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The Economic and Demographic Transition, Mortality, and Comparative Development

Matteo Cervellati and Uwe Sunde

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Matteo Cervellati
University of Bologna
IZA, Bonn

Uwe Sunde
University of Munich
IZA, Bonn
CEPR

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Abstract

We propose a unified growth theory to investigate the mechanics generating the economic and demographic transition, and the role of mortality differences for comparative development. The framework can replicate the quantitative patterns in historical time series data and in contemporaneous cross-country panel data, including the bi-modal distribution of the endogenous variables across countries. The results suggest that differences in extrinsic mortality might explain a substantial part of the observed differences in the timing of the take-off across countries and the worldwide density distribution of the main variables of interest.

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European countries have experienced fundamental changes in living conditions since the mid 19th century. Within a few generations mortality and fertility rates dropped to unprecedented levels. At roughly the same time of this demographic transition, an economic transition led to widespread education and sustained income growth after a nearly stagnant development during the entire previous history. The stylized patterns are very similar across countries and times, including countries that entered their demographic and economic transition much later than the European forerunners. Demographers such as Kirk (1996) notice that also "in non-European countries undergoing the demographic transition in the mid 20th century, the regularities are impressive". By 1970 about half of all countries in the world had not yet experienced the onset of the transition, however, and in 2000 still 40 percent of these countries were trapped in underdevelopment.[1]

Several important questions regarding these long-run development patterns still remain unresolved: What are the underlying forces behind the different dimensions of economic and demographic development, and how are they linked? Why have some countries developed early on, others only with a delay, and why do many countries still remain trapped in poor living conditions today? What is the role of mortality, in light of the fact that today's underdeveloped countries are predominantly located in areas with a high exposure to infectious diseases? This paper addresses these questions by providing a unified growth theory of the economic and demographic transition that allows for a systematic quantitative investigation of the role of the mortality environment for long-run growth and comparative development.

This paper contributes to the literature in several ways. Existing unified growth theories have modelled the endogenous transition as outcome of technological progress that changes the education-fertility trade-off, or as the result of a reduction in mortality.[2] The first contribution of this paper is the development of a tractable prototype

[1] Demographers conventionally identify the onset of the demographic transition with life expectancy at birth increasing above 50-55 years and a sustained drop in fertility, see Chesnais (1992). Nonetheless, in these countries life expectancy at birth was less than 55 years in 1970, average total fertility was around six children per woman and the share of population with completed secondary education was less than one out of five.

[2] Galor and Weil (2000), Galor and Moav (2002), Doepke (2004) and Strulik and Weisdorf (2008) investigate the role of technology for fertility. The role of mortality is studied by de la Croix and

general equilibrium unified theory that delivers analytical predictions that fit the stylized facts in terms of both the economic take-off (education and income) and demographic transition (child mortality, adults longevity and fertility). Individuals decide about acquiring skilled or unskilled human capital and fertility (in terms of number and quality of children). Endogenous improvements in life expectancy and skill-biased technological change eventually trigger a transition to sustained growth and widespread education. A demographic transition reduces mortality, and net fertility declines since skilled workers have fewer children due to a differential fertility effect.[3]

A second contribution is the application of the prototype unified growth model to a systematic quantitative analysis of the mechanics of long term development. Our calibration strategy involves fixing the parameters of the model for all generations and then simulating the economy over a long period of time (from year 0 to year 2000). This simulation period includes the onset of the transition and the convergence to the balanced growth path. The model is calibrated using data for Sweden, the textbook case of long-run economic and demographic development, by targeting data moments before the onset of the transition and on the balanced growth path.[4]

Licandro (1999), Kalemli-Ozcan, Ryder and Weil (2000), and Boucekkine, de la Croix and Licandro (2002, 2003), Cervellati and Sunde (2005) and Voigtländer and Voth (2012b). See Galor (2005) for a survey.

[3]The prototype model nests, and extends, the frameworks by Galor and Weil (2000), Cervellati and Sunde (2005) and Soares (2005). The evolution of longevity, which is modelled by considering dynamic externalities as in Cervellati and Sunde (2005), can be microfounded along the lines of de la Croix and Licandro (2012), where endogenous health investments affect longevity, or Dalgaard and Strulik (2012), where investments by parents affect the body size (and health) of their children. The differential fertility effect is similar to that in de la Croix and Doepke (2003). The fertility reduction is obtained in an occupational choice framework without the need for restrictive assumptions on the utility function, in line with the results by Mookherjee, Prina and Ray (2012).

[4]Doekpe (2004) and Lagerlof (2003) simulate unified growth models in a time series perspective. Related works, that however do not calibrate a unified growth occupational choice framework and that involve different quantitative strategies, include Eckstein, Mira and Wolpin (1999), Kalemli-Ozcan (2002), de la Croix, Lindh and Malmberg (2008), and Bar and Leukhina (2010). More traditional strategies involving comparative statics around the balanced growth path of Barro-Becker models (see Jones, Schoonbroodt, and Tertilt (2011) for a survey), are unsuitable for the purposes of this paper which requires studying the endogenous take-off of the transition and its different timing across countries.

A third contribution is the extension of the scope of the quantitative analysis of the unified growth framework to the investigation of comparative development and cross-country data patterns. The calibrated model is used to investigate the role of mortality for the onset of the transition, for its different timing across countries, and for cross-country comparative development. The model produces coherent and quantitatively testable predictions about cross-section patterns in addition to time series patterns. The simulation of the calibrated model is compared to actual time series data from Sweden for the period 1750-2000 as well as to cross-country panel data for the period 1960-2000. The paper therefore provides the first attempt to study the implications of the unified growth framework and the role of mortality from both a time series and a cross-country perspective.

The results deliver novel insights regarding the underlying mechanics of the long-run development process and the patterns of comparative development. The model features a non-linear path of long-run development in all central dimensions, including education, fertility, longevity and income per capita. These development paths match closely the historical patterns. Despite the non-linear development dynamics, the simulated data display monotonic, and almost linear, cross country correlations between the equilibrium share of educated individuals and all other central variables. When interpreted in a cross-sectional perspective, the simulated data match the correlations emerging from cross-country panel data for 1960 and 2000. The calibrated model also reproduces the well documented concave relationship between income per capita and life expectancy known as the "Preston Curve". A further implication of the model, which has not been investigated previously in the literature, is a hump-shaped relationship between life expectancy and subsequent changes in the education composition of the population. This prediction is shown to be consistent with patterns in cross-country data. The results thereby contribute to the literature on comparative development by showing for the first time that the observed patterns in time series, cross-section correlations, and in the cross-country distributions of the variables of interest can all be generated by the same non-linear development process.

As fourth contribution, the paper investigates the role of differences in the extrinsic mortality environment for development, thereby providing a link between the unified growth literature and the literature on the fundamental determinants of long-run development. The quantitative role of differences in mortality is studied by simulating a counterfactual economy that differs from the benchmark calibration for Sweden only in terms of the exogenous mortality environment, which is calibrated by targeting data moments for the highest mortality countries in 2000 (rather than for Sweden in 1800).[5] The results document that cross-country differences in baseline longevity can result in substantial delays of the economic and demographic transition. Differences in baseline longevity are shown to leave both the cross-sectional relationships, including the Preston Curve, essentially unaffected. This can help explaining why the empirical role of mortality differences for long-run development is difficult to identify with linear estimation frameworks.[6]

Finally, the analysis contributes to the debate on the determinants of the cross-country distributions of the variables of interest. Since the results suggest that countries mainly differ in terms of the timing of the take-off, we simulate an artificial world composed of countries that differ in terms of baseline longevity, but that are otherwise identical to the benchmark model. The results deliver cross-country distributions of all variables of interest that match the actual worldwide distributions in 1960 and 2000, which are bi-modal in 1960 and rather uni-modal in 2000. The framework provides an explanation for the changing bi-modality of the distributions in the different central variables due to the acceleration in the changes of all variables during the transition to the balanced growth path.[7]

The results support the view that developing countries follow similar development

[5]Unlike in most other unified growth models, the acceleration in income is not generated by the transition from corner solutions to interior solutions. This technical feature permits conducting smooth comparative statics on the main parameters of interest such as baseline mortality, and allows for the derivation of cross-sectional predictions.

[6]Cervellati and Sunde (2011) show that allowing for non-monotonic effects of life expectancy on economic growth can reconcile contradictory empirical findings by Acemoglu and Johnson (2007) and Lorentzen, McMillan and Wacziarg (2008). The analysis also complements evidence by Andersen, Daalgard and Selaya (2011) on the role of diseases for comparative development.

[7]The results can explain the twin peaks in the world income distribution (Azariadis and Stachurski, 2005), life expectancy (Bloom and Canning, 2007) and fertility, and provide an explanation for the similar distributions in all these dimensions and how they are linked.

processes as the Western forerunners, although with a substantial delay. The findings suggest that differences in the mortality environment across countries can explain a large part of the observed heterogeneity in the timing of the take-off across countries, and in the cross-country distribution of the main variables of interest.

The paper is organized as follows. Sections 1 and 2 introduce the model and derive the analytical predictions, respectively. Section 3 presents the quantitative analysis. Section 4 provides a discussion and Section 5 concludes. The proofs, data sources and details of the calibration are presented in the Appendix.

1 The Model

This section presents the theoretical framework. Even though the functional form assumptions are not needed for the analytical results in Section 2, the model is presented using the specific functional forms that are applied in the calibration in Section 3. The functional forms are specified in line with the previous literature and the available evidence, and to minimize the number of parameters.

1.1 Set up

The economy is populated by a discrete number of generations of individuals denoted by $t \in \mathbb{N}^+$. There are two relevant subperiods in the life of an individual: childhood and adulthood. The duration of childhood is denoted by $K_t = k$ while the duration of adulthood is denoted by T_t. Each individual of generation t survives to age k with probability $\pi_t \in (0, 1)$. Surviving children become adults, survive with certainty until age $k + T_t$, and then die. The variable T_t therefore represents both life expectancy at age k and the maximum duration of adulthood.[8] In the model, T_t is a summary

[8]This modeling of child survival and adult longevity follows Soares (2005). It is formally isomorphic to a "perpetual youth" modeling, where longevity is just one over the age independent adult survival probability. Considering a deterministic longevity serves the role of simplifying the set up of the model by abstracting from uncertainty and, as discussed below, by allowing for a direct match between the simulated and empirical data (that are available in terms of child mortality and life expectancy at age five). In the quantitative analysis, $1 - \pi_t$ corresponds to child mortality, and T_t corresponds to life expectancy at age five (so that $k = 5$). Assuming a constant death rate before age

statistic of the effective time available during adulthood. An alternative interpretation of T_t would be as a "health augmented" time endowment of adults.

Reproduction is asexual and takes place at age $m \geq k$, which therefore represents the length of a generation. A generation of adults consists of a mass of agents of size $N_{t+1} = N_t \pi_t n_t$ where n_t is the average (gross) fertility of the parent generation. Every individual of generation t is endowed with an innate ability $a \in [0, 1]$, which is randomly drawn from a distribution $f(a)$ that does not change over the course of generations. For the calibration of the model we assume a (truncated) normal distribution of ability with mean μ and standard deviation σ.

1.2 Preferences and Production

During childhood individuals are fed by their parents and make no choices. At the beginning of adulthood, those individuals that survive childhood make decisions about their own education and their fertility to maximize their (remaining) lifetime utility. Individuals derive utility from own consumption, c, and the quality, q, of their (surviving) offsprings πn. As in Soares (2005), the lifetime utility of an individual i of generation t is additively separable and given by,

$$\int_0^{T_t} \ln c_t^i(\tau)\, d\tau + \gamma \ln\left(\pi_t n_t^i q_t^i\right) \tag{1}$$

where $\gamma > 0$ is the weight of the utility that parents derive from their surviving children relative to their own adults lifetime consumption.[9]

The inputs of production are *skilled* human capital, denoted by s, and *unskilled* human capital, denoted by u. We follow Cervellati and Sunde (2005) and treat human capital as inherently heterogenous across generations. In line with the literature on vintage human capital, this reflects the view that individuals acquire their skills in environments characterized by the availability of a particular technology. The aggregate stocks of human capital of each type, H_t^u and H_t^s, supplied by generation t are used in a constant returns to scale technology

$$Y_t = A_t \left[(1 - x_t)\left(H_t^u\right)^\eta + x_t \left(H_t^s\right)^\eta\right]^{\frac{1}{\eta}} \ , \tag{2}$$

5, life expectancy at birth is $\pi_t(5 + T_t)$.

[9]As in Becker and Lewis (1973) parents derive utility directly from the quality of their children, which allows studying the change in the quantity-quality trade-off in the simplest way.

where $\eta \in (0,1)$. Generation t only operates the technological vintage t, which is characterized by the relative productivity of the two types of human capital, $x_t \in (0,1)$, and total factor productivity (TFP) A_t. The production function (2) is a specialized (CES) formulation of the vintage production function by Chari and Hopenhayn (1991). As in Boucekkine, de la Croix, and Licandro (2002), the vintage of technology is linked to generation-specific knowledge in terms of skilled and unskilled human capital. The returns to human capital are determined in general equilibrium on competitive markets and equal marginal productivity,

$$w_t^s = \frac{\partial Y_t}{\partial H_t^s} \quad , \quad w_t^u = \frac{\partial Y_t}{\partial H_t^u} \ . \tag{3}$$

Vintage models, which relax the assumption that human capital is perfectly homogenous across different age cohorts, are empirically appealing in the context of long term development where cohorts of workers of different age acquire knowledge of different technologies, and they have convenient technical properties. This vintage structure is not needed for the main mechanism and the analytical results, but it allows for a tractable and transparent quantitative analysis.[10]

The level of human capital acquired by each individual is increasing in the level of innate ability, a, $h^j(a)$ with $dh^j(a)/da \geq 0$ for each $j = \{u, s\}$. Individual ability is relatively more important in producing skilled human capital. As studied below, this delivers a natural equilibrium sorting of the population into skilled and unskilled. For simplicity, we make the assumption that ability only matters for skilled human capital. An individual with ability a acquires $h^s(a) = e^{\alpha a}$ units of human capital if he decides to become skilled, and $h^u(a) = e^{\alpha\mu}$ if he decides to be unskilled.[11] An individual that

[10] As discussed below, the dynamic system does not involve a forward looking component since the optimal choices of acquiring human capital by generation t do not depend on the optimal choices of the (unborn) generations of workers that will enter in the labor market in the future. The model can be simulated by setting initial conditions and iterating the solution of the general equilibrium of the economy forward across generations until the balanced growth path is reached. The vintage structure therefore allows the computation of a general equilibrium model with changing longevity, without having to impose problematic assumptions like, in particular, restricting attention to the balanced growth path as in, e.g., Jones and Schoonbroodt (2010). See Boucekkine, de la Croix and Licandro (2011) for a survey of the vintage literature.

