A Bayesian Monte Carlo Markov Chain Method for the Analysis of GPS Position Time Series

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Overview

- Introduction – Noise in GPS Position Time Series
  - Some geophysical applications (e.g. sea level) aim for a target accuracy of 0.1 mm/yr for Up component velocities
- Monte Carlo Markov Chain (MCMC) vs. Maximum Likelihood Estimation (MLE)
- Data set
- Results of the comparison
- Conclusions

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Noise in GPS Position Time Series

- It is widely accepted that GPS position time series are better modelled by a combination of white and coloured noise (i.e. time-correlated) rather than just white noise (e.g. Zhang et al., 1997; Mao et al., 1999; Caporali, 2003; Williams, 2003; Williams et al., 2004; Langbein, 2008; Hackl et al., 2011) .
- Not accounting for the time-correlated noise component leads to an underestimation (i.e. too optimistic) of the parameter uncertainties.
- Velocity uncertainties, in particular, have been reported to be too optimistic for white noise only models by
  - e.g ranges of factors of 3-6 (Zhang et al., 1997) and of 5-11 (Mao et al., 1999) for GPS daily time series
  - and of 3-4 (Williams and Willis, 2006) for DORIS weekly time series
- A number of methods for characterising time-correlated noise have been developed of which Maximum Likelihood Estimation (MLE) has become the standard method as implemented in CATS (Williams, 2008).
- Recently a method using a Bayesian Monte Carlo Markov Chain Method (MCMC) has been developed (Olivares and Teferle, 2013)

Noise in GPS Position Time Series (2)

- CATS (Williams, 2008) numerically computes MLE
  - it estimates stochastic and deterministic parameters
  - and computes the uncertainties of all estimates except for the spectral index
- MCMC (Olivares and Teferle, 2013) numerically computes a sample of the a posteriori distribution of the parameters and
  - it estimates stochastic and deterministic parameters and their uncertainties simultaneously, also for the spectral index

What is the difference (if any) when the uncertainty of the spectral index estimate is also computed?

↓

Comparison between MLE (CATS) and a Bayesian Monte Carlo Markov Chain method, which estimates simultaneously all parameters and their uncertainties.

MCMC vs. MLE

For both methods we use the following:

- **Likelihood**: $L(y|\theta) = \frac{1}{(2\pi)^{N/2}|C(\beta)|^{1/2}} e^{-\frac{1}{2}(y-\hat{y})^T C(\beta)^{-1}(y-\hat{y})}$
- **Model**: $\hat{y} = vt + y_0 + \sum_{i=1}^{2}[C_i \cos(\omega_i t) + S_i \sin(\omega_i t)]$
- $\omega_i \equiv {}^{2i\pi}/_{T_{year}}$, N time series length.
- Stochastic Parameter: $\beta = (\alpha, \sigma_{pl}, \sigma_{wn})$
- Covariance Matrix: $C(\beta)$
- All parameters: $\theta = (\beta, v, y_0, C_i, S_i)$
- Data: y, GPS weekly residual time series (ITRF2008) for IGS core network.

MCMC vs. MLE (2)

- **Noise modelling:**

The Covariance Matrix: We assume a power-law process plus white noise model (Zhang et al. 1997, Mao et al. 1999, Williams et al. 2004):

$$r_i = \sum_{j=0}^{i} h_j u_{i-j} + w_i,$$

where $h_j = \frac{(j+\alpha/2-1)!}{j!(\alpha/2-1)!}$, α the spectral index, $j \in \mathbb{N} + \{0\}$, $u \in \mathcal{N}\left(0, \sigma_{pl}\right)$ and

$w \in \mathcal{N}(0, \sigma_{wn})$. Thus, the covariance matrix is:

$$C(\beta) = \sigma_{wn}^2 \boldsymbol{I} + \sigma_{pl}^2 L L^T, \text{ with } L_{ij} = \begin{cases} h_{i-j}, i \leq j \\ 0, i > j \end{cases}$$

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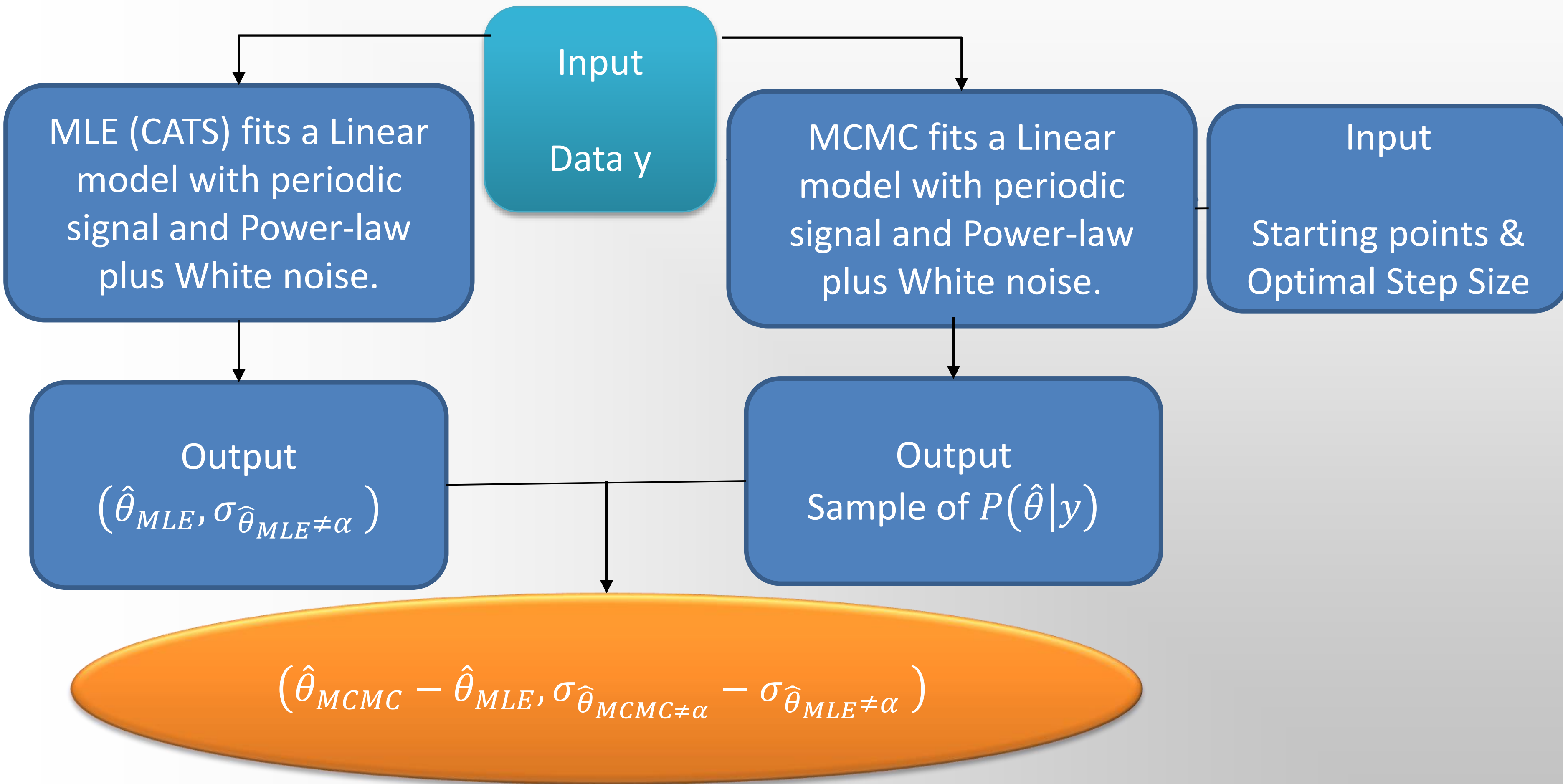
MCMC vs. MLE (3)

