et al.*[7]'. As indicated earlier, in the notched plate under increasing tensile load, damage is assumed to grow in a self-similar manner along the plane of net tension (reduced area). Hence, only crack opening (mode 1) needs to be considered. In this work, two methods for modelling the damage have been used, namely XFEM and cohesive zone modelling (CZM). All of the data were modelled using XFEM and some of Belmonte *et al.*'s[5,6] data were also modelled using CZM. Both methods use the same traction–separation damage model, illustrated schematically in Fig. 3. This material model behaves in a linear manner until the traction reaches a critical value (strength parameter) and then unloads with increasing displacement dissipating the fracture energy G_c in the process. The strength parameter has been taken as the unnotched strength σ_o of the laminate. The values of strength parameter (unnotched strength σ_o) and fracture toughness G_c are obtained from literature[12] and independently from experimental work[5,6] and are shown in Tables 1–3. Both damage models have been embedded within a mesh of 2D continuum elements having a linear elastic plane stress response. Both damage models are discussed briefly below.

In conventional FE modeling, a discontinuity such as crack requires accurate modelling and hence mesh refinement. Furthermore, crack propagation usually

Table 1 Material properties of woven fabric system conducted by Kim *et al.*[7]*

Materials type	Thickness t/mm	v_f/%	E_{xx}/GPa	E_{yy}/GPa	G_{xy}	v_{xy}	Width/mm	Unnotched strength σ_o/MPa	Fracture energy G_c/kJ m^{-2}
Glass–epoxy	2·0	62	23·6	23·6	4·0	0·11	10	351	12·6
							20	319	21·2
							40	275	33·1
Glass–polyester	2·3	60	21·6	21·6	3·9	0·16	10	309	14·9
							30	296	17·5
	2·3	44·5	14·7	14·7	2·7	0·17	10	218	7·50
							30	198	8·00
Carbon–epoxy	1·2	60	56·7	56·7	8·7	0·22	10	596	30·5
							20	581	45·0

*v_f=fibre volume fraction; E_{xx}=longitudinal Young's modulus; E_{yy}=transverse Young's modulus; v_{xy}=Poisson's ratio; t=laminate thickness.

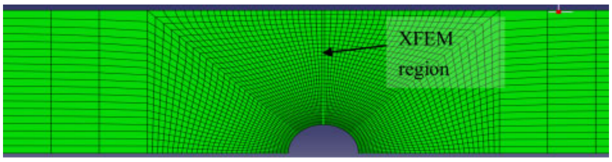
Table 2 Material properties of woven GFRP from Belmonte *et al.*[5]

Stacking sequence	t/mm	E_x/GPa	E_y/GPa	v_{xy}	G_{xy}/GPa	σ_o/MPa	G_c/kJ m^{-2}
$(0°/90°/\pm45°)_s$	1·25	15·9	15·9	0·3	6·10	291	20·3
$(90°/0°/\pm45°)_s$	1·25	16·0	16·0	0·3	6·10	291	20·3

*The unnotched strength and toughness values for the $(90°/0°/\pm45°)_s$ stacking sequence were assumed to be the same as for the $(90°/0°/\pm45°)_s$ stacking sequence.

Table 3 Material properties of all woven CFRP from Belmonte *et al.*[6]

Series	Laminate code	t/mm	E_x/GPa	E_y/GPa	v_{xy}	G_{xy}/GPa	σ_o/MPa	G_c/kJ m^{-2}
Plain weave cross-ply (PX)	PX2	0·51	50·4	50·4	0·103	4·42	481	26·0
	PX4	1·03	51·4	51·4	0·092	4·42	527	27·7
	PX8	2·03	53·1	53·1	0·083	4·42	538	22·7
Plain weave quasi-isotropic (PQ)	PQ4	1·02	37·2	37·2	0·353	13·8	390	21·6
	PQ8	2·03	36·8	36·8	0·328	13·9	428	17·9
	PQ12	3·17	35·2	35·2	0·297	13·6	372	18·3
Five harness satin cross-ply (5X)	5X2	0·81	45·1	45·1	0·077	3·78	419	28·8
	5X4	1·60	47·0	47·0	0·062	3·78	535	20·0
	5X8	3·15	47·4	47·4	0·053	3·78	456	17·6
Plain weave quasi-isotropic (5Q)	5Q4	1·53	34·1	34·1	0·296	13·2	375	19·2
	5Q8	3·17	33·5	33·5	0·320	12·7	347	16·8
	5Q12	4·59	34·8	34·8	0·322	13·2	370	12·9