[11] Assuming that individuals with average ability produce the same quantities of skilled or unskilled

decides to become skilled, respectively unskilled, pays a fix cost, measured in term of adult time, of $\underline{e}^s > \underline{e}^u \geq 0$.[12]

As in Galor and Weil (2000), raising a child involves a time cost $r_t = \widetilde{r}_t + \underline{r}$ where $\underline{r} > 0$ is a fix time cost that needs to be spent and $\widetilde{r}_t \geq 0$ is the extra time that can be spent voluntarily in addition.[13] The time spent with a child increases the child's quality according to,

$$q_t\left(\underline{r}, \widetilde{r}_t, g_{t+1}\right) = \left[\widetilde{r}_t \delta \left(1 + g_{t+1}\right) + \underline{r}\right]^{\beta} \tag{4}$$

where $g_{t+1} = (A_{t+1} - A_t)/A_t$, $\beta \in (0,1)$, and $\delta > 0$. The functional form (4) implies a complementarity between technical progress and the effectiveness of the extra time invested in children's (the quality time $\widetilde{r}_t$). As discussed in more details below, this formulation captures in the simplest way that faster technological progress increases the incentives to invest more time in raising children, as in Galor and Weil (2000).

1.3 Mortality and Technological Change

Adult Life Expectancy and Child Survival. A large body of evidence documents that environmental factors, in particular macroeconomic conditions, are crucial determinants of individual health. Child and adult mortality appear to be affected by the macro environment in different ways, however. The evidence reported by Cut-

human capital is a natural benchmark. This also implies that the average quantity of human capital of each type would be the same if acquired by the entire population (so that there are no scale effects associated with the acquisition skilled human capital during the process of development). The relative importance of ability for the level of skilled and unskilled human is irrelevant for the results, however. As discussed below, irrespective of the relative productivity of ability for the two skills, the economy passes from an equilibrium where almost all individuals are unskilled to an equilibrium where almost all individuals are skilled during the process of development.

[12]More complex skills may involve more costly processes of skill acquisition and maintenance. The crucial feature for the mechanism is that workers who decide to be skilled face a lower effective lifetime that is available for market work during their adulthood. To capture this feature in the simplest way, we follow Cervellati and Sunde (2005) and consider a fix cost in terms of time.

[13]Both increase quality but with different relative intensity. The cost $\underline{r}$ can be interpreted as the minimum investment required for the children to survive to adulthood and may include feeding (or dressing) the child. The extra investment $\widetilde{r}_t$ can be interpreted as pure quality time that is not needed for survival like, e.g., talking, playing or reading a book with the child.

ler et al. (2006) suggests that human capital is more important for adult longevity than per capita income since adult longevity depends on the ability to cure diseases and is related to the level of medical knowledge. Better living conditions in terms of higher incomes, but also in terms of access to water and electricity, are relatively more important for increasing the survival probability of children, see Wang (2003) for a survey.

In line with this evidence, we consider a differential impact of human capital and income on adult and child mortality. Adult longevity of generation t is assumed to be increasing in the share of skilled individuals in the parent generation,

$$T_t = \Upsilon(\lambda_{t-1}) = \underline{T} + \rho\lambda_{t-1} \tag{5}$$

where $\underline{T}$ is the baseline longevity that would be observed in the economy in the absence of any skilled human capital, and $\rho > 0$ reflects the scope for improvement.[14] Since $\lambda \in (0,1)$, the maximum level of adult longevity is given by $\overline{T} = \underline{T} + \rho$.

The child survival probability π_t depends on living conditions at the time of birth, as reflected by per capita income and parental education,

$$\pi_t = \Pi(\lambda_{t-1}, y_{t-1}) = 1 - \frac{1 - \underline{\pi}}{1 + \kappa\lambda_{t-1}y_{t-1}} \tag{6}$$

with $\kappa > 0$ and where $1 > \underline{\pi} > 0$ is the baseline child survival that would be observed in an economy with $\lambda_{t-1}y_{t-1} = 0$.[15]

Technology. Technological progress is reflected in the emergence of a new vintage of technology characterized by TFP, A_t, and a higher relative weight of skilled human capital in the production process, x_t. We consider technological progress that is biased

[14]This reduced form modeling of the change in longevity follows Cervellati and Sunde (2005) and allows going beyond the assumption that changes in mortality are fully exogenous (as in, e.g. Jones and Schoonbroodt (2010)) in the simplest and most parsimonious way. The evolution of longevity could be made endogenous to human capital by extending the model to the consideration of optimal investments in health along the lines of de la Croix and Licandro (2012).

[15]Larger total income Y_{t-1} improves the probability of children reaching adulthood while population size N_{t-1} deteriorates living conditions and reduces child survival rates. Considerable evidence documents the negative effect of population density and urbanization on child mortality, especially during the early stages of the demographic transition, see Galor (2005).

towards skill intensive production and that depends on the available skilled human capital. The relative productivity of skilled human capital in production, x_t, increases with the share of skilled workers in the previous generation, λ_{t-1}, and with the scope for further improvement, $1 - x_{t-1}$,

$$\frac{x_t - x_{t-1}}{x_{t-1}} = X\left(\lambda_{t-1}, x_{t-1}\right) = \lambda_{t-1}(1 - x_{t-1})\,. \tag{7}$$

For any λ_t, improvements are smaller as x_t converges to its upper limit at $x = 1$.

Finally, improvements in total factor productivity, A_t, are increasing with the share of skilled workers in the previous generation,[16]

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = G\left(\lambda_t\right) = \phi\lambda_t \quad , \quad \phi > 0\,. \tag{8}$$

2 Analytical Results

This section derives the analytical results starting from the optimal fertility and education decisions in partial equilibrium. We then characterize the intra-generational general equilibrium and the dynamic evolution of the economy over time.

2.1 Individual Decision Problem

Following Soares (2005), we set the subjective discount rate to zero and assume that individuals perfectly smooth consumption within their adult period of life, $c_t^i\left(\tau\right) = c_t^i$ for all τ. These assumptions allow to abstract from the path of consumption during the life cycle which is irrelevant to study the long term evolution of the economy.[17] The utility can then be expressed as,

$$U(c_t^i, \pi_t n_t^i q_t^i) = T_t \ln c_t^i + \gamma \ln\left(\pi_t n_t^i q_t^i\right)\,. \tag{9}$$

The key feature of this formulation is that individuals can smooth consumption over their adult life, but they cannot perfectly substitute the utility from their own con-

[16] This specification can be seen as a reduced form of endogenous growth models such as (Aghion and Howitt 1992) where ϕ can be interpreted as the average size of an innovation and the labor involved in research is increasing in λ_t.

[17] See, e.g., Rogerson and Wallenius (2009) for a similar assumption, which is equivalent to assuming a small open economy facing a zero discount rate.

sumption with utility derived from their offspring.[18]

The total time available during adulthood is limited by adult longevity T_t, or by some exogenous limit to the number of years in the labor market (e.g., due to retirement), $R > 0$.[19] The effective time available for productive activities during adulthood is therefore bounded from above by $\overline{T}_t = \min\{T_t, R\}$. An individual i with education $j = \{u, s\}$ cannot use more than the available time and cannot spend more than the total earnings for total consumption, so that

$$\overline{T}_t \geq l_t^i + \underline{e}^j + \pi_t n_t^i r_t^i \,, \tag{10}$$

and

$$l_t^i w_t^j h_t^j(a) \geq T_t c_t^i \,, \tag{11}$$

where l_t^i is the total time spent working. Given the utility function (9) both constraints will be binding at the optimum. Combining (10) and (11) delivers the budget constraint conditional on being skilled or unskilled, $j = \{u, s\}$,

$$T_t c_t^i = \left(\overline{T}_t - \underline{e}^j - \pi_t n_t^i r_t^i\right) w_t^j h_t^j(a) \ . \tag{12}$$

The problem of an individual i with ability a born in generation t is to choose the type of human capital to be acquired, $j \in \{u, s\}$, the number of children, n_t^i, and the time invested in raising each child, r_t^i, so as to maximize utility (9) subject to (12). This is equivalent to maximizing

$$T_t \ln\left[(1/T_t)\left(\overline{T}_t - \underline{e}^j - \pi_t n_t^i r_t^i\right) w_t^j h_t^j(a)\right] + \gamma \ln\left(\pi_t n_t^i q_t^i\right) \ . \tag{13}$$

Optimal Fertility and Time Spent in Children. The optimization problem is strictly concave and the first order conditions uniquely identify optimal fertility and the time spent raising children conditional on the type of human capital.

[18]The actual formulation of the utility function, and the fact that longevity implicitly affects the weight of the utility from consumption and children is irrelevant for the results. As shown in a previous version of the paper, one could equivalently assume that individuals derive utility from average per period lifetime consumption and children as in Galor and Weil (2000).

[19]The assumption of a limit R, which may be due to compulsory retirement or some other effective limitation to labor force participation at old ages is not needed for the main results but adds a realistic feature for the analysis of the quantitative role of bounds to productive life when longevity increases to old ages. In the quantitative analysis, the parameter R is calibrated exogenously to match the effective retirement age.

Lemma 1. *For any* $\left\{w_t^j, T_t, \pi_t, g_{t+1}\right\}$, *the optimal fertility of an individual acquiring human capital* $j = \{u, s\}$ *is given by,*

$$n_t^{ij} = \frac{\gamma\left(\overline{T}_t - \underline{e}^j\right)}{\left(T_t + \gamma\right) r_t^{ij} \pi_t} \tag{14}$$

where r_t^{ij} *is given by,*

$$r_t^{ij} = r_t^* = max\left\{\underline{r}, \frac{1 - \left[1/\left(\delta(1 + g_{t+1})\right)\right]}{1 - \beta}\underline{r}\right\} \tag{15}$$

Fertility decreases with child survival (since individuals only care about surviving children) but increases with life expectancy (through a positive income effect) as long as $T_t < R$. Further increases in longevity above R (so that $\overline{T}_t = R$) lead to a reduction in fertility, however. The reason is that a longer old age life increases the marginal utility of lifetime income since more income is needed to deliver a constant consumption over the life cycle, thereby lowering fertility as a longer expected time in retirement increases the weight on private consumption. Fertility is decreasing with the time invested in children, in line with a standard quantity-quality trade-off. The optimal time spent raising each child does not depend on the type of human capital acquired by parents and displays the same features as the mechanism of Galor and Weil (2000). When technical progress is too low parents may optimally decide not to invest any extra time in raising their children beyond the minimum level, so that $r_t^{ij} = \underline{r}$. If a positive extra time is invested in raising children then a larger level technological progress g_t increases r_t^* and reduces optimal fertility for unskilled and skilled individuals.[20] The quantity-quality trade-off and the optimal investment in children's quality does not depend on adult longevity and child mortality.

Optimal Type of Skills. Agents with higher ability have a comparative advantage in acquiring skilled human capital. For any vector of wages there exists a unique ability threshold for which the indirect utilities from acquiring the two types of human capital are equal.

[20] From (15) there is a unique $\underline{g} > 0$ (implicitly given by $r_t^*(\underline{g}) = \underline{r}$) such that for any $g_{t+1} > \underline{g}$ then $r_t^* > \underline{r}$ and $dr_t^*/dg_{t+1} > 0$.

Lemma 2. *For any* $\{w_t^s, w_t^u, T_t, \pi_t\}$ *there exists a unique* $\widetilde{a}_t$ *implicitly defined by*

$$\frac{h_t^s(\widetilde{a}_t)}{h_t^u} = \left(\frac{\overline{T}_t - \underline{e}^u}{\overline{T}_t - \underline{e}^s}\right)^{\frac{T_t+\gamma}{T_t}} \frac{w_t^u}{w_t^s} \tag{16}$$

such that all individuals with $a \leq \widetilde{a}_t$ *optimally choose to acquire unskilled human capital* $j = u$ *while all individuals with* $a > \widetilde{a}_t$ *acquire skilled human capital* $j = s$.

Consequently, for any distribution of abilities $f(a)$, there is a unique share of individuals λ_t that optimally acquire skilled human capital which is given by,

$$\lambda_t := \int_{\widetilde{a}_t}^{1} f(a)\, da \tag{17}$$

From (16) and (17), λ_t is increasing in the relative wage w_t^s/w_t^u, decreasing in $\underline{e}^s$, increasing in adult longevity T_t, and is unaffected by child mortality π_t.

The Effects of Mortality on Education Composition and Fertility. As a consequence of Lemma 2, the average fertility in the population is given by

$$n_t^* = N(T_t, \lambda_t, \pi_t) = \frac{\gamma}{(T_t+\gamma)\, r_t^* \pi_t}\left[(1-\lambda_t)(\overline{T}_t - \underline{e}^u) + \lambda_t(\overline{T}_t - \underline{e}^s)\right] \tag{18}$$

where r_t^* is given by (15).

Gross fertility is decreasing in π_t through a *substitution effect* but net fertility is independent of π_t. From (16), child mortality does not affect individuals' choices regarding their own skill acquisition choices either. The effect of adult longevity on fertility is more complex. From (14), higher adult longevity T_t increases gross fertility as long as $T_t < R$ due to a positive *income effect*, but decreases gross fertility when $T_t \geq R$ as the income effect turns negative. In addition, a higher T_t reduces fertility by a *differential fertility* effect since it increases the share of skilled workers (reflected by a higher λ_t) who, everything else equal, have fewer children. If the composition effect is large enough the average gross and net fertility decrease following an increase in adult life expectancy even if $T_t < R$. A further indirect effect arises through the externality of the share of skilled individuals on growth. Although the quantity-quality trade-off is not directly affected by adult longevity and child mortality, parents substitute quantity for quality in the face of technological progress, which depends on λ_t. Higher longevity reduces fertility also by indirectly changing the future parental investments in the quality of children.