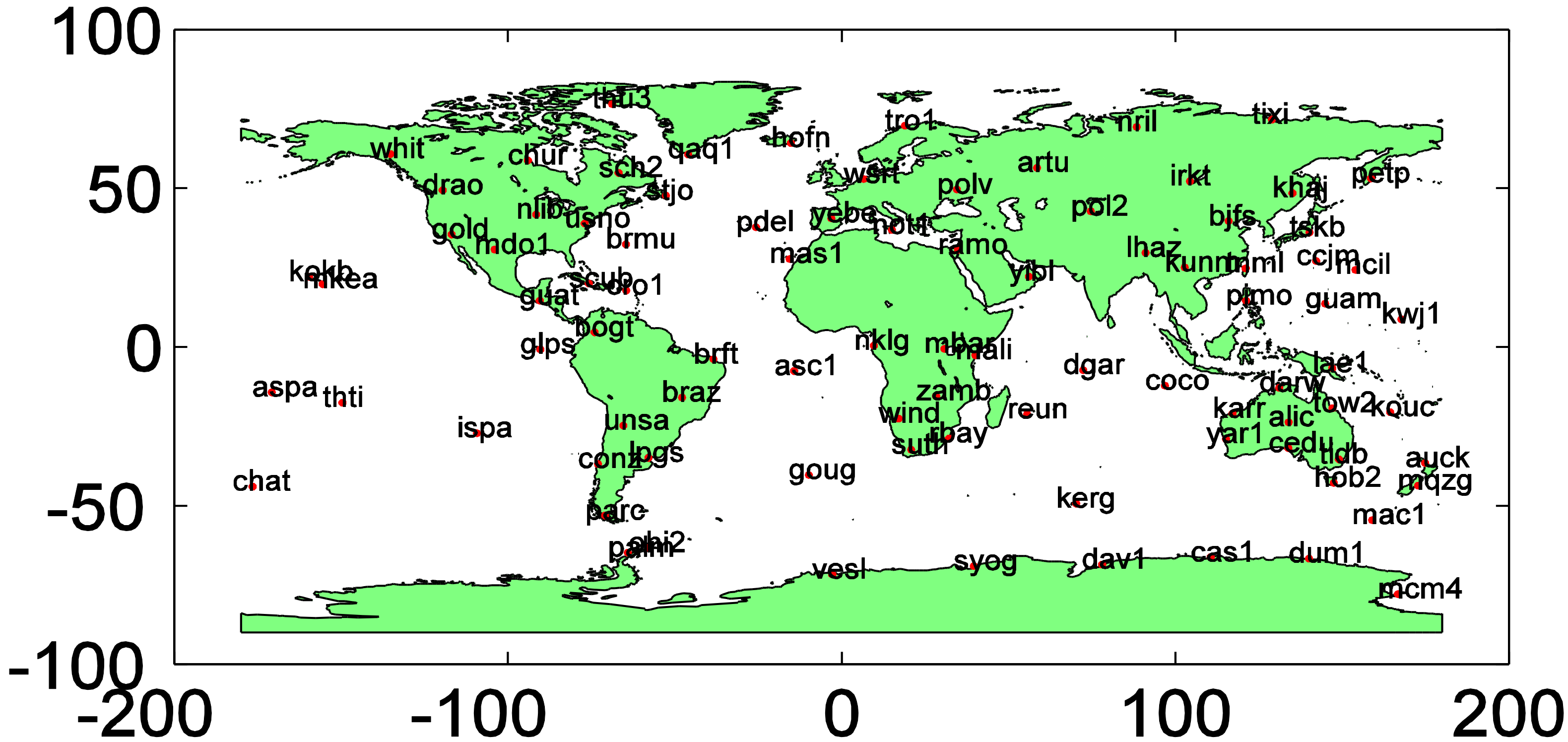
1. MCMC
   - Bayesian Theorem: $P(\theta|y) = \frac{L(y|\theta)P(\theta)}{P(y)}$
   - $P(\theta), P(y)$ are the a priori distributions of parameters and data, respectively.
2. MLE
   - $\theta_{MLE} = arg\left[max\left(L(y|\theta)\right)\right]$

MCMC vs. MLE (4)



Input
Data y
MLE (CATS) fits a Linear model with periodic signal and Power-law plus White noise.
MCMC fits a Linear model with periodic signal and Power-law plus White noise.
Input
Starting points & Optimal Step Size
Output
$(\hat{\theta}_{MLE}, \sigma_{\hat{\theta}_{MLE \neq \alpha}})$
Output
Sample of $P(\hat{\theta}|y)$
$(\hat{\theta}_{MCMC} - \hat{\theta}_{MLE}, \sigma_{\hat{\theta}_{MCMC \neq \alpha}} - \sigma_{\hat{\theta}_{MLE \neq \alpha}})$

