# On kinematical constraints in boson-fermion systems

**Kinematische Beschränkungen in Boson-Fermion-Systemen**
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genehmigte Dissertation von Yonggoo Heo, B.Sc. aus Seoul (Südkorea)
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# Erklärung zur Dissertation

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Darmstadt, den 23. April 2013

(Y. Heo)

# Zusammenfassung

Die starke Wechselwirkung der Elementarteilchen wird erfolgreich durch die Quantenchromodynamik (QCD) beschrieben. Die QCD ist eine nichtabelsche Theorie, die auf der SU(3)-Eichgruppe basiert. Sie beinhaltet die Eichfelder und die Felder der Spin-$\frac{1}{2}$-Teilchen in Form der Gluonen beziehungsweise in Form der Quarks. Quarks sind Fermionen in der Fundamentaldarstellung der SU(3)-Farbeichgruppe. Gluonen sind Bosonenfelder in der adjungierten Darstellung. Ferner tragen die Quarks Flavour-Freiheitsgrade, die unabhängig von der Farbe sind. Eine herausragende Eigenschaft der QCD ist die asymptotische Freiheit. Politzer, Gross, Wilczek und 't Hooft entdeckten die Eigenschaft der asymptotischen Freiheit in nichtabelschen Eichfeldtheorien. Sie erlaubt es, die QCD bei hohen Energien störungstheoretisch zu behandeln. Demgegenüber kann die starke Wechselwirkung bei niedrigen Energien nicht störungstheoretisch behandelt werden, da die Kopplungskonstante der QCD dort zu groß ist. Da wir uns hier für die extrem reichhaltige Phänomenologie der QCD bei niedrigen Energien interessieren, stellt sich sofort die Frage nach den relevanten Freiheitsgraden mit Hilfe derer diese Phänomenologie zu beschreiben ist.

Einen bemerkenswert erfolgreichen Ansatz für die niederenergetische QCD liefert die chirale Störungstheorie ($\chi$PT). Die effektiven Freiheitsgrade der $\chi$PT sind hier mit Hadronen an Stelle der Quarks und Gluonen zu identifizieren. Diese effektive Feldtheorie gründet auf der Beobachtung, daß die QCD im Grenzfall verschwindender Up- und Down-Stromquarkmassen, d. h. $m_{u,d} = 0$, chiral symmetrisch ist. Das hat zur Folge, daß die Händigkeit der Quarks in diesem Grenzfall eine Erhaltungsgröße darstellt. Jedoch bricht das QCD-Vakuum diese Symmetrie spontan. Die daraus hervorgehenden Goldstonebosonen können mit den Pionen, den leichtesten Anregungen des QCD-Grundzustandes, identifiziert werden. Die SU(2)-$\chi$PT beruht auf den Grundlagen der Quantenfeldtheorie und auf den Symmetrien der QCD. Eine Verallgemeinerung des chiralen SU(2)-Schemas hin zur SU(3)-Flavourgruppe, die den Strangenesssektor beinhaltet, ist mathematisch unkompliziert und wurde bereits durchgeführt. Obwohl die Masse des Strange-Quarks wesentlich größer als die Up- und Down-Quarkmasse ist, ist sie auf der typischen chiralen Skala von 1 GeV weiterhin klein. Diese Verallgemeinerung führt zu 5 weiteren Goldstonebosonen, den Kaonen und dem Eta-Meson.

Der Gültigkeitsbereich der $\chi$PT ist allerdings auf sehr kleine Anregungsenergien beschränkt. Eine Verallgemeinerung zu höheren Energien, bei denen die Resonanzdynamik der QCD sichtbar wird, ist erwünscht. In dieser Arbeit untersuchen wir die Hadrogenesis-Vermutung, in der das Anregungsspektrum der QCD als Folge von hadronischer Endzustandswechselwirkung beschreibbar sein sollte. Sie beruht auf einer Auswahl von wenigen grundlegenden hadronischen Freiheitsgraden mit den Quantenzahlen $J^P = 0^-, 1^-$ und $J^P = \frac{1}{2}^+, \frac{3}{2}^+$. Die Auswahl wird durch Eigenschaften der QCD im Grenzfall einer großen Anzahl von Farbfreiheitsgraden ($N_C$) bzw. großer Quarkmassen motiviert. Eine systematische Berechnung der Streu- und Reaktionsamplituden der grundlegenden hadronischen Freiheitsgrade ist erforderlich, wobei die Unitaritätsbedingung als auch die Konsequenzen der Mikrokausalität zu beachten sind.

In dieser Arbeit ebnen wir den Weg hin zu einer systematischen Berechnung des Baryonspektrums basierend auf der Hadrogenesis-Vermutung. In einem ersten Schritt untersuchen wir die analytische Struktur der Streu- und Reaktionsamplituden. Diese werden zunächst in eine geeignete Basis von Lorentz-Dirac-Tensoren entwickelt, welche zu invarianten Amplituden führen, die den von Mandelstam aufgesetzten Dispersionsrelationen genügen. Für die Berechnung der invarianten Amplituden wird ein Projektionsformalismus entwickelt, der in einen Mathematica-Code implementiert wurde. Die Unitaritätsbedingung läßt sich mit Hilfe einer Partialwellenprojektion effizient darstellen. Wiederum untersuchen wir die analytische Struktur dieser Amplituden. Eine Transformation von den Helizitäts-

Partialwellenamplituden hin zu kovarianten Partialwellenamplituden wird vorgeschlagen. Letztere genügen nicht-korrelierten Dispersionrelationen, die als Ausgangspunkt unserer Anwendungen herhalten werden.

In einer ersten Anwendung untersuchen wir die Formation von Baryonresonanzen mit den Quantenzahlen $J^P = \frac{1}{2}^-$. Hierfür ziehen wir eine relativistische chirale SU(3)-Lagrangedichte heran. Neben den Goldstonebosonen und den Baryongrundzuständen wird das Nonett der Vektormesonen berücksichtigt. Wir betrachten den Streuprozeß eines Goldstonebosons an einem Baryon mit $J^P = \frac{1}{2}^+$, wobei wir uns auf S-Wellenstreuung beschränken. Basierend auf den führenden Wechselwirkungstermen der chiralen Lagrangedichte berechnen wir die entsprechenden Partialwellenamplituden. Hierbei kommt eine neuartige Methode zum Einsatz, mit der eine systematische analytische Fortsetzung der chiralen Amplituden im Niederenergiebereich hin in den Resonanzbereich gelingt. Eine Reihe von Baryonresonanzen wird dynamisch erzeugt, wobei der Effekt der verschiedenen Wechselwirkungsbeiträge untersucht wird. Die Resonanzen werden als Pole in den Partialwellenamplituden auf den verschiedenen Riemannblättern gefunden.

# Contents

# 1 Introduction

The strong interactions of matter is successfully described by Quantum ChromoDynamics (QCD). It is a non-Abelian gauge theory, based on a SU(3) gauge group. QCD involves the gauge fields and the fields of spin $\frac{1}{2}$ particles as gluons and quarks, respectively. Quarks are fermions in the fundamental representation of the color SU(3) gauge group. Gluons are boson fields in the adjoint (octet) representation. Moreover, quarks carry flavour degrees of freedom which are independent of the color. There are six flavours, the up, charm, and top quarks having charge $\frac{2}{3}e$ and the down, strange, and bottom quarks having charge $-\frac{1}{3}e$. Together with the Glashow-Weinberg-Salam theory [1, 2, 3], QCD is a part of the standard model of particle physics. A distinguished feature of QCD is its asymptotic freedom. Politzer [4], Gross and Wilczek [5], and 't Hooft [6] discovered the property of asymptotic freedom in non-Abelian gauge field theories. It allows QCD to be treated perturbatively at high energies. On the other hand, the strong interaction becomes nonperturbative at low energies, because the coupling constant of QCD is too large there. This is unfortunate since the most interesting phenomena of QCD are at low energies.

A remarkably successful effective Lagrangian approach to low-energy QCD is Chiral Perturbation Theory ($\chi$PT). The effective degrees of freedom of $\chi$PT are hadrons rather than quarks and gluons. $\chi$PT has been applied extensively in the flavour SU(2) sector of low-energy QCD. This effective field theory is based on the observation that QCD is chirally symmetric in the limit where the up and down current quark masses vanish, i.e. $m_{u,d} = 0$. This implies that the handedness of quarks is a conserved property in that limit. However, the QCD vacuum brakes that chiral symmetry spontaneously. The Goldstone bosons of the spontaneously broken chiral symmetry [7] are identified with the pions, the lightest excitations of the QCD ground state. $\chi$PT relies on the principles of quantum field theory and on the symmetries of QCD. It permits systematic computations applying formal power counting rules [8, 9, 10, 11]. A generalization of the chiral SU(2) scheme to the SU(3) flavour group, which includes the strangeness sector, is mathematically straightforward and has been developed, see e.g. [12, 13, 14, 11]. Though the mass of the strange quark is much larger than the up and down quark masses, it is still small on the typical chiral scale of 1 GeV [15]. The required approximate Goldstone boson octet is readily found with the pions, the kaons, and the eta-meson. Though strict $\chi$PT is a powerful and predictive tool to analyze the interaction of Goldstone bosons with any hadron, its domain of validity is restricted to energies close to threshold. A generalization to higher energies is desired.

The resonance physics of QCD is one of the key challenges in hadron physics. Upon the discovery of the isobar resonance in a pion induced reaction, the first hadronic resonance discovered, Chew and Low suggested that it could be generated dynamically by pion-nucleon interactions [16]. Later on it was suggested by Wyld [17, 18] and also by Dalitz, Wong, and Rajasekaran [19] that a $t$-channel vector meson exchange model for the s-wave meson-baryon scattering problem has the potential to dynamically generate an octet of s-wave baryon resonances upon solving a coupled channel Schrödinger equation. This was before the quark-model of hadrons was suggested by Gell-Mann and Zweig [20, 21, 22]. The remarkable vision of the quark model was its assumption of the existence of constituent quarks with their fractional electric charges that turned out useful to describe may hadronic properties qualitatively. However, the quark model lacks a systematic link to QCD in the sense of an effective field theory. Moreover, so far any attempt to describe hadronic reaction data quantitatively in terms of constituent-quark degrees of freedom appears futile. In the last decade the coupled-channel approach to hadronic physics, as pioneered by Wyld, Dalitz, Wong, and Rajasekaran, experienced a remarkable revival. Based on the flavour SU(3) chiral Lagrangian many studies of the $J^P = \frac{1}{2}^-$ resonances have been performed, see e.g. [23, 24, 25, 26, 27, 28].

In this work we follow the hadrogenesis conjecture [29, 30, 31, 32, 33, 34, 35]. Selecting a few quasi-fundamental hadronic degrees of freedom, the zoo of resonances is conjectured to be a result of coupled-channel interactions. The identification of the proper set of degrees of freedom is guided by properties of QCD in the large-$N_C$ limit [36, 37, 38]. Here we would depart from Chew and Low, since in that limit the isobar resonance is a partner of the nucleon and therefore it should not be generated dynamically in terms of pion-nucleon interactions. The quasi-fundamental fields requested are the Goldstone bosons with $J^P = 0^-$, the lightest vector mesons with $J^P = 1^-$ together with the baryon octet and decuplet fields with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$. Early applications of the hadrogenesis are meson and baryon resonances with $J^P = 0^+, 1^+$ and $J^P = \frac{1}{2}^-, \frac{3}{2}^-$ quantum numbers. For example, an axial-vector spectrum was dynamically generated by the leading order chiral interaction of the Goldstone boson octet with the nonet of light vector mesons in [32]. The existence of various baryon resonances was predicted by the chiral interaction of the baryon octet or decuplet with the Goldstone boson octet in [31, 39, 40].

Given some effective degrees of freedom micro-causality and coupled-channel unitarity are crucial constraints that help to establish coupled-channel reaction amplitudes based on a suitable effective chiral Lagrangian. The partial-wave decomposition is useful for an analysis of scattering processes. In particular, the unitarity condition can be easily realized in terms of partial-wave amplitudes. Though it is straightforward to introduce partial-wave scattering amplitudes in the helicity formalism of Jacob and Wick [41] it is much less trivial to derive partial-wave amplitudes that are consistent with the implications of micro-causality. A problem is caused by the fact that helicity partial-wave scattering amplitudes are kinematically constrained. It is a nontrivial task to derive transformations that lead to amplitudes that are kinematically unconstrained. Only such amplitudes are useful in an application of partial-wave dispersion-integral representations [42, 43, 44, 45, 46, 47, 48]. The purpose of the thesis is a derivation of such amplitudes by suitable transformations of the helicity partial-wave scattering amplitudes for two-body reactions of a boson with $J^P = 0^-, 1^-$ and a fermion with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$, the degrees of freedom that are requested by the hadrogenesis conjecture. Recently reactions in fermion-antifermion and boson-boson systems have been studied in [49] and [50], respectively. We shall apply the technique used previously in studies of two-body scattering systems with photons, pions, and nucleons [42, 51, 52, 53, 54, 55, 56, 57, 58].

Our goal is to pave the way for systematic coupled-channel computations as requested by the hadrogenesis conjecture. We establish partial-wave amplitudes with convenient analytic properties that justify the use of uncorrelated integral-dispersion relations. In a fist step, two-body reactions of a boson with $J^P = 0^-, 1^-$ and a fermion with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ are considered. We parameterize an on-shell scattering amplitude in terms of invariant functions that are free of kinematical constraints. Such amplitudes are expected to satisfy a Mandelstam's integral representation of dispersion relation [59, 51]. In a second step, we construct projection algebras to calculate the invariant functions directly for a given reaction. These allow us to illustrate the well-known fact that different helicity partial-wave amplitudes are correlated at various kinematical conditions. The kinematical constraints in the helicity partial-wave amplitudes are eliminated by means of non-unitary transformation matrices that map the helicity states to new covariant states. The mapping procedure is based on the exclusive use of on-shell matrix elements.

While the emphasis of this thesis is of formal nature we do provide a physics application. The formation of baryon resonances with $J^P = \frac{1}{2}^-$ is reconsidered. All previous studies relied on the on-shell factorization or the on-shell reduction scheme can be justified only in the presence of short range forces [60, 31]. Thus the effect of the u-channel baryon exchange in the formation of the $J^P = \frac{1}{2}^-$ resonances has not been studied reliably so. In application of the novel unitarization scheme [45, 46, 61, 48] such a study will be provided in this thesis for the first time.

The thesis is organized as follows. In Chapter 2, we decompose the on-shell scattering amplitudes into sets of invariant amplitudes free of kinematical singularities. The sets of Lorentz-Dirac tensors reflect the MacDowell symmetry. Convenient projection algebras for the derivation of invariant amplitudes are constructed not only for spin-one-half fermions, but also spin-three-half fermions in the following chapter. The complicated cases are collected in Appendices A and B. Chapter 4 introduces the conventions used for the kinematics and the helicity wave functions. We introduce suitable partial-wave amplitudes free of kinematical constraints in Section 4.2. Technical details can be found in Appendix C. Using the relativistic chiral Lagrangian with the baryon octet and decuplet fields the formation of baryon resonances with $J^P = \frac{1}{2}^-$ is investigated in Chapter 5. Appendix D contains detailed calculations of tree-level interactions.

# 2 Analytic properties of scattering amplitudes

In quantum field theory, a two-body scattering process is described by the scattering amplitude that follows from the solution of the Bethe-Salpeter equation [62]. The scattering amplitude is given by matrix elements of the quantum mechanical scattering operator $T$.

For instance, consider the on-shell pion-nucleon scattering amplitude

$$\langle \pi(\bar{q})\, N(\bar{p},\lambda_{\bar{p}})|\, T\, |\pi(q)\, N(p,\lambda_p)\rangle = (2\,\pi)^4\, \delta^4(\bar{p}+\bar{q}-p-q)\, \bar{u}(\bar{p},\lambda_{\bar{p}})\, T_{\pi N\to \pi N}(\bar{q},q;w)\, u(p,\lambda_p)\,, \tag{2.1}$$

where the four-dimensional delta function guarantees energy-momentum conservation and $u(p,\lambda_p)$ is the nucleon isospin-doublet spinor with its helicity projections $\lambda_p$. For simplicity we do not resolve isospin degrees of freedom here.

The scattering amplitude $T_{\pi N\to\pi N}$ is obtained by solving the Bethe-Salpeter matrix equation

$$\begin{aligned} T_{\pi N\to \pi N}(\bar{k},k;w) &= K_{\pi N\to \pi N}(\bar{k},k;w) + \int \frac{d^4 l}{(2\pi)^4}\, K_{\pi N\to \pi N}(\bar{k},l;w)\, G(l;w)\, T_{\pi N\to \pi N}(l,k;w)\,, \\ G(l;w) &= -i\, \frac{1}{(\frac{1}{2}\, w - l)^2 - m_\pi^2 + i\,\epsilon}\, \frac{1}{\frac{1}{2}\, \not{w} + \not{l} - M_N + i\,\epsilon}\,, \end{aligned} \tag{2.2}$$

where $m_\pi$ and $M_N$ are the pion and nucleon masses respectively. Self energy corrections in the propagators are suppressed and hereby not considered in the thesis. The interaction kernel $K(\bar{k},k;w)$ is the sum of all two-particle irreducible diagrams deduced from a given Lagrangian. In (2.2) convenient kinematical variables are used:

$$w^\mu = q^\mu + p^\mu = \bar{q}^\mu + \bar{p}^\mu\,, \qquad k^\mu = \tfrac{1}{2}(p^\mu - q^\mu)\,, \qquad \bar{k}^\mu = \tfrac{1}{2}(\bar{p}^\mu - \bar{q}^\mu)\,, \tag{2.3}$$

where $q$, $p$, $\bar{q}$, and $\bar{p}$ are the initial and final pion and nucleon four-momenta. Here we do not want to specify the form of the scattering kernel $K(\bar{k},k;w)$, since the discussion is generic and independent on the particular form of the interaction. The Bethe-Salpeter equation (2.2) implements Lorentz invariance and unitarity for the two-body scattering process. It involves the off-shell continuation of the on-shell scattering amplitude introduced in (2.1). Only the on-shell limit with $\bar{p}^2 = p^2 = M_N^2$ and $\bar{q}^2 = q^2 = m_\pi^2$ carries direct physical information. In quantum field theory the off-shell form of the scattering amplitude reflects the particular choice of the pion and nucleon interpolating fields chosen in the Lagrangian density and therefore can be altered by a redefinition of the fields [63, 64]. Such a dependence can be removed by an appropriate on-shell reduction scheme for the Bethe-Salpeter equation [29, 31, 32, 39]. However, the on-shell reduction scheme is practical in the presence of short range forces only.

The four dimensional Bethe-Salpeter equation can be numerically solved with approximated interaction kernels and phenomenological form factors [65]. This method ensures that two-body unitarity is respected. However, a direct solution of a Bethe-Salpeter equation gives rise to an almost always impossible task of eliminating off-shell effects. In particular, when t- and u-channel exchanges of light particles are involved. Therefore, in our work, we will perform an analytic extrapolation of partial-wave amplitudes instead, where we insist on basic principles of micro-causality and unitarity.

The scattering operator $T$ can be partially diagonalized by using eigenstates of the total angular momentum operator $J$. This leads to partial-wave scattering amplitudes. The unitarity condition below the inelastic threshold takes a particularly simple form when expressed in terms of partial-wave amplitudes. For a two-body scattering process, this can be achieved by using the helicity formalism of Jacob and Wick [41]. We will discuss this issue in more detail later in Chapter 4. The key issue is a separation of left- and right-hand singularities of partial-wave amplitudes [66, 67]. The separation is achieved by means of the once-subtracted dispersion relation for a partial-wave amplitude:

$$T^J(\sqrt{s}) = U^J(\sqrt{s}) + \int_{\mu_{\text{thrs}}}^{\infty} \frac{\mathrm{d}w}{\pi} \frac{\sqrt{s} - \mu_M}{w - \mu_M} \frac{\Delta T^J(w)}{w - \sqrt{s} - i\epsilon}, \tag{2.4}$$

where the generalized potential $U^J(\sqrt{s})$ contains left-hand cuts only, by definition. The discontinuity along the right-hand cut is given by the unitary condition

$$\begin{aligned}\Delta T^J(\sqrt{s}) &= \frac{1}{2i}\left[T^J(\sqrt{s}+i\epsilon) - T^J(\sqrt{s}-i\epsilon)\right] \\ &= T^J(\sqrt{s}+i\epsilon)\,\rho^J(\sqrt{s})\,T^J(\sqrt{s}-i\epsilon),\end{aligned} \tag{2.5}$$

where $\rho^J(\sqrt{s})$ is a phase-space matrix and the summation over all possible intermediate states is implied. The relation (2.4) illustrates that the amplitude possesses a unitarity cut along the positive real $\sqrt{s}$ axis starting from the lowest s-channel threshold. Without specifying a particular structure of the left-hand singularities, Eq. (2.4) defines the generalized potential $U^J(\sqrt{s})$.

The matching point in (2.4) is determined by the condition that a scattering amplitude remains perturbative in the close vicinity of $\mu_M$. For instance for case of elastic pion-nucleon scattering, $\mu_M = M_N$ was chosen in [45]. With (2.4) and (2.5), one arrives at the following non-linear integral equation for the partial-wave scattering amplitudes

$$T^J(\sqrt{s}) = U^J(\sqrt{s}) + \int_{\mu_{\text{thrs}}}^{\infty} \frac{\mathrm{d}w}{\pi} \frac{\sqrt{s} - \mu_M}{w - \mu_M} \frac{T^J(w)\,\rho^J(w)\,T^{J*}(w)}{w - \sqrt{s} - i\epsilon}. \tag{2.6}$$

We will solve the non-linear integral equation in Chapter 5 for a simple model interaction.

For such an analysis of partial-wave amplitudes, it is required to have a profound control of the analytic properties of invariant and partial-wave amplitudes. In a first step we study in this and the following chapters the analytic structure of the on-shell scattering amplitudes by parameterizing them by an appropriate set of invariant amplitudes with convenient analytic properties. In a second step in Chapter 4 the invariant amplitudes are used to arrive at partial-wave amplitudes with suitable analytic properties.

## 2.1 Invariant amplitudes

We consider two-body reactions of a boson with $J^P = 0^-, 1^-$ and a fermion with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$. A two-body reaction is characterized by the three Mandelstam variables $s$, $t$, and $u$ with

$$s + t + u = m^2 + M^2 + \bar{m}^2 + \bar{M}^2, \tag{2.7}$$

where $m$ and $M$ are incoming masses and $\bar{m}$ and $\bar{M}$ are outgoing masses of boson and fermion, respectively. The momenta of the incoming and outgoing bosons will be denoted by $q$ and $\bar{q}$ and those of the incoming and outgoing fermions by $p$ and $\bar{p}$. Due to the conservation of the total momentum, out of the four momenta there will be only three independent momenta which determine the kinematics of the process. In this work we use the following notational convention:

$$s = w^2, \quad t = (\bar{q} - q)^2 = (\bar{p} - p)^2, \quad u = (p - \bar{q})^2 = (q - \bar{p})^2. \tag{2.8}$$

On-shell scattering amplitudes are defined in terms of plane-wave matrix elements of the scattering operator $T$ where we suppress internal degrees of freedom like isospin or strangeness quantum numbers for simplicity. The scattering amplitude can be decomposed into sets of Lorentz invariant functions $G_n(s,t)$. Invariant amplitudes free of kinematical constraints have been discussed by many authors (see e.g. S.W. MacDowell [68], Asim O. Barut, Ivan Muzinich, and David N. Williams [69], and Alan Douglas Martin and Thomas D Spearman [70]). Most detailed results are available for pion-nucleon scattering, pion photoproduction, and nucleon-nucleon scattering in M.L. Goldberger, Marcus T. Grisaru, S.W. MacDowell, and David Y. Wong [71], James Stutsman Ball [51], Yasuo Hara [52], and J. D. Jackson and G. E. Hite [53]. More complicated systems involving higher spin systems involving spin-three-half and spin-one states were investigated recently in [49, 50]. For the systems of interest in this work systematic studies and practical results are not available.

The number of invariant on-shell amplitudes follows from the number of independent helicity amplitudes. Since we assume parity conservation, the total number of independent helicity amplitudes is generally given by

$$\tfrac{1}{2}(2S_q+1)(2S_p+1)(2S_{\bar{q}}+1)(2S_{\bar{p}}+1)\,, \tag{2.9}$$

where $S_q, S_p$ and $S_{\bar{q}}, S_{\bar{p}}$ are the spins of the initial and final particles. In order to count how many independent amplitudes exist for a given spin configuration, one has to work out the relations among different partial-wave amplitudes under a parity transformation [70, 72]. For boson-boson systems, it is not always so straightforward, but it is for boson-fermion and fermion-antifermion systems [49, 50].

The merit of a decomposition of the scattering matrix into invariant functions $G_n(s,t)$ lies in their transparent analytic properties, which are expected to satisfy Mandelstam's dispersion integral representation [59, 68, 51]. For reactions involving non-zero spin particles it is not straightforward to identify such amplitudes. Once we identify a suitable set of invariant functions $G_n(s,t)$ each element is an analytic function of the Mandelstam variables with the exception of dynamical singularities. The latter have a one-to-one correspondence to physical processes, like s-, t-, and u-channel exchange processes:

$$\begin{aligned} G_n(s,t) = &\int\frac{ds'}{\pi}\int\frac{dt'}{\pi}\frac{\rho_{st}(s',t')}{(s'-s)(t'-t)} + \int\frac{dt'}{\pi}\int\frac{du'}{\pi}\frac{\rho_{tu}(t',u')}{(t'-t)(u'-u)} + \int\frac{ds'}{\pi}\int\frac{du'}{\pi}\frac{\rho_{su}(s',u')}{(s'-s)(u'-u)} \\ &+ \int\frac{ds'}{\pi}\frac{\rho_s(s')}{s'-s} + \int\frac{dt'}{\pi}\frac{\rho_t(t')}{t'-t} + \int\frac{du'}{\pi}\frac{\rho_u(u')}{u'-u}\,, \end{aligned} \tag{2.10}$$

with the spectral functions $\rho_{st}$, $\rho_{tu}$, and $\rho_{su}$. The locations of branch points are determined by the thresholds of possible intermediate states. Landau and Cutkosky have derived a general criterion for determining the positions of branch points of an arbitrary Feynman graph in [73, 74]. In the case when $|G_n(s,t)|$ does not tend to zero as $|s| \to \infty$, the representation (2.10) will not be true as it stands, but will require suitable subtractions.

A priori further kinematical singularities or constraints can not be ruled out in the case of higher spin systems as considered in our work. However, such kinematical boundary conditions can be removed by constructing a suitable set invariant functions that are associated with a set of Lorentz-Dirac tensors for a given reaction. The construction of which will be discussed in this chapter in great detail.

We begin with the elastic scattering of a pseudoscalar boson off a spin-one-half fermion. According to Eq. (2.9) only two invariant functions can be introduced here. The scattering amplitude may be written as

$$T_{0\frac{1}{2}\to 0\frac{1}{2}}(\bar{q},q,w) = \bar{u}(\bar{p},\lambda_{\bar{p}})\,\left[G_1(s,t) + G_2(s,t)\,\not{w}\right]\,u(p,\lambda_p)\,, \tag{2.11}$$

where $u(p,\lambda_p)$ and $\bar{u}(\bar{p},\lambda_{\bar{p}})$ denote the baryon wave functions with their helicity projections $\lambda_p$ and $\lambda_{\bar{p}}$. Under conserved parity, Lorentz invariance, and hermiticity, the $G_n$ in (2.11) are functions of only invariants $s$ and $t$ [75, 76, 68, 72]. Contributions of terms like $\bar{\not{p}}$ or $\not{p}$ to Eq. (2.11) are absorbed into $G_1(s,t)$ in application of the *Dirac equation* for the baryon wave functions

$$\bar{u}(\bar{p},\lambda_{\bar{p}})(\bar{\not{p}} - \bar{M}) = 0\,, \quad (\not{p} - M)\,u(p,\lambda_p) = 0\,. \tag{2.12}$$

This leads to a set of on-shell identities

$$\bar{\not p} \overset{\text{on-shell}}{=} \bar{M}\,, \qquad \bar{\not q} \overset{\text{on-shell}}{=} -\bar{M} + \not w\,,$$
$$\not p \overset{\text{on-shell}}{=} M\,, \qquad \not q \overset{\text{on-shell}}{=} -M + \not w\,. \tag{2.13}$$

where with $\overset{\text{on-shell}}{=}$ we imply the presence of the on-shell wave functions and on-shell kinematics. With Eqs. (2.13) it is then concluded that further Dirac structures in Eq. (2.11) are all on-shell redundant.

Now we move to the next case that involves a vector particle in the final state. According to Eq. (2.9) only six independent on-shell invariant amplitudes $G_n$ should exist. Problems in choosing an appropriate basis come up here. Let us start from an over-complete set of Lorentz-Dirac tensors to illustrate the problem and show how to solve it:

$$T_{0\,\frac{1}{2}\to 1\,\frac{1}{2}}(\bar q, q, w) = \epsilon^{\dagger\bar\mu}(\bar q, \lambda_{\bar q})\,\bar u(\bar p, \lambda_{\bar p})\Big[G_1\,\gamma_{\bar\mu}\,i\,\gamma_5 + G_2\,w_{\bar\mu}\,i\,\gamma_5 + G_3\,q_{\bar\mu}\,i\,\gamma_5 + G_4\,\epsilon_{\bar\mu[\bar q][w][q]}$$
$$+ G_5\,\gamma_{\bar\mu}\,\not w\,i\,\gamma_5 + G_6\,w_{\bar\mu}\,\not w\,i\,\gamma_5 + G_7\,q_{\bar\mu}\,\not w\,i\,\gamma_5 + G_8\,\epsilon_{\bar\mu[\bar q][w][q]}\,\not w\Big]\,u(p, \lambda_p)\,, \tag{2.14}$$

where $\epsilon^{\dagger\bar\mu}(\bar q, \lambda_{\bar q})$ denotes an outgoing vector-meson wave function with its helicity projection $\lambda_{\bar q}$. For notational convenience we introduce

$$\epsilon_{\mu[\bar q][w][q]} \equiv \epsilon_{\mu\alpha\gamma\beta}\,\bar q^{\alpha} w^{\gamma} q^{\beta} \equiv v_\mu\,, \tag{2.15}$$

in terms of the Levi-Civita tensor

$$-4\,i\,\epsilon_{\mu\nu\rho\sigma} = \mathbf{tr}\,\gamma_5\,\gamma_\mu\,\gamma_\nu\,\gamma_\rho\,\gamma_\sigma\,. \tag{2.16}$$

The over-complete set (2.14) is a generalization attempt of Eq. (2.11) to the presence of a further Lorentz structure. Terms including $\bar q_{\bar\mu}$ do not contribute to the on-shell scattering amplitude because of the on-shell condition of the outgoing vector meson

$$\bar q_{\bar\mu}\,\epsilon^{\bar\mu}(\bar q, \lambda_{\bar q}) = 0\,. \tag{2.17}$$

The redundancy of two out of the eight structures in Eq. (2.14) is not so immediately verified. It is a combined consequence of the energy-momentum conservation, the on-shell condition, the *Chisholm identity* [77, 78, 79]

$$\gamma^\mu\,\gamma^\nu\,\gamma^\rho = g^{\mu\nu}\,\gamma^\rho - g^{\mu\rho}\,\gamma^\nu + g^{\nu\rho}\,\gamma^\mu + i\,\epsilon^{\mu\nu\rho\sigma}\,\gamma_\sigma\,\gamma_5\,, \tag{2.18}$$

and the *Schouten identity* [80, 81][1]

$$g^{\zeta\eta}\epsilon^{\mu\nu\rho\sigma} - g^{\zeta\mu}\epsilon^{\eta\nu\rho\sigma} + g^{\zeta\nu}\epsilon^{\eta\mu\rho\sigma} - g^{\zeta\rho}\epsilon^{\eta\mu\nu\sigma} + g^{\zeta\sigma}\epsilon^{\eta\mu\nu\rho} = 0\,. \tag{2.20}$$

Once a basis is chosen the redundant Lorentz-Dirac structures can be expressed in terms of the basis structures. If the set $G_{1-3}$ and $G_{5-7}$ is chosen as a basis in Eq. (2.14), the structure $\epsilon_{\bar\mu[\bar q][q][w]}$ can be expressed as a linear combination of the corresponding basis tensors. We derived the following identity

$$\epsilon_{\bar\mu[\bar q][w][q]} \overset{\text{on-shell}}{=} \frac{1}{2}\,\gamma_{\bar\mu}\,i\,\gamma_5\Big[-\bar m^2 M + \bar M\,[M(\bar M + M) - m^2] + s(\bar M + M)\Big]$$
$$+\frac{1}{2}\,w_{\bar\mu}\,i\,\gamma_5\Big[-\bar m^2 - 2M(\bar M + M) + m^2 + t\Big] + \frac{1}{2}\,q_{\bar\mu}\,i\,\gamma_5\Big[\bar m^2 - \bar M^2 - s\Big]$$
$$+\frac{1}{2}\,\gamma_{\bar\mu}\,\not w\,i\,\gamma_5\Big[(\bar M + M)^2 - t\Big] - w_{\bar\mu}\,\not w\,i\,\gamma_5\Big[\bar M + M\Big] + q_{\bar\mu}\,\not w\,i\,\gamma_5\Big[\bar M\Big]\,. \tag{2.21}$$

[1] The general form of *the Schouten identity* is

$$g^{\mu_1\mu_2}\mathbf{tr}\left(\gamma_5\gamma^{\mu_3}\cdots\gamma^{\mu_n}\right) - g^{\mu_1\mu_3}\mathbf{tr}\left(\gamma_5\gamma^{\mu_2}\gamma^{\mu_4}\cdots\gamma^{\mu_n}\right) + \cdots + g^{\mu_1\mu_n}\mathbf{tr}\left(\gamma_5\gamma^{\mu_2}\cdots\gamma^{\mu_{n-1}}\right) = 0\,, \tag{2.19}$$

where $n$ is even and greater than 2.

Similarly if the set $G_{2-4}$ and $G_{6-8}$ is chosen, the structure $\gamma_{\bar{\mu}}\, i\,\gamma_5$ can be shown to have the following on-shell decomposition:

$$
\begin{aligned}
\gamma_{\bar{\mu}}\, i\,\gamma_5 \stackrel{\text{on-shell}}{=} \frac{w_{\bar{\mu}}\, i\,\gamma_5}{4\,v^2} \Big[ &-\bar{m}^4 M + \bar{m}^2 \left(\bar{M}^2 M - \bar{M}(m^2+M^2) + M(m^2-2M^2)\right) + s(\bar{m}^2-m^2)(\bar{M}+M) \\
&+ t\left(\bar{m}^2 M - \bar{M}[M(\bar{M}-M)+m^2] + s(M-\bar{M})\right) + \bar{M} m^2 \left((2\bar{M}-M)(\bar{M}+M)+m^2\right) \Big] \\
+ \frac{q_{\bar{\mu}}\, i\,\gamma_5}{4\,v^2} \Big[ & s^2(\bar{M}+M) + 2\bar{M}ts + s\left(-\bar{m}^2(\bar{M}+2M) - \bar{M}[(\bar{M}+M)^2+m^2]\right) \\
&+ (\bar{m}^2-\bar{M}^2)\left(\bar{m}^2 M - \bar{M}[M(\bar{M}+M)-m^2]\right) \Big] \\
- \frac{\epsilon_{\bar{\mu}[\bar{q}][q][w]}}{2\,v^2} \Big[ & \left(\bar{M}[M(\bar{M}+M)-m^2] - \bar{m}^2 M\right) + s(\bar{M}+M) \Big] \\
+ \frac{w_{\bar{\mu}}\, \not{w}\, i\,\gamma_5}{4\,v^2} \Big[ & t(-\bar{m}^2 - \bar{M}^2 - m^2 - M^2 + 2s) + (\bar{m}^2-m^2)(\bar{M}^2-M^2) + t^2 \Big] \\
+ \frac{q_{\bar{\mu}}\, \not{w}\, i\,\gamma_5}{4\,v^2} \Big[ & -\bar{m}^2(\bar{M}^2+M^2) + t(\bar{m}^2-\bar{M}^2-s) + \bar{M}^2(\bar{M}^2+2m^2-M^2) + s(M^2-\bar{M}^2) \Big] \\
- \frac{\epsilon_{\bar{\mu}[\bar{q}][q][w]}\, \not{w}}{2\,v^2} \Big[ & t - (\bar{M}+M)^2 \Big] \,,
\end{aligned}
\tag{2.22}
$$

where the vector $v_\mu$ was introduced in Eq. (2.15). Note that $v^2$ becomes zero at

$$
\begin{aligned}
s = {} & \frac{(m^2-\bar{m}^2)(\bar{M}^2-M^2)}{2t} + \frac{1}{2}(\bar{m}^2+\bar{M}^2+m^2+M^2) - \frac{t}{2} \\
& + \frac{1}{2\,t}\sqrt{[t-(\bar{m}-m)^2][t-(\bar{m}+m)^2]}\sqrt{[t-(\bar{M}-M)^2][t-(\bar{M}+M)^2]}\,.
\end{aligned}
\tag{2.23}
$$

Our result (2.22) shows that the particular basis choice $G_{2-4}$ and $G_{6-8}$ leads to invariant amplitudes that have kinematical singularities at $v^2 = 0$ (see Eq. (2.23)). This disqualifies such a choice. On the other hand the basis choice $G_{1-3}$ and $G_{5-7}$ does not lead to kinematical singularities as illustrated by the result (2.21). We affirm that indeed in terms of the latter basis choice any Lorentz-Dirac tensors can be expressed as a linear combination of the basis tensors with regular expansion coefficients. Note that a kinematically unconstrained basis is not necessarily unique. In general several choices may be possible.

We have seen in the previous discussion that when the invariant amplitudes are free of kinematical singularities, the complete set of suitable Lorentz-Dirac structures appears to involve the *minimal number of momenta* only. The eight tensor structures in (2.14) come with different numbers of momenta as summarized with

$$
\begin{array}{llll}
G_1 & \longleftrightarrow & \text{zero} & \text{momentum} \\
G_{2,3,5} & \longleftrightarrow & \text{one} & \text{momentum} \\
G_{6,7} & \longleftrightarrow & \text{two} & \text{momenta} \\
G_4 & \longleftrightarrow & \text{three} & \text{momenta} \\
G_8 & \longleftrightarrow & \text{four} & \text{momenta}
\end{array}
\;. \tag{2.24}
$$

Our proper basis follows from (2.24) by dropping the two tensor structures with largest number of momenta involved, i.e. the terms involving the Levi-Civita tensor are ruled out. Though such a condition is a useful guide to identify proper sets of basis tensors, this condition by itself is not always sufficient. For instance, in the decomposition (2.14) we could have allowed for another redundant structure

$$
\epsilon_{\bar{\mu}\mu[\bar{q}][q]}\,\gamma^\mu\,, \tag{2.25}
$$

which involves two momenta only. However, if we included the latter into our basis a kinematical singularity would arise. This illustrates that the condition of *the minimal number of momenta* is not sufficient to determine a complete set of Lorentz-Dirac structures that are free kinematical constraints. Nevertheless the number of momenta involved in a given basis tensor is a useful quantity to be observed in the construction of basis sets for systems with non-trivial spins.

We turn to the scattering of spin-one bosons off spin-one-half fermions. There exist eighteen independent on-shell invariant amplitudes by virtue of Eq. (2.9). Following the condition of the *minimal number of momenta* an over-complete decomposition of the scattering amplitudes may take the form

$$\begin{aligned}
T_{1\frac{1}{2}\to 1\frac{1}{2}}(\bar q, q, w) = \epsilon^{\dagger\bar\mu}(\bar q,\lambda_{\bar q})\,\bar u(\bar p,\lambda_{\bar p}) \Big[ & G_1\, g_{\bar\mu\mu} + G_2\,\gamma_{\bar\mu}\,\gamma_\mu + G_3\,\gamma_{\bar\mu}\,w_\mu + G_4\,w_{\bar\mu}\,\gamma_\mu + G_5\,\gamma_{\bar\mu}\,\bar q_\mu + G_6\,q_{\bar\mu}\,\gamma_\mu \\
& + G_7\,w_{\bar\mu}\,\bar q_\mu + G_8\,q_{\bar\mu}\,w_\mu + G_9\,w_{\bar\mu}\,w_\mu + G_{10}\,q_{\bar\mu}\,\bar q_\mu + G_{11}\,\epsilon_{\bar\mu\mu[\bar q][q]}\,i\,\gamma_5 + G_{12}\,\epsilon_{\bar\mu\mu[\bar q][w]}\,i\,\gamma_5 + G_{13}\,\epsilon_{\bar\mu\mu[q][w]}\,i\,\gamma_5 \\
& + G_{14}\,g_{\bar\mu\mu}\,\not{w} + G_{15}\,\gamma_{\bar\mu}\,\not{w}\,\gamma_\mu + G_{16}\,\gamma_{\bar\mu}\,\not{w}\,w_\mu + G_{17}\,w_{\bar\mu}\,\not{w}\,\gamma_\mu + G_{18}\,\gamma_{\bar\mu}\,\not{w}\,\bar q_\mu + G_{19}\,q_{\bar\mu}\,\not{w}\,\gamma_\mu + G_{20}\,w_{\bar\mu}\,\not{w}\,\bar q_\mu \\
& + G_{21}\,q_{\bar\mu}\,\not{w}\,w_\mu + G_{22}\,w_{\bar\mu}\,\not{w}\,w_\mu + G_{23}\,q_{\bar\mu}\,\not{w}\,\bar q_\mu + G_{24}\,\epsilon_{\bar\mu\mu[\bar q][q]}\,\not{w}\,i\,\gamma_5 + G_{25}\,\epsilon_{\bar\mu\mu[\bar q][w]}\,\not{w}\,i\,\gamma_5 \\
& + G_{26}\,\epsilon_{\bar\mu\mu[q][w]}\,\not{w}\,i\,\gamma_5 \Big]\, u(p,\lambda_p)\,\epsilon^\mu(q,\lambda_q)\,,
\end{aligned} \tag{2.26}$$

where $g_{\bar\mu\mu}$ is the metric tensor, $\epsilon^\mu(q,\lambda_q)$ denotes an incoming vector meson wave function with its helicity projection $\lambda_q$, and $\epsilon_{\bar\mu\mu[a][b]}$ follows the convention in Eq. (2.15). In a first step we characterize the various structures according to the number of momenta involved

$$\begin{array}{llll}
G_{1,2} & \longleftrightarrow & \text{zero} & \text{momentum} \\
G_{3-6,14-15} & \longleftrightarrow & \text{one} & \text{momentum} \\
G_{7-13,16-19} & \longleftrightarrow & \text{two} & \text{momenta} \\
G_{20-26} & \longleftrightarrow & \text{three} & \text{momenta}
\end{array}\,. \tag{2.27}$$

A proper basis should be obtainable by eliminating eight out of the 26 terms in Eq. (2.26). Initially one may be tempted to drop at least the 7 structures $G_{20-26}$ involving 3 momenta. However, tedious algebraic computations reveal that this strategy fails and does not lead to any basis. The remaining terms are linear dependent. Instead we found that it is useful to eliminate *all* structures involving the Levi-Civita tensor, i.e. the terms with $G_{11-13,24-26}$ even though some of them involve only two momenta. If a basis includes the first structure $g_{\bar\mu\mu}$ any of the terms with a Levi-Civita tensor can not be part of such a basis. Again a linear dependence would arise. A basis that excludes $G_{1,14}$ but keeps for instance $G_{11,24}$ generates kinematical singularities at

$$s = \frac{\pm\bar m^2 m - \bar m m^2 + \bar m M^2 \mp \bar M^2 m}{\bar m \mp m}\,. \tag{2.28}$$

A complete and almost proper basis follows upon the elimination of the terms involving a Levi-Civita tensor together with the two structures with $G_{10}$ and $G_{23}$. If we eliminated instead $G_1$ and $G_{10}$ a kinematical singularity at $v^2 = 0$ would arise. We exemplify the on-shell decompositions of the tensors, which we consider redundant:

$$\begin{aligned}
\epsilon_{\bar\mu\mu[\bar q][q]}\,i\,\gamma_5 \overset{\text{on-shell}}{=} g_{\bar\mu\mu} & \left[\frac{s(w\cdot\bar q)+s(w\cdot q)-2(w\cdot\bar q)(w\cdot q)}{s}\right] + \gamma_{\bar\mu}\,\gamma_\mu\left[-\frac{s(w\cdot\bar q)+s(w\cdot q)-2(w\cdot\bar q)(w\cdot q)}{s}\right] \\
& + \gamma_{\bar\mu}\,w_\mu\left[-\frac{-\bar M s + M s - 2M(w\cdot\bar q)}{s}\right] + w_{\bar\mu}\,\gamma_\mu\left[\frac{-\bar M s + 2\bar M(w\cdot q) + M s}{s}\right] \\
& + w_{\bar\mu}\,\bar q_\mu\left[-\frac{s-2(w\cdot q)}{s}\right] + q_{\bar\mu}\,w_\mu\left[-\frac{s-2(w\cdot\bar q)}{s}\right] + w_{\bar\mu}\,w_\mu\left[-\frac{2\left(-\bar M M + (\bar q\cdot q) + s\right)}{s}\right] \\
& + g_{\bar\mu\mu}\,\not{w}\left[-\frac{\bar M(w\cdot q)+M(w\cdot\bar q)}{s}\right] + \gamma_{\bar\mu}\,\not{w}\,\gamma_\mu\left[-\frac{\bar M(w\cdot q)+M(w\cdot\bar q)}{s}\right] \\
& + \gamma_{\bar\mu}\,\not{w}\,w_\mu\left[\frac{-\bar M M + (\bar q\cdot q)+s}{s}\right] + w_{\bar\mu}\,\not{w}\,\gamma_\mu\left[\frac{-\bar M M + (\bar q\cdot q)+s}{s}\right] \\
& + \gamma_{\bar\mu}\,\not{w}\,\bar q_\mu\left[-\frac{(w\cdot q)}{s}\right] + q_{\bar\mu}\,\not{w}\,\gamma_\mu\left[-\frac{(w\cdot\bar q)}{s}\right] + w_{\bar\mu}\,\not{w}\,\bar q_\mu\left[\frac{M}{s}\right] + q_{\bar\mu}\,\not{w}\,w_\mu\left[\frac{\bar M}{s}\right]\,,
\end{aligned} \tag{2.29}$$

and

$$
\begin{aligned}
q_{\bar\mu}\,\bar q_\mu \overset{\text{on-shell}}{=} g_{\bar\mu\mu} &\left[-\frac{\bar M M s-(\bar q\cdot q)s+s^2-s(w\cdot\bar q)-s(w\cdot q)+2(w\cdot\bar q)(w\cdot q)}{s}\right] \\
&+\gamma_{\bar\mu}\gamma_\mu\left[\frac{\bar M M s-(\bar q\cdot q)s+s^2-s(w\cdot\bar q)-s(w\cdot q)+2(w\cdot\bar q)(w\cdot q)}{s}\right] \\
&+\gamma_{\bar\mu}w_\mu\left[-\frac{\bar M s+Ms-2M(w\cdot\bar q)}{s}\right]+w_{\bar\mu}\gamma_\mu\left[-\frac{\bar M s-2\bar M(w\cdot q)+Ms}{s}\right]+\gamma_{\bar\mu}\bar q_\mu\left[-M\right]+q_{\bar\mu}\gamma_\mu\left[-\bar M\right] \\
&+w_{\bar\mu}\bar q_\mu\left[-\frac{s-2(w\cdot q)}{s}\right]+q_{\bar\mu}w_\mu\left[-\frac{s-2(w\cdot\bar q)}{s}\right]+w_{\bar\mu}w_\mu\left[\frac{2\left(\bar M M-(\bar q\cdot q)+s\right)}{s}\right] \\
&+g_{\bar\mu\mu}\not{w}\left[\frac{\bar M s-\bar M(w\cdot q)+Ms-M(w\cdot\bar q)}{s}\right]+\gamma_{\bar\mu}\not{w}\gamma_\mu\left[\frac{\bar M s-\bar M(w\cdot q)+Ms-M(w\cdot\bar q)}{s}\right] \\
&+\gamma_{\bar\mu}\not{w}w_\mu\left[-\frac{\bar M M-(\bar q\cdot q)+s}{s}\right]+w_{\bar\mu}\not{w}\gamma_\mu\left[-\frac{\bar M M-(\bar q\cdot q)+s}{s}\right] \\
&+\gamma_{\bar\mu}\not{w}\bar q_\mu\left[\frac{s-(w\cdot q)}{s}\right]+q_{\bar\mu}\not{w}\gamma_\mu\left[\frac{s-(w\cdot\bar q)}{s}\right]+w_{\bar\mu}\not{w}\bar q_\mu\left[\frac{M}{s}\right]+q_{\bar\mu}\not{w}w_\mu\left[\frac{\bar M}{s}\right],
\end{aligned}
\tag{2.30}
$$

where in both cases the expansion coefficients are regular with the exception of the particular point $s=0$.

We did not succeed in the construction of a basis that avoids a kinematical singularity at $s=0$. This implies that the invariant amplitudes $G_n(s,t)$ are correlated at that specific point $s=0$. In the following section we study the so-called MacDowell symmetry, which will provide a further powerful construction guide for the identification of proper basis tensors and singles out the particular point $s=0$. This will turn instrumental for the more complicated cases involving spin there-half fermions.

## 2.2 MacDowell symmetry

Let us return to the simplest case of the $0\,\frac{1}{2}\to 0\,\frac{1}{2}$ scattering process as described already in Eq. (2.11). There, the only Dirac structure $\not{w}$ is involved. In order to decompose the scattering amplitude into invariants we may have used alternative Dirac structure like ($\not{\bar q}$ or $\not{q}$) that are on-shell equivalent to our choice. At first there is no theoretical preference which Dirac structure to use. However, the implementation of the *MacDowell symmetry* will set a specific preference. In order to unravel that kinematical symmetry we introduce the following convenient projection matrices

$$
P_\pm=\frac{1}{2\sqrt{s}}\left(\sqrt{s}\pm\not{w}\right),\qquad P_\pm P_\pm=P_\pm\,,\qquad P_\pm P_\mp=0\,, \tag{2.31}
$$

in terms of the Dirac matrices used in the expansion of the scattering amplitude (2.11). We find

$$
T_{0\frac{1}{2}\to 0\frac{1}{2}}(\bar q,q,w)=\bar u(\bar p,\lambda_{\bar p})\left[F_1^+(\sqrt{s},t)P_+ + F_1^-(\sqrt{s},t)P_-\right]u(p,\lambda_p)\,, \tag{2.32}
$$

where the invariant amplitudes $F_1^\pm(\sqrt{s},t)$ can be expressed in terms of the previous $G_n(s,t)$ as

$$
F_1^\pm(\sqrt{s},t)=G_1(s,t)\pm\sqrt{s}\,G_2(s,t)\,. \tag{2.33}
$$

As a consequence the amplitude $F^-(\sqrt{s},t)$ can be computed directly from the amplitude $F^+(\sqrt{s},t)$. More specifically it holds:

$$
F_1^-(+\sqrt{s},t)=F_1^+(-\sqrt{s},t)\,. \tag{2.34}
$$

Similar relations were first introduced by MacDowell [68] for partial-wave amplitudes in pion-nucleon scattering and more recently for nucleon-Compton scattering and pion photoproduction in [45]. The *MacDowell symmetry* is useful since it reduces the number of independent invariant amplitudes by half. Furthermore it provides a useful and convenient guide how to identify proper sets of basis tensors. A further advantage will arise later in Chapter 4 when deriving partial-wave amplitudes. Typically the use of invariant functions subject to the MacDowell relation (2.34) leads to more concise and transparent expressions.

We continue by returning to the $0\,\frac{1}{2} \to 1\,\frac{1}{2}$ reaction. A basis set of tensors reflecting the *MacDowell symmetry* is readily identified

$$T_{0\,\frac{1}{2}\to 1\,\frac{1}{2}}(\bar{q}, q, w) = \epsilon^{\dagger\bar{\mu}}(\bar{q},\lambda_{\bar{q}})\,\bar{u}(\bar{p},\lambda_{\bar{p}}) \sum_{\pm} \Big[ G_1^{\pm}\,\gamma_{\bar{\mu}}\,P_{\pm}\,i\,\gamma_5 + G_2^{\pm}\,w_{\bar{\mu}}\,P_{\pm}\,i\,\gamma_5 + G_3^{\pm}\,q_{\bar{\mu}}\,P_{\pm}\,i\,\gamma_5 \Big]\, u(p,\lambda_p)\,, \tag{2.35}$$

where $G_n^{\pm}(\sqrt{s},t)$ can be expressed in terms of the $G_n(s,t)$ introduced in Eq. (2.14) as

$$G_n^{\pm}(\sqrt{s},t) = G_n^{\mp}(-\sqrt{s},t) = G_n(s,t) \pm \sqrt{s}\,G_{n+4}(s,t) \qquad (\text{for } n = 1,2,3)\,. \tag{2.36}$$

The invariant amplitudes $G_n^{\pm}(\sqrt{s},t)$ are free of kinematical constraints except at $s = 0$. We observe that our result (2.35) follows directly if the condition of *the minimal number of momenta* is combined with the request of an explicit realization of the *MacDowell relations*. Note that even though this combined condition will turn useful in the process of identifying proper basis sets for the more complicated spin systems, it still will turn out to be an insufficient condition.

We proceed by reinvestigating the $1\,\frac{1}{2} \to 1\,\frac{1}{2}$ scattering process. A set of suitable basis tensors that is reflecting the *MacDowell relations* is obtained with:

$$\begin{aligned} T_{1\,\frac{1}{2}\to 1\,\frac{1}{2}}(\bar{q}, q, w) = \epsilon^{\dagger\bar{\mu}}(\bar{q},\lambda_{\bar{q}})\,\bar{u}(\bar{p},\lambda_{\bar{p}}) \sum_{\pm} \Big[ & G_1^{\pm}\,g_{\bar{\mu}\mu}\,P_{\pm} + G_2^{\pm}\,\gamma_{\bar{\mu}}\,P_{\pm}\,\gamma_{\mu} + G_3^{\pm}\,\gamma_{\bar{\mu}}\,P_{\pm}\,w_{\mu} + G_4^{\pm}\,w_{\bar{\mu}}\,P_{\pm}\,\gamma_{\mu} + G_5^{\pm}\,\gamma_{\bar{\mu}}\,P_{\pm}\,\bar{q}_{\mu} \\ & + G_6^{\pm}\,q_{\bar{\mu}}\,P_{\pm}\,\gamma_{\mu} + G_7^{\pm}\,w_{\bar{\mu}}\,P_{\pm}\,\bar{q}_{\mu} + G_8^{\pm}\,q_{\bar{\mu}}\,P_{\pm}\,w_{\mu} + G_9^{\pm}\,w_{\bar{\mu}}\,P_{\pm}\,w_{\mu} \Big]\, u(p,\lambda_p)\,\epsilon^{\mu}(q,\lambda_q)\,. \end{aligned} \tag{2.37}$$

Like in Eq. (2.26) tensors involving a Levi-Civita tensor are eliminated. The presence of the structure $q_{\bar{\mu}}\,P_{\pm}\,\bar{q}_{\mu}$ or any other tensors involving more than two momenta would lead to kinematical singularities. The invariant functions $G_n^{\pm}(\sqrt{s},t)$ can be expressed in terms of the $G_n(s,t)$ introduced in Eq. (2.26) as

$$G_n^{\pm}(\sqrt{s},t) = G_n^{\mp}(-\sqrt{s},t) = G_n(s,t) \pm \sqrt{s}\,G_{n+13}(s,t) \qquad (\text{for } n = 1,2,\cdots,9)\,. \tag{2.38}$$

Before detailing our final results for the decomposition of the scattering amplitudes of interest in this work we introduce yet a further notation, which will simplify the algebraic computation of the invariant amplitudes significantly (see Chapter 3). We introduce vectors

$$\begin{aligned} r_{\mu} &= k_{\mu} - \frac{1}{2}\,\frac{q^2 - p^2}{s}\,w_{\mu} = q_{\mu} - \frac{(q\cdot w)}{s} w_{\mu}\,, \\ \bar{r}_{\mu} &= \bar{k}_{\mu} - \frac{1}{2}\,\frac{\bar{q}^2 - \bar{p}^2}{s}\,w_{\mu} = \bar{q}_{\mu} - \frac{(\bar{q}\cdot w)}{s} w_{\mu}\,, \\ \hat{\gamma}_{\mu} &= \gamma_{\mu} - \frac{1}{s}\not{w}\,w_{\mu}\,, \qquad \hat{\gamma}\cdot w = 0 = w\cdot\hat{\gamma}\,, \qquad \bar{r}\cdot w = 0 = w\cdot r\,, \end{aligned} \tag{2.39}$$

which are orthogonal to $w_{\mu}$. The momenta $k_{\mu}$ and $\bar{k}_{\mu}$ were already introduced in Eq. (2.3). Similarly we will use the transverse tensor

$$\hat{g}_{\mu\nu} = g_{\mu\nu} - \frac{w_{\mu}\,w_{\nu}}{s}\,, \qquad \hat{g}_{\mu\nu}\,w^{\nu} = 0 = w^{\mu}\,\hat{g}_{\mu\nu}\,. \tag{2.40}$$

Note that the use of the orthogonal vectors $\bar{r}_{\mu}$, $r_{\mu}$, and $\hat{\gamma}_{\mu}$ instead of the vectors $\bar{q}_{\mu}$, $q_{\mu}$, and $\gamma_{\mu}$ amounts to a simple rearrangement of the tensors introduced in Eq. (2.35) and Eq. (2.37). The amplitudes introduced with respect to the orthogonal vectors we denote by $F_n^{\pm}(\sqrt{s},t)$ as compared to our previous amplitude $G_n^{\pm}(\sqrt{s},t)$. The *MacDowell relation* will hold with

$$F_n^-(+\sqrt{s},t) = F_n^+(-\sqrt{s},t)\,. \tag{2.41}$$

More specifically we obtain for the vector meson production

$$\begin{aligned}
&T_{0\,\frac{1}{2}\to 1\,\frac{1}{2}}(\bar q,q,w) = \sum_{\pm,n} F_n^{\pm}(\sqrt{s},t)\,\Big[\epsilon^{\dagger\bar\mu}(\bar q,\lambda_{\bar q})\,\bar u(\bar p,\lambda_{\bar p})\,T^{(n)}_{\pm,\bar\mu}\,u(p,\lambda_p)\Big]\,,\\
&T^{(1)}_{\pm,\bar\mu} = \hat\gamma_{\bar\mu}\,P_\pm\,i\,\gamma_5\,, \qquad T^{(2)}_{\pm,\bar\mu} = w_{\bar\mu}\,P_\pm\,i\,\gamma_5\,, \qquad T^{(3)}_{\pm,\bar\mu} = r_{\bar\mu}\,P_\pm\,i\,\gamma_5\,,
\end{aligned} \tag{2.42}$$

where the invariant amplitudes $F_n^{\pm}(\sqrt{s},t)$ are free of kinematical singularities and relate to the $G_n^{\pm}$ introduced in Eq. (2.35) as

$$F_1^{\pm} = G_1^{\pm}\,, \qquad F_2^{\pm} = G_2^{\pm} \pm \frac{G_1^{\pm}}{\sqrt{s}} + \frac{(w\cdot q)\,G_3^{\pm}}{s}\,, \qquad F_3^{\pm} = G_3^{\pm}\,. \tag{2.43}$$

Similarly the on-shell scattering of spin-one off spin-one-half states can be described in the same compact manner as

$$\begin{aligned}
&T_{1\,\frac{1}{2}\to 1\,\frac{1}{2}}(\bar q,q,w) = \sum_{\pm,n} F_n^{\pm}(\sqrt{s},t)\,\Big[\epsilon^{\dagger\bar\mu}(\bar q,\lambda_{\bar q})\,\bar u(\bar p,\lambda_{\bar p})\,T^{(n)}_{\pm,\bar\mu\mu}\,u(p,\lambda_p)\,\epsilon^{\mu}(q,\lambda_q)\Big]\,,\\
&T^{(1)}_{\pm,\bar\mu\mu} = \hat g_{\bar\mu\mu}\,P_\pm\,, \qquad T^{(2)}_{\pm,\bar\mu\mu} = \hat\gamma_{\bar\mu}\,P_\pm\,\hat\gamma_\mu\,, \qquad T^{(3)}_{\pm,\bar\mu\mu} = \hat\gamma_{\bar\mu}\,P_\pm\,w_\mu\,,\\
&T^{(4)}_{\pm,\bar\mu\mu} = w_{\bar\mu}\,P_\pm\,\hat\gamma_\mu\,, \qquad T^{(5)}_{\pm,\bar\mu\mu} = \hat\gamma_{\bar\mu}\,P_\pm\,\bar r_\mu\,, \qquad T^{(6)}_{\pm,\bar\mu\mu} = r_{\bar\mu}\,P_\pm\,\hat\gamma_\mu\,,\\
&T^{(7)}_{\pm,\bar\mu\mu} = w_{\bar\mu}\,P_\pm\,\bar r_\mu\,, \qquad T^{(8)}_{\pm,\bar\mu\mu} = r_{\bar\mu}\,P_\pm\,w_\mu\,, \qquad T^{(9)}_{\pm,\bar\mu\mu} = w_{\bar\mu}\,P_\pm\,w_\mu\,,
\end{aligned} \tag{2.44}$$

where the kinematically unconstrained $F_n^{\pm}(\sqrt{s},t)$ amplitudes can be expressed in terms of the $G_n^{\pm}(\sqrt{s},t)$ amplitudes introduced in Eq. (2.37) as

$$\begin{aligned}
&F_1^{\pm} = G_1^{\pm}\,, \quad F_2^{\pm} = G_2^{\pm}\,, \quad F_3^{\pm} = G_3^{\pm} \pm \frac{G_2^{\pm}}{\sqrt{s}} + \frac{(w\cdot\bar q)\,G_5^{\pm}}{s}\,, \quad F_4^{\pm} = G_4^{\pm} \pm \frac{G_2^{\pm}}{\sqrt{s}} + \frac{(w\cdot q)\,G_6^{\pm}}{s}\,,\\
&F_5^{\pm} = G_5^{\pm}\,, \quad F_6^{\pm} = G_6^{\pm}\,, \quad F_7^{\pm} = G_7^{\pm} \pm \frac{G_5^{\pm}}{\sqrt{s}}\,, \quad F_8^{\pm} = G_8^{\pm} \pm \frac{G_6^{\pm}}{\sqrt{s}}\,,\\
&F_9^{\pm} = G_9^{\pm} \pm \frac{G_3^{\pm}+G_4^{\pm}}{\sqrt{s}} + \frac{G_1^{\pm}+G_2^{\pm}+(w\cdot\bar q)\,G_7^{\pm}+(w\cdot q)\,G_8^{\pm}}{s} \pm \frac{(w\cdot\bar q)\,G_5^{\pm}+(w\cdot q)\,G_6^{\pm}}{s\sqrt{s}}\,.
\end{aligned} \tag{2.45}$$

The basis sets given in (2.32), (2.42), and (2.44) are all free of kinematical constraints with the exception of the particular point $s=0$. For a given reaction an arbitrary Lorentz-Dirac structure can be expressed as a linear combination of our basis tensors, where all coefficients are regular at $s\neq 0$. For the $0\,\frac{1}{2}\to 1\,\frac{1}{2}$ reaction we illustrate this important property by three examples. We consider the tensor $\epsilon_{\bar\mu\mu\alpha\beta}\,\gamma^\mu$ contracted with two momenta and expand it into our basis tensors

$$\begin{aligned}
&\sqrt{s}\,\epsilon_{\bar\mu\mu[\bar r][r]}\,\gamma^\mu\,P_\pm \overset{\text{on-shell}}{=} -\sqrt{s}\,\bar E_\pm\,E_\pm\,T^{(1)}_{\pm,\bar\mu} + \sqrt{s}\,(\bar r\cdot r)\,T^{(1)}_{\mp,\bar\mu} - (\bar M\mp\sqrt{s})\,E_\pm\,T^{(2)}_{\pm,\bar\mu} \mp (\bar r\cdot r)\,T^{(2)}_{\mp,\bar\mu} \pm \sqrt{s}\,\bar E_\pm\,T^{(3)}_{\mp,\bar\mu}\,,\\
&\frac{1}{\sqrt{s}}\,\epsilon_{\bar\mu\mu[w][r]}\,\gamma^\mu\,P_\pm \overset{\text{on-shell}}{=} -E_\pm\,T^{(1)}_{\pm,\bar\mu} \pm T^{(3)}_{\mp,\bar\mu}\,, \qquad \frac{1}{\sqrt{s}}\,\epsilon_{\bar\mu\mu[\bar r][w]}\,\gamma^\mu\,P_\pm \overset{\text{on-shell}}{=} -\bar E_\mp\,T^{(1)}_{\mp,\bar\mu} \mp \tfrac{1}{2}\,(\bar\delta+1)\,T^{(2)}_{\mp,\bar\mu}\,,
\end{aligned} \tag{2.46}$$

where

$$\begin{aligned}
E_\pm &= \frac{\sqrt{s}}{2}\,(1-\delta)\pm M\,, \qquad & \delta &= \frac{m^2-M^2}{s}\,,\\
\bar E_\pm &= \frac{\sqrt{s}}{2}\,(1-\bar\delta)\pm \bar M\,, \qquad & \bar\delta &= \frac{\bar m^2-\bar M^2}{s}\,.
\end{aligned} \tag{2.47}$$

For the $1\,\frac{1}{2} \to 1\,\frac{1}{2}$ scattering process we provide one further decomposition example. A tensor consisting of two $v_\mu$ vectors as introduced in Eq. (2.15) can be decomposed into our basis tensors (2.44). We find

$$
\begin{aligned}
\frac{1}{s}\, v_{\bar\mu}\, P_\pm\, v_\mu \overset{\text{on-shell}}{=} & (\bar r\cdot r)\,\bar E_\pm\, E_\pm\, T^{(1)}_{\mp,\bar\mu\mu} - \bar E_\mp\,\bar E_\pm\, E_\mp\, E_\pm\, T^{(1)}_{\pm,\bar\mu\mu} + (\bar r\cdot r)^2\, T^{(2)}_{\mp,\bar\mu\mu} - (\bar r\cdot r)\,\bar E_\pm\, E_\pm\, T^{(2)}_{\pm,\bar\mu\mu} \\
& \pm \tfrac{1}{2}(\delta+1)(\bar r\cdot r)\,\bar E_\pm\, T^{(3)}_{\pm,\bar\mu\mu} \pm \tfrac{1}{2}(\bar\delta+1)(\bar r\cdot r)\,E_\pm\, T^{(4)}_{\pm,\bar\mu\mu} \pm (\bar r\cdot r)\,E_\pm\, T^{(5)}_{\mp,\bar\mu\mu} \pm (\bar r\cdot r)\,\bar E_\pm\, T^{(6)}_{\mp,\bar\mu\mu} \\
& + \tfrac{1}{2}(\bar\delta+1)\,E_\mp\, E_\pm\, T^{(7)}_{\pm,\bar\mu\mu} + \tfrac{1}{2}(\delta+1)\,\bar E_\mp\,\bar E_\pm\, T^{(8)}_{\pm,\bar\mu\mu} - \tfrac{1}{2}(\bar\delta+1)(\delta+1)(\bar r\cdot r)\, T^{(9)}_{\pm,\bar\mu\mu}\,,
\end{aligned}
\tag{2.48}
$$

with expansion coefficients regular with the possible exception at $s = 0$.

