A Comparison of Bayesian Monte Carlo Markov Chain and Maximum Likelihood Estimation Methods for the Statistical Analysis of Geodetic Time Series

AGU FM2013 G21A-0745

Olivares, G., Teferle, F. N.

Geophysics Laboratory, University of Luxembourg, Luxembourg

Contact: golivares74@gmail.com

Abstract

One of the objectives of TIGA is to compute precise station coordinates and velocities for GPS stations of interest. Consequently, a comprehensive knowledge of the stochastic features of the GPS time series noise is crucial, as it affects the velocity estimation for each GPS station. For that, we present a Monte Carlo Markov Chain (MCMC) method to simultaneously estimate the velocities and the stochastic parameters of the noise in GPS time series. This method allows to get a sample of the likelihood function and thereby, using Monte Carlo integration, all parameters and their uncertainties are estimated simultaneously. We propose this method as an alternative to optimization methods, such as the Maximum Likelihood Estimation (MLE) method implemented in the widely used CATS software, whenever the likelihood and the parameters of the noise are to be estimated in order to obtain more robust uncertainties for all parameters involved. Furthermore, we assess the MCMC method through comparison with the widely used CATS software using daily height time series from the Jet Propulsion Laboratory.

Maximum Likelihood Estimation (MLE)

The likelihood of getting the observational data given some parameters is defined as:

$$L(y|\theta) = \frac{1}{(2\pi)^{N/2}|C|^{1/2}} e^{\frac{1}{2}(y-\hat{y})^T C^{-1}(y-\hat{y})} \qquad (1)$$

- y: Observational data
- $\hat{y}$: Model data
- θ: Model parameter
- C: Covariance matrix
- N: Time series length

The estimated parameters are those of the argument of the maximum of the likelihood:

$$\hat{\theta} \equiv \arg(\max L) \qquad (2)$$

Very often there is no closed-form formula for the Likelihood function and numerical computation is needed. For that purpose CATS has been developed (Williams, 2008).

Monte Carlo Markov Chain (MCMC)

A Metropolis-Hasting algorithm is used to get a sample of the a posteriori distribution that, according to Bayes Theorem, is related to the Likelihood:

$$P(\theta|y) = \frac{L(y|\theta)P(\theta)}{P(y)} \qquad (3)$$

where, $P(\theta)$ and $P(y)$ are the a priori distributions of the estimated parameters and the data. Concerning the parameters we have chosen a uniform distribution for them, whereas it is not necessary to know $P(y)$for our algorithm.

Modeling

The chosen model is a linear combination of a linear and periodic (annual and semiannual) terms with residuals composed by power law and white noise as follows:

$$\hat{y}(t_i) = y_0 + v(t_i - t_0) + A^c_{1yr}cos\big(2\pi(t_i - t_0)\big) + A^s_{1yr}sin\big(2\pi(t_i - t_0)\big) + A^c_{0.5yr}cos\big(2\pi(t_i - t_0)\big) + A^s_{0.5yr}cos\big(t_i\pi(t - t_0)\big) + \varepsilon(t_i) \qquad (4)$$

with $y_0 \equiv y(t_0)$ the intercept, v the velocity, A^c_{1yr} the annual cosine amplitude, A^s_{1yr} the annual sine amplitude, $A^c_{0.5yr}$ the semi-annual cosine amplitude, $A^s_{0.5yr}$ the semi-annual sine amplitude and $\varepsilon(t)$, the residual, composed by

$$\varepsilon(t_i) = \sum_k h_{i-k}u(t_i) + w(t_i) \qquad (5)$$

where u and w are white-noise processes with amplitudes σ_{pl} and σ_{wn}, respectively; and

$$h_j = \left(\frac{\alpha}{2} + j - 1\right)\frac{h_{j-1}}{j!} \qquad (6)$$

with α being the spectral index. The first term on the right hand side of Eq. (5) is a power-law process obtained by the convolution of $u(t_i)$.

Introduction

It is widely known that not accounting for the time-correlated noise within GPS time series leads to velocity uncertainty estimates [3-11] times too optimistic when compared to a white noise only model (Zhang et al., 1997; Mao et al. 1999; Williams et al., 2003; Williams et al., 2006). Moreover, according to Bos et al. 2010, not accounting for the spectral index uncertainty also yields biased estimates for the uncertainties of deterministic parameters such as, for example, the velocity or the amplitudes.of the periodic terms.