2.2 Intra-Generational General Equilibrium

From (3), the ratio of competitively determined wages is

$$\frac{w_t^u}{w_t^s} = \frac{1-x_t}{x_t}\left(\frac{H_t^s}{H_t^u}\right)^{1-\eta} . \tag{19}$$

The aggregate levels of human capital supplied by generation t are given by,

$$H_t^u = N_t \int_0^{\widetilde{a}_t} h_t^u f(a)da \quad \text{and} \quad H_t^s = N_t \int_{\widetilde{a}_t}^1 h_t^s(a)f(a)da . \tag{20}$$

Since there is a one-to-one relationship between the share of skilled workers λ_t and the threshold ability $\widetilde{a}_t$, H_t^u is decreasing in λ_t. For any $\{T_t, \pi_t, x_t\}$, the general equilibrium of generation t is characterized by a λ_t that is compatible with the vector of market wages. From (16), λ_t is increasing in w_t^s/w_t^u while the market wage ratio (19) is decreasing in λ_t. Hence, there is a unique equilibrium share λ_t^* which is obtained by substituting (20) and the wage ratio (19) into (16). This equilibrium share is implicitly characterized by the ability threshold $\widetilde{a}_t$ in

$$\frac{h_t^u\left(\int_{\widetilde{a}_t}^1 h^s(a)f(a)da\right)^{1-\eta}}{h^s(\widetilde{a}_t)\left(\int_0^{\widetilde{a}_t} h_t^u f(a)da\right)^{1-\eta}} = \frac{x_t}{1-x_t}\left(\frac{\overline{T}_t - \underline{e}^s}{\overline{T}_t - \underline{e}^u}\right)^{\frac{T_t+\gamma}{T_t}} \tag{21}$$

as shown in the Appendix. Equation (21) implicitly identifies a unique equilibrium share of skilled workers as function of adult longevity and technology,

$$\lambda_t^* = \Lambda(T_t, x_t) . \tag{22}$$

Proposition 1. *For any generation t with $\{T_t \in (\underline{e}^s, \infty), \pi_t \in (0,1), x_t\}$ there exists a unique λ_t^*, given by* (22)*, and a unique equilibrium vector, $\left\{H_t^{j*}, w_t^{j*}, n_t^{j*}\right\}$ for $j = u, s$, for which* (16), (19) *and* (20) *jointly hold. The equilibrium share of skilled individuals λ_t^* is an increasing function of T_t, with slope zero for $T \searrow \underline{e}^s$ and $T \nearrow \infty$.*

The key state variables affecting λ_t^* are adult longevity T_t and the relative importance of human capital in the production function, x_t. An increase in T_t leads to an increase in the share of skilled individuals λ_t^*. The effect of T_t on λ_t^* is non-linear, however. For low T_t the share of population investing in skilled human capital is small due to the fix cost $\underline{e}^s > \underline{e}^u$, which prevents a large part of the population from receiving sufficient lifetime earnings when becoming skilled. When T_t is low the locus $\Lambda(T_t, x_t)$ is convex and large increases in T_t are needed induce a significant fraction of individuals to acquire skilled human capital. At the opposite extreme, if T_t is very large, the locus $\Lambda(T_t, x_t)$ is concave making large improvements in T_t necessary to induce further increases in λ_t due to the decreasing returns to human capital of either type, which drive down the relative wage w^s/w^u.[21] To shorten notation we denote by λ_t the equilibrium share of skilled workers in the following.

2.3 Dynamics

Adult longevity T_t, given in (5), is a function of the share of skilled individuals in the parents' generation. The evolution of x_t is characterized by (7). The share of skilled, λ_t is determined by the intra-generational equilibrium implied by Proposition 1. The total factor productivity, A_t, evolves as in (8). Child survival probability, π_t, evolves according to (6) and also depends on y_{t-1} and, therefore on T_{t-1}, x_{t-1}, λ_{t-1}, and A_{t-1}. Fertility is determined in (18). The dynamic path of the economy is therefore given by a sequence $\{T_t, x_t, \lambda_t, A_t, \pi_t, n_t\}$ for $t = [0, 1, .., \infty)$, which results from the evolution of the nonlinear first-order dynamic system,

$$\begin{cases} T_t &= \Upsilon(\lambda_{t-1}) \\ x_t &= X(x_{t-1}, \lambda_{t-1}) \\ \lambda_t &= \Lambda(T_t, x_t) \\ A_t &= A_{t-1}\left(1 + G(\lambda_{t-1})\right) \\ \pi_t &= \Pi(T_{t-1}, x_{t-1}, \lambda_{t-1}, A_{t-1}) \\ n_t &= N(T_t, \lambda_t, \pi_t) \end{cases} \tag{23}$$

The system is block recursive. Baseline longevity $\underline{T}$ and the past level of the share of skilled workers, λ_{t-1}, determine adult longevity T_t, which in turn affects the current share of skilled workers and technological change. Productivity A_t and child mortality

[21] That λ_t^* is flat for $T_t = \underline{e}^s$ and $T_t = \infty$ does not depend on the presence of retirement as shown in the Appendix. Characterizing analytically the second derivative of $\lambda(T_t)$ is not possible at this level of generality. That there is only one inflection point, so that $\lambda(T_t)$ is increasing and s-shaped, in the parametrization used in the calibration in Section 3 can be shown numerically and can be established analytically when imposing assumptions on the shape of the ability distribution (like, e.g., a uniform distribution).

π_t only depend on past levels of the variables and do not affect the evolution of the dynamic system (23) in terms of the crucial state variables T_t, λ_t and x_t. There are no scale effects in the dynamics so that the level of fertility, n_t, does not affect their evolution either.

The Economic and Demographic Transition. From (7), the endogenous skill biased technical change leads to an increase in the importance of skilled human capital. Also, from (8), the growth rate of technology increases with the share of individuals acquiring skilled human capital and is bounded from above.

Lemma 3. *TFP, A_t, and the relative productivity of skilled human capital x_t increase monotonically over generations with $\lim_{t\to\infty} x_t = 1$, $\lim_{t\to\infty} A_t = +\infty$ and $\lim_{t\to\infty} g_t = \phi$.*

The dynamic evolution of the economy, given by the system (23), exhibits an endogenous economic and demographic transition along the development path.

Proposition 2. [Economic and Demographic Transition] *Considering a sufficiently low x_0, the development path of the economy is characterized by:*
(i) An initial phase with few individuals acquiring skilled human capital, $\lambda \simeq 0$, low longevity, $T \simeq \underline{T}$, large child mortality $\pi \simeq \underline{\pi}$, slow income growth, and gross fertility given by,

$$n \simeq \gamma \frac{\underline{T} - \underline{e}^u}{(\underline{T} + \gamma)\, \underline{r}\ \underline{\pi}}\ . \tag{24}$$

(ii) A final phase of balanced growth in income per capita, with large life expectancy, $T \simeq \overline{T}$, low child mortality $\pi \simeq 1$, almost the entire population acquiring h^s human capital, $\lambda \simeq 1$ and

$$n \simeq \gamma \frac{min\{\overline{T}, R\} - \underline{e}^s}{\left(\overline{T} + \gamma\right) \overline{r}}\ . \tag{25}$$

where $\overline{r}$ is obtained from (15) for $g_{t+1} = \phi$ and $\overline{T}$ from (5) for $\lambda = 1$.

From Proposition 1, the equilibrium share of skilled workers $\lambda_t = \Lambda(T_t, x_t)$ is an increasing but non-linear function. The shape of this equilibrium locus depends on the relative productivity of the different types of human capital, x_t, which affects relative wages. The lower x_t, the flatter the function Λ and the lower the equilibrium share λ for any given T_t. Skill-biased technological change increases x_t across generations making the function (22) successively steeper. From (7) the importance of skilled human capital increases over the course of generations, although initially the improvements are small. This process involves reinforcing feedbacks between increases in human capital, and mortality reductions and technological progress.

The economy eventually converges endogenously to a sustained growth path, maximal adult longevity, minimal child mortality, and virtually the entire population acquiring skilled human capital. The dynamics of fertility, n_t, along the transition process results from (18) given the realized levels of T_{t-1}, x_{t-1}, λ_{t-1}, and A_{t-1}. The interaction between adult longevity and the share of skilled therefore determines the timing of the transition to the balanced growth path and may affect the patterns of comparative development across countries, whereas fertility and child mortality do not affect the dynamics of the economy. In the dynamic system (23) all variables are characterized by interior solutions although the speed of their dynamics changes vary over time until the balanced growth path is reached.[22]

3 Quantitative Analysis

We calibrate the model to match data moments for Sweden in 2000 and around 1800. The simulated data generated by the calibrated model are then compared to the historical time series for Sweden over the period 1750-2000 in order to investigate the fit of long-term development dynamics, as well as to cross-country panel data for the period 1960-2000 to analyze the relevance for cross-sectional patterns of comparative development. We perform controlled variations in baseline adult longevity to study the quantitative role of mortality for the timing of the take-off from quasi-stagnation to sustained growth, for comparative development, and for the worldwide distribution of the variables of main interest.

[22]The Appendix reports a graphical illustration of the phase transition.

3.1 Calibration

The calibration of the model requires setting the values of fifteen parameters that characterize the utility and production function $\{\gamma, \eta\}$, technological progress ϕ, adult life expectancy $\{\underline{T}, \rho\}$, child survival $\{\underline{\pi}, \kappa\}$, skill acquisition $\{\underline{e}^u, \underline{e}^s, \alpha\}$, the distribution of ability $\{\mu, \sigma\}$, and the quality of children $\{\beta, \underline{r}, \delta\}$. In addition, as discussed above we allow for the possibility that individuals retire at some exogenously given age R, which also needs to be pinned down. Finally, the age at reproduction m (corresponding to the length of one generation) and two initial conditions for technology, A_0 and x_0, need to be specified. For a given set of parameters and initial conditions, A_0 and x_0, the evolution of all the variables of interest is determined endogenously by the model along the development path in all periods $t = \{0, 1, .., \infty\}$.

The parameters m, R, and η, as well as the initial condition for x_0 are set exogenously. The other parameters and initial conditions are set endogenously to match data moments for Sweden in 2000 (ϕ, α, μ, σ, γ, A_0), or to match data moments for Sweden in 1800 and 2000 ($\underline{e}^u$, $\underline{e}^s$, β, $\underline{r}$, δ, $\underline{T}$, ρ, $\underline{\pi}$, κ). To study the role of mortality for comparative development, we calibrate an alternative scenario of baseline adult longevity, $\underline{\underline{T}}$, that reflects the worst mortality environment across countries. This calibration targets data moments for pre-transition countries with the highest observed adult mortality in 2000.[23] Finally, we simulate an artificial world composed of countries that are identical in all parameters except for baseline adult longevity, which is distributed in the interval $\{\underline{\underline{T}}, \underline{T}\}$.

For space limitations, the data sources, the details of the calibration and the discussion of the sensitivity of the parametrization are reported in the Appendix. There we also provide a table containing summary information on the data moments used as targets, the data sources and the calibrated parameters.

Benchmark Calibration. The duration of a generation, the age of retirement and the parameter of the production function are set exogenously.

Length of a generation. Across countries the average age of women at first birth before

[23] To explore the role of the endogenous cost of raising children in explaining fertility differences across countries, we also consider an alternative calibration of the quantity-quality trade-off by targeting data moments for pre-transitional countries with the highest recorded fertility in 2000.

the demographic transition is approximately 20 years. A twenty year frequency also allows for a direct match of the simulated data with cross-country panel data without the need of interpolation. We therefore set $m = 20$.

Age of retirement. The average effective retirement age was around 64 in Sweden in 2000. Since R is the number of years before retirement at age $k = 5$, we set $R = 59$.

Production Function. The elasticity of substitution between skilled and unskilled workers is taken to be $1/(1 - \eta) = 1.4$ in the literature, which implies $\eta = 0.285$.

The remaining parameters are set endogenously by matching model moments to data moments for Sweden which is the prototypical example of the economic and demographic transition and represents a natural benchmark for the calibration and for testing the quantitative fit of the model in terms of long-term development patterns. The demographic and economic data for Sweden are available since the mid 18th Century and are of comparably high quality.

From Lemma 3, $\lambda_\infty \simeq 1$. The enrolment shares in Sweden have essentially reached 100% in primary and lower secondary education after 1980 and 1995, respectively. We assume that the transition to the balanced growth path is completed by 2000 and in the calibration we take 2000 to be the year in which λ takes a value arbitrarily close to 1.[24] The determination of some parameters requires the solution of a system of simultaneous equations that target data moments in 2000 and before the onset of the transition, which in the case of Sweden occurred in the first decades of the nineteenth century. In these cases we use data for Sweden in the period around 1800 with a target for the share of skilled workers of $\lambda = 0.1$, which roughly corresponds to the enrolment rates in early 19th Century Sweden.

Technological Progress. The parameter of TFP, ϕ, is set to match the average annual growth rate of income per capita on the balanced growth path (which equals the growth rate of technological change). The average growth rate in Sweden over the period 1995-2010 has been about 2.4 percent per year. This implies targeting a growth factor of 1.61 over a twenty-year period. Given the function (8), and with $\lambda = 1$ along the balanced growth path, we set $\phi = 0.61$.

