Performance of Multiple Cabin Optimization Methods in Airline Revenue Management

by

Pierre-Olivier Lepage
B.Eng., Industrial Engineering, École Polytechnique de Montréal, 2010

Submitted to the Department of Civil and Environmental Engineering
and Sloan School of Management
in partial fulfillment of the requirements for the degrees of
Master of Science in Transportation
and
Master of Science in Operations Research
at the Massachusetts Institute of Technology
June 2013

Signature of Author: ______________________________

Department of Civil and Environmental Engineering
and Sloan School of Management
May 10, 2013

Certified by: ______________________________

Peter P. Belobaba
Principal Research Scientist, Aeronautics and Astronautics
Thesis Supervisor

Certified by: ______________________________

Cynthia Barnhart
Ford Professor, Civil and Environmental Engineering and Engineering Systems
Associate Dean, School of Engineering
Thesis Reader

Accepted by: ______________________________

Heidi Nepf
Chair, Department Committee for Graduate Students

Accepted by: ______________________________

Patrick Jaillet
Dugald C. Jackson Professor, Electrical Engineering and Computer Science
Co-Director, Operations Research Center

Performance of Multiple Cabin Optimization Methods in Airline Revenue Management

by Pierre-Olivier Lepage

Submitted to the Department of Civil and Environmental Engineering
and Sloan School of Management on May 10, 2013,
in partial fulfillment of the requirements for the degrees of
Master of Science in Transportation
and
Master of Science in Operations Research

Abstract

Although many airlines offer seats in multiple cabins (economy vs. premium classes) with different service quality, previous work on airline revenue management has focused on treating the cabins separately.

In this thesis, we develop several single-leg multiple cabin revenue management optimization algorithms. We extend two different single-leg separate cabin dynamic programming algorithms to the multiple cabin case, and also present three Expected Marginal Seat Revenue (EMSR) based heuristics and a dynamic programming decomposition heuristic. We then evaluate the revenue and passenger mix performance of the different algorithms using the Passenger Origin-Destination Simulator (PODS) which simulates competitive markets with passenger choice of fare options and cabin. We first test the methods in a simple single market network and then in a more realistic complex network.

We find that multiple cabin methods do not lead to a systematic revenue increase. Indeed, simulation results show that the performance of the different methods ranges from a decrease of 9.6% to an increase of 2.4% in revenues. The discrepancies in performance between the different methods are explained by the trade-off between revenue gains from additional economy bookings and the losses from displaced premium passengers. Further, we observe that successful methods lead to a revenue increase by accepting additional bookings in top economy classes rather than in low economy classes. Finally, the poor performance of the dynamic programming methods tested is due to a misalignment between the underlying assumptions of the algorithms and the reality of the booking and passenger choice process.

Thesis Supervisor: Peter P. Belobaba
Title: Principal Research Scientist, Aeronautics and Astronautics

Thesis Reader: Cynthia Barnhart
Title: Ford Professor, Civil and Environmental Engineering and Engineering Systems, Associate Dean, School of Engineering

Acknowledgments

This thesis marks the end of my three-year MIT journey. I feel extremely privileged to become an MIT alumni, and I believe it would not have been possible without the help and support of many people. I would like to thank some of them more specifically.

First, I would like to acknowledge my thesis supervisor Dr. Peter P. Belobaba for his insightful comments and thoughtful advice. Working with Peter has been a great pleasure and an outstanding learning experience. I would like to thank Professor Cynthia Barnhart for providing helpful feedback on this thesis. I would also like to thank Professor Amedeo R. Odoni for his advice on my career path.

I would like to thank the MIT PODS consortium member airlines who supported me financially and whose input increased the practicality of my work. More specifically, thanks to Thomas Fiig, Stefan Poelt, and Craig Hopperstad for their help with developing the different algorithms presented in this thesis.