We proceed with reactions involving spin-three-half fermions. Here we refrain from detailing the in part quite tedious derivations and restrict ourself to the presentation of the final basis sets we established. Like before we constructed basis tensors in terms of the orthogonal vectors $\bar r_\mu$, $r_\mu$, and $\hat\gamma_\mu$ as introduced in Eq. (2.39). As emphasized this choice will simplify the algebraic computation of the invariant amplitudes (see Chapter 3) and lead to more concise expressions for the partial-wave amplitudes (see Chapter 4).

The simplest reaction is a spin-three-half production process. In analogy to Eq. (2.42) its on-shell scattering amplitude can be decomposed as

$$
\begin{aligned}
& T_{0\,\frac{1}{2}\to 0\,\frac{3}{2}}(\bar q, q, w) = \sum_{\pm,n} F^\pm_n(\sqrt{s}, t)\, \Big[\bar u^{\bar\nu}(\bar p, \lambda_{\bar p})\, T^{(n)}_{\pm,\bar\nu}\, u(p,\lambda_p)\Big]\,, \\
& T^{(1)}_{\pm,\bar\nu} = w_{\bar\nu}\, P_\pm\, i\,\gamma_5\,, \qquad T^{(2)}_{\pm,\bar\nu} = r_{\bar\nu}\, P_\pm\, i\,\gamma_5\,,
\end{aligned}
\tag{2.49}
$$

with $F^\pm_n(\sqrt{s}, t)$ free of kinematical constraints at $s \neq 0$. In Eq. (2.49) the wave function of the outgoing fermion state with its helicity projection $\lambda_{\bar p}$ is denoted by $\bar u^{\bar\nu}(\bar p, \lambda_{\bar p})$. The on-shell conditions of $\bar u^{\bar\nu}(\bar p, \lambda_{\bar p})$ are given by Rarita and Schwinger [82] as

$$
\bar u^{\bar\nu}(\bar p, \lambda_{\bar p})(\bar{\not p} - \bar M) = 0\,, \qquad \bar u^{\bar\nu}(\bar p, \lambda_{\bar p})\, \bar p_{\bar\nu} = 0 = \bar u^{\bar\nu}(\bar p, \lambda_{\bar p})\, \gamma_{\bar\nu}\,.
\tag{2.50}
$$

In comparison to Eq. (2.42), the decomposition of the scattering amplitudes in the form of (2.49) can be directly obtained by dropping the term $\hat\gamma_{\bar\nu}\, P_\pm\, i\,\gamma_5$ in Eq. (2.42). By virtue of the on-shell conditions (2.50) this structure is linearly dependent on the two tensor structures kept in Eq. (2.49).

The remaining reactions of interest in this work are characterized by the number of Lorentz indices involved. There are three different cases with two open Lorentz indices. Their scattering amplitudes can all be decomposed by suitable adaptations of our previous result Eq. (2.44). The first two cases are treated by an elimination of the on-shell redundant tensor structures involving $\hat\gamma$. We find

$$
\begin{aligned}
& T_{1\,\frac{1}{2}\to 0\,\frac{3}{2}}(\bar q, q, w) = \sum_{\pm,n} F^\pm_n(\sqrt{s}, t)\, \Big[\bar u^{\bar\nu}(\bar p, \lambda_{\bar p})\, T^{(n)}_{\pm,\bar\nu\mu}\, u(p,\lambda_p)\, \epsilon^\mu(q,\lambda_q)\Big]\,, \\
& T^{(1)}_{\pm,\bar\nu\mu} = \hat g_{\bar\nu\mu}\, P_\pm\,, \qquad T^{(2)}_{\pm,\bar\nu\mu} = w_{\bar\nu}\, P_\pm\, \hat\gamma_\mu\,, \qquad T^{(3)}_{\pm,\bar\nu\mu} = r_{\bar\nu}\, P_\pm\, \hat\gamma_\mu\,, \\
& T^{(4)}_{\pm,\bar\nu\mu} = w_{\bar\nu}\, P_\pm\, \bar r_\mu\,, \qquad T^{(5)}_{\pm,\bar\nu\mu} = r_{\bar\nu}\, P_\pm\, w_\mu\,, \qquad T^{(6)}_{\pm,\bar\nu\mu} = w_{\bar\nu}\, P_\pm\, w_\mu\,,
\end{aligned}
\tag{2.51}
$$

$$
\begin{aligned}
& T_{0\,\frac{3}{2}\to 0\,\frac{3}{2}}(\bar q, q, w) = \sum_{\pm,n} F^\pm_n(\sqrt{s}, t)\, \Big[\bar u^{\bar\nu}(\bar p, \lambda_{\bar p})\, T^{(n)}_{\pm,\bar\nu\nu}\, u^\nu(p,\lambda_p)\Big]\,, \\
& T^{(1)}_{\pm,\bar\nu\nu} = \hat g_{\bar\nu\nu}\, P_\pm\,, \qquad T^{(2)}_{\pm,\bar\nu\nu} = w_{\bar\nu}\, P_\pm\, \bar r_\nu\,, \\
& T^{(3)}_{\pm,\bar\nu\nu} = r_{\bar\nu}\, P_\pm\, w_\nu\,, \qquad T^{(4)}_{\pm,\bar\nu\nu} = w_{\bar\nu}\, P_\pm\, w_\nu\,,
\end{aligned}
\tag{2.52}
$$

where $u^\nu(p, \lambda_p)$ denotes a wave function of an incoming baryon decuplet state with its helicity projection $\lambda_p$. The wave function satisfies the on-shell conditions

$$
(\not p - M)\, u^\nu(p,\lambda_p) = 0\,, \qquad p_\nu\, u^\nu(p,\lambda_p) = 0 = \gamma_\nu\, u^\nu(p,\lambda_p)\,.
\tag{2.53}
$$

The third case is more subtle and requires a replacement of one of the tensor structures. We find the decompostion

$$
\begin{aligned}
&T_{0\,\frac{1}{2}\to 1\,\frac{3}{2}}(\bar{q},q,w) = \sum_{\pm,n} F_n^{\pm}(\sqrt{s},t)\,\Big[\epsilon^{\dagger\bar{\mu}}(\bar{q},\lambda_{\bar{q}})\,\bar{u}^{\bar{\nu}}(\bar{p},\lambda_{\bar{p}})\,T^{(n)}_{\pm,\bar{\mu}\bar{\nu}}\,u(p,\lambda_p)\Big]\,,\\
&T^{(1)}_{\pm,\bar{\mu}\bar{\nu}} = \hat{g}_{\bar{\mu}\bar{\nu}}\,P_\pm\,, \qquad T^{(2)}_{\pm,\bar{\mu}\bar{\nu}} = \hat{\gamma}_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm\,, \qquad T^{(3)}_{\pm,\bar{\mu}\bar{\nu}} = \hat{\gamma}_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm\,,\\
&T^{(4)}_{\pm,\bar{\mu}\bar{\nu}} = w_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm\,, \qquad T^{(5)}_{\pm,\bar{\mu}\bar{\nu}} = w_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm\,, \qquad T^{(6)}_{\pm,\bar{\mu}\bar{\nu}} = r_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm\,,
\end{aligned}
\tag{2.54}
$$

where the tensor $r_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm$ appears rather than the tensor $r_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm$ suggested by Eq. (2.44). If $r_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm$ is chosen instead the consequent set would be linear dependent. Of course, the tensor $r_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm$ can be expressed as a linear combination of our basis tensors

$$
r_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm \stackrel{\text{on-shell}}{=} -\frac{(w\cdot\bar{q})}{s}\,T^{(4)}_{\pm,\bar{\mu}\bar{\nu}}\,,
\tag{2.55}
$$

with expansion coefficients regular at $s \neq 0$.

We are left with the three- and four-index cases. The task of finding a suitable set of amplitudes is more complicated and quite tedious. It suffices to construct Lorentz-Dirac tensors composed out of $P_\pm$ (2.31) and $w_\mu$, $\hat{\gamma}_\mu$, $\bar{r}_\mu$, $r_\mu$ (2.39), and $\hat{g}_{\mu\nu}$ (2.40). In fact we do not need to construct a suitable basis from scratch, but rather streamline the task by starting from our results involving only two open Lorentz indices.

The three index case involving two spin-one states requires the construction of thirty-six tensors. As a starting point we may use the twelve independent tensors with two Lorentz indices as established in in Eq. (2.54) properly extended to a third Lorentz index by $\hat{\gamma}_\mu\,i\gamma_5$, $w_\mu\,i\gamma_5$, and $\bar{r}_\mu\,i\gamma_5$. That leads to 36 tensor structures, but unfortunately not yet to a proper basis set. A second starting set of 36 tensors is obtained from (2.51) extended by $\hat{\gamma}_{\bar{\mu}}\,i\gamma_5$, $w_{\bar{\mu}}\,i\gamma_5$, and $r_{\bar{\mu}}\,i\gamma_5$. From those 72 two tensors we succeeded in finding a proper subset. We refrain from providing details of the tedious elimination procedure. As a result, we can express the on-shell scattering amplitude in terms of thirty-six invariant amplitudes

$$
\begin{aligned}
&T_{1\,\frac{1}{2}\to 1\,\frac{3}{2}}(\bar{q},q,w) = \sum_{\pm,n} F_n^{\pm}(\sqrt{s},t)\,\Big[\epsilon^{\dagger\bar{\mu}}(\bar{q},\lambda_{\bar{q}})\,\bar{u}^{\bar{\nu}}(\bar{p},\lambda_{\bar{p}})T^{(n)}_{\pm,\bar{\mu}\bar{\nu}\mu}\,u(p,\lambda_p)\,\epsilon^{\mu}(q,\lambda_q)\Big]\,,\\
&T^{(1)}_{\pm,\bar{\mu}\bar{\nu}\mu} = \hat{g}_{\bar{\mu}\bar{\nu}}\,P_\pm\,\hat{\gamma}_\mu\,i\gamma_5\,, \qquad T^{(2)}_{\pm,\bar{\mu}\bar{\nu}\mu} = \hat{\gamma}_{\bar{\mu}}\,\hat{g}_{\bar{\nu}\mu}\,P_\pm\,i\gamma_5\,, \qquad T^{(3)}_{\pm,\bar{\mu}\bar{\nu}\mu} = \hat{g}_{\bar{\mu}\bar{\nu}}\,P_\pm\,w_\mu\,i\gamma_5\,,\\
&T^{(4)}_{\pm,\bar{\mu}\bar{\nu}\mu} = w_{\bar{\mu}}\,\hat{g}_{\bar{\nu}\mu}\,P_\pm\,i\gamma_5\,, \qquad T^{(5)}_{\pm,\bar{\mu}\bar{\nu}\mu} = \hat{g}_{\bar{\mu}\bar{\nu}}\,P_\pm\,\bar{r}_\mu\,i\gamma_5\,, \qquad T^{(6)}_{\pm,\bar{\mu}\bar{\nu}\mu} = r_{\bar{\mu}}\,\hat{g}_{\bar{\nu}\mu}\,P_\pm\,i\gamma_5\,,\\
&T^{(7)}_{\pm,\bar{\mu}\bar{\nu}\mu} = \hat{\gamma}_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm\,\hat{\gamma}_\mu\,i\gamma_5\,, \qquad T^{(8)}_{\pm,\bar{\mu}\bar{\nu}\mu} = \hat{\gamma}_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm\,\hat{\gamma}_\mu\,i\gamma_5\,, \qquad T^{(9)}_{\pm,\bar{\mu}\bar{\nu}\mu} = \hat{\gamma}_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm\,\bar{r}_\mu\,i\gamma_5\,,\\
&T^{(10)}_{\pm,\bar{\mu}\bar{\nu}\mu} = w_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm\,\hat{\gamma}_\mu\,i\gamma_5\,, \qquad T^{(11)}_{\pm,\bar{\mu}\bar{\nu}\mu} = \hat{\gamma}_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm\,w_\mu\,i\gamma_5\,, \qquad T^{(12)}_{\pm,\bar{\mu}\bar{\nu}\mu} = w_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm\,\hat{\gamma}_\mu\,i\gamma_5\,,\\
&T^{(13)}_{\pm,\bar{\mu}\bar{\nu}\mu} = \hat{\gamma}_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm\,w_\mu\,i\gamma_5\,, \qquad T^{(14)}_{\pm,\bar{\mu}\bar{\nu}\mu} = r_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm\,\hat{\gamma}_\mu\,i\gamma_5\,, \qquad T^{(15)}_{\pm,\bar{\mu}\bar{\nu}\mu} = w_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm\,w_\mu\,i\gamma_5\,,\\
&T^{(16)}_{\pm,\bar{\mu}\bar{\nu}\mu} = w_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm\,\bar{r}_\mu\,i\gamma_5\,, \qquad T^{(17)}_{\pm,\bar{\mu}\bar{\nu}\mu} = w_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm\,w_\mu\,i\gamma_5\,, \qquad T^{(18)}_{\pm,\bar{\mu}\bar{\nu}\mu} = r_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm\,w_\mu\,i\gamma_5\,,
\end{aligned}
\tag{2.56}
$$

where the amplitudes $F_n^{\pm}(\sqrt{s},t)$ are free of kinematical constraints at $s \neq 0$.

Applying a similar strategy we managed to decompose the remaining three index amplitude. The starting points are the eight Lorentz-Dirac tensors introduced in Eq. (2.52) and the twelve tensors introduced in Eq. (2.54). We assure that the following decomposition of the on-shell reaction amplitude

$$
\begin{aligned}
&T_{0\,\frac{3}{2}\to 1\,\frac{3}{2}}(\bar{q},q,w) = \sum_{\pm,n} F_n^{\pm}(\sqrt{s},t)\,\Big[\epsilon^{\dagger\bar{\mu}}(\bar{q},\lambda_{\bar{q}})\,\bar{u}^{\bar{\nu}}(\bar{p},\lambda_{\bar{p}})T^{(n)}_{\pm,\bar{\mu}\bar{\nu}\nu}\,u^{\nu}(p,\lambda_p)\Big]\,,\\
&T^{(1)}_{\pm,\bar{\mu}\bar{\nu}\nu} = \hat{g}_{\bar{\nu}\nu}\,\hat{\gamma}_{\bar{\mu}}\,P_\pm\,i\gamma_5\,, \qquad T^{(2)}_{\pm,\bar{\mu}\bar{\nu}\nu} = \hat{g}_{\bar{\nu}\nu}\,w_{\bar{\mu}}\,P_\pm\,i\gamma_5\,, \qquad T^{(3)}_{\pm,\bar{\mu}\bar{\nu}\nu} = \hat{g}_{\bar{\nu}\nu}\,r_{\bar{\mu}}\,P_\pm\,i\gamma_5\,,\\
&T^{(4)}_{\pm,\bar{\mu}\bar{\nu}\nu} = \hat{g}_{\bar{\mu}\bar{\nu}}\,P_\pm\,w_\nu\,i\gamma_5\,, \qquad T^{(5)}_{\pm,\bar{\mu}\bar{\nu}\nu} = \hat{g}_{\bar{\mu}\bar{\nu}}\,P_\pm\,\bar{r}_\nu\,i\gamma_5\,, \qquad T^{(6)}_{\pm,\bar{\mu}\bar{\nu}\nu} = \hat{\gamma}_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm\,w_\nu\,i\gamma_5\,,\\
&T^{(7)}_{\pm,\bar{\mu}\bar{\nu}\nu} = \hat{\gamma}_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm\,\bar{r}_\nu\,i\gamma_5\,, \qquad T^{(8)}_{\pm,\bar{\mu}\bar{\nu}\nu} = \hat{\gamma}_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm\,w_\nu\,i\gamma_5\,, \qquad T^{(9)}_{\pm,\bar{\mu}\bar{\nu}\nu} = w_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm\,w_\nu\,i\gamma_5\,,\\
&T^{(10)}_{\pm,\bar{\mu}\bar{\nu}\nu} = w_{\bar{\mu}}\,w_{\bar{\nu}}\,P_\pm\,\bar{r}_\nu\,i\gamma_5\,, \qquad T^{(11)}_{\pm,\bar{\mu}\bar{\nu}\nu} = w_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm\,w_\nu\,i\gamma_5\,, \qquad T^{(12)}_{\pm,\bar{\mu}\bar{\nu}\nu} = r_{\bar{\mu}}\,r_{\bar{\nu}}\,P_\pm\,w_\nu\,i\gamma_5\,,
\end{aligned}
\tag{2.57}
$$

in terms of the twenty-four invariant functions $F_n^\pm(\sqrt{s},t)$ provides the desired representation void of kinematical constraints at $s\neq 0$.

The final and most complicated reaction of our interest is the scattering of spin-one bosons off spin-three-half fermions, where seventy-two independent Lorentz-Dirac tensors exist. To construct a suitable basis, we may partly use the basis sets of the reactions considered so far. For any candidates, it has to be assured that a set not only forms a basis, but also is free of kinematical constraints at $s\neq 0$. We parameterize the on-shell scattering amplitude in terms of the following seventy-two $F_n^\pm$ as

$$
\begin{aligned}
&T_{1\,\frac{3}{2}\to 1\,\frac{3}{2}}(\bar q, q, w) = \sum_{\pm,n} F_n^\pm(\sqrt{s},t)\left[\epsilon^{\dagger\bar\mu}(\bar q,\lambda_{\bar q})\,\bar u^{\bar\nu}(\bar p,\lambda_{\bar p})\,T^{(n)}_{\pm,\bar\mu\bar\nu\mu\nu}\,u^\nu(p,\lambda_p)\,\epsilon^\mu(q,\lambda_q)\right],\\
&T^{(1)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\mu\bar\nu}\,\hat g_{\mu\nu}\,P_\pm\,, \quad T^{(2)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\mu\mu}\,\hat g_{\bar\nu\nu}\,P_\pm\,, \quad T^{(3)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\nu\nu}\,\hat\gamma_{\bar\mu}\,P_\pm\,\hat\gamma_\mu\,,\\
&T^{(4)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\mu\bar\nu}\,P_\pm\,\hat\gamma_\mu\,w_\nu\,, \quad T^{(5)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\mu\nu}\,\hat\gamma_{\bar\mu}\,w_{\bar\nu}\,P_\pm\,, \quad T^{(6)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\mu\bar\nu}\,P_\pm\,\hat\gamma_\mu\,\bar r_\nu\,,\\
&T^{(7)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\mu\nu}\,\hat\gamma_{\bar\mu}\,r_{\bar\nu}\,P_\pm\,, \quad T^{(8)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\nu\nu}\,\hat\gamma_{\bar\mu}\,P_\pm\,w_\mu\,, \quad T^{(9)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\nu\nu}\,w_{\bar\mu}\,P_\pm\,\hat\gamma_\mu\,,\\
&T^{(10)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\nu\nu}\,\hat\gamma_{\bar\mu}\,P_\pm\,\bar r_\mu\,, \quad T^{(11)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\nu\nu}\,r_{\bar\mu}\,P_\pm\,\hat\gamma_\mu\,, \quad T^{(12)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\mu\bar\nu}\,P_\pm\,w_\mu\,w_\nu\,,\\
&T^{(13)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\mu\nu}\,w_{\bar\mu}\,w_{\bar\nu}\,P_\pm\,, \quad T^{(14)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\mu\bar\nu}\,P_\pm\,w_\mu\,\bar r_\nu\,, \quad T^{(15)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\mu\nu}\,w_{\bar\mu}\,r_{\bar\nu}\,P_\pm\,,\\
&T^{(16)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\mu\bar\nu}\,P_\pm\,\bar r_\mu\,\bar r_\nu\,, \quad T^{(17)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\mu\nu}\,r_{\bar\mu}\,r_{\bar\nu}\,P_\pm\,, \quad T^{(18)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\nu\nu}\,w_{\bar\mu}\,P_\pm\,w_\mu\,,\\
&T^{(19)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\nu\nu}\,w_{\bar\mu}\,P_\pm\,\bar r_\mu\,, \quad T^{(20)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat g_{\bar\nu\nu}\,r_{\bar\mu}\,P_\pm\,w_\mu\,, \quad T^{(21)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat\gamma_{\bar\mu}\,w_{\bar\nu}\,P_\pm\,\hat\gamma_\mu\,w_\nu\,,\\
&T^{(22)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat\gamma_{\bar\mu}\,w_{\bar\nu}\,P_\pm\,\hat\gamma_\mu\,\bar r_\nu\,, \quad T^{(23)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat\gamma_{\bar\mu}\,r_{\bar\nu}\,P_\pm\,\hat\gamma_\mu\,w_\nu\,, \quad T^{(24)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat\gamma_{\bar\mu}\,w_{\bar\nu}\,P_\pm\,w_\mu\,w_\nu\,,\\
&T^{(25)}_{\pm,\bar\mu\bar\nu\mu\nu} = w_{\bar\mu}\,w_{\bar\nu}\,P_\pm\,\hat\gamma_\mu\,w_\nu\,, \quad T^{(26)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat\gamma_{\bar\mu}\,w_{\bar\nu}\,P_\pm\,w_\mu\,\bar r_\nu\,, \quad T^{(27)}_{\pm,\bar\mu\bar\nu\mu\nu} = w_{\bar\mu}\,r_{\bar\nu}\,P_\pm\,\hat\gamma_\mu\,w_\nu\,,\\
&T^{(28)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat\gamma_{\bar\mu}\,w_{\bar\nu}\,P_\pm\,\bar r_\mu\,\bar r_\nu\,, \quad T^{(29)}_{\pm,\bar\mu\bar\nu\mu\nu} = r_{\bar\mu}\,r_{\bar\nu}\,P_\pm\,\hat\gamma_\mu\,w_\nu\,, \quad T^{(30)}_{\pm,\bar\mu\bar\nu\mu\nu} = \hat\gamma_{\bar\mu}\,r_{\bar\nu}\,P_\pm\,w_\mu\,w_\nu\,,\\
&T^{(31)}_{\pm,\bar\mu\bar\nu\mu\nu} = w_{\bar\mu}\,w_{\bar\nu}\,P_\pm\,\hat\gamma_\mu\,\bar r_\nu\,, \quad T^{(32)}_{\pm,\bar\mu\bar\nu\mu\nu} = w_{\bar\mu}\,w_{\bar\nu}\,P_\pm\,w_\mu\,w_\nu\,, \quad T^{(33)}_{\pm,\bar\mu\bar\nu\mu\nu} = w_{\bar\mu}\,w_{\bar\nu}\,P_\pm\,w_\mu\,\bar r_\nu\,,\\
&T^{(34)}_{\pm,\bar\mu\bar\nu\mu\nu} = w_{\bar\mu}\,r_{\bar\nu}\,P_\pm\,w_\mu\,w_\nu\,, \quad T^{(35)}_{\pm,\bar\mu\bar\nu\mu\nu} = w_{\bar\mu}\,w_{\bar\nu}\,P_\pm\,\bar r_\mu\,\bar r_\nu\,, \quad T^{(36)}_{\pm,\bar\mu\bar\nu\mu\nu} = r_{\bar\mu}\,r_{\bar\nu}\,P_\pm\,w_\mu\,w_\nu\,.
\end{aligned}
\tag{2.58}
$$

We summarize the main achievement of this chapter. Two-body reaction amplitudes involving either a spin-zero or spin-one state and either a spin-one-half or a spin-three-half state were decomposed into invariant amplitudes that are free of kinematical constraints at $s\neq 0$. The number of invariant amplitudes was reduced by a factor two systematically by invoking MacDowell type relations. Even or odd combinations of those amplitudes are expected to satisfy Mandelstam's dispersion integral representation (2.10). Our results are an indispensable prerequisite to establish convenient analytic properties of partial-wave amplitudes. Before studying the partial-wave amplitudes in Chapter 4 we will establish in the following Chapter 3 a convenient projection formalism in terms of which the algebraic derivation of the invariant amplitudes from a given Lagrangian is streamlined considerably.

# 3 Projection algebras

In the previous chapter we studied two-body systems of a boson with $J^P = 0^-, 1^-$ and a fermion with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$. Assuming conserved parity the on-shell scattering and reaction amplitudes were decomposed into Lorentz invariant amplitudes $F_n^{\pm}(\sqrt{s}, t)$ that are free of kinematical constraints at $s \neq 0$ and satisfy the *MacDowell relation* (2.41). Their analytic properties are governed by Mandelstam's dispersion integral representation (2.10).

In a practical application it is important to derive explicit expressions for the invariant amplitudes $F_n^{\pm}(\sqrt{s}, t)$. Though for systems with small spin such derivations can be performed by hand, this turns more and more tedious as the total spin increases. A computer algebra code for their systematic computations is highly desirable. For that purpose a derivation of a suitable projection scheme is useful. In a recent work by M.F.M. Lutz and I. Vidana [50] such a scheme was established for various boson-boson systems. In the following, we shall derive a projection algebra for boson-fermion systems. It takes the universal form

$$\frac{1}{2}\,\mathbf{tr}\big(T^{(n)}_{a,\mu\nu\cdots}\,\Lambda\,Q^{\mu\nu\cdots}_{b,k}\,\bar\Lambda\big) = \delta_{nk}\,\delta_{ab}\,, \qquad \Lambda = \not{p} + M\,, \quad \bar\Lambda = \bar{\not{p}} + \bar M\,, \tag{3.1}$$

where $T^{(n)}_{\pm,\mu\nu\cdots}$ is our basis set of Lorentz-Dirac tensors introduced in the previous chapter for the various reactions. The tensors $Q^{\mu\nu\cdots}_{\pm,k}$ denote suitable projection tensors and need to be derived in this chapter. The indices $n$ or $k$ number through the set of invariant amplitudes as determined by (2.9). The Dirac matrices $\Lambda$ and $\bar\Lambda$ are naturally implied by the on-shell conditions of Dirac spinors as recalled in Eq. (2.12). The tensors $Q^{\mu\nu\cdots}_{\pm,k}$ are subject to further constraints that reflect the particular structures of the spin-one and spin-three-half wave functions (see Eq. (2.17) and Eqs. (2.50) and (2.53)). For a given reaction, the invariant functions $F_n^{\pm}(\sqrt{s}, t)$ can be computed directly in application of the n-th projection tensor $Q^{\mu\nu\cdots}_{\pm,n}$.

## 3.1 Spin-one-half fermion

In this section we study boson-fermion systems with spin-one-half fermions only. The simplest case is the scattering of a pseudoscalar boson off a spin-one-half fermion. Its on-shell scattering amplitude is decomposed in Eq. (2.32) in terms of the two Dirac structures $P_\pm$ introduced in Eq. (2.31) only. The projection tensors $Q_\pm$ of Eq. (3.1) are readily derived. The invariant amplitudes $F_1^{\pm}(\sqrt{s}, t)$ can be computed in application of the following projection algebra

$$\frac{1}{2}\,\mathbf{tr}\big(P_\pm\,\Lambda\,Q_\pm\,\bar\Lambda\big) = 1\,, \qquad \frac{1}{2}\,\mathbf{tr}\big(P_\pm\,\Lambda\,Q_\mp\,\bar\Lambda\big) = 0\,, \tag{3.2}$$

with

$$Q_\pm = \frac{s}{v^2}\Big((\bar r\cdot r)\,P_\mp - \bar E_\mp\,E_\mp\,P_\pm\Big)\,, \qquad v_\mu = \epsilon_{\mu\alpha\gamma\beta}\,\bar r^\alpha w^\gamma r^\beta\,, \tag{3.3}$$

where the momenta $r_\mu$ and $\bar r_\mu$ were defined in Eq. (2.39). They are orthogonal to $w_\mu$. Furthermore we recall that $E_\pm$ and $\bar E_\pm$ are given in Eq. (2.47) in terms of the Mandelstam variable $s = w^2$ and the masses of the system. Our projection algebra yields the invariant amplitudes $F_1^{\pm}$ explicitly as

$$\begin{aligned}
&\frac{1}{2}\,\mathbf{tr}\Big(Q_+\,\bar\Lambda\,\Big[\sum_\pm F_1^\pm P_\pm\Big]\,\Lambda\Big) = F_1^+\,,\\
&\frac{1}{2}\,\mathbf{tr}\Big(Q_-\,\bar\Lambda\,\Big[\sum_\pm F_1^\pm P_\pm\Big]\,\Lambda\Big) = F_1^-\,.
\end{aligned} \tag{3.4}$$

We provide a simple example and consider an s-channel exchange contribution to the scattering amplitude with a pseudo-vector type vertex

$$T_{0\,\frac{1}{2}\to 0\,\frac{1}{2}}(\bar{q}, q, w) = \bar{u}(\bar{p}, \lambda_{\bar{p}}) \left[ \gamma_5 \,\bar{\not{q}}\, \frac{1}{\not{w} - M_s} \not{q}\, \gamma_5 \right] u(p, \lambda_p)\,, \tag{3.5}$$

where we do not specify the couplings strength of the vertex. The mass $M_s$ denotes the mass of the spin-one-half fermion exchanged in the s-channel. The scattering amplitude can be expressed in terms of $P_\pm$ as

$$T_{0\,\frac{1}{2}\to 0\,\frac{1}{2}}(\bar{k}, k, w) = \bar{u}(\bar{p}, \lambda_{\bar{p}}) \left[ \sum_{\pm} - \frac{(\bar{M} \mp \sqrt{s})(M \mp \sqrt{s})}{M_s \pm \sqrt{s}} \, P_\pm \right] u(p, \lambda_p)\,, \tag{3.6}$$

where the coefficients are calculated by using the projection algebra (3.2). It holds

$$\frac{1}{2}\,\mathbf{tr}\Big( Q_\pm\, \bar{\Lambda} \left[ \gamma_5 \,\bar{\not{q}}\, \frac{1}{\not{w} - M_s} \not{q}\, \gamma_5 \right] \Lambda \Big) = - \frac{(\bar{M} \mp \sqrt{s})(M \mp \sqrt{s})}{M_s \pm \sqrt{s}}\,. \tag{3.7}$$

In order to generalize the simple result (3.3) to the more complicated systems involving spin-one bosons and later spin-three-half fermions it is advantageous to follow [50] and introduce the reciprocal four-vectors $r_\perp$, $w_\rfloor$, and $w_\lfloor$. They are suitable linear combinations of $\bar{r}$, $r$, and $w$ so as to have the convenient properties

$$\begin{aligned}
r_\perp \cdot r &= 1\,, & r_\perp \cdot \bar{r} &= 0 = r_\perp \cdot w\,, \\
w_\rfloor \cdot w &= 1\,, & w_\rfloor \cdot r &= 0 = w_\rfloor \cdot \bar{q}\,, \\
w_\lfloor \cdot w &= 1\,, & w_\lfloor \cdot r &= 0 = w_\lfloor \cdot \bar{p}\,.
\end{aligned} \tag{3.8}$$

The index $\rfloor$ or $\lfloor$ of the reciprocal four-vectors indicates whether it is orthogonal to the four-momentum of outgoing boson $\bar{q}$ or that of the outgoing fermion $\bar{p}$ respectively. The patched symbol $\perp$ implies the orthogonality to both four-momenta. We recall the explicit form of the reciprocal four-vectors $r_\perp$, $w_\rfloor$, and $w_\lfloor$ from [50]. Given three four-vectors $a_\mu$, $b_\mu$, and $c_\mu$ we introduce a vector, $a^\mu_{b\,c} = a^\mu_{c\,b}$, as follows

$$\begin{aligned}
&\frac{a^\mu_{b\,c}}{a_{b\,c}\cdot a_{b\,c}} = a^\mu - \frac{a\cdot c}{c\cdot c}\, c^\mu - \frac{a\cdot (b - \frac{c\cdot b}{c\cdot c}\, c)}{(b - \frac{c\cdot b}{c\cdot c}\, c)^2} \left( b^\mu - \frac{c\cdot b}{c\cdot c}\, c^\mu \right), \\
&a^\mu_{b\,c}\, a_\mu = 1\,, \qquad a^\mu_{b\,c}\, b_\mu = 0\,, \qquad a^\mu_{b\,c}\, c_\mu = 0\,.
\end{aligned} \tag{3.9}$$

In the notation of (3.9) the desired vectors are identified with

$$\begin{aligned}
{r_\perp}^\mu &= r^\mu_{\bar{r}\,w}\,, & {w_\rfloor}^\mu &= w^\mu_{r\,\bar{q}}\,, & {w_\lfloor}^\mu &= w^\mu_{r\,\bar{p}}\,, \\
{\bar{r}_\perp}^\mu &= \bar{r}^\mu_{r\,w}\,, & {\bar{w}_\rfloor}^\mu &= w^\mu_{\bar{r}\,q}\,, & {\bar{w}_\lfloor}^\mu &= w^\mu_{\bar{r}\,p}\,,
\end{aligned} \tag{3.10}$$

where we introduced the additional vectors $\bar{r}_\perp$, $\bar{w}_\rfloor$, and $\bar{w}_\lfloor$ that will turn out to be useful. The latter vectors have properties analogous to their counter parts $r_\perp$, $w_\rfloor$, and $w_\lfloor$ characterized in Eq. (3.8). In terms of the introduced reciprocal vectors we succeeded in deriving concise results for the projection tensors $Q^{(n)}_\pm$ introduced in Eq. (3.1).

The spin-one production amplitudes $F^\pm_1$, $F^\pm_2$, and $F^\pm_3$ introduced in (2.42) can be obtained by means of the following projection algebra

$$\begin{aligned}
&\frac{1}{2}\,\mathbf{tr}(T^{(n)}_{a,\bar{\mu}}\,\Lambda\, Q^{\bar{\mu}}_{b,k}\,\bar{\Lambda}) = \delta_{nk}\,\delta_{ab} \qquad \text{with} \qquad \bar{q}_{\bar{\mu}}\, Q^{\bar{\mu}}_{\pm,k} = 0\,, \\
&Q^{\bar{\mu}}_{\pm,1} = \pm \frac{\sqrt{s}}{v^2}\, P_\pm\, v^{\bar{\mu}}\,, \\
&Q^{\bar{\mu}}_{\pm,2} = -i\,\gamma_5\, R_\pm\, {w_\rfloor}^{\bar{\mu}} - \tfrac{1}{2}(\bar{\delta} + 1)\, \frac{\sqrt{s}}{v^2}\, E_\pm\, Q_\pm\, v^{\bar{\mu}}\,, \\
&Q^{\bar{\mu}}_{\pm,3} = -i\,\gamma_5\, R_\pm\, {r_\perp}^{\bar{\mu}} - \frac{\sqrt{s}}{v^2}\, \bar{E}_\mp\, Q_\mp\, v^{\bar{\mu}}\,,
\end{aligned} \tag{3.11}$$

where we expressed our result in terms of the reciprocal vectors introduced in Eq. (3.10). The tensor $R_\pm$ in Eq. (3.11) is

$$
R_\pm = \frac{s}{v^2}\left(\bar{E}_\mp\, E_\pm\, P_\pm - (\bar{r}\cdot r)\, P_\mp\right), \tag{3.12}
$$

and $Q_\pm$ was introduced already in Eq. (3.3). The supplementary condition $\bar{q}_{\bar{\mu}}\, Q^{\bar{\mu}}_{\pm,k} = 0$ is required by the on-shell condition of the outgoing spin-one boson.

For the scattering of spin-one bosons off spin-one-half fermions eighteen invariant amplitudes $F^\pm_n$ were introduced in (2.44). We derive the following compact result

$$
\frac{1}{2}\,\mathbf{tr}\big(T^{(n)}_{a,\bar{\mu}\mu}\,\Lambda\, Q^{\bar{\mu}\mu}_{b,k}\,\bar{\Lambda}\big) = \delta_{nk}\,\delta_{ab} \qquad \text{with} \qquad \bar{q}_{\bar{\mu}}\, Q^{\bar{\mu}\mu}_{\pm,k} = 0 \qquad \text{and} \qquad q_\mu\, Q^{\bar{\mu}\mu}_{\pm,k} = 0\,, \tag{3.13}
$$

$$
\begin{aligned}
Q^{\bar{\mu}\mu}_{\pm,1} &= \frac{1}{v^2}\, v^{\bar{\mu}}\, Q_\pm\, v^\mu - Q^{\bar{\mu}\mu}_{\mp,2}\,,\\
Q^{\bar{\mu}\mu}_{\pm,2} &= r_\lfloor{}^{\bar{\mu}} \Big[P_\pm - 2(\bar{r}\cdot r)Q_\mp\Big]\, \bar{r}_\lfloor{}^{\mu} - (\bar{r}_\lfloor\cdot r_\lfloor)\,\frac{1}{v^2}\, v^{\bar{\mu}} \Big[P_\pm - 2(\bar{r}\cdot r)Q_\mp\Big]\, v^\mu\\
&\quad + \bar{E}_\pm\, i\,\gamma_5\, \frac{\sqrt{s}}{v^2}\, v^{\bar{\mu}} \Big[P_\mp + 2(\bar{r}\cdot r)R_\pm\Big]\, \bar{r}_\lfloor{}^{\mu} - E_\pm\, i\,\gamma_5\, \frac{\sqrt{s}}{v^2}\, r_\lfloor{}^{\bar{\mu}} \Big[P_\pm + 2(\bar{r}\cdot r)R_\mp\Big]\, v^\mu\,,\\
Q^{\bar{\mu}\mu}_{\pm,3} &= \mp\frac{\sqrt{s}}{v^2}\, v^{\bar{\mu}}\, i\,\gamma_5\, P_\pm\, \bar{w}_\rfloor{}^{\mu} \pm \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\Big[(\bar{r}\cdot r)\,\bar{E}_\pm\, Q^{\bar{\mu}\mu}_{\mp,2} - (\bar{r}\cdot\bar{r})\,E_\mp\, Q^{\bar{\mu}\mu}_{\pm,2}\Big]\,,\\
Q^{\bar{\mu}\mu}_{\pm,4} &= \pm\frac{\sqrt{s}}{v^2}\, w_\rfloor{}^{\bar{\mu}}\, P_\pm\, i\,\gamma_5\, v^\mu \pm \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\Big[(\bar{r}\cdot r)\,E_\pm\, Q^{\bar{\mu}\mu}_{\mp,2} - (r\cdot r)\,\bar{E}_\mp\, Q^{\bar{\mu}\mu}_{\pm,2}\Big]\,,\\
Q^{\bar{\mu}\mu}_{\pm,5} &= \mp\frac{\sqrt{s}}{v^2}\, v^{\bar{\mu}}\, i\,\gamma_5\, P_\pm\, \bar{r}_\lfloor{}^{\mu} \pm \frac{s}{v^2}\Big[(r\cdot r)\,\bar{E}_\pm\, Q^{\bar{\mu}\mu}_{\mp,2} - (\bar{r}\cdot r)\,E_\mp\, Q^{\bar{\mu}\mu}_{\pm,2}\Big]\,,\\
Q^{\bar{\mu}\mu}_{\pm,6} &= \pm\frac{\sqrt{s}}{v^2}\, r_\lfloor{}^{\bar{\mu}}\, P_\pm\, i\,\gamma_5\, v^\mu \pm \frac{s}{v^2}\Big[(\bar{r}\cdot\bar{r})\,E_\pm\, Q^{\bar{\mu}\mu}_{\mp,2} - (\bar{r}\cdot r)\,\bar{E}_\mp\, Q^{\bar{\mu}\mu}_{\pm,2}\Big]\,,\\
Q^{\bar{\mu}\mu}_{\pm,7} &= Q_\pm\left(w_\rfloor{}^{\bar{\mu}}\, \bar{r}_\lfloor{}^{\mu} - (w_\rfloor\cdot\bar{r}_\lfloor)\,\frac{1}{v^2}\, v^{\bar{\mu}}\, v^\mu\right)\\
&\quad \pm (r\cdot r)\,\bar{E}_\mp\, \frac{s}{v^2}\Big[Q^{\bar{\mu}\mu}_{\mp,4} - \tfrac{1}{2}(\bar{\delta}+1)Q^{\bar{\mu}\mu}_{\pm,5}\Big] \mp (\bar{r}\cdot r)\,E_\mp\, \frac{s}{v^2}\Big[Q^{\bar{\mu}\mu}_{\pm,4} - \tfrac{1}{2}(\bar{\delta}+1)Q^{\bar{\mu}\mu}_{\mp,5}\Big]\,,\\
Q^{\bar{\mu}\mu}_{\pm,8} &= Q_\pm\left(r_\lfloor{}^{\bar{\mu}}\, \bar{w}_\rfloor{}^{\mu} - (r_\lfloor\cdot\bar{w}_\rfloor)\,\frac{1}{v^2}\, v^{\bar{\mu}}\, v^\mu\right)\\
&\quad \pm (\bar{r}\cdot\bar{r})\,E_\mp\, \frac{s}{v^2}\Big[Q^{\bar{\mu}\mu}_{\mp,3} - \tfrac{1}{2}(\delta+1)Q^{\bar{\mu}\mu}_{\pm,6}\Big] \mp (\bar{r}\cdot r)\,\bar{E}_\mp\, \frac{s}{v^2}\Big[Q^{\bar{\mu}\mu}_{\pm,3} - \tfrac{1}{2}(\delta+1)Q^{\bar{\mu}\mu}_{\mp,6}\Big]\,,\\
Q^{\bar{\mu}\mu}_{\pm,9} &= Q_\pm\left(w_\rfloor{}^{\bar{\mu}}\, \bar{w}_\rfloor{}^{\mu} - \left(w_\rfloor\cdot\bar{w}_\rfloor - \frac{1}{s}\right)\frac{1}{v^2}\, v^{\bar{\mu}}\, v^\mu\right) - \tfrac{1}{4}(\bar{\delta}+1)(\delta+1)\,\frac{s}{v^2}\Big[(\bar{r}\cdot r)Q^{\bar{\mu}\mu}_{\mp,2} - \bar{E}_\mp\, E_\mp\, Q^{\bar{\mu}\mu}_{\pm,2}\Big]\\
&\quad \pm \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\Big[(\bar{r}\cdot r)\,E_\mp\, Q^{\bar{\mu}\mu}_{\mp,3} - (r\cdot r)\,\bar{E}_\mp\, Q^{\bar{\mu}\mu}_{\pm,3}\Big] \pm \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\Big[(\bar{r}\cdot r)\,\bar{E}_\mp\, Q^{\bar{\mu}\mu}_{\mp,4} - (\bar{r}\cdot\bar{r})\,E_\mp\, Q^{\bar{\mu}\mu}_{\pm,4}\Big]\,,
\end{aligned}
$$

where the supplementary conditions are required by the on-shell properties of the incoming and outgoing spin-one bosons.

## 3.2 Spin-three-half fermion

While the derivation of our results for the spin-one-half systems was still reasonably straightforward a more systematic procedure is required for the more complicated remaining systems involving spin-three-half fermions. In particular, the on-shell conditions (2.50) and (2.53) of the spin-three-half wave functions cause additional complications.

Let us start from the simplest case containing a spin-three-half fermion. The decomposed form of the on-shell production amplitude of a spin-three-half fermion is given in (2.49). A suitable projection algebra for the four invariant amplitudes $F_1^\pm$ and $F_2^\pm$ reads

$$\frac{1}{2}\,\mathbf{tr}\big(T^{(n)}_{a,\bar\nu}\,\Lambda\,Q^{\bar\nu}_{b,k}\,\bar\Lambda\big)=\delta_{nk}\,\delta_{ab}\qquad\text{with}\qquad \Lambda\,Q^{\bar\nu}_{\pm,k}\,\bar\Lambda\,\gamma_{\bar\nu}=0\qquad\text{and}\qquad \bar p_{\bar\nu}\,Q^{\bar\nu}_{\pm,k}=0\,,\tag{3.14}$$
$$Q^{\bar\nu}_{\pm,1}=\frac{s}{v^2}\Big[(\bar r\cdot r)\,P^{\bar\nu}_{\mp,1}-\bar E_\mp\,E_\pm\,P^{\bar\nu}_{\pm,1}\Big]\,,$$
$$Q^{\bar\nu}_{\pm,2}=\frac{s}{v^2}\Big[(\bar r\cdot r)\,P^{\bar\nu}_{\mp,2}-\bar E_\mp\,E_\pm\,P^{\bar\nu}_{\pm,2}\Big]\,,$$

where we introduced four auxiliary tensors

$$P^{\bar\nu}_{\pm,1}=w_\perp^{\bar\nu}\,i\,\gamma_5\,P_\pm-v^{\bar\nu}\,((\bar r\cdot r)\,P_\mp\pm\bar M\,E_\mp\,P_\pm)/v^2\,,$$
$$P^{\bar\nu}_{\pm,2}=r_\perp^{\bar\nu}\,i\,\gamma_5\,P_\pm-v^{\bar\nu}\,(\sqrt{s}\,\bar E_\pm\,P_\mp)/v^2\,.\tag{3.15}$$

The derivation of (3.15) goes in two steps. First we identified the four tensors $P^{\bar\nu}_{\pm,k}$ that satisfy the supplementary on-shell conditions

$$\Lambda\,P^{\bar\nu}_{\pm,k}\,\bar\Lambda\,\gamma_{\bar\nu}=0\,,\qquad \bar p_{\bar\nu}\,P^{\bar\nu}_{\pm,k}=0\,,\tag{3.16}$$

and form a linear independent set in terms of which the projection tensors $Q^{\bar\nu}_{\pm,n}$ can be expanded. While any of the auxiliary operators $v^{\bar\nu}\,P_\pm$, $w_\perp^{\bar\nu}\,i\,\gamma_5\,P_\pm$, and $r_\perp^{\bar\nu}\,i\,\gamma_5\,P_\pm$ is transverse with respect to $\bar p_{\bar\nu}$, they are not transverse to $\gamma_\nu$ in the way requested in (3.16). To achieve the latter condition suitable linear combinations are required. Moreover, the basis must be linear independent, which implies the condition that the matrix

$$P^{ab}_{cd}=\frac{1}{2}\,\mathbf{tr}\big(T^{(a)}_{c,\bar\nu}\,\Lambda\,P^{\bar\nu}_{d,b}\,\bar\Lambda\big)\qquad\text{with}\qquad \det\begin{pmatrix}P^{ab}_{++} & P^{ab}_{+-}\\ P^{ab}_{-+} & P^{ab}_{--}\end{pmatrix}\neq 0\,,\tag{3.17}$$

is invertible. As a consequence the projectors have the following decomposition

$$Q^{\bar\nu}_{\pm,n}=\sum_k c^\pm_{nk}\,P^{\bar\nu}_{\pm,k}+\sum_k d^\pm_{nk}\,P^{\bar\nu}_{\mp,k}\,,\tag{3.18}$$

with some expansion coefficients $c^\pm_{nk}$ and $d^\pm_{nk}$ which can be determined from the first condition in (3.14). This amounts to the inversion of the matrix $P^{ab}_{cd}$ introduced in (3.17).

We continue and work out projection algebras for the remaining invariant amplitudes introduced in Chapter 2. Here an order according to the complexity of the results is chosen.

For the scattering of pseudoscalar boson off a spin-three-half fermion, the eight invariant amplitudes $F^\pm_n$ introduced in (2.52) can be derived by the following projection algebra

$$\frac{1}{2}\,\mathbf{tr}\big(T^{(n)}_{a,\bar\nu\nu}\,\Lambda\,Q^{\bar\nu\nu}_{b,k}\,\bar\Lambda\big)=\delta_{nk}\,\delta_{ab}\quad\text{with}\quad \bar p_{\bar\nu}\,Q^{\bar\nu\nu}_{\pm,k}=0=\Lambda\,Q^{\bar\nu\nu}_{\pm,k}\,\bar\Lambda\,\gamma_{\bar\nu}\quad\text{and}\quad p_\nu\,Q^{\bar\nu\nu}_{\pm,k}=0=\gamma_\nu\,\Lambda\,Q^{\bar\nu\nu}_{\pm,k}\,\bar\Lambda\,,\tag{3.19}$$

$$Q^{\bar\nu\nu}_{\pm,1}=P^{\bar\nu\nu}_{\mp,1}-\frac{s}{v^2}\,2\,\bar E_\mp\,E_\mp\Big[(\bar r\cdot r)\,P^{\bar\nu\nu}_{\pm,1}-\bar E_\pm\,E_\pm\,P^{\bar\nu\nu}_{\mp,1}\Big]\,,$$
$$Q^{\bar\nu\nu}_{\pm,2}=\frac{s}{v^2}\Big[(\bar r\cdot r)\,P^{\bar\nu\nu}_{\mp,3}-\bar E_\mp\,E_\mp\,P^{\bar\nu\nu}_{\pm,3}\Big]-\frac{\sqrt{s}}{v^2}\,E_\mp\Big[(\bar r\cdot r)\,Q^{\bar\nu\nu}_{\mp,1}+\bar E_\pm\,E_\pm\,Q^{\bar\nu\nu}_{\pm,1}\Big]\,,$$
$$Q^{\bar\nu\nu}_{\pm,3}=\frac{s}{v^2}\Big[(\bar r\cdot r)\,P^{\bar\nu\nu}_{\mp,2}-\bar E_\mp\,E_\mp\,P^{\bar\nu\nu}_{\pm,2}\Big]-\frac{\sqrt{s}}{v^2}\,\bar E_\mp\Big[(\bar r\cdot r)\,Q^{\bar\nu\nu}_{\mp,1}+\bar E_\pm\,E_\pm\,Q^{\bar\nu\nu}_{\pm,1}\Big]\,,$$
$$Q^{\bar\nu\nu}_{\pm,4}=\frac{s}{v^2}\Big[(\bar r\cdot r)\,P^{\bar\nu\nu}_{\mp,4}-\bar E_\mp\,E_\mp\,P^{\bar\nu\nu}_{\pm,4}\Big]-(1/s)\,Q^{\bar\nu\nu}_{\mp,1}$$
$$\qquad+\tfrac{1}{2}(\bar\delta-1)\,\frac{\sqrt{s}}{v^2}\,E_\pm\Big[(\bar r\cdot r)\,Q^{\bar\nu\nu}_{\pm,1}+\bar E_\mp\,E_\mp\,Q^{\bar\nu\nu}_{\mp,1}\Big]\pm\tfrac{1}{2}(\delta-1)\,\frac{\sqrt{s}}{v^2}\,\bar M\Big[(\bar r\cdot r)\,Q^{\bar\nu\nu}_{\pm,1}+\bar E_\mp\,E_\mp\,Q^{\bar\nu\nu}_{\mp,1}\Big]\,,$$

with

$$
\begin{aligned}
P^{\bar\nu\nu}_{\pm,1} &= r_\perp^{\bar\nu}\,\bar r_\perp^{\nu}\,P_\pm + v^{\bar\nu}\,v^{\nu}\,(s/v^2)\,\bar E_\pm\,E_\pm\,P_\mp/v^2 - r_\perp^{\bar\nu}\,v^{\nu}\,(\sqrt{s}/v^2)\,E_\pm\,i\,\gamma_5\,P_\pm + v^{\bar\nu}\,\bar r_\perp^{\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\,i\,\gamma_5\,P_\mp\,,\\
P^{\bar\nu\nu}_{\pm,2} &= r_\perp^{\bar\nu}\,\bar w_\perp^{\nu}\,P_\pm + v^{\bar\nu}\,\bar w_\perp^{\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\,i\,\gamma_5\,P_\mp\\
&\quad - r_\perp^{\bar\nu}\,v^{\nu}\,i\,\gamma_5\,((\bar r\cdot r)\,P_\pm \pm M\,\bar E_\pm\,P_\mp)/v^2 + v^{\bar\nu}\,v^{\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\,((\bar r\cdot r)\,P_\mp \pm M\,\bar E_\mp\,P_\pm)/v^2\,,\\
P^{\bar\nu\nu}_{\pm,3} &= w_\perp^{\bar\nu}\,\bar r_\perp^{\nu}\,P_\pm - w_\perp^{\bar\nu}\,v^{\nu}\,(\sqrt{s}/v^2)\,E_\pm\,i\,\gamma_5\,P_\pm\\
&\quad + v^{\bar\nu}\,\bar r_\perp^{\nu}\,i\,\gamma_5\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\pm\,P_\pm)/v^2 + v^{\bar\nu}\,v^{\nu}\,(\sqrt{s}/v^2)\,E_\pm\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\mp\,P_\pm)/v^2\,,\\
P^{\bar\nu\nu}_{\pm,4} &= w_\perp^{\bar\nu}\,\bar w_\perp^{\nu}\,P_\pm + v^{\bar\nu}\,v^{\nu}\,\big[(1/s)\,P_\mp + \tfrac14\,(\bar\delta-1)\,(\delta-1)\,(s/v^2)\,\bar E_\pm\,E_\pm\,P_\mp\\
&\qquad \mp \tfrac12\,(\bar\delta-1)\,(\sqrt{s}/v^2)\,M\,((\bar r\cdot r)\,P_\pm - \bar E_\pm\,E_\pm\,P_\mp) \mp \tfrac12\,(\delta-1)\,(\sqrt{s}/v^2)\,\bar M\,((\bar r\cdot r)\,P_\pm - \bar E_\pm\,E_\pm\,P_\mp)\big]/v^2\\
&\quad - w_\perp^{\bar\nu}\,v^{\nu}\,i\,\gamma_5\,((\bar r\cdot r)\,P_\pm \pm M\,\bar E_\pm\,P_\mp)/v^2 + v^{\bar\nu}\,\bar w_\perp^{\nu}\,i\,\gamma_5\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\pm\,P_\pm)/v^2\,. \qquad (3.20)
\end{aligned}
$$

where the $P^{\bar\nu\nu}_{\pm,k}$ are designed to satisfy the on-shell conditions introduced in Eq. (3.19).

We proceed with the $1^- + \frac12^+ \to 0^- + \frac32^+$ reaction, which is described by the twelve invariant functions $F^\pm_n$ introduced in (2.51). Our result reads

$$
\frac12\,\mathbf{tr}\big(T^{(n)}_{a,\bar\nu\mu}\,\Lambda\,Q^{\bar\nu\mu}_{b,k}\,\bar\Lambda\big) = \delta_{nk}\,\delta_{ab} \qquad \text{with} \qquad \bar p_{\bar\nu}\,Q^{\bar\nu\mu}_{\pm,k} = 0 = \Lambda\,Q^{\bar\nu\mu}_{\pm,k}\,\bar\Lambda\,\gamma_{\bar\nu} \qquad \text{and} \qquad q_\mu\,Q^{\bar\nu\mu}_{\pm,k} = 0\,, \qquad (3.21)
$$

$$
\begin{aligned}
Q^{\bar\nu\mu}_{\pm,1} &= -\frac{\sqrt{s}}{\bar E_\pm}\Big[(\bar r\cdot r)\,P^{\bar\nu\mu}_{\pm,1} - \bar E_\pm\,E_\mp\,P^{\bar\nu\mu}_{\mp,1}\Big] \mp \frac{1}{\bar E_\pm}\Big[(\bar r\cdot r)\,Q^{\bar\nu\mu}_{\mp,3} + \bar E_\pm\,E_\mp\,Q^{\bar\nu\mu}_{\pm,3}\Big]\,,\\
Q^{\bar\nu\mu}_{\pm,2} &= \pm\,w\,P^{\bar\nu\mu}_{\mp,4} \pm \frac{1}{\sqrt{s}}\,Q^{\bar\nu\mu}_{\mp,1} \mp \tfrac12\,(\bar\delta-1)\,\frac{s}{v^2}\,E_\pm\Big[(\bar r\cdot r)\,Q^{\bar\nu\mu}_{\pm,1} + \bar E_\mp\,E_\mp\,Q^{\bar\nu\mu}_{\mp,1}\Big]\,,\\
Q^{\bar\nu\mu}_{\pm,3} &= \pm\,\frac{s}{v^2}\,\sqrt{s}\,(\bar r\cdot r)\Big[(\bar r\cdot r)\,P^{\bar\nu\mu}_{\mp,1} - \bar E_\mp\,E_\pm\,P^{\bar\nu\mu}_{\pm,1}\Big] \pm \frac{s}{v^2}\,\bar E_\mp\Big[(\bar r\cdot r)\,P^{\bar\nu\mu}_{\pm,2} - \bar E_\pm\,E_\pm\,P^{\bar\nu\mu}_{\mp,2}\Big]\,,\\
Q^{\bar\nu\mu}_{\pm,4} &= \frac{s}{v^2}\Big[(\bar r\cdot r)\,P^{\bar\nu\mu}_{\mp,5} - \bar E_\mp\,E_\mp\,P^{\bar\nu\mu}_{\pm,5}\Big] \mp \frac{s}{v^2}\,E_\mp\Big[(\bar r\cdot r)\,Q^{\bar\nu\mu}_{\pm,2} + \bar E_\mp\,E_\pm\,Q^{\bar\nu\mu}_{\mp,2}\Big] - \tfrac12\,(\bar\delta-1)\,\frac{s}{v^2}\,(r\cdot r)\,Q^{\bar\nu\mu}_{\pm,1}\,,\\
Q^{\bar\nu\mu}_{\pm,5} &= \frac{s}{v^2}\Big[(\bar r\cdot r)\,P^{\bar\nu\mu}_{\mp,3} - \bar E_\mp\,E_\mp\,P^{\bar\nu\mu}_{\pm,3}\Big] \pm \tfrac12\,(\delta+1)\,\frac{s}{v^2}\,\bar E_\mp\Big[(\bar r\cdot r)\,Q^{\bar\nu\mu}_{\mp,3} + \bar E_\pm\,E_\mp\,Q^{\bar\nu\mu}_{\pm,3}\Big] - \tfrac12\,(\delta+1)\,\frac{s}{v^2}\,(\bar r\cdot\bar r)\,Q^{\bar\nu\mu}_{\pm,1}\,,\\
Q^{\bar\nu\mu}_{\pm,6} &= \frac{s}{v^2}\Big[(\bar r\cdot r)\,P^{\bar\nu\mu}_{\mp,6} - \bar E_\mp\,E_\mp\,P^{\bar\nu\mu}_{\pm,6}\Big] \pm \tfrac12\,(\delta+1)\,\frac{s}{v^2}\,\bar E_\mp\Big[(\bar r\cdot r)\,Q^{\bar\nu\mu}_{\mp,2} + \bar E_\pm\,E_\mp\,Q^{\bar\nu\mu}_{\pm,2}\Big]\\
&\qquad - \tfrac14\,(\bar\delta-1)\,(\delta+1)\,\frac{s}{v^2}\,(\bar r\cdot r)\,Q^{\bar\nu\mu}_{\pm,1}\,,
\end{aligned}
$$

with

$$
\begin{aligned}
P^{\bar\nu\mu}_{\pm,1} &= \big[r_\perp^{\bar\nu}\,i\,\gamma_5\,P_\pm - v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\,P_\mp\big]\,v^\mu/v^2\,,\\
P^{\bar\nu\mu}_{\pm,2} &= \big[r_\perp^{\bar\nu}\,P_\pm + v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\,i\,\gamma_5\,P_\mp\big]\,\bar r_\perp^{\mu}\,,\\
P^{\bar\nu\mu}_{\pm,3} &= \big[r_\perp^{\bar\nu}\,P_\pm + v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\,i\,\gamma_5\,P_\mp\big]\,\bar w_\perp^{\mu}\,,\\
P^{\bar\nu\mu}_{\pm,4} &= \big[w_\perp^{\bar\nu}\,i\,\gamma_5\,P_\pm - v^{\bar\nu}\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\mp\,P_\pm)/v^2\big]\,v^\mu/v^2\,,\\
P^{\bar\nu\mu}_{\pm,5} &= \big[w_\perp^{\bar\nu}\,P_\pm + v^{\bar\nu}\,i\,\gamma_5\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\pm\,P_\pm)/v^2\big]\,\bar r_\perp^{\mu}\,,\\
P^{\bar\nu\mu}_{\pm,6} &= \big[w_\perp^{\bar\nu}\,P_\pm + v^{\bar\nu}\,i\,\gamma_5\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\pm\,P_\pm)/v^2\big]\,\bar w_\perp^{\mu}\,. \qquad (3.22)
\end{aligned}
$$

There remain four more cases to be considered. The final result shown here is for the $0^- + \frac12^+ \to 1^- + \frac32^+$ reaction. For the amplitudes of Eq. (2.54), we derive the following projection algebra

$$\frac{1}{2}\,\mathbf{tr}\big(T^{(n)}_{a,\bar\mu\bar\nu}\,\Lambda\,Q^{\bar\mu\bar\nu}_{b,k}\,\bar\Lambda\big) = \delta_{nk}\,\delta_{ab} \qquad \text{with} \qquad \bar q_{\bar\mu}\,Q^{\bar\mu\bar\nu}_{\pm,k} = 0 \qquad \text{and} \qquad \bar p_{\bar\nu}\,Q^{\bar\mu\bar\nu}_{\pm,k} = 0 = \Lambda\,Q^{\bar\mu\bar\nu}_{\pm,k}\,\bar\Lambda\,\gamma_{\bar\nu}\,, \tag{3.23}$$

$$\begin{aligned}
Q^{\bar\mu\bar\nu}_{\pm,1} &= \pm\frac{\sqrt{s}}{\bar M}\left[P^{\bar\mu\bar\nu}_{\mp,5} + \sqrt{s}\,\bar E_\mp\,P^{\bar\mu\bar\nu}_{\pm,4}\right],\\
Q^{\bar\mu\bar\nu}_{\pm,2} &= \pm(1/\bar E_\mp)\,P^{\bar\mu\bar\nu}_{\mp,5} \pm \tfrac{1}{2}(\bar\delta-1)\,\frac{s}{v^2}\,\frac{(\bar r\cdot r)}{\bar E_\mp}\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu}_{\pm,1} + \bar E_\mp\,E_\mp\,Q^{\bar\mu\bar\nu}_{\mp,1}\right],\\
Q^{\bar\mu\bar\nu}_{\pm,3} &= \mp\sqrt{s}\,P^{\bar\mu\bar\nu}_{\pm,1} \mp \frac{s}{v^2}\,\bar E_\pm\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu}_{\pm,1} + \bar E_\mp\,E_\mp\,Q^{\bar\mu\bar\nu}_{\mp,1}\right],\\
Q^{\bar\mu\bar\nu}_{\pm,4} &= \frac{s}{v^2}\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu}_{\mp,6} - \bar E_\mp\,E_\mp\,P^{\bar\mu\bar\nu}_{\pm,6}\right] \pm \tfrac{1}{2}(\bar\delta+1)\,\frac{s}{v^2}\,E_\mp\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu}_{\mp,2} + \bar E_\mp\,E_\pm\,Q^{\bar\mu\bar\nu}_{\pm,2}\right] + \tfrac{1}{4}(\bar\delta+1)(\bar\delta-1)\,\frac{s}{v^2}\,(r\cdot r)\,Q^{\bar\mu\bar\nu}_{\pm,1}\,,\\
Q^{\bar\mu\bar\nu}_{\pm,5} &= \frac{s}{v^2}\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu}_{\mp,3} - \bar E_\mp\,E_\mp\,P^{\bar\mu\bar\nu}_{\pm,3}\right] \pm \tfrac{1}{2}(\bar\delta+1)\,\frac{s}{v^2}\,E_\mp\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu}_{\mp,3} + \bar E_\mp\,E_\pm\,Q^{\bar\mu\bar\nu}_{\pm,3}\right] + \tfrac{1}{2}(\bar\delta+1)\,\frac{s}{v^2}\,(\bar r\cdot r)\,Q^{\bar\mu\bar\nu}_{\pm,1}\,,\\
Q^{\bar\mu\bar\nu}_{\pm,6} &= \frac{s}{v^2}\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu}_{\mp,2} - \bar E_\mp\,E_\mp\,P^{\bar\mu\bar\nu}_{\pm,2}\right] \mp \frac{s}{v^2}\,\bar E_\mp\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu}_{\pm,3} + \bar E_\pm\,E_\mp\,Q^{\bar\mu\bar\nu}_{\mp,3}\right] + \frac{s}{v^2}\,(\bar r\cdot\bar r)\,Q^{\bar\mu\bar\nu}_{\pm,1}\,,
\end{aligned}$$

with

$$\begin{aligned}
P^{\bar\mu\bar\nu}_{\pm,1} &= \left[r_\perp{}^{\bar\nu}\,i\,\gamma_5\,P_\pm - v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\,P_\mp\right]\,v^{\bar\mu}/v^2\,,\\
P^{\bar\mu\bar\nu}_{\pm,2} &= \left[r_\perp{}^{\bar\nu}\,P_\pm + v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\,i\,\gamma_5\,P_\mp\right]\,r_\perp{}^{\bar\mu}\,,\\
P^{\bar\mu\bar\nu}_{\pm,3} &= \left[r_\perp{}^{\bar\nu}\,P_\pm + v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\,i\,\gamma_5\,P_\mp\right]\,w_\lrcorner{}^{\bar\mu}\,,\\
P^{\bar\mu\bar\nu}_{\pm,4} &= \left[w_\llcorner{}^{\bar\nu}\,i\,\gamma_5\,P_\pm - v^{\bar\nu}\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\mp\,P_\pm)/v^2\right]\,v^{\bar\mu}/v^2\,,\\
P^{\bar\mu\bar\nu}_{\pm,5} &= \left[w_\llcorner{}^{\bar\nu}\,P_\pm + v^{\bar\nu}\,i\,\gamma_5\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\pm\,P_\pm)/v^2\right]\,r_\perp{}^{\bar\mu}\,,\\
P^{\bar\mu\bar\nu}_{\pm,6} &= \left[w_\llcorner{}^{\bar\nu}\,P_\pm + v^{\bar\nu}\,i\,\gamma_5\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\pm\,P_\pm)/v^2\right]\,w_\lrcorner{}^{\bar\mu}\,.
\end{aligned} \tag{3.24}$$

The remaining three and most tedious projection algebras for reactions involving a spin-three-half fermion are summarized in Appendix A. The most complicated case involves 72 projectors.