For this, we propose a Monte Carlo Markov Chain (MCMC) method that simultaneously estimates all parameters and their uncertainties, the spectral index included, as an alternative to the start-of-the-art method: the Maximum Likelihood Estimation (MLE) as implemented in CATS software (Williams, 2008). We have carried out a comparison of the performance of both methods by analyzing 90 GPS time series from the International GNSS (Global Navigation Satellite System) Service (IGS) tracking network.

In general, the parameter estimates from both methods are in good agreement. Nevertheless, unlike CATS software, which is a numerical solution of the Maximum Likelihood Estimation (MLE) that better fits the data, i.e. the daily GPS position time series from the Jet Propulsion Laboratory (JPL), the MCMC method provides larger uncertainties for all parameters except for the intercept, and more robust estimates of the stochastic parameters at low spectral index values.

The JPL data set

The JPL provides 2381 daily position time series processed using the Precise Point Positioning (PPP) strategy in the GIPSY-OASIS II software (Zumberge et al. 1997). Out of them, 90 stations of the IGS core network (shown in Fig. 1) were selected and analyzed with the MCMC and MLE methods.

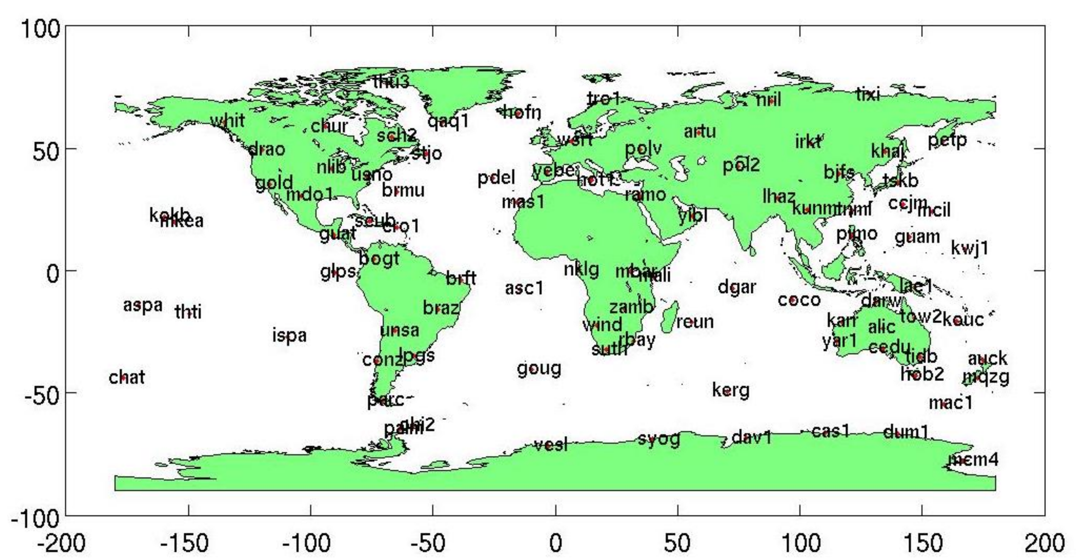

Figure 1: Map of the GPS stations of the IGS core network

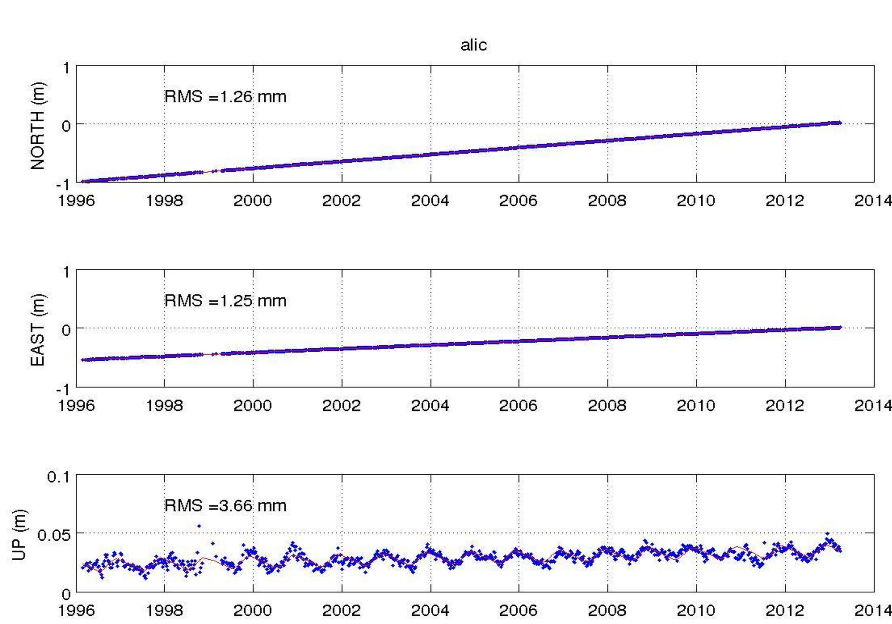

Figure 2: GPS time series for ALIC. From top to bottom: North, East and Up components.