Thanks to my colleagues and friends who made my MIT experience a lot more enjoyable. Thanks to my fellow PODS students Claire Cizaire, Himanshu Jain, Alyona Michel, Vincent Surges, and Michael Abramovich for the discussions and the fun. Thanks to my friends at the Consulting Club at MIT for helping me improve on my business skills. Thanks to my friends involved in the Sidney-Pacific Graduate Community who definitely made me feel home away from home. A special thank to George H. Chen, Jennifer Jarvis, Steve Morgan, and Stephanie Nam for our awesome year serving on SPEC. Thanks also to Professor Roger G. Mark, to his wife Dorothy, to Professor Annette Kim, and her husband Roland Tang for helping me grow both personally and professionally.

Finally, I want to recognize the contribution of those who, directly or indirectly, gave me the opportunities I needed to end up at the Institute. Thanks to the Province of Quebec for giving me an affordable and high quality education. Thanks to Professor Michel Gamache of École Polytechinique de Montréal for introducing me to Operations Research. Thanks to Jean-François Pagé at Air Canada for giving me my first opportunity in the airline industry.

Finalement, j'aimerais dédier ce mémoire à ma mère, mon père et ma soeur. Merci pour les fous rires, merci de vous assurer que je garde toujours les deux pieds sur terre, merci de faire de mon monde un endroit où il fait bon vivre.

Contents

Abstract **3**

Acknowledgments **4**

List of Figures **9**

List of Tables **11**

1 Introduction **13**
- 1.1 Airline revenue management concepts . . . 13
  - 1.1.1 Differential pricing . . . 13
  - 1.1.2 Revenue management . . . 16
- 1.2 Problem definition . . . 18
  - 1.2.1 Separate vs multiple cabin . . . 18
  - 1.2.2 Passenger decision process . . . 19
  - 1.2.3 Airline decision process . . . 20
- 1.3 Contribution . . . 22
- 1.4 Structure of the thesis . . . 22

2 Literature Review **23**
- 2.1 Separate cabin revenue management problem . . . 23
  - 2.1.1 Lautenbacher-Stidham dynamic programming . . . 23
  - 2.1.2 Higher demand variance in dynamic programming . . . 25
  - 2.1.3 Expected Marginal Seat Revenue (EMSR) . . . 27
- 2.2 Multiple cabin revenue management problem . . . 28
  - 2.2.1 Multiple product upgrades . . . 28
  - 2.2.2 Shared nesting . . . 29
- 2.3 Chapter summary . . . 30

3 Dynamic Formulations and Heuristics **31**
- 3.1 Multiple cabin dynamic programming (DP) formulations . . . 31
  - 3.1.1 Multiple cabin DP . . . 31

3.1.2 Multiple Cabin DP with Variance . . . 33
3.2 Heuristics . . . 34
3.2.1 Shared nesting with EMSR . . . 34
3.2.2 Multiple cabin DP heuristic . . . 39
3.3 Chapter summary . . . 40

4 Simulation and Results **41**
4.1 Simulator description . . . 41
4.1.1 Passenger Choice Model . . . 43
4.1.2 Revenue Management System . . . 44
4.2 Performance indicators . . . 45
4.3 Results . . . 47
4.3.1 Network and fare structure . . . 47
4.3.2 Base case scenarios . . . 49
4.3.3 Performance evaluation . . . 58
4.4 Chapter summary . . . 78

5 Conclusion **81**
5.1 Contribution and findings . . . 81
5.2 Directions for further research . . . 82

Bibliography **85**

List of Figures

1.1 Potential revenue with a single fare (A) and multiple fares (differential pricing) (B) . . . 14
1.2 Nested booking limits [1] . . . 17
1.3 Airline operation planning process [5] . . . 17
1.4 Passenger itinerary choice . . . 19
1.5 Airline information structure . . . 20