Human Capital and Ability Distribution. The calibration of the time cost associated

[24] In the simulation, 2000 corresponds to the first generation for which λ exceeds 0.999.

with the different skills requires setting the values of $\{\underline{e}^u, \underline{e}^s\}$. For these parameters we target the average years of schooling in Sweden (for the cohort age 25-35) that was 12 years in 2000. The earliest available data suggest around 1 year of schooling on average before or around the onset of the transition. This implies setting $\underline{e}^s = 12$ and $\underline{e}^u = 0$. The parameter α, relating to the importance of ability for individual human capital, and the moments of the ability distribution $\{\mu, \sigma\}$ are calibrated by targeting the income distribution in Sweden by 2000. By 2000, the income distribution in the model results from the income distribution of skilled workers, since $\lambda \simeq 1$. The individual (per period) income earned by a skilled worker is given by $w_t^s \cdot e^{\alpha a}$, which implies that individual log income is given by $\ln w_t^s + \alpha a$. The assumption of a normal distribution of ability (truncated to lie within a finite interval) and the exponential production function of human capital together imply that for $\lambda = 1$ the distribution of income in the model is also approximately log-normal with thicker tails due to the truncation. With $a \in [0, 1]$, the observed difference between the lowest and the highest income in the data, is matched by setting $\alpha = 6.1$. Matching the other data moments therefore requires setting $\alpha\mu = 3$ and $\alpha\sigma = 0.4$, which for $\alpha = 6.1$, implies $\mu = 0.49$ and $\sigma = 0.066$.

Adult longevity. The baseline mortality parameter, $\underline{T}$, and the parameter linking adult life expectancy to human capital, ρ, are calibrated targeting the levels of adult longevity in 2000 and 1800. Life expectancy at age five in Sweden was approximately 76 in 2000 and 48 around 1800. With these targets, the parameters of the function (5) are set to $\underline{T} = 45$ and $\rho = 31$.

Child survival probability. Child mortality in Sweden fluctuated around one third in the period 1760-1800 and was about 0.004 in 2000. Targeting a child survival probability 0.67 and 0.996 for 1800 and 2000, respectively, and using condition (6) delivers a baseline child survival $\underline{\pi} = 0.5$ and $\kappa = 0.005$.

Preferences. The parameter γ is calibrated by targeting gross fertility $n = 1$ along the balanced growth path, which is also equivalent to targeting the net reproduction rates approximately at replacement levels, with child survival at $\pi = 0.996$. The time spent in raising children is determined endogenously in the model and changes overtime with the growth rate of income and technology. We set a target for the number of years spent raising a child in 2000 of $r = 5$. Solving for γ from (14) with $\lambda = 1$, $\pi = 0.996$, $R = 59$, and $r = 5$ delivers $\gamma = 9$.

Production function of children's quality. To calibrate the parameters of the function that determines the quality of children as outcome of parental investments, (4), we use the optimal time investment by parents in children, (15), and the minimum growth rate of technology $\underline{g}$ for which parents spend no extra time in raising children (see footnote 20). The parameters $\{\beta, \underline{r}, \delta\}$ are calibrated targeting the levels of gross fertility for Sweden in 1800 and 2000, and the growth rate of technology around the period of the exit from the corner solution of zero investments in children's quality. We take 1900 to be the period of the exit from the corner solution of the quantity-quality trade-off. We therefore target the growth level of productivity in 1900 to set the level of $\underline{g}$. An average of 1.2 percent delivers a corresponding growth over a 20-year generation of 0.27. With these targets we get $\{\beta = 0.23, \underline{r} = 4.7, \delta = 3.54\}$.

Sweden, like the other European countries, displays pre-transitional fertility levels that are particularly low in a cross-country perspective.[25] The average total fertility rates of the highest fertility countries was around 7, or above, in 2000 as compared to about 5 for pre-transitional Europe. To explore the role of the cost of raising children for the high fertility countries, we calibrate an alternative ("low fertility cost") quantity-quality function. Changing the target for the pre-transitional fertility to $n = 3.5$ and re-calibrating the parameters accordingly delivers $\{\beta' = 0.75, \underline{r}' = 3, \delta' = 1.06\}$.

Initial Conditions and Time Conventions. The initial importance of skilled human capital in the production function, x_0, is a free parameter that only affects the timing of the take-off in the simulation. Choosing x_0 sufficiently low, the simulation starts in the phase of stagnation. We set $x_0 = 0.04$, which implies that the simulation covers the period from 0 A.D. until 2000 A.D. The initial level of TFP is a shift parameter that is set in order to match as target the level of log GDP per capita in 2000. This implies setting $A_0 = 15$.

The relevant parameters for the timing of the onset of the transition, with the no-

[25]The comparatively low fertility levels in Europe compared to other regions is well documented and the reasons are a matter of debate. Voigtländer and Voth (2012a) trace the roots of the European marriage pattern back to the shock on population following the black death. See also Moav (2005) and Strulik and Weisdorf (2012) on the role of the cost of children for fertility.

table of exception of $\underline{T}$, are calibrated targeting data moments for Sweden in 2000. The parameters governing child mortality and fertility, according to (6) and (15), are calibrated targeting moments in 1800 as well as in 2000. Since the system (23) does not involve any scale effect the calibration of these parameters only affects levels of the child survival probability and fertility, but not the dynamic evolution of the central endogenous variables (share of skilled workers, adult longevity and technological progress) and the timing of the take-off.

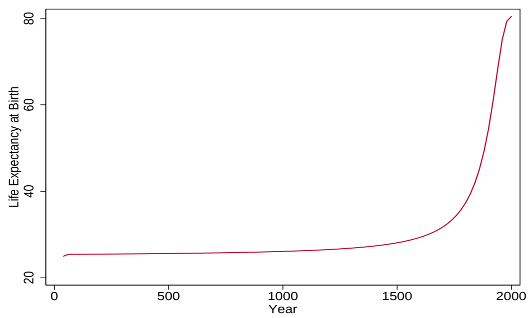
Cross-country differences in baseline life expectancy. The calibrated model is used to investigate the quantitative role of the mortality environment for comparative development. Sweden (and generally European countries) have a comparably favorable mortality environment which is reflected in a relatively low exposure to infectious diseases, whereas the less developed countries of today are often located in areas with a harsher mortality environment. A permanently higher exogenous exposure to infectious diseases implies faster aging and lower life expectancy under similar (economic) living conditions. As alternative scenario, we target a life expectancy at age five of 45 years (compared to 48 years reflecting Sweden around 1800 just before the transition). This target is in line with the lowest available measure in 2000 and life expectancy at age five is not much higher in several Sub-Saharan African and Latin American countries. This implies setting a $\underline{\underline{T}}$=40.

The last element of the calibration that is needed to simulate a cross-sectional distribution of the variables of interest is the world-wide distribution of baseline mortality. To create a meaningful distribution, the calibration is based on data about differences in historical disease prevalence across 113 countries. These data have been collected from historical sources from the early 20th century and therefore reflect extrinsic mortality across the world before major health innovations occurred in most countries. The calibration exploits information on whether a particular infectious disease was detected in a country (i.e., not on the spread of the disease or the number of infected cases, which were potentially endogenous to development already in the 19th century). We simulate a world of 113 countries that only differ in terms of their baseline adult longevity, which lies between the two extremes $\left[\underline{\underline{T}}, \underline{T}\right]$.

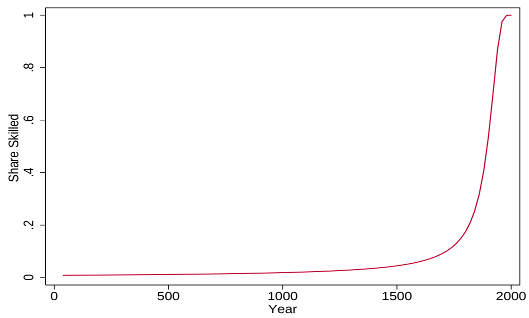
3.2 Results

Time Series Analysis. Figure 1 depicts the simulated data for the equilibrium share of individuals acquiring skilled human capital and of life expectancy at birth that is obtained from the benchmark calibration. The figure plots the evolution of these variables over the entire simulation period and illustrates the lengthy phase of slow development followed by the endogenous take-off.

Figure 1: Long-Run Development: Simulation of benchmark calibration



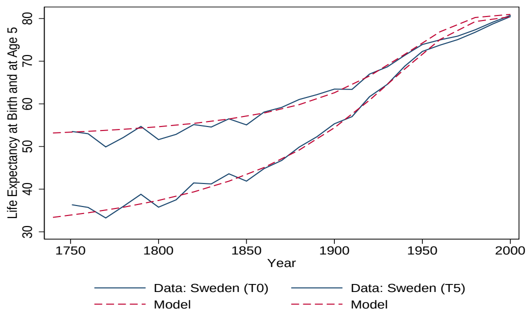
(a) Life Expectancy at Birth



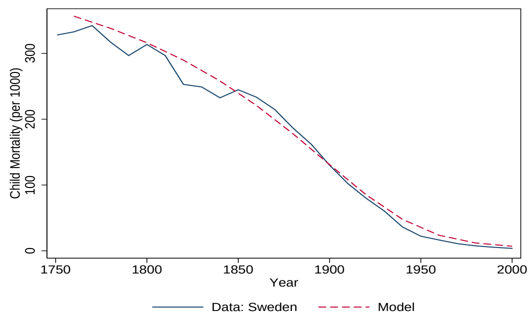
(b) Share Skilled λ

Although the transition appears sudden in a long term perspective, it actually takes place over a time horizon of about 200 years. Figure 2 restricts attention to the period 1750-2000 and compares the simulated data to the corresponding time series of historical data from Sweden. Panels (a) and (b) report the evolution of life expectancy [at birth (T0) and at age five (plus five years, T5)] and child mortality rates, respectively. The calibration targets life expectancy at age five as well as child mortality at two points in time (in 1800 and 2000). The model performs well in matching the evolution of adult longevity over the entire period, both in terms of levels and in terms of the duration of the transition. Also life expectancy at birth (which was not targeted) is matched well. Figures 2(c) and (d) plot the share of skilled individuals, λ, against the primary school enrolment rate and against the (shorter) series of average school years, respectively. Neither data series constitutes a perfect empirical counterpart for λ, but both reflect the education acquisition in the population. The model dynamics resemble the evolution of the enrolment rates in primary education and tend to lead slightly the dynamics of average school years. Given that the model does not account for institutional changes, like compulsory schooling legislation or school systems, the model's dynamics fit the data well.

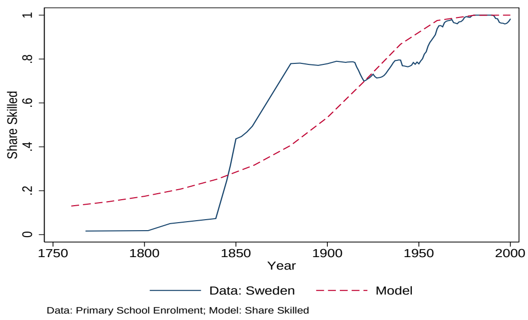
Figure 2: Long-Run Development: Simulation of Benchmark Calibration of the Model and Historical Data for Sweden 1750-2000



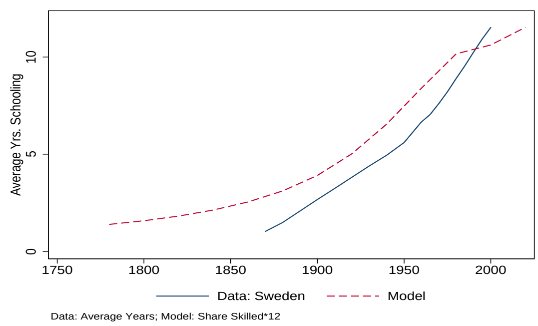
(a) Life Expectancy at Birth



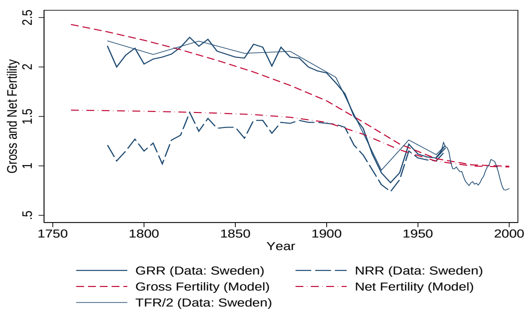
(b) Child Mortality Rate



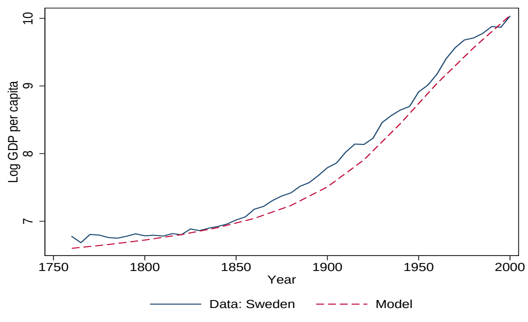
(c) Primary School Enrolment, λ



(d) Average Years of Schooling, λ



(e) Gross and Net Reproduction Rates



(f) log GDP per capita

Figure 2(e) depicts gross and net fertility. The model was calibrated by targeting three moments that are apparent in this figure: the levels of gross fertility before and after the transition (1800 and 2000) as well as the exit from the corner solution of zero additional investment in child quality around 1900. The simulation matches not only the initial and terminal levels and the timing of the drop in gross fertility, however, but also the duration, the level and the dynamic evolution of net fertility, which were not explicitly targeted in the calibration. The presence of differential fertility explains the eventual reduction in net fertility following the reduction in mortality, which has been difficult to rationalize in models based on the quantity-quality trade-off (Kalemli-Ozcan, 2003, and Doepke, 2005). The change in the quantity-quality trade-off is small and the observed drop in gross and net fertility is mainly due to the differential fertility effect and the negative income effect that emerges when life expectancy reaches old ages.[26]

Figure 2(f) depicts the evolution of income per capita. The elasticity of technological progress and the level of initial technology were calibrated to match the growth rate and level of income per capita in 2000. The evolution of income per capita matches closely the data series over the entire period, however, including the pre-transitional level and the acceleration during the transition.