Our formal results have been implemented into a Mathematica code using the FeynCalc package [83]. With this code analytic and compact results for invariant amplitudes from any tree-level process can be computed efficiently.

# 4 Analytic properties of partial-wave amplitudes

In this chapter we perform a partial-wave decomposition of the scattering amplitudes studied in the previous chapter by using the sets of invariant amplitudes $F_{\pm}^{(n)}(\sqrt{s},t)$. The merit of partial-wave amplitudes is that they provide a convenient realization of the unitarity condition. In Chapter 2 we already introduced a framework, in terms of which the combined consequences of micro-causality and unitarity can be systematically implemented. It requires the solution of the non-linear integral equation (2.6). We will apply the helicity formalism of Jacob and Wick [41]. In a first step helicity partial-wave amplitudes are expressed in terms of the invariant functions. Such a representation shows that helicity partial-wave amplitudes suffer from various kinematical constraints, which would prohibit their efficient application in the non-linear integral equation (2.6).

Kinematical constraints of helicity partial-wave amplitudes have been discussed by many authors; for example, J. D. Jackson and G. E. Hite [53], T.L. Trueman [84], and Alan Douglas Martin and Thomas D Spearman [70]. Kinematical constraints can be partially eliminated by introducing partial-wave amplitudes with respects to angular momentum $(L)$ states. A complete elimination requires the introduction of *covariant partial-wave amplitudes* $T_{\pm}^{J}(\sqrt{s})$ [45, 49, 50]. They are associated to covariant states and a covariant projector algebra which diagonalizes the Bethe-Salpeter two-body scattering equation for local interactions [31, 85, 32]. We will derive suitable transformations that eliminate the kinematical constraints. In this chapter we shall focus on two-body system of a boson with $J^P=0^-$ (or $J^P=1^-$) and a fermion with $J^P=\frac{1}{2}^+$, for which such transformations are not available so far.

## 4.1 Partial-wave decomposition

Helicity partial-wave amplitudes are introduced in the center-of-mass frame in terms of matrix elements of the scattering operator in plane-wave eigenfunctions of the helicity operator [41]. Such matrix elements take the general form

$$\langle \lambda_{\bar q}, \lambda_{\bar p} |\, T \,| \lambda_q, \lambda_p \rangle\,, \tag{4.1}$$

where $\lambda_q, \lambda_p$ and $\lambda_{\bar q}, \lambda_{\bar p}$ specify the helicity projections of the incoming and outgoing wave functions. The number of independent amplitudes under conserved parity is determined by Eq. (2.9). The phase conventions assumed in this thesis imply the relations

$$\langle -\lambda_{\bar q}, -\lambda_{\bar p} |\, T \,| -\lambda_q, -\lambda_p \rangle = (-1)^{S_{\bar q}+S_{\bar p}-S_q-S_p}\,\langle \lambda_{\bar q}, \lambda_{\bar p} |\, T \,| \lambda_q, \lambda_p \rangle\,. \tag{4.2}$$

In the center-of-mass frame a two-body reaction is characterized by the scattering angle $\theta$ and the magnitudes of the initial and final three-momenta

$$p_{\rm cm} = \frac{\sqrt{[s-(M-m)^2][s-(M+m)^2]}}{2\sqrt{s}}\,, \qquad \bar p_{\rm cm} = \frac{\sqrt{[s-(\bar M-\bar m)^2][s-(\bar M+\bar m)^2]}}{2\sqrt{s}}\,, \tag{4.3}$$

which can be expressed in terms of the Mandelstam variable $s$ and the masses of the system. We write the four-momenta of the incoming- and outgoing-particles using these variables

$$\begin{aligned} q^\mu &= \{q_0, 0, 0, +p_{\rm cm}\}\,, \qquad & \bar q^\mu &= \{\bar q_0, +\bar p_{\rm cm}\sin\theta, 0, +\bar p_{\rm cm}\cos\theta\}\,,\\ p^\mu &= \{p_0, 0, 0, -p_{\rm cm}\}\,, \qquad & \bar p^\mu &= \{\bar p_0, -\bar p_{\rm cm}\sin\theta, 0, -\bar p_{\rm cm}\cos\theta\}\,, \end{aligned} \tag{4.4}$$

where the zeroth components of four-vectors in Eq. (4.4) can be written as

$$
\begin{aligned}
&q_0=\sqrt{m^2+p_{\rm cm}^2}=\frac{\sqrt{s}}{2}(1+\delta)\,, && \bar q_0=\sqrt{\bar m^2+\bar p_{\rm cm}^2}=\frac{\sqrt{s}}{2}(1+\bar\delta)\,,\\
&p_0=\sqrt{M^2+p_{\rm cm}^2}=\frac{\sqrt{s}}{2}(1-\delta)\,, && \bar p_0=\sqrt{\bar M^2+\bar p_{\rm cm}^2}=\frac{\sqrt{s}}{2}(1-\bar\delta)\,,
\end{aligned}
\tag{4.5}
$$

in terms of $\delta$ and $\bar\delta$ introduced in Eq. (2.47).

Given this convention for the center-of-mass frame we specify the boson and fermion helicity-wave functions such that the phase convention of Eq. (4.2) holds. The outgoing Dirac spinors are introduced with

$$
u(\bar p,\pm\tfrac12)=\begin{pmatrix}\sqrt{\frac{\bar E_+}{2\bar M}}\,e^{-i\theta\frac{\sigma_2}{2}}\,\chi_\pm\\ \pm\sqrt{\frac{\bar E_-}{2\bar M}}\,e^{-i\theta\frac{\sigma_2}{2}}\,\chi_\pm\end{pmatrix},\quad \chi_+=\begin{pmatrix}0\\1\end{pmatrix},\quad \chi_-=\begin{pmatrix}1\\0\end{pmatrix},
\tag{4.6}
$$

where $E_\pm$ and $\bar E_\pm$ are given in (2.47). The spinors of the incoming states are obtained by setting $\theta=0$ and removing the bars in (4.6). The Dirac spinors (4.6) are constructed to be eigenstates of the helicity-operator

$$
\frac{n_p\cdot W}{M}\,u(p,\pm\tfrac12)=\pm\tfrac12\,u(p,\pm\tfrac12)\,,\qquad \frac{n_{\bar p}\cdot W}{\bar M}\,u(\bar p,\pm\tfrac12)=\pm\tfrac12\,u(\bar p,\pm\tfrac12)\,,
\tag{4.7}
$$

where $W^\mu$ is the Pauli-Lubanski vector for $S=\frac12$ and $n^\mu_p$ is a normalized space-like four-vector orthogonal to $p^\mu$ [72, 86]. We use

$$
n_p^\mu=-\frac{1}{\sqrt{p^2}}\left(|\vec p|,\frac{p^0}{|\vec p|}\vec p\right).
\tag{4.8}
$$

The helicity projection operator $n\cdot W/M$ is explicitly given in (C.26). The completeness relation over positive energy states holds in the form

$$
\sum_{\lambda=\pm\frac12}u(p,\lambda)\otimes\bar u(p,\lambda)=\frac{\not p+M}{2M}\,.
\tag{4.9}
$$

We continue and specify the wave functions of the outgoing spin-one states

$$
\epsilon^\mu(\bar q,\pm1)=\left\{0,\mp\frac{\cos\theta}{\sqrt2},-\frac{i}{2},\pm\frac{\sin\theta}{\sqrt2}\right\},\quad \epsilon^\mu(\bar q,0)=\left\{\frac{\bar p_{\rm cm}}{\bar m},\frac{\bar q_0}{\bar m}\sin\theta,0,\frac{\bar q_0}{\bar m}\cos\theta\right\},
\tag{4.10}
$$

where $\bar q_0$ is the energy of the outgoing vector meson (4.5). The wave functions of the corresponding initial states are obtained by $\theta=0$ and the removal of the bars from (4.10). The spin-one wave functions are eigenstates of the helicity operator with

$$
\frac{n_q^\alpha\,[W_\alpha]^\mu{}_\nu}{m}\,\epsilon^\nu(q,\pm1)=\pm\,\epsilon^\mu(q,\pm1)\,,\qquad \frac{n_q^\alpha\,[W^\alpha]^\mu{}_\nu}{m}\,\epsilon^\nu(q,0)=0\,,
\tag{4.11}
$$

with the Pauli-Lubanski vector $W^\alpha$ for the $S=1$ case (see Appendix C). The 4-vector $n_q^\alpha$ was introduced already in Eq. (4.8). The wave functions satisfy the on-shell condition, the space-like property, and the completeness respectively as

$$
q^\mu\,\epsilon_\mu(q,\lambda)=0\,,\qquad g^{\mu\nu}\,\epsilon^\dagger_\mu(q,\lambda)\,\epsilon_\nu(q,\lambda')=-\delta_{\lambda\lambda'}\,,
\tag{4.12}
$$

$$
\sum_{\lambda=\pm1,0}\epsilon_\mu(q,\lambda)\otimes\epsilon^\dagger_\nu(q,\lambda)=-g_{\mu\nu}+\frac{q_\mu q_\nu}{m^2}\,.
\tag{4.13}
$$

The helicity matrix elements of the scattering operator, $T$, are decomposed into partial-wave amplitudes characterized by the total angular momentum $J$. Given a specific process together with our convention for the helicity wave functions it suffices to specify the helicity projection $\lambda_q, \lambda_p$ and $\lambda_{\bar{q}}, \lambda_{\bar{p}}$. We write

$$\langle \lambda_{\bar{q}} \lambda_{\bar{p}} | T | \lambda_q \lambda_p \rangle = \sum_J (2J+1) \langle \lambda_{\bar{q}} \lambda_{\bar{p}} | \, T_J | \lambda_q \lambda_p \rangle \, d^{(J)}_{\lambda, \bar{\lambda}}(\theta) \,, \tag{4.14}$$

with $\lambda = \lambda_q - \lambda_p$ and $\bar{\lambda} = \lambda_{\bar{q}} - \lambda_{\bar{p}}$. Wigner's rotation functions, $d^{(J)}_{\lambda,\bar{\lambda}}(\theta)$, are used in a convention with

$$d^{(J)}_{\lambda,\bar{\lambda}}(\theta) = (-)^{\lambda - \bar{\lambda}} \, d^{(J)}_{-\lambda,-\bar{\lambda}}(\theta) = (-)^{\lambda-\bar{\lambda}} \, d^{(J)}_{\bar{\lambda},\lambda}(\theta) = d^{(J)}_{-\bar{\lambda},-\lambda}(\theta) \,. \tag{4.15}$$

Further properties of the Wigner functions and helicity partial-wave amplitudes are discussed in Appendix C. The helicity partial-wave amplitudes follow from the plane-wave matrix elements of the scattering operator by a suitable projection:

$$\langle \lambda_{\bar{q}} \lambda_{\bar{p}} | \, T_J | \lambda_q \lambda_p \rangle = \int_{-1}^{1} \frac{d\cos\theta}{2} \langle \lambda_{\bar{q}} \lambda_{\bar{p}} | T | \lambda_q \lambda_p \rangle \, d^{(J)}_{\lambda,\bar{\lambda}}(\theta) \,. \tag{4.16}$$

In line with Eq. (4.16) we introduce the angular momentum projection of the helicity state $|\lambda_q, \lambda_p\rangle$. We write

$$|\lambda_q, \lambda_p \rangle_J \qquad \text{with} \quad T \, |\lambda_q, \lambda_p \rangle_J = T_J |\lambda_q, \lambda_p \rangle \,. \tag{4.17}$$

It is useful to decouple the two parity sectors by introducing parity eigenstates of good total angular momentum $J$. We introduce a set, $|n_\pm, J\rangle$, of parity eigenstates with

$$P \, |n_\pm, J \rangle = \pm (-1)^{J - \frac{1}{2}} \, |n_\pm, J \rangle \,, \tag{4.18}$$

in terms of even or odd combinations of the $|+\lambda_q, +\lambda_p\rangle_J$ and $|-\lambda_q, -\lambda_p\rangle_J$ states introduced in Eq. (4.17). In general for a given parity sector there are

$$\frac{1}{2} (2S_q + 1)(2S_p + 1) \,, \tag{4.19}$$

distinct states to be considered. We finally can introduce the helicity partial-wave amplitudes $t^J_\pm(\sqrt{s})$ that carry good angular momentum $J$ and good parity and formulate the unitarity condition in a concise manner

$$[t^J_\pm]_{ab} = \langle a_\pm, J | T | b_\pm, J \rangle \,, \qquad [\Im t^J_\pm]_{ab} = \sum_c \, [t^{*,J}_\pm]_{ac} \, \frac{p_{\mathrm{cm},c}}{8\pi \sqrt{s}} \, 2M_c \, [t^J_\pm]_{cb} \,, \tag{4.20}$$

where the indices $a$ and $b$ span the bases of outgoing and incoming two-particle helicity states. The unitarity condition in Eq. (4.20) resembles the condition (2.5) already discussed in Chapter 2 with the particular phase-space function

$$\rho^J_{ab}(\sqrt{s}) = \frac{p_{\mathrm{cm},a}}{8\pi \sqrt{s}} \, 2M_a \, \delta_{ab} \,. \tag{4.21}$$

For sufficiently large $s$ the unitarity condition takes the even simpler form

$$\Im \, [t^J_\pm]^{-1}_{ab} = -\frac{p_{\mathrm{cm},a}}{8\pi \sqrt{s}} \, 2M_a \, \delta_{ab} \,. \tag{4.22}$$

Let us consider a simple example. For the scattering of a spin-zero boson off a spin-one-half fermion, the parity eigenstates are written as

$$|1_\pm, J\rangle_{0\frac{1}{2}} = \frac{1}{\sqrt{2}}\left(|0,-\tfrac{1}{2}\rangle_J \pm |0,+\tfrac{1}{2}\rangle_J\right) . \tag{4.23}$$

Using the decomposition (2.32), the helicity partial-wave amplitude can be written as

$$\begin{aligned} t^J_\pm(\sqrt{s}) = &\pm \sqrt{\frac{\bar{E}_\pm E_\pm}{4\bar{M}M}} \int_{-1}^{1} \frac{\mathrm{d}\cos\theta}{2} F_1^\pm(\sqrt{s},\cos\theta) P_{J-\frac{1}{2}}(\cos\theta) \\ &\mp \sqrt{\frac{\bar{E}_\mp E_\mp}{4\bar{M}M}} \int_{-1}^{1} \frac{\mathrm{d}\cos\theta}{2} F_1^\mp(\sqrt{s},\cos\theta) P_{J+\frac{1}{2}}(\cos\theta) , \end{aligned} \tag{4.24}$$

where $E_\pm$ and $\bar{E}_\pm$ are given in (2.47) and $P_l(z)$ is the Legendre polynomial. In (4.24), we used the representation of the Wigner function:

$$\begin{aligned} \cos\frac{\theta}{2}\, d^{(J)}_{+\frac{1}{2},\frac{1}{2}}(\theta) &= \frac{1}{2}\left[P_{J-\frac{1}{2}}(\cos\theta) + P_{J+\frac{1}{2}}(\cos\theta)\right] , \\ \sin\frac{\theta}{2}\, d^{(J)}_{-\frac{1}{2},\frac{1}{2}}(\theta) &= \frac{1}{2}\left[P_{J-\frac{1}{2}}(\cos\theta) - P_{J+\frac{1}{2}}(\cos\theta)\right] . \end{aligned} \tag{4.25}$$

From the result (4.24) together with the MacDowell relation (2.34) for the invariant amplitudes $F_1^\pm(\sqrt{s},t) = F_1^\mp(-\sqrt{s},t)$ it follows the MacDowell relation

$$t^J_\pm(-\sqrt{s}) = -t^J_\mp(+\sqrt{s}) , \tag{4.26}$$

for the helicity partial-wave amplitude.

We turn to more complicated sectors. For the parity eigenstates of a two-body state composed of a spin-one and a spin-one-half we use the following convention

$$\begin{aligned} |1_\pm, J\rangle_{1\frac{1}{2}} &= \frac{1}{\sqrt{2}}\left(|0,-\tfrac{1}{2}\rangle_J \mp |0,+\tfrac{1}{2}\rangle_J\right) , \\ |2_\pm, J\rangle_{1\frac{1}{2}} &= \frac{1}{\sqrt{2}}\left(|+1,+\tfrac{1}{2}\rangle_J \mp |-1,-\tfrac{1}{2}\rangle_J\right) , \\ |3_\pm, J\rangle_{1\frac{1}{2}} &= \frac{1}{\sqrt{2}}\left(|+1,-\tfrac{1}{2}\rangle_J \mp |-1,+\tfrac{1}{2}\rangle_J\right) . \end{aligned} \tag{4.27}$$

As a further example we consider the reaction $0^- + \frac{1}{2}^+ \to 1^- + \frac{1}{2}^+$. Using Eq. (2.42) we find

$$\begin{aligned} \left[t^J_\pm(\sqrt{s})\right]_{11} = \frac{\sqrt{s}}{2\bar{m}} \int_{-1}^{1} \frac{\mathrm{d}x}{2} \Bigg[ & P_{J-\frac{1}{2}}(x) \Bigg( \mp \frac{\bar{\delta}+1}{2} \sqrt{\frac{\bar{E}_\mp E_\mp}{\bar{M}M}}\, E_\pm F_3^\mp(\sqrt{s},x)\, x - \sqrt{\frac{\bar{E}_\pm E_\pm}{\bar{M}M}} \left[ \frac{\bar{\delta}+1}{2} F_1^\mp(\sqrt{s},x) \mp \bar{E}_\mp F_2^\mp(\sqrt{s},x) \right] \Bigg) \\ &+ P_{J+\frac{1}{2}}(x) \Bigg( \pm \frac{\bar{\delta}+1}{2} \sqrt{\frac{\bar{E}_\pm E_\pm}{\bar{M}M}}\, E_\mp F_3^\pm(\sqrt{s},x)\, x - \sqrt{\frac{\bar{E}_\mp E_\mp}{\bar{M}M}} \left[ \frac{\bar{\delta}+1}{2} F_1^\pm(\sqrt{s},x) \pm \bar{E}_\pm F_2^\pm(\sqrt{s},x) \right] \Bigg) \Bigg] , \\ \left[t^J_\pm(\sqrt{s})\right]_{21} = \int_{-1}^{1} \frac{\mathrm{d}x}{2} \Bigg[ & P_{J-\frac{1}{2}}(x) \Bigg( \sqrt{\frac{\bar{E}_\mp E_\mp}{8\bar{M}M}}\, E_\pm F_3^\mp(\sqrt{s},x)\, x \pm \sqrt{\frac{\bar{E}_\pm E_\pm}{8\bar{M}M}} \left[ 2F_1^\mp(\sqrt{s},x) \mp E_\mp F_3^\pm(\sqrt{s},x) \right] \Bigg) \\ &+ P_{J+\frac{1}{2}}(x) \Bigg( \sqrt{\frac{\bar{E}_\pm E_\pm}{8\bar{M}M}}\, E_\mp F_3^\pm(\sqrt{s},x)\, x \mp \sqrt{\frac{\bar{E}_\mp E_\mp}{8\bar{M}M}} \left[ 2F_1^\pm(\sqrt{s},x) \mp E_\pm F_3^\mp(\sqrt{s},x) \right] \Bigg) \Bigg] \\ \left[t^J_\pm(\sqrt{s})\right]_{31} = \pm \int_{-1}^{1} \frac{\mathrm{d}x}{2} \Bigg[ & P_{J-\frac{1}{2}}(x) \Bigg( \sqrt{\frac{2J-1}{2J+3}} \sqrt{\frac{\bar{E}_\pm E_\pm}{8\bar{M}M}}\, E_\mp F_3^\pm(\sqrt{s},x) - \sqrt{\frac{2J+3}{2J-1}} \sqrt{\frac{\bar{E}_\mp E_\mp}{8\bar{M}M}}\, E_\pm F_3^\mp(\sqrt{s},x)\, x \Bigg) \\ &+ P_{J+\frac{1}{2}}(x) \Bigg( \sqrt{\frac{2J+3}{2J-1}} \sqrt{\frac{\bar{E}_\mp E_\mp}{8\bar{M}M}}\, E_\pm F_3^\mp(\sqrt{s},x) - \sqrt{\frac{2J-1}{2J+3}} \sqrt{\frac{\bar{E}_\pm E_\pm}{8\bar{M}M}}\, E_\mp F_3^\pm(\sqrt{s},x)\, x \Bigg) \Bigg] . \end{aligned} \tag{4.28}$$

From the MacDowell relation (2.41) for the invariant amplitudes $F_n^{\pm}(\sqrt{s},t)=F_n^{\mp}(-\sqrt{s},t)$ there follow the MacDowell relations

$$\left[t^J_{\pm}(-\sqrt{s})\right]_{n1}=(-1)^{n+1}\left[t^J_{\mp}(+\sqrt{s})\right]_{n1}\,, \tag{4.29}$$

for the helicity partial-wave amplitudes.

The derivation of analogous expressions for the remaining reactions considered in this work is straightforward. We refrain from providing explicit results here.

## 4.2 Kinematical constraints

The helicity partial-wave amplitudes $t^J_{\pm}(\sqrt{s})$ suffer from various kinematical constraints that make their application in Eq. (2.6) cumbersome. This is readily seen by expanding the invariant amplitudes $F_n^{\pm}(\sqrt{s},\cos\theta)$ in terms of Legendre polynomials

$$F_n^{\pm}(\sqrt{s},\cos\theta)=\sum_{l=0}^{\infty}(2l+1)\,c^{\pm}_{n,l}(\sqrt{s})\,\left(\bar p_{\rm cm}\,p_{\rm cm}\right)^l\,P_l(\cos\theta)\,, \tag{4.30}$$

where we factored out the term $\left(\bar p_{\rm cm}\,p_{\rm cm}\right)^l$ from the expansion coefficients $c^{\pm}_{n,l}(\sqrt{s})$. This factor is required, because otherwise a singular behavior would arise from the behavior of the Legendre polynomial as

$$\begin{aligned}P_l(\cos\theta)&\to P_l\left(-\frac{\bar r\cdot r}{\sqrt{\bar r^2\,r^2}}\right)\,,\\ P_l(x)&=\sum_{k=0}^{[l/2]}(-1)^k\frac{(2\,l-2\,k)!}{2^l\,k!\,(l-k)!\,(l-2\,k)!}\,x^{l-2\,k}\,,\end{aligned} \tag{4.31}$$

where $r^2=-p^2_{\rm cm}$ or $\bar r^2=-\bar p^2_{\rm cm}$. Since the invariant amplitudes are free of kinematical constraints the expansion coefficients $c^{\pm}_{n,l}(\sqrt{s})$ are kinematically regular functions except for possible dynamical singularities.

We return to the simplest application $0^-+\frac{1}{2}^+\to 0^-+\frac{1}{2}^+$, for which the partial-wave amplitudes were expressed in (4.24) in terms of their invariant amplitudes. Applying (4.30) and the orthogonality of Legendre polynomials to Eq. (4.24), we can express the helicity partial-wave amplitudes $t^J_{\pm}(\sqrt{s})$ as

$$t^J_{\pm}(\sqrt{s})=\pm\sqrt{\frac{\bar E_{\pm}E_{\pm}}{4\bar M M}}\,c^{\pm}_{1,J-\frac{1}{2}}(\sqrt{s})\,\left(\bar p_{\rm cm}\,p_{\rm cm}\right)^{J-\frac{1}{2}}\mp\sqrt{\frac{\bar E_{\mp}E_{\mp}}{4\bar M M}}\,c^{\mp}_{1,J+\frac{1}{2}}(\sqrt{s})\,\left(\bar p_{\rm cm}\,p_{\rm cm}\right)^{J+\frac{1}{2}}\,. \tag{4.32}$$

The result (4.32) shows that $t^J_{\pm}(\sqrt{s})$ vanish not only at the thresholds, $\sqrt{s}=\pm(M+m)$ and $\sqrt{s}=\pm(\bar M+\bar m)$, but also at the pseudo-thresholds, $\sqrt{s}=\pm|M-m|$ and $\sqrt{s}=\pm|\bar M-\bar m|$ (see Eq. (4.3)). While $p^2_{\rm cm}=E_+\,E_-$ vanishes at $|\sqrt{s}|=|M\pm m|$ the energy $E_+$ at $\sqrt{s}=-M\pm m$ and $E_-$ at $\sqrt{s}=+M\pm m$ only. As a consequence we derive the threshold behavior of the helicity partial-wave amplitudes. We find

$$t^J_{+}(\sqrt{s})\propto\left(\bar p_{\rm cm}\,p_{\rm cm}\right)^{J-\frac{1}{2}}\,,\qquad t^J_{-}(\sqrt{s})\propto\left(\bar p_{\rm cm}\,p_{\rm cm}\right)^{J+\frac{1}{2}}\,, \tag{4.33}$$

for energies close to $\sqrt{s}=M\pm m$ or $\sqrt{s}=\bar M\pm\bar m$. Similarly we find

$$t^J_{+}(\sqrt{s})\propto\left(\bar p_{\rm cm}\,p_{\rm cm}\right)^{J+\frac{1}{2}}\,,\qquad t^J_{-}(\sqrt{s})\propto\left(\bar p_{\rm cm}\,p_{\rm cm}\right)^{J-\frac{1}{2}}\,, \tag{4.34}$$

for energies close to $\sqrt{s}=-M\pm m$ or $\sqrt{s}=-\bar M\pm\bar m$. The results (4.33) and (4.34) are kinematical constraints that hold irrespective of the detailed form of the interaction. They can be summarized into the condition

$$t^J_{\pm}(\sqrt{s})\propto\sqrt{\bar E_{\pm}\,E_{\pm}}\,\left(\bar p_{\rm cm}\,p_{\rm cm}\right)^{J-\frac{1}{2}}\,, \tag{4.35}$$

which holds at any threshold or pseudo-threshold. All kinematical constraints at $|\sqrt{s}| = |M \pm m|$ and $|\sqrt{s}| = |\bar{M} \pm \bar{m}|$ are removed by the following transformation

$$T^J_{\pm,0\frac{1}{2}\to 0\frac{1}{2}}(\sqrt{s}) \equiv \sqrt{s}\left(\frac{s}{\bar{p}_{\rm cm}\,p_{\rm cm}}\right)^{J-\frac{1}{2}}\sqrt{\frac{4\,\bar{M}\,M}{\bar{E}_\pm\,E_\pm}}\;t^J_\pm(\sqrt{s})\,, \tag{4.36}$$

where the transformation (4.36) implies a change in the phase-space distribution:

$$\rho^J_{\pm,0\frac{1}{2}\to 0\frac{1}{2}}(\sqrt{s}) = -\,\Im\left[T^J_\pm(\sqrt{s})\right]^{-1} = \frac{p_{\rm cm}}{8\,\pi\,s}\,E_\pm\left(\frac{p^2_{\rm cm}}{s}\right)^{J-\frac{1}{2}}. \tag{4.37}$$

According to Eq. (4.37) the phase-space function is asymptotically bounded at large $\sqrt{s}$. Note that the latter condition requires the factor $s^J$ in the transformation (4.36). The *covariant partial-wave amplitudes* $T^J_\pm(\sqrt{s})$ can be computed from the invariant amplitudes as follows

$$T^J_{\pm,0\frac{1}{2}\to 0\frac{1}{2}}(\sqrt{s}) = \pm\sqrt{s}\left[A^{J-\frac{1}{2}}_{\pm,1}(\sqrt{s}) - \sqrt{\frac{\bar{E}_\mp\,E_\mp}{\bar{E}_\pm\,E_\pm}}\left(\frac{\bar{p}_{\rm cm}\,p_{\rm cm}}{s}\right)A^{J+\frac{1}{2}}_{\mp,1}(\sqrt{s})\right], \tag{4.38}$$

where the coefficients

$$A^l_{\pm,n}(\sqrt{s}) \equiv \left(\frac{s}{\bar{p}_{\rm cm}\,p_{\rm cm}}\right)^l\int_{-1}^{1}\frac{{\rm d}\cos\theta}{2}F^\pm_n(\sqrt{s},\cos\theta)\,P_l(\cos\theta)\,. \tag{4.39}$$

are free of kinematical constraints with the exception at $\sqrt{s} = 0$. The regularity of the expansion coefficients is a direct consequence of standard properties of the Legendre polynomials

$$\int_{-1}^{1}{\rm d}x\,x^n\,P_l(x) = \begin{cases} 0 & \text{for } n < l \\ \dfrac{2^{l+1}\,l!\,l!}{(2\,l+1)!} & \text{for } n = l \end{cases}, \tag{4.40}$$

and the regularity of the invariant amplitudes $F^\pm_n(\sqrt{s},\cos\theta)$. From (4.36) with the MacDowell relations (4.26) for the helicity partial-wave amplitudes it leads to the MacDowell relations

$$T^J_\pm(+\sqrt{s}) = T^J_\mp(-\sqrt{s})\,, \tag{4.41}$$

for the *covariant partial-wave amplitudes*.

We assure that not only the expression of $T^J_\pm(\sqrt{s})$ in terms of $F^\pm_n(\sqrt{s},\cos\theta)$, but also the inverse result where $F^\pm_n(\sqrt{s},\cos\theta)$ is expressed in terms of $T^J_\pm(\sqrt{s})$ are free of kinematical constraints at $s \neq 0$. In order to reconstruct the invariant amplitudes $F^\pm_1(\sqrt{s},\cos\theta)$ in terms of $T^J_\pm(\sqrt{s})$, an alternative representation of the Wigner function is useful:

$$\begin{aligned}(J+\frac{1}{2})\,d^{(J)}_{+\frac{1}{2},\frac{1}{2}}(\theta) &= \cos\frac{\theta}{2}\left[P'_{J+\frac{1}{2}}(\cos\theta) - P'_{J-\frac{1}{2}}(\cos\theta)\right],\\ (J+\frac{1}{2})\,d^{(J)}_{-\frac{1}{2},\frac{1}{2}}(\theta) &= \sin\frac{\theta}{2}\left[P'_{J+\frac{1}{2}}(\cos\theta) + P'_{J-\frac{1}{2}}(\cos\theta)\right],\end{aligned} \tag{4.42}$$

which can be driven from (4.25) in application of the Appendix relations (C.14). Using (4.42) and the orthogonality of Wigner functions (see (C.1)), we derive the explicit result

$$F^\pm_1(\sqrt{s},\cos\theta) = \pm\sqrt{\frac{4\bar{M}M}{\bar{E}_\pm E_\pm}}\sum_J\left[t^J_\pm(\sqrt{s})\,P'_{J+\frac{1}{2}}(\cos\theta) - t^J_\mp(\sqrt{s})\,P'_{J-\frac{1}{2}}(\cos\theta)\right]. \tag{4.43}$$

Eq. (4.43) shows that the regularity of the invariant amplitude $F_1^{\pm}(\sqrt{s},\cos\theta)$ at the thresholds and pseudo-thresholds is a consequence of a cancellation of the singular behaviour of the Legendre polynomials (see Eq. (4.31)) and the vanishing of the helicity partial-wave amplitudes $t_{\pm}^{J}(\sqrt{s})$ (see Eq. (4.35)). In turn, any violation of the kinematical constraint (4.31) would imply a behaviour of the invariant amplitudes $F_1^{\pm}(\sqrt{s},\cos\theta)$ that is necessarily at odds with its Mandelstam representation as required by micro-causality. In contrast, if the invariant amplitudes are reconstructed in terms of the *covariant partial-wave amplitudes* $T_{\pm}^{J}(\sqrt{s})$ an artificial behaviour at the thresholds and pseudo-thresholds is avoided irrespective of any details of the approximation scheme applied to the partial-wave amplitudes. We derive the result

$$F_1^{\pm}(\sqrt{s},\cos\theta) = \pm\frac{1}{\sqrt{s}}\sum_J\left[Y'_{J+\frac{1}{2}}(\sqrt{s},\cos\theta)\,T_{\pm}^{J}(\sqrt{s}) - \frac{\bar{E}_{\mp}\,E_{\mp}}{s}\,Y'_{J-\frac{1}{2}}(\sqrt{s},\cos\theta)\,T_{\mp}^{J}(\sqrt{s})\right], \tag{4.44}$$

in terms of the regular combinations

$$Y'_n(\sqrt{s},\cos\theta) = \left(\frac{\bar{p}_{\rm cm}\,p_{\rm cm}}{s}\right)^{n-1} P'_n(\cos\theta)\,. \tag{4.45}$$

While the transformation Eq. (4.36) is well known in the literature (see e.g. [87]), corresponding transformations for the remaining processes studied in this work are not available. We continue with a detailed analysis of the kinematical constraints in the production process $0^- + \frac{1}{2}^+ \to 1^- + \frac{1}{2}^+$. Inserting the decomposition (4.30) into Eq. (4.28) and applying the recurrence and the orthogonality relations of Legendre polynomials as recalled in the Appendix relations (C.14), we can find

$$\begin{aligned}
\big[t_{\pm}^{J}(\sqrt{s})\big]_{11} &= \frac{\sqrt{s}}{2\,\bar{m}}\,\frac{\bar{\delta}+1}{2}\left[\mp\frac{(2J-1)}{2(2J)}\sqrt{\frac{\bar{E}_{\mp}\,E_{\mp}}{\bar{M}\,M}}\,E_{\pm}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J-\frac{3}{2}}\,c^{\mp}_{3,J-\frac{3}{2}}(\sqrt{s})\right.\\
&\quad-\sqrt{\frac{\bar{E}_{\pm}\,E_{\pm}}{\bar{M}\,M}}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J-\frac{1}{2}}\left(c^{\mp}_{1,J-\frac{1}{2}}(\sqrt{s})\mp\frac{(2J+1)}{2(2J+2)}\,E_{\mp}\,c^{\pm}_{3,J-\frac{1}{2}}(\sqrt{s})\right)\\
&\quad-\sqrt{\frac{\bar{E}_{\mp}\,E_{\mp}}{\bar{M}\,M}}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J+\frac{1}{2}}\left(c^{\pm}_{1,J+\frac{1}{2}}(\sqrt{s})\pm\frac{(2J+1)}{2(2J)}\,E_{\pm}\,c^{\mp}_{3,J+\frac{1}{2}}(\sqrt{s})\right)\\
&\quad\left.\pm\frac{(2J+3)}{2(2J+2)}\sqrt{\frac{\bar{E}_{\pm}\,E_{\pm}}{\bar{M}\,M}}\,E_{\mp}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J+\frac{3}{2}}\,c^{\pm}_{3,J+\frac{3}{2}}(\sqrt{s})\right]\\
&\quad\pm\frac{\sqrt{s}}{2\,\bar{m}}\left[\sqrt{\frac{\bar{E}_{\pm}\,E_{\pm}}{\bar{M}\,M}}\,\bar{E}_{\mp}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J-\frac{1}{2}}\,c^{\mp}_{2,J-\frac{1}{2}}(\sqrt{s})-\sqrt{\frac{\bar{E}_{\mp}\,E_{\mp}}{\bar{M}\,M}}\,\bar{E}_{\pm}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J+\frac{1}{2}}\,c^{\pm}_{2,J+\frac{1}{2}}(\sqrt{s})\right],\\
\big[t_{\pm}^{J}(\sqrt{s})\big]_{21} &= \frac{(2J-1)}{2(2J)}\sqrt{\frac{\bar{E}_{\mp}\,E_{\mp}}{8\,\bar{M}\,M}}\,E_{\pm}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J-\frac{3}{2}}\,c^{\mp}_{3,J-\frac{3}{2}}(\sqrt{s})\\
&\quad\pm2\sqrt{\frac{\bar{E}_{\pm}\,E_{\pm}}{8\,\bar{M}\,M}}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J-\frac{1}{2}}\left(c^{\mp}_{1,J-\frac{1}{2}}(\sqrt{s})\mp\frac{(2J+3)}{4(2J+2)}\,E_{\mp}\,c^{\pm}_{3,J-\frac{1}{2}}(\sqrt{s})\right)\\
&\quad\mp2\sqrt{\frac{\bar{E}_{\mp}\,E_{\mp}}{8\,\bar{M}\,M}}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J+\frac{1}{2}}\left(c^{\pm}_{1,J+\frac{1}{2}}(\sqrt{s})\pm\frac{(2J-1)}{4(2J)}\,E_{\pm}\,c^{\mp}_{3,J+\frac{1}{2}}(\sqrt{s})\right)\\
&\quad+\frac{(2J+3)}{2(2J+2)}\sqrt{\frac{\bar{E}_{\pm}\,E_{\pm}}{8\,\bar{M}\,M}}\,E_{\mp}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J+\frac{3}{2}}\,c^{\pm}_{3,J+\frac{3}{2}}(\sqrt{s}),\\
\big[t_{\pm}^{J}(\sqrt{s})\big]_{31} &= \pm\frac{\sqrt{2J-1}\,\sqrt{2J+3}}{2J\,(2J+2)}\left[-(J+1)\sqrt{\frac{\bar{E}_{\mp}\,E_{\mp}}{8\,\bar{M}\,M}}\,E_{\pm}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J-\frac{3}{2}}\,c^{\mp}_{3,J-\frac{3}{2}}(\sqrt{s})\right.\\
&\quad+J\sqrt{\frac{\bar{E}_{\pm}\,E_{\pm}}{8\,\bar{M}\,M}}\,E_{\mp}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J-\frac{1}{2}}\,c^{\pm}_{3,J-\frac{1}{2}}(\sqrt{s})+(J+1)\sqrt{\frac{\bar{E}_{\mp}\,E_{\mp}}{8\,\bar{M}\,M}}\,E_{\pm}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J+\frac{1}{2}}\,c^{\mp}_{3,J+\frac{1}{2}}(\sqrt{s})\\
&\quad\left.-J\sqrt{\frac{\bar{E}_{\pm}\,E_{\pm}}{8\,\bar{M}\,M}}\,E_{\mp}\,(\bar{p}_{\rm cm}\,p_{\rm cm})^{J+\frac{3}{2}}\,c^{\pm}_{3,J+\frac{3}{2}}(\sqrt{s})\right].
\end{aligned} \tag{4.46}$$

From Eq. (4.46) we conclude that the helicity partial-wave amplitudes $[t^J_\pm(\sqrt{s})]_{n1}$ vanish at the thresholds and pseudo-thresholds. A detailed analysis reveals the conditions

$$
\begin{aligned}
\mp\big[t^J_\pm(\sqrt{s})\big]_{31} &\propto \sqrt{\frac{E_\pm}{\bar E_\pm}}\,(\bar p_{\rm cm}\,p_{\rm cm})^{J-\frac{1}{2}}\,,\\
\left(\pm\frac{\big[t^J_\pm(\sqrt{s})\big]_{21}}{\bar E_\pm}+\sqrt{\frac{2J-1}{2J+3}}\,\frac{\big[t^J_\pm(\sqrt{s})\big]_{31}}{\bar E_\pm}\right) &\propto \sqrt{\frac{E_\pm}{\bar E_\pm}}\,(\bar p_{\rm cm}\,p_{\rm cm})^{J-\frac{1}{2}}\,,\\
\frac{1}{\sqrt{s}}\left(\pm\bar m\,\frac{\big[t^J_\pm(\sqrt{s})\big]_{11}}{\bar p^2_{\rm cm}}+\frac{\bar q_0}{\sqrt{2}}\,\frac{\big[t^J_\pm(\sqrt{s})\big]_{21}}{\bar p^2_{\rm cm}}\mp\frac{\bar q_0}{\sqrt{2}}\sqrt{\frac{2J-1}{2J+3}}\,\frac{\big[t^J_\pm(\sqrt{s})\big]_{31}}{\bar p^2_{\rm cm}}\right) &\propto \sqrt{\frac{E_\pm}{\bar E_\pm}}\,(\bar p_{\rm cm}\,p_{\rm cm})^{J-\frac{1}{2}}\,,
\end{aligned}
\tag{4.47}
$$

valid close to $|\sqrt{s}| = |m \pm M|$ or $|\sqrt{s}| = |\bar m \pm \bar M|$. The kinematical constraints can be eliminated by a non-unitary transformation only. We make the ansatz

$$
T^J_{\pm,0\frac{1}{2}\to1\frac{1}{2}}(\sqrt{s}) \equiv \sqrt{s}\left(\frac{s}{\bar p_{\rm cm}\,p_{\rm cm}}\right)^{J-\frac{1}{2}}\sqrt{\frac{4\bar M\,M}{\bar E_\pm\,E_\pm}}\,\left[\bar U^J_{\pm,1\frac{1}{2}}(\sqrt{s})\right]^T t^J_{\pm,0\frac{1}{2}\to1\frac{1}{2}}(\sqrt{s})\;U^J_{\pm,0\frac{1}{2}}(\sqrt{s})\,,
\tag{4.48}
$$

where the factors $s^J$ is introduced to arrive at an asymptotically bounded phase-space matrix for the $1^-\frac{1}{2}^+$ state. In (4.48), the nontrivial matrices $U^J_\pm(\sqrt{s})$ and $\bar U^J_\pm(\sqrt{s})$ characterize the transformation for the initial and final two-body states from the helicity basis to the new kinematical-free basis. For a two-body state, $U^J_\pm(\sqrt{s})$ can be recovered from $\bar U^J_\pm(\sqrt{s})$ by removing the bars. Using the condition (4.47), we find

$$
U^J_{\pm,1\frac{1}{2}}(\sqrt{s}) = \begin{pmatrix} \frac{\sqrt{s}\,m}{E_\mp} & 0 & 0\\ \pm\frac{\sqrt{s}\,q_0}{\sqrt{2}\,E_\mp} & 1 & 0\\ -\sqrt{\frac{2J-1}{2J+3}}\,\frac{\sqrt{s}\,q_0}{\sqrt{2}\,E_\mp} & \pm\sqrt{\frac{2J-1}{2J+3}} & \mp\frac{E_\pm}{\sqrt{s}} \end{pmatrix}\,, \qquad U^J_{\pm,0\frac{1}{2}}(\sqrt{s}) = 1\,.
\tag{4.49}
$$

It follows

$$
\begin{aligned}
\left[T^J_{\pm,0\frac{1}{2}\to1\frac{1}{2}}(\sqrt{s})\right]_{11} &= \pm\,s\sqrt{s}\left(A^{J-\frac{1}{2}}_{\mp,2}(\sqrt{s})-\frac{\bar E_\pm\,E_\mp}{s}\,A^{J+\frac{1}{2}}_{\pm,2}(\sqrt{s})\right)\\
&\quad-(\bar\delta+1)\sqrt{s}\,E_\mp A^{J+\frac{1}{2}}_{\pm,1}(\sqrt{s})\mp\frac{\bar\delta+1}{2}\,\sqrt{s}\,E_\pm\,E_\mp\left(A^{J+\frac{1}{2}}_{\mp,3}(\sqrt{s})-\frac{(\bar p_{\rm cm}\,p_{\rm cm})^2}{s\,\bar E_\mp\,E_\pm}\,A^{J+\frac{3}{2}}_{\pm,3}(\sqrt{s})\right),\\
\left[T^J_{\pm,0\frac{1}{2}\to1\frac{1}{2}}(\sqrt{s})\right]_{21} &= \pm\,\sqrt{2s}\left(A^{J-\frac{1}{2}}_{\mp,1}(\sqrt{s})-\frac{\bar E_\mp\,E_\mp}{s}\,A^{J+\frac{1}{2}}_{\pm,1}(\sqrt{s})\right)-\frac{\sqrt{s}\,E_\mp}{\sqrt{2}\,(J+1)}\left(A^{J-\frac{1}{2}}_{\mp,3}(\sqrt{s})-\frac{(\bar p_{\rm cm}\,p_{\rm cm})^2}{s^2}\,A^{J+\frac{3}{2}}_{\pm,3}(\sqrt{s})\right),\\
\left[T^J_{\pm,0\frac{1}{2}\to1\frac{1}{2}}(\sqrt{s})\right]_{31} &= \frac{\sqrt{2J-1}\,\sqrt{2J+3}}{2J\,(2J+2)}\,\frac{s}{\sqrt{2}}\left[(J+1)A^{J-\frac{3}{2}}_{\mp,3}(\sqrt{s})-J\,\frac{\bar E_\pm\,E_\mp}{s}\,A^{J-\frac{1}{2}}_{\pm,3}(\sqrt{s})\right.\\
&\quad\left.-\frac{(\bar p_{\rm cm}\,p_{\rm cm})^2}{s^2}\left((J+1)A^{J+\frac{1}{2}}_{\mp,3}(\sqrt{s})-J\,\frac{\bar E_\pm\,E_\mp}{s}\,A^{J+\frac{3}{2}}_{\pm,3}(\sqrt{s})\right)\right],
\end{aligned}
\tag{4.50}
$$

where the functions $A^l_{\pm,n}(\sqrt{s})$ were introduced in (4.39). Our result (4.50) proves that the expansion coefficients are regular at $s \neq 0$. We note that also the inverse relation analogous to Eq. (4.44) is regular at $s \neq 0$ in this case. Each sign inside of $U^J_\pm(\sqrt{s})$ (4.49) is deliberately taken for the MacDowell relations

$$
\big[T^J_\pm(+\sqrt{s})\big]_{n1} = \big[T^J_\mp(-\sqrt{s})\big]_{n1}\,,
\tag{4.51}
$$

for the *covariant partial-wave amplitudes*. It is analogous to Eq. (4.41) but different from Eq. (4.29).

An analogous analysis is performed for the scattering of a spin-one off a spin-one-half states. Here we consider the ansatz

$$
T^J_{\pm,1\frac{1}{2}\to1\frac{1}{2}}(\sqrt{s}) \equiv \sqrt{s}\left(\frac{s}{\bar p_{\rm cm}\,p_{\rm cm}}\right)^{J-\frac{1}{2}}\sqrt{\frac{4\bar M\,M}{\bar E_\pm\,E_\pm}}\,\left[\bar U^J_{\pm,1\frac{1}{2}}(\sqrt{s})\right]^T t^J_{\pm,1\frac{1}{2}\to1\frac{1}{2}}(\sqrt{s})\;U^J_{\pm,1\frac{1}{2}}(\sqrt{s})\,,
\tag{4.52}
$$

with the transformation matrix $U^J_{\pm,1\frac{1}{2}}(\sqrt{s})$ already introduced in Eq. (4.49). Explicit computations reveal that the covariant partial-wave amplitudes introduced in Eq. (4.52) are unconstrained at $s \neq 0$.

# 5 SU(3) coupled-channel dynamics

In this chapter we consider the scattering of pseudoscalar mesons off spin-one-half baryons based on the chiral Lagrangian [31, 32, 39, 40]. Besides the octet of Goldstone bosons we consider the baryon octet and decuplet ground states with $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ respectively. The leading order counter terms are estimated by the inclusion of vector meson fields with $J^P = 1^-$ in the tensor representation. We collect the tree-level coupled-channel interaction terms and compute the invariant amplitudes of Chapter 2 applying the projection method developed in Chapter 3. Following [45, 46, 61, 48] we solve the non-linear dispersion integral equation (2.6) for the partial-wave amplitudes, where an analytic continuation of the generalized potential is performed.

The main focus of our study in this chapter is the formation of baryon resonances with $J^P = \frac{1}{2}^-$. It is well known that such resonances can be dynamically generated by the leading order chiral interaction, the Weinberg-Tomozawa term [17, 19, 23, 88]. Though a qualitative reproduction of the empirical resonance states is possible, a quantitative description is still an open challenge. The details of the generated resonance spectrum depend on the particular unitarization scheme applied [89, 90, 91, 31]. Also the effect of the long-range forces implied by the u-channel exchange processes has not been investigated systematically so far. So far either an on-shell factorization [60] or an on-shell reduction scheme [31] has been applied. Both schemes break down in the presence of long-range forces. Given the novel scheme introduced in [45, 46, 61, 48] studies of the effect of the u-channel exchange processes are possible now. Here we apply this scheme for the first time to the formation of $J^P = \frac{1}{2}^-$ resonances via coupled-channel dynamics based on the chiral Lagrangian.

A further interesting issue is the role played by vector-meson degrees of freedom. This depends on the way the vector mesons are introduced in the chiral Lagrangian. In the phenomenology of the hidden local gauge approach the t-channel vector meson exchange process contains the leading order Weinberg-Tomozawa interaction of the systematic chiral Lagrangian approach. If the vector mesons are introduced as heavy fields they start to contribute at subleading order only, i.e. they do not affect the Weinberg-Tomozawa theorem but may be used to estimate the counter terms at subleading order. We will scrutinize this picture by computing the effect of the t-channel exchange process on the resonance spectrum.

## 5.1 Chiral SU(3) Lagrangian

We recall the interaction terms of the relativistic chiral SU(3) Lagrangian density relevant for the meson-baryon scattering process. For details on the systematic construction principle, see, for example, [12, 13, 92, 93, 14]. The basic building blocks of the chiral Lagrangian are

$$U_\mu = \tfrac{1}{2}\, e^{-i\Phi/2f} \left(\partial_\mu e^{i\Phi/f}\right) e^{-i\Phi/2f} \,, \qquad V_{\mu\nu}\,, \qquad B\,, \qquad B_\nu\,, \tag{5.1}$$

where we include the pseudoscalar meson octet field $\Phi(J^P = 0^-)$, the vector meson nonet field $V_{\mu\nu}(J^P = 1^-)$, the baryon octet field $B(J^P = \frac{1}{2}^+)$, and the baryon decuplet field $B_\nu(J^P = \frac{3}{2}^+)$. The vector mesons are represented by anti-symmetric tensor fields $V_{\mu\nu} = -V_{\nu\mu}$ [94, 95, 96, 97]. The parameter $f$ in Eq. (5.1) is the chiral limit value of the pion-decay constant [98, 40].

We assume perfect isospin symmetry and decompose the fields into their isospin multiplets

$$\begin{aligned}
&\Phi = \Phi^a \lambda_a = \vec{\tau}\cdot\vec{\pi}(140) + \alpha^\dagger \cdot K(494) + K^\dagger(494)\cdot\alpha + \eta(547)\lambda_8\,,\\
&V_{\mu\nu} = \vec{\tau}\cdot\vec{\rho}_{\mu\nu}(770) + \alpha^\dagger\cdot \mathbf{K}_{\mu\nu}(892) + \mathbf{K}^\dagger_{\mu\nu}(892)\cdot\alpha + (\tfrac{2}{3} + \tfrac{1}{\sqrt{3}}\lambda_8)\omega_{\mu\nu}(782) + (\tfrac{\sqrt{2}}{3} - \tfrac{\sqrt{2}}{\sqrt{3}}\lambda_8)\phi_{\mu\nu}(1020)\,,\\
&\sqrt{2}B = \sqrt{2}B^a\lambda_a = \alpha^\dagger\cdot N(939) + \lambda_8\Lambda(1115) + \vec{\tau}\cdot\vec{\Sigma}(1195) + \Xi^T(1315)i\sigma^2\cdot\alpha\,,\\
&\sqrt{2}\,\alpha^\dagger = (\lambda_4 + i\lambda_5,\, \lambda_6 + i\lambda_7)\,, \qquad \vec{\tau} = (\lambda_1,\, \lambda_1,\, \lambda_3)\,,
\end{aligned} \tag{5.2}$$

with the isospin singlet fields $\eta$, $\omega_{\mu\nu}$, and $\phi_{\mu\nu}$, the doublet fields, e.g. $K=(K^+,K^0)^T$ and $\Xi=(\Xi^0,\Xi^-)^T$, and the triplet fields, e.g. $\vec{\pi}=(\pi^1,\pi^2,\pi^3)$. The matrices $\lambda_a$ are the $SU(3)$ generators, also known as the Gell-Mann matrices. The numbers in brackets recall the approximate masses of the particles in units of MeV [32, 39]. The completely symmetric baryon decuplet fields $B_\nu^{abc}$ are related to the physical states as

$$
\begin{aligned}
&B_\nu^{111}=\Delta_\nu^{++}, \quad && B_\nu^{112}=\Delta_\nu^{+}/\sqrt{3}, \quad && B_\nu^{122}=\Delta_\nu^{0}/\sqrt{3}, \quad && B_\nu^{222}=\Delta_\nu^{-},\\
&B_\nu^{113}=\Sigma_\nu^{+}/\sqrt{3}, && B_\nu^{123}=\Sigma_\nu^{0}/\sqrt{6}, && B_\nu^{223}=\Sigma_\nu^{-}/\sqrt{3}, && \\
&B_\nu^{133}=\Xi_\nu^{0}/\sqrt{3}, && B_\nu^{233}=\Xi_\nu^{-}/\sqrt{3}, && && \\
&B_\nu^{333}=\Omega_\nu^{-}, && && &&
\end{aligned}
\tag{5.3}
$$

which combine to the isospin multiplet fields $\Delta(1232)$, $\vec{\Sigma}(1385)$, $\Xi(1530)$, and $\Omega(1672)$ [40].

In QCD the chiral $G=SU(3)_L\otimes SU(3)_R$ group is spontaneously broken down to the subgroup $H=SU(3)_V\subset G$. This implies a non-linear transformation for massive fields coupled to the Goldstone bosons [12, 13, 92, 93, 14]. The transformation depends only the SU(3) flavour structure of the field considered. For $h\in H$, the transformations of the fields are

$$
U_\mu \to h\,U_\mu\,h^{-1}, \qquad V_{\mu\nu}\to h\,V_{\mu\nu}\,h^{-1}, \qquad B\to h\,B\,h^{-1}, \qquad B_\nu^{abc}\to h^a_{\bar a}\,h^b_{\bar b}\,h^c_{\bar c}\,B_\nu^{\bar a\bar b\bar c}\,. \tag{5.4}
$$

Covariant derivatives of any of the fields are constructed

$$
\begin{aligned}
&D_\lambda B=\partial_\lambda B+[\Gamma_\lambda,B]_-\,, \qquad && D_\lambda V_{\mu\nu}=\partial_\lambda V_{\mu\nu}+[\Gamma_\lambda,V_{\mu\nu}]_-\,,\\
&(D_\lambda B_\nu)^{abc}=\partial_\lambda B_\nu^{abc}+\Gamma^a_{\lambda,n}B_\nu^{nbc}+\Gamma^b_{\lambda,n}B_\nu^{anc}+\Gamma^c_{\lambda,n}B_\nu^{abn}\,, &&
\end{aligned}
\tag{5.5}
$$

in terms of the chiral connection

$$
\Gamma_\lambda=\tfrac{1}{2}\left(e^{-i\Phi/2f}\,(\partial_\lambda e^{i\Phi/2f})+e^{i\Phi/2f}\,(\partial_\lambda e^{-i\Phi/2f})\right)\,, \tag{5.6}
$$

such that they again have the transformation properties depending on their SU(3) flavour structures only (5.4). Note that we use the notations $[A,B]_\pm=AB\pm BA$ for SU(3) matrices $A$ and $B$ throughout this work.

The chiral Lagrangian contains all possible interaction terms, formed with the fields $U_\mu$, $V_{\mu\nu}$, $B$, $B_\nu$, and their covariant derivatives. Further terms in the chiral Lagrangian that are proportional to the light quark-masses of QCD are not discussed here explicitly. The relative importance of the various terms is estimated by counting the number of 'small' derivatives in a given vertex [12, 13, 92, 93, 14]. The form of the kinetic part of the chiral Lagrangian is

$$
\begin{aligned}
\mathscr{L}_{\rm kin}=\;&\mathbf{tr}\,\bar B\,(i\,\partial\!\!\!/-M_{[8]})\,B-f^2\,\mathbf{tr}\,U_\mu\,U^\mu-\frac{1}{4}\,\mathbf{tr}\,(D_\mu V^{\mu\lambda})\,(D^\nu V_{\nu\lambda})+\frac{1}{8}\,m^2_{[9]}\,\mathbf{tr}\,V_{\mu\nu}\,V^{\mu\nu}\\
&-\mathbf{tr}\,\bar B_\mu\cdot\Big((i\,\partial\!\!\!/-M_{[10]})g^{\mu\nu}-i(\gamma^\mu D^\nu+\gamma^\nu D^\mu)+\gamma^\mu(i\,\partial\!\!\!/+M_{[10]})\gamma^\nu\Big)\,B_\nu\,,
\end{aligned}
\tag{5.7}
$$

where $m_{[9]}$, $M_{[8]}$, and $M_{[10]}$ are the masses of nonet vector meson, octet baryon, and decuplet baryon in the chiral limit, respectively. In Eq. (5.7) we apply the convenient *dot-notation* suggested in [31]. The objects $\bar B_{\bar\nu}\cdot B_\nu$, $\bar B_{\bar\nu}\cdot\Phi$, and $\Phi\cdot B_\nu$ transform as an SU(3) octet

$$
(\bar B^{\bar\nu}\cdot B_\nu)^a_b=\bar B^{\bar\nu}_{bcd}B_\nu^{acd}\,, \quad (\bar B^{\bar\nu}\cdot\Phi)^a_b=\epsilon^{kla}\bar B^{\bar\nu}_{knb}\Phi^n_l\,, \quad (\Phi\cdot B_\nu)^a_b=\epsilon_{klb}\Phi^l_n B_\nu^{kna}\,, \tag{5.8}
$$

where $\bar B_{\bar\nu}$ and $B_\nu$ are anti-decuplet and decuplet fields, $\Phi$ is an octet field, and $\epsilon_{abc}$ is the completely antisymmetric pseudo-tensor.

Expanding the chiral connection $\Gamma_\mu$ in powers of the Goldstone boson fields, we can extract the leading order two-body interaction, the Weinberg-Tomozawa terms, as

$$
\begin{aligned}
\mathscr{L}_{\rm WT}=\;&\frac{i}{8\,f^2}\,\mathbf{tr}\,\bar B\,\gamma^\mu[[\Phi,(\partial_\mu\Phi)]_-,B]_-+\frac{3\,i}{8\,f^2}\,\mathbf{tr}\,(\bar B^\nu\cdot\gamma^\mu B_\nu)\,[\Phi,(\partial_\mu\Phi)]_-\\
&-\frac{1}{16\,f^2}\,\mathbf{tr}\,(\partial_\mu V^{\mu\lambda})\,[[\Phi,(\partial^\nu\Phi)]_-,V_{\nu\lambda}]_-+\frac{1}{48\,f^2}\,\mathbf{tr}\,[\Phi,(\partial_\mu\Phi)]_-\,[\Phi,(\partial^\mu\Phi)]_-\,.
\end{aligned}
\tag{5.9}
$$

The first term in (5.9) reproduces the low-energy theorems of meson-nucleon scattering derived first by Weinberg and Tomazawa applying current-algebra techniques [99, 100].

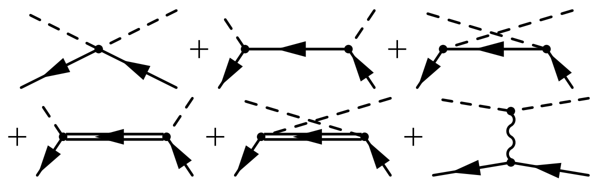
We collect all interaction terms with at most one derivative involved that contribute to the scattering of a pseudoscalar meson off a baryon octet state at tree-level. The terms are sorted according to the number of fields involved:

$$
\begin{aligned}
\mathscr{L}^{(2)} = &\, \mathbf{tr}\,\bar{B}\,(i\slashed{\partial} - M_{[8]})B + \frac{1}{4}\mathbf{tr}\,(\partial_\mu\Phi)(\partial^\mu\Phi) - \frac{1}{4}\mathbf{tr}\,(\partial_\mu V^{\mu\lambda})(\partial^\nu V_{\nu\lambda}) + \frac{1}{8}m^2_{[9]}\,\mathbf{tr}\,V_{\mu\nu}\,V^{\mu\nu} \\
& - \mathbf{tr}\,\bar{B}_\mu \cdot \Big((i\slashed{\partial} - M_{[10]})g^{\mu\nu} - i(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu) + \gamma^\mu(i\slashed{\partial} + M_{[10]})\gamma^\nu\Big)\,B_\nu\,, \\
\mathscr{L}^{(3)} = & - \frac{F_A}{2\,f}\,\mathbf{tr}\,\bar{B}\,\gamma_\mu\gamma_5\,[(\partial^\mu\Phi),B]_- - \frac{D_A}{2\,f}\,\mathbf{tr}\,\bar{B}\,\gamma_\mu\gamma_5\,[(\partial^\mu\Phi),B]_+ \\
& - \frac{C_A}{2\,f}\,\mathbf{tr}\,\Big((\bar{B}_\mu\cdot(\partial^\mu\Phi))B + \bar{B}\,((\partial^\mu\Phi)\cdot B_\mu)\Big) \\
& - \frac{F_V}{2\,f_V}\,\mathbf{tr}\,\bar{B}\gamma_\mu[(\partial_\nu V^{\mu\nu}),B]_- - \frac{D_V}{2\,f_V}\,\mathbf{tr}\,\bar{B}\gamma_\mu[(\partial_\nu V^{\mu\nu}),B]_+ - \frac{G_V}{2\,f_V}\,\mathbf{tr}\,\bar{B}\gamma_\mu B\,\mathbf{tr}\,(\partial_\nu V^{\mu\nu}) \\
& - \frac{F_T\,m_V}{8\,f_V}\,\mathbf{tr}\,\bar{B}\sigma_{\mu\nu}[V^{\mu\nu},B]_- - \frac{D_T\,m_V}{8\,f_V}\,\mathbf{tr}\,\bar{B}\sigma_{\mu\nu}[V^{\mu\nu},B]_+ - \frac{G_T\,m_V}{8\,f_V}\,\mathbf{tr}\,\bar{B}\sigma_{\mu\nu}B\,\mathbf{tr}\,V^{\mu\nu} \\
& - i\frac{f_V\,h_P}{8\,f^2}\,\mathbf{tr}\,(\partial_\mu\Phi)\,V^{\mu\nu}\,(\partial_\nu\Phi)\,, \\
\mathscr{L}^{(4)} = &\, \frac{i}{8\,f^2}\,\mathbf{tr}\,\bar{B}\,\gamma^\mu[[\Phi,(\partial_\mu\Phi)]_-,B]_-\,,
\end{aligned}
\tag{5.10}
$$

where the $F_A$, $D_A$, $C_A$, $F_V$, $D_V$, $G_V$, $F_T$, $D_T$, $G_T$, $f_V$, and $h_P$ are dimensionless and the parameter $m_V$ carries dimension of mass. Here we assume exact charge conjugation symmetry and parity invariance. Note that the chiral Lagrangian that would lead to the terms in (5.10) is easily constructed by replacing $\partial_\mu\Phi \to -2if\,U_\mu$ (or $2if\,U_\mu^\dagger$). Derivatives acting on baryon fields in (5.10) must be understood as covariant derivatives $\partial_\mu B \to D_\mu B$ and $\partial_\mu B_\nu \to D_\mu B_\nu$.