3.1 Cabin EMSR values . . . 37
3.2 Three different separate cabin DP problems . . . 40

4.1 PODS Architecture representation [4] . . . 42
4.2 Passenger arrival curves . . . 43
4.3 Airline 1 class closure rates in single market scenario at low demand . . 52
4.4 Airline 1 class closure rates in single market scenario at medium demand 53
4.5 Airline 1 class closure rates in single market scenario at high demand . . 53
4.6 Airline 1 class closure rates in realistic network scenario at low demand 56
4.7 Airline 1 class closure rates in realistic network scenario at medium demand 57
4.8 Airline 1 class closure rates in realistic network scenario at high demand 57
4.9 Relative revenue change over base case in single market scenario at low demand . . . 58
4.10 Relative revenue change over base case in single market scenario at medium demand . . . 59
4.11 Relative revenue change over base case in single market scenario at high demand . . . 59
4.12 Load factors in single market scenario at low demand . . . 60
4.13 Load factors in single market scenario at medium demand . . . 61
4.14 Load factors in single market scenario at high demand . . . 61
4.15 Cabin load factors in single market scenario at low demand . . . 62
4.16 Cabin load factors in single market scenario at medium demand . . . 63
4.17 Cabin load factors in single market scenario at high demand . . . 63
4.18 Fare class mix in single market scenario at low demand . . . 64

4.19 Fare class mix in single market scenario at medium demand 64
4.20 Fare class mix in single market scenario at high demand 65
4.21 Closure rates in single market scenario at low demand 66
4.22 Closure rates in single market scenario at medium demand 66
4.23 Closure rates in single market scenario at high demand 67
4.24 Relative revenue change over base case in realistic network scenario at low demand 69
4.25 Relative revenue change over base case in realistic network scenario at medium demand 70
4.26 Relative revenue change over base case in realistic network scenario at high demand 70
4.27 Load factor in realistic network scenario at low demand 71
4.28 Load factor in realistic network scenario at medium demand 71
4.29 Load factor in realistic network scenario at high demand 72
4.30 Cabin load factor in realistic network scenario at low demand 72
4.31 Cabin load factor in realistic network scenario at medium demand . . . 73
4.32 Cabin load factor in realistic network scenario at high demand 73
4.33 Fare class mix in realistic network scenario at low demand 74
4.34 Fare class mix in realistic network scenario at medium demand 75
4.35 Fare class mix in realistic network scenario at high demand 75
4.36 Closure rates in realistic network scenario at low demand 76
4.37 Closure rates in realistic network scenario at medium demand 76
4.38 Closure rates in realistic network scenario at high demand 77

List of Tables

1.1 Example of a fare structure 15

3.1 Calculation example for “Shared nesting Full EMSR” 35
3.2 Calculation example for “Shared nesting Economy EMSR” 36
3.3 Expected Marginal Seat Revenue for different cabin capacities 38
3.4 Example for “Shared nesting EMSRc bidprice control” (0 seat shared) . 38
3.5 Example for “Shared nesting EMSRc bidprice control” (2 seats shared) 39

4.1 Single market fare structure 47
4.2 Realistic network restricted fare structure 48
4.3 Realistic network semi-restricted fare structure 48
4.4 Distinct EMSR results in single market scenario at low demand 49
4.5 Distinct EMSR results in single market scenario at medium demand . . 49
4.6 Distinct EMSR results in single market scenario at high demand 50
4.7 Distinct EMSR fare class mix in single market scenario at low demand . 50
4.8 Distinct EMSR fare class mix in single market scenario at medium demand 51
4.9 Distinct EMSR fare class mix in single market scenario at high demand 51
4.10 Distinct EMSR results in realistic network scenario at low demand . . . 54
4.11 Distinct EMSR results in realistic network scenario at medium demand 54
4.12 Distinct EMSR results in realistic network scenario at high demand . . 54
4.13 Distinct EMSR fare class mix in realistic network scenario at low demand 55
4.14 Distinct EMSR fare class mix in realistic network scenario at medium demand 55
4.15 Distinct EMSR fare class mix in realistic network scenario at high demand 56

Chapter 1

Introduction

IN an effort to maximize their profit, airlines have relied on quantitative approaches to assist their decision making process. In this thesis, we focus on the single-leg multiple-cabin revenue management problem. More specifically, on a given flight-leg providing non-stop service between an origin and a destination, airlines have to decide at what fare they are willing to sell their seats by taking into account historical bookings, time remaining until departure, and remaining capacity in the different cabins.