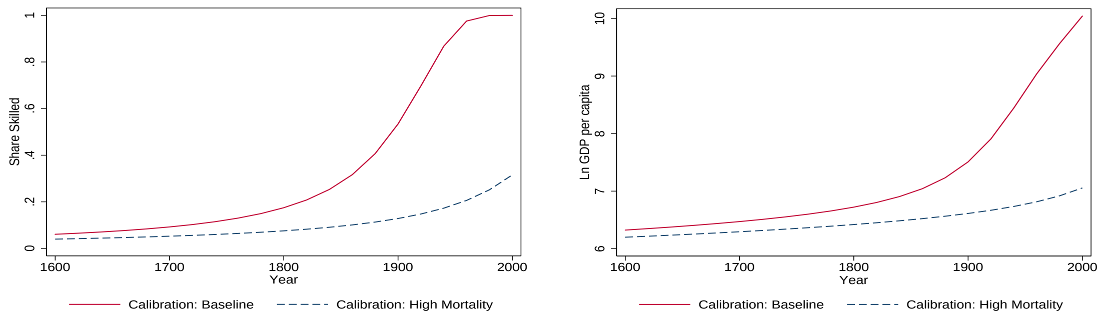
Mortality and Comparative Development. From the system (23), a lower baseline adult longevity implies a lower equilibrium share of skilled for any given level of technology and education of the previous generation, and therefore a delayed take-off. To investigate the quantitative importance of this prediction, we replicate the quantitative analysis with the baseline adult longevity re-calibrated for the countries with the highest baseline mortality, while keeping the remaining parameters of the benchmark calibration unchanged. This counterfactual exercise isolates the role of adult longevity by simulating the same model that has been calibrated for data moments of Sweden and investigating the effects of changes in the baseline longevity to levels that reflect those of the highest mortality countries in sub-Saharan Africa.

Figure 3, plots the share of skilled individuals and income per capita for the benchmark calibration and contrasts them with the counterfactual with $\underline{\underline{T}} = 40$ rather than $\underline{T} = 45$. The take-off is delayed by about 140 years, or 7 generations.[27]

[26]The endogenous cost of raising children is actually very similar before the onset of the transition and on the balanced growth path, with levels of 4.7 and 5, respectively. Assuming a fixed cost of raising children at post-transition levels leaves the benchmark time series of fertility essentially unchanged. This is not the case for the quantity-quality function calibrated targeting data moments for the high fertility countries (see below).

[27]This finding complements the results by Chakraborty et al. (2010) that suggest that the disease environment can have an important impact on economic development.

Figure 3: The Role of Lower Baseline Longevity for Comparative Development



(a) Share Skilled

(b) log Income per capita

The results suggest that differences in extrinsic mortality environment that are compatible with the observed differences in pre-transitional adult longevity of about three years (48 years in Sweden in 1800 and 45 years in sub-Saharan Africa in 2000) can explain an important part of the cross-country differences in comparative development by substantially delaying the timing of the take-off to sustained growth.

Cross-Country Analysis. We now turn to evaluate the ability of the model (calibrated for Sweden) to account for comparative development patterns. If the mechanism driving the transition process is generally valid one would expect that, at each point in time, different countries are in different phases of their (otherwise similar) development process. Notice that no data moment of the cross-country analysis that follows has been explicitly used as target for the calibration of the model. The results can therefore be used to judge the ability of the model to fit the data.

Figure 4 presents the data generated by the baseline model (as depicted in Figure 2), but plotted against the key variable driving the transition, the share of skilled workers λ, at the respective point in time (rather than as time series) using cross-country panel data for the period 1960-2000.[28] Figure 4 panels (a), (b) and (c), plot the simulated data for life expectancy at birth, child mortality and income per capita against λ. These simulated data, which have been generated with the benchmark calibration, are plotted together with corresponding cross country data for 1960 and 2000. The cross-sectional interpretation of the calibrated data fits the cross-country

[28] As empirical counterpart of λ across countries we consider the share of the total population with some formal education, generated as one minus the fraction of the population with "no schooling education" in the total population. See the Appendix for further information.

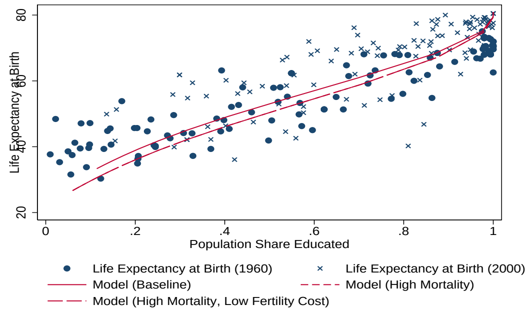
data patterns quantitatively well. For each level of λ the predicted adult longevity and child mortality roughly correspond to the respective average levels in the data. The relation appears remarkably stable over the 40-year horizon and no substantial shift can be seen in the data pattern from 1960 to 2000. The results also document that the simulated data for the alternative, high mortality, calibration are essentially identical to the benchmark calibration and the correlations between λ. Only the cross-sectional relationship between life expectancy at birth and education is slightly different. This further supports the hypothesis that, apart from the timing of the take-off, the different countries experience a very similar development process. The joint consideration of Figures 2, 3 and 4 suggests that differences in baseline mortality may be relevant to explain the delay in comparative development, but their effect is difficult to detect with cross-country panel data.

Figure 4(d) plots the share of skilled against the value of the same variable 40 years (two generations) earlier. In the data, this corresponds to plotting the share of educated individuals in 2000 against that in 1960. The calibration performs comparably better for countries with a larger lagged share of educated individuals while it underestimates the improvements in education for countries with low λ in 1960. This provides a first indication that, compared to Sweden or other European countries for the same level of initial share of educated individuals, the developing countries appear to have experienced an acceleration in education acquisition in the last forty years. Also in this case higher baseline mortality leaves the evolution of education overtime completely unchanged.

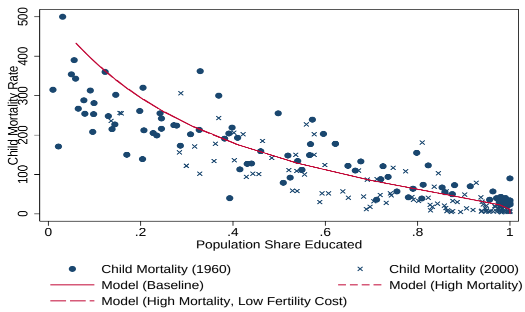
Figure 5(a) and (b) present the respective results for gross and net fertility.[29] The benchmark model matches the fertility levels for the more developed countries (the ones with a relatively large λ) that have undergone the demographic transition around, or shortly after, the period of the demographic transition in Sweden. The benchmark model underestimates the fertility levels for pre-transitional countries with low levels of λ, which in the data correspond to underdeveloped high mortality countries, however. The match between simulated model and data is substantially better for the alternative

[29]Since reproduction in the model is asexual, the level n refers to the gross reproduction rate (the number of daughters for each woman). In order to compare this number to the data on total fertility rates, we multiply the gross reproduction rate n by two.

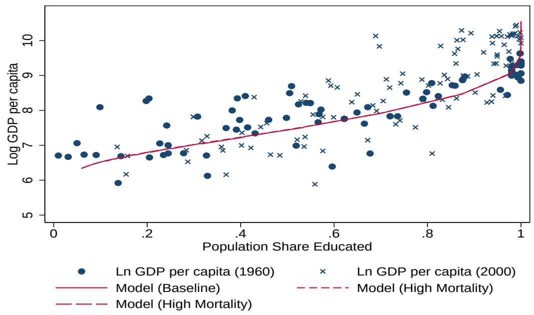
Figure 4: Education, Mortality and Income [Simulation and Data (1960 and 2000)]



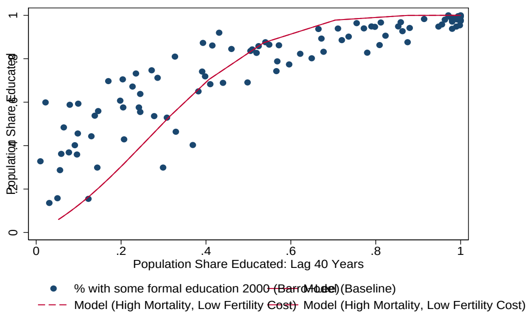
(a) Life Expectancy at Birth



(b) Child Mortality



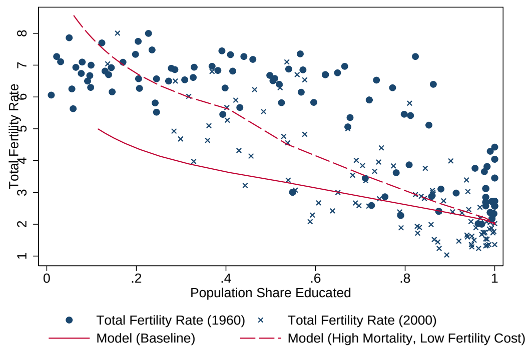
(c) Log GDP per capita



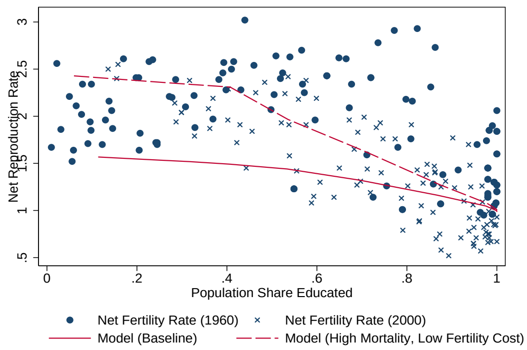
(d) λ 1960 and 2000

parametrization of the quantity-quality trade-off that was calibrated targeting data moments for the highest fertility countries.[30]

Figure 5: Education and Fertility [Simulation and Data (1960 and 2000)]



(a) Total Fertility Rate



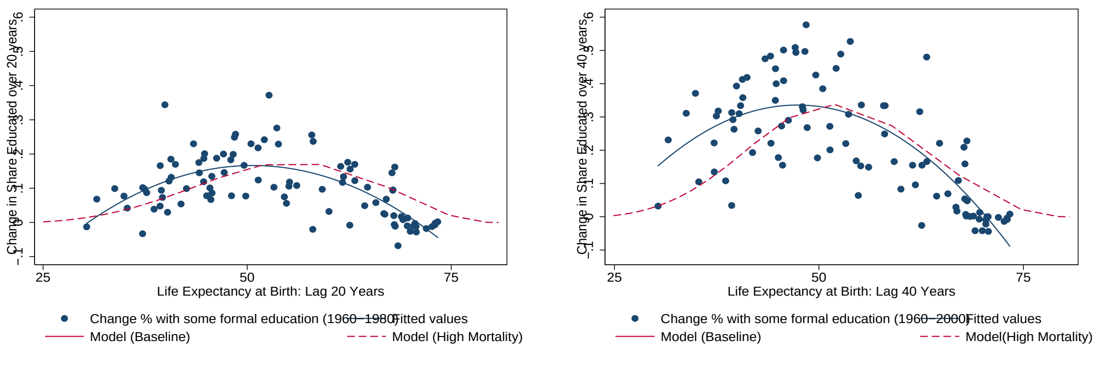
(b) Net Reproduction Rate

The non-linearity of the equilibrium locus Λ, characterized in Proposition 1, im-

[30]The kink in the simulated data corresponds to the exit from the corner solution of the extra time invested in children. Recall that the cross-sectional patterns depicted in Figure 4 are unaffected by the actual calibration of the quantity-quality trade-off and the kink in fertility because the dynamic system (23) is block recursive and does not involve any scale effect.

plies that the changes in λ are largest in the intermediate range where the slope is steepest. The increase in the share of skilled workers is relatively large in countries with intermediate levels of adult longevity, but relatively small in pre-transitional and post-transitional countries. Furthermore, from the dynamic simulation of Figure 2, the countries with largest increases in λ in the short run display lower increases in the future due to the convergence process, while the countries with lowest longevity display the largest overall improvements. The model therefore predicts a non-monotonic relationship between life expectancy and changes in λ at each point in time. Panels (a) and (b) of Figure 6 depict the relationship between life expectancy in 1960 and the change in the share of individuals with no formal education over the following twenty and forty years in the data (including a quadratic regression line), and compares them to the respective data from the benchmark calibration. The model matches the data well but somewhat underestimates the improvements in the change in education in countries with lower initial life expectancy. This again suggests that, compared to the historical experience of Sweden, education improvements in the poorest countries were comparatively large in the period 1960-2000. The model also generates the, Preston Curve (Preston, 1975), see Figure 7.

Figure 6: Life Expectancy and Changes in Education [Simulation and Data]

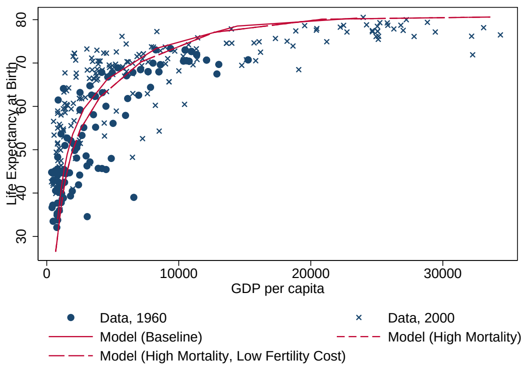


(a) Change in λ over 20 years

(b) Change in λ over 40 years

Cross-Country Distributions. Figure 1 illustrates that the dynamic evolution of the economy is characterized by a very long period of slow development followed by a (comparatively) rapid transition to a sustained growth path. This feature holds irrespective of the level of baseline adult longevity and of the actual timing of the take-off. A direct implication is that even if different countries have a different timing

Figure 7: The Preston Curve



of the take-off, at each point in time relatively few countries should be observed during the transition (since for most of its history each country is either pre-transitional or post-transitional). We would expect the cross-sectional distribution of all variables of interest to display two modes corresponding to the mass of countries that are still pre-transitional or on the balanced growth path, as characterized in Proposition 2.[31] While intuitive, this cross-sectional implication of the non-linear development process has not been pointed out and investigated in the existing unified growth literature.

We simulate the evolution of the countries differing in terms of their baseline adult longevity in the range $[40, 45]$ using the calibrated distribution described above. The data obtained from this artificial world are then pooled, used to estimate the cross-country distribution of all variables of interest in 1960 and in 2000, and compared to the corresponding distributions obtained from cross-county data.

