GOVERNANCE AND THE EFFICIENCY OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 444

Good news and bad news in subjective performance evaluation

Jörg Budde *

* University of Bonn

September 2013

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

Sonderforschungsbereich/Transregio 15 · www.sfbtr15.de
Universität Mannheim · Freie Universität Berlin · Humboldt-Universität zu Berlin · Ludwig-Maximilians-Universität München
Rheinische Friedrich-Wilhelms-Universität Bonn · Zentrum für Europäische Wirtschaftsforschung Mannheim

Speaker: Prof. Dr. Klaus M. Schmidt · Department of Economics · University of Munich · D-80539 Munich,
Phone: +49(89)2180 2250 · Fax: +49(89)2180 3510

Good news and bad news in subjective performance evaluation [‡]

Jörg Budde[§]

University of Bonn

September 2013

Abstract

Earlier studies show that contracts under subjective performance evaluation are dichotomous and punish only worst performance. I show that with limited liability payments need not be binary. More importantly, if the agent earns a rent from limited liability, the optimal contract distinguishes only signals of good news and bad news of the agent's action.

Keywords: bonus, monotone likelihood ratio, wage compression

JEL classification numbers: D82, M52, M54

[‡]Financial support by the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

[§]Address of Correspondence: Jörg Budde, Department of Economics, University of Bonn, Adenauerallee 24-42, D-53113 Bonn; phone: +49-228-739247; email: Joerg.Budde@uni-bonn.de

1 Introduction

When outputs are nonverifiable in a principal-agent relation, credibility of compensation payments may be achieved by the principal's commitment to pay a constant amount to all agent and a third party. MacLeod (2003) shows that in a single-agent case this leads to wage compression in the sense that only under the poorest outcome the agent's wage is reduced. This wage compression result also holds when an additional verifiable performance signal is available (Rajan and Reichelstein 2009, Ederhof 2010)

I show that this extreme form of bonus payments in general only applies if limited liability is not an issue. With binding liability constraints the set of outcomes for which the agent is punished is enlarged up to the case where a bonus is paid only under good news in the sense of Milgrom (1981). Thus, while the wage compression result seems to comply with some empirical evidence, there also exist situations which are in line with a more straightforward economic intuition to pay rewards only if a positive signal on the agent action is observed. The paper therefore provides a rationale for the existence of a whole range of different bonus agreements, including both lenient and more demanding requirements.

2 The model

The model generalizes MacLeod's (2003) model of subjective performance evaluation, and mainly builds on the assumptions of Rogerson's (1985) analysis of the standard moral hazard problem with verifiable information.

So consider a principal (she) who hires an agent to fulfil a certain task on her behalf. If the agent (he) accepts the contract offer, he chooses an action a from a real interval $A = [\underline{a}, \overline{a}]$. The principal's gross benefit from the action is $B(a)$ with $B' > 0$ and $B'' \leq 0$. Rather than B the principal observes a signal $Y \in \{y_1, \ldots, y_n\}$ of the agent's action, the probability $p_i(a) = \text{Prob}\{Y = y_i \mid a\}$, $i = 1, \ldots, n$ of which depends on the agent's action choice. Assume that p_i is strictly positive and twice continuously differentiable for every $i \in \{1, \ldots, n\}$ and that the strict monotone likelihood ratio property (MLRP) is fulfilled such that $p_i'(a)/p_i(a) < p_{i+1}'(a)/p_{i+1}(a)$ holds for $i = 1, \ldots, n-1$ (see Milgrom 1981). To exclude message sending games, I assume that the agent does not observe Y.[1] Furthermore, I assume that the convexity of the distribution function condition (CDFC) holds, i.e. that $P_i''(a) > 0$, where $P_i(a) = \sum_{j=1}^{i} p_j(a)$ is the cumulative distribution function of Y under action a. Since most contracts derived here will have the structure of a simple bonus contract, it will be convenient to introduce the survival function $R_i(a) = 1 - P_i(a)$.

The contract between the principal and the agent fixes payments $w_i = w(y_i), i = 1, \ldots, n$ contingent on the observed realization of the signal Y. To make the payment credible, the principal commits to pay a certain sum $\overline{w} \geq \max_i\{w_i\}_{i=1,\ldots,n}$ and to transfer the residual amount $\overline{w} - w_i$ to a third party. By this arrangement, he has no incentive to misreport the observed signal.