Fig. 2 shows the North, East and Up components of ALIC in Australia as a representative GPS time series from this data set. Also shown is the RMS for each component, which is 1.26 mm, 1.25 mm and 3.66 mm for the North, East and Up components, respectively.

These are typical values for the RMS of state-of-the-art GPS time series and in this case, but also in general, the RMS for the Up component is ~3 times larger than for the North and East components.

The MCMC Algorithm

Fig. 3 shows the flow chart of the MCMC algorithm, namely, a Metropolis-Hastings algorithm.

The initial inputs are the starting values of all parameters $\theta_0 = \big(\alpha, \sigma_{pl}, \sigma_{wn}, v, y_0, A^c_{1yr}, A^s_{1yr}, A^c_{0.5yr}, A^s_{0.5yr}\big)$, the previously estimated optimal step size ρ, and the Markov Chain length N.

At the i^{th} step, the Markov Chain moves towards the maximum of the likelihood if $\frac{L(\theta_{i+1})}{L(\theta_i)} > 1$, or around the last Markov Chain point if $\frac{L(\theta_{i+1})}{L(\theta_i)} > r \in [0,1]$; otherwise, the computation of a new parameter vector θ_{i+1} is repeated.

This loop is performed 10^5times.

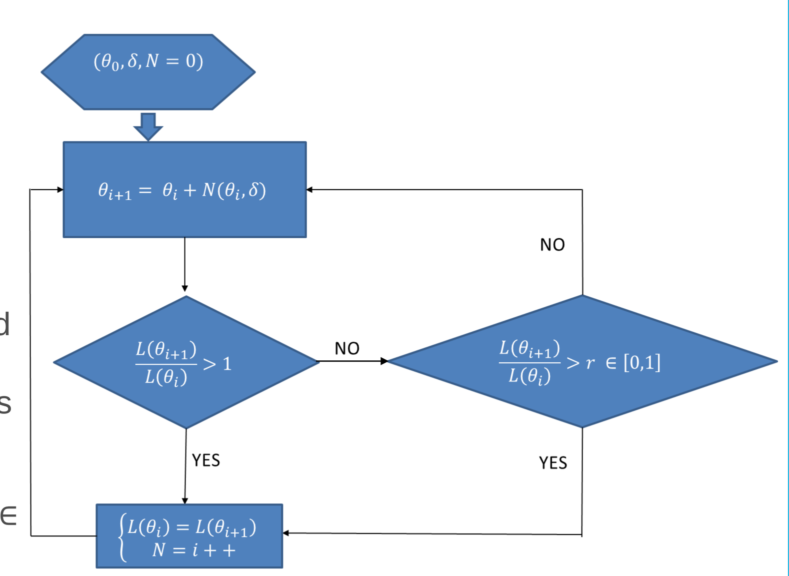

Figure 3: Flow chart of the Metropolis-Hasting algorithm used in this study to get a sample of the a posteriori distribution function of all parameters.

Methodology

Figure 4 shows the methodology for the comparison between the MCMC method and the MLE method (with CATS v3.1.2 software).

The common input is the data used y (the JPL data set).

There is an extra input for the MCMC method only: starting points θ_0 and optimal step size ρ.

Both methods fit data with a linear and periodic terms plus the residual composed by a power-law and a white-noise process, i. e. according to Eq. (6).