We collect the tree-level interaction terms contributing to the scattering of pseudoscalar mesons off octet baryons: the Weinberg-Tomazawa interaction, the s- and u-channel octet baryon exchanges, the s- and u-channel decuplet baryon exchanges, and the vector meson t-channel exchange. We write

$$
i\,K^{(I,S)}_{0\frac{1}{2}\to 0\frac{1}{2}}(\bar{k},k,w) =
$$



$$
\begin{aligned}
= &\, \frac{C^{(I,S)}_{\mathrm{WT}}}{4f^2}\,(\bar{\slashed{q}} + \slashed{q}) \\
& + \sum_{[8]} \frac{1}{4f^2}\,C^{(I,S)}_{[8],AA}\,\gamma_5\bar{\slashed{q}}S_{[8]}(p+q)\slashed{q}\gamma_5 \\
& + \sum_{[8]} \frac{1}{4f^2}\,\tilde{C}^{(I,S)}_{[8],AA}\,\gamma_5\slashed{q}S_{[8]}(p-\bar{q})\bar{\slashed{q}}\gamma_5 \\
& - \sum_{[10]} \frac{1}{4f^2}\,C^{(I,S)}_{[10],AA}\,\bar{\Gamma}^{(A)}_\rho(\bar{q})S^{\rho\sigma}_{[10]}(p+q)\Gamma^{(A)}_\sigma(q) \\
& - \sum_{[10]} \frac{1}{4f^2}\,\tilde{C}^{(I,S)}_{[10],AA}\,\bar{\Gamma}^{(A)}_\rho(-q)S^{\rho\sigma}_{[10]}(p-\bar{q})\Gamma^{(A)}_\sigma(-\bar{q}) \\
& + i\sum_{[9],X} \frac{1}{4f^2}\,C^{(I,S)}_{[9],PX}\,\Gamma^{(P)}_{\rho\sigma}(\bar{q},q)G^{\rho\sigma,\zeta\eta}_{[9]}(\bar{q}-q)\,\Gamma^{(X)}_{\zeta\eta}(\bar{q}-q)\,,
\end{aligned}
\tag{5.11}
$$

where the sums run first over the vector (V) and tensor (T) couplings for the vector mesons, $X \in \{V, T\}$, and then over all baryon-octet and baryon-decuplet states with $[8] \in \{\Sigma, N, \Xi, \Lambda\}$ and $[10] \in \{\Delta_\mu, \Sigma_\mu, \Xi_\mu, \Omega_\mu\}$ for the s- and u-channel baryon exchanges, but $[9] \in \{\rho_{\mu\nu}, K_{\mu\nu}, \omega_{\mu\nu}, \phi_{\mu\nu}\}$ in the case of t-channel vector meson exchange. In Eq. (5.11) we use the generic baryonic vertices

$$
\begin{aligned}
&\Gamma^{(A)}_\mu(q) = q_\mu\,, \\
&\Gamma^{(V)}_{\mu\nu}(q) = \frac{i}{2}\left(\gamma_\mu q_\nu - q_\mu \gamma_\nu\right), \qquad \Gamma^{(T)}_{\mu\nu}(q) = \frac{i\,m_V}{8}\,[\gamma_\mu, \gamma_\nu]_-\,, \\
&\bar\Gamma^{(A)}_\mu(q) = \gamma_0\left[\Gamma^{(A)}_\mu(q)\right]^\dagger \gamma_0\,, \qquad \bar\Gamma^{(X)}_{\mu\nu}(q) = \gamma_0\left[\Gamma^{(X)}_{\nu\mu}(q)\right]^\dagger \gamma_0\,,
\end{aligned}
\tag{5.12}
$$

where $X \in \{V, T\}$ and the mesonic vertex

$$
\Gamma^{\mu\nu}_{(P)}(\bar q, q) = \frac{1}{2}\left(\bar q^\mu q^\nu - q^\mu \bar q^\nu\right). \tag{5.13}
$$

The $\Gamma^{(X)}_{\mu\nu}(q)$ and $\Gamma^{\mu\nu}_{(P)}(\bar q, q)$ are anti-symmetric under exchange of the Lorentz indices. The vertices have different dimensions, but it is easy to see their dimensions by counting the number of momenta in the arguments of the vertices. The propagators of the octet and decuplet baryons and the nonet of vector meson can be directly derived from $\mathscr{L}^{(2)}$ in (5.10) as

$$
S_{[8]}(p) = \frac{1}{\not p - M_{[8]}}\,, \tag{5.14}
$$

$$
S^{\mu\nu}_{[10]}(p) = \frac{\not p + M_{[10]}}{p^2 - M^2_{[10]}}\left(-g^{\mu\nu} + \frac{2}{3\,M^2_{[10]}}p^\mu p^\nu + \frac{1}{3\,M_{[10]}}\left(\gamma^\mu p^\nu - \gamma^\nu p^\mu\right) + \frac{1}{3}\gamma^\mu\gamma^\nu\right), \tag{5.15}
$$

$$
G^{\mu\nu,\rho\sigma}_{[9]}(k) = \frac{1}{k^2 - m^2_{[9]}}\left[-\frac{1}{m^2_{[9]}}\begin{pmatrix} g^{\mu\rho}k^\nu k^\sigma - g^{\nu\rho}k^\mu k^\sigma \\ -g^{\mu\sigma}k^\nu k^\rho + g^{\nu\sigma}k^\mu k^\rho \end{pmatrix} + \frac{k^2 - m^2_{[9]}}{m^2_{[9]}}\left(g^{\mu\rho}g^{\nu\sigma} - g^{\nu\rho}g^{\mu\sigma}\right)\right]. \tag{5.16}
$$

It remains to specify the coupled-channel structure of the diagrams. This is done in terms of the coefficients $C^{(I,S)}_{\rm WT}$, $C^{(I,S)}_{[8],AA}$, $\tilde C^{(I,S)}_{[8],AA}$, $C^{(I,S)}_{[10],AA}$, $\tilde C^{(I,S)}_{[10],AA}$, and $C^{(I,S)}_{[9],PX}$ that are introduced in Eq. (5.11) and detailed in Appendix D. We use a convention for the coupled-channel states as implied by Tab. (5.1) and explained in [85, 31]. In the $SU(3)$ limit the meson-baryon scattering channels can be decomposed into flavour invariants according to the following decomposition

$$
\begin{aligned}
&8\otimes 8 = 1\oplus 8\oplus 8\oplus 10\oplus \bar{10}\oplus 27\,,\\
&(I,S)_1 = (0,-1), \quad (I,S)_8 = \begin{pmatrix} (\frac{1}{2},0) \\ (1,-1),\,(0,-1) \\ (\frac{1}{2},-2)\end{pmatrix}, \quad (I,S)_{10} = \begin{pmatrix} (\frac{3}{2},0) \\ (1,-1) \\ (\frac{1}{2},-2) \\ (0,-3)\end{pmatrix},\\
&(I,S)_{\bar{10}} = \begin{pmatrix} (0,1) \\ (\frac{1}{2},0) \\ (1,-1) \\ (\frac{3}{2},-2)\end{pmatrix}, \quad (I,S)_{27} = \begin{pmatrix} (1,1) \\ (\frac{3}{2},0),\,(\frac{1}{2},0) \\ (2,-1),\,(1,-1),\,(0,-1) \\ (\frac{3}{2},-2),\,(\frac{1}{2},-2) \\ (1,-3)\end{pmatrix}.
\end{aligned}
\tag{5.17}
$$

Consequently, the scattering problem decouples into eleven orthogonal channels specified by isospin $(I)$ and strangeness $(S)$ quantum numbers

$$
(I,S) \in \{(0,1),\,(1,1),\,(\tfrac{1}{2},0),\,(\tfrac{3}{2},0),\,(0,-1),\,(1,-1),\,(2,-1),\,(\tfrac{1}{2},-2),\,(\tfrac{3}{2},-2)(0,-3),\,(1,-3)\}\,. \tag{5.18}
$$

The various channels contributions in a given sector $(I,S)$ are listed in Tab. (5.1).

| $(0,1)$ | $(1,1)$ | $(\frac{1}{2},0)$ | $(\frac{3}{2},0)$ |
|---|---|---|---|
| $\left(\frac{1}{\sqrt{2}}(K^T i\sigma_2 N)\right)$ | $\left(\frac{1}{\sqrt{2}}(K^T i\sigma_2 \vec{\sigma} N)\right)$ | $\begin{pmatrix} \frac{1}{\sqrt{3}}(\vec{\pi}\cdot\vec{\sigma}N) \\ (\eta N) \\ (K\Lambda) \\ \frac{1}{\sqrt{3}}(\vec{\Sigma}\cdot\vec{\sigma}K) \end{pmatrix}$ | $\begin{pmatrix} (\vec{\pi}\cdot\vec{T}N) \\ (\vec{\Sigma}\cdot\vec{T}K) \end{pmatrix}$ |
| $(0,-1)$ | $(1,-1)$ | $(2,-1)$ | $(\frac{1}{2},-2)$ |
| $\begin{pmatrix} \frac{1}{\sqrt{3}}(\vec{\pi}\cdot\vec{\Sigma}) \\ \frac{1}{\sqrt{2}}(\bar{K}N) \\ (\eta\Lambda) \\ \frac{1}{\sqrt{2}}(K^T i\sigma_2 \Xi) \end{pmatrix}$ | $\begin{pmatrix} (\vec{\pi}\Lambda) \\ \frac{1}{i\sqrt{2}}(\vec{\pi}\times\vec{\Sigma}) \\ \frac{1}{\sqrt{2}}(\bar{K}\vec{\sigma}N) \\ (\eta\vec{\Sigma}) \\ \frac{1}{\sqrt{2}}(K^T i\sigma_2\vec{\sigma}\Xi) \end{pmatrix}$ | $\left(\frac{1}{2}(\pi_i\Sigma_j+\pi_j\Sigma_i)-\frac{1}{3}\delta_{ij}\vec{\pi}\cdot\vec{\Sigma}\right)$ | $\begin{pmatrix} \frac{1}{\sqrt{3}}(\vec{\pi}\cdot\vec{\sigma}\Xi) \\ (i\sigma_2\bar{K}^T\Lambda) \\ \frac{1}{\sqrt{3}}(\vec{\Sigma}\cdot\vec{\sigma}i\sigma_2\bar{K}^T) \\ (\eta\Xi) \end{pmatrix}$ |
| $(\frac{3}{2},-2)$ | $(0,-3)$ | $(1,-3)$ | $(\frac{1}{2},-4)$ |
| $\begin{pmatrix} (\vec{\pi}\cdot\vec{T}\Xi) \\ (\vec{\Sigma}\cdot\vec{T}i\sigma_2\bar{K}^T) \end{pmatrix}$ | $\left(\frac{1}{\sqrt{2}}(\bar{K}\Xi)\right)$ | $\left(\frac{1}{\sqrt{2}}(\bar{K}\vec{\sigma}\Xi)\right)$ | |

**Table 5.1.:** The coupled-channel states of $8\otimes 8$ characterized by isospin (I) and strangeness (S). The Pauli matrices $\sigma_i$ act on isospin doublet fields, e.g. $N$ and $K$. The $4\times 2$ matrices $T_i$ describe the transition from isospin $\frac{1}{2}$ to $\frac{3}{2}$ states. We use the normalization implied by $\vec{T}\cdot\vec{T}^\dagger = \mathbb{1}_4$ and $T_i^\dagger T_j = \delta_{ij}\mathbb{1}_2 - \frac{1}{3}\sigma_i\sigma_j$.

## 5.2 Unitarity and causality

In [45, 46], the authors show how to improve the convergence of strict chiral perturbation theory by an analytic continuation of subthreshold partial-wave scattering amplitudes. As compared to strict chiral perturbation theory the applicability domain is extended towards higher energies beyond the threshold region into the resonance region. The extrapolation of the amplitudes is performed insisting on the basic principles of analyticity and unitarity. A non-linear integral equation (2.6) is imposed on the covariant partial-wave amplitudes $T^J_\pm(\sqrt{s})$ introduced in Section 4.1,

$$T^J_\pm(\sqrt{s}) = U^J_\pm(\sqrt{s}) + \int_{\mu_{\rm thrs}}^{\infty} \frac{dw}{\pi}\,\frac{\sqrt{s}-\mu_M}{w-\mu_M}\,\frac{T^J_\pm(w)\,\rho^J_\pm(w)\,T^{J*}_\pm(w)}{w-\sqrt{s}-i\epsilon}\,. \tag{5.19}$$

which separates contributions from left- and right-hand singularities. In Eq. (5.19) coupled-channel indices are suppressed and a summation over all possible two-body intermediate states is implied. The phase-space matrices $\rho^J_\pm(\sqrt{s})$ are given in (4.37). The generalized potential $U^J_\pm(\sqrt{s})$ contains left-hand cuts only. The relation (5.19) illustrates that the amplitude possesses a unitarity cut along the positive real $\sqrt{s}$ axis starting from the lowest s-channel threshold. The structure of the left-hand cuts in $U^J_\pm(\sqrt{s})$ can be analyzed by assuming the Mandelstam representation [59, 51] for the invariant amplitudes $F^\pm_n(\sqrt{s},t)$ introduced in Chapter 2. The same result would follow by an examination of the structure of Feynman diagrams in perturbation theory [101]. The form of the analyticity domain of partial-wave amplitudes as implied by basic principles of quantum field theory is discussed in [102]. For a given approximation of the generalized potential any solution of the non-linear integral equation (5.19) satisfies the coupled-channel unitarity constraint exactly.

The key issue of our approach is an analytic continuation of the generalized potential by means of suitable conformal variables. The potential is split into two parts, an inside and an outside parts:

$$U^J_\pm(\sqrt{s}) = U^J_{\pm,\rm inside}(\sqrt{s}) + U^J_{\pm,\rm outside}(\sqrt{s}), \qquad U^J_{\pm,\rm outside}(\sqrt{s}) = \sum_{k=0}^{\infty} \frac{d^k U^J_{\pm,\rm outside}(\xi^{-1}(\xi))}{k!\,d\xi^k}\bigg|_{\xi=0} \Big[\xi(\sqrt{s})\Big]^k. \tag{5.20}$$

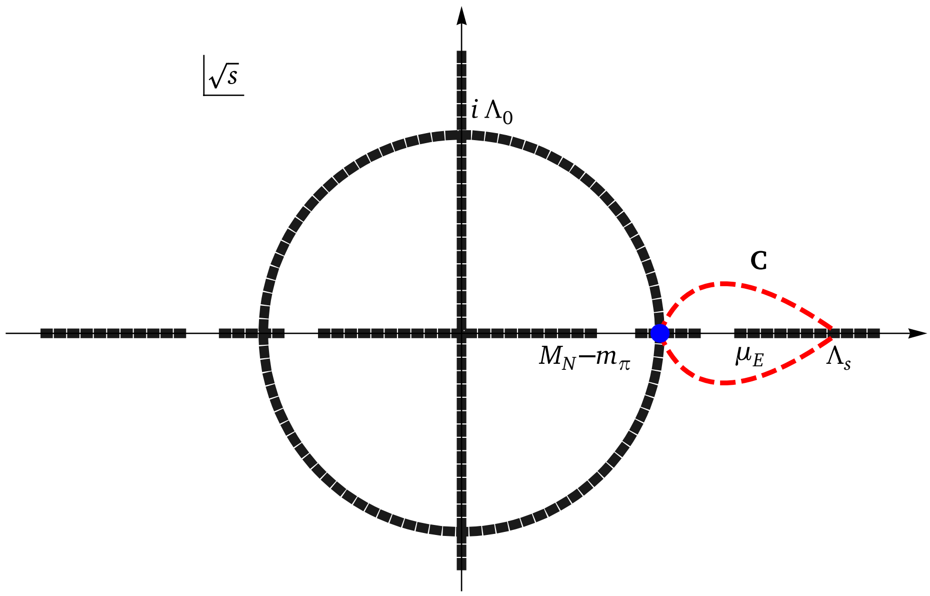



**Figure 5.1.:** Singularities in the covariant partial-wave amplitude for $\pi N \to \pi N$.

The inside and outside parts are separated by a closed contour $\Omega$ as exemplified in Fig. 5.1 for the case of pion-nucleon scattering. While contributions that are characterized by cut lines lying inside the domain $\Omega$ are computed explicitly, contributions that are determined by cut lines outside the domain $\Omega$ are expanded in terms of a conformal variable. The latter contributions can not be resolved within an approach based on the chiral Lagrangian. They probe short-range physics that can not be predicted by the chiral Lagrangian. Nevertheless this physics can be treated in a model-independent manner using conformal variables. The convergence boundary of the conformal mapping $\xi(\sqrt{s})$ is identified with the closed contour $C$ in Fig. 5.1.

It is emphasized that an analytic extrapolation of the generalized potential is an indispensable part of the unitarization scheme applied. An attempt to compute the generalized potential in strict chiral perturbation theory is futile, since such an expansion breaks down at too low energies and therefore is unable to describe the generation of baryon resonance in a controlled manner.

To be specific we consider the case of pion-nucleon scattering [68, 103], for which we recall the analytic structure of its partial-wave amplitudes. In the center-of-mass frame, the Mandelstam variables $t$ and $u$ can be simply expressed in terms of the $s$, the scattering angle $\theta$, and the masses as

$$
\begin{aligned}
t &= (\cos\theta - 1)\,\frac{M_N^4 - 2\,M_N^2\,(m_\pi^2 + s) + (m_\pi^2 - s)^2}{2\,s}\,,\\
u &= 2\,M_N^2 + 2\,m_\pi^2 - s - t\,.
\end{aligned} \tag{5.21}
$$

Apart from the s-channel unitarity cut, i.e. $(M_N + m_\pi)^2 < s < \infty$, there are the u-channel and t-channel cuts in the complex $s$ plane. The leading u-channel branch points are determined by the one nucleon and the pion-nucleon two-body exchanges. According to the Landau condition [73] branch points may arise through the endpoint singularities at $\cos\theta = \pm 1$ and $u = M_N^2$ or $u = (M_N + m_\pi)^2$. This leads to the u-channel cuts on the real $s$ axis as follows

$$
\left(\frac{M_N^2 - m_\pi^2}{M_N}\right)^2 < s < M_N^2 + 2\,m_\pi^2\,, \qquad -\infty < s < (M_N - m_\pi)^2\,. \tag{5.22}
$$

The leading t-channel branch point is determined by the two-pion exchange process. Here the Landau condition suggests $\cos\theta = \pm 1$ together with $t = 4\,m_\pi^2$. This leads to a branch point at

$$
s = \Lambda_0^2 = M_N^2 - m_\pi^2\,. \tag{5.23}
$$

When multi-pion exchanges are considered in the t-channel the corresponding branch points become complex, but its modulus is unchanged. Consequently, the t-channel cuts form a circle in the complex $s$ plane as follows

$$s = \Lambda_0^2\, e^{i\,\phi}\,, \tag{5.24}$$

where $\phi \in (-\pi, +\pi]$ is a polar angle. The singularity structure of the $\pi N$ scattering amplitude is explicitly shown in Fig. 5.1. Note that in the $\sqrt{s}$ plane as used in Fig. 5.1 each cut has its mirror partner ($\sqrt{s} \to -\sqrt{s}$), because they originate from a Mandelstam representation written in terms of s, t, and u.

In the case of $\pi N \to \pi N$, our particular choice of the domain $\Omega$ is shown in Fig. 5.1 [45]. The associated conformal mapping is specified with

$$\xi(\sqrt{s}) = \frac{a_1\,(\Lambda_s - \sqrt{s})^2 - 1}{(a_1 - 2\,a_2)\,(\Lambda_s - \sqrt{s})^2 + 1}\,, \qquad a_1 = \frac{1}{(\Lambda_s - \mu_E)^2}\,, \qquad a_2 = \frac{1}{(\Lambda_s - \Lambda_0)^2}\,, \tag{5.25}$$

with the expansion point $\mu_E = M_N + m_\pi$ and $\Lambda_0 = \sqrt{M_N^2 - m_\pi^2}$. The parameter $\Lambda_s$ plays the role of a cutoff, it determines to which energies the conformal expansion converges on the real $\sqrt{s}$ axis. On the real axis convergence is guaranteed in the interval $\sqrt{s} \in (\Lambda_0, \Lambda_s)$. The conformal mapping (5.25) was suggested in [45] as a superposition of the function $(\Lambda_s - \sqrt{s})^2$ and a Möbius transformation. The inverse mapping is

$$\xi^{-1}(x) = \Lambda_s - \frac{\sqrt{1+x}}{\sqrt{a_1 + (2\,a_2 - a_1)\,x}}\,. \tag{5.26}$$

The boundary line $\Omega$ in Fig. (5.1) is determined by the condition

$$\sqrt{s} \in \Omega \qquad \text{for} \qquad \sqrt{s} = \xi^{-1}(e^{i\,\phi}) \qquad \text{with} \qquad \phi \in (-\pi, +\pi]\,. \tag{5.27}$$

For energies $\sqrt{s} > \Lambda_s$, we could simply cut off the integral in (5.19). However, that would lead to a pronounced cutoff effect in the scattering amplitude at $\Lambda_s$. Since the scattering amplitudes should not depend on the precise value of the cutoff scale we follow [45] and eliminate such a dependence. The outside part of generalized potential is continued by a constant for $\sqrt{s} > \Lambda_s$. Since it holds $\xi'(\Lambda_s) = 0$ a smooth behavior of $\xi(\sqrt{s})$ at $\sqrt{s} = \Lambda_s$ arises. Given this prescription for the generalized potential the integral in Eq. (5.19) can be extended to infinity. Note that owing to the boundedness of the phase-space function $\rho^J_\pm(\sqrt{s})$ and the presence of the matching scale $\mu_M$ in Eq. (5.19) the influence of the region $\Lambda_s < w < \infty$ is largely suppressed.

For a given channel the parameters $\mu_E$ and $\Lambda_0$ have to be determined. The expansion point $\mu_E$ is identified to be the mean of the two threshold masses

$$\mu_E = \frac{1}{2}\,(M + m + \bar{M} + \bar{m})\,. \tag{5.28}$$

The parameter $\Lambda_0$ is set by the condition that all loop contributions to the generalized potential contribute to the outside part of the potential only. This unambiguously defines the value for $\Lambda_0$ for any considered channel, though a detailed analysis of the singularity structure of each channel needs to be performed. In the case of the pion-nucleon potential $\Lambda_0$ is determined by the t-channel branch point implied by the two-pion exchange contribution in the t-channel as it arises at the one-loop level. As a consequence the inside part of the potential is fully determined by the tree-level expressions Eq. (5.11). To this extent we may say that left-hand cut structures implied by loop effects are integrated out systematically by being moved into $U^J_{\pm,\text{outside}}(\sqrt{s})$. Note that the latter receives contributions from Eq. (5.11) as well.

The value for the matching scale $\mu_M$ and cutoff scale $\Lambda_s$ should be universal in a given sector specified by isospin and strangeness $(I, S)$. Following previous works [31] we identify the matching scale with

$$\mu_M^2 = m^2 + M^2\,, \tag{5.29}$$

where $m$ and $M$ are the meson and baryon masses of the lightest channel in the given sector. At leading order the results may show a small residual dependence on the choice of the matching scale. Large dependencies are ruled out since the scattering amplitude is expected to have a perturbative representation in the close vicinity of the matching scale $\mu_M$ [31]. The particular choice of $\mu_M$ turns fully irrelevant at a precision level where local counter terms allow to shift the strength of the generalized potential by a small constant. A typical value for the cutoff scale $\Lambda_s$ is

$$\Lambda_s = m + M + 2\,m_\pi\,, \tag{5.30}$$

where in this case $m$ and $M$ are the meson and baryon masses of the heaviest channel in the given sector. In Eq. (5.30) the size of the Hilbert space is set to be two pion-mass units larger than the largest channel mass considered. This is a typical value used also in previous studies [47, 104, 105, 48]. In any case the dependence on that particular choice has shown so far to cause very minor effects [45, 46, 47, 104, 105, 48].

Given an approximated generalized potential (5.20), a solution of the non-linear integral equation (5.19) can be constructed by the N/D technique [106, 70]. We represent the covariant partial-wave amplitude by a quotient

$$T^J_\pm(\sqrt{s}) = [D^J_\pm(\sqrt{s})]^{-1}\, N^J_\pm(\sqrt{s})\,, \tag{5.31}$$

where analytic functions $N^J_\pm(\sqrt{s})$ and $D^J_\pm(\sqrt{s})$ contain the left-hand cut and the right-hand cut on the real $\sqrt{s}$ axis respectively. These two functions can be numerically solved by the following system of linear integral equations

$$\begin{aligned}
D^J_\pm(\sqrt{s}) &= \mathbb{1} - \int_{\mu_{\rm thrs}}^{\infty} \frac{dw}{\pi}\, \frac{\sqrt{s}-\mu_M}{w-\mu_M}\, \frac{N^J_\pm(w)\,\rho^J_\pm(w)}{w-\sqrt{s}}\,,\\
N^J_\pm(\sqrt{s}) &= U^J_\pm(\sqrt{s}) + \int_{\mu_{\rm thrs}}^{\infty} \frac{dw}{\pi}\, \frac{\sqrt{s}-\mu_M}{w-\mu_M}\, \frac{N^J_\pm(w)\,\rho^J_\pm(w)\,[U^J_\pm(w)-U^J_\pm(\sqrt{s})]}{w-\sqrt{s}}\,.
\end{aligned} \tag{5.32}$$

Standard matrix inversion algorithms suffice to obtain stable numerical solutions to (5.32).

## 5.3 Parameters and results

Our main goal of this section is the study of the dynamical generation of baryon resonances with $J^P = \frac{1}{2}^-$. The existence of the s-wave resonances $N(1535)$, $\Lambda(1405)$, $\Lambda(1670)$, and $\Xi(1690)$[1] was predicted by previous coupled-channel computations based on the leading order chiral interaction [39, 108]. The leading order interaction, Weinberg-Tomozawa term, is parameter free in the sense that the value of $f$ may be determined by other processes not considered in this work. Following [31] we use

$$f \simeq 90\,\text{MeV}\,. \tag{5.33}$$

For the mass parameters which enter the computation we take the isospin averaged values from the Particle Data Group [107].

Within our unitarization scheme various $J^P = \frac{1}{2}^-$ resonances close to their empirical masses are obtained. The pole position on some higher Riemann sheet defines the mass and half-width of the corresponding resonance through its real and imaginary parts, respectively [109, 39, 108, 110]. The results for the pole masses based on the Weinberg-Tomozawa interaction only are collected in Tab. 5.2. The

[1] The quantum number $J^P$ assignment for the $\Xi(1690)$ resonance needs a confirmation [107].

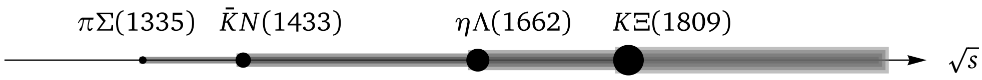



**Figure 5.2.:** Unitarity cut for $(I,S)=(0,-1)$.

table provides in addition the modulus of the resonance coupling strengths $g_i$ to the various channels. Close to the pole the scattering amplitude can be factorized into

$$T_{ij}(\sqrt{s}) = -\frac{g_i\, M_R\, g_j}{\sqrt{s}-M_R} + \cdots . \tag{5.34}$$

Signals of the four stars $N(1535)$ and $N(1650)$ resonances are found in the $(I,S)=(\frac{1}{2},0)$ sector. Both states couple most strongly to the closed $K\Sigma$ channel. Their masses are within 50 MeV of the empirical estimates. Since the masses of the resonances are not accurately reproduced at this leading order the widths of the two states are not realistically described here. It is interesting to observe that, in contrast to previous studies [111, 112, 109, 39, 108, 113, 114], our improved unitarization scheme does predict the $N(1650)$ resonance with reasonable properties at leading order already. The three star $\Xi(1690)$ resonance shows up in the $(I,S)=(\frac{1}{2},-2)$ sector. It provides a clean and narrow peak in the $\bar{K}\Sigma \to \bar{K}\Sigma$ amplitude. We find also a pole associated with the three star $\Sigma(1750)$ resonance in $(I,S)=(1,-1)$. Its existence is not strongly seen in the scattering amplitudes since the pole is found on the 4th Riemann sheet only. A further pole on the 5th Riemann sheet at $1.940-i\,0.152$ GeV, not included in Tab. 5.2, has an even smaller influence on the physical scattering region.

Of particular interest is the $(I,S)=(0,-1)$ sector with two resonances, the $\Lambda(1405)$ and $\Lambda(1670)$. Both states are obtained with masses quite close to their empirical values. While the $\Lambda(1405)$ pole is located on the 2nd Riemann sheet the $\Lambda(1670)$ pole is on the 3rd sheet. The corresponding unitarity cuts are shown in Fig. 5.2. In Fig. 5.3 the resonance parts of all coupled-channel partial-wave amplitudes

$$R(\sqrt{s}) = T(\sqrt{s}) - U(\sqrt{s}), \tag{5.35}$$

are shown. The scattering amplitudes $\bar{K}N \to \bar{K}N$ and $K\Xi \to K\Xi$ show particularly strong evidence of the $\Lambda(1405)$ and $\Lambda(1670)$ resonances, respectively. In the $\bar{K}N \to K\Xi$ and $\eta\Lambda \to K\Xi$ reaction amplitudes, both resonances are clearly visible [25, 115]. For the $\Lambda(1670)$ resonance the most relevant channel in a future comparison with empirical data is the $\bar{K}N \to \eta\Lambda$ amplitude. In contrast to previous studies [28, 116, 117, 118, 119] we do not confirm the existence of a second resonance to the $\Lambda(1405)$ state. As already been

| $(I,S)$ | Resonance | $M_R$ [MeV] | $\lvert g_i\rvert$ | | | | |
|---|---|---|---|---|---|---|---|
| $(\frac{1}{2},0)$ | | | $\lvert g_{\pi N}\rvert$ | $\lvert g_{\eta N}\rvert$ | $\lvert g_{K\Lambda}\rvert$ | $\lvert g_{K\Sigma}\rvert$ | |
| | $N(1535)$ | $1485.0-0.1\,i$ | 0.096 | 1.447 | 1.180 | 2.547 | |
| | $N(1650)$ | $1707.5-143.5\,i$ | 1.440 | 1.986 | 1.456 | 4.895 | |
| (0,-1) | | | $\lvert g_{\pi\Sigma}\rvert$ | $\lvert g_{\bar{K}N}\rvert$ | $\lvert g_{\eta\Lambda}\rvert$ | $\lvert g_{K\Xi}\rvert$ | |
| | $\Lambda(1405)$ | $1417.1-6.6\,i$ | 0.795 | 2.832 | 2.011 | 0.894 | |
| | $\Lambda(1670)$ | $1638.0-17.5\,i$ | 0.488 | 0.809 | 1.661 | 5.004 | |
| (1,-1) | | | $\lvert g_{\pi\Lambda}\rvert$ | $\lvert g_{\pi\Sigma}\rvert$ | $\lvert g_{\bar{K}N}\rvert$ | $\lvert g_{\eta\Sigma}\rvert$ | $\lvert g_{K\Xi}\rvert$ |
| | $\Sigma(1750)$ | $1806.7-39.1\,i$ | 1.018 | 0.993 | 0.995 | 1.945 | 2.912 |
| $(\frac{1}{2},-2)$ | | | $\lvert g_{\pi\Xi}\rvert$ | $\lvert g_{\bar{K}\Lambda}\rvert$ | $\lvert g_{\bar{K}\Sigma}\rvert$ | $\lvert g_{\eta\Xi}\rvert$ | |
| | $\Xi(1690)$ | $1667.3-3.6\,i$ | 0.274 | 0.506 | 2.766 | 2.107 | |

**Table 5.2.:** Pole positions and coupling constants of $J^P=\frac{1}{2}^-$ resonances with isospin ($I$) and strangeness ($S$). The results are based on the leading order Weinberg-Tomozawa interaction only.

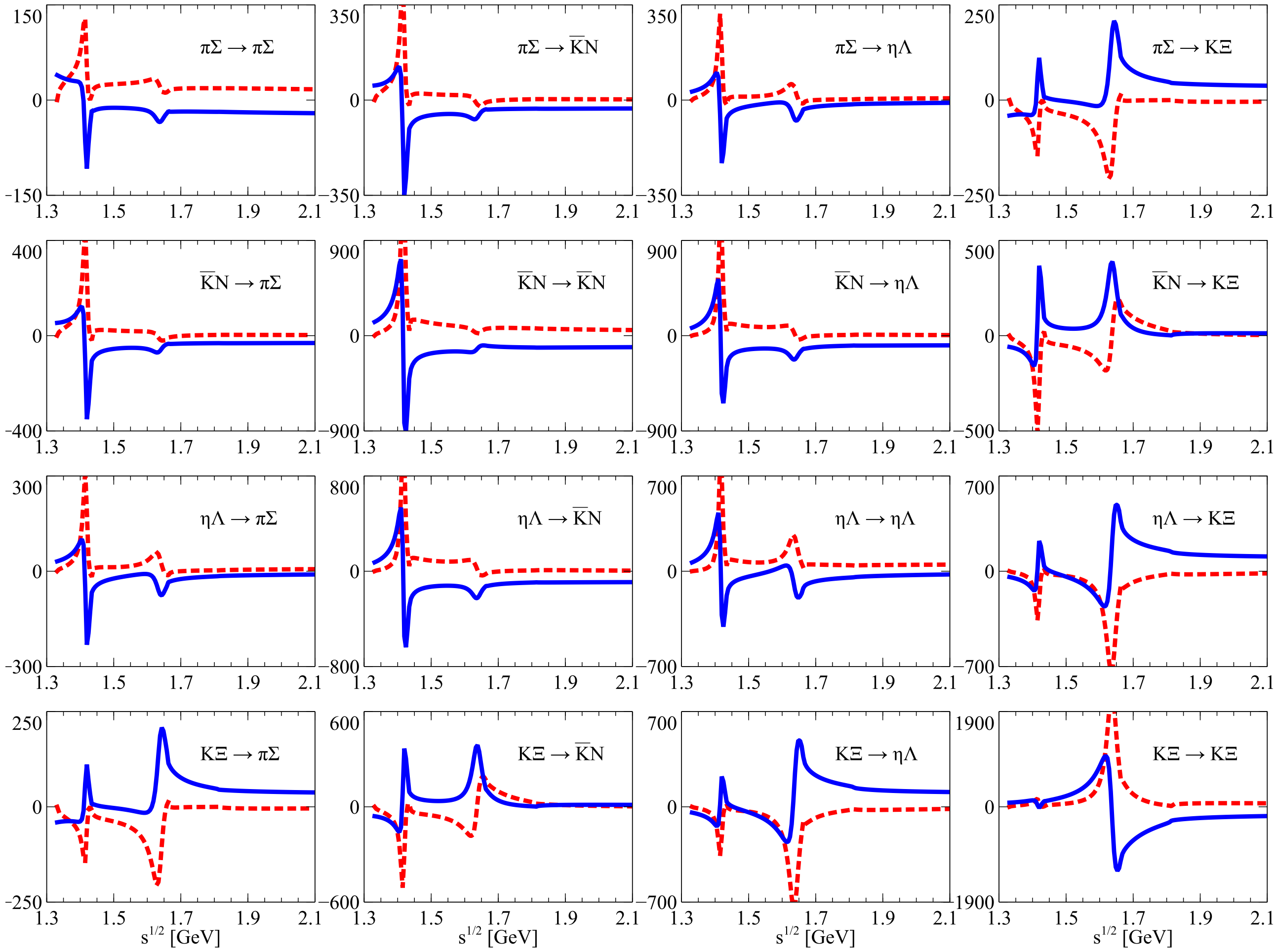



**Figure 5.3.:** The resonance amplitudes $R(\sqrt{s})$ with $J^P = \frac{1}{2}^-$ and $(I,S) = (0,-1)$ as introduced in Eq. (5.35). The solid and dotted lines show the real and imaginary parts of the amplitudes, respectively. The results are obtained with the Weinberg-Tomozawa interaction term only.

emphasized in [120] the fate of such a second resonance depends decisively on the details of the chosen approach.

In the following we provide a detailed analysis of subleading effects in the formation of the $\Lambda(1405)$ and $\Lambda(1670)$ resonances. The s-, t-, and u-channel exchange terms lead to several parameters which need to be estimated. They are in part well constrained by empirical data. There are ten dimensionless parameters $F_A$, $D_A$, $C_A$, $F_V$, $D_V$, $G_V$, $F_T$, $D_T$, $G_T$, and $h_P$ and three parameters $m_V$, $f$, and $f_V$ of dimension of mass. The parameter $h_P$ in (5.10) is determined from the $\rho \to \pi\pi$ decay process (see e.g. [121, 122, 123, 124])

$$h_P \simeq 1.93\,, \tag{5.36}$$

where the positivity of the coupling constant defines our phase convention for the vector fields. Given the convention introduced in [125] the size of $h_P$ depends on the particular value used for parameter $f$ and $f_V$. Since the parameter $f_V$ was introduced in [125] as the transition matrix elements of the light vector mesons to the vector current of QCD the value of $f_V$ is determined

$$f_V \simeq 140\,\mathrm{MeV}\,, \tag{5.37}$$

from electromagnetic properties of the light vector mesons [125]. In our work only the particular combination $f_V\, h_P/f^2$ that determines the $\rho \to \pi\pi$ decay width enters.

The parameters $F_A \simeq 0.45$ and $D_A \simeq 0.80$ are deduced from the weak decay widths of the baryon octet states [126] and $C_A \simeq 1.6$ can be derived from the hadronic decay width of the baryon decuplet states [127]. The choice $C_A > 0$ defines our phase convention for the decuplet fields.

We turn to the parameters $F_V$, $D_V$, $G_V$, $F_T$, $D_T$, and $G_T$ that are least well known. Constraints can be deduced from phenomenological estimates of the $\rho$ and $\omega$ meson coupling strengths to the nucleon. The $\rho NN$ and $\omega NN$ coupling constants introduced in [122, 128] can be related to our coupling constants $F_V$, $D_V$, and $G_V$ as

$$\begin{aligned} g_{\rho NN} &= \frac{m_V}{f_V}\frac{D_V+F_V}{2} \simeq 3.25\,, \\ g_{\omega NN} &= \frac{m_V}{f_V}\frac{D_V+F_V+2\,G_V}{2} \simeq 11.7\,, \end{aligned} \tag{5.38}$$

where we recall the phenomenological values obtained in [122, 128]. The three parameters

$$D_V \simeq -0.18\,, \qquad F_V \simeq 1.35\,, \qquad G_V \simeq 1.53\,, \tag{5.39}$$

are obtained from (5.38) supplemented by the OZI rule [129, 21, 22, 130]. The latter implies the approximated vanishing of the $\phi NN$ coupling constant

$$g_{\phi NN} = \frac{m_V}{f_V}\frac{D_V-F_V+G_V}{\sqrt{2}} \simeq 0\,. \tag{5.40}$$

The positivity of the coupling constant $F_V > 0$ may be inferred from the phenomenological assumption of universally coupled light vector mesons [123, 124, 131, 96].

It remains to arrive at an estimate of the tensor coupling constants $D_T$, $F_T$, and $G_T$. They always appear in combination with the mass parameter $m_V$. Following Ref. [125] we identify $m_V = 776\,\mathrm{MeV}$ with the averaged mass of the $\rho$ and $\omega$ mesons. The tensor coupling strengths $\kappa_{\rho NN}$ and $\kappa_{\omega NN}$ of [122, 132, 128] suggest

$$\begin{aligned} \kappa_{\rho NN} &= \frac{M_N}{m_V}\frac{D_T+F_T}{D_V+F_V} \simeq 6.6\,, \\ \kappa_{\omega NN} &= \frac{M_N}{m_V}\frac{D_T+F_T+2\,G_T}{D_V+F_V+2\,G_V} \simeq 0\,, \end{aligned} \tag{5.41}$$

where $M_N$ is the mass of nucleon. Together with (5.38) and (5.40) they lead to

$$D_T \simeq 4.83\,, \qquad F_T \simeq 1.56\,, \qquad G_T \simeq -3.27\,, \tag{5.42}$$

where we enforced the OZI rule for the tensor coupling constants.

Given our estimate for the parameters relevant at subleading order we return to our study of the $\Lambda(1405)$ and $\Lambda(1670)$ resonances. Besides the pole masses we will show results on the two reaction amplitudes, $\bar{K}N \to K\Xi$ and $\eta\Lambda \to K\Xi$ which clearly exhibit both $\Lambda$ resonances.

We consider four parameter sets, see Tab. 5.3. While parameter Set 1 corresponds to the leading order ansatz, where the Weinberg-Tomozawa interaction is considered only, the last Set 4 takes all processes as described in Eq. (5.11) into account. In Set 2 the effect of the Weinberg-Tomozawa term together with the s- and u-channel baryon octet exchanges is considered. In Set 3 all terms but the t-channel vector meson exchange term are included. As compared to Set 2 in Set 3 the s- and u-channel exchanges of the baryon decuplet are incorporated in addition. The implications of the four parameter sets are collected in Tab. 5.4 and Fig. 5.4, where the resonance amplitudes for the two reactions $\bar{K}N \to K\Xi$ and $\eta\Lambda \to K\Xi$ are shown. We discuss the consequences of the four choices. Switching on the s- and u-channel exchange terms of the baryon octet states leads to a minor effect. As seen in Tab. 5.4 the pole positions

| set # | $F_A$ | $D_A$ | $C_A$ | $h_P$ |
|---|---|---|---|---|
| set 1 | 0 | 0 | 0 | 0 |
| set 2 | 0.45 | 0.80 | 0 | 0 |
| set 3 | 0.45 | 0.80 | 1.60 | 0 |
| set 4 | 0.45 | 0.80 | 1.60 | 1.93 |

**Table 5.3.:** Parameter sets characterizing the various contributions of tree-level interactions (5.11). The contribution of the Weinberg-Tomozawa interaction only to resonance plot is presented as the set 1. $F_A \neq 0 \neq D_A$ and $C_A \neq 0$ indicate the contributions of the Weinberg-Tomozawa term together with the octet and decuplet baryon exchanges, respectively. The set 4 takes all tree-level interactions (5.11) including nonet vector meson exchange.

of the two $\Lambda$ resonances move by about 5 MeV only. The coupling constants change by less than 2% as compared to those of the Weinberg-Tomozawa scenario of the parameter set 1. In Fig. 5.4 the solid blue and dotted red lines are barely distinguishable. We turn to the parameter set 3, for which we see sizeable differences as compared to the sets 1 and 2. The mass and width of the $\Lambda(1405)$ state are pulled towards their empirical values. While the set 1 underestimates the empirical width of the $\Lambda(1405)$ with about 50 MeV, by about a factor 4, the consideration of the baryon decuplet brings the total width to about 74 MeV. The resonance coupling constant for the $\Lambda(1405)$ changes most significantly when switching on the effect of the baryon decuplet exchanges. Only minor effects are seen with the parameter set 4 where the vector meson exchange is considered in addition. For the $\Lambda(1690)$ all four parameter sets are consistent with the empirical width of about 25-50 MeV, however, the resonance mass is underestimated systematically. Again the effect of baryon decuplet exchanges is most important.

The present study is partial and requires a substantial extension. So far the effect of the symmetry breaking counter terms has not been considered. All results are based on a leading order truncation in Eq. (5.20). Once the residual symmetry conserving parameters that are available at the subleading chiral order is included it is justified to consider subleading terms in the conformal expansion. A detailed comparison with scattering data is required to arrive at an accurate determination of the parameter set.

| (I,S) | Resonance | $M_R$ [MeV] | $\lvert g_{\pi\Sigma}\rvert$ | $\lvert g_{\bar{K}N}\rvert$ | $\lvert g_{\eta\Lambda}\rvert$ | $\lvert g_{K\Xi}\rvert$ | set # |
|---|---|---|---|---|---|---|---|
| (0,-1) | $\Lambda(1405)$ | $1417.1 - 6.6\,i$ | 0.795 | 2.832 | 2.011 | 0.894 | set 1 |
| | | $1423.3 - 7.5\,i$ | 0.841 | 2.578 | 1.924 | 0.916 | set 2 |
| | | $1412.9 - 37.9\,i$ | 2.213 | 3.930 | 3.016 | 2.921 | set 3 |
| | | $1412.2 - 39.9\,i$ | 2.301 | 4.052 | 3.111 | 3.144 | set 4 |
| | $\Lambda(1670)$ | $1638.0 - 17.5\,i$ | 0.488 | 0.809 | 1.661 | 5.004 | set 1 |
| | | $1633.9 - 17.1\,i$ | 0.396 | 0.855 | 1.679 | 5.082 | set 2 |
| | | $1594.0 - 23.1\,i$ | 0.518 | 0.993 | 1.981 | 5.673 | set 3 |
| | | $1595.9 - 24.7\,i$ | 0.566 | 1.002 | 1.952 | 5.658 | set 4 |

**Table 5.4.:** Pole positions and coupling constants of the $\Lambda(1405)$ and $\Lambda(1670)$ resonances according to the four sets of parameters in Tab. 5.3.

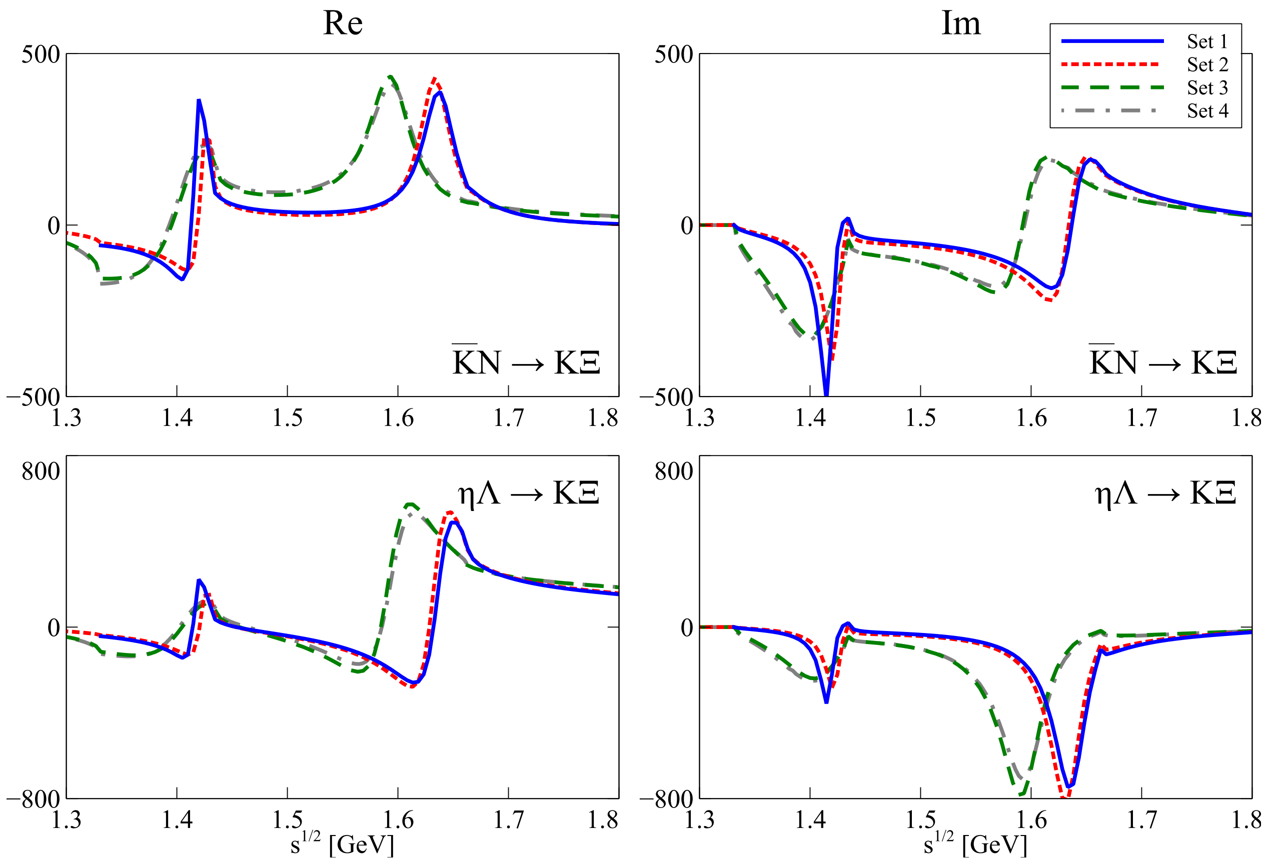



**Figure 5.4.:** The resonant amplitudes for the $\bar{K}N \to K\Xi$ and $\eta\Lambda \to K\Xi$ reactions with $(I,S) = (0,-1)$. The l.h. and r.h. panels show the real and imaginary parts of the amplitudes, respectively. The results for the parameter sets 1, 2, 3, and 4 are indicated by solid blue, dotted red, dashed greed, and dot-dashed gray lines.

# 6 Summary

In this thesis we have paved the way for a systematic test of the hadrogenesis conjecture. Hadrogenesis gives a systematic framework for resonances in QCD via coupled-channel dynamics. It relies on a selection of quasi-fundamental hadronic degrees of freedom in QCD with $J^P = 0^-, 1^-$ and $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ quantum numbers. The selection is guided by properties of QCD in the large-$N_C$ and the heavy quark-mass limit. Systematic computations of scattering and reaction amplitudes involving the quasi-fundamental hadronic degrees of freedom are asked for that are consistent with the coupled-channel unitarity condition and the consequences of micro-causality. Such computations have not been performed systematically so far. The micro-causality condition implies specific analytic properties of scattering and reaction amplitudes. Mandelstam formulated appropriate dispersion-integral representations for invariant amplitudes that are consistent with the micro-causality condition. For two-body boson-fermion scattering and reaction amplitudes involving fields with $J^P = 0^-, 1^-$ and $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ quantum numbers their generic analytic structure has not been studied systematically before. In this thesis we provided a first comprehensive study. Suitable Lorentz-Dirac tensors were identified in terms of which the scattering and reaction amplitudes can be decomposed. The challenge here was the finding of bases sets that imply invariant amplitudes free of kinematical constraints and therefore are expected to satisfy Mandelstam's dispersion integral representation.

In practical computations it is crucial to derive explicit results for the invariant amplitudes. As the spins of the involved particles increase the number of invariant amplitudes increases rapidly and therefore a computer algebra approach is indispensable. For instance, assuming parity conservation the scattering of a spin-one boson off a spin-three-half fermion is characterized by 72 distinct invariant amplitudes that have to be computed. For that purpose we developed a projection algebra that permits an efficient derivation of the invariant amplitudes. A Mathematica code based on the FeynCalc package was written, so that any tree-level Feynman diagram can be decomposed into its invariant amplitudes.

Coupled-channel unitarity is most efficiently realized in terms of partial-wave amplitudes with good total angular momentum $J$. Though it is straightforward to introduce such amplitudes applying the helicity formalism of Jacob and Wick, it is much less trivial to respect the consequences of micro-causality as implemented with partial-wave dispersion relations. The challenges are kinematical constraints in the helicity partial-wave amplitudes that have to be eliminated systematically. In a non-relativistic framework this is achieved using angular momentum $L$ states rather than the helicity eigenstates. However, for relativistic systems this turns into a more complicated task. Kinematical constraints at thresholds and pseudo-thresholds occur and both must be eliminated. In this thesis we exemplified this problem and resolved it for two-body states composed out of a $J^P = \frac{1}{2}^+$ fermion and a $J^P = 0^-$ or $J^P = 1^-$ boson. A non-unitary transformation of the helicity partial-wave amplitudes to covariant partial-wave amplitudes, which is well suited to be used in partial-wave integral-dispersion relations, was suggested.

In the final part of the thesis we worked out a physics application. The formation of $J^P = \frac{1}{2}^-$ resonances was reinvestigated. Our approach is based on the relativistic chiral Lagrangian with the baryon octet and decuplet fields. In order to estimate the size of the leading order counter terms the nonent of light vector mesons with $J^P = 1^-$ was included in the tensor field representation. Based on that chiral Lagrangian the coupled-channel partial-wave scattering amplitudes with $J^P = \frac{1}{2}^-$ were analytically extrapolated from the threshold region into the resonance region where the extrapolation was stabilized by the unitarity condition. The extrapolation was performed in the following way. In a first step, the covariant partial-wave scattering amplitudes were separated into parts with left- and right-hand cuts. The left-hand cut part defines a generalized potential, where two types of contributions were taken into account. Contributions from close-by left-hand cuts were derived from the chiral Lagrangian directly. In our case they stem from the baryon octet u-channel exchange process. The contributions from distant left-hand cuts were extrapolated to higher energies using a conformal expansion. The expansion coefficients were determined from the chiral Lagrangian. Finally the right-hand cut part was derived as a solution of a set of non-linear integral equations. The latter combines the constraints of micro-causality and coupled-channel unitarity. In the resulting partial-wave amplitudes we searched for resonance poles on higher Riemnan sheets. In particular the role of the long-range forces implied by the u-channel exchange of the baryon octet states was investigated.

# A Completion of projection algebras

In this appendix, we summarize the projection algebras $Q^{\mu\nu\cdots}_{\pm,n}$ for the reactions of a boson with $J^P = 0^-$ or $1^-$ and a fermion with $J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$ that were not given already in Chapter 3. The results are expressed in terms of auxiliary tensors $P^{\mu\nu\cdots}_{\pm,n}$ that satisfy the same on-shell conditions as the projection tensors $Q^{\mu\nu\cdots}_{\pm,n}$. In the derivation of our results, the following kinematical relations are useful

$$
\begin{aligned}
&v^2 = s\left((\bar r\cdot r)^2 - \bar r^2\, r^2\right),\\
&v^2\,(r_\perp\cdot \bar r_\perp) = s\,(\bar r\cdot r)\,, \quad v^2\,(r_\perp\cdot r_\perp) = -s\,\bar r^2\,,\\
&s\,(w_\lfloor\cdot w_\rfloor) = 1 + \tfrac{1}{4}(\bar\delta+1)(\bar\delta-1)\,s\,(\bar r_\perp\cdot\bar r_\perp)\,,\\
&s\,(w_\lfloor\cdot \bar w_\rfloor) = 1 + \tfrac{1}{4}(\bar\delta-1)(\delta+1)\,s\,(r_\perp\cdot\bar r_\perp)\,,\\
&v^2\,(r_\perp\cdot w_\rfloor) = -\tfrac{1}{2}(\bar\delta+1)\,s\,(\bar r\cdot r)\,,\\
&v^2\,(r_\perp\cdot w_\lfloor) = -\tfrac{1}{2}(\bar\delta-1)\,s\,(\bar r\cdot r)\,,\\
&(w_\rfloor\cdot\bar r) = -\tfrac{1}{2}(\bar\delta+1)\,, \quad (w_\lfloor\cdot\bar r) = -\tfrac{1}{2}(\bar\delta-1)\,,\\
&(\bar w_\rfloor\cdot\bar r) = -\tfrac{1}{2}(\delta+1)\,, \quad (\bar w_\lfloor\cdot\bar r) = -\tfrac{1}{2}(\delta-1)\,.
\end{aligned}
\tag{A.1}
$$

The results are sorted according to the compelxity of the expressions.

## A.1 $0^- + \frac{3}{2}^+ \to 1^- + \frac{3}{2}^+$

For the twenty-four invariant amplitudes $F^\pm_n$ introduced in (2.57), we find the projection algebra

$$
\frac{1}{2}\,\mathbf{tr}\big(T^{(n)}_{a,\bar\mu\bar\nu\nu}\,\Lambda\,Q^{\bar\mu\bar\nu\nu}_{b,k}\,\bar\Lambda\big) = \delta_{nk}\,\delta_{ab}\,,
\tag{A.2}
$$

$$
\text{with}\qquad \bar q_{\bar\mu}\,Q^{\bar\mu\bar\nu\nu}_{\pm,k} = 0\,,\qquad \bar p_{\bar\nu}\,Q^{\bar\mu\bar\nu\nu}_{\pm,k} = 0 = \Lambda\,Q^{\bar\mu\bar\nu\nu}_{\pm,k}\,\bar\Lambda\,\gamma_{\bar\nu}\,,\ \text{and}\qquad p_\nu\,Q^{\bar\mu\bar\nu\nu}_{\pm,k} = 0 = \gamma_\nu\,\Lambda\,Q^{\bar\mu\bar\nu\nu}_{\pm,k}\,\bar\Lambda\,,
$$

$$
\begin{aligned}
Q^{\bar\mu\bar\nu\nu}_{\pm,1} &= -(\bar\delta-1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{\bar M}\,(r\cdot r)\,\bar E_\pm\left(\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\nu}_{\mp,5} - \bar E_\mp\,E_\pm\,P^{\bar\mu\bar\nu\nu}_{\pm,5}\right] - \sqrt{s}\,\bar E_\mp\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\nu}_{\pm,1} - \bar E_\pm\,E_\pm\,P^{\bar\mu\bar\nu\nu}_{\mp,1}\right]\right)\\
&\quad - 2\,\frac{s}{v^2}\,\frac{\sqrt{s}}{\bar M}\,(\bar r\cdot\bar r)\,E_\pm\left(\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\nu}_{\pm,7} - \bar E_\pm\,E_\mp\,P^{\bar\mu\bar\nu\nu}_{\mp,7}\right] - \sqrt{s}\,\bar E_\pm\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\nu}_{\mp,3} - \bar E_\mp\,E_\mp\,P^{\bar\mu\bar\nu\nu}_{\pm,3}\right]\right)\\
&\quad + \tfrac{1}{2}(\bar\delta-1)\,\frac{s}{\bar M}\,\bar E_\pm\,E_\pm\,P^{\bar\mu\bar\nu\nu}_{\mp,1} \pm \sqrt{s}\,(\bar r\cdot r)\,P^{\bar\mu\bar\nu\nu}_{\pm,1} + \frac{1}{\bar M}\,\bar E_\pm\left(s\,\bar E_\mp\,P^{\bar\mu\bar\nu\nu}_{\pm,3} - \sqrt{s}\,P^{\bar\mu\bar\nu\nu}_{\mp,7} + E_\pm\,P^{\bar\mu\bar\nu\nu}_{\pm,5}\right),\\
Q^{\bar\mu\bar\nu\nu}_{\pm,2} &= -2\,\frac{s}{v^2}\,\bar E_\mp\,E_\pm\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\nu}_{\pm,9} - \bar E_\pm\,E_\mp\,P^{\bar\mu\bar\nu\nu}_{\mp,9}\right] + P^{\bar\mu\bar\nu\nu}_{\mp,9}\\
&\quad \pm \tfrac{1}{2}(\bar\delta+1)\,\frac{s}{v^2}\,E_\pm\left(\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\nu}_{\mp,1} + \bar E_\mp\,E_\mp\,Q^{\bar\mu\bar\nu\nu}_{\pm,1}\right] \mp \bar E_\mp\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\nu}_{\mp,5} - \bar E_\pm\,E_\mp\,Q^{\bar\mu\bar\nu\nu}_{\pm,5}\right]\right) + \tfrac{1}{2}(\bar\delta+1)\,Q^{\bar\mu\bar\nu\nu}_{\pm,5}\,,\\
Q^{\bar\mu\bar\nu\nu}_{\pm,3} &= -2\,\frac{s}{v^2}\,\bar E_\mp\,E_\pm\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\nu}_{\pm,5} - \bar E_\pm\,E_\mp\,P^{\bar\mu\bar\nu\nu}_{\mp,5}\right] + P^{\bar\mu\bar\nu\nu}_{\mp,5}\\
&\quad \mp \frac{s}{v^2}\,\bar E_\mp\left(\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\nu}_{\pm,1} + \bar E_\pm\,E_\pm\,Q^{\bar\mu\bar\nu\nu}_{\mp,1}\right] \pm \bar E_\pm\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\nu}_{\pm,5} - \bar E_\mp\,E_\pm\,Q^{\bar\mu\bar\nu\nu}_{\mp,5}\right]\right),\\
Q^{\bar\mu\bar\nu\nu}_{\pm,4} &= \pm\,\frac{\sqrt{s}}{\bar M}\left[P^{\bar\mu\bar\nu\nu}_{\mp,8} - \sqrt{s}\,\bar E_\mp\,P^{\bar\mu\bar\nu\nu}_{\pm,4}\right] \mp \frac{(\bar r\cdot r)}{\bar M\,w}\,Q^{\bar\mu\bar\nu\nu}_{\mp,3} \pm \frac{M}{\sqrt{s}}\,Q^{\bar\mu\bar\nu\nu}_{\pm,3} \pm \frac{2}{\sqrt{s}}\,Q^{\bar\mu\bar\nu\nu}_{\mp,1}\\
&\quad \mp \tfrac{1}{2}(\bar\delta-1)(\delta-1)\,\frac{\sqrt{s}}{\bar M}\left(Q^{\bar\mu\bar\nu\nu}_{\pm,3} + \frac{s}{v^2}\,\bar E_\mp\,E_\pm\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\nu}_{\mp,3} + \bar E_\pm\,E_\mp\,Q^{\bar\mu\bar\nu\nu}_{\pm,3}\right]\right)
\end{aligned}
$$

$$
\begin{aligned}
&\mp (\delta - 1)\, \frac{s}{v^2}\, \bar{E}_\mp \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,1} + \bar{E}_\pm\, E_\pm\, Q^{\bar\mu\bar\nu\nu}_{\mp,1} \right] ,\\
Q^{\bar\mu\bar\nu\nu}_{\pm,5} &= \frac{\sqrt{s}}{\bar{E}_\pm} \left[ (\bar{r}\cdot r)\, P^{\bar\mu\bar\nu\nu}_{\pm,1} - \bar{E}_\pm\, E_\pm\, P^{\bar\mu\bar\nu\nu}_{\mp,1} \right] \mp \frac{1}{\bar{E}_\pm}\, Q^{\bar\mu\bar\nu\nu}_{\pm,1} ,\\
Q^{\bar\mu\bar\nu\nu}_{\pm,6} &= \pm \sqrt{s}\, P^{\bar\mu\bar\nu\nu}_{\pm,4} + \frac{1}{s}\, Q^{\bar\mu\bar\nu\nu}_{\mp,1} \mp \frac{1}{\sqrt{s}}\, Q^{\bar\mu\bar\nu\nu}_{\pm,4} \pm \tfrac{1}{2}\, (\bar\delta + 1)\, \frac{\sqrt{s}}{v^2}\, M \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,1} + \bar{E}_\pm\, E_\pm\, Q^{\bar\mu\bar\nu\nu}_{\mp,1} \right]\\
&\quad \pm \tfrac{1}{2}\, (\bar\delta - 1)\, \frac{s}{v^2}\, E_\pm \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\mp,4} + \bar{E}_\pm\, E_\mp\, Q^{\bar\mu\bar\nu\nu}_{\pm,4} \right] + \tfrac{1}{2}\, (\delta - 1)\, \frac{\sqrt{s}}{v^2}\, \bar{E}_\pm \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,1} - \bar{E}_\mp\, E_\pm\, Q^{\bar\mu\bar\nu\nu}_{\mp,1} \right]\\
&\quad \mp M\, \frac{\sqrt{s}}{v^2} \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,1} + \bar{E}_\pm\, E_\pm\, Q^{\bar\mu\bar\nu\nu}_{\mp,1} \right] ,\\
Q^{\bar\mu\bar\nu\nu}_{\pm,7} &= \pm \sqrt{s}\, P^{\bar\mu\bar\nu\nu}_{\pm,3} \mp \frac{1}{\sqrt{s}}\, Q^{\bar\mu\bar\nu\nu}_{\pm,5} \pm \tfrac{1}{2}\, (\bar\delta - 1)\, \frac{s}{v^2}\, E_\pm \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\mp,5} + \bar{E}_\pm\, E_\mp\, Q^{\bar\mu\bar\nu\nu}_{\pm,5} \right] + \frac{\sqrt{s}}{v^2}\, E_\pm \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\mp,1} - \bar{E}_\pm\, E_\mp\, Q^{\bar\mu\bar\nu\nu}_{\pm,1} \right] ,\\
Q^{\bar\mu\bar\nu\nu}_{\pm,8} &= \pm \sqrt{s}\, P^{\bar\mu\bar\nu\nu}_{\pm,2} \mp \frac{s}{v^2}\, \bar{E}_\pm \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,4} + \bar{E}_\mp\, E_\pm\, Q^{\bar\mu\bar\nu\nu}_{\mp,4} \right] + \frac{\sqrt{s}}{v^2}\, \bar{E}_\pm \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\mp,1} - \bar{E}_\mp\, E_\pm\, Q^{\bar\mu\bar\nu\nu}_{\pm,1} \right] ,\\
Q^{\bar\mu\bar\nu\nu}_{\pm,9} &= \frac{s}{v^2} \left[ (\bar{r}\cdot r)\, P^{\bar\mu\bar\nu\nu}_{\mp,12} - \bar{E}_\mp\, E_\pm\, P^{\bar\mu\bar\nu\nu}_{\pm,12} \right] - \frac{1}{s}\, Q^{\bar\mu\bar\nu\nu}_{\mp,2} \pm \frac{\sqrt{s}}{v^2}\, M \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,2} + \bar{E}_\mp\, E_\pm\, Q^{\bar\mu\bar\nu\nu}_{\mp,2} \right]\\
&\quad + \tfrac{1}{4}\, (\bar\delta - 1)(\bar\delta + 1)\, \frac{s}{v^2}\, (r\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,4} + \tfrac{1}{2}\, (\delta - 1)\, \frac{\sqrt{s}}{v^2}\, \bar{E}_\pm \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,2} + \bar{E}_\mp\, E_\pm\, Q^{\bar\mu\bar\nu\nu}_{\mp,2} \right]\\
&\quad \pm \tfrac{1}{2}\, (\bar\delta + 1)\, \frac{1}{v^2} \Big( s\, E_\pm \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\mp,6} + \bar{E}_\mp\, E_\mp\, Q^{\bar\mu\bar\nu\nu}_{\pm,6} \right] - \sqrt{s}\, M \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,2} + \bar{E}_\mp\, E_\pm\, Q^{\bar\mu\bar\nu\nu}_{\mp,2} \right]\\
&\qquad - E_\mp \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,1} - \bar{M}\, M\, \frac{E_\pm}{E_\mp}\, Q^{\bar\mu\bar\nu\nu}_{\mp,1} \right] \Big) ,\\
Q^{\bar\mu\bar\nu\nu}_{\pm,10} &= \frac{s}{v^2} \left[ (\bar{r}\cdot r)\, P^{\bar\mu\bar\nu\nu}_{\mp,11} - \bar{E}_\mp\, E_\pm\, P^{\bar\mu\bar\nu\nu}_{\pm,11} \right] - \frac{\sqrt{s}}{v^2}\, E_\pm \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\mp,2} + \bar{E}_\pm\, E_\mp\, Q^{\bar\mu\bar\nu\nu}_{\pm,2} \right]\\
&\quad + \tfrac{1}{4}\, (\bar\delta - 1)(\bar\delta + 1)\, \frac{s}{v^2}\, (r\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,5} \pm \tfrac{1}{2}\, (\bar\delta + 1)\, \frac{s}{v^2}\, E_\pm \Big( - \frac{E_\mp}{\sqrt{s}}\, Q^{\bar\mu\bar\nu\nu}_{\pm,1} + \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\mp,7} + \bar{E}_\mp\, E_\mp\, Q^{\bar\mu\bar\nu\nu}_{\pm,7} \right] \Big) ,\\
Q^{\bar\mu\bar\nu\nu}_{\pm,11} &= \frac{s}{v^2} \left[ (\bar{r}\cdot r)\, P^{\bar\mu\bar\nu\nu}_{\mp,10} - \bar{E}_\mp\, E_\pm\, P^{\bar\mu\bar\nu\nu}_{\pm,10} \right] - \frac{\sqrt{s}}{v^2}\, \bar{E}_\mp \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\mp,2} + \bar{E}_\pm\, E_\mp\, Q^{\bar\mu\bar\nu\nu}_{\pm,2} \right]\\
&\quad + \tfrac{1}{2}\, (\bar\delta + 1)\, \frac{s}{v^2} \Big( \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,4} \mp \frac{1}{\sqrt{s}}\, \bar{E}_\mp\, E_\mp\, Q^{\bar\mu\bar\nu\nu}_{\pm,1} \right] \pm E_\pm \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\mp,8} + \bar{E}_\mp\, E_\mp\, Q^{\bar\mu\bar\nu\nu}_{\pm,8} \right] \Big) ,\\
Q^{\bar\mu\bar\nu\nu}_{\pm,12} &= \frac{s}{v^2} \left[ (\bar{r}\cdot r)\, P^{\bar\mu\bar\nu\nu}_{\mp,6} - \bar{E}_\mp\, E_\pm\, P^{\bar\mu\bar\nu\nu}_{\pm,6} \right] + \frac{s}{v^2}\, (\bar{r}\cdot\bar{r})\, Q^{\bar\mu\bar\nu\nu}_{\pm,4} \mp \frac{\sqrt{s}}{v^2}\, (\bar{r}\cdot\bar{r})\, Q^{\bar\mu\bar\nu\nu}_{\mp,1}\\
&\quad \mp \frac{s}{v^2}\, \bar{E}_\mp \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\pm,8} + \bar{E}_\pm\, E_\pm\, Q^{\bar\mu\bar\nu\nu}_{\mp,8} \right] - \frac{\sqrt{s}}{v^2}\, \bar{E}_\mp \left[ (\bar{r}\cdot r)\, Q^{\bar\mu\bar\nu\nu}_{\mp,3} + \bar{E}_\pm\, E_\mp\, Q^{\bar\mu\bar\nu\nu}_{\pm,3} \right] ,
\end{aligned}
$$