The agent's preferences can be described by the utility function $V(w_i, a) = U(w_i) - C(a)$, where $U(w_i)$ is his utility from money and $C(a)$ is his cost of effort. The agent is risk averse and increasingly effort averse, i.e. $U' > 0, U'' < 0$, $C' > 0$ and $C'' > 0$. His outside options are described by his reservation utility

[1]This assumption is similar to that in section II.b of MacLeod (2003) and is also made in Rajan and Reichelstein (2009).

V^R. Moreover, the agent is of limited wealth such that wage payments w_i must not fall short of a certain minimum level $w^{min} \geq 0$.[2] Limited liability will be the main difference to the analysis of McLeod (2003), who assumes that wages have to be nonnegative, but precludes corner solutions by assuming $\lim_{w \to 0} U(w) = -\infty$.

The principal is risk neutral and wants to maximize her gross benefit net of wage payments. I focus on the principal's problem to implement a certain action $a^0 \in A$ at minimal cost (cf. Grossman and Hart 1983). This can be stated as follows

$$\min_{w_1, \ldots w_n} \overline{w} \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} p_i(a^0) U(w_i) - C(a^0) \geq V^R \tag{2}$$

$$a^0 \in \arg\max_{a \in A} \left\{ \sum_{i=1}^{n} p_i(a) U(w_i) - C(a) \right\} \tag{3}$$

$$w_i \leq \overline{w} \tag{4}$$

$$w_i \geq w^{min}. \tag{5}$$

The principal wants to minimize her total payment $\overline{w}$, subject to the familiar participation constraint (2) and incentive compatibility constraint (3). The budget constraint (4) requires that all wages have to be smaller than the principal's total payment $\overline{w}$, and the liability constraint (5) demands that the agent's wage must not fall short of his liability level w^{min}.

Under MLRP and CDFC, the agent's incentive compatibility constraint (3)

[2] Nonnegativity of w^{min} is only assumed for ease of exposition. $w^{min} < 0$ would require upfront payments of the agent.

can be replaced by a first order condition[3], which takes the form

$$\sum_{i=1}^{n} p_i'(a^0) w_i - C'(a^0) = 0, \tag{6}$$

stating that a^0 is a stationary point of the agent's action choice problem.

3 Results

Without a binding liability constraint, MacLeod (2003) derives a wage compression result in the sense that the optimal contract is dichotomous and pays a bonus for all but the worst outcome. In the present model, however, this in general is no longer the case when the agent's liability constraint becomes binding. To see this, consider the agent's incentive constraint (6). Due to the MLRP and the assumption of a non-moving support of Y, $p_i'(a^0)$ is negative at least for the worst realization $i = 1$. Any wage payment for this outcome hinders incentives. Consequently, also in MacLeod's (2003) model no bonus is paid. The compensation under y_1 only serves to provide some expected utility for the agent, helping to meet his participation constraint. If the agent's liability constraint is binding, however, this is no longer necessary. If the agent earns a rent from his limited liability, for instance, the principal will preclude bonus payments for all realizations of Y for which $p_i'(a^0) < 0$ in order to increase incentives. For notational convenience, denote by $Y_-(a^0) = \{y_i \mid p_i'(a^0) < 0\}$ the set of realizations for which this is the case, and its complement by $Y_+(a^0) = Y \backslash Y_-(a^0)$, and let $P_-(a^0) = \sum_{Y_-} p_i(a^0)$ and $P_+(a^0) = \sum_{Y_+} p_i(a^0)$ be the probabilities of these sets. Using this notation, the

[3] A proof is in the appendix

above arguments are formalized in the following proposition.

Proposition 1 *The solution to the principal's implementation problem given by (1), (2), (6), (4) and (5) has the following structure:*

1. *If none of the liability constraints (5) is binding, the agent's wage is two-step with*

$$w_i = \begin{cases} U^{-1}\left(V^R + C(a^0) - \frac{R_1(a^0)}{R_1'(a^0)}C'(a^0)\right) & \textit{if} \quad i = 1 \\ U^{-1}\left(V^R + C(a^0) + \frac{1-R_1(a^0)}{R_1'(a^0)}C'(a^0)\right) & \textit{else}. \end{cases} \tag{7}$$

