A Bayesian Monte Carlo Markov Chain Method for the Statistical Analysis of Geodetic Time Series

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Overview

- Introduction – Noise in GPS Position Time Series
  - Some geophysical applications (e.g. sea level) aim for a target accuracy of 0.1 mm/yr for Up component velocities
- Monte Carlo Markov Chain (MCMC) vs. Maximum Likelihood Estimation (MLE)
- Data set
- Results of the comparison
- Conclusions

Noise in GPS Position Time Series

- It is widely accepted that GPS position time series are better modelled by a combination of white and coloured noise (i.e. time-correlated) rather than just white noise (e.g. Zhang et al., 1997; Mao et al., 1999; Caporali, 2003; Williams, 2003; Williams et al., 2004; Langbein, 2008; Hackl et al., 2011) .
- Not accounting for the time-correlated noise component leads to an underestimation (i.e. too optimistic) of the parameter uncertainties.
- Velocity uncertainties, in particular, have been reported to be too optimistic for white noise only models by
  - e.g ranges of factors of 3-6 (Zhang et al., 1997) and of 5-11 (Mao et al., 1999) for GPS daily time series
  - and of 3-4 (Williams and Willis, 2006) for DORIS weekly time series
- A number of methods for characterising time-correlated noise have been developed of which Maximum Likelihood Estimation (MLE) has become the standard method as implemented in CATS (Williams, 2008).
- Recently a method using a Bayesian Monte Carlo Markov Chain Method (MCMC) has been developed (Olivares and Teferle, 2013)

Noise in GPS Position Time Series (2)

- CATS (Williams, 2008) numerically computes MLE
  - it estimates stochastic and deterministic parameters
  - and computes the uncertainties of all estimates except for the spectral index
- MCMC (Olivares and Teferle, 2013) numerically computes a sample of the a posteriori distribution of the parameters and
  - it estimates stochastic and deterministic parameters and their uncertainties simultaneously, also for the spectral index

What is the difference (if any) when the uncertainty of the spectral index estimate is also computed?

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Comparison between MLE (CATS) and a Bayesian Monte Carlo Markov Chain method, which estimates simultaneously all parameters and their uncertainties.

MCMC vs. MLE

For both methods we use the following:

- **Likelihood**: $L(y|\theta) = \frac{1}{(2\pi)^{N/2}|C(\beta)|^{1/2}} e^{-\frac{1}{2}(y-\hat{y})^T C(\beta)^{-1}(y-\hat{y})}$
- **Model**: $\hat{y} = vt + y_0 + \sum_{i=1}^{2}[C_i \cos(\omega_i t) + S_i \sin(\omega_i t)]$
- $\omega_i \equiv {}^{2i\pi}/_{T_{year}}$, N time series length.
- Stochastic Parameter: $\beta = (\alpha, \sigma_{pl}, \sigma_{wn})$
- Covariance Matrix: $C(\beta)$
- All parameters: $\theta = (\beta, v, y_0, C_i, S_i)$
- Data: y, GPS daily position time series (synthetic and JPL).

MCMC vs. MLE (2)

- **Noise modelling:**

The Covariance Matrix: We assume a power-law process ($P(f) \propto f^{-\alpha}$) plus white noise model (Zhang et al. 1997, Mao et al. 1999, Williams et al. 2004):

$$r_i = \sum_{j=0}^{i} h_j u_{i-j} + w_i,$$

where $h_j = \frac{(j+\alpha/2-1)!}{j!(\alpha/2-1)!}$, $\alpha \geq 0$ the spectral index, $j \in \mathbb{N} + \{0\}$, $u \in \mathcal{N}\left(0, \sigma_{pl}\right)$ and