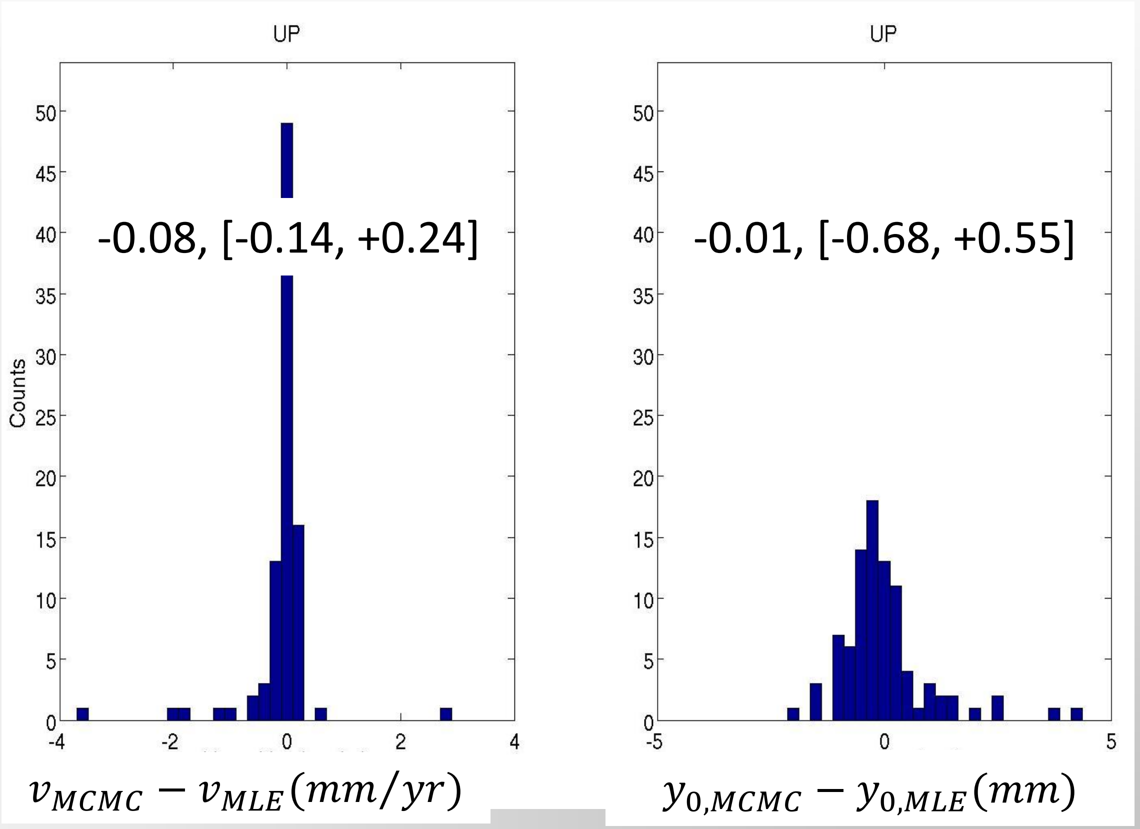
Data Set. ITRF 2008 Solution



- ITRF 2008 solution for IGS core network. GPS time series are position residuals from http://itrf.ensg.ign.fr/.
- 91 stations with time series lengths ranging from 6 to 12 years.

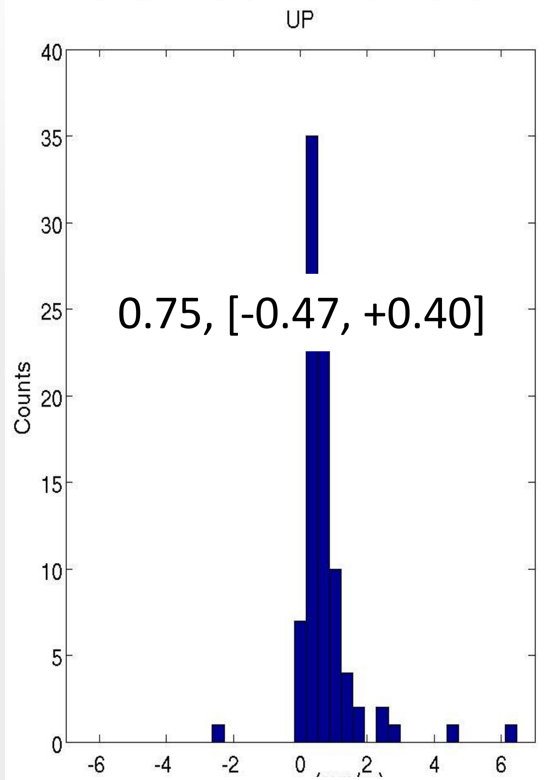
Results: Velocity and Intercept

- All differences are within the sub-millimetre per year level at 1σ Confidence Level (CL).
- Up velocity and intercept estimates are marginally larger from MLE
- N and E components show similar results, though even smaller, $[-0.04^{+0.14}_{-0.11}, 0.00^{+0.14}_{-0.13}]$, respectively.
- All results beyond 5σ Confidence Level (CL) are considered to be outliers.

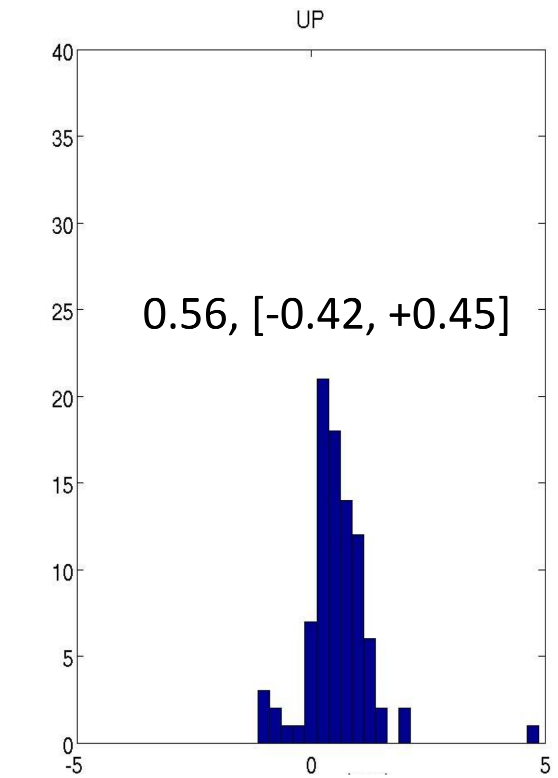


Results: Velocity and Intercept Uncertainties

- Up component velocity uncertainty from MCMC is around one millimetre per year larger at 1σ CL.
- The Up component velocity uncertainties ratio ${}^{\sigma_{V_{mcmc}}}/_{\sigma_{V_{mle}}}$ ranges from 1.7 to 4.6 at 1σ CL.
- N and E component velocity uncertainties (not shown) from MLE are larger, $\left[-0.14^{+0.10}_{-0.10}, -0.18^{+0.14}_{-0.15}\right]$, respectively.
- N and E component velocity ratios range from 1.1 to 3.5, and from 1.2 to 3.2, respectively.