where

$$
\begin{aligned}
P^{\bar\mu\bar\nu\nu}_{\pm,1} &= v^{\bar\mu} \left[ {r_\perp}^{\bar\nu}\, {\bar{r}_\perp}{}^{\nu}\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (s/v^2)\, \bar{E}_\pm\, E_\pm\, P_\mp/v^2 - {r_\perp}^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\pm\, i\gamma_5\, P_\pm + v^{\bar\nu}\, {\bar{r}_\perp}{}^{\nu}\, (\sqrt{s}/v^2)\, \bar{E}_\pm\, i\gamma_5\, P_\mp \right] / v^2 ,\\
P^{\bar\mu\bar\nu\nu}_{\pm,2} &= v^{\bar\mu} \left[ {r_\perp}^{\bar\nu}\, {\bar{w}_\perp}{}^{\nu}\, P_\pm + v^{\bar\nu}\, {\bar{w}_\perp}{}^{\nu}\, (\sqrt{s}/v^2)\, \bar{E}_\pm\, i\gamma_5\, P_\mp - {r_\perp}^{\bar\nu}\, v^{\nu}\, i\gamma_5\, ((\bar{r}\cdot r)\, P_\pm \pm M\, \bar{E}_\pm\, P_\mp)/v^2 \right.\\
&\quad \left. + v^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, \bar{E}_\pm\, ((\bar{r}\cdot r)\, P_\mp \pm M\, \bar{E}_\mp\, P_\pm)/v^2 \right] / v^2 ,\\
P^{\bar\mu\bar\nu\nu}_{\pm,3} &= v^{\bar\mu} \left[ {w_\perp}^{\bar\nu}\, {\bar{r}_\perp}{}^{\nu}\, P_\pm - {w_\perp}^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\pm\, i\gamma_5\, P_\pm + v^{\bar\nu}\, {\bar{r}_\perp}{}^{\nu}\, i\gamma_5\, ((\bar{r}\cdot r)\, P_\mp \pm \bar{M}\, E_\pm\, P_\pm)/v^2 \right.\\
&\quad \left. + v^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\pm\, ((\bar{r}\cdot r)\, P_\mp \pm \bar{M}\, E_\mp\, P_\pm)/v^2 \right] / v^2 ,\\
P^{\bar\mu\bar\nu\nu}_{\pm,4} &= v^{\bar\mu} \left[ {w_\perp}^{\bar\nu}\, {\bar{w}_\perp}{}^{\nu}\, P_\pm + v^{\bar\nu}\, v^{\nu}\, \{ (1/s)\, P_\mp + \tfrac{1}{4}\, (\bar\delta - 1)(\delta - 1)(s/v^2)\, \bar{E}_\pm\, E_\pm\, P_\mp \right.\\
&\qquad \mp \tfrac{1}{2}\, (\bar\delta - 1)(\sqrt{s}/v^2)\, M\, ((\bar{r}\cdot r)\, P_\pm - \bar{E}_\pm\, E_\pm\, P_\mp) \mp \tfrac{1}{2}\, (\delta - 1)(\sqrt{s}/v^2)\, \bar{M}\, ((\bar{r}\cdot r)\, P_\pm - \bar{E}_\pm\, E_\pm\, P_\mp) \} / v^2\\
&\quad \left. - {w_\perp}^{\bar\nu}\, v^{\nu}\, i\gamma_5\, ((\bar{r}\cdot r)\, P_\pm \pm M\, \bar{E}_\pm\, P_\mp)/v^2 + v^{\bar\nu}\, {\bar{w}_\perp}{}^{\nu}\, i\gamma_5\, ((\bar{r}\cdot r)\, P_\mp \pm \bar{M}\, E_\pm\, P_\pm)/v^2 \right] / v^2 ,\\
P^{\bar\mu\bar\nu\nu}_{\pm,5} &= {r_\perp}^{\bar\mu} \left[ {r_\perp}^{\bar\nu}\, {\bar{r}_\perp}{}^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (s/v^2)\, \bar{E}_\pm\, E_\mp\, i\gamma_5\, P_\mp/v^2 + {r_\perp}^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\mp\, P_\pm - v^{\bar\nu}\, {\bar{r}_\perp}{}^{\nu}\, (\sqrt{s}/v^2)\, \bar{E}_\pm\, P_\mp \right] ,\\
P^{\bar\mu\bar\nu\nu}_{\pm,6} &= {r_\perp}^{\bar\mu} \left[ {r_\perp}^{\bar\nu}\, {\bar{w}_\perp}{}^{\nu}\, i\gamma_5\, P_\pm - v^{\bar\nu}\, {\bar{w}_\perp}{}^{\nu}\, (\sqrt{s}/v^2)\, \bar{E}_\pm\, P_\mp + {r_\perp}^{\bar\nu}\, v^{\nu}\, ((\bar{r}\cdot r)\, P_\pm \mp M\, \bar{E}_\pm\, P_\mp)/v^2 \right.\\
&\quad \left. + v^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, \bar{E}_\pm\, i\gamma_5\, ((\bar{r}\cdot r)\, P_\mp \mp M\, \bar{E}_\mp\, P_\pm)/v^2 \right] ,
\end{aligned}
$$

$$
\begin{aligned}
P^{\bar\mu\bar\nu\nu}_{\pm,7} &= r_\perp{}^{\bar\mu}\,\big[w_\llcorner{}^{\bar\nu}\,\bar r_\perp{}^{\nu}\,i\gamma_5\,P_\pm + w_\llcorner{}^{\bar\nu}\,v^\nu\,(\sqrt{s}/v^2)\,E_\mp\,P_\pm - v^{\bar\nu}\,\bar r_\perp{}^{\nu}\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\mp\,P_\pm)/v^2 \\
&\quad + v^{\bar\nu}\,v^\nu\,(\sqrt{s}/v^2)\,E_\mp\,i\gamma_5\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\pm\,P_\pm)/v^2\big]\,, \\
P^{\bar\mu\bar\nu\nu}_{\pm,8} &= r_\perp{}^{\bar\mu}\,\big[w_\llcorner{}^{\bar\nu}\,\bar w_\llcorner{}^{\nu}\,i\gamma_5\,P_\pm + v^{\bar\nu}\,v^\nu\,\big\{(1/s)\,i\gamma_5\,P_\mp + \tfrac{1}{4}(\bar\delta-1)(\delta-1)(s/v^2)\,\bar E_\pm\,E_\mp\,i\gamma_5\,P_\mp \\
&\qquad \pm\,\tfrac{1}{2}(\bar\delta-1)(\sqrt{s}/v^2)\,M\,i\gamma_5\,((\bar r\cdot r)\,P_\pm - \bar E_\pm\,E_\mp\,P_\mp) \mp \tfrac{1}{2}(\delta-1)(\sqrt{s}/v^2)\,\bar M\,i\gamma_5\,((\bar r\cdot r)\,P_\pm - \bar E_\pm\,E_\mp\,P_\mp)\big\}/v^2 \\
&\quad + w_\llcorner{}^{\bar\nu}\,v^\nu\,((\bar r\cdot r)\,P_\pm \mp M\,\bar E_\pm\,P_\mp)/v^2 - v^{\bar\nu}\,\bar w_\llcorner{}^{\nu}\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\mp\,P_\pm)/v^2\big]\,, \\
P^{\bar\mu\bar\nu\nu}_{\pm,9} &= w_\lrcorner{}^{\bar\mu}\,\big[r_\perp{}^{\bar\nu}\,\bar r_\perp{}^{\nu}\,i\gamma_5\,P_\pm + v^{\bar\nu}\,v^\nu\,(s/v^2)\,\bar E_\pm\,E_\mp\,i\gamma_5\,P_\mp/v^2 + r_\perp{}^{\bar\nu}\,v^\nu\,(\sqrt{s}/v^2)\,E_\mp\,P_\pm - v^{\bar\nu}\,\bar r_\perp{}^{\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\,P_\mp\big]\,, \\
P^{\bar\mu\bar\nu\nu}_{\pm,10} &= w_\lrcorner{}^{\bar\mu}\,\big[r_\perp{}^{\bar\nu}\,\bar w_\llcorner{}^{\nu}\,i\gamma_5\,P_\pm - v^{\bar\nu}\,\bar w_\llcorner{}^{\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\,P_\mp + r_\perp{}^{\bar\nu}\,v^\nu\,((\bar r\cdot r)\,P_\pm \mp M\,\bar E_\pm\,P_\mp)/v^2 \\
&\quad + v^{\bar\nu}\,v^\nu\,(\sqrt{s}/v^2)\,\bar E_\pm\,i\gamma_5\,((\bar r\cdot r)\,P_\mp \mp M\,\bar E_\mp\,P_\pm)/v^2\big]\,, \\
P^{\bar\mu\bar\nu\nu}_{\pm,11} &= w_\lrcorner{}^{\bar\mu}\,\big[w_\llcorner{}^{\bar\nu}\,\bar r_\perp{}^{\nu}\,i\gamma_5\,P_\pm + w_\llcorner{}^{\bar\nu}\,v^\nu\,(\sqrt{s}/v^2)\,E_\mp\,P_\pm - v^{\bar\nu}\,\bar r_\perp{}^{\nu}\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\mp\,P_\pm)/v^2 \\
&\quad + v^{\bar\nu}\,v^\nu\,(\sqrt{s}/v^2)\,E_\mp\,i\gamma_5\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\pm\,P_\pm)/v^2\big]\,, \\
P^{\bar\mu\bar\nu\nu}_{\pm,12} &= w_\lrcorner{}^{\bar\mu}\,\big[w_\llcorner{}^{\bar\nu}\,\bar w_\llcorner{}^{\nu}\,i\gamma_5\,P_\pm + v^{\bar\nu}\,v^\nu\,\big\{(1/s)\,i\gamma_5\,P_\mp + \tfrac{1}{4}(\bar\delta-1)(\delta-1)(s/v^2)\,\bar E_\pm\,E_\mp\,i\gamma_5\,P_\mp \\
&\qquad \pm\,\tfrac{1}{2}(\bar\delta-1)(\sqrt{s}/v^2)\,M\,i\gamma_5\,((\bar r\cdot r)\,P_\pm - \bar E_\pm\,E_\mp\,P_\mp) \mp \tfrac{1}{2}(\delta-1)(\sqrt{s}/v^2)\,\bar M\,i\gamma_5\,((\bar r\cdot r)\,P_\pm - \bar E_\pm\,E_\mp\,P_\mp)\big\}/v^2 \\
&\quad + w_\llcorner{}^{\bar\nu}\,v^\nu\,((\bar r\cdot r)\,P_\pm \mp M\,\bar E_\pm\,P_\mp)/v^2 - v^{\bar\nu}\,\bar w_\llcorner{}^{\nu}\,((\bar r\cdot r)\,P_\mp \pm \bar M\,E_\mp\,P_\pm)/v^2\big]\,.
\end{aligned}
\tag{A.3}
$$

## A.2 $1^- + \frac{1}{2}^+ \to 1^- + \frac{3}{2}^+$

For the thirty-six invariant amplitudes $F_n^\pm$ introduced in (2.56), we derive

$$
\frac{1}{2}\,\mathbf{tr}\big(T^{(n)}_{a,\bar\mu\bar\nu\mu}\,\Lambda\,Q^{\bar\mu\bar\nu\mu}_{b,k}\,\bar\Lambda\big) = \delta_{nk}\,\delta_{ab}\,,
\tag{A.4}
$$

$$
\text{with}\qquad \bar q_{\bar\mu}\,Q^{\bar\mu\bar\nu\mu}_{\pm,k} = 0\,,\qquad q_\mu\,Q^{\bar\mu\bar\nu\mu}_{\pm,k} = 0\,,\ \text{and}\qquad \bar p_{\bar\nu}\,Q^{\bar\mu\bar\nu\mu}_{\pm,k} = 0 = \Lambda\,Q^{\bar\mu\bar\nu\mu}_{\pm,k}\,\bar\Lambda\,\gamma_{\bar\nu}\,,
$$

$$
\begin{aligned}
Q^{\bar\mu\bar\nu\mu}_{\pm,1} &= 3\,(\bar\delta-1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{\bar M}\,(r\cdot r)\,\bar E_\mp\,\Big(s\,\bar E_\pm\,E_\mp\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,1} - \bar E_\mp\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\pm,1}\Big] + \sqrt{s}\,E_\mp\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\pm,2} - \bar E_\pm\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\mp,2}\Big] \\
&\qquad - \sqrt{s}\,\bar E_\pm\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,7} - \bar E_\mp\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\pm,7}\Big] + \Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\pm,8} - \bar E_\pm\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\mp,8}\Big]\Big) \\
&\quad + 4\,\frac{s}{v^2}\,\frac{\sqrt{s}}{\bar M}\,(\bar r\cdot\bar r)\,E_\mp\,\Big(s\,\bar E_\mp\,E_\pm\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\pm,4} - \bar E_\pm\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\mp,4}\Big] + \sqrt{s}\,E_\pm\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,5} - \bar E_\mp\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\pm,5}\Big] \\
&\qquad - \sqrt{s}\,\bar E_\mp\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\pm,10} - \bar E_\pm\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\mp,10}\Big] + \Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,11} - \bar E_\mp\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\pm,11}\Big]\Big) \\
&\quad + 2\,(\bar\delta-1)\,\frac{\sqrt{s}}{\bar M}\,E_\mp\,\Big(s\,\bar E_\mp\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\pm,1} + \sqrt{s}\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\mp,2} - \sqrt{s}\,\bar E_\mp\,P^{\bar\mu\bar\nu\mu}_{\pm,7} + P^{\bar\mu\bar\nu\mu}_{\mp,8}\Big) \\
&\quad \pm E_\mp\,\Big(-s\,\bar E_\mp\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\pm,1} + \sqrt{s}\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\mp,2} + \sqrt{s}\,\bar E_\mp\,P^{\bar\mu\bar\nu\mu}_{\pm,7} + P^{\bar\mu\bar\nu\mu}_{\mp,8}\Big) \\
&\quad + (\bar r\cdot r)\,\frac{1}{\bar M}\,\Big(s\,\bar E_\mp\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\mp,1} + \sqrt{s}\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\pm,2} - \sqrt{s}\,\bar E_\mp\,P^{\bar\mu\bar\nu\mu}_{\mp,7} + P^{\bar\mu\bar\nu\mu}_{\pm,8}\Big) \\
&\quad - \frac{s}{\bar M}\,\sqrt{s}\,\bar E_\mp\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\pm,4} - 3\,\bar E_\pm\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\mp,4}\Big] - \frac{s}{\bar M}\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,5} - 3\,\bar E_\mp\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\pm,5}\Big] + 2\,\frac{\sqrt{s}}{\bar M}\,\bar E_\mp\,\Big[P^{\bar\mu\bar\nu\mu}_{\pm,11} - \sqrt{s}\,\bar E_\pm\,P^{\bar\mu\bar\nu\mu}_{\mp,10}\Big]\,, \\
Q^{\bar\mu\bar\nu\mu}_{\pm,2} &= -(\bar\delta-1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{\bar M}\,(r\cdot r)\,\bar E_\pm\,\Big(s\,\bar E_\mp\,E_\pm\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\pm,1} - \bar E_\pm\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\mp,1}\Big] + \sqrt{s}\,E_\pm\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,2} - \bar E_\mp\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\pm,2}\Big] \\
&\qquad - \sqrt{s}\,\bar E_\mp\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\pm,7} - \bar E_\pm\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\mp,7}\Big] + \Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,8} - \bar E_\mp\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\pm,8}\Big]\Big) \\
&\quad - 2\,\frac{s}{v^2}\,\frac{\sqrt{s}}{\bar M}\,(\bar r\cdot\bar r)\,E_\pm\,\Big(s\,\bar E_\pm\,E_\mp\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,4} - \bar E_\mp\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\pm,4}\Big] + \sqrt{s}\,E_\mp\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\pm,5} - \bar E_\pm\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\mp,5}\Big] \\
&\qquad - \sqrt{s}\,\bar E_\pm\,\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,10} - \bar E_\mp\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\pm,10}\Big] + \Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\pm,11} - \bar E_\pm\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\mp,11}\Big]\Big) \\
&\quad + \tfrac{1}{2}(\bar\delta-1)\,\frac{s}{\bar M}\,\bar E_\pm\,E_\pm\,\Big[P^{\bar\mu\bar\nu\mu}_{\mp,7} - \sqrt{s}\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\mp,1}\Big] \pm \sqrt{s}\,(\bar r\cdot r)\,\Big[P^{\bar\mu\bar\nu\mu}_{\pm,7} - \sqrt{s}\,E_\pm\,P^{\bar\mu\bar\nu\mu}_{\pm,1}\Big] + \frac{1}{\bar M}\,\bar E_\pm\,E_\pm\,\Big[P^{\bar\mu\bar\nu\mu}_{\pm,8} + \sqrt{s}\,E_\mp\,P^{\bar\mu\bar\nu\mu}_{\pm,2}\Big]
\end{aligned}
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\begin{aligned}
&-\frac{\sqrt{s}}{\bar{M}}\,\bar{E}_\pm\left(s\,\bar{E}_\mp\,E_\pm\,P^{\bar{\mu}\bar{\nu}\mu}_{\pm,4}+\sqrt{s}\,E_\pm\,P^{\bar{\mu}\bar{\nu}\mu}_{\mp,5}-\sqrt{s}\,\bar{E}_\mp\,P^{\bar{\mu}\bar{\nu}\mu}_{\pm,10}+P^{\bar{\mu}\bar{\nu}\mu}_{\mp,11}\right),\\
Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,3}&=\pm\frac{\sqrt{s}}{\bar{M}}\left[P^{\bar{\mu}\bar{\nu}\mu}_{\mp,17}-\sqrt{s}\,\bar{E}_\mp\,P^{\bar{\mu}\bar{\nu}\mu}_{\pm,16}\right]\\
&\mp\tfrac{1}{4}(\bar{\delta}-1)(\delta+1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{\bar{M}}\,(\bar{r}\cdot r)\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,6}+\bar{E}_\mp\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,6}\right]\pm\tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\,\bar{E}_\mp\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}+\bar{E}_\pm\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,1}\right],\\
Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,4}&=\frac{\sqrt{s}}{\bar{E}_\pm}\left[(\bar{r}\cdot r)P^{\bar{\mu}\bar{\nu}\mu}_{\pm,3}-\bar{E}_\pm\,E_\pm\,P^{\bar{\mu}\bar{\nu}\mu}_{\mp,3}\right]\mp\frac{1}{\bar{E}_\pm}\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,12}+\bar{E}_\pm\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,12}\right]\\
&+\tfrac{1}{2}(\bar{\delta}+1)\Big(-\frac{E_\pm}{\bar{E}_\pm}\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,8}\pm\frac{s}{v^2}\,\frac{(\bar{r}\cdot r)}{\bar{E}_\pm}\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}+\bar{E}_\pm\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,1}\right]\pm\frac{s}{v^2}\,E_\pm\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,2}+\bar{E}_\mp\,E_\mp\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,2}\right]\Big),\\
Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,5}&=\frac{\sqrt{s}}{\bar{E}_\pm}\left[(\bar{r}\cdot r)P^{\bar{\mu}\bar{\nu}\mu}_{\pm,7}-\bar{E}_\pm\,E_\pm\,P^{\bar{\mu}\bar{\nu}\mu}_{\mp,7}\right]\mp\frac{1}{\bar{E}_\pm}\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,2}-\frac{E_\pm}{\bar{E}_\pm}\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,8}\\
&\mp\frac{s}{v^2}\,E_\pm\left[(\bar{r}\cdot r)(Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,1}+Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,2})+\bar{E}_\mp\,E_\mp\,(Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}+Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,2})\right],\\
Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,6}&=\frac{2}{(\bar{\delta}-1)}\,\frac{1}{(r\cdot r)}\left[(\bar{r}\cdot r)P^{\bar{\mu}\bar{\nu}\mu}_{\mp,11}-\bar{E}_\mp\,E_\pm\,P^{\bar{\mu}\bar{\nu}\mu}_{\pm,11}\right]+\frac{2}{(\bar{\delta}-1)}\,\frac{\bar{E}_\mp}{E_\mp}\left(Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,7}\pm\frac{1}{E_\pm}\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,9}+\bar{E}_\pm\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,9}\right]\right)\\
&\mp\frac{s}{v^2}\,\bar{E}_\mp\left[(\bar{r}\cdot r)(Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}+Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,2})+\bar{E}_\pm\,E_\pm\,(Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,1}+Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,2})\right]\mp\frac{1}{E_\mp}\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,1}+\frac{(\bar{r}\cdot r)}{(r\cdot r)}\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,5},\\
Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,7}&=s\left[(\bar{r}\cdot r)P^{\bar{\mu}\bar{\nu}\mu}_{\pm,4}-\bar{E}_\pm\,E_\mp\,P^{\bar{\mu}\bar{\nu}\mu}_{\mp,4}\right]\\
&\pm\tfrac{1}{2}(\bar{\delta}-1)\,\frac{s}{v^2}\,E_\mp\left[(\bar{r}\cdot r)(Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}+Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,2})+\bar{E}_\pm\,E_\pm\,(Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,1}+Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,2})\right]\mp\frac{1}{\sqrt{s}}\,(Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,1}+Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,2}),\\
Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,8}&=s\left[(\bar{r}\cdot r)P^{\bar{\mu}\bar{\nu}\mu}_{\pm,1}-\bar{E}_\pm\,E_\mp\,P^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}\right]\mp\frac{s}{v^2}\,\bar{E}_\pm\left[(\bar{r}\cdot r)(Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,1}+Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,2})+\bar{E}_\mp\,E_\mp\,(Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}+Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,2})\right],\\
Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,9}&=\pm\sqrt{s}\,P^{\bar{\mu}\bar{\nu}\mu}_{\pm,10}\mp\frac{1}{\sqrt{s}}\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,5}+\tfrac{1}{2}(\bar{\delta}-1)\,\frac{s}{v^2}\,E_\pm\left(E_\mp\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,2}\pm\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,5}+\bar{E}_\pm\,E_\mp\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,5}\right]\right)\\
&-\frac{\sqrt{s}}{v^2}\,E_\pm\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,1}\mp\bar{M}\,E_\mp\,Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}\right]\mp\frac{s}{v^2}\,E_\pm\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,7}+\bar{E}_\pm\,E_\mp\,Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,7}\right],\\
Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,10}&=\mp\sqrt{s}\,P^{\bar{\mu}\bar{\nu}\mu}_{\mp,6}\pm\frac{1}{\sqrt{s}}\,Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,4}+\tfrac{1}{4}(\bar{\delta}+1)(\bar{\delta}-1)\,\frac{s}{v^2}\,(r\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}\mp\tfrac{1}{2}(\bar{\delta}-1)\,\frac{s}{v^2}\,E_\mp\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,4}+\bar{E}_\mp\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,4}\right]\\
&\pm\tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\,E_\mp\left(\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,7}+\bar{E}_\mp\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,7}\right]\pm\frac{1}{\sqrt{s}}\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,2}\pm\bar{M}\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,2}\right]\right),\\
Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,11}&=\pm\sqrt{s}\,P^{\bar{\mu}\bar{\nu}\mu}_{\pm,13}-\tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\left((\bar{r}\cdot\bar{r})(Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}+Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,2})\mp\bar{E}_\pm\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,8}+\bar{E}_\mp\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,8}\right]\right)\\
&\mp\frac{s}{v^2}\,\bar{E}_\pm\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,3}+\bar{E}_\mp\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,3}\right],\\
Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,12}&=\mp\frac{s}{v^2}\,\sqrt{s}\,(\bar{r}\cdot r)\left[(\bar{r}\cdot r)P^{\bar{\mu}\bar{\nu}\mu}_{\mp,3}-\bar{E}_\mp\,E_\mp\,P^{\bar{\mu}\bar{\nu}\mu}_{\pm,3}\right]\pm\frac{s}{v^2}\,\bar{E}_\mp\left[(\bar{r}\cdot r)P^{\bar{\mu}\bar{\nu}\mu}_{\pm,9}-\bar{E}_\pm\,E_\mp\,P^{\bar{\mu}\bar{\nu}\mu}_{\mp,9}\right]\\
&+\tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\left((\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,1}\pm(\bar{r}\cdot r)\bar{E}_\mp\,Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,5}\pm E_\mp\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,8}+\bar{E}_\mp\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,8}\right]\right),\\
Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,13}&=\pm\sqrt{s}\,P^{\bar{\mu}\bar{\nu}\mu}_{\pm,16}\mp\frac{1}{\sqrt{s}}\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,3}-\tfrac{1}{4}(\bar{\delta}-1)(\delta+1)\,\frac{s}{v^2}\,(\bar{r}\cdot r)(Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}+Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,2})\\
&\pm\tfrac{1}{2}(\bar{\delta}-1)\,\frac{s}{v^2}\,E_\pm\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,3}+\bar{E}_\pm\,E_\mp\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,3}\right]\\
&\pm\tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\left(\bar{E}_\pm\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,7}+\bar{E}_\mp\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,7}\right]-\frac{\bar{M}}{\sqrt{s}}\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}+\bar{E}_\pm\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,1}\right]\right),\\
Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,14}&=\pm\frac{1}{E_\pm}\,P^{\bar{\mu}\bar{\nu}\mu}_{\mp,8}\mp\frac{1}{E_\pm}\,Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,6}\mp\frac{s}{v^2}\,\bar{E}_\mp\left[(\bar{r}\cdot r)(Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,6}+Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,8})+\bar{E}_\pm\,E_\mp\,(Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,6}+Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,8})\right]\\
&-\frac{s}{v^2}\,\frac{\bar{E}_\mp}{E_\pm}\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,2}+\bar{E}_\pm\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,1}\right]\pm\frac{s}{v^2}\,\frac{(\bar{r}\cdot\bar{r})}{E_\pm}\left[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu}_{\pm,5}+\bar{E}_\mp\,E_\pm\,Q^{\bar{\mu}\bar{\nu}\mu}_{\mp,5}\right],
\end{aligned}
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\begin{aligned}
Q^{\bar\mu\bar\nu\mu}_{\pm,15} &= \frac{s}{v^2}\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,18} - \bar E_\mp\, E_\pm\, P^{\bar\mu\bar\nu\mu}_{\pm,18}\right] \\
&\quad \mp \tfrac{1}{8}(\bar\delta+1)(\bar\delta-1)(\delta+1)\left(\frac{s}{v^2}\right)^2 \Big((r\cdot r)\,\bar E_\mp\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,1} + \bar E_\pm\, E_\pm\, Q^{\bar\mu\bar\nu\mu}_{\pm,1}\right] \\
&\qquad\qquad\qquad - (\bar r\cdot r)\,E_\pm\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,2} + \bar E_\mp\, E_\mp\, Q^{\bar\mu\bar\nu\mu}_{\pm,2}\right]\Big) \\
&\quad + \tfrac{1}{4}(\bar\delta+1)(\bar\delta-1)\,\frac{s}{v^2}\,(r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\pm,3} - \tfrac{1}{4}(\bar\delta-1)(\delta+1)\,\frac{s}{v^2}\,(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\pm,4} \\
&\quad - \tfrac{1}{4}(\bar\delta+1)(\delta+1)\,\frac{s}{v^2}\left[2\,(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,7} - \bar E_\mp\, E_\pm\, Q^{\bar\mu\bar\nu\mu}_{\pm,7}\right] \\
&\quad \pm \tfrac{1}{2}(\bar\delta+1)\,\frac{s}{v^2}\,E_\pm\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,13} + \bar E_\mp\, E_\mp\, Q^{\bar\mu\bar\nu\mu}_{\pm,13}\right] \pm \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\,\bar E_\mp\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,10} + \bar E_\pm\, E_\pm\, Q^{\bar\mu\bar\nu\mu}_{\pm,10}\right], \\
Q^{\bar\mu\bar\nu\mu}_{\pm,16} &= \frac{s}{v^2}\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,12} - \bar E_\mp\, E_\pm\, P^{\bar\mu\bar\nu\mu}_{\pm,12}\right] \mp \frac{s}{v^2}\,E_\pm\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\pm,10} + \bar E_\mp\, E_\mp\, Q^{\bar\mu\bar\nu\mu}_{\mp,10}\right] \\
&\quad + \tfrac{1}{4}(\bar\delta+1)(\bar\delta-1)\,\frac{s}{v^2}\,(r\cdot r)\left(Q^{\bar\mu\bar\nu\mu}_{\pm,5} \pm \frac{s}{v^2}\,E_\pm\left[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu}_{\pm,1} + Q^{\bar\mu\bar\nu\mu}_{\mp,2}) + \bar E_\mp\, E_\mp\,(Q^{\bar\mu\bar\nu\mu}_{\mp,1} + Q^{\bar\mu\bar\nu\mu}_{\pm,2})\right]\right) \\
&\quad - \tfrac{1}{2}(\bar\delta-1)\,\frac{s}{v^2}\,(r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\pm,4} - \tfrac{1}{2}(\bar\delta+1)\,\frac{s}{v^2}\left((r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,7} \mp E_\pm\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,9} + \bar E_\mp\, E_\mp\, Q^{\bar\mu\bar\nu\mu}_{\pm,9}\right]\right), \\
Q^{\bar\mu\bar\nu\mu}_{\pm,17} &= \frac{s}{v^2}\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,15} - \bar E_\mp\, E_\pm\, P^{\bar\mu\bar\nu\mu}_{\pm,15}\right] \\
&\quad - \tfrac{1}{4}(\bar\delta+1)(\delta+1)\,\frac{s}{v^2}\Big(\left[2\,(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,8} - \bar E_\mp\, E_\pm\, Q^{\bar\mu\bar\nu\mu}_{\pm,8}\right] \\
&\qquad \pm \frac{s}{v^2}\,\bar E_\mp\left[(\bar r\cdot r)((\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,1} + \bar E_\pm\, E_\pm\, Q^{\bar\mu\bar\nu\mu}_{\pm,1}) + \bar E_\pm\, E_\pm\,((\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,2} + \bar E_\mp\, E_\mp\, Q^{\bar\mu\bar\nu\mu}_{\pm,2})\right]\Big) \\
&\quad + \tfrac{1}{2}(\bar\delta+1)\,\frac{s}{v^2}\left((\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\pm,3} \pm E_\pm\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,11} + \bar E_\mp\, E_\mp\, Q^{\bar\mu\bar\nu\mu}_{\pm,11}\right]\right) \\
&\quad - \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\left((\bar r\cdot\bar r)\,Q^{\bar\mu\bar\nu\mu}_{\pm,4} \mp \bar E_\mp\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,12} + \bar E_\pm\, E_\pm\, Q^{\bar\mu\bar\nu\mu}_{\pm,12}\right]\right), \\
Q^{\bar\mu\bar\nu\mu}_{\pm,18} &= \frac{s}{v^2}\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu}_{\mp,14} - \bar E_\mp\, E_\pm\, P^{\bar\mu\bar\nu\mu}_{\pm,14}\right] + \frac{s}{v^2}\,(\bar r\cdot\bar r)\,Q^{\bar\mu\bar\nu\mu}_{\pm,3} \mp \frac{s}{v^2}\,\bar E_\mp\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\pm,11} + \bar E_\pm\, E_\pm\, Q^{\bar\mu\bar\nu\mu}_{\mp,11}\right] \\
&\quad - \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\Big((\bar r\cdot\bar r)(Q^{\bar\mu\bar\nu\mu}_{\mp,8} + Q^{\bar\mu\bar\nu\mu}_{\pm,6}) \mp \bar E_\mp\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu}_{\mp,14} + \bar E_\pm\, E_\pm\, Q^{\bar\mu\bar\nu\mu}_{\pm,14}\right] \\
&\qquad \pm \frac{s}{v^2}\,(\bar r\cdot\bar r)\,\bar E_\mp\left[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu}_{\mp,1} + Q^{\bar\mu\bar\nu\mu}_{\pm,2}) + \bar E_\pm\, E_\pm\,(Q^{\bar\mu\bar\nu\mu}_{\pm,1} + Q^{\bar\mu\bar\nu\mu}_{\mp,2})\right]\Big),
\end{aligned}
$$

where

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\begin{aligned}
P^{\bar\mu\bar\nu\mu}_{\pm,1} &= \left[r_\perp{}^{\bar\nu}\, i\gamma_5\, P_\pm - v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\, P_\mp\right] v^{\bar\mu}\, v^\mu/v^2/v^2, \\
P^{\bar\mu\bar\nu\mu}_{\pm,2} &= \left[r_\perp{}^{\bar\nu}\, P_\pm + v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\, i\gamma_5\, P_\mp\right] r_\perp{}^{\bar\mu}\, v^\mu/v^2, \\
P^{\bar\mu\bar\nu\mu}_{\pm,3} &= \left[r_\perp{}^{\bar\nu}\, P_\pm + v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\, i\gamma_5\, P_\mp\right] w_\lrcorner{}^{\bar\mu}\, v^\mu/v^2, \\
P^{\bar\mu\bar\nu\mu}_{\pm,4} &= \left[w_\llcorner{}^{\bar\nu}\, i\gamma_5\, P_\pm - v^{\bar\nu}\,((\bar r\cdot r)\,P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\right] v^{\bar\mu}\, v^\mu/v^2/v^2, \\
P^{\bar\mu\bar\nu\mu}_{\pm,5} &= \left[w_\llcorner{}^{\bar\nu}\, P_\pm + v^{\bar\nu}\, i\gamma_5\,((\bar r\cdot r)\,P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\right] r_\perp{}^{\bar\mu}\, v^\mu/v^2, \\
P^{\bar\mu\bar\nu\mu}_{\pm,6} &= \left[w_\llcorner{}^{\bar\nu}\, P_\pm + v^{\bar\nu}\, i\gamma_5\,((\bar r\cdot r)\,P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\right] w_\lrcorner{}^{\bar\mu}\, v^\mu/v^2, \\
P^{\bar\mu\bar\nu\mu}_{\pm,7} &= \left[r_\perp{}^{\bar\nu}\, P_\pm + v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\, i\gamma_5\, P_\mp\right] v^{\bar\mu}\, \bar r_\perp{}^\mu/v^2, \\
P^{\bar\mu\bar\nu\mu}_{\pm,8} &= \left[r_\perp{}^{\bar\nu}\, i\gamma_5\, P_\pm - v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\, P_\mp\right] r_\perp{}^{\bar\mu}\, \bar r_\perp{}^\mu, \\
P^{\bar\mu\bar\nu\mu}_{\pm,9} &= \left[r_\perp{}^{\bar\nu}\, i\gamma_5\, P_\pm - v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\, P_\mp\right] w_\lrcorner{}^{\bar\mu}\, \bar r_\perp{}^\mu, \\
P^{\bar\mu\bar\nu\mu}_{\pm,10} &= \left[w_\llcorner{}^{\bar\nu}\, P_\pm + v^{\bar\nu}\, i\gamma_5\,((\bar r\cdot r)\,P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\right] v^{\bar\mu}\, \bar r_\perp{}^\mu/v^2, \\
P^{\bar\mu\bar\nu\mu}_{\pm,11} &= \left[w_\llcorner{}^{\bar\nu}\, i\gamma_5\, P_\pm - v^{\bar\nu}\,((\bar r\cdot r)\,P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\right] r_\perp{}^{\bar\mu}\, \bar r_\perp{}^\mu, \\
P^{\bar\mu\bar\nu\mu}_{\pm,12} &= \left[w_\llcorner{}^{\bar\nu}\, i\gamma_5\, P_\pm - v^{\bar\nu}\,((\bar r\cdot r)\,P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\right] w_\lrcorner{}^{\bar\mu}\, \bar r_\perp{}^\mu, \\
P^{\bar\mu\bar\nu\mu}_{\pm,13} &= \left[r_\perp{}^{\bar\nu}\, P_\pm + v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\, i\gamma_5\, P_\mp\right] v^{\bar\mu}\, \bar w_\lrcorner{}^\mu/v^2, \\
P^{\bar\mu\bar\nu\mu}_{\pm,14} &= \left[r_\perp{}^{\bar\nu}\, i\gamma_5\, P_\pm - v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\, P_\mp\right] r_\perp{}^{\bar\mu}\, \bar w_\lrcorner{}^\mu,
\end{aligned}
$$

$$P^{\bar\mu\bar\nu\mu}_{\pm,15} = \big[r_\lfloor^{\bar\nu}\, i\,\gamma_5\, P_\pm - v^{\bar\nu}\,(\sqrt{s}/v^2)\,\bar E_\pm\, P_\mp\big]\, w_\rfloor^{\bar\mu}\, \bar w_\rfloor^{\mu}\,,$$
$$P^{\bar\mu\bar\nu\mu}_{\pm,16} = \big[w_\lfloor^{\bar\nu}\, P_\pm + v^{\bar\nu}\, i\,\gamma_5\,((\bar r\cdot r)\,P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\big]\, v^{\bar\mu}\, \bar w_\rfloor^{\mu}/v^2\,,$$
$$P^{\bar\mu\bar\nu\mu}_{\pm,17} = \big[w_\lfloor^{\bar\nu}\, i\,\gamma_5\, P_\pm - v^{\bar\nu}\,((\bar r\cdot r)\,P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\big]\, r_\rfloor^{\bar\mu}\, \bar w_\rfloor^{\mu}\,,$$
$$P^{\bar\mu\bar\nu\mu}_{\pm,18} = \big[w_\lfloor^{\bar\nu}\, i\,\gamma_5\, P_\pm - v^{\bar\nu}\,((\bar r\cdot r)\,P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\big]\, w_\rfloor^{\bar\mu}\, \bar w_\rfloor^{\mu}\,. \tag{A.5}$$

## A.3 $1^- + \frac{3}{2}^+ \to 1^- + \frac{3}{2}^+$

Finally, we find the projection algebra for the scattering of spin-one bosons off spin-three-half fermions. The seventy-two invariant amplitudes $F^\pm_n$ introduced in (2.58) can be derived from the following projection algebra

$$\frac{1}{2}\,\mathbf{tr}\big(T^{(n)}_{a,\bar\mu\bar\nu\mu\nu}\,\Lambda\, Q^{\bar\mu\bar\nu\mu\nu}_{b,k}\,\bar\Lambda\big) = \delta_{nk}\,\delta_{ab}\,, \tag{A.6}$$
$$\text{with}\quad \bar q_{\bar\mu}\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,k} = 0\,,\quad q_\mu\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,k} = 0\,,\quad \bar p_{\bar\nu}\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,k} = 0 = \Lambda\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,k}\,\bar\Lambda\,\gamma_{\bar\nu}\,,\ \text{and}\quad p_\nu\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,k} = 0 = \gamma_\nu\,\Lambda\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,k}\,\bar\Lambda\,,$$

$$\begin{aligned}
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} = {}& 8\,(\bar\delta-1)\,(\delta-1)\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\,\frac{s}{v^2}\,(\bar r\cdot\bar r)\,(r\cdot r)\Big(s\,\bar E_\mp\, E_\mp\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - \bar E_\pm\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,1}\big]\\
&\quad + \sqrt{s}\,\bar E_\mp\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,13} - \bar E_\pm\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,13}\big] - \sqrt{s}\,E_\mp\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} - \bar E_\mp\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,5}\big]\\
&\quad + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} - \bar E_\mp\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\big]\Big)\\
& - 6\,(\bar\delta-1)\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\,\frac{s}{v^2}\,(r\cdot r)\,\bar E_\mp\, E_\mp\Big(s\,\bar E_\pm\, E_\pm\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,2} - \bar E_\mp\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,2}\big]\\
&\quad + \sqrt{s}\,\bar E_\pm\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,14} - \bar E_\mp\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,14}\big] - \sqrt{s}\,E_\pm\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,6} - \bar E_\pm\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,6}\big]\\
&\quad + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,18} - \bar E_\pm\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,18}\big]\Big)\\
& - 6\,(\delta-1)\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\,\frac{s}{v^2}\,(\bar r\cdot\bar r)\,\bar E_\mp\, E_\mp\Big(s\,\bar E_\pm\, E_\pm\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,3} - \bar E_\mp\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,3}\big]\\
&\quad + \sqrt{s}\,\bar E_\pm\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,15} - \bar E_\mp\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,15}\big] - \sqrt{s}\,E_\pm\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,7} - \bar E_\pm\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,7}\big]\\
&\quad + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,19} - \bar E_\pm\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,19}\big]\Big)\\
& + 8\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\,\frac{s}{v^2}\,(\bar r\cdot\bar r)\,(r\cdot r)\Big(s\,\bar E_\mp\, E_\mp\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} - \bar E_\pm\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,4}\big]\\
&\quad + \sqrt{s}\,\bar E_\mp\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,16} - \bar E_\pm\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,16}\big] - \sqrt{s}\,E_\mp\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,8} - \bar E_\mp\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,8}\big]\\
&\quad + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,20} - \bar E_\mp\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,20}\big]\Big)\\
& + \tfrac{1}{4}\,(\bar\delta+1)\,(\delta+1)\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\Big(s\,\bar E_\mp\, E_\mp\,\big[9\,(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - 25\,\bar E_\pm\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,1}\big]\\
&\quad - \sqrt{s}\,E_\mp\,\big[9\,(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} - 25\,\bar E_\mp\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,5}\big] + \sqrt{s}\,\bar E_\mp\,\big[9\,(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} - 25\,\bar E_\pm\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,13}\big]\\
&\quad + \big[9\,(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} - 25\,\bar E_\mp\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\big]\Big)\\
& - \tfrac{1}{2}\,(\delta+1)\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\Big(s\,\bar E_\mp\, E_\mp\,\big[(9\pm 3\,(\bar M/\sqrt{s}))\,(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - (25\mp 3\,(\bar M/\sqrt{s}))\,\bar E_\pm\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,1}\big]\\
&\quad - \sqrt{s}\,E_\mp\,\big[(\bar r\cdot r)\,(9\mp 3\,(\bar M/\sqrt{s}))\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} - (25\pm 3\,(\bar M/\sqrt{s}))\,\bar E_\mp\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,5}\big]\\
&\quad + \sqrt{s}\,\bar E_\mp\,\big[(\bar r\cdot r)\,(9\pm 3\,(\bar M/\sqrt{s}))\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} - (25\mp 3\,(\bar M/\sqrt{s}))\,\bar E_\pm\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,13}\big]\\
&\quad + \big[(\bar r\cdot r)\,(9\mp 3\,(\bar M/\sqrt{s}))\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} - (25\pm 3\,(\bar M/\sqrt{s}))\,\bar E_\mp\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\big]\\
&\quad - 8\,(\bar r\cdot\bar r)\,(s\,\bar E_\mp\, E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,3} - \sqrt{s}\,E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,7} + \sqrt{s}\,\bar E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,15} + P^{\bar\mu\bar\nu\mu\nu}_{\mp,19})\Big)\\
& - \tfrac{1}{2}\,(\bar\delta+1)\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\Big(s\,\bar E_\mp\, E_\mp\,\big[(\bar r\cdot r)\,(9\pm 3\,(M/\sqrt{s}))\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - (25\mp 3\,(M/\sqrt{s}))\,\bar E_\pm\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,1}\big]
\end{aligned}$$

$$
\begin{aligned}
&- \sqrt{s}\, E_\mp \left[ (\bar r\cdot r)(9 \pm 3(M/\sqrt{s}))\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} - (25 \mp 3(M/\sqrt{s}))\, \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} \right] \\
&+ \sqrt{s}\, \bar E_\mp \left[ (\bar r\cdot r)(9 \mp 3(M/\sqrt{s}))\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} - (25 \pm 3(M/\sqrt{s}))\, \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,13} \right] \\
&+ \left[ (\bar r\cdot r)(9 \mp 3(M/\sqrt{s}))\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} - (25 \pm 3(M/\sqrt{s}))\, \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,17} \right] \\
&- 8\,(r\cdot r)\left( s\, \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,2} - \sqrt{s}\, E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,6} + \sqrt{s}\, \bar E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,14} + P^{\bar\mu\bar\nu\mu\nu}_{\mp,18} \right)\Big) \\
+ \frac{\sqrt{s}}{\bar M}\frac{\sqrt{s}}{M} \Big( &+ \bar E_\mp E_\mp \left[ (2\bar M M \pm 3\bar M\sqrt{s} \pm 3M\sqrt{s} + 9s)(\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} \right. \\
&\qquad \left. - (2\bar M M \mp 3\bar M\sqrt{s} \mp 3M\sqrt{s} + 25s)\, \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,1} \right] \\
&+ 2\sqrt{s}\,(r\cdot r)\, \bar E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,2} - (4 \pm (\bar M/\sqrt{s}))\sqrt{s}\, E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,2} \right] \\
&+ 2\sqrt{s}\,(\bar r\cdot \bar r)\, E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,3} - (4 \pm (M/\sqrt{s}))\sqrt{s}\, \bar E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,3} \right] \\
&+ s\, \bar E_\mp E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} - 5\, \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,4} \right] \\
&+ (E_\mp/\sqrt{s}) \left[ (2\bar M M \pm 3\bar M\sqrt{s} \mp 3M\sqrt{s} - 9s)(\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} \right. \\
&\qquad \left. - (2\bar M M \mp 3\bar M\sqrt{s} \pm 3M\sqrt{s} - 25s)\, \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} \right] \\
&- 2\,(r\cdot r) \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,6} - (4 \mp (\bar M/\sqrt{s}))\sqrt{s}\, E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,6} \right] \\
&+ 2\, \bar E_\mp E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,7} - (4 \pm (M/\sqrt{s}))\sqrt{s}\, \bar E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,7} \right] - \sqrt{s}\, E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,8} - 5\, \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,8} \right] \\
&- (\bar E_\mp/\sqrt{s}) \left[ (2\bar M M \mp 3\bar M\sqrt{s} \pm 3M\sqrt{s} - 9s)(\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} \right. \\
&\qquad \left. - (2\bar M M \pm 3\bar M\sqrt{s} \mp 3M\sqrt{s} - 25s)\, \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,13} \right] \\
&- 2\, \bar E_\mp E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,14} - (4 \pm (\bar M/\sqrt{s}))\sqrt{s}\, E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,14} \right] \\
&+ 2\,(\bar r\cdot \bar r) \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,15} - (4 \mp (M/\sqrt{s}))\sqrt{s}\, \bar E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,15} \right] + \sqrt{s}\, \bar E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,16} - 5\, \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,16} \right] \\
&+ (1/s) \left[ (2\bar M M \mp 3\bar M\sqrt{s} \mp 3M\sqrt{s} + 9s)(\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} \right. \\
&\qquad \left. - (2\bar M M \pm 3\bar M\sqrt{s} \pm 3M\sqrt{s} + 25s)\, \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,17} \right] \\
&- 2\,(E_\mp/\sqrt{s}) \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,18} - (4 \mp (\bar M/\sqrt{s}))\sqrt{s}\, E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,18} \right] \\
&- 2\,(\bar E_\mp/\sqrt{s}) \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,19} - (4 \mp (M/\sqrt{s}))\sqrt{s}\, \bar E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,19} \right] + \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,20} - 5\, \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,20} \right] \Big) \\
- \frac{v^2}{\bar M M} &\left( \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,1} - (E_\mp/\sqrt{s})\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} + (\bar E_\mp/\sqrt{s})\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,13} + (1/s)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,17} \right),
\end{aligned}
$$

$$
\begin{aligned}
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2} = 4\,\frac{s}{v^2}\,(\bar r\cdot\bar r)(r\cdot r) &\Big( s\, \bar E_\mp E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,1} \right] + \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,17} \right] \\
&- \sqrt{s}\, E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} \right] + \sqrt{s}\, \bar E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,13} \right] \Big) \\
+ s\, \bar E_\mp E_\mp &\left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - 3\, \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,1} \right] - \sqrt{s}\, E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} - 3\, \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} \right] \\
+ \sqrt{s}\, \bar E_\mp &\left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} - 3\, \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,13} \right] + \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} - 3\, \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,17} \right],
\end{aligned}
$$

$$
\begin{aligned}
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3} = -\tfrac{3}{2}(\bar\delta - 1)(\delta - 1)\, \frac{\sqrt{s}}{\bar M}\frac{\sqrt{s}}{M}\frac{s}{v^2}\,(\bar r\cdot\bar r)(r\cdot r) &\Big( s\, \bar E_\pm E_\pm \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,1} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} \right] \\
&- \sqrt{s}\, E_\pm \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} \right] + \sqrt{s}\, \bar E_\pm \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,13} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} \right] \\
&+ \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,17} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} \right] \Big) \\
+ (\bar\delta - 1)\, \frac{\sqrt{s}}{\bar M}\frac{\sqrt{s}}{M}\frac{s}{v^2}\,(r\cdot r)\, \bar E_\pm E_\pm &\Big( s\, \bar E_\mp E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,2} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,2} \right] \\
&- \sqrt{s}\, E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,6} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,6} \right] + \sqrt{s}\, \bar E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,14} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,14} \right] \\
&+ \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,18} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,18} \right] \Big) \\
+ (\delta - 1)\, \frac{\sqrt{s}}{\bar M}\frac{\sqrt{s}}{M}\frac{s}{v^2}\,(\bar r\cdot\bar r)\, \bar E_\pm E_\pm &\Big( s\, \bar E_\mp E_\mp \left[ (\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,3} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,3} \right]
\end{aligned}
$$

$$
\begin{aligned}
&\quad - \sqrt{s}\,E_\mp \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,7} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,7}\right] + \sqrt{s}\,\bar E_\mp \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,15} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,15}\right] \\
&\quad + \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,19} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,19}\right]\Big) \\
&- 2\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\,\frac{s}{v^2}\,(\bar r\cdot\bar r)(r\cdot r)\Big(2\bar M M\,\bar E_\pm E_\pm \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,1} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,1}\right] + \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,20} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,20}\right] \\
&\quad - 2\bar M M\,(E_\pm/\sqrt{s})\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,5}\right] + 2\bar M M\,(\bar E_\pm/\sqrt{s})\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,13} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,13}\right] \\
&\quad + 2\bar M M\,(1/s)\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,17} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,17}\right] + s\,\bar E_\pm E_\pm \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,4} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,4}\right] \\
&\quad - \sqrt{s}\,E_\pm \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,8} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,8}\right] + \sqrt{s}\,\bar E_\pm \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,16} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,16}\right]\Big) \\
&- \tfrac{1}{4}(\bar\delta-1)(\delta-1)\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\Big(s\,\bar E_\pm E_\pm \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,1} - 4\,\bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,1}\right] \\
&\quad - 4\sqrt{s}\,\bar E_\pm (r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} + 4\sqrt{s}\,E_\pm (\bar r\cdot\bar r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} - \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,17} + 4\,\bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,17}\right]\Big) \\
&- \tfrac{1}{2}(\delta-1)\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\Big(s\,(\bar r\cdot\bar r)\,\bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,3} - \sqrt{s}\,(\bar r\cdot\bar r)\,E_\pm \left[P^{\bar\mu\bar\nu\mu\nu}_{\pm,7} \pm \bar M E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,1}\right] \\
&\quad + \bar E_\pm E_\pm \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} \pm \bar M \bar E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,13}\right] - (\bar r\cdot r)(\bar E_\pm/\sqrt{s})\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\Big) \\
&- \tfrac{1}{2}(\bar\delta-1)\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\Big(s\,(r\cdot r)\,\bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,2} + \sqrt{s}\,(r\cdot r)\,\bar E_\pm \left[P^{\bar\mu\bar\nu\mu\nu}_{\mp,14} \mp M \bar E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,1}\right] \\
&\quad - \bar E_\pm E_\pm \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,13} \pm M E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,5}\right] - (\bar r\cdot r)(E_\pm/\sqrt{s})\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\Big) \\
&+ \frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\Big(-2\bar M M\,\bar E_\pm E_\pm \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,1} - 2\,\bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,1}\right] \\
&\quad + \bar M M\,(E_\pm/\sqrt{s})\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} - 3\,\bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,5}\right] - \bar M M\,(\bar E_\pm/\sqrt{s})\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,13} - 3\,\bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,13}\right] \\
&\quad - 2\bar M M\,(1/s)\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,17} - 2\,\bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,17}\right] \pm \bar M\,(\bar r\cdot r)\,E_\pm \left[P^{\bar\mu\bar\nu\mu\nu}_{\pm,14} + \sqrt{s}\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,2}\right] \\
&\quad \mp M\,(\bar r\cdot r)\,\bar E_\pm \left[P^{\bar\mu\bar\nu\mu\nu}_{\mp,7} - \sqrt{s}\,\bar E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,3}\right] + \bar E_\pm E_\pm \left[s\,\bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} - (r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,6}\right] \\
&\quad + \sqrt{s}\,\bar E_\pm \left[\bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,16} + (r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,8}\right] + \bar E_\pm E_\pm \left[P^{\bar\mu\bar\nu\mu\nu}_{\mp,20} + (\bar r\cdot\bar r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,15}\right] \\
&\quad + (\bar E_\pm/\sqrt{s})\left[(r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,18} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,19}\right]\Big) + v^2\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,1}\,,
\end{aligned}
$$

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\begin{aligned}
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,4} &= \frac{1}{(\bar r\cdot r)^2 - \bar M^2 (r\cdot r)}\Big(\pm v^2\sqrt{s}\left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} \mp \bar M E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,4}\right] \\
&\quad + \tfrac{1}{8}(\bar\delta-1)^2(\delta-1)\,\frac{s}{v^2}\,\bar M s\,(r\cdot r)\left[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}) \pm \bar M E_\pm (Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2} + Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3})\right] \\
&\quad \pm \tfrac{1}{2}((\delta-1)\bar M - (\bar\delta-1)M)\sqrt{s}\left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu\nu}_{\mp,5} \mp \bar M E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,5}\right] \\
&\quad \pm \tfrac{1}{2}(\bar\delta-1)\,s\,E_\pm \left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu\nu}_{\mp,21} \mp \bar M E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,21}\right]\Big) \\
&\pm \tfrac{1}{4}(\bar\delta-1)^2(\delta-1)\,\frac{s}{v^2}\,s\,E_\pm (Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2}) \mp \tfrac{1}{2}(\bar\delta-1)(\delta-1)\,\frac{s}{v^2}\,\sqrt{s}\,(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}) \\
&- \tfrac{1}{2}(\delta-1)\,\frac{s}{v^2}\,\bar M\left[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}) \pm \bar M E_\pm (Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2})\right] \\
&\pm \tfrac{1}{2}(\delta-1)\,\frac{s}{v^2}\,\bar E_\mp \left[(\bar r\cdot r)\,Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} + \bar E_\pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1}\right] - Q^{\bar\mu\bar\nu\mu\nu}_{\pm,5} \mp \sqrt{s}\,Q^{\bar\mu\bar\nu\mu\nu}_{\pm,21} \mp (Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2} + Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3})/\sqrt{s}\,,
\end{aligned}
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\begin{aligned}
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,5} &= \mp(\bar\delta-1)^2(\delta-1)\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\,\frac{s}{v^2}\,(r\cdot r)\,\bar E_\pm\Big(s\,\bar E_\mp E_\mp \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,1}\right] \\
&\quad - \sqrt{s}\,E_\mp \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,5}\right] + \sqrt{s}\,\bar E_\mp \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,13}\right] \\
&\quad + \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\right]\Big) \\
&\mp (\bar\delta-1)^2\,\frac{\sqrt{s}}{\bar M}\,\frac{\sqrt{s}}{M}\,\frac{s}{v^2}\,(r\cdot r)\,E_\mp\Big(s\,\bar E_\pm E_\pm \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,2} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,2}\right] \\
&\quad - \sqrt{s}\,E_\pm \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,6} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,6}\right] + \sqrt{s}\,\bar E_\pm \left[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,14} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,14}\right]
\end{aligned}
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\begin{aligned}
&\quad + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,18} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,18}\big]\Big)\\
&+(\bar\delta-1)(\delta-1)\,\frac{s}{v^2}\,\bar E_\pm E_\mp\Big(\pm\tfrac12\,\frac{\sqrt s}{\bar M}\,\frac{\sqrt s}{M}\,\bar E_\mp\Big\{s\,\bar E_\pm E_\pm\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,3} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,3}\big]\\
&\qquad - \sqrt s\,E_\pm\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,7} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,7}\big] + \sqrt s\,\bar E_\pm\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,15} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,15}\big]\\
&\qquad + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,19} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,19}\big]\Big\}\\
&\quad +4\,\frac{\sqrt s}{M}\,E_\pm\Big\{s\,\bar E_\mp E_\mp\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,1}\big] - \sqrt s\,E_\mp\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,5}\big]\\
&\qquad + \sqrt s\,\bar E_\mp\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,13}\big] + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\big]\Big\}\Big)\\
&-3(\delta-1)\,\frac{\sqrt s}{M}\,\frac{s}{v^2}\,(\bar r\cdot\bar r)\,E_\mp\Big(s\,\bar E_\pm E_\pm\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,3} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,3}\big] + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,19} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,19}\big]\\
&\quad - \sqrt s\,E_\pm\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,7} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,7}\big] + \sqrt s\,\bar E_\pm\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,15} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,15}\big]\Big)\\
&+(\bar\delta-1)\,\frac{s}{v^2}\,E_\pm E_\mp\Big(\pm\frac{\sqrt s}{\bar M}\,\frac{\sqrt s}{M}\,\bar E_\pm\Big\{s\,\bar E_\mp E_\mp\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,4}\big] + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,20} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,20}\big]\\
&\qquad - \sqrt s\,E_\mp\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,8} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,8}\big] + \sqrt s\,\bar E_\mp\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,16} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,16}\big]\Big\}\\
&\quad +3\,\frac{\sqrt s}{M}\,E_\mp\Big\{s\,\bar E_\pm E_\pm\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,2} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,2}\big] - \sqrt s\,E_\pm\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,6} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,6}\big]\\
&\qquad + \sqrt s\,\bar E_\pm\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,14} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,14}\big] + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,18} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,18}\big]\Big\}\Big)\\
&+4\,\frac{\sqrt s}{M}\,\frac{s}{v^2}\,\bar E_\pm E_\mp E_\pm\Big(s\,\bar E_\mp E_\mp\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,4}\big] - \sqrt s\,E_\mp\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,8} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,8}\big]\\
&\quad + \sqrt s\,\bar E_\mp\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,16} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,16}\big] + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,20} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,20}\big]\Big)\\
&\pm\tfrac18(\bar\delta-1)^2(\delta-1)\,\frac{\sqrt s}{\bar M}\,\frac{\sqrt s}{M}\Big(s\,E_\mp\big[5(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - 9\,\bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,1}\big]\\
&\quad + \sqrt s\,\big[5(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} - 9\,\bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,13}\big] + 9\,E_\mp\big(\sqrt s\,E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} - P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\big)\Big)\\
&+\tfrac14(\bar\delta-1)^2\,\frac{\sqrt s}{\bar M}\,\frac{\sqrt s}{M}\Big(\frac{M}{\sqrt s}\Big\{-s\,E_\mp\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,1}\big]\\
&\qquad + \sqrt s\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} + \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,13}\big] + \sqrt s\,E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} + E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\Big\}\\
&\quad \mp(4\sqrt s\,E_\mp E_\pm)\big(\sqrt s\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,2} + P^{\bar\mu\bar\nu\mu\nu}_{\pm,14}\big)\Big)\\
&+\tfrac14(\bar\delta-1)(\delta-1)\,\frac{\sqrt s}{\bar M}\,\frac{\sqrt s}{M}\Big(\sqrt s\,\bar M\,E_\mp\big[4(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - 13\,\bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,1}\big]\\
&\quad \pm E_\mp\big[5(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} \pm 12\,\bar M\,E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,5}\big] + \bar M\big[4(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} - 13\,\bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,13}\big]\\
&\quad \mp(1/\sqrt s)\big[5(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} \pm 12\,\bar M\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\big]\\
&\quad \pm 2\,\bar E_\pm\big(\sqrt s\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,7} - P^{\bar\mu\bar\nu\mu\nu}_{\mp,19} - s\,\bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,3} - \sqrt s\,\bar E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,15}\big)\Big)\\
&-\tfrac12(\bar\delta-1)\,\frac{1}{\bar M\,M}\Big(\pm\bar M\,M\,\sqrt s\Big\{\sqrt s\,E_\mp\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + 2\,\bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,1}\big] - \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} + 2\,\bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,13}\big]\Big\}\\
&\quad + M\Big\{\sqrt s\,E_\mp\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} \mp \bar M\,E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,5}\big] + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} \mp \bar M\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\big]\Big\}\\
&\quad \pm 2s\,E_\mp\Big\{\sqrt s\,E_\pm\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,2} \pm 2\,\bar M\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,2}\big] + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,14} \pm 2\,\bar M\,E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,14}\big]\Big\}\\
&\quad \mp\sqrt s\big((\bar r\cdot r)\,s\sqrt s\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} + (\bar r\cdot r)\,s\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,16} + 2s\,E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,8}\\
&\qquad - 4\sqrt s\,E_\mp E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,6} - 2\sqrt s\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,20} + 4\,E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,18}\big)\Big)\\
&+\tfrac12(\delta-1)\,\frac{1}{M}\Big(4s\,\bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,7} - 4\sqrt s\,\bar E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,19} \mp 3\,\bar M\,(\bar r\cdot r)\,s\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - 4s\sqrt s\,\bar E_\mp \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,3}
\end{aligned}
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\begin{aligned}
&\qquad \mp 3\bar{M}\sqrt{s}\,E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} \mp 3\bar{M}\,(\bar r\cdot r)\sqrt{s}\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} - 4s\,\bar E_\mp \bar E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,15} \pm 3\bar M\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\Big) \\
&+\bar M\sqrt{s}\,E_\mp\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,1}\right] \pm s\,\frac{1}{\bar M\,M}\,\bar E_\mp \bar E_\pm E_\mp\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\mp,3} + \bar M\,M\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,3}\right] \\
&\mp\left[(\bar r\cdot r)E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,5} \pm \bar M\,E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,5}\right] \mp \sqrt{s}\,\frac{1}{\bar M\,M}\,\bar E_\mp E_\mp\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,7} + \bar M\,M\,(\bar E_\pm/\bar E_\mp)P^{\bar\mu\bar\nu\mu\nu}_{\mp,7}\right] \\
&\pm s\,\frac{1}{\bar M\,M}\,E_\mp\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\mp,8} \pm 3\bar M\,E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,8}\right] - \bar M\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,13}\right] \\
&\pm\sqrt{s}\,\frac{1}{\bar M\,M}\,\bar E_\mp \bar E_\pm\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\mp,15} - \bar M\,M\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,15}\right] \mp (1/\sqrt{s})\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\mp,17} \pm \bar M\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\right] \\
&\pm\frac{1}{\bar M\,M}\,\bar E_\mp\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,19} - \bar M\,M\,(\bar E_\pm/\bar E_\mp)P^{\bar\mu\bar\nu\mu\nu}_{\mp,19}\right] \mp \sqrt{s}\,\frac{1}{\bar M\,M}\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\mp,20} \pm 3\bar M\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,20}\right] \\
&\pm 2E_\mp\Big(\frac{1}{\bar M\,M}\Big\{-\sqrt{s}\,E_\pm\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,6} - s\,\bar E_\mp \bar E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,4}\right] + \left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,18} + s\,\bar E_\mp \bar E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,16}\right]\Big\} \\
&\qquad + \sqrt{s}\,(\bar M/M)\,E_\pm\,(\sqrt{s}\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,2} + P^{\bar\mu\bar\nu\mu\nu}_{\pm,14})\Big) \\
&\mp\frac{v^2}{\sqrt{s}}\,\frac{1}{\bar M\,M}\Big(\tfrac12(\bar\delta-1)(P^{\bar\mu\bar\nu\mu\nu}_{\mp,13} + \sqrt{s}\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,1}) - (1/s)(P^{\bar\mu\bar\nu\mu\nu}_{\pm,17} - \sqrt{s}\,E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,5})\Big)\,,
\end{aligned}
$$