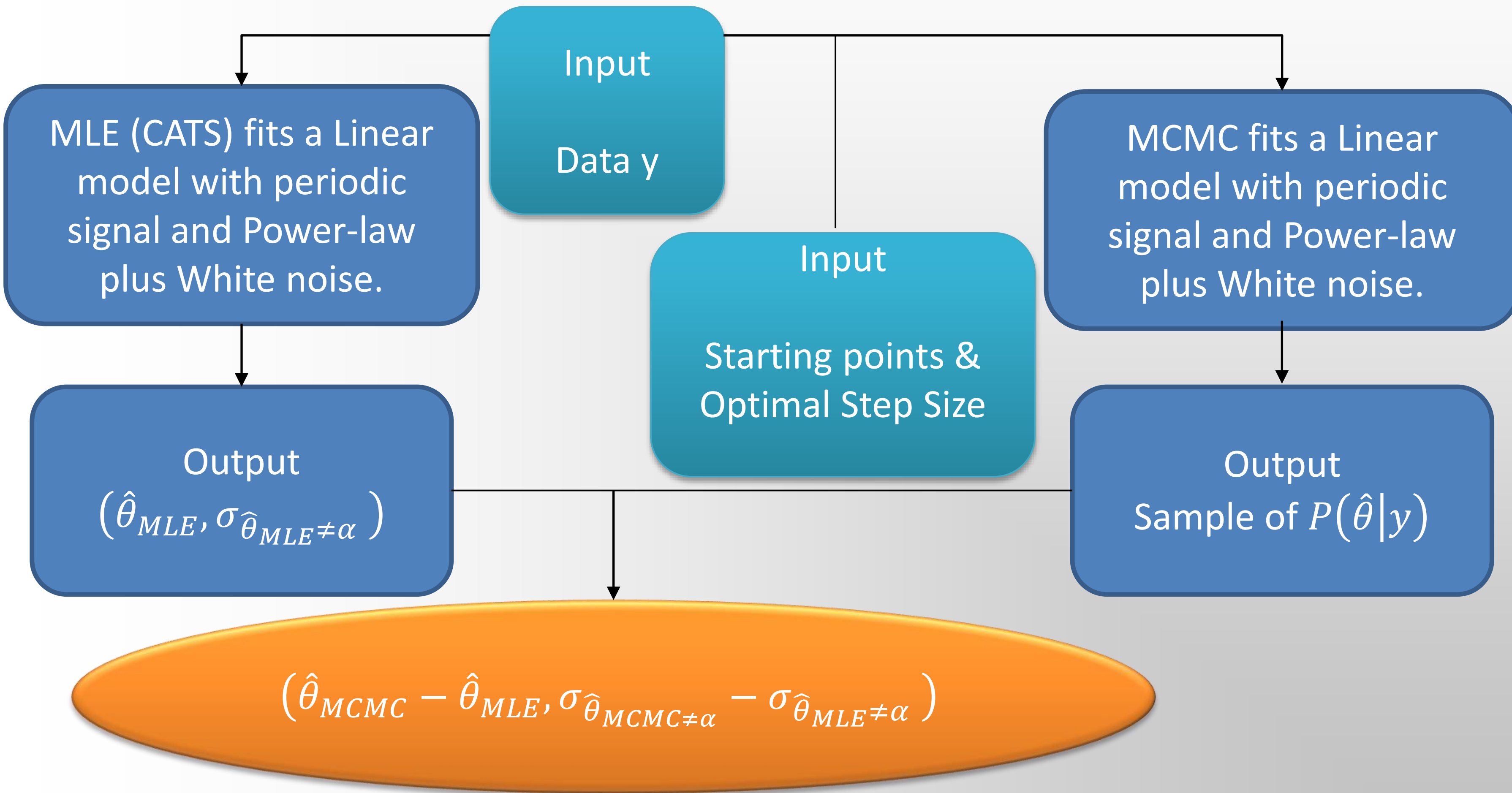
$w \in \mathcal{N}(0, \sigma_{wn})$. Thus, the covariance matrix is:

$$C(\beta) = \sigma_{wn}^2 \boldsymbol{I} + \sigma_{pl}^2 L L^T, \text{ with } L_{ij} = \begin{cases} h_{i-j}, i \leq j \\ 0, i > j \end{cases}$$

MCMC vs. MLE (3)