2. *If some liability constraint (5) is binding and the agent earns a rent such that his participation constraint (2) is not binding, the agent's wage is two-step with*

$$w_i = \begin{cases} w^{min} & \textit{if} \quad y_i \in Y_-(a^0) \\ U^{-1}\left(U(s^{min}) + \frac{C'(a^0)}{P_+'(a^0)}\right) & \textit{if} \quad y_i \in Y_+(a^0). \end{cases} \tag{8}$$

3. *If some liability constraint (5) and the agent's participation constraint (2) are binding, the agent's wage is three-step with*

$$w_i = \begin{cases} w^{min} & \textit{if} \quad i < \hat{i} \\ \hat{w} \in [w^{min}, \overline{w}] & \textit{if} \quad i = \hat{i} \\ \overline{w} & \textit{if} \quad i > \hat{i} \end{cases} \tag{9}$$

for some $\hat{i}$ with $y_{\hat{i}} \in Y_-(a^0)$.

Proposition 1 describes how the optimal contract changes with liability matters. Only if the liability constraint is not binding, the contract proposed by

MacLeod (2003) persists, and a bonus is refused only for the worst possible performance. If on the other hand the liability constraint binds but the participation constraint does not, a bonus payment is refused for all signal realizations which convey bad news about the agent's action. If both the liability constraint and the participation constraint are binding, the optimal contract is "between" these two extremes, refusing the bonus payment only for some lower levels of the signal conveying bad news. For higher level of the signal, conveying news which are "bad, but not too bad", the same bonus is paid as under good news. The extend of this leniency depends on the severity of the liability constraint. The more severe the liability constraint is, the less freehanded the principal will be in distributing the bonus. Therefore, all aspiration levels from "no complete failure" up to "good news" may be observed in practice.

This property of the optimal contract remains even if a verifiable performance measure is available. For this situation and unlimited liability, Rajan and Reichelstein (2009) derive a "super wage compression" result in the sense that the unverifiable measure is only used for the worst outcome of the verifiable measure, and for this worst verifiable performance the compensation is only cut for the worst outcome of the unverifiable measure. With binding liability limits, however, the optimal contract again may vary from this wage compression contract to a "rewarding only good news" contract as described above. To formalize this, let $X \in \{x_1, \ldots, x_m\}$ be the verifiable measure and denote by $p_{ij}(a)$ the probability of (y_i, x_j) under action a. Similar to the notation above, let $XY_-(a^0) = \big\{(y_i, x_j) \mid p'_{ij}(a^0) < 0\big\}$ the set of outcomes conveying bad news about the agent's action, and $XY_+(a^0) = \big\{(y_i, x_j) \mid p'_{ij}(a^0) \geq< 0\big\}$ be the set with good news. Then, the optimal contract specifies wages $w_{ij} = w(y_i, x_j)$ and bonus pools

$\overline{w}_j = \overline{w}(x_j)$ as follows:

Corrollary 1 *The solution to the principal's implementation problem under verifiable and unverifiable information has the following structure:*

1. *If none of the liability constraints is binding, the agent's wage is*

$$w_{ij} = \begin{cases} w_{11} > 0 & \textit{if} \quad j = 1, i = 1 \\ \overline{w}_j & \textit{else} \end{cases} \tag{10}$$

2. *If some liability constraint is binding and the agent earns a rent such that his participation constraint is not binding, the agent's wage is*

$$w_i = \begin{cases} w^{min} & \textit{if} \quad (y_i, x_j) \in XY_{-}(a^0) \\ \overline{w}_j & \textit{if} \quad (y_i, x_j) \in XY_{+}(a^0). \end{cases} \tag{11}$$

3. *If some liability constraint and the agent's participation constraint are binding, the agent's wage is*

$$w_i = \begin{cases} w^{min} & \textit{if} \quad \frac{p'_{ij}(a^0)}{p_{ij}(a^0)} < \frac{p'_{\hat{i}\hat{j}}(a^0)}{p_{\hat{i}\hat{j}}(a^0)} \\ \hat{w} \in [w^{min}, \overline{w}_j] & \textit{if} \quad i = \hat{i}, j = \hat{j} \\ \overline{w}_j & \textit{if} \quad \frac{p'_{ij}(a^0)}{p_{ij}(a^0)} > \frac{p'_{\hat{i}\hat{j}}(a^0)}{p_{\hat{i}\hat{j}}(a^0)} \end{cases} \tag{12}$$

for some $(\hat{i}, \hat{j})$ with $(y_{\hat{i}}, x_{\hat{j}}) \in XY_{-}(a^0)$.