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\begin{aligned}
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6} &= \pm(v^2/\bar E_\pm)P^{\bar\mu\bar\nu\mu\nu}_{\pm,1} \mp \frac{s}{v^2}\,E_\pm\Big[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}) + \bar E_\mp E_\mp(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3})\Big] \\
&\quad \mp(1/\bar E_\pm)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3} \pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,7})\,, \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,7} &= -\frac{s}{M}\Big(\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,14} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,14}\right] + E_\mp\sqrt{s}\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,2} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,2}\right]\Big) \\
&\quad + \frac{s}{M\,v^2}\,\bar E_\pm E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2} + \bar E_\mp E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2}\right] \\
&\quad \mp\frac{s}{v^2}\,\bar E_\pm\Big[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}) + \bar E_\mp E_\mp(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3})\Big] \pm \bar E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\mp,10}/M\,, \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,8} &= +2\,\frac{s}{\bar M\,v^2}\,\bar E_\pm E_\mp\Big(\tfrac12(\bar\delta-1)\sqrt{s}\,E_\pm\Big\{\sqrt{s}\,\bar E_\mp\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,25} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,25}\right] \\
&\qquad + \left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\mp,29} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,29}\right]\Big\} + \sqrt{s}\,\bar E_\mp\Big\{\sqrt{s}\,\bar E_\pm\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\mp,27} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,27}\right] \\
&\qquad + \left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,31} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,31}\right]\Big\}\Big) \\
&\quad - \frac{1}{\bar M}\Big(\tfrac12(\bar\delta-1)s\,\bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,25} + s\,\bar E_\mp \bar E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,27} + \sqrt{s}\,\bar E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,31} \pm \bar M\sqrt{s}(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,25} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,29}\Big) \\
&\quad - \tfrac12(\delta+1)\frac{s}{v^2}\Big((\bar r\cdot\bar r)(r\cdot r)\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,17} - \bar E_\pm E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\mp,17}\right]/\bar M + \bar E_\pm E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,7} - \bar E_\mp E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,7}\right] \\
&\qquad \mp \bar E_\pm\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} + \bar E_\mp E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}\right]\Big) \\
&\quad + \tfrac12(\delta+1)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,7} + \bar E_\pm E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\mp,17}/\bar M)\,, \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,9} &= -2\,\frac{s}{M\,v^2}\,\bar E_\mp E_\pm\Big(\tfrac12(\delta-1)\sqrt{s}\,\bar E_\pm\Big\{\sqrt{s}\,E_\mp\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\mp,9} - \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,9}\right] \\
&\qquad - \left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\mp,21} - \bar E_\mp E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\pm,21}\right]\Big\} + \sqrt{s}\,E_\mp\Big\{\sqrt{s}\,E_\pm\left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,10} - \bar E_\pm E_\mp P^{\bar\mu\bar\nu\mu\nu}_{\mp,10}\right] \\
&\qquad - \left[(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\pm,22} - \bar E_\pm E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,22}\right]\Big\}\Big) \\
&\quad + \frac{1}{M}\Big(\tfrac12(\delta-1)s\,\bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,9} + s\,E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,10} - \sqrt{s}\,E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\mp,22} \pm M\sqrt{s}(\bar r\cdot r)P^{\bar\mu\bar\nu\mu\nu}_{\mp,9} + \bar E_\mp E_\pm P^{\bar\mu\bar\nu\mu\nu}_{\pm,21}\Big) \\
&\quad - \tfrac12(\bar\delta+1)\frac{s}{v^2}\Big((\bar r\cdot\bar r)(r\cdot r)\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,16} - \bar E_\mp E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\mp,16}\right]/M + \bar E_\mp E_\pm\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,6} - \bar E_\pm E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6}\right] \\
&\qquad \mp E_\pm\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} + \bar E_\mp E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}\right]\Big) \\
&\quad + \tfrac12(\bar\delta+1)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6} + \bar E_\mp E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\mp,16}/M)\,, \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,10} &= \mp s\sqrt{s}\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,16} + \frac{s}{(\bar r\cdot r)^2 - \bar M^2 M^2}\Big(\tfrac14 s\left[(\bar r\cdot r)\big(2((\delta-1)\bar M - (\bar\delta-1)M) \pm (\bar\delta-1)(\delta-1)\sqrt{s}\big)P^{\bar\mu\bar\nu\mu\nu}_{\pm,16}\right.
\end{aligned}
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\begin{aligned}
&\qquad\qquad - \bar{M} M \left( -2((\delta-1)\bar{M} - (\bar{\delta}-1)M) \pm (\bar{\delta}-1)(\delta-1)\sqrt{s} \right) P^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,16} \Big] \\
&\quad + \tfrac{1}{8}(\delta-1)\frac{s}{v^2}\bar{M} \Big\{ \Big[ (\bar{r}\cdot r)\left(2((\delta-1)\bar{M} + (\bar{\delta}-1)M) \pm (\bar{\delta}-1)(\delta-1)\sqrt{s}\right)\sqrt{s}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,4} \\
&\qquad\qquad + \bar{M} M \left(2((\delta-1)\bar{M} + (\bar{\delta}-1)M) \mp (\bar{\delta}-1)(\delta-1)\sqrt{s}\right)\sqrt{s}\,E_{\mp}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,4} \Big] \\
&\qquad \pm M\sqrt{s}\Big[ (\bar{r}\cdot r)\left(2((\delta-1)\bar{M} - (\bar{\delta}-1)M) \mp (\bar{\delta}-1)(\delta-1)\sqrt{s}\right) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,5} \\
&\qquad\qquad - \bar{M} M \left(2((\delta-1)\bar{M} - (\bar{\delta}-1)M) \pm (\bar{\delta}-1)(\delta-1)\sqrt{s}\right) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,5} \Big] \Big\} \\
&\quad - \tfrac{1}{4}(\delta-1)^2\frac{s}{v^2}\bar{M}^2 M \left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,1} + \bar{M} M\, Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1} \right] \mp E_{\pm}\left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,21} - \bar{M} M\,(E_{\mp}/E_{\pm}) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,21} \right] \Big) \\
&+ \tfrac{1}{4}\frac{s}{v^2}\left(4\bar{M} M \mp 2(\bar{\delta}-1)M\sqrt{s} + (\bar{\delta}-1)(\delta-1)s\right) E_{\mp}\sqrt{s}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,4} \\
&- \tfrac{1}{8}(\delta-1)\frac{s}{v^2}\left(\pm 2((\delta-1)\bar{M}\sqrt{s} - (\bar{\delta}-1)M\sqrt{s}) + (\bar{\delta}-1)(\delta-1)s\right) s\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,5} \\
&\pm \tfrac{1}{2}(\delta-1)\frac{s}{v^2}\sqrt{s}\left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,1} \mp \tfrac{1}{2}(\delta-1)\bar{M}\sqrt{s}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1} \right] \\
&- \tfrac{1}{2}(\delta-1)\frac{s}{v^2} s\,(\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,5} + \frac{s}{v^2}(\bar{r}\cdot r) E_{\pm}\sqrt{s}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,4} \mp \frac{s}{v^2} E_{\mp}\left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3} + \bar{E}_{\pm} E_{\pm} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,3} \right],
\end{aligned}
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\begin{aligned}
Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,11} &= \pm s\sqrt{s}\,P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,8} + \tfrac{1}{4}\frac{s^2}{(\bar{r}\cdot r)^2 - \bar{M}^2 M^2}\Big( (\bar{r}\cdot r)\left(2((\delta-1)\bar{M} - (\bar{\delta}-1)M) \mp (\bar{\delta}-1)(\delta-1)\sqrt{s}\right) P^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,8} \\
&\quad + \bar{M} M \left(2((\delta-1)\bar{M} - (\bar{\delta}-1)M) \pm (\bar{\delta}-1)(\delta-1)\sqrt{s}\right) P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,8} \Big) \\
&- \tfrac{1}{2}(\bar{\delta}-1)\frac{s^2}{(\bar{r}\cdot r)^2 - \bar{M}^2 M^2}\frac{s}{v^2}\Big( +\tfrac{1}{2}(\bar{\delta}-1)\bar{M} M \left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,1} + \bar{M} M\, Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1} \right] \\
&\quad \pm \tfrac{1}{4}\sqrt{s}\,\bar{M}\Big[ (\bar{r}\cdot r)\left(2((\delta-1)\bar{M} - (\bar{\delta}-1)M) \pm (\bar{\delta}-1)(\delta-1)\sqrt{s}\right) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,4} \\
&\qquad - \bar{M} M \left(2((\delta-1)\bar{M} - (\bar{\delta}-1)M) \mp (\bar{\delta}-1)(\delta-1)\sqrt{s}\right) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,4} \Big] \\
&\quad - \tfrac{1}{4}\Big[ (\bar{r}\cdot r)\left(2((\delta-1)\bar{M} + (\bar{\delta}-1)M) \pm (\bar{\delta}-1)(\delta-1)\sqrt{s}\right) \bar{E}_{\pm}\sqrt{s}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,5} \\
&\qquad + \bar{M} M \left(2((\delta-1)\bar{M} + (\bar{\delta}-1)M) \mp (\bar{\delta}-1)(\delta-1)\sqrt{s}\right) \bar{E}_{\mp}\sqrt{s}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,5} \Big] \Big) \\
&\mp \frac{s}{(\bar{r}\cdot r)^2 - \bar{M}^2 M^2}\left[ (\bar{r}\cdot r)\bar{E}_{\pm} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,21} - \bar{M} M\,\bar{E}_{\mp} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,21} \right] \\
&\pm \tfrac{1}{8}\frac{s}{v^2}\sqrt{s}\Big( (\bar{\delta}-1)s\left(2((\delta-1)\bar{M} - (\bar{\delta}-1)M) \mp (\bar{\delta}-1)(\delta-1)\sqrt{s}\right) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,4} \\
&\qquad \pm 2\left(4\bar{M} M \mp 2(\delta-1)\bar{M}\sqrt{s} + (\bar{\delta}-1)(\delta-1)s\right) \bar{E}_{\mp} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,5} \Big) \\
&+ \frac{s}{v^2}(\bar{r}\cdot r)\left[ \sqrt{s}\,\bar{E}_{\pm} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,5} - \tfrac{1}{2}(\bar{\delta}-1)s\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,4} \right] \\
&\pm \tfrac{1}{2}(\bar{\delta}-1)\frac{s}{v^2}\sqrt{s}\left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,1} \mp \tfrac{1}{2}(\bar{\delta}-1)M\sqrt{s}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1} \right] \mp \frac{s}{v^2}\bar{E}_{\mp}\left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3} + \bar{E}_{\pm} E_{\pm} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,3} \right],
\end{aligned}
$$

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\begin{aligned}
Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,12} &= \pm\frac{\sqrt{s}}{\bar{M}}\left[ P^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,32} + \sqrt{s}\,\bar{E}_{\mp} P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,28} \right] \\
&+ \tfrac{1}{8}(\bar{\delta}-1)(\delta-1)(\delta+1)\frac{\sqrt{s}}{\bar{M}}\frac{s}{v^2}\bar{E}_{\mp}\Big( \left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,11} - \bar{E}_{\pm} E_{\mp} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,11} \right] \\
&\qquad \pm 2\bar{E}_{\pm} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,2} \mp 2\frac{s}{v^2}(\bar{r}\cdot\bar{r}) E_{\mp}\left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,2} + \bar{E}_{\pm} E_{\pm} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,2} \right] \Big) \\
&\pm \tfrac{1}{4}(\bar{\delta}+1)(\delta+1)\frac{M}{\bar{M}}\frac{s}{v^2}\bar{E}_{\mp}\left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,11} + \bar{E}_{\pm} E_{\mp} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,11} \right] \\
&+ \tfrac{1}{4}(\delta-1)(\delta+1)\frac{s}{v^2}\Big( (\bar{r}\cdot\bar{r}) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1} \pm \bar{E}_{\mp}\left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,11} + \bar{E}_{\pm} E_{\mp} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,11} \right] \Big) \\
&\mp \tfrac{1}{2}(\bar{\delta}-1)(\delta-1)\frac{\sqrt{s}}{\bar{M}}\frac{s}{v^2}\bar{E}_{\mp} E_{\mp}\left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,20} + \bar{E}_{\pm} E_{\pm} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,20} \right] \\
&\mp (\delta-1)\frac{s}{v^2}\bar{E}_{\mp}\left[ (\bar{r}\cdot r) Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,8} + \bar{E}_{\pm} E_{\mp} Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,8} \right]
\end{aligned}
$$

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\begin{aligned}
&\pm \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\,\bar{E}_\mp\Big(\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,4}+\bar{E}_\pm E_\mp Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,4}\big]-M\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,11}+\bar{E}_\pm E_\mp Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,11}\big]/\bar{M}\\
&\qquad +(\bar{E}_\pm/(\sqrt{s}\,\bar{M}))\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,2}+\bar{M}\,M\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,2}\big]\pm(2/\sqrt{s})\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3}\pm M\,\bar{E}_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,3}\big]\Big)\\
&\mp \tfrac{1}{2}(\bar{\delta}+1)(\delta-1)\,\frac{\sqrt{s}}{\bar{M}}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,20}\pm(\delta+1)\,\frac{\sqrt{s}}{\bar{M}}\,\big[Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,20}\mp(\bar{E}_\pm/(2s))Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,11}\big]\\
&\pm(1/(\bar{M}\sqrt{s}))\,\big[2\bar{M}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,8}-(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,20}-(\bar{M}\,M+2s)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,20}\big]\,,
\end{aligned}
$$

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\begin{aligned}
Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,13}=\pm\frac{\sqrt{s}}{M}\,&\Big[P^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,24}-\sqrt{s}\,E_\mp P^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,12}\Big]\\
&+\tfrac{1}{8}(\bar{\delta}-1)(\bar{\delta}+1)(\delta-1)\,\frac{\sqrt{s}}{M}\,\frac{s}{v^2}\,E_\mp\Big(\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,10}-\bar{E}_\mp E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,10}\big]\\
&\qquad\pm 2E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,2}\mp 2\,\frac{s}{v^2}\,(r\cdot r)\bar{E}_\mp\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,2}+\bar{E}_\pm E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,2}\big]\Big)\\
&+\tfrac{1}{4}(\bar{\delta}-1)(\bar{\delta}+1)\,\frac{s}{v^2}\Big((r\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1}\pm E_\mp\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,10}+\bar{E}_\mp E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,10}\big]\Big)\\
&\pm\tfrac{1}{4}(\bar{\delta}-1)(\delta-1)\,\frac{s}{v^2}\,\frac{E_\mp}{M}\Big(\bar{M}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,10}+\bar{E}_\mp E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,10}\big]-2\sqrt{s}\,\bar{E}_\mp\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,19}+\bar{E}_\pm E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,19}\big]\Big)\\
&+\tfrac{1}{2}(\bar{\delta}+1)\,\frac{\sqrt{s}}{v^2}\,E_\mp\Big(2\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3}\pm\bar{M}\,E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,3}\big]\pm\sqrt{s}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,5}+\bar{E}_\mp E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,5}\big]\\
&\qquad\pm(E_\pm/M)\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,2}+\bar{M}\,M\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,2}\big]\Big)\\
&\mp(\bar{\delta}-1)\,\frac{s}{v^2}\,E_\mp\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,9}+\bar{E}_\mp E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,9}\big]\pm\tfrac{1}{2}(\delta-1)\,\frac{\bar{M}}{M}\,\frac{s}{v^2}\,E_\mp\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,10}+\bar{E}_\mp E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,10}\big]\\
&\mp\tfrac{1}{2}(\bar{\delta}-1)(\delta-1)\,\frac{\sqrt{s}}{M}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,19}-\tfrac{1}{2}(\bar{\delta}+1)(E_\pm/(M\sqrt{s}))Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,10}\\
&\mp(1/(M\sqrt{s}))\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,19}+\bar{M}\,M\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,19}\big]\pm(2/\sqrt{s})Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,9}\,,
\end{aligned}
$$

$$
\begin{aligned}
Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,14}=-\frac{\sqrt{s}}{\bar{E}_\pm}\,&\Big[(\bar{r}\cdot r)P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,25}-\bar{E}_\pm E_\mp P^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,25}\Big]\\
&+\tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\Big(\pm E_\mp\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,7}+\bar{E}_\mp E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,7}\big]\pm\bar{E}_\mp\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,6}+\bar{E}_\pm E_\mp Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,6}\big]\\
&\qquad+\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,3}+\bar{E}_\mp E_\mp Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3}\big]+(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1}\Big)\\
&\mp Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,8}/\bar{E}_\pm\pm\tfrac{1}{2}(\delta+1)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,7}/\bar{E}_\pm\,,
\end{aligned}
$$

$$
\begin{aligned}
Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,15}=\frac{\sqrt{s}}{E_\pm}\,&\Big[(\bar{r}\cdot r)P^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,9}-\bar{E}_\mp E_\pm P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,9}\Big]\\
&+\tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\Big(\pm E_\mp\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,7}+\bar{E}_\mp E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,7}\big]\pm\bar{E}_\mp\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,6}+\bar{E}_\pm E_\mp Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,6}\big]\\
&\qquad+\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,3}+\bar{E}_\mp E_\mp Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3}\big]+(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1}\Big)\\
&\mp Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,9}/E_\pm\pm\tfrac{1}{2}(\bar{\delta}+1)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,6}/E_\pm\,,
\end{aligned}
$$

$$
\begin{aligned}
Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,16}=-\frac{\sqrt{s}}{\bar{E}_\pm}\,&\Big[(\bar{r}\cdot r)P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,13}-\bar{E}_\pm E_\mp P^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,13}\Big]\\
&+\frac{s}{v^2}\Big((r\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1}-(E_\mp/\bar{E}_\pm)\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3}+\bar{E}_\pm E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,3}\big]\Big)\\
&\mp\frac{s}{v^2}\,E_\mp\Big(\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,6}+\bar{E}_\mp E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,6}\big]+(E_\pm/\bar{E}_\pm)\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,7}+\bar{E}_\pm E_\mp Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,7}\big]\Big)\mp(1/\bar{E}_\pm)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,10}\,,
\end{aligned}
$$

$$
\begin{aligned}
Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,17}=\frac{\sqrt{s}}{E_\pm}\,&\Big[(\bar{r}\cdot r)P^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,5}-\bar{E}_\mp E_\pm P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,5}\Big]\\
&+\frac{s}{v^2}\Big((\bar{r}\cdot\bar{r})Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1}-(\bar{E}_\mp/E_\pm)\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3}+\bar{E}_\pm E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,3}\big]\Big)\\
&\mp\frac{s}{v^2}\,\bar{E}_\mp\Big(\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,7}+\bar{E}_\pm E_\mp Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,7}\big]+(\bar{E}_\pm/E_\pm)\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,6}+\bar{E}_\mp E_\pm Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,6}\big]\Big)\mp(1/E_\pm)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,11}\,,
\end{aligned}
$$

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\begin{aligned}
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,18} &= -2\,\frac{s}{v^2}\,\bar E_\mp E_\mp \Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,33} - \bar E_\pm E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,33}\Big] + P^{\bar\mu\bar\nu\mu\nu}_{\mp,33} \\
&\quad - \tfrac14(\bar\delta+1)(\delta+1)\left(\frac{s}{v^2}\right)^2 \Big(\big[(\bar r\cdot r)^3\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} - (\bar r\cdot\bar r)(r\cdot r)\,\bar E_\mp E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1}\big] \\
&\qquad\qquad \pm 2(\bar r\cdot\bar r)(r\cdot r)\Big\{E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,7} + \bar E_\mp E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,7}\big] + \bar E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,6} + \bar E_\pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6}\big]\Big\}\Big) \\
&\quad - \tfrac14(\bar\delta+1)(\delta+1)\,\frac{s}{v^2}\Big(\big[(\bar r\cdot r)(2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2}) - \bar E_\mp E_\mp\,(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3} + Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1})\big] \\
&\qquad\qquad \pm E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,7} + 2\,\bar E_\mp E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,7}\big] \pm \bar E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,6} + 2\,\bar E_\pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6}\big]\Big) \\
&\quad + \tfrac12(\bar\delta+1)\,\frac{s}{v^2}\Big(\pm E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,8} + \bar E_\mp E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,8}\big] - \bar E_\mp E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,14} - \bar E_\pm E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,14}\big]\Big) \\
&\quad + \tfrac12(\delta+1)\,\frac{s}{v^2}\Big(\pm \bar E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,9} + \bar E_\pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,9}\big] - \bar E_\mp E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,15} - \bar E_\pm E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,15}\big]\Big) \\
&\quad + \tfrac12(\bar\delta+1)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,14} + \tfrac12(\delta+1)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,15}\,, \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,19} &= -2\,\frac{s}{v^2}\,\bar E_\mp E_\mp \Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,21} - \bar E_\pm E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,21}\Big] + P^{\bar\mu\bar\nu\mu\nu}_{\mp,21} \\
&\quad + \tfrac12(\bar\delta+1)\left(\frac{s}{v^2}\right)^2 (r\cdot r)\Big(\bar E_\mp E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} - \bar E_\pm E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1}\big] \\
&\qquad\qquad \pm 2\,\bar E_\mp\Big\{(r\cdot r)\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,7} + \bar E_\pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,7}\big] - \bar E_\pm E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6} + \bar E_\mp E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,6}\big]\Big\}\Big) \\
&\quad - \tfrac12(\bar\delta+1)\,\frac{s}{v^2}\Big(\bar E_\mp E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,16} - \bar E_\pm E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,16}\big] \\
&\qquad\qquad \mp E_\mp\,\big[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,10} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6}) + \bar E_\mp E_\pm\,(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,10} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,6})\big] \\
&\qquad\qquad + (r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2} + Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} \pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,7})\Big) \\
&\quad + \tfrac12(\bar\delta+1)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,16} + \frac{s}{v^2}\Big((r\cdot r)\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,15} - \bar E_\mp E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,15}\big] \mp E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,9} + \bar E_\mp E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,9}\big]\Big)\,, \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,20} &= -2\,\frac{s}{v^2}\,\bar E_\mp E_\mp \Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,29} - \bar E_\pm E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,29}\Big] + P^{\bar\mu\bar\nu\mu\nu}_{\mp,29} \\
&\quad + \tfrac12(\delta+1)\left(\frac{s}{v^2}\right)^2 (\bar r\cdot\bar r)\Big(\bar E_\mp E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} - \bar E_\pm E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1}\big] \\
&\qquad\qquad \pm 2\,E_\mp\Big\{(\bar r\cdot\bar r)\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6} + \bar E_\mp E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,6}\big] - \bar E_\mp E_\pm\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,7} + \bar E_\pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,7}\big]\Big\}\Big) \\
&\quad - \tfrac12(\delta+1)\,\frac{s}{v^2}\Big(\bar E_\mp E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,17} - \bar E_\pm E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\big] \\
&\qquad\qquad \mp \bar E_\mp\,\big[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,11} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,7}) + \bar E_\pm E_\mp\,(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,11} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,7})\big] \\
&\qquad\qquad + (\bar r\cdot\bar r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2} + Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} \pm \bar E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,6})\Big) \\
&\quad + \tfrac12(\delta+1)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,17} + \frac{s}{v^2}\Big((\bar r\cdot\bar r)\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,14} - \bar E_\mp E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,14}\big] \mp \bar E_\mp\,\big[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,8} + \bar E_\pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,8}\big]\Big)\,, \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,21} &= \tfrac14(\bar\delta-1)(\delta-1)\,\frac{s}{v^2}\,\frac{\sqrt s}{\bar M}\,\frac{\sqrt s}{M}\,\bar E_\pm E_\pm\Big(s\,\bar E_\mp E_\mp\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} - \bar E_\pm E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,4}\big] \\
&\qquad\qquad - \sqrt s\, E_\mp\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,8} - \bar E_\mp E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,8}\big] + \sqrt s\,\bar E_\mp\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,16} - \bar E_\pm E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,16}\big] \\
&\qquad\qquad + \big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,20} - \bar E_\mp E_\mp\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,20}\big]\Big) \\
&\quad + \tfrac14(\bar\delta+1)(\delta+1)\,\frac{\sqrt s}{\bar M}\,\frac{\sqrt s}{M}\Big(\frac{s}{2}\,\big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} - 2\,\bar E_\pm E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,4}\big] + \sqrt s\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,8} - \sqrt s\,\bar E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,16} - P^{\bar\mu\bar\nu\mu\nu}_{\pm,20}\Big) \\
&\quad - \tfrac12(\bar\delta+1)\,\frac{\sqrt s}{\bar M}\,\frac{\sqrt s}{M}\Big(s\,\big[(1/2)(\bar r\cdot r)(1\pm M/\sqrt s)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} - \bar E_\pm E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,4}\big] \\
&\qquad\qquad + \sqrt s\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,8} - \sqrt s\,\bar E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,16} - P^{\bar\mu\bar\nu\mu\nu}_{\pm,20}\Big) \\
&\quad - \tfrac12(\delta+1)\,\frac{\sqrt s}{\bar M}\,\frac{\sqrt s}{M}\Big(s\,\big[(1/2)(\bar r\cdot r)(1\pm \bar M/\sqrt s)\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} - \bar E_\pm E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,4}\big]
\end{aligned}
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\begin{aligned}
&\qquad + \sqrt{s}\, E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,8} - \sqrt{s}\,\bar E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,16} - P^{\bar\mu\bar\nu\mu\nu}_{\pm,20} \Big) \\
&- \frac{1}{2\bar M M} \Big( s\,[(\bar r\cdot r)(\bar M M \mp \bar M\sqrt{s} \mp M\sqrt{s} - s)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} - \bar E_\pm E_\pm (\bar M M - 2s)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,4}] \\
&\qquad + \sqrt{s}\, E_\pm\, [(\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,8} + (\bar M M - 2s)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,8}] - \sqrt{s}\,\bar E_\pm\, [(\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,16} + (\bar M M - 2s)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,16}] \\
&\qquad - [(\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,20} - (\bar M M + 2s)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,20}] \Big) \\
&+ \frac{\sqrt{s}}{32\bar M M v^2} \big( - (\bar\delta - 1)^2 (\delta - 1)^2 s\sqrt{s} + 4((\delta-1)\bar M + (\bar\delta - 1) M)^2 \sqrt{s} \pm 16 \bar M M ((\delta-1)\bar M + (\bar\delta - 1) M) \\
&\qquad \big) [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2} + \bar E_\pm E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2}] \\
&+ \frac{\sqrt{s}}{8 v^2} \big( \pm 2((\delta-1)\bar M + (\bar\delta-1) M) + (\bar\delta - 1)(\delta - 1)\sqrt{s} \\
&\qquad \big) [(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}) + \bar E_\pm E_\pm (Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3})] \\
&+ (1/s)\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} - (1/(2\bar M M s))(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2} + (1/(2s))\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} \\
&- (1/(2\bar M M))(1/(4s)) \big( (\bar\delta - 1)(\delta - 1) s \mp 2(\delta - 1)\bar M\sqrt{s} \mp 2(\bar\delta - 1) M\sqrt{s} - 4\bar M M \big)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2}\,, \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,22} &= s\,[(\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,3} - \bar E_\pm E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,3}] \pm \tfrac{1}{2}(\bar\delta - 1)\frac{s}{v^2} E_\pm\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,6} + \bar E_\pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6}] \\
&- \frac{\sqrt{s}}{v^2} E_\pm \Big( [(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2} + Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}) + \bar E_\mp E_\mp (Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2} + Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3})] \\
&\qquad + [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} \mp \bar M E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1}] \pm \sqrt{s}\,[(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,5} + \bar E_\pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,5}] \Big) \mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6}/\sqrt{s}\,, \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,23} &= \mp \tfrac{1}{2}(\delta - 1)\frac{\sqrt{s}}{M} s\,(\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,2} \pm \frac{\sqrt{s}}{M}\, [(\bar r\cdot r)\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,14} \pm M\bar E_\pm\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,14}] \\
&\pm \tfrac{1}{2}(\delta - 1)^2 \left(\frac{s}{v^2}\right)^2 \frac{\sqrt{s}}{M} (\bar r\cdot \bar r)\,\bar E_\pm E_\pm\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2} + \bar E_\mp E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2}] \\
&+ \tfrac{1}{2}(\delta - 1)\frac{s}{v^2}\frac{1}{M} \Big( (\bar r\cdot\bar r)\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,10} - \bar E_\pm E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,10}] \pm \bar E_\pm\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2} - (\delta - 1)\bar E_\mp \sqrt{s}\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2}] \Big) \\
&+ \frac{\sqrt{s}}{v^2}\bar E_\pm \Big( [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} - \bar E_\mp E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}] \mp \sqrt{s}\,[(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,4} + \bar E_\mp E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,4}] \Big) - (\bar E_\pm/(M\sqrt{s}))\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,10}\,, \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,24} &= \mp\sqrt{s}\, P^{\bar\mu\bar\nu\mu\nu}_{\pm,28} + Q^{\bar\mu\bar\nu\mu\nu}_{\mp,8}/s \mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,12}/\sqrt{s} \\
&\mp \tfrac{1}{8}(\bar\delta - 1)(\delta - 1)(\delta + 1)\left(\frac{s}{v^2}\right)^2 (\bar r\cdot\bar r)\, E_\mp\, [(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}) + \bar E_\pm E_\pm (Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3})] \\
&\pm \tfrac{1}{4}(\bar\delta - 1)(\delta - 1)(\delta + 1)\frac{s}{v^2}\bar E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} + \tfrac{1}{4}(\delta - 1)(\delta + 1)\frac{s}{v^2}(\bar r\cdot\bar r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,5} \pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1}/\sqrt{s}) \\
&- \tfrac{1}{4}(\bar\delta - 1)(\delta - 1)\frac{s}{v^2}\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,8} - \bar E_\pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,8}] \\
&\mp \tfrac{1}{2}(\bar\delta + 1)\frac{\sqrt{s}}{v^2} M\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,8} + \bar E_\pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,8}] \pm \tfrac{1}{2}(\bar\delta - 1)\frac{s}{v^2} E_\mp\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,12} + \bar E_\pm E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,12}] \\
&+ \tfrac{1}{2}(\delta - 1)\frac{1}{v^2} \Big( \sqrt{s}\,\bar E_\mp\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,4} \mp \bar M (\bar E_\pm/\bar E_\mp) E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,4}] \pm \bar E_\mp\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3} - \bar M M (\bar E_\pm/\bar E_\mp)\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}] \\
&\qquad \pm s\,\bar E_\pm\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,21} + \bar E_\mp E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,21}] \pm \sqrt{s}\,\bar M\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,8} + \bar E_\pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,8}] \Big) \\
&+ \frac{1}{v^2} \Big( \sqrt{s}\,\bar E_\mp\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,4} \mp \bar M (\bar E_\pm/\bar E_\mp) E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,4}] \pm \bar E_\mp\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3} - \bar M M (\bar E_\pm/\bar E_\mp)\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}] \\
&\qquad \pm s\,\bar E_\pm\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,21} + \bar E_\mp E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,21}] \pm \sqrt{s} M\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,8} + \bar E_\pm E_\mp\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,8}] \Big)\,, \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,25} &= \pm\sqrt{s}\, P^{\bar\mu\bar\nu\mu\nu}_{\mp,12} \mp \frac{\sqrt{s}}{v^2} M\, [(\bar r\cdot r)\, Q^{\bar\mu\bar\nu\mu\nu}_{\pm,9} + \bar E_\mp E_\pm\, Q^{\bar\mu\bar\nu\mu\nu}_{\mp,9}] + Q^{\bar\mu\bar\nu\mu\nu}_{\mp,9}/s \mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,13}/\sqrt{s} \\
&\mp \tfrac{1}{8}(\bar\delta - 1)(\bar\delta + 1)(\delta - 1)\left(\frac{s}{v^2}\right)^2 (r\cdot r)\,\bar E_\mp\, [(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}) + \bar E_\pm E_\pm (Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3})]
\end{aligned}
$$

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\begin{aligned}
&\pm \tfrac14(\bar\delta-1)(\bar\delta+1)(\delta-1)\frac{s}{v^2}E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} + \tfrac14(\bar\delta-1)(\bar\delta+1)\frac{1}{v^2}(r\cdot r)(s Q^{\bar\mu\bar\nu\mu\nu}_{\pm,4} \pm \sqrt{s}Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1})\\
&- \tfrac14(\bar\delta-1)(\delta-1)\frac{s}{v^2}\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,9} - \bar E_\mp E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\mp,9}\right]\\
&\pm \tfrac12(\delta-1)\frac{\sqrt{s}}{v^2}\left(\sqrt{s}\,\bar E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,13} + \bar E_\pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,13}\right] - \bar M\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,9} + \bar E_\mp E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\mp,9}\right]\right)\\
&+ \tfrac12(\bar\delta+1)\frac{1}{v^2}\Big(\sqrt{s}\,E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,5} \mp M\,\bar E_\mp(E_\pm/E_\mp)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,5}\right] \pm s\,E_\pm\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,21} + \bar E_\mp E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,21}\right]\\
&\qquad \pm \sqrt{s}\,M\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,9} + \bar E_\mp E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\mp,9}\right] \pm E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3} - \bar M M(E_\pm/E_\mp)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}\right]\Big),\\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,26} &= \mp\sqrt{s}\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,27} + \frac{\sqrt{s}}{v^2}E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,8} - \bar E_\pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,8}\right] \mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,14}/\sqrt{s}\\
&\pm \tfrac14(\bar\delta-1)(\delta+1)\left(\frac{s}{v^2}\right)^2(r\cdot r)\bar E_\pm\left[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}) + \bar E_\mp E_\mp(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3})\right]\\
&\mp \tfrac14(\bar\delta-1)(\delta+1)\frac{s}{v^2}E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} \pm \tfrac12(\bar\delta-1)\frac{s}{v^2}E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,14} + \bar E_\pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,14}\right]\\
&+ \tfrac12(\delta+1)\frac{1}{v^2}\Big(s(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,5} \pm s\,\bar E_\pm\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,22} + \bar E_\mp E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,22}\right]\\
&\qquad \pm \sqrt{s}\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + \bar E_\mp E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}\right] + \sqrt{s}\,\bar E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,6} \mp \bar M(\bar E_\pm/\bar E_\mp)E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6}\right]\Big),\\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,27} &= \pm\sqrt{s}\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,10} + \frac{\sqrt{s}}{v^2}\bar E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,9} - \bar E_\pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,9}\right] \mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,15}/\sqrt{s}\\
&\pm \tfrac14(\bar\delta+1)(\delta-1)\left(\frac{s}{v^2}\right)^2(\bar r\cdot\bar r)E_\pm\left[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}) + \bar E_\mp E_\mp(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3})\right]\\
&\mp \tfrac14(\bar\delta+1)(\delta-1)\frac{s}{v^2}\bar E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} \pm \tfrac12(\delta-1)\frac{s}{v^2}\bar E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,15} + \bar E_\pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,15}\right]\\
&+ \tfrac12(\bar\delta+1)\frac{1}{v^2}\Big(s(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,4} \pm s\,E_\pm\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,23} + \bar E_\mp E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,23}\right]\\
&\qquad \pm \sqrt{s}\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + \bar E_\mp E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}\right] + \sqrt{s}\,E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,7} \mp M\,\bar E_\mp(E_\pm/E_\mp)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,7}\right]\Big),\\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,28} &= \mp\sqrt{s}\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,15} \pm \frac{\sqrt{s}}{v^2}(r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} \pm \sqrt{s}Q^{\bar\mu\bar\nu\mu\nu}_{\pm,5}) \mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,16}/\sqrt{s}\\
&\mp \tfrac12(\bar\delta-1)\left(\frac{s}{v^2}\right)^2(r\cdot r)E_\mp\left[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}) + \bar E_\pm E_\pm(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3})\right]\\
&\pm \tfrac12(\bar\delta-1)\frac{s}{v^2}E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,16} + \bar E_\pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,16}\right] \mp \frac{s}{v^2}E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,22} + \bar E_\pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\mp,22}\right]\\
&+ \frac{\sqrt{s}}{v^2}E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,10} - \bar E_\pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,10}\right] - \frac{\sqrt{s}}{v^2}E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6} \mp \bar M E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\mp,6}\right],\\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,29} &= \pm\sqrt{s}\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,6} \pm \frac{\sqrt{s}}{v^2}(\bar r\cdot\bar r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3} \pm \sqrt{s}Q^{\bar\mu\bar\nu\mu\nu}_{\pm,4}) \mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,17}/\sqrt{s}\\
&\mp \tfrac12(\delta-1)\left(\frac{s}{v^2}\right)^2(\bar r\cdot\bar r)\bar E_\mp\left[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}) + \bar E_\pm E_\pm(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3})\right]\\
&\pm \tfrac12(\delta-1)\frac{s}{v^2}\bar E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,17} + \bar E_\pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,17}\right] \mp \frac{s}{v^2}\bar E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,23} + \bar E_\pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\mp,23}\right]\\
&+ \frac{\sqrt{s}}{v^2}\bar E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,11} - \bar E_\pm E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\pm,11}\right] - \frac{\sqrt{s}}{v^2}\bar E_\mp\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,7} \mp M\,\bar E_\pm Q^{\bar\mu\bar\nu\mu\nu}_{\mp,7}\right],\\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,30} &= \mp\sqrt{s}\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,26}\\
&+ \tfrac14(\delta-1)(\delta+1)\frac{s}{v^2}(\bar r\cdot\bar r)\left(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,7} \pm \frac{s}{v^2}\bar E_\pm\left[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}) + \bar E_\mp E_\mp(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1} + 2Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3})\right]\right)\\
&+ \tfrac12(\delta+1)\frac{\sqrt{s}}{v^2}\bar E_\pm\left(\sqrt{s}\,\bar E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\mp,4} \pm \bar E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3} \pm \sqrt{s}\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,23} + \bar E_\mp E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,23}\right]\right)\\
&+ \frac{\sqrt{s}}{v^2}\bar E_\pm\left(\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,8} - \bar E_\mp E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\pm,8}\right] \mp \sqrt{s}\left[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,12} + \bar E_\mp E_\mp Q^{\bar\mu\bar\nu\mu\nu}_{\mp,12}\right]\right),
\end{aligned}
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\begin{aligned}
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,31} &= \pm\sqrt{s}\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,11} \\
&\quad + \tfrac{1}{4}(\bar\delta-1)(\bar\delta+1)\,\frac{s}{v^2}\,(r\cdot r)\Big(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6} \pm \frac{s}{v^2}\,E_\pm\,[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1}+2Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}) + \bar E_\mp\,E_\mp\,(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,1}+2Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3})]\Big) \\
&\quad + \tfrac{1}{2}(\bar\delta+1)\,\frac{\sqrt{s}}{v^2}\,E_\pm\Big(\sqrt{s}\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\mp,5} \pm E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3} \pm \sqrt{s}\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,22} + \bar E_\mp\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\pm,22}]\Big) \\
&\quad + \frac{\sqrt{s}}{v^2}\,E_\pm\Big([(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,9} - \bar E_\mp\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\pm,9}] \mp \sqrt{s}\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,13} + \bar E_\mp\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\mp,13}]\Big), \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,32} &= \frac{s}{v^2}\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,36} - \bar E_\mp\,E_\mp\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,36}\Big] \\
&\quad - \tfrac{1}{16}(\bar\delta-1)(\bar\delta+1)(\delta-1)(\delta+1)\left(\frac{s}{v^2}\right)^2\bar E_\mp\,E_\mp\Big(\bar E_\pm\,E_\pm\,Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} \\
&\qquad\qquad + [(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2}+Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}) + \bar E_\pm\,E_\pm\,(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2}+Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3})]\Big) \\
&\quad - \tfrac{1}{16}(\bar\delta-1)(\bar\delta+1)(\delta-1)(\delta+1)\,\frac{s}{v^2}\,(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2}+Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}) \\
&\quad \pm \tfrac{1}{8}(\bar\delta-1)(\bar\delta+1)(\delta+1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{v^2}\,\bar E_\mp\,E_\mp\Big(\sqrt{s}\,E_\pm\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,4} + \bar E_\pm\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\pm,4}] \\
&\qquad\qquad + M\,[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2}+Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}) + \bar E_\pm\,E_\pm\,(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2}+Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3})]\Big) \\
&\quad \pm \tfrac{1}{8}(\bar\delta+1)(\delta-1)(\delta+1)\,\frac{\sqrt{s}}{v^2}\,\bar M\,(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2}+Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}) \\
&\quad \pm \tfrac{1}{8}(\bar\delta+1)(\delta-1)(\delta+1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{v^2}\,\bar E_\mp\,E_\mp\Big(\sqrt{s}\,\bar E_\pm\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,5} + \bar E_\mp\,E_\pm\,Q^{\bar\mu\bar\nu\mu\nu}_{\pm,5}] \\
&\qquad\qquad + \bar M\,[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2}+Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3}) + \bar E_\pm\,E_\pm\,(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2}+Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3})]\Big) \\
&\quad \pm \tfrac{1}{8}(\bar\delta-1)(\bar\delta+1)(\delta+1)\,\frac{\sqrt{s}}{v^2}\,M\,(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2}+Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}) \\
&\quad + \tfrac{1}{4}(\bar\delta-1)(\bar\delta+1)\,\frac{s}{v^2}\,(r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,12} + \tfrac{1}{4}(\delta-1)(\delta+1)\,\frac{s}{v^2}\,(\bar r\cdot\bar r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,13} \\
&\quad - \tfrac{1}{4}(\bar\delta+1)(\delta+1)\,\frac{1}{v^2}\Big(s\,[2(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,21} - \bar E_\mp\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\pm,21}] + s\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,18} + \bar E_\mp\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\mp,18}] \\
&\qquad\qquad + [2(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3} - \bar M\,M\,Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}] + (\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2}\Big) \\
&\quad \pm \tfrac{1}{2}(\bar\delta+1)\,\frac{\sqrt{s}}{v^2}\Big(\sqrt{s}\,E_\mp\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,24} + \bar E_\mp\,E_\pm\,Q^{\bar\mu\bar\nu\mu\nu}_{\pm,24}] + (M\pm\sqrt{s})\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,18} + \bar E_\mp\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\mp,18}] \\
&\qquad\qquad - (E_\pm/\sqrt{s})\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,8} + \bar M\,M\,(E_\mp/E_\pm)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,8}]\Big) \\
&\quad \pm \tfrac{1}{2}(\delta+1)\,\frac{\sqrt{s}}{v^2}\Big(\sqrt{s}\,\bar E_\mp\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,25} + \bar E_\pm\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\pm,25}] + (\bar M\pm\sqrt{s})\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,18} + \bar E_\mp\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\mp,18}] \\
&\qquad\qquad - (\bar E_\pm/\sqrt{s})\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,9} + \bar M\,M\,(\bar E_\mp/\bar E_\pm)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,9}]\Big) \\
&\quad - (\pm\bar M\sqrt{s} \pm M\sqrt{s} + s)\,\frac{1}{v^2}\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,18} + \bar E_\mp\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\mp,18}] - Q^{\bar\mu\bar\nu\mu\nu}_{\mp,18}/s, \\
Q^{\bar\mu\bar\nu\mu\nu}_{\pm,33} &= \frac{s}{v^2}\Big[(\bar r\cdot r)\,P^{\bar\mu\bar\nu\mu\nu}_{\mp,35} - \bar E_\mp\,E_\mp\,P^{\bar\mu\bar\nu\mu\nu}_{\pm,35}\Big] - \frac{\sqrt{s}}{v^2}\,E_\mp\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,18} + \bar E_\pm\,E_\pm\,Q^{\bar\mu\bar\nu\mu\nu}_{\pm,18}] \\
&\quad - \tfrac{1}{8}(\bar\delta-1)(\bar\delta+1)(\delta+1)\left(\frac{s}{v^2}\right)^2(r\cdot r)\Big((\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,1} \pm \bar E_\mp\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\mp,6} + \bar E_\pm\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\pm,6}]\Big) \\
&\quad + \tfrac{1}{4}(\bar\delta-1)(\bar\delta+1)\,\frac{s}{v^2}\,(r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,14} \\
&\quad \pm \tfrac{1}{4}(\bar\delta+1)(\delta+1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{v^2}\,(r\cdot r)\Big(\sqrt{s}\,\bar E_\mp\,[(\bar r\cdot r)Q^{\bar\mu\bar\nu\mu\nu}_{\pm,5} + \bar E_\pm\,E_\mp\,Q^{\bar\mu\bar\nu\mu\nu}_{\mp,5}] \\
&\qquad\qquad \pm \bar E_\pm\,[(\bar r\cdot r)(Q^{\bar\mu\bar\nu\mu\nu}_{\pm,2}+Q^{\bar\mu\bar\nu\mu\nu}_{\mp,3}) + \bar E_\mp\,E_\mp\,(Q^{\bar\mu\bar\nu\mu\nu}_{\mp,2}+Q^{\bar\mu\bar\nu\mu\nu}_{\pm,3})]\Big)
\end{aligned}
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\begin{aligned}
&- \tfrac{1}{4}(\bar{\delta}+1)(\delta+1)\,\frac{1}{v^2}\Big(s\,\big[2(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,22} - \bar{E}_{\mp}\,E_{\mp}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,22}\big] + \sqrt{s}\,E_{\mp}\,(Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,2} + 2\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3} \pm \sqrt{s}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,5})\Big)\\
&\pm \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\,E_{\mp}\Big(\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,26} + \bar{E}_{\mp}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,26}\big] - E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,8}/\sqrt{s}\Big)\\
&+ \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\Big((\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,13} \mp \bar{E}_{\pm}\,E_{\mp}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,9}/\sqrt{s} \pm \bar{E}_{\mp}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,31} + \bar{E}_{\pm}\,E_{\mp}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,31}\big]\Big)\,,
\end{aligned}
$$

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\begin{aligned}
Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,34} &= \frac{s}{v^2}\,\Big[(\bar{r}\cdot r)\,P^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,34} - \bar{E}_{\mp}\,E_{\mp}\,P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,34}\Big] - \frac{\sqrt{s}}{v^2}\,\bar{E}_{\mp}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,18} + \bar{E}_{\pm}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,18}\big]\\
&- \tfrac{1}{8}(\bar{\delta}+1)(\delta-1)(\delta+1)\,\Big(\frac{s}{v^2}\Big)^2\,(\bar{r}\cdot\bar{r})\Big((\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1} \pm E_{\mp}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,7} + \bar{E}_{\mp}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,7}\big]\Big)\\
&+ \tfrac{1}{4}(\delta-1)(\delta+1)\,\frac{s}{v^2}\,(\bar{r}\cdot\bar{r})\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,15}\\
&\pm \tfrac{1}{4}(\bar{\delta}+1)(\delta+1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{v^2}\,(\bar{r}\cdot\bar{r})\Big(\sqrt{s}\,E_{\mp}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,4} + \bar{E}_{\mp}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,4}\big]\\
&\qquad \pm E_{\pm}\,\big[(\bar{r}\cdot r)(Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,2} + Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,3}) + \bar{E}_{\mp}\,E_{\mp}\,(Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,2} + Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3})\big]\Big)\\
&- \tfrac{1}{4}(\bar{\delta}+1)(\delta+1)\,\frac{1}{v^2}\Big(s\,\big[2(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,23} - \bar{E}_{\mp}\,E_{\mp}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,23}\big] + \sqrt{s}\,\bar{E}_{\mp}\,(Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,2} + 2\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3} \pm \sqrt{s}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,4})\Big)\\
&\pm \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\,\bar{E}_{\mp}\Big(\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,27} + \bar{E}_{\pm}\,E_{\mp}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,27}\big] - \bar{E}_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,9}/\sqrt{s}\Big)\\
&+ \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\Big((\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,12} \mp \bar{E}_{\mp}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,8}/\sqrt{s} \pm E_{\mp}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,30} + \bar{E}_{\mp}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,30}\big]\Big)\,,
\end{aligned}
$$

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\begin{aligned}
Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,35} &= \frac{s}{v^2}\,\Big[(\bar{r}\cdot r)\,P^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,23} - \bar{E}_{\mp}\,E_{\mp}\,P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,23}\Big]\\
&+ \tfrac{1}{4}(\bar{\delta}-1)(\bar{\delta}+1)\,\frac{s}{v^2}\,(r\cdot r)\Big(Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,16} - \frac{s}{v^2}\,\Big\{(r\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1} \mp E_{\mp}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,6} + \bar{E}_{\mp}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,6}\big]\Big\}\Big)\\
&- \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{v^2}\,(r\cdot r)\,E_{\mp}\Big(\big[(\bar{r}\cdot r)(Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,2} + Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3}) + \bar{E}_{\pm}\,E_{\pm}\,(Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,2} + Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,3})\big]\\
&\qquad \pm \sqrt{s}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,5} + \bar{E}_{\mp}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,5}\big]\Big)\\
&\pm \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\,E_{\mp}\Big(\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,28} + \bar{E}_{\mp}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,28}\big] - E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,10}/\sqrt{s} \pm E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,22}\Big)\\
&- \frac{\sqrt{s}}{v^2}\,E_{\mp}\Big(\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,19} + \bar{E}_{\pm}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,19}\big] \pm \sqrt{s}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,31} + \bar{E}_{\mp}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,31}\big] \mp E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,9} + \sqrt{s}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,13}\Big)\,,
\end{aligned}
$$

$$
\begin{aligned}
Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,36} &= \frac{s}{v^2}\,\Big[(\bar{r}\cdot r)\,P^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,30} - \bar{E}_{\mp}\,E_{\mp}\,P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,30}\Big]\\
&+ \tfrac{1}{4}(\delta-1)(\delta+1)\,\frac{s}{v^2}\,(\bar{r}\cdot\bar{r})\Big(Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,17} - \frac{s}{v^2}\,\Big\{(\bar{r}\cdot\bar{r})Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1} \mp \bar{E}_{\mp}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,7} + \bar{E}_{\pm}\,E_{\mp}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,7}\big]\Big\}\Big)\\
&- \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{v^2}\,(\bar{r}\cdot\bar{r})\,\bar{E}_{\mp}\Big(\big[(\bar{r}\cdot r)(Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,2} + Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3}) + \bar{E}_{\pm}\,E_{\pm}\,(Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,2} + Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,3})\big]\\
&\qquad \pm \sqrt{s}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,4} + \bar{E}_{\pm}\,E_{\mp}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,4}\big]\Big)\\
&\pm \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\,\bar{E}_{\mp}\Big(\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,29} + \bar{E}_{\pm}\,E_{\mp}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,29}\big] - \bar{E}_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,11}/\sqrt{s} \pm \bar{E}_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,23}\Big)\\
&- \frac{\sqrt{s}}{v^2}\,\bar{E}_{\mp}\Big(\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,20} + \bar{E}_{\pm}\,E_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,20}\big] \pm \sqrt{s}\,\big[(\bar{r}\cdot r)Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,30} + \bar{E}_{\pm}\,E_{\mp}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,30}\big]\\
&\qquad \mp \bar{E}_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\mp,8} + \sqrt{s}\,\bar{E}_{\pm}\,Q^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,12}\Big)\,,
\end{aligned}
$$

where

$$
\begin{aligned}
P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,1} &= v^{\bar{\mu}}\,\big[r_{\lfloor}{}^{\bar{\nu}}\,\bar{r}_{\lfloor}{}^{\nu}\,P_{\pm} + v^{\bar{\nu}}\,v^{\nu}\,(s/v^2)\,\bar{E}_{\pm}\,E_{\pm}\,P_{\mp}/v^2 - r_{\lfloor}{}^{\bar{\nu}}\,v^{\nu}\,(\sqrt{s}/v^2)\,E_{\pm}\,i\,\gamma_5\,P_{\pm} + v^{\bar{\nu}}\,\bar{r}_{\lfloor}{}^{\nu}\,(\sqrt{s}/v^2)\,\bar{E}_{\pm}\,i\,\gamma_5\,P_{\mp}\big]\,v^{\mu}/(v^2)^2\,,\\
P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,2} &= v^{\bar{\mu}}\,\big[r_{\lfloor}{}^{\bar{\nu}}\,\bar{w}_{\lfloor}{}^{\nu}\,P_{\pm} + v^{\bar{\nu}}\,v^{\nu}\,(\sqrt{s}/v^2)\,\bar{E}_{\pm}\,((\bar{r}\cdot r)\,P_{\mp} \pm M\,\bar{E}_{\mp}\,P_{\pm})/v^2\\
&\quad - r_{\lfloor}{}^{\bar{\nu}}\,v^{\nu}\,i\,\gamma_5\,((\bar{r}\cdot r)\,P_{\pm} \pm M\,\bar{E}_{\pm}\,P_{\mp})/v^2 + v^{\bar{\nu}}\,\bar{w}_{\lfloor}{}^{\nu}\,(\sqrt{s}/v^2)\,\bar{E}_{\pm}\,i\,\gamma_5\,P_{\mp}\big]\,v^{\mu}/(v^2)^2\,,\\
P^{\bar{\mu}\bar{\nu}\mu\nu}_{\pm,3} &= v^{\bar{\mu}}\,\big[w_{\lfloor}{}^{\bar{\nu}}\,\bar{r}_{\lfloor}{}^{\nu}\,P_{\pm} + v^{\bar{\nu}}\,v^{\nu}\,(\sqrt{s}/v^2)\,E_{\pm}\,((\bar{r}\cdot r)\,P_{\mp} \pm \bar{M}\,E_{\mp}\,P_{\pm})/v^2
\end{aligned}
$$

$$
\begin{aligned}
&- w_\lfloor^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\pm\, i\gamma_5\, P_\pm + v^{\bar\nu}\, \bar r_\perp^{\nu}\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\big]\, v^{\mu}/(v^2)^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,4} &= v^{\bar\mu}\,\big[w_\lfloor^{\bar\nu}\, \bar w_\lfloor^{\nu}\, P_\pm + v^{\bar\nu}\, v^{\nu}\, \{(1/s)\, P_\mp + \tfrac{1}{4}(\bar\delta-1)(\delta-1)(s/v^2)\, \bar E_\pm\, E_\pm\, P_\mp\\
&\qquad \mp \tfrac{1}{2}(\bar\delta-1)(\sqrt{s}/v^2)\, M\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\pm\, P_\mp) \mp \tfrac{1}{2}(\delta-1)(\sqrt{s}/v^2)\, \bar M\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\pm\, P_\mp)\}/v^2\\
&\quad - w_\lfloor^{\bar\nu}\, v^{\nu}\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm \pm M\, \bar E_\pm\, P_\mp)/v^2 + v^{\bar\nu}\, \bar w_\lfloor^{\nu}\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\big]\, v^{\mu}/(v^2)^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,5} &= r_\perp^{\bar\mu}\,\big[r_\perp^{\bar\nu}\, \bar r_\perp^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (s/v^2)\, \bar E_\pm\, E_\mp\, i\gamma_5\, P_\mp/v^2 + r_\perp^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\mp\, P_\pm - v^{\bar\nu}\, \bar r_\perp^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, P_\mp\big]\, v^{\mu}/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,6} &= r_\perp^{\bar\mu}\,\big[r_\perp^{\bar\nu}\, \bar w_\lfloor^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \mp M\, \bar E_\mp\, P_\pm)/v^2\\
&\quad + r_\perp^{\bar\nu}\, v^{\nu}\, ((\bar r\cdot r)\, P_\pm \mp M\, \bar E_\pm\, P_\mp)/v^2 - v^{\bar\nu}\, \bar w_\lfloor^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, P_\mp\big]\, v^{\mu}/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,7} &= r_\perp^{\bar\mu}\,\big[w_\lfloor^{\bar\nu}\, \bar r_\perp^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\mp\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\\
&\quad + w_\lfloor^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\mp\, P_\pm - v^{\bar\nu}\, \bar r_\perp^{\nu}\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\big]\, v^{\mu}/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,8} &= r_\perp^{\bar\mu}\,\big[w_\lfloor^{\bar\nu}\, \bar w_\lfloor^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, \{(1/s)\, i\gamma_5\, P_\mp + \tfrac{1}{4}(\bar\delta-1)(\delta-1)(s/v^2)\, \bar E_\pm\, E_\mp\, i\gamma_5\, P_\mp\\
&\qquad \pm \tfrac{1}{2}(\bar\delta-1)(\sqrt{s}/v^2)\, M\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\mp\, P_\mp) \mp \tfrac{1}{2}(\delta-1)(\sqrt{s}/v^2)\, \bar M\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\mp\, P_\mp)\}/v^2\\
&\quad + w_\lfloor^{\bar\nu}\, v^{\nu}\, ((\bar r\cdot r)\, P_\pm \mp M\, \bar E_\pm\, P_\mp)/v^2 - v^{\bar\nu}\, \bar w_\lfloor^{\nu}\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\big]\, v^{\mu}/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,9} &= w_\lfloor^{\bar\mu}\,\big[r_\perp^{\bar\nu}\, \bar r_\perp^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (s/v^2)\, \bar E_\pm\, E_\mp\, i\gamma_5\, P_\mp/v^2 + r_\perp^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\mp\, P_\pm - v^{\bar\nu}\, \bar r_\perp^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, P_\mp\big]\, v^{\mu}/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,10} &= w_\lfloor^{\bar\mu}\,\big[r_\perp^{\bar\nu}\, \bar w_\lfloor^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \mp M\, \bar E_\mp\, P_\pm)/v^2\\
&\quad + r_\perp^{\bar\nu}\, v^{\nu}\, ((\bar r\cdot r)\, P_\pm \mp M\, \bar E_\pm\, P_\mp)/v^2 - v^{\bar\nu}\, \bar w_\lfloor^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, P_\mp\big]\, v^{\mu}/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,11} &= w_\lfloor^{\bar\mu}\,\big[w_\lfloor^{\bar\nu}\, \bar r_\perp^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\mp\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\\
&\quad + w_\lfloor^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\mp\, P_\pm - v^{\bar\nu}\, \bar r_\perp^{\nu}\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\big]\, v^{\mu}/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,12} &= w_\lfloor^{\bar\mu}\,\big[w_\lfloor^{\bar\nu}\, \bar w_\lfloor^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, \{(1/s)\, i\gamma_5\, P_\mp + \tfrac{1}{4}(\bar\delta-1)(\delta-1)(s/v^2)\, \bar E_\pm\, E_\mp\, i\gamma_5\, P_\mp\\
&\qquad \pm \tfrac{1}{2}(\bar\delta-1)(\sqrt{s}/v^2)\, M\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\mp\, P_\mp) \mp \tfrac{1}{2}(\delta-1)(\sqrt{s}/v^2)\, \bar M\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\mp\, P_\mp)\}/v^2\\
&\quad + w_\lfloor^{\bar\nu}\, v^{\nu}\, ((\bar r\cdot r)\, P_\pm \mp M\, \bar E_\pm\, P_\mp)/v^2 - v^{\bar\nu}\, \bar w_\lfloor^{\nu}\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\big]\, v^{\mu}/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,13} &= v^{\bar\mu}\,\big[r_\perp^{\bar\nu}\, \bar r_\perp^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (s/v^2)\, \bar E_\pm\, E_\mp\, i\gamma_5\, P_\mp/v^2 + r_\perp^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\mp\, P_\pm - v^{\bar\nu}\, \bar r_\perp^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, P_\mp\big]\, \bar r_\perp^{\mu}/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,14} &= v^{\bar\mu}\,\big[r_\perp^{\bar\nu}\, \bar w_\lfloor^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \mp M\, \bar E_\mp\, P_\pm)/v^2\\
&\quad + r_\perp^{\bar\nu}\, v^{\nu}\, ((\bar r\cdot r)\, P_\pm \mp M\, \bar E_\pm\, P_\mp)/v^2 - v^{\bar\nu}\, \bar w_\lfloor^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, P_\mp\big]\, \bar r_\perp^{\mu}/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,15} &= v^{\bar\mu}\,\big[w_\lfloor^{\bar\nu}\, \bar r_\perp^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\mp\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\\
&\quad + w_\lfloor^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\mp\, P_\pm - v^{\bar\nu}\, \bar r_\perp^{\nu}\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\big]\, \bar r_\perp^{\mu}/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,16} &= v^{\bar\mu}\,\big[w_\lfloor^{\bar\nu}\, \bar w_\lfloor^{\nu}\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^{\nu}\, \{(1/s)\, i\gamma_5\, P_\mp + \tfrac{1}{4}(\bar\delta-1)(\delta-1)(s/v^2)\, \bar E_\pm\, E_\mp\, i\gamma_5\, P_\mp\\
&\qquad \pm \tfrac{1}{2}(\bar\delta-1)(\sqrt{s}/v^2)\, M\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\mp\, P_\mp) \mp \tfrac{1}{2}(\delta-1)(\sqrt{s}/v^2)\, \bar M\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\mp\, P_\mp)\}/v^2\\
&\quad + w_\lfloor^{\bar\nu}\, v^{\nu}\, ((\bar r\cdot r)\, P_\pm \mp M\, \bar E_\pm\, P_\mp)/v^2 - v^{\bar\nu}\, \bar w_\lfloor^{\nu}\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\big]\, \bar r_\perp^{\mu}/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,17} &= r_\perp^{\bar\mu}\,\big[r_\perp^{\bar\nu}\, \bar r_\perp^{\nu}\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (s/v^2)\, \bar E_\pm\, E_\pm\, P_\mp/v^2 - r_\perp^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\pm\, i\gamma_5\, P_\pm + v^{\bar\nu}\, \bar r_\perp^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, i\gamma_5\, P_\mp\big]\, \bar r_\perp^{\mu}\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,18} &= r_\perp^{\bar\mu}\,\big[r_\perp^{\bar\nu}\, \bar w_\lfloor^{\nu}\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, ((\bar r\cdot r)\, P_\mp \pm M\, \bar E_\mp\, P_\pm)/v^2\\
&\quad - r_\perp^{\bar\nu}\, v^{\nu}\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm \pm M\, \bar E_\pm\, P_\mp)/v^2 + v^{\bar\nu}\, \bar w_\lfloor^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, i\gamma_5\, P_\mp\big]\, \bar r_\perp^{\mu}\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,19} &= r_\perp^{\bar\mu}\,\big[w_\lfloor^{\bar\nu}\, \bar r_\perp^{\nu}\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\pm\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\\
&\quad - w_\lfloor^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\pm\, i\gamma_5\, P_\pm + v^{\bar\nu}\, \bar r_\perp^{\nu}\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\big]\, \bar r_\perp^{\mu}\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,20} &= r_\perp^{\bar\mu}\,\big[w_\lfloor^{\bar\nu}\, \bar w_\lfloor^{\nu}\, P_\pm + v^{\bar\nu}\, v^{\nu}\, \{(1/s)\, P_\mp + \tfrac{1}{4}(\bar\delta-1)(\delta-1)(s/v^2)\, \bar E_\pm\, E_\pm\, P_\mp\\
&\qquad \mp \tfrac{1}{2}(\bar\delta-1)(\sqrt{s}/v^2)\, M\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\pm\, P_\mp) \mp \tfrac{1}{2}(\delta-1)(\sqrt{s}/v^2)\, \bar M\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\pm\, P_\mp)\}/v^2\\
&\quad - w_\lfloor^{\bar\nu}\, v^{\nu}\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm \pm M\, \bar E_\pm\, P_\mp)/v^2 + v^{\bar\nu}\, \bar w_\lfloor^{\nu}\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\big]\, \bar r_\perp^{\mu}\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,21} &= w_\lfloor^{\bar\mu}\,\big[r_\perp^{\bar\nu}\, \bar r_\perp^{\nu}\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (s/v^2)\, \bar E_\pm\, E_\pm\, P_\mp/v^2 - r_\perp^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, E_\pm\, i\gamma_5\, P_\pm + v^{\bar\nu}\, \bar r_\perp^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, i\gamma_5\, P_\mp\big]\, \bar r_\perp^{\mu}\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,22} &= w_\lfloor^{\bar\mu}\,\big[r_\perp^{\bar\nu}\, \bar w_\lfloor^{\nu}\, P_\pm + v^{\bar\nu}\, v^{\nu}\, (\sqrt{s}/v^2)\, \bar E_\pm\, ((\bar r\cdot r)\, P_\mp \pm M\, \bar E_\mp\, P_\pm)/v^2
\end{aligned}
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\begin{aligned}
&- r_\perp{}^{\bar\nu}\, v^\nu\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm \pm M\, \bar E_\pm\, P_\mp)/v^2 + v^{\bar\nu}\, \bar w_\llcorner{}^\nu\, (\sqrt{s}/v^2)\, \bar E_\pm\, i\gamma_5\, P_\mp\big]\, \bar r_\perp{}^\mu\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,23} &= w_\lrcorner{}^{\bar\mu}\, \big[w_\llcorner{}^{\bar\nu}\, \bar r_\perp{}^\nu\, P_\pm + v^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, E_\pm\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\\
&- w_\llcorner{}^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, E_\pm\, i\gamma_5\, P_\pm + v^{\bar\nu}\, \bar r_\perp{}^\nu\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\big]\, \bar r_\perp{}^\mu\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,24} &= w_\lrcorner{}^{\bar\mu}\, \big[w_\llcorner{}^{\bar\nu}\, \bar w_\llcorner{}^\nu\, P_\pm + v^{\bar\nu}\, v^\nu\, \{(1/s)\, P_\mp + \tfrac{1}{4}(\bar\delta - 1)(\delta - 1)(s/v^2)\, \bar E_\pm\, E_\pm\, P_\mp\\
&\quad \mp \tfrac{1}{2}(\bar\delta - 1)(\sqrt{s}/v^2)\, M\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\pm\, P_\mp) \mp \tfrac{1}{2}(\delta - 1)(\sqrt{s}/v^2)\, \bar M\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\pm\, P_\mp)\}/v^2\\
&- w_\llcorner{}^{\bar\nu}\, v^\nu\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm \pm M\, \bar E_\pm\, P_\mp)/v^2 + v^{\bar\nu}\, \bar w_\llcorner{}^\nu\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\big]\, \bar r_\perp{}^\mu\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,25} &= v^{\bar\mu}\, \big[r_\perp{}^{\bar\nu}\, \bar r_\perp{}^\nu\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^\nu\, (s/v^2)\, \bar E_\pm\, E_\mp\, i\gamma_5\, P_\mp/v^2 + r_\perp{}^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, E_\mp\, P_\pm - v^{\bar\nu}\, \bar r_\perp{}^\nu\, (\sqrt{s}/v^2)\, \bar E_\pm\, P_\mp\big]\, \bar w_\lrcorner{}^\mu/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,26} &= v^{\bar\mu}\, \big[r_\perp{}^{\bar\nu}\, \bar w_\llcorner{}^\nu\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, \bar E_\pm\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \mp M\, \bar E_\mp\, P_\pm)/v^2\\
&+ r_\perp{}^{\bar\nu}\, v^\nu\, ((\bar r\cdot r)\, P_\pm \mp M\, \bar E_\pm\, P_\mp)/v^2 - v^{\bar\nu}\, \bar w_\llcorner{}^\nu\, (\sqrt{s}/v^2)\, \bar E_\pm\, P_\mp\big]\, \bar w_\lrcorner{}^\mu/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,27} &= v^{\bar\mu}\, \big[w_\llcorner{}^{\bar\nu}\, \bar r_\perp{}^\nu\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, E_\mp\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\\
&+ w_\llcorner{}^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, E_\mp\, P_\pm - v^{\bar\nu}\, \bar r_\perp{}^\nu\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\big]\, \bar w_\lrcorner{}^\mu/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,28} &= v^{\bar\mu}\, \big[w_\llcorner{}^{\bar\nu}\, \bar w_\llcorner{}^\nu\, i\gamma_5\, P_\pm + v^{\bar\nu}\, v^\nu\, \{(1/s)\, i\gamma_5\, P_\mp + \tfrac{1}{4}(\bar\delta - 1)(\delta - 1)(s/v^2)\, \bar E_\pm\, E_\mp\, i\gamma_5\, P_\mp\\
&\quad \pm \tfrac{1}{2}(\bar\delta - 1)(\sqrt{s}/v^2)\, M\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\mp\, P_\mp) \mp \tfrac{1}{2}(\delta - 1)(\sqrt{s}/v^2)\, \bar M\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\mp\, P_\mp)\}/v^2\\
&+ w_\llcorner{}^{\bar\nu}\, v^\nu\, ((\bar r\cdot r)\, P_\pm \mp M\, \bar E_\pm\, P_\mp)/v^2 - v^{\bar\nu}\, \bar w_\llcorner{}^\nu\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\big]\, \bar w_\lrcorner{}^\mu/v^2\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,29} &= r_\perp{}^{\bar\mu}\, \big[r_\perp{}^{\bar\nu}\, \bar r_\perp{}^\nu\, P_\pm + v^{\bar\nu}\, v^\nu\, (s/v^2)\, \bar E_\pm\, E_\pm\, P_\mp/v^2 - r_\perp{}^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, E_\pm\, i\gamma_5\, P_\pm + v^{\bar\nu}\, \bar r_\perp{}^\nu\, (\sqrt{s}/v^2)\, \bar E_\pm\, i\gamma_5\, P_\mp\big]\, \bar w_\lrcorner{}^\mu\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,30} &= r_\perp{}^{\bar\mu}\, \big[r_\perp{}^{\bar\nu}\, \bar w_\llcorner{}^\nu\, P_\pm + v^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, \bar E_\pm\, ((\bar r\cdot r)\, P_\mp \pm M\, \bar E_\mp\, P_\pm)/v^2\\
&- r_\perp{}^{\bar\nu}\, v^\nu\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm \pm M\, \bar E_\pm\, P_\mp)/v^2 + v^{\bar\nu}\, \bar w_\llcorner{}^\nu\, (\sqrt{s}/v^2)\, \bar E_\pm\, i\gamma_5\, P_\mp\big]\, \bar w_\lrcorner{}^\mu\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,31} &= r_\perp{}^{\bar\mu}\, \big[w_\llcorner{}^{\bar\nu}\, \bar r_\perp{}^\nu\, P_\pm + v^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, E_\pm\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\\
&- w_\llcorner{}^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, E_\pm\, i\gamma_5\, P_\pm + v^{\bar\nu}\, \bar r_\perp{}^\nu\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\big]\, \bar w_\lrcorner{}^\mu\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,32} &= r_\perp{}^{\bar\mu}\, \big[w_\llcorner{}^{\bar\nu}\, \bar w_\llcorner{}^\nu\, P_\pm + v^{\bar\nu}\, v^\nu\, \{(1/s)\, P_\mp + \tfrac{1}{4}(\bar\delta - 1)(\delta - 1)(s/v^2)\, \bar E_\pm\, E_\pm\, P_\mp\\
&\quad \mp \tfrac{1}{2}(\bar\delta - 1)(\sqrt{s}/v^2)\, M\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\pm\, P_\mp) \mp \tfrac{1}{2}(\delta - 1)(\sqrt{s}/v^2)\, \bar M\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\pm\, P_\mp)\}/v^2\\
&- w_\llcorner{}^{\bar\nu}\, v^\nu\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm \pm M\, \bar E_\pm\, P_\mp)/v^2 + v^{\bar\nu}\, \bar w_\llcorner{}^\nu\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\big]\, \bar w_\lrcorner{}^\mu\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,33} &= w_\lrcorner{}^{\bar\mu}\, \big[r_\perp{}^{\bar\nu}\, \bar r_\perp{}^\nu\, P_\pm + v^{\bar\nu}\, v^\nu\, (s/v^2)\, \bar E_\pm\, E_\pm\, P_\mp/v^2 - r_\perp{}^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, E_\pm\, i\gamma_5\, P_\pm + v^{\bar\nu}\, \bar r_\perp{}^\nu\, (\sqrt{s}/v^2)\, \bar E_\pm\, i\gamma_5\, P_\mp\big]\, \bar w_\lrcorner{}^\mu\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,34} &= w_\lrcorner{}^{\bar\mu}\, \big[r_\perp{}^{\bar\nu}\, \bar w_\llcorner{}^\nu\, P_\pm + v^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, \bar E_\pm\, ((\bar r\cdot r)\, P_\mp \pm M\, \bar E_\mp\, P_\pm)/v^2\\
&- r_\perp{}^{\bar\nu}\, v^\nu\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm \pm M\, \bar E_\pm\, P_\mp)/v^2 + v^{\bar\nu}\, \bar w_\llcorner{}^\nu\, (\sqrt{s}/v^2)\, \bar E_\pm\, i\gamma_5\, P_\mp\big]\, \bar w_\lrcorner{}^\mu\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,35} &= w_\lrcorner{}^{\bar\mu}\, \big[w_\llcorner{}^{\bar\nu}\, \bar r_\perp{}^\nu\, P_\pm + v^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, E_\pm\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\mp\, P_\pm)/v^2\\
&- w_\llcorner{}^{\bar\nu}\, v^\nu\, (\sqrt{s}/v^2)\, E_\pm\, i\gamma_5\, P_\pm + v^{\bar\nu}\, \bar r_\perp{}^\nu\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\big]\, \bar w_\lrcorner{}^\mu\,,\\
P^{\bar\mu\bar\nu\mu\nu}_{\pm,36} &= w_\lrcorner{}^{\bar\mu}\, \big[w_\llcorner{}^{\bar\nu}\, \bar w_\llcorner{}^\nu\, P_\pm + v^{\bar\nu}\, v^\nu\, \{(1/s)\, P_\mp + \tfrac{1}{4}(\bar\delta - 1)(\delta - 1)(s/v^2)\, \bar E_\pm\, E_\pm\, P_\mp\\
&\quad \mp \tfrac{1}{2}(\bar\delta - 1)(\sqrt{s}/v^2)\, M\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\pm\, P_\mp) \mp \tfrac{1}{2}(\delta - 1)(\sqrt{s}/v^2)\, \bar M\, ((\bar r\cdot r)\, P_\pm - \bar E_\pm\, E_\pm\, P_\mp)\}/v^2\\
&- w_\llcorner{}^{\bar\nu}\, v^\nu\, i\gamma_5\, ((\bar r\cdot r)\, P_\pm \pm M\, \bar E_\pm\, P_\mp)/v^2 + v^{\bar\nu}\, \bar w_\llcorner{}^\nu\, i\gamma_5\, ((\bar r\cdot r)\, P_\mp \pm \bar M\, E_\pm\, P_\pm)/v^2\big]\, \bar w_\lrcorner{}^\mu\,.
\end{aligned}
\tag{A.7}
$$