2 Mesh of model implemented

requires a predefined crack path allowing the propagation of cracks along element boundaries. Extended finite element model, which is an extension to conventional FE methods, is based on an enrichment function that allows a displacement jump between crack faces to occur during crack propagation. Crack path and crack location are not required to be specified *a priori* and the elements effectively split to allow arbitrary crack propagation. The response at this split is defined by the traction–separation response discussed above.

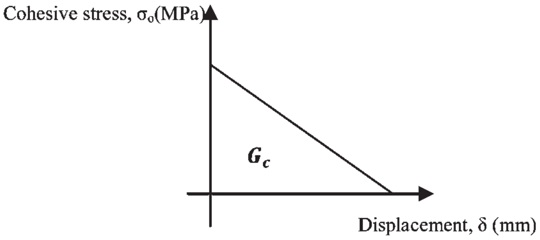
Damage initiation and propagation are simulated at regions experiencing principal stresses greater than corresponding values specified in traction–separation law and always take place orthogonally to the maximum principal stresses. Damage initiation is triggered when the maximum principal stress reaches the critical traction. Damage evolution is controlled by a damage parameter D which is determined from the current separation and the release separation (determined from G_c and σ_o). Fracture makes the structural response non-linear and numerical methods can experience difficulty converging to a solution. Viscous regularisation (a form of damping) has been used to facilitate convergence and a parametric study was undertaken to determine the optimum value (small enough not to influence the solution values but large enough to allow convergence to be obtained). The XFEM region (i.e. the region capable of sustaining damage) was assigned to a band of the model beside the notch edge (*see* Fig. 2). Four noded 2D plane stress elements (CPS4) were used as the current implementation of XFEM only worked in conjunction with the first order elements.

Unlike XFEM, CZM requires that the failure path is specified beforehand and seeded with cohesive elements.[14] As already discussed, the method of modelling the damage in the CZM is the same as in the XFEM approach (a traction–separation response). The cohesive zone elements were located along the net section plane. The elements are required to be thin (0·01 mm was used in the current work) and were assigned a high initial stiffness. Cohesive elements (COH2D4) were used within cohesive zone region and eight noded quadratic plane stress elements (CPS8) were used in uncracked region. Along common interfaces, the elements were connected using a tie constraint.

Comparison of strength prediction with experimental data and other models

Woven composite systems tested by Kim *et al.*[7]

Comparison between XFEM results and experimental data for the notched woven laminates reported by Kim *et al.*[7] are shown in Fig. 4. In general, the agreement is good with discrepancies no more than 20% (and generally much less than this) for any system/notch size. In all cases apart from the 30 mm wide glass–polyester samples, the XFEM approach underestimates the experimental strength. A slightly larger error overall is observed for the glass–polyester composites than the other systems, but no particular significance is attached to this, especially given the uncertainty regarding the toughness values. Figure 5 compares the XFEM results to ECGM approach adopted by Afaghi-Khatibi *et al.*[12] The agreement between the ECGM results and XFEM is very good, not unexpectedly as both represent fracture mechanics formulations of the problem, and this provides validation for the XFEM formulation used here.