We define the single-leg multiple cabin revenue management problem, present relevant previous work, describe multiple algorithms that we developed, and test these algorithms in a realistic simulation environment. Our contribution is to extend existing separate cabin algorithms to the multiple cabin problem and highlight the different trade-offs that need to be taken into account when solving the multiple cabin revenue management problem in a competitive passenger choice environment.

In this chapter, we provide some background and explain the motivations behind our work. We review relevant airline revenue management concepts, define the single-leg multiple-cabin revenue management problem, explicitly state the contribution of our work, and present the structure of this thesis.

■ 1.1 Airline revenue management concepts

In this section, we provide some background relevant to the multiple cabin revenue management problem. More specifically, we review the concept of differential pricing and define the logic behind revenue management.

■ 1.1.1 Differential pricing

Following US airline industry deregulation in 1978, airlines were allowed more flexibility in pricing and route choice. In an effort to increase revenues, airlines extended the practice of selling seats at different fares by implementing multiple fare classes. As a result, two identical seats can be sold at different fares in a strategy referred to as "differential pricing." Consequently, airlines charge higher fares and extract more revenues from consumers with high willingness-to-pay (WTP) while accepting low fare bookings requests from low-WTP passengers. For the airlines, the advantage is that they can capture a higher proportion of consumers' WTP and increase their revenues. Passengers also gain from this strategy because more passengers can afford to buy tickets and airlines, motivated by increased revenue opportunities, offer more traveling options.

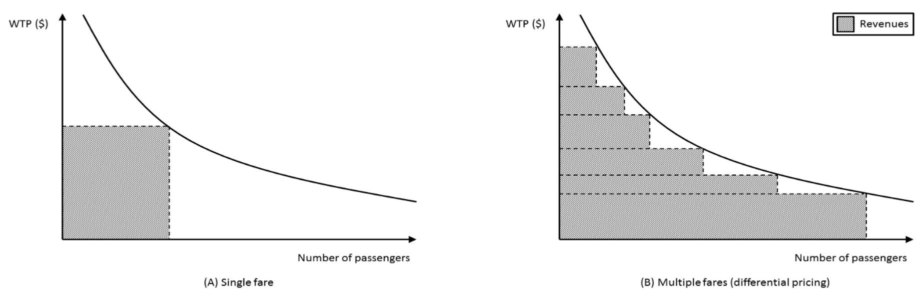

Figure 1.1. Potential revenue with a single fare (A) and multiple fares (differential pricing) (B)

Figure 1.1 shows a typical demand curve and potential revenues for two different pricing strategies. In Figure 1.1(A), the airline offers a single fare. In this strategy, passengers with a WTP lower than the offered fare cannot afford to buy a ticket; all passengers that have a WTP greater than or equal to the offered fare buy tickets at this single offered fare. As illustrated by the shaded area under the curve in Figure 1.1(A), the potential revenue for this strategy is the product of the fare and the number of passengers with WTP greater than or equal to the offered fare. The second strategy, differential pricing, which is presented in Figure 1.1(B) is to offer multiple fares. Here, more people are able to buy tickets than with the first strategy because the lowest fare offered is lower than the single fare offered in the first strategy. The potential revenue of differential pricing is represented by the shaded area under the curve in Figure 1.1(B) and, as one can see, is greater than the potential revenue for the single fare strategy.

This theory assumes that each and every passenger is forced to buy the highest fare available that is less than or equal to their WTP. For obvious reasons, there is no way airlines can know somebody's WTP and, even if they were able, they could not force

Table 1.1. Example of a fare structure

Cabin	Class	Fare	Adv. Purchase	Min. Stay	Change fee	Cancel fee
Premium	F	\$1000	0	No	No	No
	P	\$850	7	No	No	\$ 150
Economy	Y	\$700	0	No	No	No
	M	\$500	5	No	No	\$150
	B	\$300	14	No	\$ 100	\$ 150
	Q	\$150	21	Sat. Stay	\$ 200	\$ 150

people to buy a specific fare. That said, there exists a strategy airlines use to prevent high-WTP passengers from buying cheaper fares; airlines add different restrictions on low fare classes that make these low fare classes unattractive to high-WTP passengers.