# B Amplitudes and projection algebras for inverse reactions

So far we have discussed on-shell scattering amplitudes $T_{i\to f}$ and their associated projection algebras as summarized in Tab. (B.1). The reactions not treated so far are related to already considered processes by a hermitian conjugation and an interchange of indices between incoming and outgoing, i.e.

$$T_{i\to f} = [T_{f\to i}]^{\dagger}(\mathrm{f} \longleftrightarrow \mathrm{i})\,. \tag{B.1}$$

A suitable set of Lorentz-Dirac tensors can be directly identified, when we write its on-shell scattering amplitudes by applying Eq. (B.1). For a basis of a given reaction $T^{(n)}_{\pm}$ a basis for its inverse reaction $\bar{T}^{(n)}_{\pm}$ is introduced with

$$\bar{T}^{(n)}_{\pm} = \gamma_0\,[T^{(n)}_{\pm}]^{\dagger}\,\gamma_0\Bigg|_{\substack{r \;\longleftrightarrow\; \bar{r} \\ \{\mu,\nu\} \;\longleftrightarrow\; \{\bar{\mu},\bar{\nu}\}}}\,. \tag{B.2}$$

In this appendix we specify all inverse reactions where results for the projection algebras are expressed in terms of auxiliary tensors $P^{\mu\nu\cdots}_{\pm,n}$ that satisfy the same on-shell conditions as the projection tensors $Q^{\mu\nu\cdots}_{\pm,n}$.

| $\mathrm{i}\setminus\mathrm{f}$ | $0^-\frac{1}{2}^+$ | $1^-\frac{1}{2}^+$ | $0^-\frac{3}{2}^+$ | $1^-\frac{3}{2}^+$ |
|---|---|---|---|---|
| $0^-\frac{1}{2}^+$ | $\surd$ | $\surd$ | $\surd$ | $\surd$ |
| $1^-\frac{1}{2}^+$ | | $\surd$ | $\surd$ | $\surd$ |
| $0^-\frac{3}{2}^+$ | | | $\surd$ | $\surd$ |
| $1^-\frac{3}{2}^+$ | | | | $\surd$ |

**Table B.1.:** A summary of reactions considered so far.

## B.1 $1^- + \frac{1}{2}^+ \to 0^- + \frac{1}{2}^+$

The on-shell scattering amplitude can be obtained from (2.42) by using (B.1) as follows

$$T_{1\,\frac{1}{2}\to 0\,\frac{1}{2}}(\bar{k},k,w) = \sum_{\pm,n} \bar{F}^{\pm}_{n}(\sqrt{s},t)\,\Big[\bar{u}(\bar{p},\lambda_{\bar{p}})\,\bar{T}^{(n)}_{\pm,\mu}\,u(p,\lambda_p)\,\epsilon^{\mu}(q,\lambda_q)\Big]\,,$$

$$\bar{T}^{(1)}_{\pm,\mu} = \gamma_5\,i\,P_{\pm}\,\hat{\gamma}_{\mu}\,, \qquad \bar{T}^{(2)}_{\pm,\mu} = \gamma_5\,i\,P_{\pm}\,w_{\mu}\,, \qquad \bar{T}^{(3)}_{\pm,\mu} = \gamma_5\,i\,P_{\pm}\,\bar{r}_{\mu}\,. \tag{B.3}$$

Similarly, we find its projection algebra from (3.11) as follows

$$\frac{1}{2}\,\mathbf{tr}\big(\bar{T}^{(n)}_{a,\mu}\,\Lambda\,\bar{Q}^{\mu}_{b,k}\,\bar{\Lambda}\big) = \delta_{nk}\,\delta_{ab} \qquad \text{with} \qquad q_{\mu}\,\bar{Q}^{\mu}_{\pm,n} = 0\,, \tag{B.4}$$

$$\bar{Q}^{\mu}_{\pm,1} = \mp\,\frac{\sqrt{s}}{v^2}\,P_{\pm}\,v^{\mu}\,,$$

$$\bar{Q}^{\mu}_{\pm,2} = -\bar{R}_{\pm}\,i\,\gamma_5\,\bar{w}_{\perp}{}^{\mu} + \tfrac{1}{2}(\delta+1)\,\frac{\sqrt{s}}{v^2}\,\bar{E}_{\pm}\,Q_{\pm}\,v^{\mu}\,,$$

$$\bar{Q}^{\mu}_{\pm,3} = -\bar{R}_{\pm}\,i\,\gamma_5\,\bar{r}_{\perp}{}^{\mu} + \frac{\sqrt{s}}{v^2}\,E_{\mp}\,Q_{\mp}\,v^{\mu}\,,$$

where $\bar{R}_{\pm} = \frac{s}{v^2}\Big(E_{\mp}\,\bar{E}_{\pm}\,P_{\pm} - (\bar{r}\cdot r)\,P_{\mp}\Big)$.

## B.2 $0^- + \frac{3}{2}^+ \to 0^- + \frac{1}{2}^+$

Applying (B.1) to (2.49), the on-shell scattering amplitude can be written as

$$\begin{aligned} T_{0\,\frac{3}{2}\to 0\,\frac{1}{2}}(\bar{k},k,w) = \sum_{\pm,n} \bar{F}_n^{\pm}(\sqrt{s},t)\, \Big[\bar{u}(\bar{p},\lambda_{\bar{p}})\, \bar{T}^{(n)}_{\pm,\nu}\, u^{\nu}(p,\lambda_p)\Big]\,, \\ \bar{T}^{(1)}_{\pm,\nu} = \gamma_5\, i\, P_\pm\, w_\nu\,, \qquad \bar{T}^{(2)}_{\pm,\nu} = \gamma_5\, i\, P_\pm\, \bar{r}_\nu\,. \end{aligned} \tag{B.5}$$

For the associated projection algebra, from (3.14) we derive

$$\begin{aligned} &\frac{1}{2}\,\mathbf{tr}\big(\bar{T}^{(n)}_{a,\nu}\,\Lambda\,\bar{Q}^{\nu}_{b,k}\,\bar{\Lambda}\big) = \delta_{nk}\,\delta_{ab} \qquad \text{with} \qquad \gamma_\nu\,\Lambda\,\bar{Q}^{\nu}_{\pm,k}\,\bar{\Lambda} = 0 \qquad \text{and} \qquad p_\nu\,\bar{Q}^{\nu}_{\pm,k} = 0\,, \\ &\bar{Q}^{\nu}_{\pm,1} = \frac{s}{v^2}\,\Big[(\bar{r}\cdot r)\,\bar{P}^{\nu}_{\mp,1} - E_\mp\,\bar{E}_\pm\,\bar{P}^{\nu}_{\pm,1}\Big]\,, \\ &\bar{Q}^{\nu}_{\pm,2} = \frac{s}{v^2}\,\Big[(\bar{r}\cdot r)\,\bar{P}^{\nu}_{\mp,2} - E_\mp\,\bar{E}_\pm\,\bar{P}^{\nu}_{\pm,2}\Big]\,, \end{aligned} \tag{B.6}$$

where

$$\begin{aligned} \bar{P}^{\nu}_{\pm,1} &= \bar{w}_{\perp}{}^{\nu}\, P_\pm\, i\,\gamma_5 + v^{\nu}\,\big((\bar{r}\cdot r)\,P_\mp \pm M\,\bar{E}_\mp\,P_\pm\big)/v^2\,, \\ \bar{P}^{\nu}_{\pm,2} &= \bar{r}_{\perp}{}^{\nu}\, P_\pm\, i\,\gamma_5 + v^{\nu}\,\big(\sqrt{s}\,E_\pm\,P_\mp\big)/v^2\,. \end{aligned} \tag{B.7}$$

## B.3 $0^- + \frac{3}{2}^+ \to 1^- + \frac{1}{2}^+$

For $0\frac{3}{2} \to 1\frac{1}{2}$, the on-shell scattering amplitude can be obtained from (2.51) by using (B.1) as follows

$$\begin{aligned} &T_{0\,\frac{3}{2}\to 1\,\frac{1}{2}}(\bar{k},k,w) = \sum_{\pm,n} \bar{F}_n^{\pm}(\sqrt{s},t)\, \Big[\epsilon^{\bar{\mu}}(\bar{q},\lambda_{\bar{q}})\,\bar{u}(\bar{p},\lambda_{\bar{p}})\, \bar{T}^{(n)}_{\pm,\bar{\mu}\nu}\, u^{\nu}(p,\lambda_p)\Big]\,, \\ &\bar{T}^{(1)}_{\pm,\bar{\mu}\nu} = P_\pm\,\hat{g}_{\bar{\mu}\nu}\,, \qquad \bar{T}^{(2)}_{\pm,\bar{\mu}\nu} = \gamma_{\bar{\mu}}\,P_\pm\,w_\nu\,, \qquad \bar{T}^{(3)}_{\pm,\bar{\mu}\nu} = \gamma_{\bar{\mu}}\,P_\pm\,\bar{r}_\nu\,, \\ &\bar{T}^{(4)}_{\pm,\bar{\mu}\nu} = r_{\bar{\mu}}\,P_\pm\,w_\nu\,, \qquad \bar{T}^{(5)}_{\pm,\bar{\mu}\nu} = w_{\bar{\mu}}\,P_\pm\,\bar{r}_\nu\,, \qquad \bar{T}^{(6)}_{\pm,\bar{\mu}\nu} = w_{\bar{\mu}}\,P_\pm\,w_\nu\,. \end{aligned} \tag{B.8}$$

From (3.21) we derive the projection algebra :

$$\frac{1}{2}\,\mathbf{tr}\big(\bar{T}^{(n)}_{a,\bar{\mu}\nu}\,\Lambda\,\bar{Q}^{\bar{\mu}\nu}_{b,k}\,\bar{\Lambda}\big) = \delta_{nk}\,\delta_{ab} \qquad \text{with} \qquad p_\nu\,\bar{Q}^{\bar{\mu}\nu}_{\pm,k} = 0 = \gamma_\nu\,\Lambda\,\bar{Q}^{\bar{\mu}\nu}_{\pm,k}\,\bar{\Lambda} \qquad \text{and} \qquad \bar{q}_{\bar{\mu}}\,\bar{Q}^{\bar{\mu}\nu}_{\pm,k} = 0\,, \tag{B.9}$$

$$\begin{aligned} \bar{Q}^{\bar{\mu}\nu}_{\pm,1} &= -\frac{\sqrt{s}}{E_\pm}\,\Big[(\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\nu}_{\pm,1} - E_\pm\,\bar{E}_\mp\,\bar{P}^{\bar{\mu}\nu}_{\mp,1}\Big] \mp \frac{1}{E_\pm}\,\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\mu}\nu}_{\mp,3} + E_\pm\,\bar{E}_\mp\,\bar{Q}^{\bar{\mu}\nu}_{\pm,3}\Big]\,, \\ \bar{Q}^{\bar{\mu}\nu}_{\pm,2} &= \pm\sqrt{s}\,\bar{P}^{\bar{\mu}\nu}_{\mp,4} \mp \tfrac{1}{2}(\delta-1)\,\frac{s}{v^2}\,\bar{E}_\pm\,\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\mu}\nu}_{\pm,1} + E_\mp\,\bar{E}_\mp\,\bar{Q}^{\bar{\mu}\nu}_{\mp,1}\Big] \pm \frac{1}{\sqrt{s}}\,\bar{Q}^{\bar{\mu}\nu}_{\mp,1}\,, \\ \bar{Q}^{\bar{\mu}\nu}_{\pm,3} &= \pm\frac{s}{v^2}\,\sqrt{s}\,(\bar{r}\cdot r)\,\Big[(\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\nu}_{\mp,1} - E_\mp\,\bar{E}_\pm\,\bar{P}^{\bar{\mu}\nu}_{\pm,1}\Big] \pm \frac{s}{v^2}\,E_\mp\,\Big[(\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\nu}_{\pm,2} - E_\pm\,\bar{E}_\pm\,\bar{P}^{\bar{\mu}\nu}_{\mp,2}\Big]\,, \\ \bar{Q}^{\bar{\mu}\nu}_{\pm,4} &= \frac{s}{v^2}\,\Big[(\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\nu}_{\mp,5} - E_\mp\,\bar{E}_\mp\,\bar{P}^{\bar{\mu}\nu}_{\pm,5}\Big] \mp \frac{s}{v^2}\,\bar{E}_\mp\,\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\mu}\nu}_{\pm,2} + E_\mp\,\bar{E}_\pm\,\bar{Q}^{\bar{\mu}\nu}_{\mp,2}\Big] - \tfrac{1}{2}(\delta-1)\,\frac{s}{v^2}\,(\bar{r}\cdot\bar{r})\,\bar{Q}^{\bar{\mu}\nu}_{\pm,1}\,, \\ \bar{Q}^{\bar{\mu}\nu}_{\pm,5} &= \frac{s}{v^2}\,\Big[(\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\nu}_{\mp,3} - E_\mp\,\bar{E}_\mp\,\bar{P}^{\bar{\mu}\nu}_{\pm,3}\Big] \pm \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\,E_\mp\,\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\mu}\nu}_{\mp,3} + E_\pm\,\bar{E}_\mp\,\bar{Q}^{\bar{\mu}\nu}_{\pm,3}\Big] - \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\,(r\cdot r)\,\bar{Q}^{\bar{\mu}\nu}_{\pm,1}\,, \\ \bar{Q}^{\bar{\mu}\nu}_{\pm,6} &= \frac{s}{v^2}\,\Big[(\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\nu}_{\mp,6} - E_\mp\,\bar{E}_\mp\,\bar{P}^{\bar{\mu}\nu}_{\pm,6}\Big] \pm \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\,E_\mp\,\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\mu}\nu}_{\mp,2} + E_\pm\,\bar{E}_\mp\,\bar{Q}^{\bar{\mu}\nu}_{\pm,2}\Big] \\ &\quad - \tfrac{1}{4}(\delta-1)(\bar{\delta}+1)\,\frac{s}{v^2}\,(\bar{r}\cdot r)\,\bar{Q}^{\bar{\mu}\nu}_{\pm,1}\,, \end{aligned}$$

where

$$
\begin{aligned}
\bar P^{\bar\mu\nu}_{\pm,1} &= -v^{\bar\mu}\,[\bar r_\perp{}^\nu\,P_\pm\,i\gamma_5 + v^\nu\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp]/v^2\,,\\
\bar P^{\bar\mu\nu}_{\pm,2} &= r_\perp{}^{\bar\mu}\,[\bar r_\perp{}^\nu\,P_\pm - v^\nu\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp\,i\gamma_5]\,,\\
\bar P^{\bar\mu\nu}_{\pm,3} &= w_\perp{}^{\bar\mu}\,[\bar r_\perp{}^\nu\,P_\pm - v^\nu\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp\,i\gamma_5]\,,\\
\bar P^{\bar\mu\nu}_{\pm,4} &= -v^{\bar\mu}\,[\bar w_\perp{}^\nu\,P_\pm\,i\gamma_5 + v^\nu\,((\bar r\cdot r)\,P_\mp \pm M\,\bar E_\mp\,P_\pm)/v^2]/v^2\,,\\
\bar P^{\bar\mu\nu}_{\pm,5} &= r_\perp{}^{\bar\mu}\,[\bar w_\perp{}^\nu\,P_\pm - v^\nu\,((\bar r\cdot r)\,P_\mp \pm M\,\bar E_\pm\,P_\pm)\,i\gamma_5/v^2]\,,\\
\bar P^{\bar\mu\nu}_{\pm,6} &= w_\perp{}^{\bar\mu}\,[\bar w_\perp{}^\nu\,P_\pm - v^\nu\,((\bar r\cdot r)\,P_\mp \pm M\,\bar E_\pm\,P_\pm)\,i\gamma_5/v^2]\,.
\end{aligned}
\tag{B.10}
$$

## B.4 $1^- + \frac{3}{2}^+ \to 0^- + \frac{1}{2}^+$

Applying (B.1) to (2.54), the on-shell scattering amplitude can be written as

$$
\begin{aligned}
&T_{1\,\frac{3}{2}\to 0\,\frac{1}{2}}(\bar k, k, w) = \sum_{\pm,n} \bar F_n^\pm(\sqrt{s},t)\,\Big[\bar u(\bar p,\lambda_{\bar p})\,\bar T^{(n)}_{\pm,\mu\nu}\,u^\nu(p,\lambda_p)\,\epsilon^\mu(q,\lambda_q)\Big]\,,\\
&\bar T^{(1)}_{\pm,\mu\nu} = P_\pm\,\hat g_{\mu\nu}\,, \qquad \bar T^{(2)}_{\pm,\mu\nu} = P_\pm\,\hat\gamma_\mu\,w_\nu\,, \qquad \bar T^{(3)}_{\pm,\mu\nu} = P_\pm\,\hat\gamma_\mu\,\bar r_\nu\,,\\
&\bar T^{(4)}_{\pm,\mu\nu} = P_\pm\,w_\mu\,w_\nu\,, \qquad \bar T^{(5)}_{\pm,\mu\nu} = P_\pm\,w_\mu\,\bar r_\nu\,, \qquad \bar T^{(6)}_{\pm,\mu\nu} = P_\pm\,\bar r_\mu\,\bar r_\nu\,.
\end{aligned}
\tag{B.11}
$$

For the associated projection algebra, from (3.23) we derive

$$
\frac{1}{2}\,\mathbf{tr}\big(\bar T^{(n)}_{a,\mu\nu}\,\Lambda\,\bar Q^{\mu\nu}_{b,k}\,\bar\Lambda\big) = \delta_{nk}\,\delta_{ab} \qquad \text{with} \qquad q_\mu\,\bar Q^{\mu\nu}_{\pm,k} = 0 \qquad \text{and} \qquad p_\nu\,\bar Q^{\mu\nu}_{\pm,k} = 0 = \gamma_\nu\,\Lambda\bar Q^{\mu\nu}_{\pm,k}\,\bar\Lambda\,,
\tag{B.12}
$$

$$
\begin{aligned}
\bar Q^{\mu\nu}_{\pm,1} &= \pm\,\frac{\sqrt{s}}{M}\,\Big[\bar P^{\mu\nu}_{\mp,5} + \sqrt{s}\,E_\mp\,\bar P^{\mu\nu}_{\pm,4}\Big]\,,\\
\bar Q^{\mu\nu}_{\pm,2} &= \pm\,\frac{1}{E_\mp}\,\bar P^{\mu\nu}_{\mp,5} \pm \tfrac{1}{2}(\delta-1)\,\frac{s}{v^2}\,\frac{(\bar r\cdot r)}{E_\mp}\,\Big[(\bar r\cdot r)\bar Q^{\mu\nu}_{\pm,1} + E_\mp\,\bar E_\mp\,\bar Q^{\mu\nu}_{\mp,1}\Big]\,,\\
\bar Q^{\mu\nu}_{\pm,3} &= \mp\sqrt{s}\,\bar P^{\mu\nu}_{\pm,1} \mp \frac{s}{v^2}\,E_\pm\,\Big[(\bar r\cdot r)\bar Q^{\mu\nu}_{\pm,1} + E_\mp\,\bar E_\mp\,\bar Q^{\mu\nu}_{\mp,1}\Big]\,,\\
\bar Q^{\mu\nu}_{\pm,4} &= \frac{s}{v^2}\,\Big[(\bar r\cdot r)\,\bar P^{\mu\nu}_{\mp,6} - E_\mp\,\bar E_\mp\,\bar P^{\mu\nu}_{\pm,6}\Big] \pm \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\,\bar E_\mp\,\Big[(\bar r\cdot r)\bar Q^{\mu\nu}_{\mp,2} + E_\mp\,\bar E_\pm\,\bar Q^{\mu\nu}_{\pm,2}\Big] + \tfrac{1}{4}(\delta^2-1)\,\frac{s}{v^2}\,(\bar r\cdot\bar r)\bar Q^{\mu\nu}_{\pm,1}\,,\\
\bar Q^{\mu\nu}_{\pm,5} &= \frac{s}{v^2}\,\Big[(\bar r\cdot r)\,\bar P^{\mu\nu}_{\mp,3} - E_\mp\,\bar E_\mp\,\bar P^{\mu\nu}_{\pm,3}\Big] \pm \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\,\bar E_\mp\,\Big[(\bar r\cdot r)\bar Q^{\mu\nu}_{\mp,3} + E_\mp\,\bar E_\pm\,\bar Q^{\mu\nu}_{\pm,3}\Big] + \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\,(\bar r\cdot r)\bar Q^{\mu\nu}_{\pm,1}\,,\\
\bar Q^{\mu\nu}_{\pm,6} &= \frac{s}{v^2}\,\Big[(\bar r\cdot r)\,\bar P^{\mu\nu}_{\mp,2} - E_\mp\,\bar E_\mp\,\bar P^{\mu\nu}_{\pm,2}\Big] \mp \frac{s}{v^2}\,E_\mp\,\Big[(\bar r\cdot r)\bar Q^{\mu\nu}_{\pm,3} + E_\pm\,\bar E_\mp\,\bar Q^{\mu\nu}_{\mp,3}\Big] + \frac{s}{v^2}\,(r\cdot r)\bar Q^{\mu\nu}_{\pm,1}\,,
\end{aligned}
$$

where

$$
\begin{aligned}
\bar P^{\mu\nu}_{\pm,1} &= -v^{\mu}\,\big[\bar r_\perp{}^\nu\,P_\pm\,i\gamma_5 + v^\nu\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp\big]/v^2\,,\\
\bar P^{\mu\nu}_{\pm,2} &= \bar r_\perp{}^{\mu}\,\big[\bar r_\perp{}^\nu\,P_\pm - v^\nu\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp i\gamma_5\big]\,,\\
\bar P^{\mu\nu}_{\pm,3} &= \bar w_\perp{}^{\mu}\,\big[\bar r_\perp{}^\nu\,P_\pm - v^\nu\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp i\gamma_5\big]\,,\\
\bar P^{\mu\nu}_{\pm,4} &= -v^{\mu}\,\big[\bar w_\perp{}^\nu\,P_\pm\,i\gamma_5 + v^\nu\,((\bar r\cdot r)\,P_\mp \pm M\,\bar E_\mp\,P_\pm)/v^2\big]/v^2\,,\\
\bar P^{\mu\nu}_{\pm,5} &= \bar r_\perp{}^{\mu}\,\big[\bar w_\perp{}^\nu\,P_\pm - v^\nu\,((\bar r\cdot r)\,P_\mp \pm M\,\bar E_\pm\,P_\pm)\,i\gamma_5/v^2\big]\,,\\
\bar P^{\mu\nu}_{\pm,6} &= \bar w_\perp{}^{\mu}\,\big[\bar w_\perp{}^\nu\,P_\pm - v^\nu\,((\bar r\cdot r)\,P_\mp \pm M\,\bar E_\pm\,P_\pm)\,i\gamma_5/v^2\big]\,.
\end{aligned}
\tag{B.13}
$$

## B.5 $1^- + \frac{3}{2}^+ \to 1^- + \frac{1}{2}^+$

For $1\frac{3}{2} \to 1\frac{1}{2}$, the on-shell scattering amplitude can be obtained from (2.56) by using (B.1) as follows

$$
\begin{aligned}
&T_{1\frac{3}{2}\to 1\frac{1}{2}}(\bar{k},k,w) = \sum_{\pm,n} \bar{F}_n^{\pm}(\sqrt{s},t)\left[\epsilon^{\dagger\bar{\mu}}(\bar{q},\lambda_{\bar{q}})\,\bar{u}(\bar{p},\lambda_{\bar{p}})\,\bar{T}^{(n)}_{\pm,\bar{\mu}\mu\nu}\,u^{\nu}(p,\lambda_p)\,\epsilon^{\mu}(q,\lambda_q)\right],\\
&\bar{T}^{(1)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,\hat{\gamma}_{\bar{\mu}}\,P_{\pm}\,\hat{g}_{\mu\nu}\,, \quad \bar{T}^{(2)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,P_{\pm}\,\hat{\gamma}_{\mu}\,\hat{g}_{\bar{\mu}\nu}\,, \quad \bar{T}^{(3)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,w_{\bar{\mu}}\,P_{\pm}\,\hat{g}_{\mu\nu}\,,\\
&\bar{T}^{(4)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,P_{\pm}\,w_{\mu}\,\hat{g}_{\bar{\mu}\nu}\,, \quad \bar{T}^{(5)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,r_{\bar{\mu}}\,P_{\pm}\,\hat{g}_{\mu\nu}\,, \quad \bar{T}^{(6)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,P_{\pm}\,\bar{r}_{\mu}\,\hat{g}_{\bar{\mu}\nu}\,,\\
&\bar{T}^{(7)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,\hat{\gamma}_{\bar{\mu}}\,P_{\pm}\,\hat{\gamma}_{\mu}\,w_{\nu}\,, \quad \bar{T}^{(8)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,\hat{\gamma}_{\bar{\mu}}\,P_{\pm}\,\hat{\gamma}_{\mu}\,\bar{r}_{\nu}\,, \quad \bar{T}^{(9)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,r_{\bar{\mu}}\,P_{\pm}\,\hat{\gamma}_{\mu}\,w_{\nu}\,,\\
&\bar{T}^{(10)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,\hat{\gamma}_{\bar{\mu}}\,P_{\pm}\,w_{\mu}\,w_{\nu}\,, \quad \bar{T}^{(11)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,w_{\bar{\mu}}\,P_{\pm}\,\hat{\gamma}_{\mu}\,\bar{r}_{\nu}\,, \quad \bar{T}^{(12)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,\hat{\gamma}_{\bar{\mu}}\,P_{\pm}\,w_{\mu}\,r_{\nu}\,,\\
&\bar{T}^{(13)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,w_{\bar{\mu}}\,P_{\pm}\,\hat{\gamma}_{\mu}\,w_{\nu}\,, \quad \bar{T}^{(14)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,\hat{\gamma}_{\bar{\mu}}\,P_{\pm}\,\bar{r}_{\mu}\,\bar{r}_{\nu}\,, \quad \bar{T}^{(15)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,w_{\bar{\mu}}\,P_{\pm}\,w_{\mu}\,w_{\nu}\,,\\
&\bar{T}^{(16)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,r_{\bar{\mu}}\,P_{\pm}\,w_{\mu}\,w_{\nu}\,, \quad \bar{T}^{(17)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,w_{\bar{\mu}}\,P_{\pm}\,w_{\mu}\,\bar{r}_{\nu}\,, \quad \bar{T}^{(18)}_{\pm,\bar{\mu}\mu\nu} = \gamma_5\, i\,w_{\bar{\mu}}\,P_{\pm}\,\bar{r}_{\mu}\,\bar{r}_{\nu}\,.
\end{aligned}
\tag{B.14}
$$

For the associated projection algebra, from (A.4) we derive

$$
\begin{aligned}
&\frac{1}{2}\,\mathbf{tr}\big(\bar{T}^{(n)}_{a,\bar{\mu}\mu\nu}\,\Lambda\,\bar{Q}^{\bar{\mu}\mu\nu}_{b,k}\,\bar{\Lambda}\big) = \delta_{nk}\,\delta_{ab}\,,\\
&\quad\text{with}\quad \bar{q}_{\bar{\mu}}\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,k} = 0\,,\quad q_{\mu}\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,k} = 0\,,\ \text{and}\quad p_{\nu}\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,k} = 0 = \gamma_{\nu}\,\Lambda\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,k}\,\bar{\Lambda}\,,
\end{aligned}
\tag{B.15}
$$

$$
\begin{aligned}
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1} &= 3(\delta-1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{M}\,(\bar{r}\cdot\bar{r})\,E_{\mp}\Big[s\,E_{\pm}\,\bar{E}_{\mp}\,((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,1} - E_{\mp}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,1}) + \sqrt{s}\,\bar{E}_{\mp}\,((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,2} - E_{\pm}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,2})\\
&\qquad - \sqrt{s}\,E_{\pm}\,((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,7} - E_{\mp}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,7}) + ((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,8} - E_{\pm}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,8})\Big]\\
&\quad + 4\,\frac{s}{v^2}\,\frac{\sqrt{s}}{M}\,(r\cdot r)\,\bar{E}_{\mp}\Big[s\,E_{\mp}\,\bar{E}_{\pm}\,((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,4} - E_{\pm}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,4}) + \sqrt{s}\,\bar{E}_{\pm}\,((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,5} - E_{\mp}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,5})\\
&\qquad - \sqrt{s}\,E_{\mp}\,((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,10} - E_{\pm}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,10}) + ((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,11} - E_{\mp}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,11})\Big]\\
&\quad + 2(\delta-1)\,\frac{\sqrt{s}}{M}\,\bar{E}_{\mp}\Big[s\,E_{\mp}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,1} + \sqrt{s}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,2} - \sqrt{s}\,E_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,7} + \bar{P}^{\bar{\mu}\mu\nu}_{\mp,8}\Big]\\
&\quad \pm \bar{E}_{\mp}\Big[-s\,E_{\mp}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,1} + \sqrt{s}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,2} + \sqrt{s}\,E_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,7} + \bar{P}^{\bar{\mu}\mu\nu}_{\mp,8}\Big]\\
&\quad + \frac{(\bar{r}\cdot r)}{M}\Big[s\,E_{\mp}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,1} + \sqrt{s}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,2} - \sqrt{s}\,E_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,7} + \bar{P}^{\bar{\mu}\mu\nu}_{\pm,8}\Big]\\
&\quad - \frac{s}{M}\,\sqrt{s}\,E_{\mp}\Big[(\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,4} - 3\,E_{\pm}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,4}\Big] - \frac{s}{M}\Big[(\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,5} - 3\,E_{\mp}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,5}\Big] + 2\,\frac{\sqrt{s}}{M}\,E_{\mp}\Big[\bar{P}^{\bar{\mu}\mu\nu}_{\pm,11} - \sqrt{s}\,E_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,10}\Big]\,,\\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2} &= -(\delta-1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{M}\,(\bar{r}\cdot\bar{r})\,E_{\pm}\Big[s\,E_{\mp}\,\bar{E}_{\pm}\,((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,1} - E_{\pm}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,1}) + \sqrt{s}\,\bar{E}_{\pm}\,((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,2} - E_{\mp}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,2})\\
&\qquad - \sqrt{s}\,E_{\mp}\,((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,7} - E_{\pm}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,7}) + ((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,8} - E_{\mp}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,8})\Big]\\
&\quad - 2\,\frac{s}{v^2}\,\frac{\sqrt{s}}{M}\,(r\cdot r)\,\bar{E}_{\pm}\Big[s\,E_{\pm}\,\bar{E}_{\mp}\,((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,4} - E_{\mp}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,4}) + \sqrt{s}\,\bar{E}_{\mp}\,((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,5} - E_{\pm}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,5})\\
&\qquad - \sqrt{s}\,E_{\pm}\,((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,10} - E_{\mp}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,10}) + ((\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,11} - E_{\pm}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,11})\Big]\\
&\quad + \tfrac{1}{2}(\delta-1)\,\frac{s}{M}\,E_{\pm}\,\bar{E}_{\pm}\Big[\bar{P}^{\bar{\mu}\mu\nu}_{\mp,7} - \sqrt{s}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,1}\Big] \pm \sqrt{s}\,(\bar{r}\cdot r)\Big[\bar{P}^{\bar{\mu}\mu\nu}_{\pm,7} - \sqrt{s}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,1}\Big] + \frac{1}{M}\,E_{\pm}\,\bar{E}_{\pm}\Big[\bar{P}^{\bar{\mu}\mu\nu}_{\pm,8} + \sqrt{s}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,2}\Big]\\
&\quad - \frac{\sqrt{s}}{M}\,E_{\pm}\Big[+s\,E_{\mp}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,4} + \sqrt{s}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,5} - \sqrt{s}\,E_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,10} + \bar{P}^{\bar{\mu}\mu\nu}_{\mp,11}\Big]\,,\\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,3} &= \pm\,\frac{\sqrt{s}}{M}\Big[\bar{P}^{\bar{\mu}\mu\nu}_{\mp,17} - \sqrt{s}\,E_{\mp}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,16}\Big] \mp \tfrac{1}{4}(\delta-1)(\bar{\delta}+1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{M}\,(\bar{r}\cdot r)\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,6} + E_{\mp}\,\bar{E}_{\pm}\,\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,6}\Big]\\
&\quad \pm \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\,E_{\mp}\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + E_{\pm}\,\bar{E}_{\pm}\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1}\Big]\,,
\end{aligned}
$$

$$
\begin{aligned}
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,4} &= \frac{\sqrt{s}}{E_\pm}\Big[(\bar{r}\cdot r)\bar{P}^{\bar{\mu}\mu\nu}_{\pm,3} - E_\pm \bar{E}_\pm \bar{P}^{\bar{\mu}\mu\nu}_{\mp,3}\Big] \mp \frac{1}{E_\pm}\Big[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,12} + E_\pm \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,12}\Big] \\
&\quad + \tfrac{1}{2}(\delta+1)\Big[\pm\frac{s}{v^2}\frac{(\bar{r}\cdot r)}{E_\pm}((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + E_\pm \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1}) \pm \frac{s}{v^2}\bar{E}_\pm((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,2} + E_\mp \bar{E}_\mp \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2}) - \frac{\bar{E}_\pm}{E_\pm}\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,8}\Big], \\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,5} &= \frac{\sqrt{s}}{E_\pm}\Big[(\bar{r}\cdot r)\bar{P}^{\bar{\mu}\mu\nu}_{\pm,7} - E_\pm \bar{E}_\pm \bar{P}^{\bar{\mu}\mu\nu}_{\mp,7}\Big] \\
&\quad \mp \frac{s}{v^2}\bar{E}_\pm\Big[(\bar{r}\cdot r)(\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,2}) + E_\mp \bar{E}_\mp(\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2})\Big] \mp \frac{1}{E_\pm}\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2} - \frac{\bar{E}_\pm}{E_\pm}\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,8}, \\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,6} &= \frac{2}{\delta-1}\frac{1}{(\bar{r}\cdot\bar{r})}\Big[(\bar{r}\cdot r)\bar{P}^{\bar{\mu}\mu\nu}_{\mp,11} - E_\mp \bar{E}_\pm \bar{P}^{\bar{\mu}\mu\nu}_{\pm,11}\Big] + \frac{2}{\delta-1}\frac{E_\mp}{\bar{E}_\mp}\Big[\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,7} \pm \frac{1}{\bar{E}_\pm}((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,9} + E_\pm \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,9})\Big] \\
&\quad \mp \frac{s}{v^2}E_\mp\Big[(\bar{r}\cdot r)(\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2}) + E_\pm \bar{E}_\pm(\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,2})\Big] \mp \frac{1}{\bar{E}_\mp}\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1} + \frac{(\bar{r}\cdot r)}{(\bar{r}\cdot\bar{r})}\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,5}, \\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,7} &= s\Big[(\bar{r}\cdot r)\bar{P}^{\bar{\mu}\mu\nu}_{\pm,4} - E_\pm \bar{E}_\mp \bar{P}^{\bar{\mu}\mu\nu}_{\mp,4}\Big] \mp \frac{1}{\sqrt{s}}\Big[\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,2}\Big] \\
&\quad \pm \tfrac{1}{2}(\delta-1)\frac{s}{v^2}\bar{E}_\mp\Big[(\bar{r}\cdot r)(\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2}) + E_\pm \bar{E}_\pm(\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,2})\Big], \\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,8} &= s\Big[(\bar{r}\cdot r)\bar{P}^{\bar{\mu}\mu\nu}_{\pm,1} - E_\pm \bar{E}_\mp \bar{P}^{\bar{\mu}\mu\nu}_{\mp,1}\Big] \mp \frac{s}{v^2}E_\pm\Big[(\bar{r}\cdot r)(\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,2}) + E_\mp \bar{E}_\mp(\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2})\Big], \\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,9} &= \pm\sqrt{s}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,10} + \tfrac{1}{2}(\delta-1)\frac{s}{v^2}\bar{E}_\pm\Big[\bar{E}_\mp \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2} \pm ((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,5} + E_\pm \bar{E}_\mp \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,5})\Big] \\
&\quad - \frac{\sqrt{s}}{v^2}\bar{E}_\pm\Big[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1} \mp M\,\bar{E}_\mp \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1}\Big] \mp \frac{s}{v^2}\bar{E}_\pm\Big[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,7} + E_\pm \bar{E}_\mp \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,7}\Big] \mp \frac{1}{\sqrt{s}}\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,5}, \\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,10} &= \mp\sqrt{s}\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,6} + \tfrac{1}{4}(\delta+1)(\delta-1)\frac{s}{v^2}(\bar{r}\cdot\bar{r})\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1} \mp \tfrac{1}{2}(\delta-1)\frac{s}{v^2}\bar{E}_\mp\Big[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,4} + E_\mp \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,4}\Big] \\
&\quad \pm \tfrac{1}{2}(\delta+1)\frac{s}{v^2}\bar{E}_\mp\Big[((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,7} + E_\mp \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,7}) \pm \frac{1}{\sqrt{s}}((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2} \pm M\,\bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,2})\Big] \pm \frac{1}{\sqrt{s}}\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,4}, \\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,11} &= \pm\sqrt{s}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,13} \mp \frac{s}{v^2}E_\pm\Big[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,3} + E_\mp \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,3}\Big] \\
&\quad - \tfrac{1}{2}(\bar{\delta}+1)\frac{s}{v^2}\Big[(r\cdot r)(\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2}) \mp E_\pm((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,8} + E_\mp \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,8})\Big], \\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,12} &= \mp\frac{s}{v^2}\sqrt{s}(\bar{r}\cdot r)\Big[(\bar{r}\cdot r)\bar{P}^{\bar{\mu}\mu\nu}_{\mp,3} - E_\mp \bar{E}_\mp \bar{P}^{\bar{\mu}\mu\nu}_{\pm,3}\Big] \pm \frac{s}{v^2}E_\mp\Big[(\bar{r}\cdot r)\bar{P}^{\bar{\mu}\mu\nu}_{\pm,9} - E_\pm \bar{E}_\mp \bar{P}^{\bar{\mu}\mu\nu}_{\mp,9}\Big] \\
&\quad + \tfrac{1}{2}(\delta+1)\frac{s}{v^2}\Big[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1} \pm (\bar{r}\cdot r)E_\mp \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,5} \pm \bar{E}_\mp((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,8} + E_\mp \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,8})\Big], \\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,13} &= \pm\sqrt{s}\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,16} - \tfrac{1}{4}(\delta-1)(\bar{\delta}+1)\frac{s}{v^2}(\bar{r}\cdot r)(\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2}) \pm \tfrac{1}{2}(\delta-1)\frac{s}{v^2}\bar{E}_\pm\Big[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,3} + E_\pm \bar{E}_\mp \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,3}\Big] \\
&\quad \pm \tfrac{1}{2}(\bar{\delta}+1)\frac{s}{v^2}\Big[E_\pm((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,7} + E_\mp \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,7}) - \frac{M}{\sqrt{s}}((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + E_\pm \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1})\Big] \mp \frac{1}{\sqrt{s}}\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,3}, \\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,14} &= \pm\frac{1}{\bar{E}_\pm}\bar{P}^{\bar{\mu}\mu\nu}_{\mp,8} \mp \frac{s}{v^2}E_\mp\Big[(\bar{r}\cdot r)(\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,6} + \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,8}) + E_\pm \bar{E}_\mp(\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,6} + \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,8})\Big] \\
&\quad - \frac{s}{v^2}\frac{E_\mp}{\bar{E}_\pm}\Big[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2} + E_\pm \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1}\Big] \pm \frac{s}{v^2}\frac{(r\cdot r)}{\bar{E}_\pm}\Big[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,5} + E_\mp \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,5}\Big] \mp \frac{1}{\bar{E}_\pm}\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,6}, \\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,15} &= \frac{s}{v^2}\Big[(\bar{r}\cdot r)\bar{P}^{\bar{\mu}\mu\nu}_{\mp,18} - E_\mp \bar{E}_\pm \bar{P}^{\bar{\mu}\mu\nu}_{\pm,18}\Big] \\
&\quad \mp \tfrac{1}{8}(\delta+1)(\delta-1)(\bar{\delta}+1)\left(\frac{s}{v^2}\right)^2\Big[(\bar{r}\cdot\bar{r})E_\mp((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + E_\pm \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1}) \\
&\qquad\qquad - (\bar{r}\cdot r)\bar{E}_\pm((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,2} + E_\mp \bar{E}_\mp \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2})\Big] \\
&\quad + \tfrac{1}{4}(\delta+1)(\delta-1)\frac{s}{v^2}(\bar{r}\cdot\bar{r})\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,3} - \tfrac{1}{4}(\delta-1)(\bar{\delta}+1)\frac{s}{v^2}(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,4} \\
&\quad - \tfrac{1}{4}(\delta+1)(\bar{\delta}+1)\frac{s}{v^2}\Big[2(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,7} - E_\mp \bar{E}_\pm \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,7}\Big]
\end{aligned}
$$

$$
\begin{aligned}
&\pm \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\,\bar{E}_\pm \left[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,13} + E_\mp\,\bar{E}_\mp\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,13}\right] \pm \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\,E_\mp \left[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,10} + E_\pm\,\bar{E}_\pm\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,10}\right],\\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,16} &= \frac{s}{v^2}\left[(\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,12} - E_\mp\,\bar{E}_\pm\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,12}\right] \mp \frac{s}{v^2}\,\bar{E}_\pm \left[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,10} + E_\mp\,\bar{E}_\mp\,\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,10}\right]\\
&+ \tfrac{1}{4}(\delta+1)(\delta-1)\,\frac{s}{v^2}\,(\bar{r}\cdot\bar{r})\left[\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,5} \pm \frac{s}{v^2}\,\bar{E}_\pm\,((\bar{r}\cdot r)(\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,2}) + E_\mp\,\bar{E}_\mp\,(\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2}))\right]\\
&- \tfrac{1}{2}(\delta-1)\,\frac{s}{v^2}\,(\bar{r}\cdot\bar{r})\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,4} - \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\left[(\bar{r}\cdot\bar{r})\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,7} \mp \bar{E}_\pm\,((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,9} + E_\mp\,\bar{E}_\mp\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,9})\right],\\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,17} &= \frac{s}{v^2}\left[(\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,15} - E_\mp\,\bar{E}_\pm\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,15}\right]\\
&- \tfrac{1}{4}(\delta+1)(\bar{\delta}+1)\,\frac{s}{v^2}\left[(2(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,8} - E_\mp\,\bar{E}_\pm\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,8})\right.\\
&\qquad \left.\pm \frac{s}{v^2}\,E_\mp\,((\bar{r}\cdot r)((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + E_\pm\,\bar{E}_\pm\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1}) + E_\pm\,\bar{E}_\pm\,((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,2} + E_\mp\,\bar{E}_\mp\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2}))\right]\\
&+ \tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\left[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,3} \pm \bar{E}_\pm\,((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,11} + E_\mp\,\bar{E}_\mp\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,11})\right]\\
&- \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\left[(r\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,4} \mp E_\mp\,((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,12} + E_\pm\,\bar{E}_\pm\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,12})\right],\\
\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,18} &= \frac{s}{v^2}\left[(\bar{r}\cdot r)\,\bar{P}^{\bar{\mu}\mu\nu}_{\mp,14} - E_\mp\,\bar{E}_\pm\,\bar{P}^{\bar{\mu}\mu\nu}_{\pm,14}\right] + \frac{s}{v^2}\,(r\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,3} \mp \frac{s}{v^2}\,E_\mp \left[(\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,11} + E_\pm\,\bar{E}_\pm\,\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,11}\right]\\
&- \tfrac{1}{2}(\bar{\delta}+1)\,\frac{s}{v^2}\left[(r\cdot r)(\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,8} + \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,6}) \mp E_\mp\,((\bar{r}\cdot r)\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,14} + E_\pm\,\bar{E}_\pm\,\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,14})\right.\\
&\qquad \left.\pm \frac{s}{v^2}\,(r\cdot r)\,E_\mp\,((\bar{r}\cdot r)(\bar{Q}^{\bar{\mu}\mu\nu}_{\mp,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\pm,2}) + E_\pm\,\bar{E}_\pm\,(\bar{Q}^{\bar{\mu}\mu\nu}_{\pm,1} + \bar{Q}^{\bar{\mu}\mu\nu}_{\mp,2}))\right],
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,1} &= v^{\bar{\mu}}\,v^{\mu}\left[\bar{r}_\perp{}^{\nu}\,P_\pm\,i\gamma_5 + v^{\nu}\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp\right]/((v^2)^2),\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,2} &= -v^{\bar{\mu}}\,\bar{r}_\perp{}^{\mu}\left[\bar{r}_\perp{}^{\nu}\,P_\pm - v^{\nu}\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp\,i\gamma_5\right]/v^2,\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,3} &= -v^{\bar{\mu}}\,\bar{w}_\lrcorner{}^{\mu}\left[\bar{r}_\perp{}^{\nu}\,P_\pm - v^{\nu}\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp\,i\gamma_5\right]/v^2,\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,4} &= v^{\bar{\mu}}\,v^{\mu}\left[\bar{w}_\llcorner{}^{\nu}\,P_\pm\,i\gamma_5 + v^{\nu}\,((\bar{r}\cdot r)\,P_\mp \pm M\,\bar{E}_\mp\,P_\pm)/v^2\right]/((v^2)^2),\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,5} &= -v^{\bar{\mu}}\,\bar{r}_\perp{}^{\mu}\left[\bar{w}_\llcorner{}^{\nu}\,P_\pm - v^{\nu}\,((\bar{r}\cdot r)\,P_\mp \pm M\,\bar{E}_\pm\,P_\pm)\,i\gamma_5/v^2\right]/v^2,\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,6} &= -v^{\bar{\mu}}\,\bar{w}_\lrcorner{}^{\mu}\left[\bar{w}_\llcorner{}^{\nu}\,P_\pm - v^{\nu}\,((\bar{r}\cdot r)\,P_\mp \pm M\,\bar{E}_\pm\,P_\pm)\,i\gamma_5/v^2\right]/v^2,\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,7} &= -r_\perp{}^{\bar{\mu}}\,v^{\mu}\left[\bar{r}_\perp{}^{\nu}\,P_\pm - v^{\nu}\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp\,i\gamma_5\right]/v^2,\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,8} &= r_\perp{}^{\bar{\mu}}\,\bar{r}_\perp{}^{\mu}\left[\bar{r}_\perp{}^{\nu}\,P_\pm\,i\gamma_5 + v^{\nu}\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp\right],\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,9} &= r_\perp{}^{\bar{\mu}}\,\bar{w}_\lrcorner{}^{\mu}\left[\bar{r}_\perp{}^{\nu}\,P_\pm\,i\gamma_5 + v^{\nu}\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp\right],\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,10} &= -r_\perp{}^{\bar{\mu}}\,v^{\mu}\left[\bar{w}_\llcorner{}^{\nu}\,P_\pm - v^{\nu}\,((\bar{r}\cdot r)\,P_\mp \pm M\,\bar{E}_\pm\,P_\pm)\,i\gamma_5/v^2\right]/v^2,\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,11} &= r_\perp{}^{\bar{\mu}}\,\bar{r}_\perp{}^{\mu}\left[\bar{w}_\llcorner{}^{\nu}\,P_\pm\,i\gamma_5 + v^{\nu}\,((\bar{r}\cdot r)\,P_\mp \pm M\,\bar{E}_\mp\,P_\pm)/v^2\right],\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,12} &= r_\perp{}^{\bar{\mu}}\,\bar{w}_\lrcorner{}^{\mu}\left[\bar{w}_\llcorner{}^{\nu}\,P_\pm\,i\gamma_5 + v^{\nu}\,((\bar{r}\cdot r)\,P_\mp \pm M\,\bar{E}_\mp\,P_\pm)/v^2\right],\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,13} &= -w_\lrcorner{}^{\bar{\mu}}\,v^{\mu}\left[\bar{r}_\perp{}^{\nu}\,P_\pm - v^{\nu}\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp\,i\gamma_5\right]/v^2,\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,14} &= w_\lrcorner{}^{\bar{\mu}}\,\bar{r}_\perp{}^{\mu}\left[\bar{r}_\perp{}^{\nu}\,P_\pm\,i\gamma_5 + v^{\nu}\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp\right],\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,15} &= w_\lrcorner{}^{\bar{\mu}}\,\bar{w}_\lrcorner{}^{\mu}\left[\bar{r}_\perp{}^{\nu}\,P_\pm\,i\gamma_5 + v^{\nu}\,(\sqrt{s}/v^2)\,E_\pm\,P_\mp\right],\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,16} &= -w_\lrcorner{}^{\bar{\mu}}\,v^{\mu}\left[\bar{w}_\llcorner{}^{\nu}\,P_\pm - v^{\nu}\,((\bar{r}\cdot r)\,P_\mp \pm M\,\bar{E}_\pm\,P_\pm)\,i\gamma_5/v^2\right]/v^2,\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,17} &= w_\lrcorner{}^{\bar{\mu}}\,\bar{r}_\perp{}^{\mu}\left[\bar{w}_\llcorner{}^{\nu}\,P_\pm\,i\gamma_5 + v^{\nu}\,((\bar{r}\cdot r)\,P_\mp \pm M\,\bar{E}_\mp\,P_\pm)/v^2\right],\\
\bar{P}^{\bar{\mu}\mu\nu}_{\pm,18} &= w_\lrcorner{}^{\bar{\mu}}\,\bar{w}_\lrcorner{}^{\mu}\left[\bar{w}_\llcorner{}^{\nu}\,P_\pm\,i\gamma_5 + v^{\nu}\,((\bar{r}\cdot r)\,P_\mp \pm M\,\bar{E}_\mp\,P_\pm)/v^2\right].
\end{aligned}
\tag{B.16}
$$

## B.6 $1^- + \frac{3}{2}^+ \to 0^- + \frac{3}{2}^+$

Applying (B.1) to (2.57), the on-shell scattering amplitude can be written as

$$T_{1\,\frac{3}{2}\to 0\,\frac{3}{2}}(\bar{k},k,w) = \sum_{\pm,n} \bar{F}_n^{\pm}(\sqrt{s},t)\,\Big[\bar{u}^{\bar{\nu}}(\bar{p},\lambda_{\bar{p}})\,\bar{T}^{(n)}_{\pm,\bar{\nu}\mu\nu}\,u^{\nu}(p,\lambda_p)\,\epsilon^{\mu}(q,\lambda_q)\Big]\,,$$

$$\begin{aligned}
\bar{T}^{(1)}_{\pm,\bar{\nu}\mu\nu} &= \gamma_5\, i\,\hat{g}_{\bar{\nu}\nu}\,P_{\pm}\,\hat{\gamma}_{\mu}\,, & \bar{T}^{(2)}_{\pm,\bar{\nu}\mu\nu} &= \gamma_5\, i\,\hat{g}_{\bar{\nu}\nu}\,P_{\pm}\,w_{\mu}\,, & \bar{T}^{(3)}_{\pm,\bar{\nu}\mu\nu} &= \gamma_5\, i\,\hat{g}_{\bar{\nu}\nu}\,P_{\pm}\,\bar{r}_{\mu}\,,\\
\bar{T}^{(4)}_{\pm,\bar{\nu}\mu\nu} &= \gamma_5\, i\,w_{\bar{\nu}}\,P_{\pm}\,\hat{g}_{\mu\nu}\,, & \bar{T}^{(5)}_{\pm,\bar{\nu}\mu\nu} &= \gamma_5\, i\,r_{\bar{\nu}}\,P_{\pm}\,\hat{g}_{\mu\nu}\,, & \bar{T}^{(6)}_{\pm,\bar{\nu}\mu\nu} &= \gamma_5\, i\,w_{\bar{\nu}}\,P_{\pm}\,\hat{\gamma}_{\mu}\,w_{\nu}\,,\\
\bar{T}^{(7)}_{\pm,\bar{\nu}\mu\nu} &= \gamma_5\, i\,r_{\bar{\nu}}\,P_{\pm}\,\hat{\gamma}_{\mu}\,w_{\nu}\,, & \bar{T}^{(8)}_{\pm,\bar{\nu}\mu\nu} &= \gamma_5\, i\,w_{\bar{\nu}}\,P_{\pm}\,\hat{\gamma}_{\mu}\,\bar{r}_{\nu}\,, & \bar{T}^{(9)}_{\pm,\bar{\nu}\mu\nu} &= \gamma_5\, i\,w_{\bar{\nu}}\,P_{\pm}\,w_{\mu}\,w_{\nu}\,,\\
\bar{T}^{(10)}_{\pm,\bar{\nu}\mu\nu} &= \gamma_5\, i\,r_{\bar{\nu}}\,P_{\pm}\,w_{\mu}\,w_{\nu}\,, & \bar{T}^{(11)}_{\pm,\bar{\nu}\mu\nu} &= \gamma_5\, i\,w_{\bar{\nu}}\,P_{\pm}\,w_{\mu}\,\bar{r}_{\nu}\,, & \bar{T}^{(12)}_{\pm,\bar{\nu}\mu\nu} &= \gamma_5\, i\,w_{\bar{\nu}}\,P_{\pm}\,\bar{r}_{\mu}\,\bar{r}_{\nu}\,.
\end{aligned} \tag{B.17}$$