$\sigma_{v,MCMC} - \sigma_{v,MLE}\ (mm/yr)$

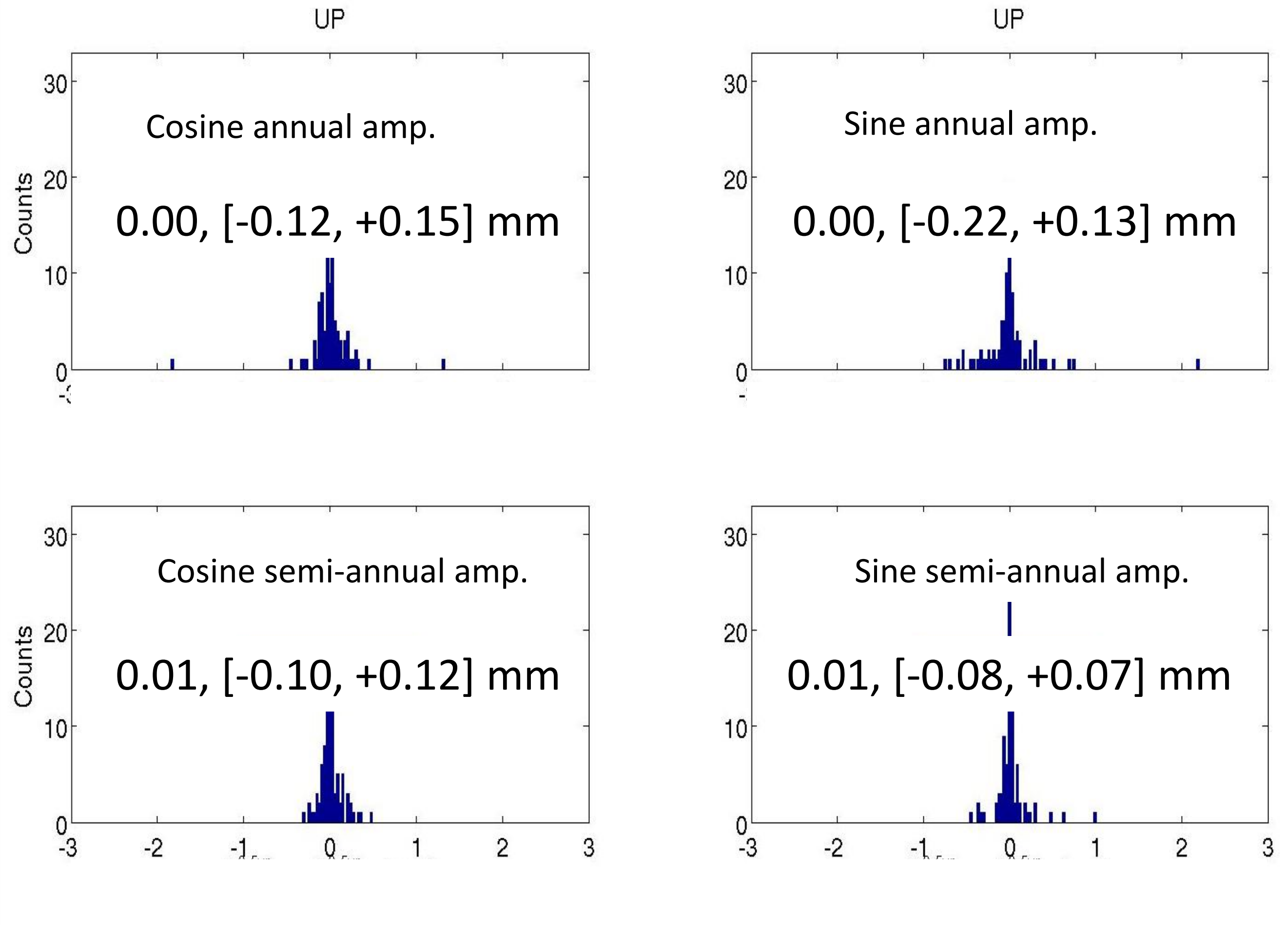


$\sigma_{y_0,MCMC} - \sigma_{y_0,MLE}\ (mm)$

- The intercept uncertainties from MCMC are larger for all three components, namely, $\left[0.12^{+0.17}_{-0.16}, 0.16^{+0.22}_{-0.19}, 0.56^{+0.45}_{-0.42}\right]$ for the N, E and Up components, respectively (N and E not shown).

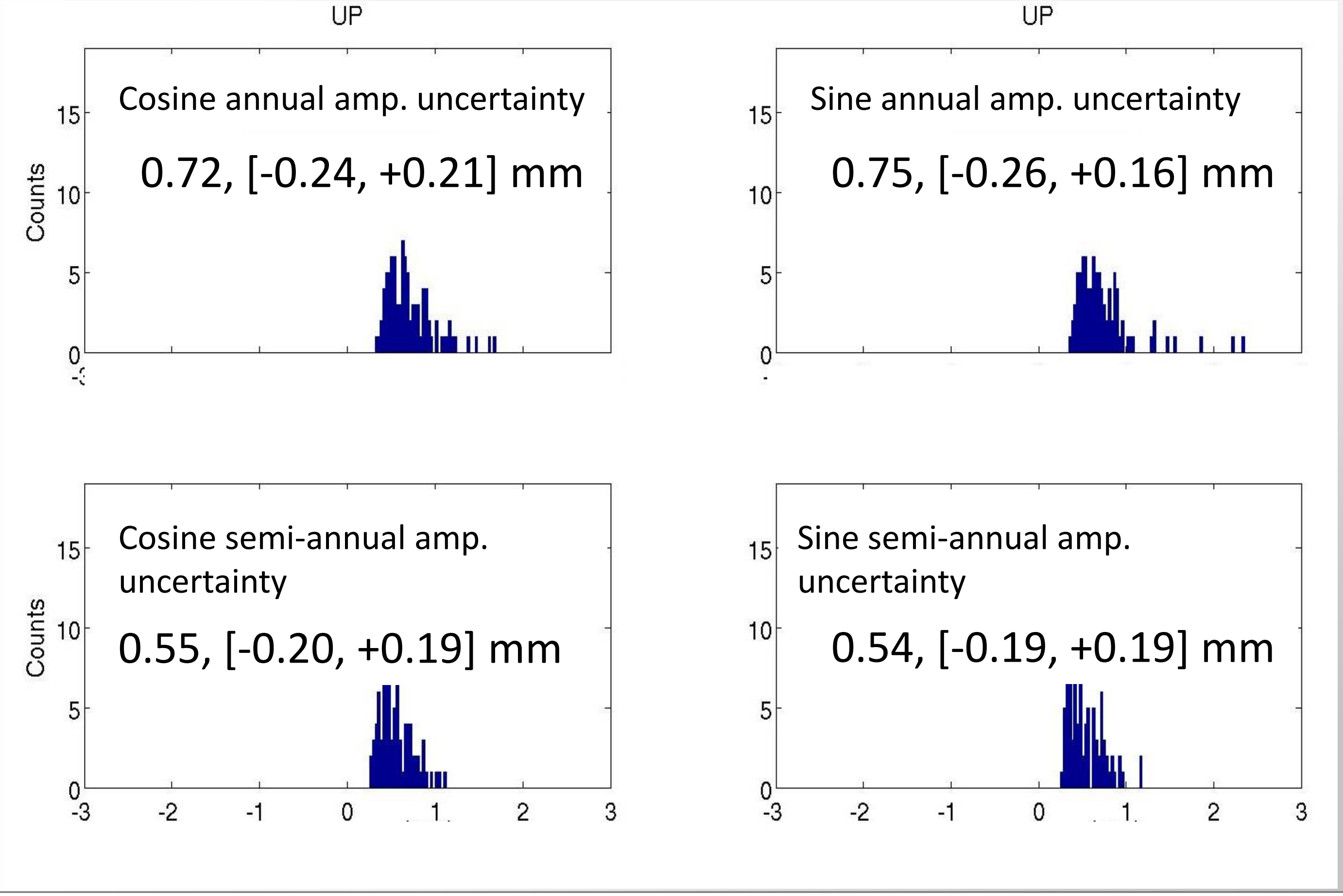
Results: Periodic Amplitudes

- For all three components all estimates agree at sub-millimeter level within 1σ CL (N and E not shown).