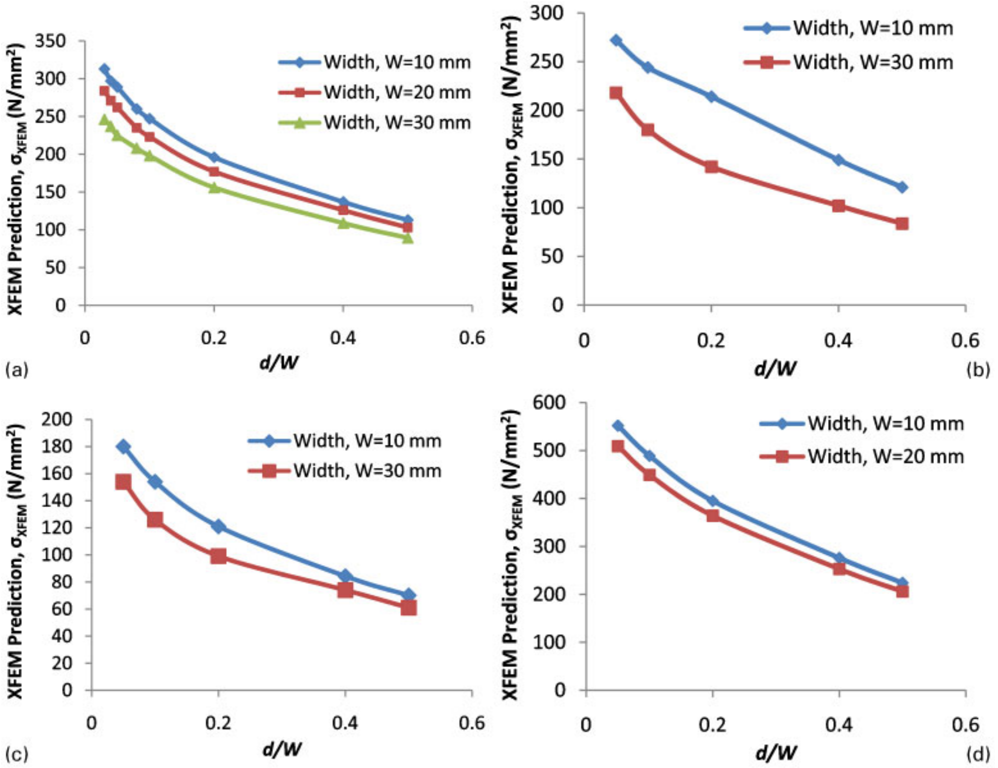


3 Physically based constitutive model used in current analysis

Typical load–displacement plots from the XFEM are shown in Fig. 6, giving results from 10 mm wide glass/epoxy lay-up plates with hole diameter of 0·4 mm and close-up view damage plots at specific locations in the graph were illustrated in Fig. 7. Owing to large stress concentration at the vicinity of hole as tension load is applied, the onset of damage will be initiated at the hole edge as expected. After initiation, the crack was still able to carry increased load until it reached about one hole diameter in length at which point ultimate failure occurred. It would appear that the length of the process zone is critical in determining ultimate failure. This process continued until the plate separated completely associated with catastrophic failure after maximum load had been achieved. Two types of parametric studies were carried out to determine the sensitivity of the strength prediction to key model parameters (10 mm wide glass/epoxy with 0·4 mm hole diameter was chosen for this purpose). The first study was to determine an acceptable viscous regularisation value to use to ensure that the strength prediction results were independent from the viscosity constant. It was found that it was difficult to obtain converged solutions if the viscosity was too small; however, as can be seen, large viscosity values gave excessive and non-physical results. From Fig. 8, a viscosity of 0·0001 is sufficient as lower values produce similar results. The second study assessed mesh sensitivity and the results are shown in Fig. 9. The number of elements refers to the region surrounding the hole, and the mesh surrounding the hole was refined in a radial direction. It can be seen that there is no significant mesh sensitivity; indeed one of the basic features of XFEM formulation is that a high level of mesh refinement at the crack tip is not required.[13]

Quasi-isotropic woven GFRP tested by Belmonte *et al.*[5]