Again, there exists a critical outcome $(y_{\hat{i}}, x_{\hat{j}})$. If the likelihood ratio $p'_{ij}(a^0)/p_{ij}(a^0)$ falls short of that of $(y_{\hat{i}}, x_{\hat{j}})$, only the minimum wage is paid. If the likelihood ratio is higher, the bonus pool is fully distributed to the agent. Different to the setting with only unverifiable information, the size of this bonus pool $\overline{w}_j$ may vary for different levels of the verifiable signal X.

4 Discussion and conclusion

The preceding analysis has shown that under subjective performance evaluation, wage compression as well as rewarding only good news and solutions in between may occur. Since liability limits are not an issue in situations where the agent's reservation utility is high, wage compression is more likely in situations where the agent has rather attractive outside options. Then, the harder punishments under the "rewarding only good news" regime would prevent the agent from signing a contract that is not too expensive for the principal. With poor outside options, however, the agent is willing to swallow this pill as long as it is not too bitter. Therefore, compressed bonus schemes should predominantly be observed in industries where competition for workers is high. Empirical evidence can be found in the banking sector, where bonuses are almost obligatory. Wage floors, on the other hand, are likely to result in more demanding targets and less compressed bonus schemes.

A Proof of the validity of the first-order approach

This section proofs that in the principal's optimization problem (1)–(5), the agent's incentive constraint (3) can be substituted by the first-order condition (6). To that purpose, I follow the arguments of Rogerson (1985) to show that under the optimal compensation scheme, the agent's expected utility is a concave function of his effort if the MLRP and the CDFC are fulfilled. So denote (1)–(5) as the *unrelaxed program*. In the *relaxed program*, (3) is replaced by (6). In addition, it is useful to define the *doubly relaxed program* in which the incentive constraint (3) is replaced by the inequality

$$\sum_{i=1}^{n} p_i'(a^0)w_i - C'(a^0) \geq 0. \tag{13}$$

The second relaxation mainly serves as a technical device to proof the monotonicity of the compensation scheme:

Lemma 1 *If $w = (w_1, \ldots, w_n)$ is a solution to the doubly relaxed program, it is nondecreasing in Y, i.e. $w_i \leq w_{i+1}$ for $i = 1, \ldots, n-1$.*

Proof The proof is by contradiction. So suppose that $s_i > s_{i+1}$ for some i. With that in mind, inspect the first-order conditions of the doubly relaxed program with respect to w_i and w_{i+1},

$$\lambda U'(w_i)p_i(a^0) + \mu U'(w_i)p_i'(a^0) - \nu_i + \eta_i$$

and

$$\lambda U'(w_{i+1})p_{i+1}(a^0) + \mu U'(w_{i+1})p_{i+1}'(a^0) - \nu_{i+1} + \eta_{i+1}.$$

λ and μ denote the multipliers of the participation constraint (2) and the incentive compatibility constraint (13), whereas ν_i and η_i are the multipliers of the bonus pool constraints (4) and the liability constraints (5). By definition, all multipliers are non-negative. Since $s_i > s_{i+1}$, the liability constraint will not be binding for w_i, therefore $\eta_i = 0$. From the same relation, it follows that the budget constraint will be slack under w_{i+1}, thus $\nu_{i+1} = 0$. By the nonnegativity of ν_i and η_{i+1}, it follows that

$$\lambda U'(w_i)p_i(a^0) + \mu U'(w_i)p_i'(a^0) \geq 0$$

and

$$\lambda U'(w_{i+1})p_{i+1}(a^0) + \mu U'(w_{i+1}) \leq 0$$

or, by $U' > 0$ and $p_i(a^0) > 0$,

$$\lambda + \mu \frac{p_i'(a^0)}{p_i(a^0)} \geq 0$$

and

$$\lambda + \mu \frac{p_{i+1}'(a^0)}{p_{i+1}(a^0)} \leq 0$$

Due to the MLRP, both conditions can only be fulfilled if $\mu = 0$. Then, however, the principal would do best by offering a flat wage, which contradicts the initial assumption.