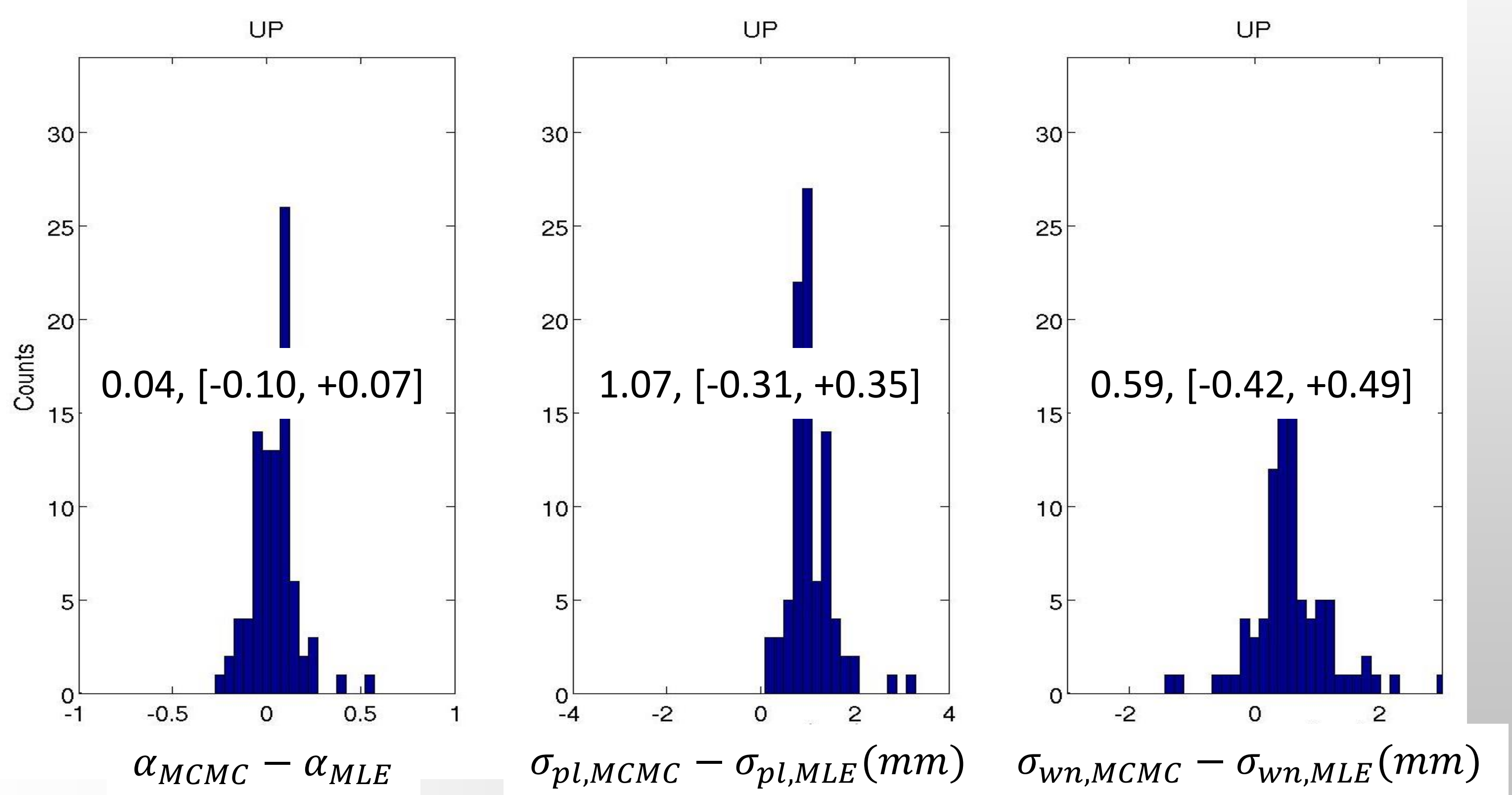
Results: Periodic Amplitudes Uncertainties

- For all periodic parameters and all components (N and E not shown) the uncertainties of the amplitudes are larger from MCMC.
- It holds: $0 < (\sigma_{MCMC} - \sigma_{MLE}) \leq 0.93\ mm$ at 1σ CL.



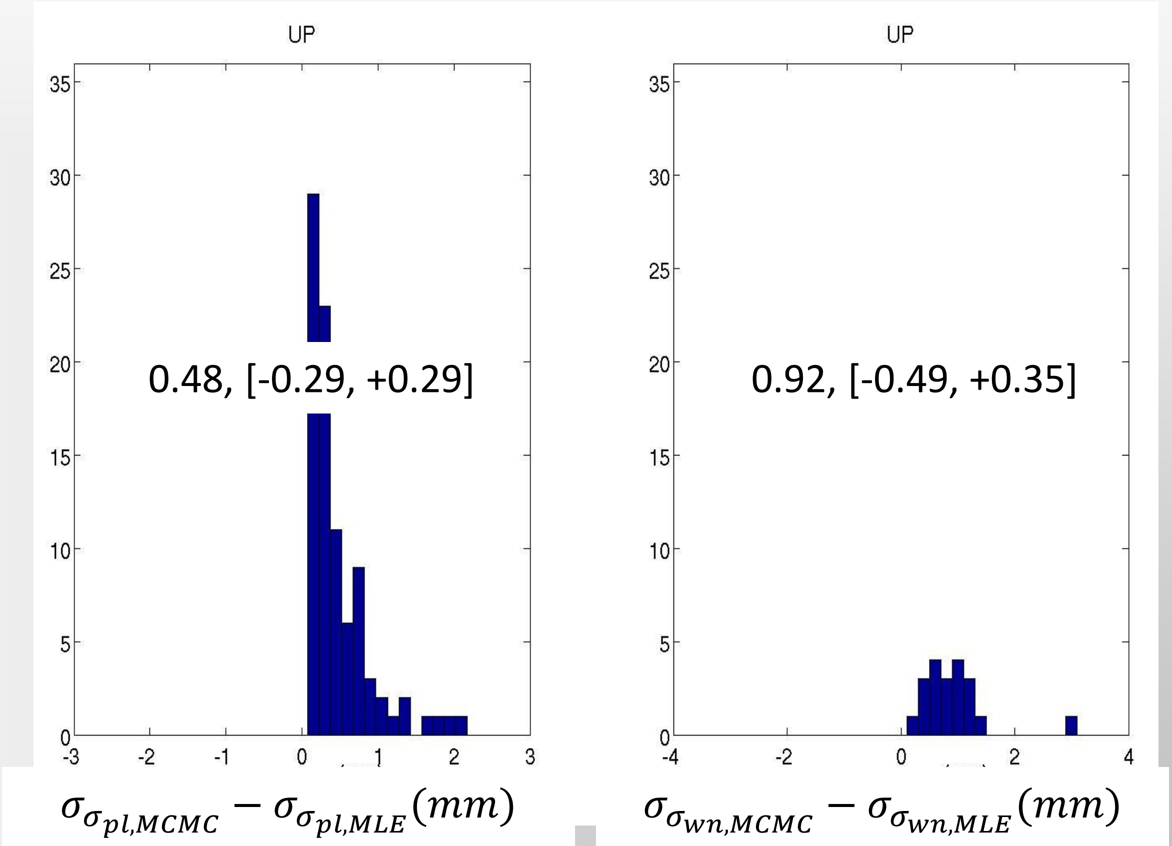
Results: Stochastic Parameters

- All MCMC estimates are larger for all components (N and E not shown), i.e. there is more stochasticity according to this method.