Figure 8: Distributions: Education [Simulation and Data (1960 and 2000)]

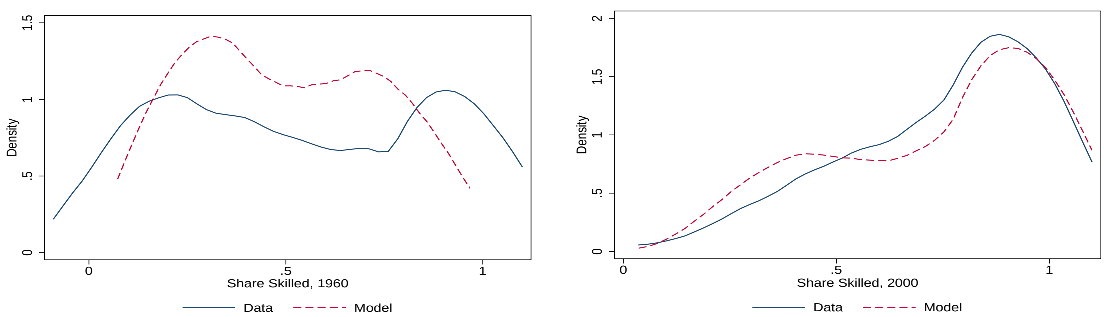
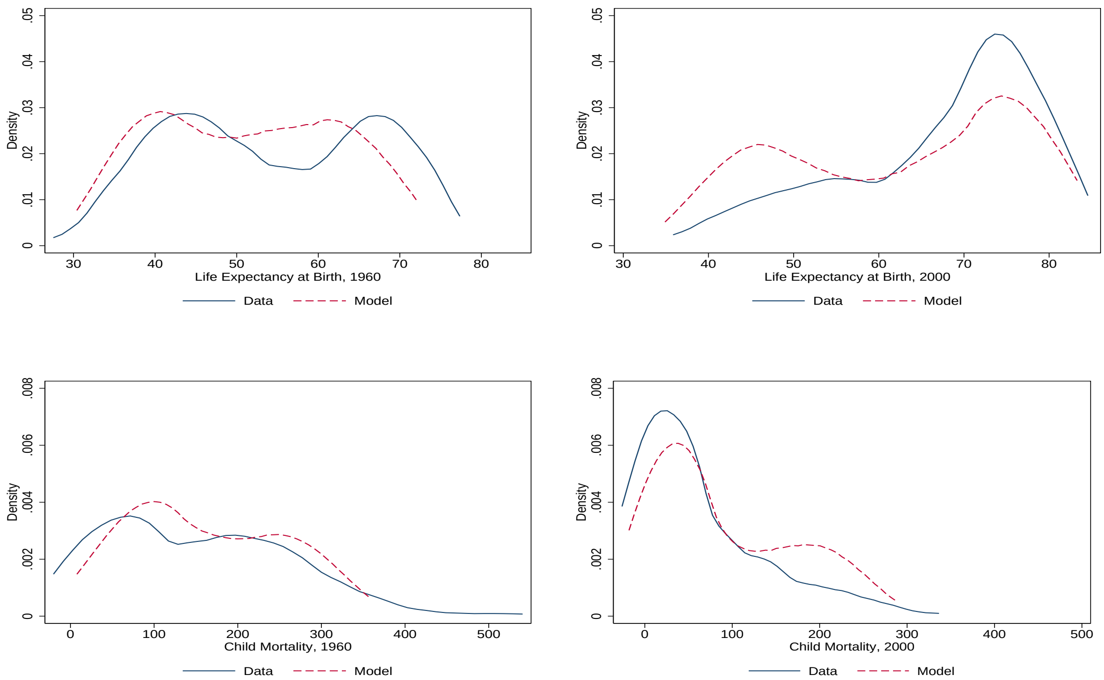


Figure 8 plots the simulated distributions of education for the years 1960 and 2000,

[31]The precise shape of the distribution depends on the actual distributions of the underlying variables, like baseline mortality, that drive the delay in the take-off. Nonetheless, the bi-modality should be detectable as long as sufficiently many countries are still pre-transitional.

and contrasts them to the respective distributions of the actual cross-country data by ways of kernel density estimates. Figure 9 does the same for the distributions of life expectancy and child mortality.

Figure 9: Distributions: Mortality [Simulation and Data (1960 and 2000)]



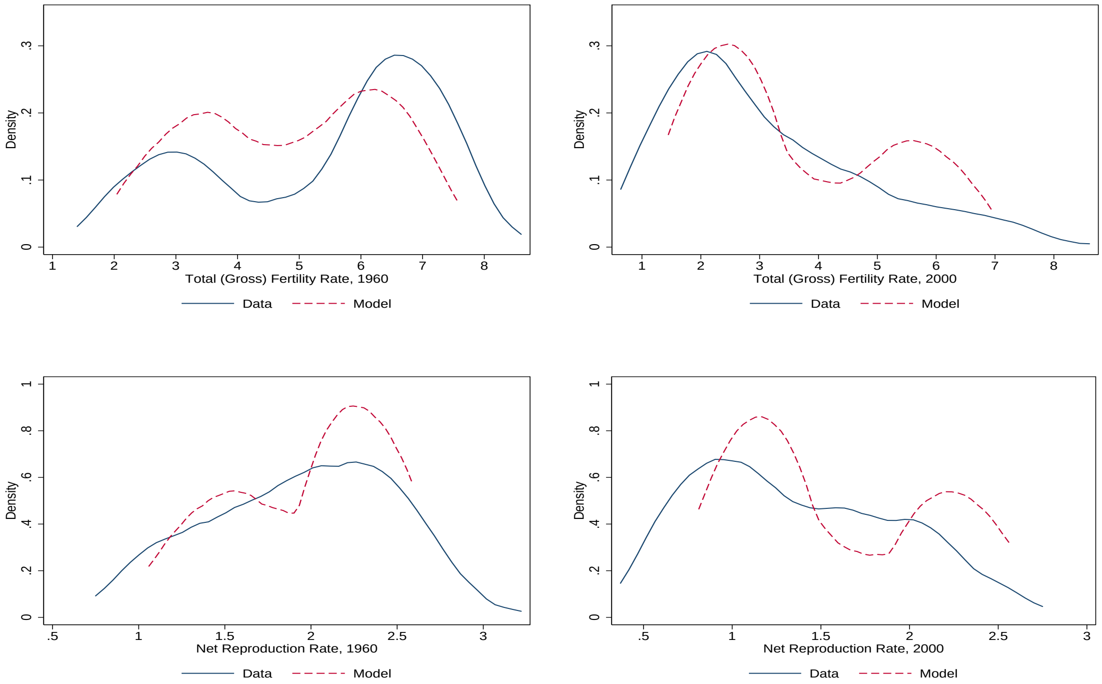
For all variables the expected bi-modality is apparent in 1960 both in the simulated and the actual data, while the distributions tend to be more unimodal by 2000 (when most countries have undergone the transition).[32] The patterns of the actual data are matched in terms of the support, the location of the modes, and the shape of the distribution.

Most of the countries display fertility patterns resembling the high fertility countries, rather than Europe. We therefore simulate the artificial world by considering as benchmark the parametrization of the quantity-quality function that was calibrated targeting data moments for these countries. Figure 10 presents the results for total fertility rates and net reproduction rates for 1960 and 2000. The simulation fits the data by roughly capturing the peaks at low and high levels of fertility, as well as the shape of the distribution and its change over the 40-year horizon.[33]

[32]The bi-modality of the simulated distribution is not due the actual calibrated distribution of baseline mortality. We have performed the exercise also considering a uniform distribution of baseline mortality, with the same pattern of bi-modality and a shift over time.

[33]The actual calibration of the function (15) is irrelevant for the kernel distributions of all variables

Figure 10: Density Distributions of Fertility [Simulation and Data (1960 and 2000)]



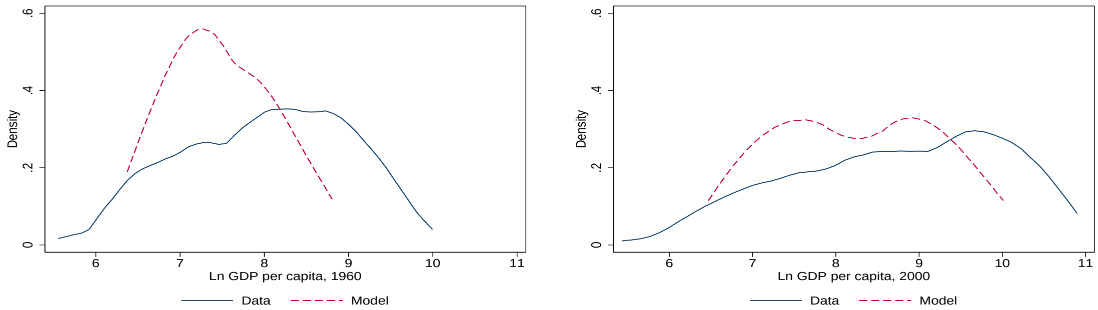
Finally, Figure 11 depicts the world-wide distribution of incomes per capita, which the model fits reasonably well especially for 2000. Notice that the model is limited in capturing the world income distribution by construction, since the model is calibrated to Sweden as the most developed country.[34]

The counterfactual exercise of comparing an artificial world (in which all countries are identical except for the baseline adult longevity) to the actual data suggests a potentially important quantitative role of differences in mortality environment for comparative development patterns. This role has been difficult to identify empirically. The quantitative results presented above suggest that differences in baseline mortality alone

apart from gross and net fertility. Unreported kernel distributions generated with the calibration for Sweden display a similar fit to the actual data for the most developed countries, but underestimate the location of the peak for high fertility. This also implies that differences in the cost of raising children across countries are potentially more important for the cross-country differences in pre-transitional fertility levels than differences in mortality.

[34]The model does not consider other determinants of cross country income differences, like e.g. differences in physical capital, natural resources or institutions that have been shown to be empirically relevant, nor does it consider possible cross-country spill-overs or transfers of technology and innovations. Also, while the sample for GDP corresponds to the 90 countries used for the density plots in Figures 9 and 10, the sample for 1960 only contains 72 countries due to data availability, and is therefore not perfectly comparable.

Figure 11: Distribution of Income per Capita [Simulation and Data (1960 and 2000)]



can potentially explain a substantial share of the observed cross-country differences in the distribution of the variables of main interest.

4 Discussion

We briefly comment on some relevant assumptions and possible extensions that could help improving the qualitative predictions and the quantitative fit.

Differential Fertility. The existence of a fertility differential by education that emerges from the model is one of the most robust stylized facts in demography, see Skirbekk (2008). Before 1750 higher social status (or income, wealth, or social class) was often associated with higher fertility. A reversal in differential fertility can be rationalized by the existence of subsistence levels in consumption.

Differential Mortality. Despite being small compared to the changes in average mortality, differential mortality related to education has been observed in the last decades in countries that have completed the demographic transition. Its consideration would reinforce the role of adult life expectancy for the incentives to acquire skilled human capital.[35]

Complementary Channels of the Fertility Transition. The cost of child raising appears key to explain high fertility in non-European countries. Reproduction is asexual in the model. In reality most of the time cost of raising children is provided by mothers at least before the demographic transition. Studying the differential participation of females in labor markets could help explaining the cross-country differences in the cost of raising children, see Falcao and Soares (2008). We considered an exogenous age of

[35]The locus (22) would be steeper for any level of technology since a mark-up in longevity for the skilled is equivalent to a reduction in the fixed cost $\underline{e}^s$.

reproduction. Postponement would imply a temporarily stronger differential fertility associated with education acquisition.

Health and Labor Supply. Higher life expectancy and education were associated with a lower lifetime labor supply in data from the U.S. over the past 150 years, see Hazan (2009). The predictions of the model are compatible with this evidence. In our occupational choice model increases in T_t increase in the share of skilled. Given the higher wages, becoming skilled is associated with a larger total lifetime income and utility, despite the lower effective lifetime labor supply due to the increasing time spent in acquiring human capital, $\underline{e}^s$.

Triggers of the transition. Skill-biased technical change monotonically increases the importance of human capital. This feature is necessary as long as productivity eventually increases enough to induce a sufficiently large fraction of the population to acquire skilled human capital.[36] Other variables can trigger the transition, with potentially important policy implications. The incentives for the acquisition of skilled human capital depend on the relative effectiveness of the time invested in acquiring education, which may be affected also by public schooling, see Galor et al. (2009), or endogenous investments in health, see de la Croix and Licandro (2012).[37]

Mechanics of Stagnation. Despite the continuous technical progress, the slow development before the take-off is due to the fact that it is not optimal to acquire skilled human capital until the returns are sufficiently large. The Malthusian phase is typically modeled as a corner solution of the dynamic system. Abstracting from such subsistence effects allows performing smooth comparative statics on parameters of interest, most notably for baseline adult longevity, which is key for the cross-sectional analysis.[38] The consideration of corner solutions could nonetheless help improving the quantitative fit of the model in the time series dimension in terms of fertility levels at the onset of the transition, see de la Croix and Doepke (2003) and de la Croix and Licandro (2012).

Sources of Stagnation. The analysis has abstracted from relevant microeconomic

[36]A monotonic technical change may not be realistic if, for instance, appropriate human capital is needed to innovate or adopt innovations, see Aiyar, Dalgaard, and Moav (2008).

[37]The relative importance of human capital in production of goods and the productivity of education are isomorphic in inducing a larger fraction of skilled individuals λ for any T.

[38]Even small changes in baseline parameters change the dynamic evolution of the system, which would not necessarily be the case in the presence of corner solutions in the main state variables.

sources of underdevelopment related to, e.g., capital market imperfections, inequality and limited access to education, and from the political economy of development like lobbying on public policies and social conflicts, see Ray (2010) for a broad perspective. Considering these issues in a unified framework appears a natural direction of future research on the long run determinants of comparative development.

5 Concluding Remarks

This paper has proposed a prototype unified theory of the economic and demographic transition. The dynamic equilibrium path is characterized by the endogenous evolution of mortality, fertility, education and income. The model is calibrated to historical data for Sweden and matches closely the historical time series. In a cross-sectional perspective, it produces out of sample predictions that can account for correlation and distribution patterns of the demographic and economic variables observed in cross-country panel data for 1960-2000. Taken together, the results provide a demonstration of the ability of the unified growth framework to explain the stylized facts in terms of the dynamics, cross-sectional patterns and distributions in the central variables of the economic and demographic transition.