1. MCMC
   - Bayesian Theorem: $P(\theta|y) = \frac{L(y|\theta)P(\theta)}{P(y)}$
   - $P(\theta), P(y)$ are the a priori distributions of parameters and data, respectively.
2. MLE
   - $\theta_{MLE} = arg\left[max\left(L(y|\theta)\right)\right]$

MCMC vs. MLE (4)



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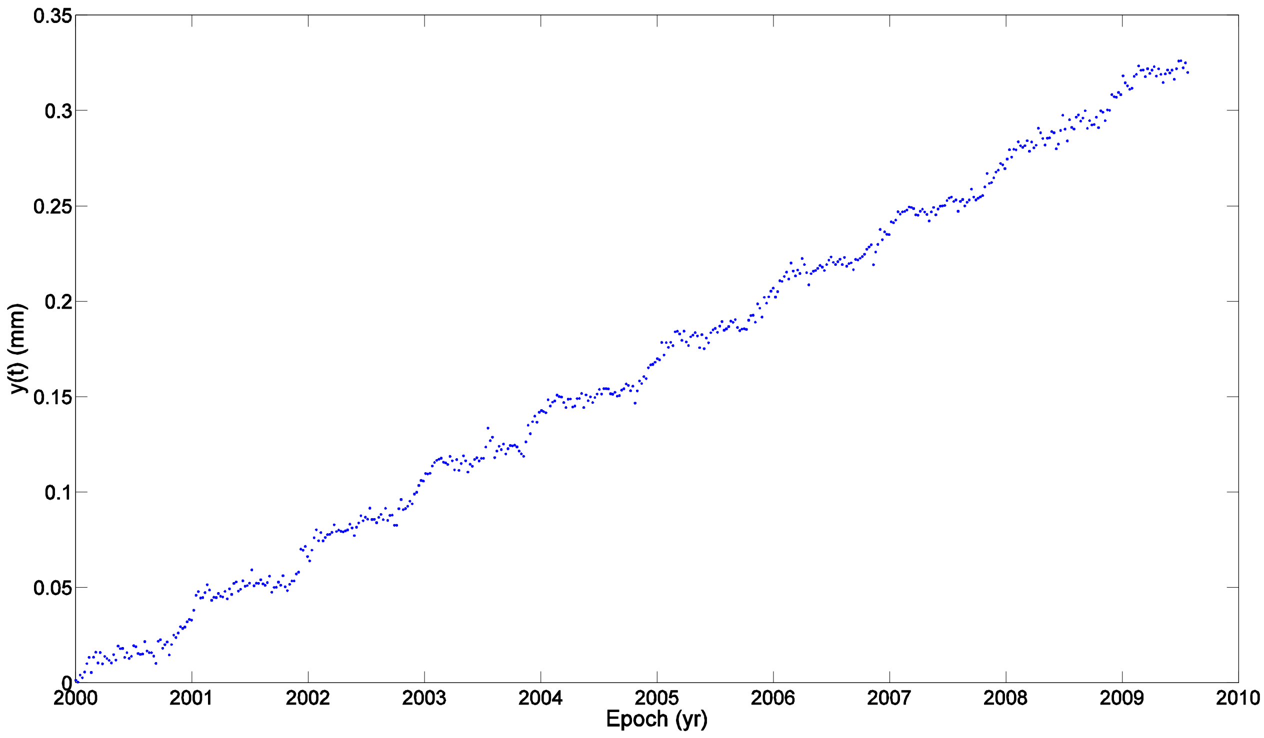
Data Set (I). Synthetic Data

- 300 synthetic GPS position time series generated with

$$y(t) = y_0 + v(t - t_0) + \sum_{i=annual,\,semi} A_{c,i}\cos(\omega_i t) + A_{s,i}\sin(\omega_i t)$$



Results: Parameters Differences (synth)

- α estimates from MCMC are larger, i.e. there is more time-correlation according to this method.
- CATS sets σ_{wn} to zero for 21% of the time series. According to the simulations, σ_{pl} estimates from MLE are larger to account for the null value of σ_{wn}.
- The velocity differences are negligible, and the estimated y_0 from MCMC are slightly larger, though sub-millimetre at 1σ confidence level (CL).