Results: Stochastic Parameters Uncertainties

- CATS (standard version) does not provide the uncertainty of the estimated spectral index α.
- MCMC σ_{wn} and σ_{pl} uncertainties are larger for all components than from MLE (N and E not shown).
- CATS considers for 70 GPS stations $\sigma_{wn} = 0$ when the estimated white noise amplitude is smaller than a threshold, hence the shorter histograms on the right .



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Conclusions

- A new Bayesian Monte Carlo Markov Chain method for parameter estimation in GPS position time series has been compared to MLE.
- Overall, both methods agree well, but there are some differences:
  - MLE (CATS) yields larger (more positive/less negative) velocity estimates, i.e. $\Delta v < 0$. The differences are sub-milimetre at 1σ CL for the Up component. Some differences are above the 0.1 mm/yr target accuracy threshold.
  - MCMC yields Up component velocity uncertainties which are around [1.7,4.6] times larger than from CATS, and within the millimetre per year range at 1σ CL, well above the target accuracy of 0.1 mm/yr.
    - Considering the velocity uncertainty from MLE ranges from 5 to 11 in Mao et al. (1999), the worst-escenario case with MCMC would yield an Up component velocity uncertainty 50 times greater at 1σ CL than white noise only models.
  - Differences in the periodic components are insignificant. The differences of their uncertainties are marginally larger for MCMC (sub-milimetre at 1σ CL).
  - According to MCMC the stochastic noise is more important:
    - $\Delta\alpha, \Delta\sigma_{pl}, \Delta\sigma_{wn} > 0$.
  - MCMC estimates the uncertainty of the spectral index estimate.
  - Some differences are not Gaussian, i.e. quantiles are not symmetric

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Thank you for your attention!

References

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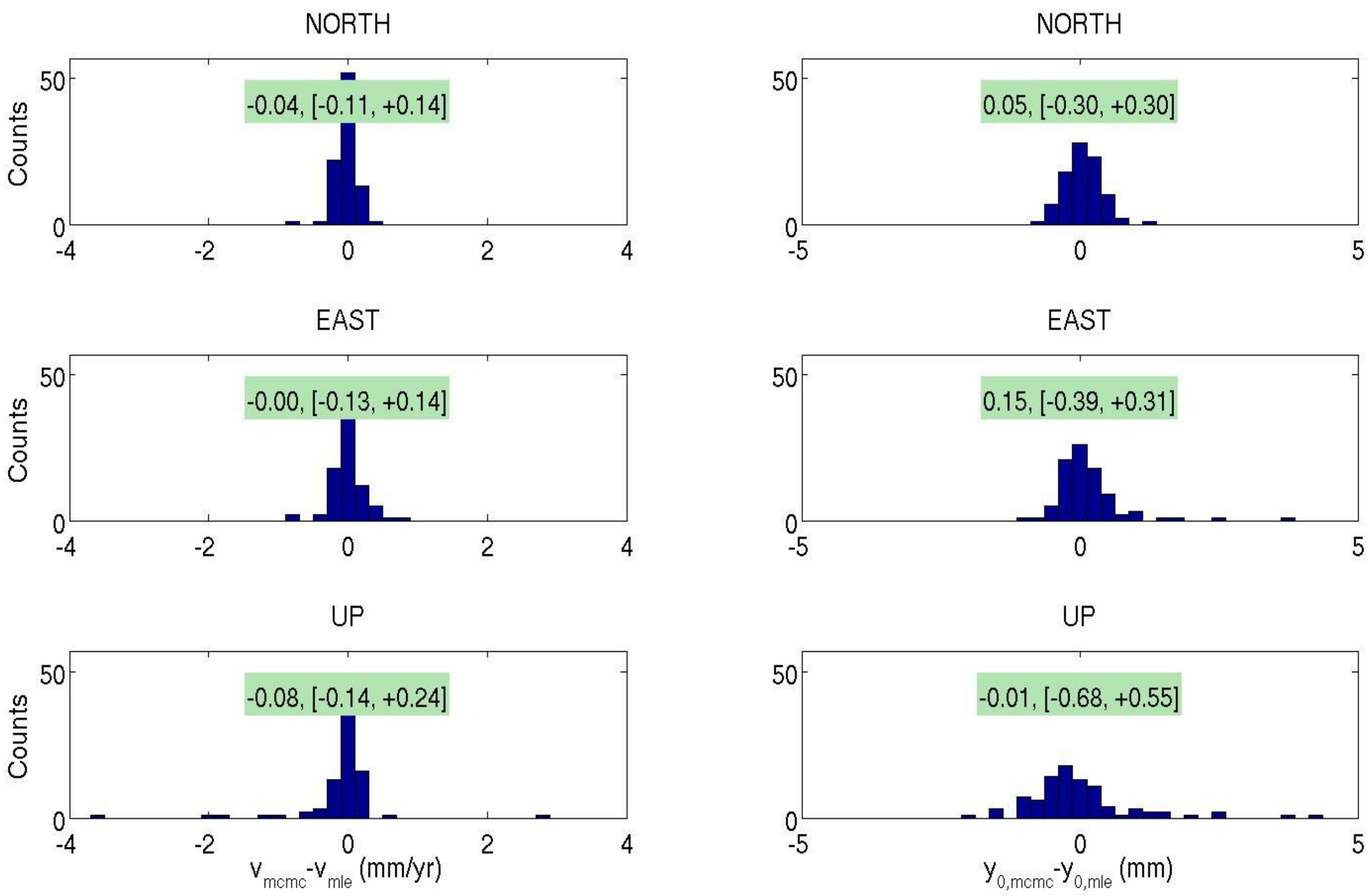
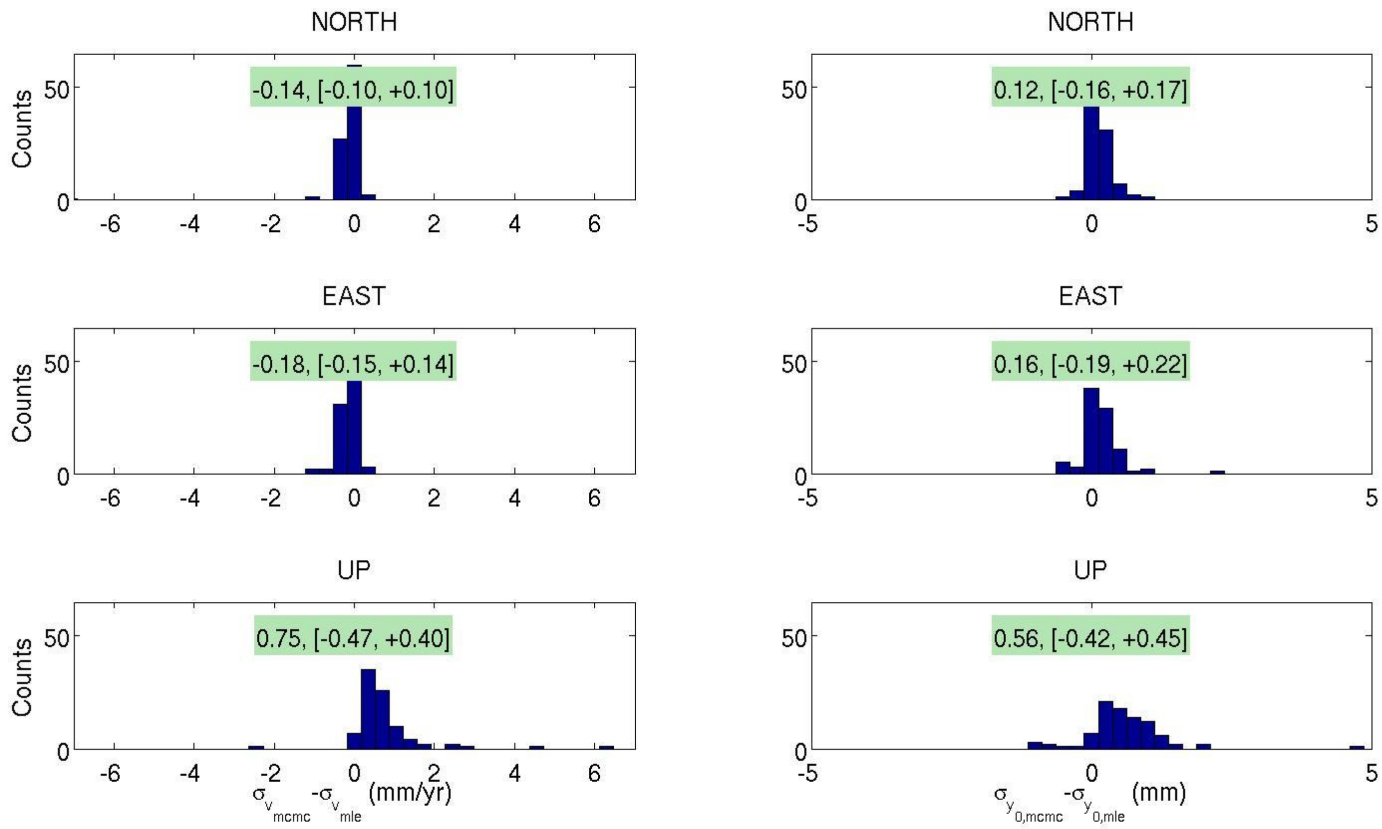
Appendix. Velocity and Intercept