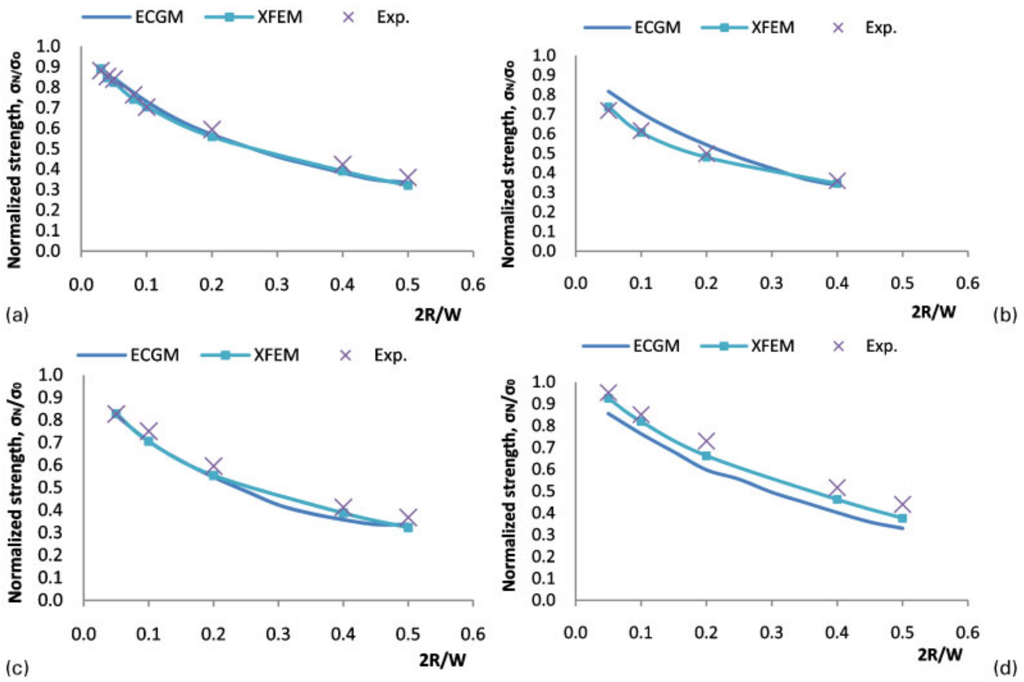
As expected, all of the models showed failure along the net tension plane, which is within the enriched XFEM region specified in the model. The XFEM predicted notched strengths are compared with experimental data, CZM predictions and various analytical approaches[5] in Fig. 10. Typical load–displacement plot is given in



a glass–epoxy; *b* glass–polyester (V_{1f}=60%); *c* glass–polyester (V_{1f}=44·5%); *d* carbon–epoxy

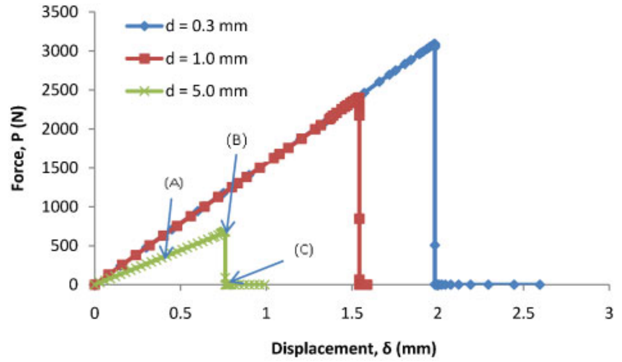
4 Comparison of experimental notched strength with XFEM modelling of Kim *et al.*'s work[7]

Fig. 11, with associated enclosed view at crack initiation and propagation point displayed in Fig. 12. Similar fashion of XFEM results of load–displacement plot was observed as shown in earlier case (Fig. 6). The XFEM gives very reasonable agreement with experimental data, to within <6% and with the CZM. Minor discrepancies between the XFEM and CZM are most likely due to the different element types (the elements used in CZM were quadratic while in XFEM only a linear elements could be used as suggested in the relevant ABAQUS manual[13]). For the $(0°90°/+45°/-45°)_s$ lay-up, the Whitney–Nuismer point and average stress criterion (PSC and