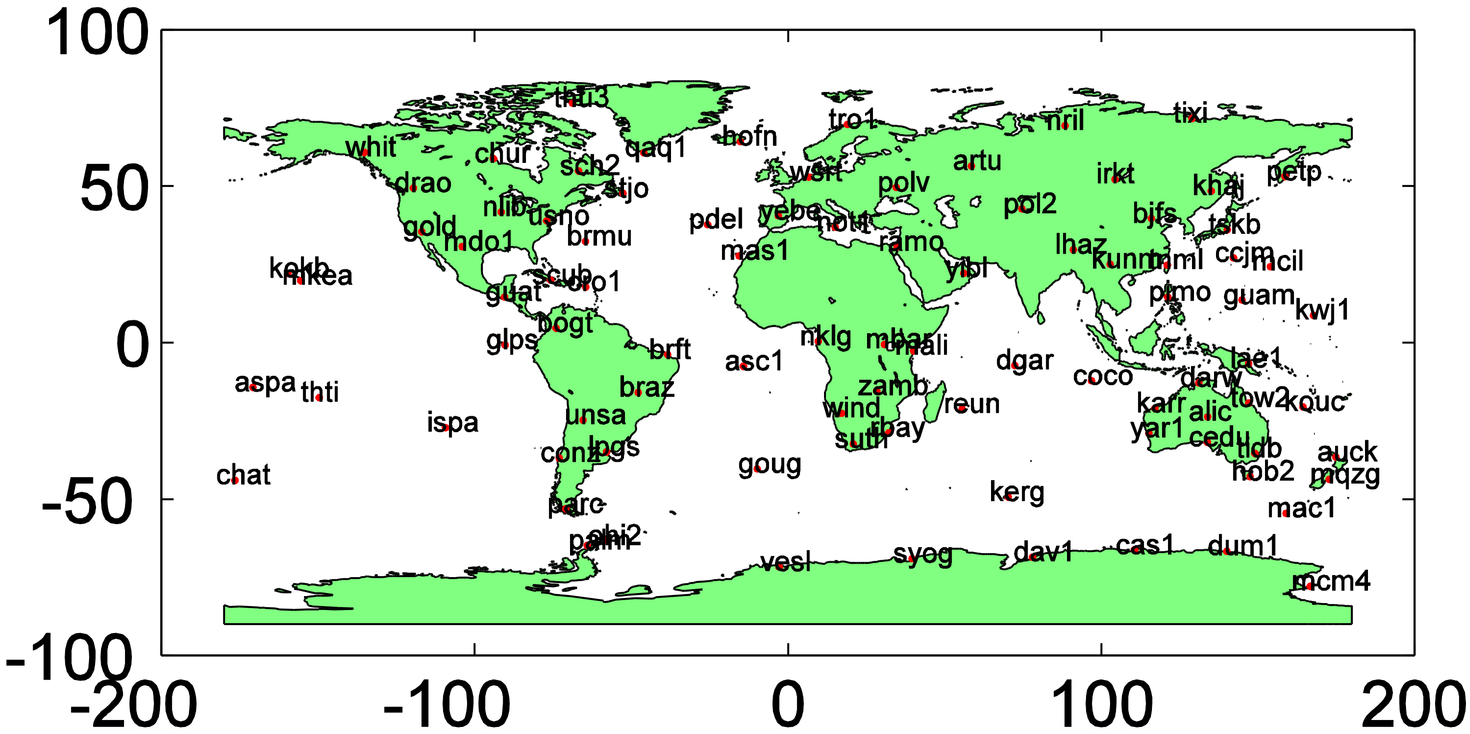
Par. Difference	Median and quantiles [16% , 84%]
Spectral Index α	0.14 [-0.05 , +0.10]
σ_{pl} (mm)	-0.13 [-0.24 , +0.16]
σ_{wn}(mm)	0.06 [-0.15 , +1.04]
v (mm/yr)	0.00 [-0.09 , +0.06]
y_0 (mm)	0.08 [-0.51 , +0.69]

Results: Parameters Uncertainties Differences (synth)

- Velocity uncertainties from MCMC are 41% times larger and sub-millimetre at 1σ CL.
- MLE y_0 estimates are larger and millimetre at 1σ CL.