The findings support the view that all countries follow similar development processes, characterized by a long period of stagnation, a rapid take-off, and a convergence to a balanced growth path, even though they differ substantially in terms of the timing of the take-off. The inherently non-linear dynamic development process generates remarkably stable and essentially linear cross-sectional relationships between demographic and economic variables, such as correlations of education with mortality, fertility or income per capita, even among countries with very different levels of development or at different points of their development process. Differences in mortality environments across countries can explain the delay in development of those countries that are permanently exposed to harsher disease environments.

The results suggest some interesting directions for further research. The focus of the paper is on the study of the macroeconomic mechanics of unified theories for long run growth and comparative development. To isolate the role of mortality, the cross-section implications are derived in a world consisting of countries that are identical except their baseline longevity. The analysis of could be extended beyond extrinsic mortality differences to compare the role of alternative macroeconomic channels like, for instance, the role of institutions and other relevant cross-differences for explaining the delay in development. Finally, while instructive regarding the main mechanism, the analysis has completely abstracted from spill-overs (of e.g. technology and medical knowledge) and the interactions between countries at different stages of development. The results suggest that extending the analysis to their consideration can be relevant also for deriving implications for development policies.

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6 Appendix

6.1 Derivations and Proofs

Proof of Lemma 1. Consider an individual acquiring human capital of type $j = u, s$. Taking the first order condition of (13) with respect to n_t^i and restricting to an interior solution gives (14), while taking the first order condition with respect to r_t^i gives,

$$-T_t \pi_t n_t^i r_t^{ij} + \gamma \left(\overline{T}_t - \pi_t n_t^i r_t^{ij} - \underline{e}^j\right) \left(q_r(\cdot) r_t^{ij}\right) / q(\cdot) \geq 0. \tag{26}$$

Using (14) to simplify (26) implies $\left[q_r(r_t^{ij}, g_{t+1}) r_t^{ij}\right] / q(r_t^{ij}, g_{t+1}) \geq 1$. Given the functional form (4) this implies (15).

Proof of Lemma 2. The optimal type of human capital maximizes the indirect utility obtained from $j = u, s$. Evaluating the indirect utility substituting for n_t^{ij} with $j = u, s$ from (14) and noting that $r_t^{iu} = r_t^{is} = r_t^*$ from (15) implies that the optimal type of skill depends on,

$$\left(\overline{T}_t - \underline{e}^u\right)^{(T_t+\gamma)} \left(w_t^u h_t^u\right)^{T_t} \gtrless \left(\overline{T}_t - \underline{e}^s\right)^{(T_t+\gamma)} \left(w_t^s h_t^s(a)\right)^{T_t} . \tag{27}$$

Since the indirect utility obtained by acquiring skilled human capital increases with ability, there exists a unique $\widetilde{a}_t$ such that all individuals with $a < \widetilde{a}_t$ optimally choose to acquire u, while those with $a > \widetilde{a}$ optimally choose to obtain s. Solving (27) as equality gives (16).

Proof of Proposition 1. The wage ratio is given by,

$$\frac{w_t^u}{w_t^s} = \frac{1 - x_t}{x_t} \left(\frac{\int_{\widetilde{a}_t}^1 h^s(a) f(a) da}{\int_0^{\widetilde{a}_t} h^u f(a) da} \right)^{1-\eta} . \tag{28}$$

Substituting (28) into (16) gives the general equilibrium ability threshold (21). Rearrange (21) to get the equilibrium relationship between $\widetilde{a}_t$ and T_t expressed as

$$\mathcal{G}\left(\widetilde{a}_t\right)^{1-\eta} F\left(x_t\right) - \left(\frac{\overline{T}_t - \underline{e}^s}{\overline{T}_t - \underline{e}^u} \right)^{\frac{T_t+\gamma}{T_t}} = 0 \tag{29}$$

where $\overline{T}_t := min\{T_t, R\}$, $\mathcal{F}(x) := ((1-x_t)/x_t)$ and

$$\mathcal{G}(\widetilde{a}_t) = \frac{(h^u)^{\frac{1}{1-\eta}} \int_{\widetilde{a}_t}^{1} h^s(a) f(a) \, da}{h^s(\widetilde{a}_t)^{\frac{1}{1-\eta}} \int_0^{\widetilde{a}_t} h^u f(a) \, da} \tag{30}$$

with $\mathcal{G}'(\widetilde{a}_t) < 0$. Notice that $\left[\left(\overline{T}_t - \underline{e}^s\right) / \left(\overline{T}_t - \underline{e}^u\right)\right] \in (0,1)$ for $T_t \in (\underline{e}^s, \infty)$. For any x_t, the function (29) is therefore defined over the range $\widetilde{a} \in (\underline{a}(x_t), 1]$ where[39]

$$\underline{a}(x_t) : \mathcal{G}(\underline{a}(x_t))^{1-\eta} \mathcal{F}(x_t) = 1 \tag{31}$$

Applying calculus it follows that $\partial \underline{a}(x_t)/\partial x_t < 0$ with $\lim_{x\to 0} \underline{a}(x) = 1$ and $\lim_{x\to 1} \underline{a}(x) = 0$. Accordingly for any x_t there exists a level $\overline{\lambda}(x_t) < 1$ which represents the maximum share of the population that for each generation t would acquire skilled human capital in the case in which $T_t \to \infty$. By totally differentiating (29) we have,

$$\frac{d\widetilde{a}_t}{dT_t} = \frac{d\left(\left(\frac{\overline{T}_t - \underline{e}^s}{\overline{T}_t - \underline{e}^u}\right)^{\frac{T_t+\gamma}{T_t}}\right)/dT_t}{\left[(1-\eta)\mathcal{G}(\widetilde{a}_t)^{-\eta} \mathcal{G}'(\widetilde{a}_t) F(x_t)\right]} < 0 \tag{32}$$

which is negative since $\mathcal{G}'(\widetilde{a}_t) < 0$ and for $T_t < R$, the numerator is,[40]

$$-\frac{\gamma}{T_t^2} \ln\left(\frac{T_t - \underline{e}^s}{T_t - \underline{e}^u}\right) e^{\ln\left(\frac{T_t - \underline{e}^s}{T_t - \underline{e}^u}\right)} e^{\frac{T_t+\gamma}{T_t}} + \frac{T_t+\gamma}{T_t}\left(\frac{T_t - \underline{e}^s}{T_t - \underline{e}^u}\right)^{\frac{\gamma}{T_t}} \frac{\underline{e}^s - \underline{e}^u}{(T_t - \underline{e}^u)^2} > 0 \tag{33}$$

For $T_t = \underline{e}^s$ we have $\widetilde{a}_t = 1$ which implies $\mathcal{G}(\widetilde{a}_t) = 0$ so that $\mathcal{G}(\widetilde{a}_t)^{-\eta} = \infty$. Since $\mathcal{G}'(1)$ is a finite number we have that the denominator of (32) goes to infinity as $T_t \to \underline{e}^s$. In turns the numerator has a limit at zero. For $T_t \to \infty$ we have $\widetilde{a}_t \to \underline{a} < 1$ so that the denominator of (32) is a finite number while the numerator has a limit at zero.[41] Hence $\lim_{T_t \to \underline{e}\frac{d\widetilde{a}_t}{dT_t}}{}^s = \lim_{T_t\to\infty} \frac{d\widetilde{a}_t}{dT_t} = 0$ which also implies that the equilibrium locus (22) is convex for $T_t \to \underline{e}^s$ and concave for $T_t \to \infty$.

Proof of Lemma 3. From Proposition 1 for any $T_t > \underline{e}^s$ and any $x_t > 0$, we have $\lambda_t > 0$. From (7) this implies $x_t > x_{t-1}$ for all t with $\lim_{t\to\infty} x_t = 1$; from (8), $g_t > 0$ and $\lim_{t\to\infty} A_t = \infty$ for any $A_0 > 0$. In the limit as $\lambda_t \to 1$, $g_t = \phi$ from (8).

[39] Since the denominator of (29) has a discontinuity at $\underline{a}$ and the function takes negative values for any $a \leq \underline{a}(x_t)$.

[40] If $\overline{T}_t = R$ then equation (33) reads as $-\frac{\gamma}{T_t^2} \ln\left(\frac{R-\underline{e}^s}{R-\underline{e}^u}\right) e^{\ln\left(\frac{R-\underline{e}^s}{R-\underline{e}^u}\right)} e^{\frac{T_t+\gamma}{T_t}} > 0$.

[41] The same is true if $T_t > R$ since $\lim_{T_t\to\infty}\left[-\frac{\gamma}{T_t^2} \ln\left(\frac{R-\underline{e}^s}{R-\underline{e}^u}\right) e^{\ln\left(\frac{R-\underline{e}^s}{R-\underline{e}^u}\right)} e^{\frac{T_t+\gamma}{T_t}}\right] = 0$.

Proof of Proposition 2. The equilibrium relationship linking $\widetilde{a}_t$ and T_t is given in (29). For any T_t, $\widetilde{a}_t$ is an implicit function of x_t. Recall that by implicit differentiation of (21) $\partial\widetilde{a}_t/\partial x_t < 0$ which implies that the equilibrium share of skilled individuals is increasing in $x_t : \partial\lambda_t/\partial x_t > 0$ for any T_t. Consider part *(i)*. If $x_0 \simeq 0$ and $A_0 \simeq 0$ then $\underline{a}\,(0) \simeq 1$; for all $T \in (\underline{e}^s, \infty)$ which implies $\widetilde{a} \simeq 1$ and $\lambda \simeq 0$. In this case the two loci Λ and Υ cross only once for $\lambda \simeq 0$ and $T \simeq \underline{T}$ and the average fertility is given by n^u as implied by (14) evaluated at $T = \underline{T}$. Under these conditions, from (2) the level of income per capita is (arbitrarily) low which, from (6) and (24) implies $\pi_0 \simeq \underline{\pi}$. Phase *(ii)* follows directly from Lemma 3, where $A_\infty \to \infty$, $x_\infty \to 1$, $\lambda_\infty \simeq 1$, $T = \overline{T}$. From (8) this also implies that $g_\infty = \phi$. Finally, since $A_\infty \to \infty$, it follows that $y_\infty \to \infty$ and from (6), $\pi_\infty \simeq 1$ so that fertility is given as in (25).

Figure 12 depicts the evolution of the conditional system given by equations (5) and (22) for the case in which the latter function has a unique inflection point. From *(i)* and *(ii)* the conditional system has a unique steady state for x_0 and x_∞ as illustrated in Figure 12 Panels (a) and (c).

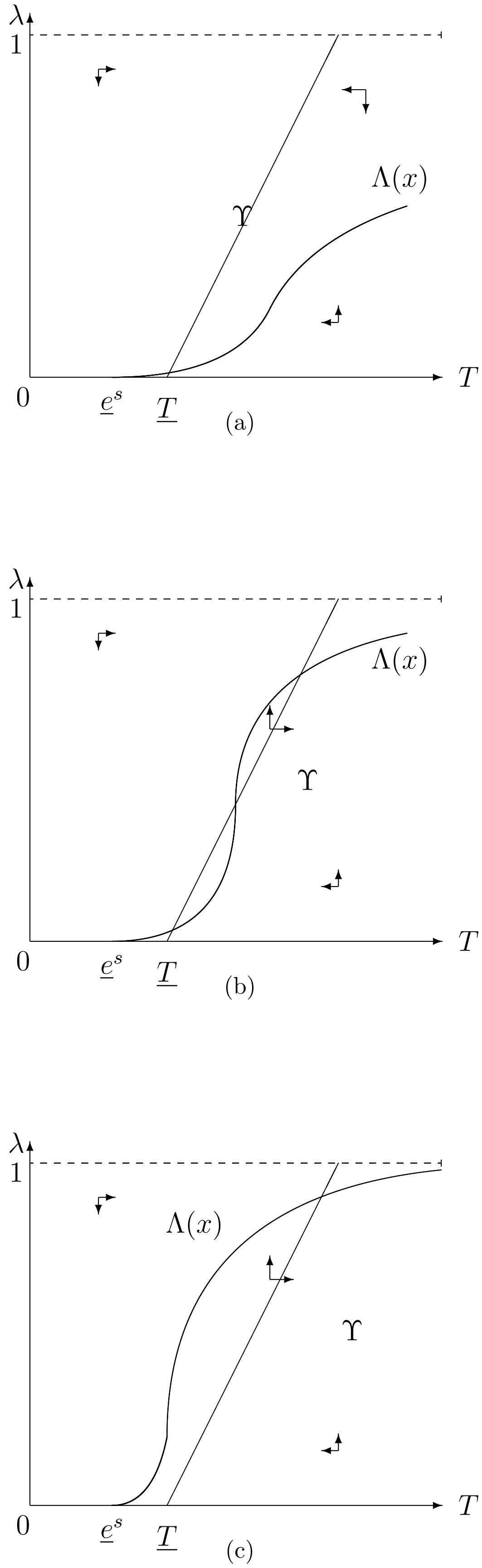


Figure 12: The Process of Development

6.2 Calibration: Data Sources and Details

Length of a generation. The mean age at first birth for the average country is set to 20 years. in Sweden around 1800 was slightly higher, see Dribe (2004), while age at first birth is still below 20 in pre-transitional countries in Africa nowadays, see Mturi and Hinde (2007);

Age of retirement. Data from `http://www.oecd.org/dataoecd/3/1/39371913.xls`;

Technological Progress. Data sources: ERS Dataset (`www.ers.usda.gov`) or historical statistics from the Bank of Sweden (`www.historicalstatistics.org`);

Production Function. The elasticity of substitution between skilled and unskilled workers is set following the literature. See for instance Acemoglu (2002);

Human Capital. To calibrate some parameters we target a 10 percent pre-transitional share of skilled individuals. The alternative available data sources provide slightly heterogeneous information on enrolment rates in the early 19th Century Sweden, with estimates ranging from about 5 to about 15 percent, see de la Croix et al. (2008) and Ljungberg and Nilsson (2009). The target of the precise value of λ before the transition used for the computation is of little importance for the the obtained parameters, however. The results for alternative parameters obtained by targeting levels of λ up to 0.3 are essentially the same. The average years of schooling in Sweden was 12 years in 2000 (for the cohort age 25-35). Data from Lutz et al. (2007). The earliest available data suggest around 1 year of schooling on average before or around the onset of the transition. The estimates are slightly lower when referring to the entire population alive in Sweden in 2000 since older cohorts are included (for instance 11.4 in the data of Barro and Lee, 2001 and 11.5 years in Ljungberg and Nilsson (2009)). Regarding pre-transitional education levels, the estimates differ somewhat more. Ljungberg and Nilsson (2009) report 1.03 years of schooling in the total Swedish population aged 15-65 in 1870, and 0.1 average standard school years of the population aged 7-14 around 1810-1820, considering absenteeism and length of school years.