For the associated projection algebra, from (A.2) we derive

$$\frac{1}{2}\,\mathbf{tr}\big(\bar{T}^{(n)}_{a,\bar{\nu}\mu\nu}\,\Lambda\,\bar{Q}^{\bar{\nu}\mu\nu}_{b,k}\,\bar{\Lambda}\big) = \delta_{nk}\,\delta_{ab}\,, \tag{B.18}$$

$$\text{with}\qquad q_{\mu}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,k} = 0\,,\qquad \bar{p}_{\bar{\nu}}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,k} = 0 = \Lambda\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,k}\,\bar{\Lambda}\,\gamma_{\bar{\nu}}\,,\ \text{and}\qquad p_{\nu}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,k} = 0 = \gamma_{\nu}\,\Lambda\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,k}\,\bar{\Lambda}\,,$$

$$\begin{aligned}
\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,1} &= -(\delta-1)\,\frac{s}{v^2}\,\frac{\sqrt{s}}{M}\,(\bar{r}\cdot\bar{r})\,E_{\pm}\Big[\big((\bar{r}\cdot r)\,\bar{P}^{\bar{\nu}\mu\nu}_{\mp,5} - E_{\mp}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,5}\big) - \sqrt{s}\,E_{\mp}\big((\bar{r}\cdot r)\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,1} - E_{\pm}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\nu}\mu\nu}_{\mp,1}\big)\Big]\\
&\quad -2\,\frac{s}{v^2}\,\frac{\sqrt{s}}{M}\,(r\cdot r)\,\bar{E}_{\pm}\Big[\big((\bar{r}\cdot r)\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,7} - E_{\pm}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\nu}\mu\nu}_{\mp,7}\big) - \sqrt{s}\,E_{\pm}\big((\bar{r}\cdot r)\,\bar{P}^{\bar{\nu}\mu\nu}_{\mp,3} - E_{\mp}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,3}\big)\Big]\\
&\quad +\tfrac{1}{2}(\delta-1)\,\frac{s}{M}\,E_{\pm}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\nu}\mu\nu}_{\mp,1} \pm \sqrt{s}\,(\bar{r}\cdot r)\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,1} + \frac{1}{M}\,E_{\pm}\Big[s\,E_{\mp}\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,3} - \sqrt{s}\,\bar{P}^{\bar{\nu}\mu\nu}_{\mp,7} + \bar{E}_{\pm}\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,5}\Big]\,,\\
\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,2} &= -2\,\frac{s}{v^2}\,E_{\mp}\,\bar{E}_{\pm}\Big[(\bar{r}\cdot r)\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,9} - E_{\pm}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\nu}\mu\nu}_{\mp,9}\Big] + \bar{P}^{\bar{\nu}\mu\nu}_{\mp,9}\\
&\quad \pm\tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\,\bar{E}_{\pm}\Big[\big((\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,1} + E_{\mp}\,\bar{E}_{\mp}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,1}\big) \mp E_{\mp}\big((\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,5} - E_{\pm}\,\bar{E}_{\mp}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,5}\big)\Big] + \tfrac{1}{2}(\delta+1)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,5}\,,\\
\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,3} &= -2\,\frac{s}{v^2}\,E_{\mp}\,\bar{E}_{\pm}\Big[(\bar{r}\cdot r)\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,5} - E_{\pm}\,\bar{E}_{\mp}\,\bar{P}^{\bar{\nu}\mu\nu}_{\mp,5}\Big] + \bar{P}^{\bar{\nu}\mu\nu}_{\mp,5}\\
&\quad \mp\frac{s}{v^2}\,E_{\mp}\Big[\big((\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,1} + E_{\pm}\,\bar{E}_{\pm}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,1}\big) \pm E_{\pm}\big((\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,5} - E_{\mp}\,\bar{E}_{\pm}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,5}\big)\Big]\,,\\
\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,4} &= \pm\frac{\sqrt{s}}{M}\,\big(\bar{P}^{\bar{\nu}\mu\nu}_{\mp,8} - \sqrt{s}\,E_{\mp}\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,4}\big) \mp \tfrac{1}{2}(\delta-1)(\bar{\delta}-1)\,\frac{\sqrt{s}}{M}\Big[\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,3} + \frac{s}{v^2}\,E_{\mp}\,\bar{E}_{\pm}\big((\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,3} + E_{\pm}\,\bar{E}_{\mp}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,3}\big)\Big]\\
&\quad \mp(\bar{\delta}-1)\,\frac{s}{v^2}\,E_{\mp}\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,1} + E_{\pm}\,\bar{E}_{\pm}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,1}\Big] \mp \frac{(\bar{r}\cdot r)}{M\,\sqrt{s}}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,3} \pm \frac{\bar{M}}{\sqrt{s}}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,3} \pm \frac{2}{\sqrt{s}}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,1}\,,\\
\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,5} &= \frac{\sqrt{s}}{E_{\pm}}\Big[(\bar{r}\cdot r)\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,1} - E_{\pm}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\nu}\mu\nu}_{\mp,1}\Big] \mp \frac{1}{E_{\pm}}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,1}\,,\\
\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,6} &= \pm\sqrt{s}\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,4} \pm \tfrac{1}{2}(\delta+1)\,\frac{\sqrt{s}}{v^2}\,\bar{M}\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,1} + E_{\pm}\,\bar{E}_{\pm}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,1}\Big]\\
&\quad \pm\tfrac{1}{2}(\delta-1)\,\frac{s}{v^2}\,\bar{E}_{\pm}\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,4} + E_{\pm}\,\bar{E}_{\mp}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,4}\Big] + \tfrac{1}{2}(\bar{\delta}-1)\,\frac{\sqrt{s}}{v^2}\,E_{\pm}\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,1} - E_{\mp}\,\bar{E}_{\pm}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,1}\Big]\\
&\quad \mp\bar{M}\,\frac{\sqrt{s}}{v^2}\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,1} + E_{\pm}\,\bar{E}_{\pm}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,1}\Big] + \frac{1}{s}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,1} \mp \frac{1}{\sqrt{s}}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,4}\,,\\
\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,7} &= \pm\sqrt{s}\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,3} \pm \tfrac{1}{2}(\delta-1)\,\frac{s}{v^2}\,\bar{E}_{\pm}\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,5} + E_{\pm}\,\bar{E}_{\mp}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,5}\Big] + \frac{\sqrt{s}}{v^2}\,\bar{E}_{\pm}\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,1} - E_{\pm}\,\bar{E}_{\mp}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,1}\Big] \mp \frac{1}{\sqrt{s}}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,5}\,,\\
\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,8} &= \pm\sqrt{s}\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,2} \mp \frac{s}{v^2}\,E_{\pm}\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,4} + E_{\mp}\,\bar{E}_{\mp}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,4}\Big] + \frac{\sqrt{s}}{v^2}\,E_{\pm}\Big[(\bar{r}\cdot r)\,\bar{Q}^{\bar{\nu}\mu\nu}_{\mp,1} - E_{\mp}\,\bar{E}_{\pm}\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,1}\Big]\,,\\
\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,9} &= \frac{s}{v^2}\Big[(\bar{r}\cdot r)\,\bar{P}^{\bar{\nu}\mu\nu}_{\mp,12} - E_{\mp}\,\bar{E}_{\pm}\,\bar{P}^{\bar{\nu}\mu\nu}_{\pm,12}\Big] + \tfrac{1}{4}(\delta-1)(\delta+1)\,\frac{s}{v^2}\,(\bar{r}\cdot\bar{r})\,\bar{Q}^{\bar{\nu}\mu\nu}_{\pm,4}
\end{aligned}$$

$$
\begin{aligned}
&+\tfrac{1}{2}(\bar\delta-1)\,\frac{\sqrt{s}}{v^2}\,E_\pm\left[(\bar r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\pm,2}+E_\mp\,\bar E_\pm\,\bar Q^{\bar\nu\mu\nu}_{\mp,2}\right]\pm\tfrac{1}{2}(\delta+1)\,\frac{1}{v^2}\left[s\,\bar E_\pm\,((\bar r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\mp,6}+E_\mp\,\bar E_\mp\,\bar Q^{\bar\nu\mu\nu}_{\pm,6})\right.\\
&\qquad\left.-\sqrt{s}\,\bar M\,((\bar r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\pm,2}+E_\mp\,\bar E_\pm\,\bar Q^{\bar\nu\mu\nu}_{\mp,2})-\bar E_\mp\,((\bar r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\pm,1}-M\,\bar M\,\frac{\bar E_\pm}{\bar E_\mp}\,\bar Q^{\bar\nu\mu\nu}_{\mp,1})\right]\\
&\pm\frac{\sqrt{s}}{v^2}\,\bar M\left[(\bar r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\pm,2}+E_\mp\,\bar E_\pm\,\bar Q^{\bar\nu\mu\nu}_{\mp,2}\right]-\frac{1}{s}\,\bar Q^{\bar\nu\mu\nu}_{\mp,2}\,,\\
\bar Q^{\bar\nu\mu\nu}_{\pm,10}&=\frac{s}{v^2}\left[(\bar r\cdot r)\bar P^{\bar\nu\mu\nu}_{\mp,11}-E_\mp\,\bar E_\pm\,\bar P^{\bar\nu\mu\nu}_{\pm,11}\right]+\tfrac{1}{4}(\delta-1)(\delta+1)\,\frac{s}{v^2}\,(\bar r\cdot\bar r)\bar Q^{\bar\nu\mu\nu}_{\pm,5}\\
&\pm\tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\,\bar E_\pm\left[((\bar r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\mp,7}+E_\mp\,\bar E_\mp\,\bar Q^{\bar\nu\mu\nu}_{\pm,7})-\frac{\bar E_\mp}{\sqrt{s}}\,\bar Q^{\bar\nu\mu\nu}_{\pm,1}\right]-\frac{\sqrt{s}}{v^2}\,\bar E_\pm\left[(\bar r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\mp,2}+E_\pm\,\bar E_\mp\,\bar Q^{\bar\nu\mu\nu}_{\pm,2}\right],\\
\bar Q^{\bar\nu\mu\nu}_{\pm,11}&=\frac{s}{v^2}\left[(\bar r\cdot r)\bar P^{\bar\nu\mu\nu}_{\mp,10}-E_\mp\,\bar E_\pm\,\bar P^{\bar\nu\mu\nu}_{\pm,10}\right]-\frac{\sqrt{s}}{v^2}\,E_\mp\left[(\bar r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\mp,2}+E_\pm\,\bar E_\mp\,\bar Q^{\bar\nu\mu\nu}_{\pm,2}\right]\\
&+\tfrac{1}{2}(\delta+1)\,\frac{s}{v^2}\left[((\bar r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\pm,4}\mp\frac{1}{\sqrt{s}}\,E_\mp\,\bar E_\mp\,\bar Q^{\bar\nu\mu\nu}_{\pm,1})\pm\bar E_\pm\,((\bar r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\mp,8}+E_\mp\,\bar E_\mp\,\bar Q^{\bar\nu\mu\nu}_{\pm,8})\right],\\
\bar Q^{\bar\nu\mu\nu}_{\pm,12}&=\frac{s}{v^2}\left[(\bar r\cdot r)\bar P^{\bar\nu\mu\nu}_{\mp,6}-E_\mp\,\bar E_\pm\,\bar P^{\bar\nu\mu\nu}_{\pm,6}\right]\mp\frac{s}{v^2}\,E_\mp\left[(\bar r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\pm,8}+E_\pm\,\bar E_\pm\,\bar Q^{\bar\nu\mu\nu}_{\mp,8}\right]\\
&-\frac{\sqrt{s}}{v^2}\,E_\mp\left[(\bar r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\mp,3}+E_\pm\,\bar E_\mp\,\bar Q^{\bar\nu\mu\nu}_{\pm,3}\right]+\frac{s}{v^2}\,(r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\pm,4}\mp\frac{\sqrt{s}}{v^2}\,(r\cdot r)\bar Q^{\bar\nu\mu\nu}_{\mp,1}\,,
\end{aligned}
$$

where

$$
\begin{aligned}
\bar P^{\bar\nu\mu\nu}_{\pm,1}&=-v^\mu\left[\bar r_\lrcorner{}^\nu\,r_\lrcorner{}^{\bar\nu}\,P_\pm+v^\nu\,v^{\bar\nu}\,(1/v^2)(s/v^2)E_\pm\,\bar E_\pm\,P_\mp+\bar r_\lrcorner{}^\nu\,v^{\bar\nu}\,(\sqrt{s}/v^2)\bar E_\pm\,P_\pm\,i\gamma_5-v^\nu\,r_\lrcorner{}^{\bar\nu}\,(\sqrt{s}/v^2)E_\pm\,P_\mp\,i\gamma_5\right]/v^2\,,\\
\bar P^{\bar\nu\mu\nu}_{\pm,2}&=-v^\mu\left[\bar r_\lrcorner{}^\nu\,w_\llcorner{}^{\bar\nu}\,P_\pm+v^\nu\,v^{\bar\nu}\,(1/v^2)(\sqrt{s}/v^2)E_\pm\,((\bar r\cdot r)P_\mp\pm\bar M\,E_\mp\,P_\pm)\right.\\
&\qquad\left.+\bar r_\lrcorner{}^\nu\,v^{\bar\nu}\,(1/v^2)((\bar r\cdot r)P_\pm\pm\bar M\,E_\pm\,P_\mp)i\gamma_5-v^\nu\,w_\llcorner{}^{\bar\nu}\,(\sqrt{s}/v^2)E_\pm\,P_\mp\,i\gamma_5\right]/v^2\,,\\
\bar P^{\bar\nu\mu\nu}_{\pm,3}&=-v^\mu\left[\bar w_\llcorner{}^\nu\,r_\lrcorner{}^{\bar\nu}\,P_\pm+v^\nu\,v^{\bar\nu}\,(1/v^2)(\sqrt{s}/v^2)\bar E_\pm\,((\bar r\cdot r)P_\mp\pm M\,\bar E_\mp\,P_\pm)\right.\\
&\qquad\left.+\bar w_\llcorner{}^\nu\,v^{\bar\nu}\,(\sqrt{s}/v^2)\bar E_\pm\,P_\pm\,i\gamma_5-v^\nu\,r_\lrcorner{}^{\bar\nu}\,(1/v^2)((\bar r\cdot r)P_\mp\pm M\,\bar E_\pm\,P_\pm)i\gamma_5\right]/v^2\,,\\
\bar P^{\bar\nu\mu\nu}_{\pm,4}&=-v^\mu\left[\bar w_\llcorner{}^\nu\,w_\llcorner{}^{\bar\nu}\,P_\pm+v^\nu\,v^{\bar\nu}\,((1/s)P_\mp+\tfrac{1}{4}(\delta-1)(\bar\delta-1)(s/v^2)E_\pm\,\bar E_\pm\,P_\mp\right.\\
&\qquad\quad\mp\tfrac{1}{2}(\delta-1)(\sqrt{s}/v^2)\bar M\,((\bar r\cdot r)P_\pm-E_\pm\,\bar E_\pm\,P_\mp)\mp\tfrac{1}{2}(\bar\delta-1)(\sqrt{s}/v^2)M\,((\bar r\cdot r)P_\pm-E_\pm\,\bar E_\pm\,P_\mp))/v^2\\
&\qquad\left.+\bar w_\llcorner{}^\nu\,v^{\bar\nu}\,(1/v^2)((\bar r\cdot r)P_\pm\pm\bar M\,E_\pm\,P_\mp)i\gamma_5-v^\nu\,w_\llcorner{}^{\bar\nu}\,(1/v^2)((\bar r\cdot r)P_\mp\pm M\,\bar E_\pm\,P_\pm)i\gamma_5\right]/v^2\,,\\
\bar P^{\bar\nu\mu\nu}_{\pm,5}&=\bar r_\lrcorner{}^\mu\left[\bar r_\lrcorner{}^\nu\,r_\lrcorner{}^{\bar\nu}\,P_\pm\,i\gamma_5+v^\nu\,v^{\bar\nu}\,(1/v^2)(s/v^2)(E_\pm\,\bar E_\mp\,P_\mp\,i\gamma_5)-\bar r_\lrcorner{}^\nu\,v^{\bar\nu}\,(\sqrt{s}/v^2)(\bar E_\mp\,P_\pm)+v^\nu\,r_\lrcorner{}^{\bar\nu}\,(\sqrt{s}/v^2)(E_\pm\,P_\mp)\right],\\
\bar P^{\bar\nu\mu\nu}_{\pm,6}&=\bar r_\lrcorner{}^\mu\left[\bar r_\lrcorner{}^\nu\,w_\llcorner{}^{\bar\nu}\,P_\pm\,i\gamma_5+v^\nu\,v^{\bar\nu}\,(1/v^2)(\sqrt{s}/v^2)E_\pm\,((\bar r\cdot r)P_\mp\mp\bar M\,E_\mp\,P_\pm)i\gamma_5\right.\\
&\qquad\left.-\bar r_\lrcorner{}^\nu\,v^{\bar\nu}\,(1/v^2)((\bar r\cdot r)P_\pm\mp\bar M\,E_\pm\,P_\mp)+v^\nu\,w_\llcorner{}^{\bar\nu}\,(\sqrt{s}/v^2)E_\pm\,P_\mp\right],\\
\bar P^{\bar\nu\mu\nu}_{\pm,7}&=\bar r_\lrcorner{}^\mu\left[\bar w_\llcorner{}^\nu\,r_\lrcorner{}^{\bar\nu}\,P_\pm\,i\gamma_5+v^\nu\,v^{\bar\nu}\,(1/v^2)(\sqrt{s}/v^2)\bar E_\mp\,((\bar r\cdot r)P_\mp\pm M\,\bar E_\pm\,P_\pm)i\gamma_5\right.\\
&\qquad\left.-\bar w_\llcorner{}^\nu\,v^{\bar\nu}\,(\sqrt{s}/v^2)\bar E_\mp\,P_\pm+v^\nu\,r_\lrcorner{}^{\bar\nu}\,(1/v^2)((\bar r\cdot r)P_\mp\pm M\,\bar E_\mp\,P_\pm)\right],\\
\bar P^{\bar\nu\mu\nu}_{\pm,8}&=\bar r_\lrcorner{}^\mu\left[\bar w_\llcorner{}^\nu\,w_\llcorner{}^{\bar\nu}\,P_\pm\,i\gamma_5+v^\nu\,v^{\bar\nu}\,((1/s)P_\mp+\tfrac{1}{4}(\delta-1)(\bar\delta-1)(s/v^2)E_\pm\,\bar E_\mp\,P_\mp\right.\\
&\qquad\quad\pm\tfrac{1}{2}(\delta-1)(\sqrt{s}/v^2)\bar M\,((\bar r\cdot r)P_\pm-E_\pm\,\bar E_\mp\,P_\mp)\mp\tfrac{1}{2}(\bar\delta-1)(\sqrt{s}/v^2)M\,((\bar r\cdot r)P_\pm-E_\pm\,\bar E_\mp\,P_\mp))i\gamma_5/v^2\\
&\qquad\left.-\bar w_\llcorner{}^\nu\,v^{\bar\nu}\,(1/v^2)((\bar r\cdot r)P_\pm\mp\bar M\,E_\pm\,P_\mp)+v^\nu\,w_\llcorner{}^{\bar\nu}\,(1/v^2)((\bar r\cdot r)P_\mp\pm M\,\bar E_\mp\,P_\pm)\right],\\
\bar P^{\bar\nu\mu\nu}_{\pm,9}&=\bar w_\llcorner{}^\mu\left[\bar r_\lrcorner{}^\nu\,r_\lrcorner{}^{\bar\nu}\,P_\pm\,i\gamma_5+v^\nu\,v^{\bar\nu}\,(1/v^2)(s/v^2)(E_\pm\,\bar E_\mp\,P_\mp\,i\gamma_5)-\bar r_\lrcorner{}^\nu\,v^{\bar\nu}\,(\sqrt{s}/v^2)(\bar E_\mp\,P_\pm)+v^\nu\,r_\lrcorner{}^{\bar\nu}\,(\sqrt{s}/v^2)(E_\pm\,P_\mp)\right],\\
\bar P^{\bar\nu\mu\nu}_{\pm,10}&=\bar w_\llcorner{}^\mu\left[\bar r_\lrcorner{}^\nu\,w_\llcorner{}^{\bar\nu}\,P_\pm\,i\gamma_5+v^\nu\,v^{\bar\nu}\,(1/v^2)(\sqrt{s}/v^2)E_\pm\,((\bar r\cdot r)P_\mp\mp\bar M\,E_\mp\,P_\pm)i\gamma_5\right.\\
&\qquad\left.-\bar r_\lrcorner{}^\nu\,v^{\bar\nu}\,(1/v^2)((\bar r\cdot r)P_\pm\mp\bar M\,E_\pm\,P_\mp)+v^\nu\,w_\llcorner{}^{\bar\nu}\,(\sqrt{s}/v^2)E_\pm\,P_\mp\right],\\
\bar P^{\bar\nu\mu\nu}_{\pm,11}&=\bar w_\llcorner{}^\mu\left[\bar w_\llcorner{}^\nu\,r_\lrcorner{}^{\bar\nu}\,P_\pm\,i\gamma_5+v^\nu\,v^{\bar\nu}\,(1/v^2)(\sqrt{s}/v^2)\bar E_\mp\,((\bar r\cdot r)P_\mp\pm M\,\bar E_\pm\,P_\pm)i\gamma_5\right.\\
&\qquad\left.-\bar w_\llcorner{}^\nu\,v^{\bar\nu}\,(\sqrt{s}/v^2)\bar E_\mp\,P_\pm+v^\nu\,r_\lrcorner{}^{\bar\nu}\,(1/v^2)((\bar r\cdot r)P_\mp\pm M\,\bar E_\mp\,P_\pm)\right],\\
\bar P^{\bar\nu\mu\nu}_{\pm,12}&=\bar w_\llcorner{}^\mu\left[\bar w_\llcorner{}^\nu\,w_\llcorner{}^{\bar\nu}\,P_\pm\,i\gamma_5+v^\nu\,v^{\bar\nu}\,((1/s)P_\mp+\tfrac{1}{4}(\delta-1)(\bar\delta-1)(s/v^2)E_\pm\,\bar E_\mp\,P_\mp\right.\\
&\qquad\quad\pm\tfrac{1}{2}(\delta-1)(\sqrt{s}/v^2)\bar M\,((\bar r\cdot r)P_\pm-E_\pm\,\bar E_\mp\,P_\mp)\mp\tfrac{1}{2}(\bar\delta-1)(\sqrt{s}/v^2)M\,((\bar r\cdot r)P_\pm-E_\pm\,\bar E_\mp\,P_\mp))i\gamma_5/v^2
\end{aligned}
$$

$$- \bar{w}_{\lfloor}{}^{\nu}\, v^{\bar{\nu}}\,(1/v^2)\big((\bar{r}\cdot r)\,P_{\pm} \mp \bar{M}\,E_{\pm}\,P_{\mp}\big) + v^{\nu}\,w_{\lfloor}{}^{\bar{\nu}}\,(1/v^2)\big((\bar{r}\cdot r)\,P_{\mp} \pm M\,\bar{E}_{\mp}\,P_{\pm}\big)\Big]\,. \tag{B.19}$$

# C Theory of angular momentum

## C.1 Wigner function

The Wigner function $d^{(J)}_{m,n}(\theta)$ satisfies the orthogonality and the completeness relation respectively as follows

$$(2J+1)\int_{-1}^{1}\frac{\mathrm{d}\cos\theta}{2}\,d^{(J)}_{m,n}(\theta)\,d^{(J')}_{m,n}(\theta)=\delta_{JJ'}\,, \tag{C.1}$$

$$\sum_J (2J+1)\,d^{(J)}_{m,n}(\theta)\,d^{(J)}_{m,n}(\theta')=2\,\delta(\cos\theta-\cos\theta')\,. \tag{C.2}$$

It relates to the Legendre polynomial as

$$d^{(l)}_{0,0}(\theta)=P_l(\cos\theta)\,,\qquad d^{(l)}_{1,0}(\theta)=-\frac{\sin\theta}{\sqrt{l(l+1)}}P'_l(\cos\theta)\,, \tag{C.3}$$

where $P'_l(z)=\mathrm{d}P_l(z)/\mathrm{d}z$. Using (C.1), (C.2), and (C.3), we recover the orthogonality and the completeness relations of the Legendre polynomials.

The differential equation that defines the Wigner functions is given in references [133, 134, 86] as

$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)-\frac{1}{\sin^2\theta}\left(m^2+n^2-2\,m\,n\,\cos\theta\right)+J\,(J+1)\right]\,d^{(J)}_{m,n}(\theta)=0\,. \tag{C.4}$$

Its general solution may be taken as

$$d^{(J)}_{m,n}(\theta)=N_F^{Jmn}\left[\cos\frac{\theta}{2}\right]^{n+m}\left[\sin\frac{\theta}{2}\right]^{n-m}F(-J+n,J+n+1,n-m+1;\frac{1-\cos\theta}{2})\,, \tag{C.5}$$

where $F(a,b,c;z)$ is the hypergeometric function with some normalization factor $N_F^{Jmn}$. Alternatively the Jacobi polynomial $P_n^{\alpha,\beta}(z)$ may be used[1]:

$$P_n^{\alpha,\beta}(x)=\binom{n+\alpha}{n}\,F(-n,n+\alpha+\beta+1,\alpha+1;\frac{1-x}{2})\,, \tag{C.6}$$

in terms of which the Wigner functions take the form

$$d^{(J)}_{m,n}(\theta)=\sqrt{\frac{\Gamma(J+n+1)\,\Gamma(J-n+1)}{\Gamma(J+m+1)\,\Gamma(J-m+1)}}\left[\sin\frac{\theta}{2}\right]^{n-m}\left[\cos\frac{\theta}{2}\right]^{n+m}P_{J-n}^{n-m,n+m}(\cos\theta)\,. \tag{C.7}$$

The normalization factor is determined by the condition (C.1) and a property of the Jacobi polynomials[2]:

$$\int_{-1}^{1}\frac{\mathrm{d}\cos\theta}{2}\left[\sin\frac{\theta}{2}\right]^{\alpha}\left[\cos\frac{\theta}{2}\right]^{\beta}P_m^{\alpha,\beta}(\cos\theta)\,P_n^{\alpha,\beta}(\cos\theta)=\frac{\delta_{mn}}{2\,n+\alpha+\beta+1}\,\frac{\Gamma(n+\alpha+1)\,\Gamma(n+\beta+1)}{\Gamma(n+1)\,\Gamma(n+\alpha+\beta+1)}\,. \tag{C.8}$$

1 See Eq. *(22.5.42)* in [135].
2 See Eqs. *(22.1.1)*, *(22.1.2)*, and *(22.2.1)* in [135].

We provide some specific example cases with $J=\frac{1}{2}$, 1, and $\frac{3}{2}$. It holds

$$d^{(\frac{1}{2})}_{m,n}=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix},\quad d^{(1)}_{m,n}=\begin{pmatrix}\frac{1+\cos\theta}{2} & -\frac{\sin\theta}{\sqrt{2}} & \frac{1-\cos\theta}{2}\\ \frac{\sin\theta}{\sqrt{2}} & \cos\theta & -\frac{\sin\theta}{\sqrt{2}}\\ \frac{1-\cos\theta}{2} & \frac{\sin\theta}{\sqrt{2}} & \frac{1+\cos\theta}{2}\end{pmatrix},$$

$$d^{(\frac{3}{2})}_{m,n}=\begin{pmatrix}\frac{3}{4}\cos\frac{\theta}{2}+\frac{1}{4}\cos\frac{3\theta}{2} & -\frac{\sqrt{3}}{4}\sin\frac{\theta}{2}-\frac{\sqrt{3}}{4}\sin\frac{3\theta}{2} & \frac{\sqrt{3}}{4}\cos\frac{\theta}{2}-\frac{\sqrt{3}}{4}\cos\frac{3\theta}{2} & \frac{1}{4}\sin\frac{3\theta}{2}-\frac{3}{4}\sin\frac{\theta}{2}\\ \frac{\sqrt{3}}{4}\sin\frac{\theta}{2}+\frac{\sqrt{3}}{4}\sin\frac{3\theta}{2} & \frac{1}{4}\cos\frac{\theta}{2}+\frac{3}{4}\cos\frac{3\theta}{2} & \frac{1}{4}\sin\frac{\theta}{2}-\frac{3}{4}\sin\frac{3\theta}{2} & \frac{\sqrt{3}}{4}\cos\frac{\theta}{2}-\frac{\sqrt{3}}{4}\cos\frac{3\theta}{2}\\ \frac{\sqrt{3}}{4}\cos\frac{\theta}{2}-\frac{\sqrt{3}}{4}\cos\frac{3\theta}{2} & \frac{3}{4}\sin\frac{3\theta}{2}-\frac{1}{4}\sin\frac{\theta}{2} & \frac{1}{4}\cos\frac{\theta}{2}+\frac{3}{4}\cos\frac{3\theta}{2} & -\frac{\sqrt{3}}{4}\sin\frac{\theta}{2}-\frac{\sqrt{3}}{4}\sin\frac{3\theta}{2}\\ \frac{3}{4}\sin\frac{\theta}{2}-\frac{1}{4}\sin\frac{3\theta}{2} & \frac{\sqrt{3}}{4}\cos\frac{\theta}{2}-\frac{\sqrt{3}}{4}\cos\frac{3\theta}{2} & \frac{\sqrt{3}}{4}\sin\frac{\theta}{2}+\frac{\sqrt{3}}{4}\sin\frac{3\theta}{2} & \frac{3}{4}\cos\frac{\theta}{2}+\frac{1}{4}\cos\frac{3\theta}{2}\end{pmatrix}. \tag{C.9}$$

It is very useful to recall the *addition theorem for the Wigner functions*

$$\begin{aligned}d^{(j)}_{m,n}(\theta)\,d^{(j')}_{m',n'}(\theta)&=(-1)^{m'-n'}\,d^{(j)}_{m,n}(\theta)\,d^{(j')}_{-m',-n'}(\theta)\\&=\sum_{J=|j-j'|}^{j+j'}(-1)^{m-n}(2J+1)\begin{pmatrix}j & j' & J\\ -m & m' & m-m'\end{pmatrix}\begin{pmatrix}j & j' & J\\ -n & n' & n-n'\end{pmatrix}d^{(J)}_{m-m',n-n'}(\theta),\end{aligned}\tag{C.10}$$

where we use Wigner three-j symbol as implemented in Mathematica 8 with ThreeJSymbol. We provide some useful applications of Eq. (C.10). For example, if we consider

$$\sqrt{J+\frac{1}{2}}\,d^{(J-\frac{1}{2})}_{0,0}(\theta)\,d^{(\frac{1}{2})}_{\frac{1}{2},\frac{1}{2}}(\theta)-\sqrt{J-\frac{1}{2}}\,d^{(J-\frac{1}{2})}_{1,0}(\theta)\,d^{(\frac{1}{2})}_{\frac{1}{2},-\frac{1}{2}}(\theta)\tag{C.11}$$

and

$$-\sqrt{J-\frac{1}{2}}\,d^{(J-\frac{1}{2})}_{1,0}(\theta)\,d^{(\frac{1}{2})}_{\frac{1}{2},\frac{1}{2}}(\theta)-\sqrt{J+\frac{1}{2}}\,d^{(J-\frac{1}{2})}_{0,0}(\theta)\,d^{(\frac{1}{2})}_{\frac{1}{2},-\frac{1}{2}}(\theta),\tag{C.12}$$

then we obtain the following two relations

$$\begin{aligned}\sqrt{J+\frac{1}{2}}\,d^{(J)}_{+\frac{1}{2},\frac{1}{2}}(\theta)&=\sqrt{J+\frac{1}{2}}\cos\frac{\theta}{2}\,d^{(J-\frac{1}{2})}_{0,0}(\theta)+\sqrt{J-\frac{1}{2}}\sin\frac{\theta}{2}\,d^{(J-\frac{1}{2})}_{1,0}(\theta),\\ \sqrt{J+\frac{1}{2}}\,d^{(J)}_{-\frac{1}{2},\frac{1}{2}}(\theta)&=\sqrt{J-\frac{1}{2}}\cos\frac{\theta}{2}\,d^{(J-\frac{1}{2})}_{-1,0}(\theta)+\sqrt{J+\frac{1}{2}}\sin\frac{\theta}{2}\,d^{(J-\frac{1}{2})}_{0,0}(\theta),\end{aligned}\tag{C.13}$$

where the property of Wigner functions (4.15) is used, if necessary. Applying the *addition theorem for the Wigner functions* with $d^{(J)}_{\frac{1}{2},\frac{1}{2}}(\theta)\,d^{(\frac{1}{2})}_{\frac{1}{2},\frac{1}{2}}(\theta)$ and $d^{(J)}_{\frac{1}{2},-\frac{1}{2}}(\theta)\,d^{(\frac{1}{2})}_{\frac{1}{2},-\frac{1}{2}}(\theta)$, we obtain Eq. (4.25).

It may be also useful to recall the recurrence relations of Legendre polynomials:

$$\begin{aligned}&(l+1)\,P_{l+1}(z)=(2l+1)\,z\,P_l(z)-l\,P_{l-1}(z),\\&(z^2-1)\,P_l'(z)=l\,z\,P_l(z)-l\,P_{l-1}(z)=(l+1)\left[P_{l+1}(z)-z\,P_l(z)\right],\\&(z\mp 1)\left[P_{l+1}'(z)\pm P_l'(z)\right]=(l+1)\left[P_{l+1}(z)\mp P_l(z)\right],\end{aligned}\tag{C.14}$$

and their integral relation:

$$(2n+1)\int_{-1}^{1}\frac{dx}{2}\,x\,P_m(x)\,P_n(x)=\frac{m+1}{2n+1}\delta_{n,m+1}+\frac{m}{2n+1}\delta_{n,m-1}.\tag{C.15}$$

## C.2 Auxiliary helicity amplitudes

We provide convenient expressions for the helicity partial-wave amplitudes $t^J_\pm$ introduced in Eq. (4.20). We recall

$$[t^J_\pm]_{\bar n n} = \langle \bar n_\pm, J|T_J|n_\pm, J\rangle = \langle \lambda_{\bar q}, \lambda_{\bar p}|T_J|\lambda_q, \lambda_p\rangle \pm (-1)^{\frac{1}{2}-S_q-S_p}\langle \lambda_{\bar q}, \lambda_{\bar p}|T_J| - \lambda_q, -\lambda_p\rangle$$
$$= \int_{-1}^{1}\frac{\mathrm{d}\cos\theta}{2}\, d^{(J)}_{\lambda,\bar\lambda}(\theta)\,\langle \lambda_{\bar q}\lambda_{\bar p}|T|\lambda_q\lambda_p\rangle \pm (-1)^{\frac{1}{2}-S_q-S_p}\int_{-1}^{1}\frac{\mathrm{d}\cos\theta}{2}\, d^{(J)}_{-\lambda,\bar\lambda}(\theta)\,\langle \lambda_{\bar q}\lambda_{\bar p}|T| - \lambda_q - \lambda_p\rangle\,, \tag{C.16}$$

where $\lambda = \lambda_q - \lambda_p$ and $\bar\lambda = \lambda_{\bar q} - \lambda_{\bar p}$ are set to the values implied by the given state $|n_\pm, J\rangle$. In Eq. (C.16) we used the parity relation Eq. (4.18). We introduce auxiliary helicity amplitudes with

$$2\,\phi^\pm_{\bar n n} = \frac{1}{d^{(J_0)}_{\lambda,\bar\lambda}(\theta)}\,\langle \lambda_{\bar q}\lambda_{\bar p}|T|\lambda_q\lambda_p\rangle \pm \frac{(-1)^{\frac{1}{2}-S_q-S_p}}{d^{(J_0)}_{-\lambda,\bar\lambda}(\theta)}\,\langle \lambda_{\bar q}\lambda_{\bar p}|T| - \lambda_q - \lambda_p\rangle\,. \tag{C.17}$$

In application of the *addition theorem for the Wigner functions* (C.10) the $t^J_\pm$ can be expressed conveniently in terms of the $\phi^\pm_{\bar n n}$, Legendre polynomials, and Wigner three-j symbols [49]. It holds

$$\left[t^J_\pm\right]_{\bar n n} = \int_{-1}^{1}\frac{\mathrm{d}\cos\theta}{2}\left[\left(\phi^+_{\bar n n} + \phi^-_{\bar n n}\right) d^{(J_0)}_{\lambda,\bar\lambda}(\theta)\, d^{(J)}_{\lambda,\bar\lambda}(\theta) \pm \left(\phi^+_{\bar n n} - \phi^-_{\bar n n}\right) d^{(J_0)}_{-\lambda,\bar\lambda}(\theta)\, d^{(J)}_{-\lambda,\bar\lambda}(\theta)\right]$$
$$= \sum_{j=|J-J_0|}^{J+J_0}(2\,j+1)\begin{pmatrix} J_0 & J & j \\ -\lambda & \lambda & 0\end{pmatrix}\begin{pmatrix} J_0 & J & j \\ -\bar\lambda & \bar\lambda & 0\end{pmatrix}$$
$$\times\int_{-1}^{1}\frac{\mathrm{d}\cos\theta}{2}\,P_j(\cos\theta)\left[(-1)^{\lambda-\bar\lambda}\left(\phi^+_{\bar n n} + \phi^-_{\bar n n}\right) \pm (-1)^{\lambda+\bar\lambda+J_0+J+j}\left(\phi^+_{\bar n n} - \phi^-_{\bar n n}\right)\right], \tag{C.18}$$

where $J_0 = \mathrm{Max}(|\lambda|, |\bar\lambda|)$ and the lower and upper bounds of the summation are determined by the properties of the Wigner three-j symbols.

## C.3 Helicity projection operators

We recall that the representations of the Poincaré group are classified according to the values of two Casimir operators $P^2 = P_\mu P^\mu$ and $W^2 = W_\mu W^\mu$. The energy-momentum operator $P^\mu$ is the infinitesimal generator of translations and the Pauli-Lubanski operator $W^\mu$ is constructed from the angular momentum operator $J^{\mu\nu}$, the infinitesimal generator of Lorentz transformations, as

$$W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\,J_{\nu\rho}\,P_\sigma\,. \tag{C.19}$$

For any given representation of the Poincaré algebra the eigenvalue of the second Casimir operator $W^2$ take the form

$$W^2 = -P^2\,S\,(S+1) \tag{C.20}$$

where the spin $S$ is integer or half integer. The general definition of the helicity projection operator referring to [72, 86] is

$$\frac{n_p\cdot W}{\sqrt{p^2}} \qquad \text{with} \qquad n^\mu_p = -\frac{1}{\sqrt{p^2}}\left(|\vec p|, \frac{p^0}{|\vec p|}\vec p\right), \tag{C.21}$$

in terms of the Pauli-Lubanski vector and $n_p^\mu$ a normalized space-like four-vector orthogonal to $p_\mu$.

For the case $S = \frac{1}{2}$ we may identify

$$J^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]_- = \frac{i}{4}(\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu), \tag{C.22}$$

in Eq. (C.19). There are different conventions used for the representations of Dirac gamma matrices in the literature. In this work the Pauli-Dirac representation is applied for the Dirac matrices with

$$\gamma^0 = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix}, \tag{C.23}$$

where $\vec{\sigma} = \{\sigma^1, \sigma^2, \sigma^3\}$ is the set of the Pauli matrices. The $\gamma$ matrices satisfy the Clifford algebra

$$[\gamma^\mu, \gamma^\nu]_+ = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\, g^{\mu\nu}, \tag{C.24}$$

where $g^{\mu\nu}$ is the metric tensor with positive signature $(+,-,-,-)$. An explicit computation confirms that the particular representation carries spin-one-half. The second Casimir operators takes the form

$$W^2 = W_\mu W^\mu = -\frac{3}{4} P^2\, \mathbb{1}_4 . \tag{C.25}$$

For Dirac spinors, the helicity projection operator takes the simple form

$$\frac{n_p \cdot W}{\sqrt{p^2}} = \frac{1}{2} \frac{\vec{\Sigma} \cdot \vec{p}}{|\vec{p}|} \qquad \text{with} \quad \vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}. \tag{C.26}$$

We turn to the case with $S = 1$ where we may use

$$J^{\mu\nu}_{\alpha\beta} = i\,(g^\mu{}_\alpha\, g^\nu{}_\beta - g^\mu{}_\beta\, g^\nu{}_\alpha), \tag{C.27}$$

in Eq. (C.19). Note that the spin one realization of the Poincare algebra leads to a 4-dimensional matrix structure. The latter can be efficiently implemented by assigning an additional pair of Lorentz indices to it. For the case of an upper and a lower Lorentz index the original matrix structure is recovered. Given this notation the Pauli-Lubanski vector becomes $[W^\mu]^{\alpha\beta} = -i\, \epsilon^{\mu\alpha\beta[p]}$, where $p$ is momentum of the spin-one state. In the center-of-mass frame with $p^\mu = \{E_{\rm cm}, p_{\rm cm} \sin\theta, 0, p_{\rm cm} \cos\theta\}$, we can write the helicity projection operator as

$$\frac{n_p^\alpha\, [W_\alpha]^\mu{}_\nu}{\sqrt{p^2}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & i\cos\theta & 0 \\ 0 & -i\cos\theta & 0 & i\sin\theta \\ 0 & 0 & -i\sin\theta & 0 \end{pmatrix}_{\mu\nu}, \tag{C.28}$$

where we use two lower indices on the r.h.s. of Eq. (C.28) as required by the applied convention.

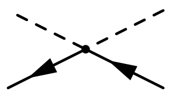
| $[C_{\mathrm{WT}}^{(1,1)}]_{a,b}$ | $[C_{\mathrm{WT}}^{(1/2,0)}]_{a,b}$ | $[C_{\mathrm{WT}}^{(3/2,0)}]_{a,b}$ | $[C_{\mathrm{WT}}^{(0,-1)}]_{a,b}$ | $[C_{\mathrm{WT}}^{(1,-1)}]_{a,b}$ |
|---|---|---|---|---|
| (1,1): $-2$ | (1,1): $2$<br>(1,3): $-\frac{3}{2}$<br>(1,4): $\frac{1}{2}$<br>(2,3): $-\frac{3}{2}$<br>(2,4): $-\frac{3}{2}$<br>(4,4): $2$ | (1,1): $-1$<br>(1,2): $-1$<br>(2,2): $-1$ | (1,1): $4$<br>(1,2): $\sqrt{\frac{3}{2}}$<br>(1,4): $-\sqrt{\frac{3}{2}}$<br>(2,2): $3$<br>(2,3): $\frac{3}{\sqrt{2}}$<br>(3,4): $-\frac{3}{\sqrt{2}}$<br>(4,4): $3$ | (1,3): $\sqrt{\frac{3}{2}}$<br>(1,5): $\sqrt{\frac{3}{2}}$<br>(2,2): $2$<br>(2,3): $1$<br>(2,5): $-1$<br>(3,3): $1$<br>(3,4): $\sqrt{\frac{3}{2}}$<br>(4,5): $\sqrt{\frac{3}{2}}$<br>(5,5): $1$ |
| $[C_{\mathrm{WT}}^{(2,-1)}]_{a,b}$ | $[C_{\mathrm{WT}}^{(1/2,-2)}]_{a,b}$ | $[C_{\mathrm{WT}}^{(3/2,-2)}]_{a,b}$ | $[C_{\mathrm{WT}}^{(1,-3)}]_{a,b}$ | |
| (1,1): $-2$ | (1,1): $2$<br>(1,2): $\frac{3}{2}$<br>(1,3): $\frac{1}{2}$<br>(2,4): $-\frac{3}{2}$<br>(3,3): $2$<br>(3,4): $\frac{3}{2}$ | (1,1): $-1$<br>(1,2): $-1$<br>(2,2): $-1$ | (1,1): $-2$ | |

**Table D.1.:** Coefficients $[C_{\mathrm{WT}}^{(I,S)}]_{b,a} = [C_{\mathrm{WT}}^{(I,S)}]_{a,b}$ specifying the contribution of Weinberg-Tomozawa term to $PB \to PB$. Only non-zero elements are shown.

# D Tree-level scattering kernel

In this appendix, we collect the expressions for the invariant amplitudes $F_1^{\pm}$ introduced in (2.32). For the tree-level scattering kernel (5.11), each of them can be calculated by the on-shell projection algebra (3.2). We shall write only $F_1^+(\sqrt{s})$ because $F_1^-(\sqrt{s}) = F_1^+(-\sqrt{s})$ under the MacDowell symmetry as discussed in Section 2.2.

## D.1 Weinberg-Tomozawa

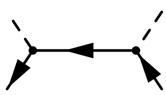


$$= \frac{C_{\mathrm{WT}}^{(I,S)}}{4f^2}(\bar{\not{q}} + \not{q}), \tag{D.1}$$

$$F_1^+ = \frac{C_{\mathrm{WT}}^{(I,S)}(2\sqrt{s} - \bar{M} - M)}{4f^2}, \tag{D.2}$$

where the coupling coefficients $[C_{\mathrm{WT}}^{(I,S)}]_{a,b} = [C_{\mathrm{WT}}^{(I,S)}]_{b,a}$ are collected in Tab. D.1.

## D.2 Octet baryon exchanges



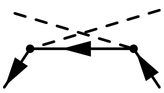
$$= \sum_{[8]} \frac{1}{4f^2} C_{[8],AA}^{(I,S)} \gamma_5 \bar{\not{q}} S_{[8]}(p+q) \not{q} \gamma_5, \tag{D.3}$$

$$F_1^+ = \sum_{[8]} \frac{C_{[8],AA}^{(I,S)}(\sqrt{s} - M)(\bar{M} - \sqrt{s})}{4f^2(M_{[8]} + \sqrt{s})}, \tag{D.4}$$

| | $[C^{(1,-1)}_{\Sigma,AA}]_{a,b}$ | | $[C^{(1/2,0)}_{N,AA}]_{a,b}$ |
|---|---|---|---|
| (1,1): | $\frac{4D_A^2}{3}$ | (1,1): | $3(D_A+F_A)^2$ |
| (1,2): | $-4\sqrt{\frac{2}{3}}D_AF_A$ | (1,2): | $-(D_A-3F_A)(D_A+F_A)$ |
| (1,3): | $2\sqrt{\frac{2}{3}}D_A(D_A-F_A)$ | (1,3): | $-(D_A+F_A)(D_A+3F_A)$ |
| (1,4): | $\frac{4D_A^2}{3}$ | (1,4): | $3(D_A-F_A)(D_A+F_A)$ |
| (1,5): | $2\sqrt{\frac{2}{3}}D_A(D_A+F_A)$ | (2,2): | $\frac{1}{3}(D_A-3F_A)^2$ |
| (2,2): | $8F_A^2$ | (2,3): | $\frac{1}{3}(D_A-3F_A)(D_A+3F_A)$ |
| (2,3): | $-4F_A(D_A-F_A)$ | (2,4): | $-(D_A-3F_A)(D_A-F_A)$ |
| (2,4): | $-4\sqrt{\frac{2}{3}}D_AF_A$ | (3,3): | $\frac{1}{3}(D_A+3F_A)^2$ |
| (2,5): | $-4F_A(D_A+F_A)$ | (3,4): | $-(D_A-F_A)(D_A+3F_A)$ |
| (3,3): | $2(D_A-F_A)^2$ | (4,4): | $3(D_A-F_A)^2$ |
| (3,4): | $2\sqrt{\frac{2}{3}}D_A(D_A-F_A)$ | | |
| (3,5): | $2(D_A-F_A)(D_A+F_A)$ | | |
| (4,4): | $\frac{4D_A^2}{3}$ | | |
| (4,5): | $2\sqrt{\frac{2}{3}}D_A(D_A+F_A)$ | | |
| (5,5): | $2(D_A+F_A)^2$ | | |
| | $[C^{(1/2,-2)}_{\Xi,AA}]_{a,b}$ | | $[C^{(0,-1)}_{\Lambda,AA}]_{a,b}$ |
| (1,1): | $3(D_A-F_A)^2$ | (1,1): | $4D_A^2$ |
| (1,2): | $(D_A-3F_A)(D_A-F_A)$ | (1,2): | $-2\sqrt{\frac{2}{3}}D_A(D_A+3F_A)$ |
| (1,3): | $3(D_A-F_A)(D_A+F_A)$ | (1,3): | $-\frac{4D_A^2}{\sqrt{3}}$ |
| (1,4): | $(D_A-F_A)(D_A+3F_A)$ | (1,4): | $2\sqrt{\frac{2}{3}}D_A(D_A-3F_A)$ |
| (2,2): | $\frac{1}{3}(D_A-3F_A)^2$ | (2,2): | $\frac{2}{3}(D_A+3F_A)^2$ |
| (2,3): | $(D_A-3F_A)(D_A+F_A)$ | (2,3): | $\frac{2}{3}\sqrt{2}D_A(D_A+3F_A)$ |
| (2,4): | $\frac{1}{3}(D_A-3F_A)(D_A+3F_A)$ | (2,4): | $-\frac{2}{3}(D_A-3F_A)(D_A+3F_A)$ |
| (3,3): | $3(D_A+F_A)^2$ | (3,3): | $\frac{4D_A^2}{3}$ |
| (3,4): | $(D_A+F_A)(D_A+3F_A)$ | (3,4): | $-\frac{2}{3}\sqrt{2}D_A(D_A-3F_A)$ |
| (4,4): | $\frac{1}{3}(D_A+3F_A)^2$ | (4,4): | $\frac{2}{3}(D_A-3F_A)^2$ |

**Table D.2.:** Matrix elements $[C^{(I,S)}_{[8],AA}]_{b,a} = [C^{(I,S)}_{[8],AA}]_{a,b}$ describing the s-channel octet baryon exchange. Vanishing elements are not shown.

where the matrix elements $[C^{(I,S)}_{[8],AA}]_{a,b} = [C^{(I,S)}_{[8],AA}]_{b,a}$ are collected in Tab. D.2.

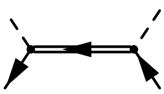


$$= \sum_{[8]} \frac{1}{4f^2}\, \tilde{C}^{(I,S)}_{[8],AA}\, \gamma_5 \not{q}\, S_{[8]}(p-\bar{q}) \not{\bar{q}}\, \gamma_5\,, \tag{D.5}$$

$$F_1^+ = \frac{\tilde{C}^{(I,S)}_{[8],AA}\left(M_{[8]}[\bar{M}^2 - \sqrt{s}(\bar{M}+M) + \bar{M}M + M^2 - u] + \bar{M}M(\bar{M}+M-\sqrt{s}) - \sqrt{s}u\right)}{4f^2(M^2_{[8]} - u)} \tag{D.6}$$

where the coupling coefficients $[\tilde{C}^{(I,S)}_{[8],AA}]_{a,b} = [\tilde{C}^{(I,S)}_{[8],AA}]_{b,a}$ are collected in Tab. D.3.

## D.3 Decuplet baryon exchanges



$$= -\sum_{[10]} \frac{1}{4f^2}\, C^{(I,S)}_{[10],AA}\, \bar{\Gamma}^{(A)}_{\rho}(\bar{q}) S^{\rho\sigma}_{[10]}(p+q) \Gamma^{(A)}_{\sigma}(q)\,, \tag{D.7}$$

| $(I, S)$ | $[\tilde{C}^{(I,S)}_{\Sigma,AA}]_{a,b}$ | $[\tilde{C}^{(I,S)}_{N,AA}]_{a,b}$ |
|---|---|---|
| (0, 1) | (1,1): $3(D_A-F_A)^2$ | |
| (1, 1) | (1,1): $(D_A-F_A)^2$ | |
| (1/2, 0) | (1,3): $2D_A(D_A-F_A)$<br>(1,4): $4F_A(D_A-F_A)$<br>(2,4): $2D_A(D_A-F_A)$ | (1,1): $-(D_A+F_A)^2$<br>(1,2): $-(D_A-3F_A)(D_A+F_A)$<br>(2,2): $\frac{1}{3}(D_A-3F_A)^2$ |
| (3/2, 0) | (1,2): $-2F_A(D_A-F_A)$ | (1,1): $2(D_A+F_A)^2$ |
| (0, -1) | (1,1): $-8F_A^2$<br>(1,3): $\frac{4D_A^2}{\sqrt{3}}$<br>(2,4): $-3(D_A-F_A)(D_A+F_A)$ | (1,2): $\sqrt{6}(D_A-F_A)(D_A+F_A)$<br>(2,3): $\frac{1}{3}\sqrt{2}(D_A-3F_A)(D_A+3F_A)$ |
| (1, -1) | (1,1): $\frac{4D_A^2}{3}$<br>(1,2): $4\sqrt{\frac{2}{3}}D_AF_A$<br>(2,2): $4F_A^2$<br>(2,4): $-4\sqrt{\frac{2}{3}}D_AF_A$<br>(3,5): $-(D_A-F_A)(D_A+F_A)$<br>(4,4): $\frac{4D_A^2}{3}$ | (1,3): $-\sqrt{\frac{2}{3}}(D_A+F_A)(D_A+3F_A)$<br>(2,3): $-2(D_A-F_A)(D_A+F_A)$<br>(3,4): $-\sqrt{\frac{2}{3}}(D_A-3F_A)(D_A-F_A)$ |
| (2, -1) | (1,1): $4F_A^2$ | |
| (1/2, -2) | (1,2): $-2D_A(D_A+F_A)$<br>(1,3): $-4F_A(D_A+F_A)$<br>(3,4): $-2D_A(D_A+F_A)$ | (2,2): $\frac{1}{3}(D_A+3F_A)^2$<br>(2,3): $(D_A-F_A)(D_A+3F_A)$<br>(3,3): $-(D_A-F_A)^2$ |
| (3/2, -2) | (1,2): $2F_A(D_A+F_A)$ | (2,2): $2(D_A-F_A)^2$ |
| (0, -3) | (1,1): $3(D_A+F_A)^2$ | |
| (1, -3) | (1,1): $(D_A+F_A)^2$ | |

| $(I, S)$ | $[\tilde{C}^{(I,S)}_{\Xi,AA}]_{a,b}$ | $[\tilde{C}^{(I,S)}_{\Lambda,AA}]_{a,b}$ |
|---|---|---|
| (0, 1) | | (1,1): $-\frac{1}{3}(D_A+3F_A)^2$ |
| (1, 1) | | (1,1): $\frac{1}{3}(D_A+3F_A)^2$ |
| (1/2, 0) | (3,3): $\frac{1}{3}(D_A-3F_A)^2$<br>(3,4): $-(D_A-3F_A)(D_A+F_A)$<br>(4,4): $-(D_A+F_A)^2$ | (1,4): $-\frac{2}{3}D_A(D_A+3F_A)$<br>(2,3): $\frac{2}{3}D_A(D_A+3F_A)$ |
| (3/2, 0) | (2,2): $2(D_A+F_A)^2$ | (1,2): $-\frac{2}{3}D_A(D_A+3F_A)$ |
| (0, -1) | (1,4): $-\sqrt{6}(D_A-F_A)(D_A+F_A)$<br>(3,4): $-\frac{1}{3}\sqrt{2}(D_A-3F_A)(D_A+3F_A)$ | (1,1): $\frac{4D_A^2}{3}$<br>(2,4): $-\frac{1}{3}(D_A-3F_A)(D_A+3F_A)$<br>(3,3): $\frac{4D_A^2}{3}$ |
| (1, -1) | (1,5): $-\sqrt{\frac{2}{3}}(D_A-3F_A)(D_A-F_A)$<br>(2,5): $2(D_A-F_A)(D_A+F_A)$<br>(4,5): $-\sqrt{\frac{2}{3}}(D_A+F_A)(D_A+3F_A)$ | (1,4): $-\frac{4D_A^2}{3}$<br>(2,2): $-\frac{4D_A^2}{3}$<br>(3,5): $\frac{1}{3}(D_A-3F_A)(D_A+3F_A)$ |
| (2, -1) | | (1,1): $\frac{4D_A^2}{3}$ |
| (1/2, -2) | (1,1): $-(D_A-F_A)^2$<br>(1,4): $(D_A-F_A)(D_A+3F_A)$<br>(4,4): $\frac{1}{3}(D_A+3F_A)^2$ | (1,3): $-\frac{2}{3}D_A(D_A-3F_A)$<br>(2,4): $\frac{2}{3}D_A(D_A-3F_A)$ |
| (3/2, -2) | (1,1): $2(D_A-F_A)^2$ | (1,2): $-\frac{2}{3}D_A(D_A-3F_A)$ |
| (0, -3) | | (1,1): $-\frac{1}{3}(D_A-3F_A)^2$ |
| (1, -3) | | (1,1): $\frac{1}{3}(D_A-3F_A)^2$ |

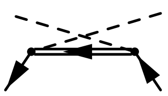
**Table D.3.:** Coefficients $[\tilde{C}^{(I,S)}_{[8],AA}]_{b,a} = [\tilde{C}^{(I,S)}_{[8],AA}]_{a,b}$ specifying the u-channel octet baryon exchange. Elements that are zero are not shown.

| $[C^{(3/2,0)}_{\Delta_\mu,AA}]_{a,b}$ | $[C^{(1,-1)}_{\Sigma_\mu,AA}]_{a,b}$ | $[C^{(1/2,-2)}_{\Xi_\mu,AA}]_{a,b}$ | $[C^{(0,-3)}_{\Omega_\mu,AA}]_{a,b}$ |
|---|---|---|---|
| (1,1): $2C_A^2$<br>(1,2): $-2C_A^2$<br>(2,2): $2C_A^2$ | (1,1): $C_A^2$<br>(1,2): $\sqrt{\frac{2}{3}}C_A^2$<br>(1,3): $-\sqrt{\frac{2}{3}}C_A^2$<br>(1,4): $-C_A^2$<br>(1,5): $\sqrt{\frac{2}{3}}C_A^2$<br>(2,2): $\frac{2C_A^2}{3}$<br>(2,3): $-\frac{2C_A^2}{3}$<br>(2,4): $-\sqrt{\frac{2}{3}}C_A^2$<br>(2,5): $\frac{2C_A^2}{3}$<br>(3,3): $\frac{2C_A^2}{3}$<br>(3,4): $\sqrt{\frac{2}{3}}C_A^2$<br>(3,5): $-\frac{2C_A^2}{3}$<br>(4,4): $C_A^2$<br>(4,5): $-\sqrt{\frac{2}{3}}C_A^2$<br>(5,5): $\frac{2C_A^2}{3}$ | (1,1): $C_A^2$<br>(1,2): $-C_A^2$<br>(1,3): $-C_A^2$<br>(1,4): $C_A^2$<br>(2,2): $C_A^2$<br>(2,3): $C_A^2$<br>(2,4): $-C_A^2$<br>(3,3): $C_A^2$<br>(3,4): $-C_A^2$<br>(4,4): $C_A^2$ | (1,1): $4C_A^2$ |

**Table D.4.:** Matrix elements $[C^{(I,S)}_{[10],AA}]_{b,a} = [C^{(I,S)}_{[10],AA}]_{a,b}$ describing the s-channel decuplet baryon exchange. Only non-zero elements are shown.

$$
\begin{aligned}
F_1^+ = \sum_{[10]} \frac{C^{(I,S)}_{[10],AA}}{24f^2 M_{[10]}^2 (M_{[10]}^2 - s)} \Big\{ & [-3m^2 - 3\bar{m}^2 + 3t + 2(s - (M+\bar{M})\sqrt{s} + M\bar{M})] M_{[10]}^3 \\
& + [-(M+2\sqrt{s})\bar{m}^2 + (M-\sqrt{s})\bar{M}^2 + \sqrt{s}(-2m^2 + M(\sqrt{s}-M) + 3t) \\
& \quad + ((M-\sqrt{s})^2 - m^2)\bar{M}] M_{[10]}^2 + [(2m^2 - 2M^2 + s + M\sqrt{s})\bar{m}^2 \\
& \quad - (\bar{M}-\sqrt{s})(2(m^2-M^2)\bar{M} + s(M+\bar{M}) + \sqrt{s}(m^2 + M(\bar{M}-M)))] M_{[10]} \\
& + \sqrt{s}(m^2 - M^2 + s)(\bar{m}^2 - \bar{M}^2 + s) \Big\},
\end{aligned}
\tag{D.8}
$$

where the matrix elements $[C^{(I,S)}_{[10],AA}]_{a,b} = [C^{(I,S)}_{[10],AA}]_{b,a}$ are collected in Tab. D.4.



$$
= -\sum_{[10]} \frac{1}{4f^2} \tilde{C}^{(I,S)}_{[10],AA} \bar{\Gamma}^{(A)}_{\rho}(-q) S^{\rho\sigma}_{[10]}(p-\bar{q}) \Gamma^{(A)}_{\sigma}(-\bar{q}),
\tag{D.9}
$$

$$
\begin{aligned}
F_1^+ = \sum_{[10]} \frac{\tilde{C}^{(I,S)}_{[10],AA}}{24f^2 M_{[10]}^2 (M_{[10]}^2 - u)} \Big\{ & M_{[10]}^3 [2\sqrt{s}(\bar{M}+M) + (\bar{M}-M)^2 - 3s - u] \\
& + M_{[10]}^2 [\bar{m}^2(M-\sqrt{s}) + 2\bar{M}^3 + \bar{M}^2(M - 2\sqrt{s}) + \bar{M}(m^2 + M^2 + 2M\sqrt{s} - 3s - 2u) \\
& \quad + \sqrt{s}(3(s+u) - m^2) + 2M^3 - 2M^2\sqrt{s} - M(3s+2u)] \\
& + M_{[10]} [\bar{m}^2(-\bar{M}(\bar{M} - M + \sqrt{s}) + 2m^2 + u) - M(\bar{M}^3 + \bar{M}(M^2 - m^2) + m^2 M) \\
& \quad + u(-\sqrt{s}(\bar{M}+M) + 2\bar{M}M + m^2) + M\sqrt{s}(\bar{M}(\bar{M}+M) - m^2)] \\
& - (\bar{m}^2 - M^2 + u)(\bar{M}^2 - m^2 - u)(\bar{M} + M - \sqrt{s}) \Big\},
\end{aligned}
\tag{D.10}
$$

where the coupling coefficients $[\tilde{C}^{(I,S)}_{[10],AA}]_{a,b} = [\tilde{C}^{(I,S)}_{[10],AA}]_{b,a}$ are collected in Tab. D.5.

| | $[\tilde{C}^{(I,S)}_{\Delta_\mu,AA}]_{a,b}$ | $[\tilde{C}^{(I,S)}_{\Sigma_\mu,AA}]_{a,b}$ | $[\tilde{C}^{(I,S)}_{\Xi_\mu,AA}]_{a,b}$ | $[\tilde{C}^{(I,S)}_{\Omega_\mu,AA}]_{a,b}$ |
|---|---|---|---|---|
| (0, 1) | | (1,1): $C_A^2$ | | |
| (1, 1) | | (1,1): $\frac{C_A^2}{3}$ | | |
| (1/2, 0) | (1,1): $\frac{8C_A^2}{3}$ | (1,3): $-C_A^2$<br>(1,4): $\frac{2C_A^2}{3}$<br>(2,4): $C_A^2$ | (3,3): $C_A^2$<br>(3,4): $-C_A^2$<br>(4,4): $-\frac{C_A^2}{3}$ | |
| (3/2, 0) | (1,1): $\frac{2C_A^2}{3}$ | (1,2): $-\frac{C_A^2}{3}$ | (2,2): $\frac{2C_A^2}{3}$ | |
| (0, -1) | (1,2): $-4\sqrt{\frac{2}{3}}C_A^2$ | (1,1): $-\frac{2C_A^2}{3}$<br>(1,3): $-\sqrt{3}C_A^2$<br>(2,4): $C_A^2$ | (1,4): $\sqrt{\frac{2}{3}}C_A^2$<br>(3,4): $\sqrt{2}C_A^2$ | (4,4): $-2C_A^2$ |
| (1, -1) | (2,3): $-\frac{4C_A^2}{3}$ | (1,1): $C_A^2$<br>(1,2): $-\sqrt{\frac{2}{3}}C_A^2$<br>(2,2): $\frac{C_A^2}{3}$<br>(2,4): $-\sqrt{\frac{2}{3}}C_A^2$<br>(3,5): $\frac{C_A^2}{3}$<br>(4,4): $C_A^2$ | (1,5): $\sqrt{\frac{2}{3}}C_A^2$<br>(2,5): $-\frac{2C_A^2}{3}$<br>(4,5): $\sqrt{\frac{2}{3}}C_A^2$ | (5,5): $2C_A^2$ |
| (2, -1) | | (1,1): $\frac{C_A^2}{3}$ | | |
| (1/2, -2) | (3,3): $\frac{8C_A^2}{3}$ | (1,2): $-C_A^2$<br>(1,3): $\frac{2C_A^2}{3}$<br>(3,4): $C_A^2$ | (1,1): $-\frac{C_A^2}{3}$<br>(1,4): $C_A^2$<br>(4,4): $C_A^2$ | |
| (3/2, -2) | (2,2): $\frac{2C_A^2}{3}$ | (1,2): $-\frac{C_A^2}{3}$ | (1,1): $\frac{2C_A^2}{3}$ | |
| (0, -3) | | (1,1): $C_A^2$ | | |
| (1, -3) | | (1,1): $\frac{C_A^2}{3}$ | | |

**Table D.5.:** Coefficients $[\tilde{C}^{(I,S)}_{[10],AA}]_{b,a} = [\tilde{C}^{(I,S)}_{[10],AA}]_{a,b}$ specifying the u-channel decuplet baryon exchange. Vanishing elements are not shown.

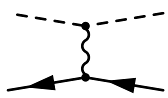
| | $[C^{(I,S)}_{\omega_{\mu\nu},PX}]_{a,b}$ | $[C^{(I,S)}_{\phi_{\mu\nu},PX}]_{a,b}$ |
|---|---|---|
| (0, 1) | (1,1): $-\frac{1}{2}h_P(D_X+F_X+2G_X)$ | (1,1): $h_P(D_X-F_X+G_X)$ |
| (1, 1) | (1,1): $-\frac{1}{2}h_P(D_X+F_X+2G_X)$ | (1,1): $h_P(D_X-F_X+G_X)$ |
| (1/2, 0) | (3,3): $-\frac{1}{3}h_P(D_X+3G_X)$<br>(4,4): $-h_P(D_X+G_X)$ | (3,3): $\frac{1}{3}h_P(4D_X+3G_X)$<br>(4,4): $G_X h_P$ |
| (3/2, 0) | (2,2): $-h_P(D_X+G_X)$ | (2,2): $G_X h_P$ |
| (0, -1) | (2,2): $\frac{1}{2}h_P(D_X+F_X+2G_X)$<br>(4,4): $-\frac{1}{2}h_P(D_X-F_X+2G_X)$ | (2,2): $-h_P(D_X-F_X+G_X)$<br>(4,4): $h_P(D_X+F_X+G_X)$ |
| (1, -1) | (3,3): $\frac{1}{2}h_P(D_X+F_X+2G_X)$<br>(5,5): $-\frac{1}{2}h_P(D_X-F_X+2G_X)$ | (3,3): $-h_P(D_X-F_X+G_X)$<br>(5,5): $h_P(D_X+F_X+G_X)$ |
| (2, -1) | | |
| (1/2, -2) | (2,2): $\frac{1}{3}h_P(D_X+3G_X)$<br>(3,3): $h_P(D_X+G_X)$ | (2,2): $-\frac{1}{3}h_P(4D_X+3G_X)$<br>(3,3): $-G_X h_P$ |
| (3/2, -2) | (2,2): $h_P(D_X+G_X)$ | (2,2): $-G_X h_P$ |
| (0, -3) | (1,1): $\frac{1}{2}h_P(D_X-F_X+2G_X)$ | (1,1): $-h_P(D_X+F_X+G_X)$ |
| (1, -3) | (1,1): $\frac{1}{2}h_P(D_X-F_X+2G_X)$ | (1,1): $-h_P(D_X+F_X+G_X)$ |

**Table D.6.:** Coefficients $[C^{(I,S)}_{[9],PX}]_{b,a} = [C^{(I,S)}_{[9],PX}]_{a,b}$ describing the t-channel vector meson exchange. Elements that are zero are not shown.

| | $[C^{(I,S)}_{\rho_{\mu\nu},PX}]_{a,b}$ | $[C^{(I,S)}_{K_{\mu\nu},PX}]_{a,b}$ |
|---|---|---|
| (0, 1) | (1,1): $\frac{3}{2}h_P(D_X+F_X)$ | |
| (1, 1) | (1,1): $-\frac{1}{2}h_P(D_X+F_X)$ | |
| (1/2, 0) | (1,1): $2h_P(D_X+F_X)$<br>(3,4): $-D_X h_P$<br>(4,4): $2F_X h_P$ | (1,3): $-\frac{1}{2}h_P(D_X+3F_X)$<br>(1,4): $-\frac{1}{2}h_P(D_X-F_X)$<br>(2,3): $-\frac{1}{2}h_P(D_X+3F_X)$<br>(2,4): $\frac{3}{2}h_P(D_X-F_X)$ |
| (3/2, 0) | (1,1): $-h_P(D_X+F_X)$<br>(2,2): $-F_X h_P$ | (1,2): $h_P(D_X-F_X)$ |
| (0, -1) | (1,1): $4F_X h_P$<br>(2,2): $\frac{3}{2}h_P(D_X+F_X)$<br>(4,4): $-\frac{3}{2}h_P(D_X-F_X)$ | (1,2): $-\sqrt{\frac{3}{2}}h_P(D_X-F_X)$<br>(1,4): $-\sqrt{\frac{3}{2}}h_P(D_X+F_X)$<br>(2,3): $\frac{1}{\sqrt{2}}h_P(D_X+3F_X)$<br>(3,4): $\frac{1}{\sqrt{2}}h_P(D_X-3F_X)$ |
| (1, -1) | (1,2): $-2\sqrt{\frac{2}{3}}D_X h_P$<br>(2,2): $2F_X h_P$<br>(3,3): $-\frac{1}{2}h_P(D_X+F_X)$<br>(5,5): $\frac{1}{2}h_P(D_X-F_X)$ | (1,3): $\frac{1}{\sqrt{6}}h_P(D_X+3F_X)$<br>(1,5): $-\frac{1}{\sqrt{6}}h_P(D_X-3F_X)$<br>(2,3): $-h_P(D_X-F_X)$<br>(2,5): $-h_P(D_X+F_X)$<br>(3,4): $-\sqrt{\frac{3}{2}}h_P(D_X-F_X)$<br>(4,5): $\sqrt{\frac{3}{2}}h_P(D_X+F_X)$ |
| (2, -1) | (1,1): $-2F_X h_P$ | |
| (1/2, -2) | (1,1): $-2h_P(D_X-F_X)$<br>(2,3): $-D_X h_P$<br>(3,3): $2F_X h_P$ | (1,2): $-\frac{1}{2}h_P(D_X-3F_X)$<br>(1,3): $\frac{1}{2}h_P(D_X+F_X)$<br>(2,4): $\frac{1}{2}h_P(D_X-3F_X)$<br>(3,4): $\frac{3}{2}h_P(D_X+F_X)$ |
| (3/2, -2) | (1,1): $h_P(D_X-F_X)$<br>(2,2): $-F_X h_P$ | (1,2): $-h_P(D_X+F_X)$ |
| (0, -3) | (1,1): $-\frac{3}{2}h_P(D_X-F_X)$ | |
| (1, -3) | (1,1): $\frac{1}{2}h_P(D_X-F_X)$ | |

**Table D.7.:** Continuation of Tab. D.6.

## D.4 Vector meson exchange



$$= i\sum_{[9],X}\frac{1}{4f^2}\,C^{(I,S)}_{[9],PX}\,\Gamma^{(P)}_{\rho\sigma}(\bar q,q)\,G^{\rho\sigma,\zeta\eta}_{[9]}(\bar q-q)\,\Gamma^{(X)}_{\zeta\eta}(\bar q-q)\,, \tag{D.11}$$

$$\begin{aligned} F_1^+ = {}& \sum_{[9]} C^{(I,S)}_{[9],PV}\,\frac{(\bar m^2-m^2)(\bar M-M)-t(M+\bar M-2\sqrt{s})}{8f^2(m^2_{[9]}-t)} \\ &+\sum_{[9]} C^{(I,S)}_{[9],PT}\,\frac{m_V\,[\bar m^2+m^2-t+2(\bar M-\sqrt{s})(\sqrt{s}-M)]}{16f^2(m^2_{[9]}-t)}\,, \end{aligned} \tag{D.12}$$

where $X\in\{V,T\}$ and the coupling coefficients $[C^{(I,S)}_{[9],PX}]_{a,b}=[C^{(I,S)}_{[9],PX}]_{b,a}$ are collected in the Tab. D.6 - D.7.

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