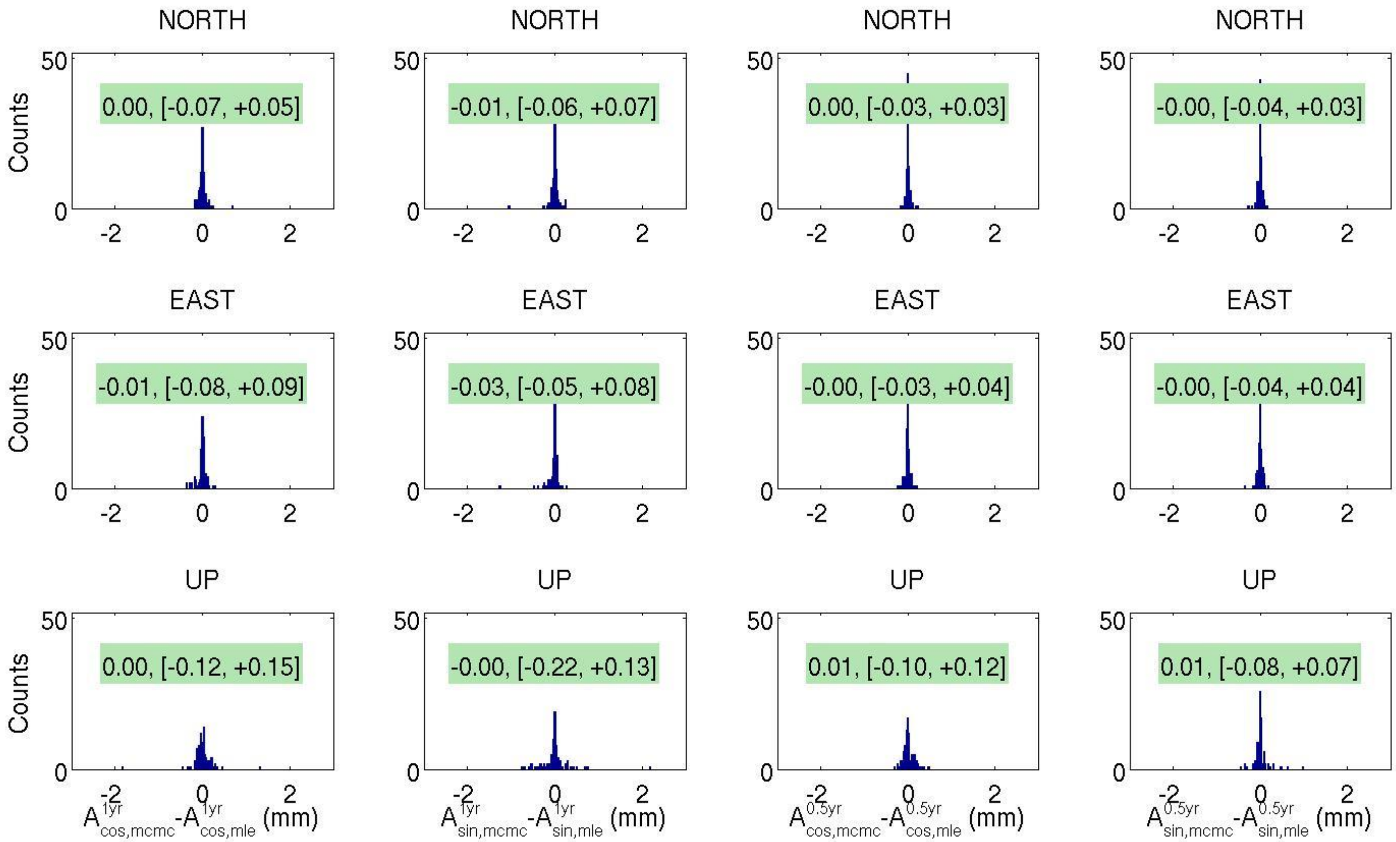


Appendix. Velocity and Intercept (2)