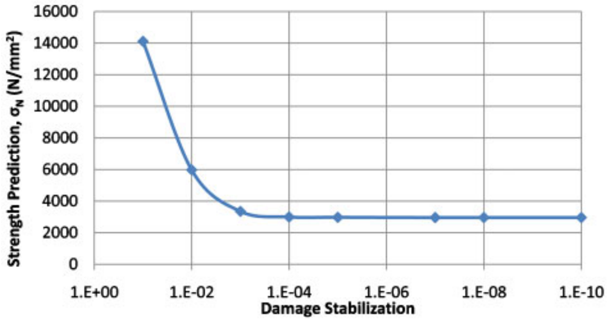


a glass–epoxy (*W*=10 mm); *b* glass–polyester (*W*=30 mm; v_f=60%); *c* glass–polyester (*W*=30 mm; v_f=44·5%); *d* carbon–epoxy (*W*=10 mm)

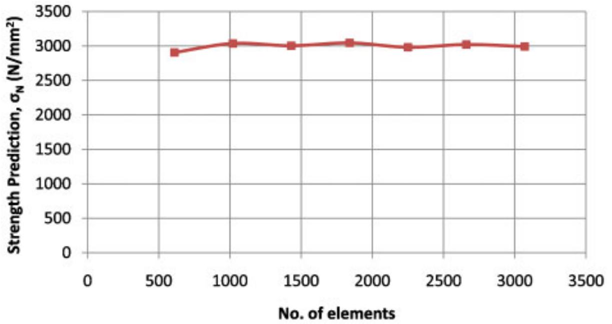
5 Normalised notched strength plotted as function of hole size for laminates tested by Kim *et al.*[7]: comparison between experiment, ECGM approach[12] and XFEM results from current study



6 Typical load–displacement plots resulting from implementing XFEM approach for laminate reported by Kim *et al.*[7]



8 Notched specimen failure load as function of damage stabilisation coefficient for woven glass–epoxy system (10 mm wide and hole diameter of 0·4 mm) following Kim *et al.*[7]



9 Notched specimen failure load as function of number of elements for woven glass–epoxy system (10 mm wide and hole diameter of 0·4 mm) following Kim *et al.*[7]

ASC), give excellent agreement as expected (these semiempirical models are calibrated at one of the notched strength results). The CDG model also gives good agreement. The strength predictions for the $(90°/0°/+45°/-45°)_s$ are reasonable. Note that for this stacking sequence, the same strength and toughness values were assigned as for the other stacking sequence. The results perhaps suggest that this stacking sequence has a lower strength and/or toughness – a lower toughness would be consistent with the surface 90° layers restricting the extent of 0° ply splitting, which seems plausible.

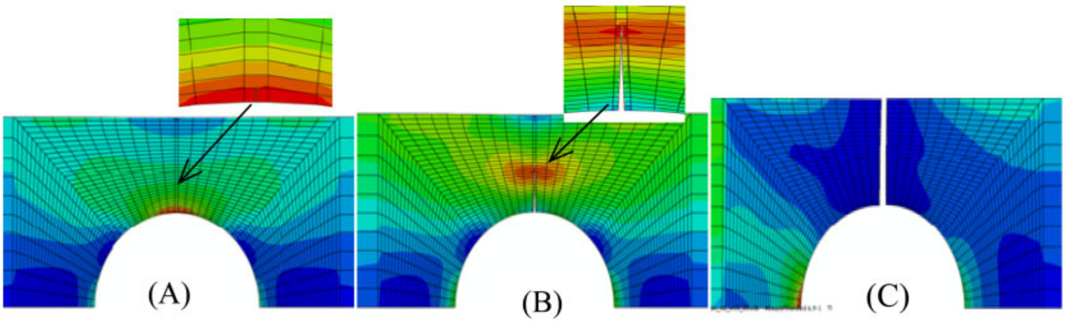
Cross-ply and quasi-isotropic woven CFRP tested by Belmonte *et al.*[6]

The results for the cross-ply and quasi-isotropic woven CFRP composite systems with different notch sizes are presented in a similar way in Fig. 13. Both CZM and XFEM of failure initiation and propagation in woven CFRP exhibited similar trends as shown in woven GFRP. Overall agreement is very good, within 10% for most systems (the range of error is 0·09–19·41%). Similar size errors were found in large width (40 mm) and small width (25 mm) specimens. In almost all cases, the XFEM tends to underestimate the experimental strength, with a higher discrepancy for the cross-ply systems than the quasi-isotropic systems. The underestimation may be a result of the simplifying assumptions within the constitutive law. For instance, the tow fractures tend to follow the crimp regions and this means that the assumption of self-similar crack growth is not entirely realistic.