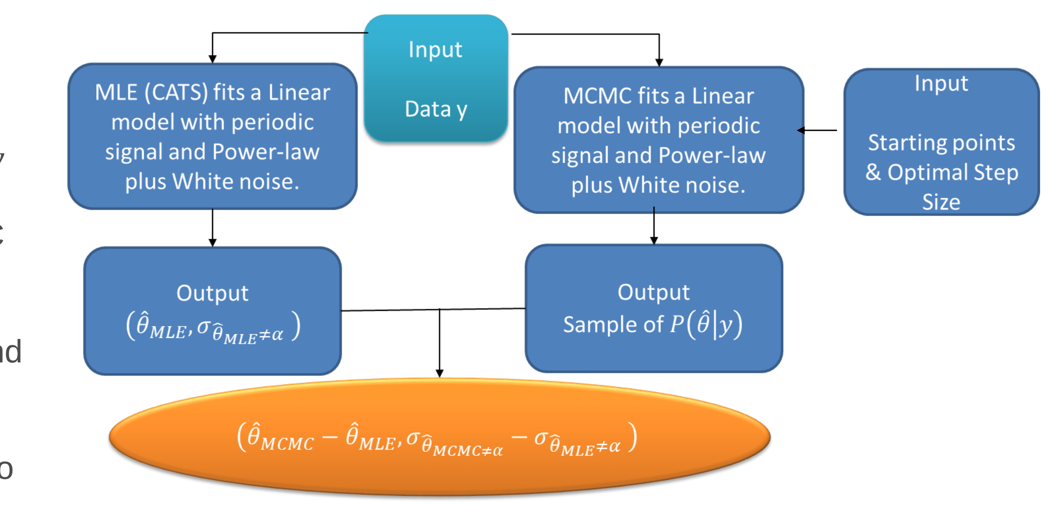

Figure 4: Flow chart of the comparison method between MCMC and MLE.

The output from MLE is that of Eq. (2), i.e. $\hat{\theta}_{MLE}$ and the uncertainties of all parameters except for α.

The MCMC method provides a sample of the a posteriori distribution function $P(\hat{\theta}|y)$. Then $\hat{\theta}_{MCMC}$ and their uncertainties are obtained and compared with those from the MLE method.

Only results for the Up component are shown.

Results

Fig. 5 shows the correlation plots of the estimated stochastic parameters from MCMC (vertical axis) and MLE (horizontal axis) for the UP component.

In Fig. 5a the spectral index from the MCMC method is systematically larger than from the MLE method.

On the other hand, Fig. 5b shows that σ_{pl} from MLE is larger than from MCMC. This is in order to account for the lack of power when CATS sets $\sigma_{wn} = 0$ in some cases (Fig. 5c).

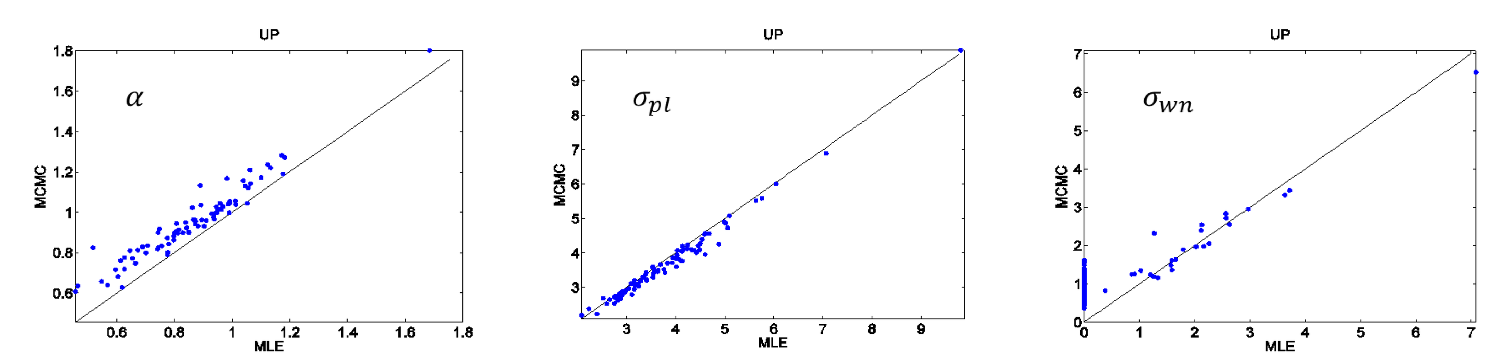

Figure 5: Correlation plots for α (a), $\sigma_{pl}(mm)$ (b) and $\sigma_{wn}(mm)$ (c).

In order to account for the lack of power when CATS sets $\sigma_{wn} = 0$, α estimates from MLE are smaller than from MCMC (Fig. 6).

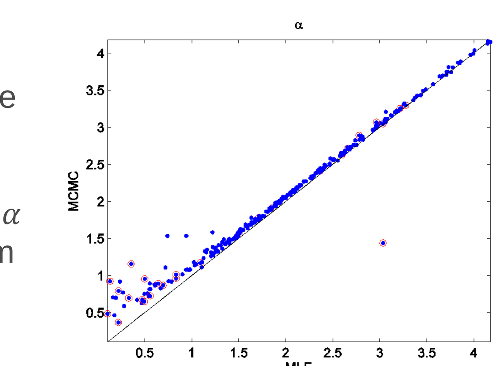

Figure 6: Correlation plots for α. Red-circled points correspond to $\sigma_{wn} = 0\ mm$.