Unc. Difference	Median and quantiles [16% , 84%]
v (mm/yr)	0.24 [-0.12 , +0.13]
y_0 (mm)	-0.89 [-0.66 , +1.02]

Data Set (II). JPL Solution



- JPL solution for IGS core network. GPS position time series are from http://sideshow.jpl.nasa.gov/post/series.html
- 90 stations with time series lengths ranging from 6 to 19 years.

Results: Stochastic Parameters Differences (JPL)

- All MCMC α estimates are larger for all components, i.e. there is more time-correlation according to this method.
- CATS sets σ_{wn} to zero 71%, 66% and 60% for the North, East and Up components, respectively. According to some simulations, σ_{pl} estimates from MLE are larger to account for the null value of σ_{wn}.

	Median and quantiles [16% , 84%]		
Par. Difference	**North**	**East**	**Up**
Spectral Index α	0.09 [-0.04 , +0.07]	0.08 [-0.04 , +0.08]	0.08 [-0.03 , +0.06]
σ_{pl} (mm)	-0.02 [-0.07 , +0.03]	-0.02 [-0.07 , +0.03]	-0.12 [-0.14 , +0.09]
σ_{wn} (mm)	0.22 [-0.08 , +0.14]	0.20 [-0.13 , +0.24]	0.71 [-0.68 , +0.48]

Results: Velocity and Intercept Differences (JPL)

- All velocity differences are negligible at 1σ CL.
- Intercept differences range from -2.53 to 14.76 mm at 1σ Cl. Estimates from MCMC are larger (smaller) for the North and East (Up) coordinates.
- All results beyond 3σ CL are considered outliers.

	Median and quantiles [16% , 84%]		
Par. Difference	**North**	**East**	**Up**
v (mm/yr)	0.00 [-0.03 , 0.03]	0.00 [-0.02 , 0.04]	0.00 [-0.06 , 0.12]
y_0 (mm)	3.38 [-4.65 , +10.94]	0.76 [-3.29 , +14.00]	-0.20 [-0.75 , +1.21]

Results: Velocity and Intercept Uncertainties Differences (JPL)

- All component velocity uncertainties from MCMC are around 0.1 millimetre per year larger at 1σ CL.

- The velocity uncertainties are 40% larger for the North and East components (consistent with results for synthetic data), whereas they are 18% larger for the Up component at 1σ CL.

- The intercept uncertainty from MLE is larger for all three components, ranging from sub-millimetre (North and East) to millimetre (Up).

	Median and quantiles [16% , 84%]		
Unc. Difference	North	East	Up
σ_v (mm/yr)	0.08 [-0.04 , 0.08]	0.08 [-0.04 , 0.10]	0.08 [-0.05 , 0.16]
σ_{y0} (mm)	-0.43 [-0.30 , +0.23]	-0.40 [-0.27 , +0.18]	-1.34 [-0.87 , +0.65]

Results: Periodic Amplitudes Differences

- For all three components all estimates agree at sub-milimetre level within 1σ CL.

	Median and quantiles [16% , 84%]		
Par. Difference	**North**	**East**	**Up**
Annual cos. (mm)	0.00 [-0.06 , +0.07]	-0.01 [-0.10 , +0.10]	0.00 [-0.12 , +0.13]
Annual sine (mm)	0.00 [-0.08 , +0.04]	0.00 [-0.12 , +0.10]	0.00 [-0.09 , +0.11]
Semi. cos. (mm)	0.00 [-0.03 , +0.06]	0.00 [-0.04 , +0.03]	0.01 [-0.08 , +0.06]
Semi. Sine (mm)	0.00 [-0.03 , +0.03]	0.01 [-0.04 , +0.06]	0.00 [-0.07 , +0.07]

Results: Periodic Amplitudes Uncertainties Differences

- For all periodic parameters and components, the uncertainties of the amplitudes are larger from MCMC, though the differences are at sub-milimetre level.
- It holds: $-0.03 < (\sigma_{MCMC} - \sigma_{MLE}) \leq 0.34\ mm$ at 1σ CL.