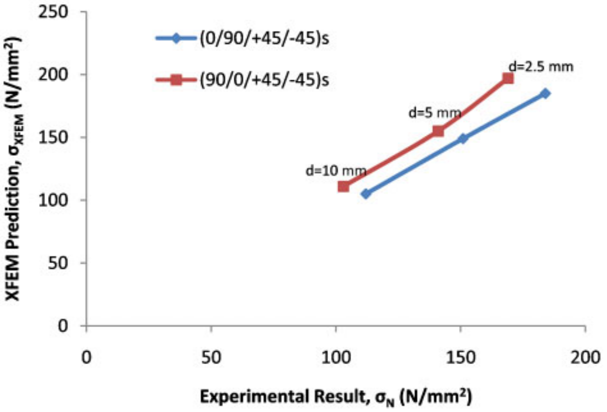
As with the GFRP results, the PSC and ASC (the Whitney–Nuismer failure criteria) give the best predictions compared with other approaches for the hole sizes of 10 mm or less, over which range they were calibrated against the experimental data. As a result, the PSC and ASC predictions are less good for the larger hole size of 20 mm where the same characteristic distance values were used. Overall the numerical results obtained here using XFEM show better prediction than the CDG model implemented by Belmonte *et al.*,[6] especially for the cross-ply systems. The CDG model was formulated based on an isotropic analysis, which could only be corrected partially for the orthotropic cross-ply systems and so the discrepancies are perhaps not surprising.

Concluding remarks

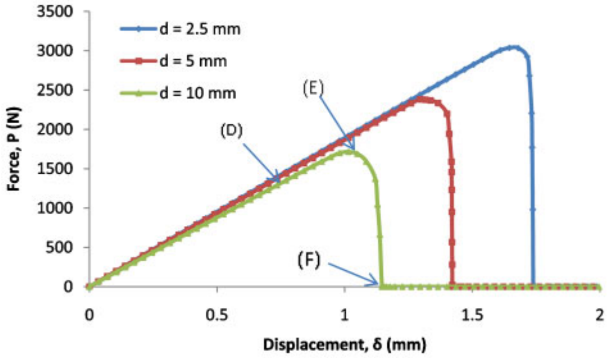
Two-dimensional modelling of notched woven GFRP and CFRP composite plates has been implemented within an FE framework. The XFEM and CZM



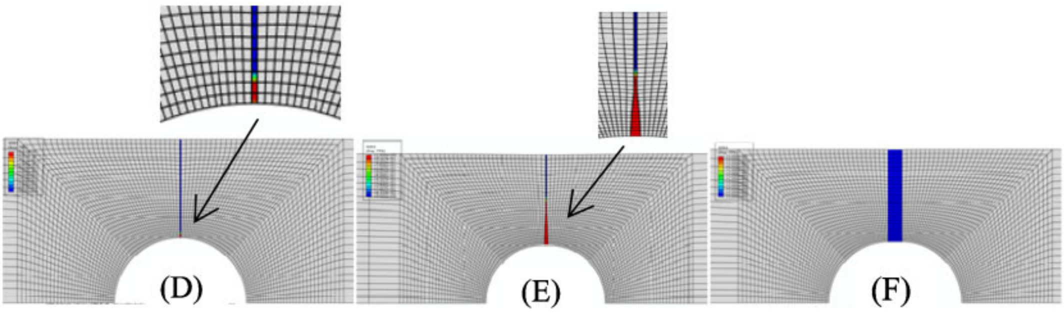
7 Damage plot of crack initiation and crack propagation using XFEM as labelled in Fig. 5 for glass/epoxy (W=10 mm; d=2 mm)