Table 1.1 shows an example of a differentiated fare structure for two different types of seats. In this example the airline offers seats in two different cabins, premium and economy, each at multiple price points. Note that differential pricing is applied within each cabin. Following industry practice, a letter designates the booking class in which tickets are sold. It is generally assumed that high-WTP passengers make their travel decisions later in the booking process than low-WTP passengers, that they spend a relatively short period of time at their destination, and that they are likely to change their travel plans at some point in the booking process. In this example, the airline uses four types of restrictions:

1. Advance Purchase

   Advance purchase requires passengers to purchase x days before departure. For example, Class B can be bought up to 14 days before departure.

2. Minimum stay (at destination)

   Minimum stay restrictions requires passengers to stay at the destination for a minimum time. For example, Class Q can be bought by passengers staying over a Saturday night at their destination.

3. Change fee

   Change fee restrictions requires passengers to pay a nominal fee for any change to the reservation. For example, changing a Class B ticket for a different Class or a different departure time costs $100.

4. Cancel fee

   Cancel fee restrictions requires passengers to pay a nominal fee upon cancellation of the booking. For example, Canceling a ticket in classes P, M, B, and Q costs $150.

These are examples of restrictions in the industry, but other restrictions can also be applied on the different fare products (e.g. Round-trip requirement, etc.). We refer interested readers in a complete discussion of differentiated fare structures to Botimer's PhD thesis [7] on pricing and fare structure differentiation.

■ 1.1.2 Revenue management

Because airlines use differential pricing and because capacity is limited, airlines have to decide the number of seats to allocate to each fare class. In fact, instead of allocating seats to a specific class, airlines define nested booking limits for each fare class. Figure 1.2 shows nested booking limits for four different classes going from the most valuable Y-class to the least valuable Q-class. As we can see, nested booking limits are such that as long as there are still seats available, airlines never reject booking requests in the highest booking class.

The advantage of differential pricing is that it increases the potential revenue that airlines can capture, while the inconvenience of differential pricing is that it forces airlines to decide whether to accept or reject each booking request. In the airline context, we define revenue management as the science of maximizing revenues by determining the number of seats available at each fare class.

Figure 1.3 shows the different operational decisions that an airline has to make from a long term horizon until the day of departure. At the strategic level, airlines decide what kind of aircraft they buy and which route they serve. At the tactical level, airlines can affect their revenues by choosing their pricing strategy, by applying revenue management. Revenue management occurs after the decisions regarding fleet planning, scheduling, and pricing have occurred. Revenue management is the last opportunity for the airline to maximize its profit through revenue maximization. Since the airline

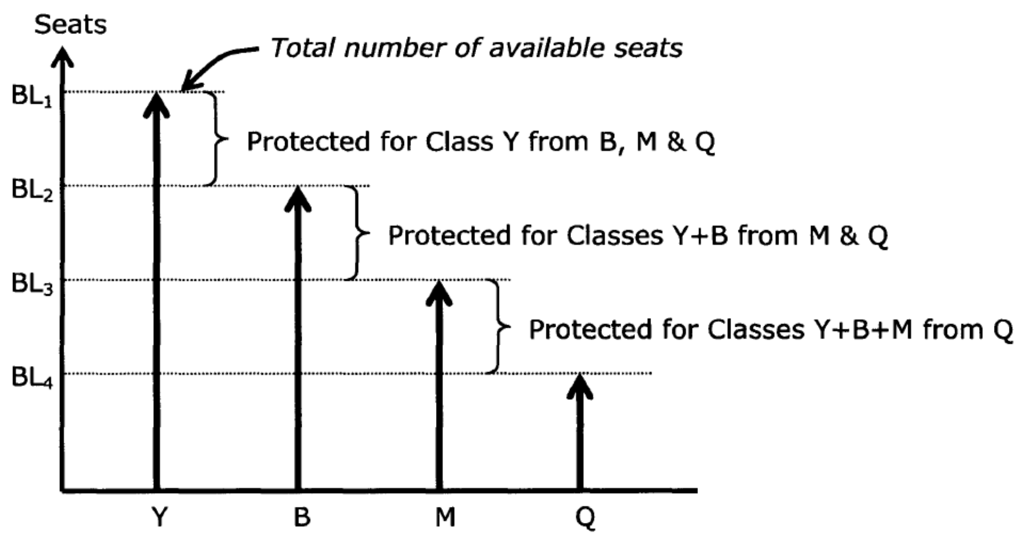

Figure 1.2. Nested booking limits [1]