Given the monotonicity of wages, it is straightforward to proof the concavity of the agent's expected utility:

Lemma 2 *If $w = (w_1, \ldots, w_n)$ is a solution to the doubly relaxed program, the agent's expected utility under w is a concave function of his action a*

Proof Consider the agent's expected utility under w,

$$\mathrm{E}\left[U(w_i) \mid a\right] = \sum_{i=1}^{n} U(w_i)p_i(a) - C(a),$$

and rewrite it in utility differences $\Delta_i = U(w_{i+1}) - U(w_i)$,

$$\mathrm{E}\left[U(w_i) \mid a\right] = U(w_1) + \sum_{i=1}^{n-1} \Delta_i (1 - P_i(a)) - C(a). \tag{14}$$

By Lemma 1, $\Delta_i \geq 0$. the monotonicity of (14) then follows by the CDFC and the convexity of the cost function C.

Next, I proof that the solutions of the relaxed program and the doubly relaxed program are identical.

Lemma 3 *If in the doubly relaxed program an action $a > \underline{a}$ is implemented, the incentive constraint (13) is binding in its optimal solution.*

Proof If The incentive constraint would not bind, it could likewise be dropped. Then, the principal would do best stipulating a flat wage. Under a flat wage, however, only the least costly action $\underline{a}$ can be implemented.

The last step is to proof that the solutions of the relaxed program and the unrelaxed program coincide:

Proposition 2 *If the MLRP and the CDFC hold, each solution to the relaxed program is also a solution to the unrelaxed program.*

Proof By lemma 3, the solutions to the relaxed program and the doubly relaxed program coincide. Consequently, lemma 1 and 2 also apply to the relaxed program. By the concavity of the agent's expected utility, the stationary point described by (6) is identical to the solution to the incetive constraint (15) of the unrelaxed program.

B Proofs of propositions

B.1 Proof of proposition 1

1. The first claim is identical to that in proposition 6 in MacLeod (2003) and can be proven similarly.

2. To prove the second claim, consider the first-order condition

$$\frac{\partial \mathcal{L}}{\partial w_i} = \lambda p_i(a^0)U'(w_i) + \mu p_i'(a^0)U'(w_i) - \nu_i + \eta_i = 0 \tag{15}$$

   with respect to w_i, where λ and μ are the multipliers of the participation constraint (2) and the incentive compatibility constraint (6), and μ_i and η_i denote the multipliers of the budget constraint (4) and the liability constraint (5) under the outcome y_i. If the agent earns a rent, $\lambda = 0$ will hold. Furthermore, the validity of the first-order approach implies that the incentive constraint (6) will be binding and thus $\mu > 0$ for all $a > \underline{a}$. From this and the nonnegativity of ν_i and η_i it follows that $\eta_i > 0$ and thus $w_i = s^{min}$ must hold whenever $p_i(a^0) < 0$. Contrary, $p_i(a^0) > 0$ implies that $\nu_i > 0$ and thus $w_i = \overline{w}$. This establishes the binary structure of the compensation scheme. The size of the bonus pool can then be derived from the binding incentive constraint.

3. As above, consider the first-order condition (15). First suppose $p_i(a) > 0$. Since $\lambda > 0$ by assumption and again $\mu > 0$ holds, $v_i > 0$ must hold and $w_i = \overline{w}$. Next, consider the case $p_i(a) < 0$ and suppose that both the budget

constraint and the liability constraint do not hold. Then,

$$\frac{\partial \mathcal{L}}{\partial w_i} = \lambda p_i(a^0)U'(w_i) + \mu p_i'(a^0)U'(w_i) = 0$$

or $p_i'(a)/p(a) = \lambda/\mu$ must hold. By strict MLRP, this can only be the case for one single realization $i = \hat{i}$. By the monotonicity of the incentive scheme it follows that $w_i = w^{min}$ for $i < \hat{i}$ and $w_i = \overline{w}$ for $i > \hat{i}$.

References

[1] Ederhof, M. (2010) Discretion in Bonus Plans. The Accounting Review 85, 1921–1949.

[2] Grossman, S. J. and O. D. Hart (1983). An Analysis of the Principal-Agent Problem. Econometrica 51, 7–45.

[3] McLeod, B.W. (2003). Optimal Contracting with Subjective Evaluation. American Economic Review 93, 216–40.

[4] Milgrom, P.R. (1981). Good news and bad news: representation theorems and applications. The Bell Journal of Economics 12, 380–391.

[5] Rajan, M.V. and S. Reichelstein (2009). Objective versus Subjective Indicators of Managerial Performance. The Accounting Review 84 (1), 209–237.

[6] Rogerson, W.P. (1985). The First-Order Approach to Principal-Agent Problems. Econometrica 53, 1357–1367.