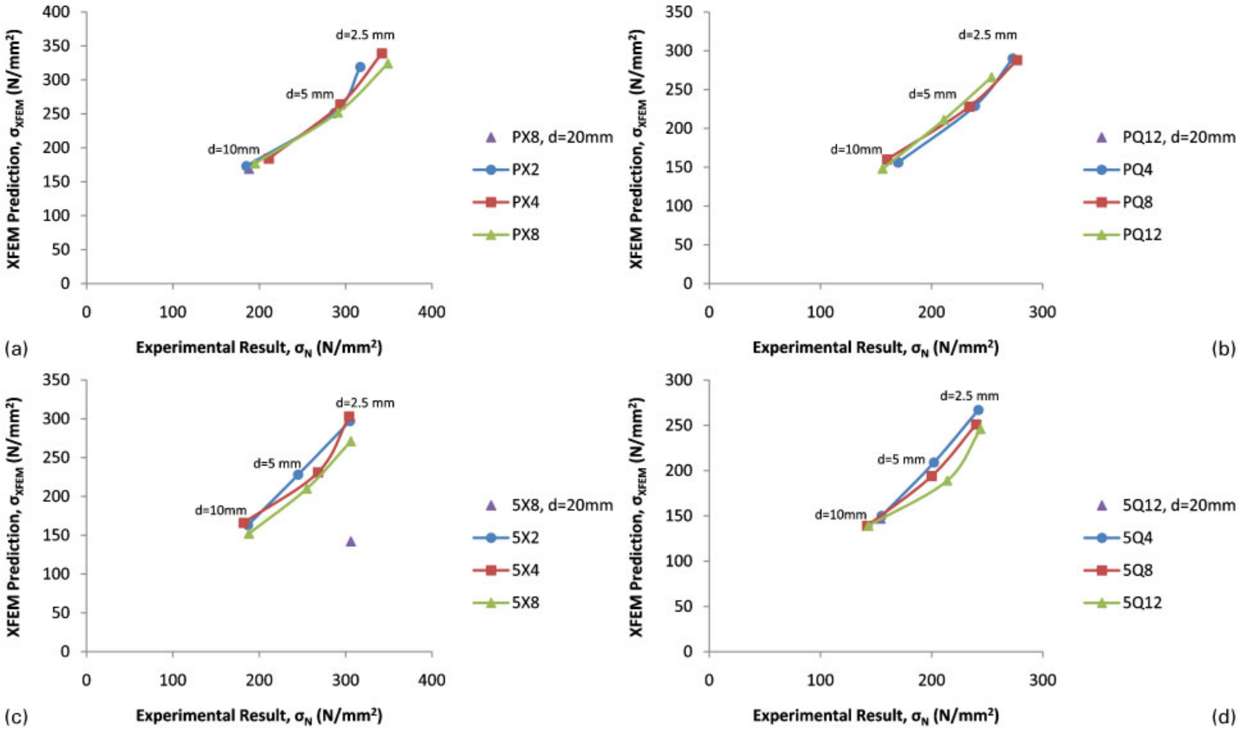
10 Comparison of woven GFRP notched strength prediction using XFEM (present work) with experimental data



11 Typical load–displacement curve from CZM (woven GFRP $(0°/90°/\pm45°)_s$



12 Damage plot for cohesive damage modelling using CZM as labelled in Fig. 9 for GFRP plate (d=10 mm). Note that failure in cohesive zones is denoted as SDEG=1 and given as red colour (D, E) and blue colour (F)



a PX series; *b* PQ series; *c* 5X series; *d* 5Q series

13 Comparison of woven CFRP notched strength prediction using XFEM (present work) with experimental data

approaches gave very reasonable agreement with the experimental data for all the systems investigated. There was also good agreement with a range of other modelling approaches. The advantages of the XFEM/CZM approach used in the present work are that, unlike the other approaches, it is physically based (and consistent with experimental observations of damage and fracture) and that it may be implemented using independently measured material properties. In turn this offers the prospect that the approach may be applicable to failure in other classes of problem (e.g. net tension failure at a bolted joint) without extensive recalibration of the model.

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