Fig. 7 shows the correlation for the velocity and the intercept estimates from the MCMC and MLE methods. Both methods agree well, even for the amplitudes periodic terms (not shown here).

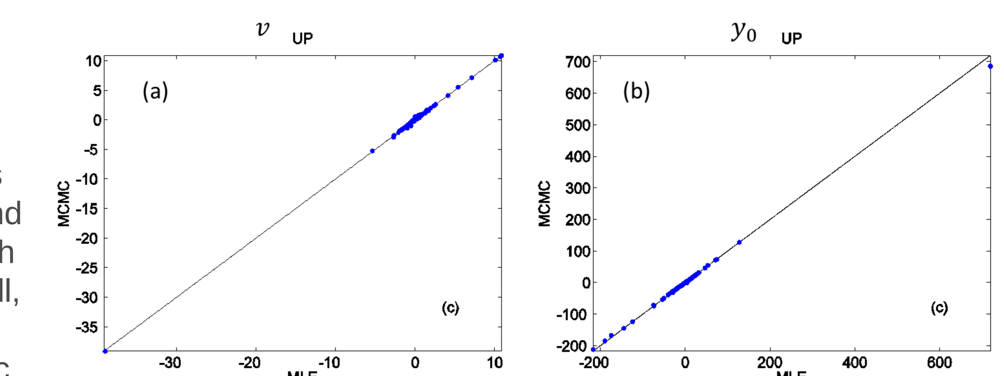

Figure 7: Correlation plots for $v\ (mm/yr)$ (a), $y_0(mm)$ (b).

Fig. 8 shows the correlation plots for the velocity and the intercept uncertainties from the MCMC and MLE methods (UP component). The MCMC method provides larger uncertainties for v, and smaller ones for y_0 than MLE.

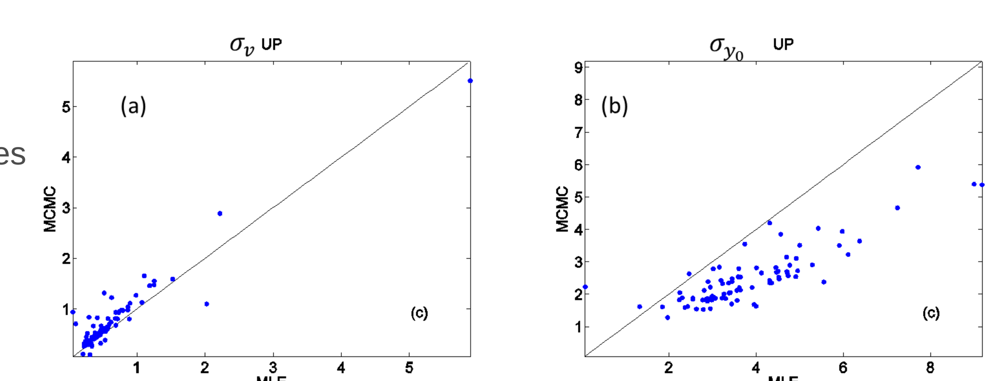

Figure 8: Correlation plots for $\sigma_v(mm/yr)$ (a),$\sigma_{y_0}(mm)$ (b).

The medians of the ratio of the MCMC and MLE uncertainties (R_p) indicate that the velocity uncertainties from MCMC are 40% larger for the North and East components, and 18% for the Up component, than from MLE (Tab. 1); whereas those for y_0 are smaller.

R_p	North	East	Up
σ_v	1.40	1.40	1.18
σ_{y_0}	0.70	0.72	0.63

Table 1: Median of the ratio of the uncertainties

Conclusions

- A new Bayesian Monte Carlo Markov Chain method for parameter estimation has been compared to MLE as implemented in CATS using the GPS position time series from the JPL.
- Overall, both methods agree well, but there are some differences:
  - MCMC estimates the uncertainty of the spectral index estimate.
  - MLE yields larger estimates for the power amplitude of the power-law noise (σ_{pl}) in order to account for zero-values of the white-noise amplitude (σ_{wn}).
  - According to MCMC there is more time-correlated noise.
  - The North and East component velocity uncertainties from MCMC are 40% larger than those from CATS. The Up component velocity uncertainties from MCMC are 18% larger. However, the differences are sub-milimetre at the 1σ CL.

Acknowledgements

The authors would like to express their gratitude to the Jet Propulsion Laboratory (JPL) for providing the daily GPS position time series. Furthermore, the simulations and solutions for this work were computed using the HPC platform hosted at the University of Luxembourg.

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Presented at: *American Geophysical Union Fall Meeting 2013, San Francisco, 9 - 13 December 2013*