Ability Distribution. We estimate the income distribution for Sweden in 2000 using micro data from the ECHP dataset for individual incomes of full-time employees aged 25 to 45, which corresponds to the two last cohorts in the dynamic simulation, and equivalently to the two first generations with $\lambda = 1$ in the data. The income used to estimate the parameters of the ability distribution are converted in US-$ using an average exchange rate of 9 Kroner for one US-$ in 2000. The income distribution is approximately log-normal between the 5th and 95th percentile of the data, with slightly thicker tails. The distribution of log incomes has mean 9.7, standard deviation 0.4, and the lowest and highest observed log-incomes are 6.7 and 12.8, respectively, which implies a maximum spread of 6.1. The moments of the income distribution for the age cohort 25-65 are essentially the same, with the lowest, mean, and highest levels of log income being 6.7, 9.7, and 12.8, respectively, and with a standard deviation of 0.41. The ECHP data are based on surveys and refer to total net income from work, which might explain the small differences between the log income per capita from macro data, which is approximately 10 in 2000, and the mean log income from the micro data that is about 9.7. The relevant data moments extracted from this data set are broadly consistent with other data sources based on register data and alternative surveys for gross earnings, see, Domeij and Floden (2010). The data moments are also close to the ones typically used for the calibration of dispersion in permanent incomes in other OECD countries. For instance, Erosa et al. (2011) match a variance of log permanent earnings in the US of 0.36. Robustness checks show that the results are fairly insensitive to varying the dispersion. It is worth noting that the distribution of cognitive ability (or IQ), which is generally measured in the literature as a truncated normal with mean 100 and standard deviation 15, see, e.g., Neisser et al. (1996), would imply a very similar parametrization when normalized for a support $a \in [0, 1]$, with $\mu = 0.5$ and $\sigma = 0.075$.

Adult longevity. The average of life expectancy at age five in the period 1760-1840 was 48.38, in the period 1790-1810 it was 48.06. Data from the Human Mortality Data Base available at `http://www.mortality.org/`. Similar figures are documented for

England, France and Italy, see Woods (1997) and Bideau et al. (1997) and Lewis and Gowland (2007). In 2000 child mortality in Sweden was around 0.004, which explains the convergence of life expectancy at 5 plus five years of 80.74, and of life expectancy at birth of 80.45.

Child survival probability. Data from `http://www.mortality.org`. The levels of income per capita needed for the computation of the parameters of the function of child survival are extracted from the database of historical statistics of the Bank of Sweden that is freely available online at `www.historicalstatistics.org`. The data are converted to US-$ using an average exchange rate in 2000 of 9 Kroner for one US-$. The income levels used for the calibration of condition (6) are 22,717 and 884 US-$, which correspond to the GDP per capita of Sweden in 2000 and 1800, respectively, in US-$ per 2000.

Preferences. Total fertility rates (TFR) in Sweden were on average 1.8 children per woman over the period 1980-2000, with substantial fluctuations. In 1990, the TFR was 2.13, whereas in 2000 it was 1.54 (World Development Indicators). A gross fertility of 1 (which would correspond to a TFR of 2) along the balanced growth path is a reasonable target. Targets in the range from 0.75 to 1.1 deliver very similar results. Concerning the cost of raising children, the target $r = 5$ in 2000 is set in line with the estimates by Haveman and Wolfe (1995). This is equivalent to setting a target for the share of work life that is spent in raising a child is about 15 percent which is in line with Doepke (2004) and de la Croix and Doepke (2003). The weight of children relative to own lifetime consumption changes with T_t, as in Soares (2005). For $\gamma = 9$ the relative weight of children compared to per period consumption, γ/T_t, drops from around 0.18 before the transition to around 0.12 in the steady state.

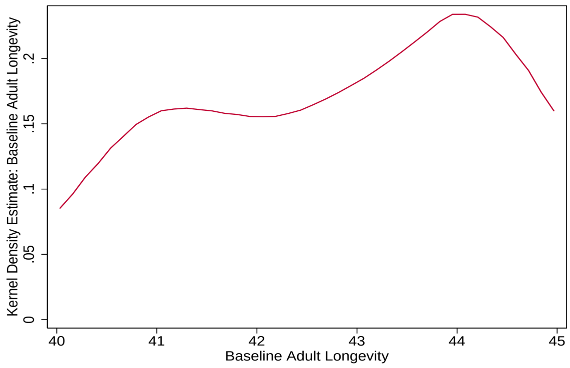
Production function of children's quality. Gross fertility in Sweden in 1800 and 2000 was $n = 2.3$ and $n = 1$. A clear drop in gross fertility occurs around 1900. The data are from Keyfitz and Flieger (1968) and World Development Indicators. The level of TFP and income per capita growth around 1900 vary between 0.7 and 1.7 percent per year. The largest estimates are based on indexed data and include land, see Krantz and Schön (2007), Schön (2008) and Greasley and Madsen (2010). Estimates of TFP and income per capita growth around 1900 vary between 0.7 and 1.7 percent per year. For the calibration we consider the average, 1.2. As an alternative calibration that does not rely on information about the growth rate of technology during the transition, one can also use information on the share of skilled around 1900 and compute the growth rate that is implied by (8). According to estimates by Ljungberg and Nilsson (2009) average years of schooling for the cohort aged 7-14 was around 4 in 1900. Given $\{\phi = 0.61, \underline{e}^u = 0, \underline{e}^s = 12\}$ this implies targeting a level of $\underline{g} = 0.2745$, which delivers essentially the same parametrization.

Initial Conditions. The time axis is set with reference to the convergence to the post-transitional balanced growth path (in terms of λ converging to 1) in 2000. This implies that the choice of $x_0 = 0.04$ determines the beginning of time in the calibration in the stagnation period. This parametrization also implies that the income share of unskilled human capital in total production is larger than 99.9% at the beginning of the simulation, and still above 95% in 1800 just before the transition. The initial level of technology is set targeting the level of GDP per capita in Sweden in 2000 equal to 10.03. Data are from `www.historicalstatistics.org`.

Cross-country differences in life expectancy. For background evidence on the role of a higher exposure to diseases in leading to a faster deficit accumulation and earlier death see, e.g., Mitnitski et al. (2001) and Searle et al. (2007). Research based on the investigation of skeletons documents that adult longevity during the Mesolithic period was lower in more difficult mortality environments, see Boldson and Paine (2000). As alternative scenario, we target a life expectancy at age five at 45 years (compared to 48 years reflecting Sweden around 1800 just before the transition). The data source is UN Population Statistics available at `www.unstats.un.org`. Data on life expectancy at five for earlier periods are missing for many countries, including most Sub-Saharan Africa countries in 1960. Alternatively, the available information on child mortality and life expectancy at birth in 1960 can be used to derive an estimate of life expectancy at age five. This delivers a very similar target for the highest mortality countries. In 1960 life expectancy at birth was as low as 33 years in some countries like Afghanistan, and child mortality one third. Assuming a constant death rate below the age of 5, these numbers imply a life expectancy at age five between 44 and 45 years. In some countries, like Swaziland life expectancy at birth is just above 30 years still today (data from the CIA World Factbook). This suggests that 45 is possibly a conservative estimate of baseline adult longevity in the worst conceivable mortality environment. Retaining a target of 76 years for life expectancy at age five on the balanced growth path, this implies setting a $\underline{\underline{\mathrm{T}}}$=40 and $\overline{\rho} = 36$ (rather than $\underline{\mathrm{T}}$=45 and $\rho = 31$ as in the benchmark calibration).

Cross-country differences in disease environment. The data in the historical disease prevalence across 113 countries is taken from Murray and Schaller (2010). For each pathogen we construct a binary indicator of whether or not a disease has been present at severe or epidemic levels at least once in the history up to the early 20th century. The diseases include leishmanias, schistosomes, trypanosomes, leprosy, malaria, typhus, filariae, dengue, and tuberculosis. Six of these diseases fall into the class of multi-host vector-transmitted diseases, which are particularly difficult to prevent or eradicate even today because the pathogens survive in multiple hosts (both humans and animals), and which are bound to specific transmission vectors, like mosquitos, which require a particular geographical habitat. The endemicity of the class of multi-host vector-transmitted diseases is fairly insensitive to economic development and globalization, and thus an informative measure of cross-country differences in the extrinsic mortality environment, see Smith et al. (2007). Cervellati, Sunde and Valmori (2012) document the health relevance of the number these pathogens in terms of predicting life expectancy and the likelihood of outbreaks of epidemics. The frequency distribution of the counts of pathogens for all countries of the world is used as distribution of baseline adult longevity within the support $[40, 45]$. The resulting distribution, depicted in Figure 13 in terms of a kernel density plot, is modestly skewed. The frequency of simulated countries with baseline longevity $\underline{T} = 45$ corresponds to the frequency of countries with the lowest observed number of multi-host vector-transmitted diseases ever diagnosed (which includes Sweden). Conversely, the frequency of simulated countries with baseline longevity $\underline{\underline{T}} = 40$ corresponds to the frequency of countries with the highest number of multi-host vector-transmitted pathogens (which include several Sub-Saharan African countries). The distribution on the full support $(40, 45)$ is created by a linear intrapolation of the frequency distribution of the counts of multi-host vector-transmitted diseases on a grid of 0.25 diseases. The figure plots the resulting distribution of baseline longevity for the 113 countries of the Murray-Schaller (2010) data.

Figure 13: Synthetic Distribution of $\underline{T}$



6.3 Data Sources for Time Series and Cross-Section

Time Series for Sweden. Life expectancy and fertility data are taken from the Human Mortality Database (`http://www.mortality.org`), Keyfitz and Flieger (1968) (up to 1960) and World Development Indicators (after 1960), respectively. The Data for GDP, population and GDP per capita is provided by the internet portal for historical Swedish statistics, `www.historia.se` and the Swedish Central Statistical Office, `www.scb.se`. The data on schooling are from de la Croix, Lindh, and Malmberg (2008) while the data on average years of schooling are from Ljungberg and Nilsson (2009).

Cross Country Panel Data. We use data from Barro and Lee as benchmark since they are used more frequently and go back to 1960. The other data sources are Human Mortality Database (`www.mortality.org`), the UN Population Statistics (different historical volumes of the UN Demographic Yearbook, `www.unstats.un.org`), the World

Development Indicators at:
(`http://data.worldbank.org/data-catalog/world-development-indicators`).

All results are qualitatively and quantitatively very similar using alternative measures like the fraction of total population with at least completed lower secondary education, or the fraction restricted to different age cohorts such as, e.g., age 20-24 years from Lutz, Goujon, and Sanderson (2007).

Kernel Distribution. For comparability, the distributions of real data are based on a homogenized sample of 90 countries, for which information on the share of skilled individuals, life expectancy at birth, child mortality, total fertility rate, and the net reproduction rate is available for 1960 and 2000. The results are similar when using unrestricted samples for the different variables.

Table 1: Summary Information on Calibration of Parameters

Parameter		Value	Matched Moment (Information Source)
Benchmark Calibration			
Parameters Set Exogenously			
Year of convergence to balanced growth path		2000	First generation with $\lambda > 0.999$
Length of one generation	m	20 years	Average age at first birth (Dribe, 2004, Mturi and Hinde, 2007)
Years before retirement (at age 5)	R	59	Average effective age of retirement in Sweden (OECD)
Production function	η	0.2857	Elasticity of Substitution between skilled and unskilled labor (Acemoglu, 2002)
Parameters Set Endogenously			
TFP growth	ϕ	0.61	Average growth GDP per capita 1995-2010 (ERS Dataset, Sweden)
Time cost for unskilled/skilled education	$\{\underline{e}^u, \underline{e}^s\}$	{0,12}	Years of schooling in 1820 and 2000 (Lutz et al. 2007/Ljungberg-Nilsson, 2009)
Productivity of ability for Human Capital	α	6.1	Spread of log income distribution 2000 (ECHP)
Mean/standard deviation of ability distribution	$\{\mu, \sigma\}$	{0.49,0.066}	Mean and variance of log income in 2000 (ECHP)
Baseline adult longevity/scope for improvement	$\{\underline{T}, \rho\}$	{45,31}	Average LE at 5 in 1760-1800 and 2000 (Human Mortality DataBase)
Minimum child survival and elasticity parameter	$\{\underline{\pi}, \kappa\}$	{0.5, 0.005}	Child survival probability in 1800 and 2000 (Human Mortality Data Base)
Preferences	γ	9	Gross (total) fertility around 2000 (World Development Indicators)
Function quality of children	$\{\beta, \underline{r}, \delta\}$	$\{0.23, 4.7, 3.54\}$	Pre/Post-transitional Fertility, TFP growth 1900 (Keyfitz and Flieger, 1968, World Development Indicators, Historical Statistics Sweden)
Initial Conditions			
Initial importance of skilled human capital	x_0	0.04	Initial year of calibration, generations before balanced growth is reached
Initial TFP	A_0	15	Level of log GDP per capita Sweden 2000 (Historical Statistics Sweden)
Cross-Country Analysis			
Parameters Set Endogenously			
Baseline adult longevity/scope for improvement	$\{\underline{\underline{T}}, \rho'\}$	{40,36}	Minimum observed life expectancy at age 5 across country in 2000 (UN)
Function quality of children (high fertility)	$\{\beta', \underline{r}', \delta'\}$	$\{0.75, 3, 1.06\}$	Highest fertility rates 1960 (World Development Indicators)
Distribution of baseline adult longevity			Worldwide distribution of Human Pathogens (Murray and Schaller, 2010)

Bibliography for Appendix

The following references refer exclusively to the data sources and articles cited in the Appendix.

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