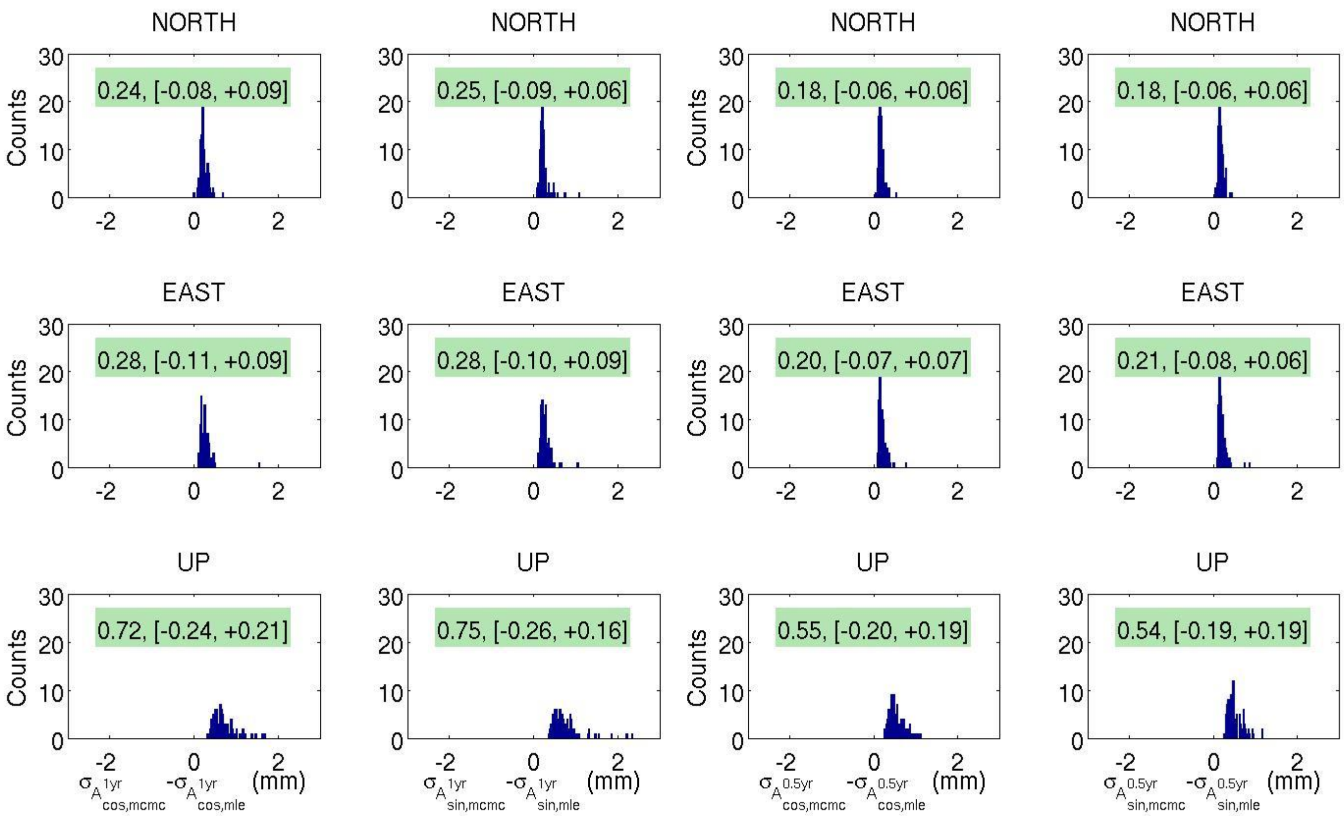
Appendix. Periodic Amplitudes

- All estimates are the same at $10^{-2}\ mm$ level within 1σ CL.



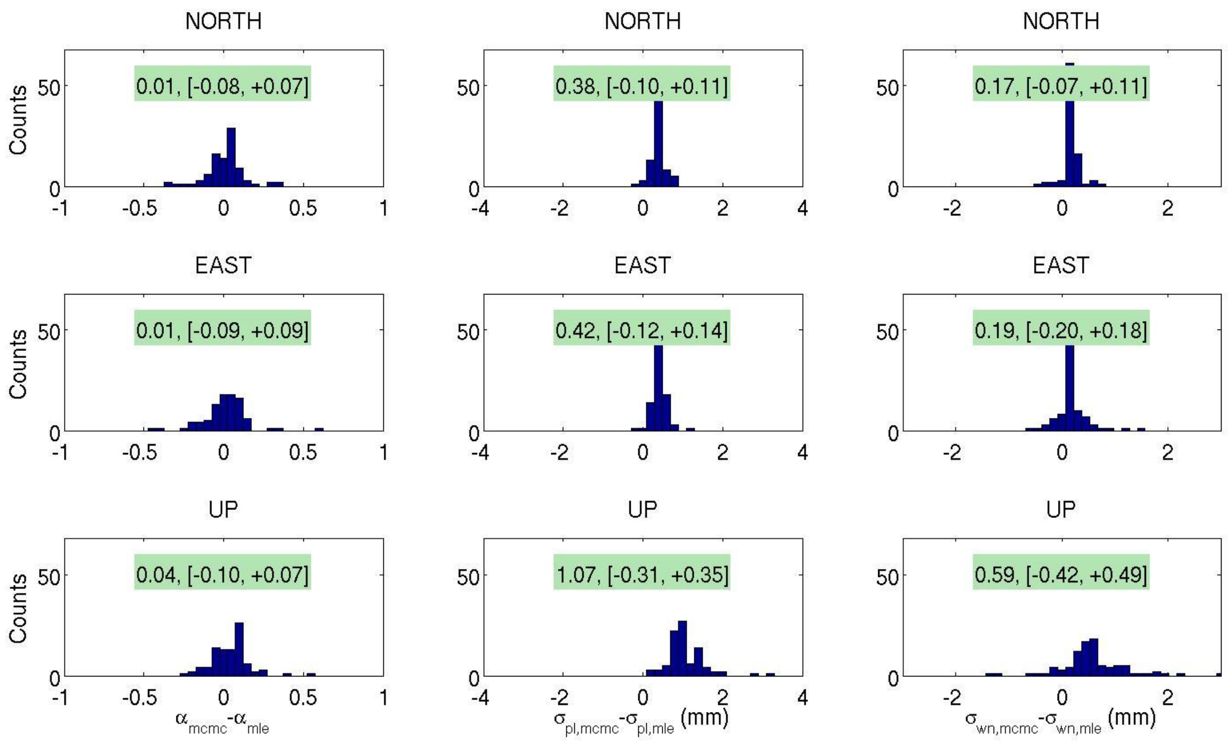
Appendix. Periodic Amplitudes (2)

- For all periodic parameters: $0 < mean(\sigma_{MCMC} - \sigma_{MLE}) < 0.93\ mm$ at 1σ confidence level (CL).



Appendix. Stochastic Parameters

- All MCMC estimates are larger, i.e. there is more stochasticity according to this method. This is consistent with .



Appendix. Stochastic Parameters (3)

- CATS (standard version) does not provide the uncertainty of the estimated spectral index α.
- Uncertities from MCMC are larger and the differences are not Gaussian (see second quantiles corresponding to 2σ CL).
- CATS considers for 70 GPS stations $\sigma_{wn} = 0$ when the estimated White noise amplitude is smaller than a threshold, hence the shorter histograms on the right .