	Median and quantiles [16% , 84%]		
Unc. Difference	**North**	**East**	**Up**
Annual cos. (mm)	0.02 [-0.03 , +0.06]	0.02 [-0.05 , +0.08]	0.05 [-0.05 , +0.12]
Annual sine (mm)	0.03 [-0.04 , +0.10]	0.03 [-0.04 , +0.15]	0.06 [-0.04 , +0.12]
Semi. cos. (mm)	0.03 [-0.03 , +0.12]	0.02 [-0.03 , +0.07]	0.05 [-0.04 , +0.29]
Semi. Sine (mm)	0.03 [-0.03 , +0.18]	0.02 [-0.03 , +0.15]	0.05 [-0.05 , +0.25]

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Conclusions

- A new Bayesian Monte Carlo Markov Chain method for parameter estimation in GPS position time series has been compared to MLE.
- Overall, both methods agree well, but there are some differences:
  - MCMC estimates the uncertainty of the spectral index estimate.
  - According to MCMC there is more time-correlated noise, i.e. $\alpha_{MCMC} - \alpha_{MLE} > 0$.
  - MLE yields larger estimates for σ_{pl} in order to account for zero-values of σ_{wn}.
  - (JPL data set) North and East component velocity uncertainties from MCMC are 40% larger tan those from CATS. Up component from MCMC is 18% larger. The differences are sub-milimetre at 1σ CL.
  - As some geophysical applications (e.g. sea level) aim for a target accuracy of 0.1 mm/yr for Up component velocities, these differences could be noticeable.

References

- Caporali, A., Average Strain Rate in the Italian crust inferred from a permanent GPS network – 1. Statistical analysis of the time series of permanent GPS stations, *Geophys. J. Int*., vol 155, 1, 241-253, (2003).
- Hackl, M., R. Malservisi, U. Hugentobler, R. Wonnacott, “Estimation of velocity uncertainties from GPS time series: Examples from the analysis of the South African TrigNet network”, J. Geophys. Res., vol. 116, no. B11, (2011).
- Langbein, J. “Noise in GPS displacement measurements from Southern California and Southern Nevada”, *J. Geophys. Res.*, vol. 113, no. B05405, (2008).
- Mao, A., C.G.A. Harrison and H.D. Timothy, “Noise in GPS coordinate time series”, *J. Geophys. Res.*, vol. 104 no. B2, pp. 27972816, (1999).
- Olivares, G., and F.N. Teferle, “A Monte Carlo Markov Chain Method for Parameter Estimation of Fractional Differenced Gaussian Processes”, *IEEE Transactions on Signal Processing*, DOI: 10.1109/TSP.2013.2245658, (2013).
- Williams, S.D.P. “CATS: GPS coordinate time series analysis software”, *GPS Solutions*, no. 12, pp. 147–153, (2008).
- Williams, S.D.P., Y. Bock and P. Fang and P. Jamason and R.M. Nikolaidis and M. Miller and D.J. Johnson, “Error analysis of continuous GPS position time series”, *J. Geophys. Res.*, vol. 109, pp. B03412, (2004).
- Williams, S.D.P. and P. Willis, “Error Analysis of Weekly Station Coordinates in the DORIS Network”, *J. Geod*., vol. 80, no. 8-11, pp. 525-539., (2006).
- Zhang, J., Y. Bock, P. Fang, S.P.D. Williams, J. Genrich, S. Wdowinski and J. Behr, “Southern California Permanent GPS Geodetic Array: Error analysis of daily position estimates and site velocities”, *J. Geophys. Res.*, vol. 102, no. B2, (1997).