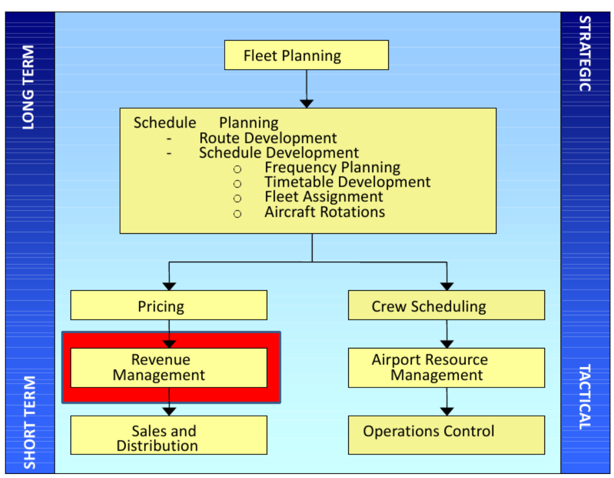

Figure 1.3. Airline operation planning process [5]

has already committed to fly on a given route and that the marginal cost of a passenger (i.e. a meal and incremental fuel) is negligible, we can assume cost to be fixed at the time of the booking request which makes revenue maximization equivalent to profit maximization.

1.2 Problem definition

In this section, we describe the problem on which this thesis focuses. The objective is to better understand the characteristics of the single-leg multiple cabin revenue management problem as it is faced by the airlines in today's air transportation industry.

We first explain the difference between separate and multiple cabin revenue management problems, describe how people make their traveling decision, and illustrate how airlines make their revenue management decision.

1.2.1 Separate vs multiple cabin

It is important to understand the difference between separate and multiple cabin revenue management problems. In the separate cabin revenue management problem, passengers with bookings in a given cabin are only allowed to use seats in their respective cabin. However, in the multiple cabin revenue management problem, passengers with bookings in the economy cabin can be upgraded to a premium cabin seat while premium bookings are always using premium capacity. It is assumed that economy passengers always accept an upgrade to a higher quality premium seat.

Here, we focus on upgrades caused by the airline's effort to better use its capacity and we are not taking any other types of upgrades into account. For example, we are not interested in upgrades due to frequent-flyer programs or credit card deals. Stated otherwise, we are only interested in cases for which the demand is such that a given flight would depart with empty seats in the premium cabin, while some passengers are rejected because the economy cabin is full. In these cases, allowing additional economy bookings to use available premium capacity can potentially increase revenue.

Although both separate and multiple cabin revenue management problems compare the immediate benefit of a booking with the decrease in future expected revenue caused by the reduction in capacity, the multiple cabin revenue management problem is more complex because the airline has to take into account the trade-off between economy and business cabin bookings.

1.2.2 Passenger decision process

Figure 1.4 shows a schematic representation of the different options available for a passenger that wants to travel from the origin “O” to the destination “D”. Because the origin lies within the catchment area of two different airports, the passenger chooses between these two different departure airports to initiate his trip and chooses between the multiple itinerary options available at each airport. We make the distinction between passenger itineraries and flight-legs. A passenger itinerary is defined as any set of flights allowing the passenger to go from his or her preferred origin to his or her desired destination. A flight-leg is the unit of operation of an airline, and passengers fly on multiple flight-legs to complete their itineraries. Therefore, a single flight-leg provides service to passengers going from different origins to different destinations. In Figure 1.4, flight-legs are represented by full arrows between any pair of airport and an itinerary would be any set of flight-leg that links the departure airport to the arrival airport.

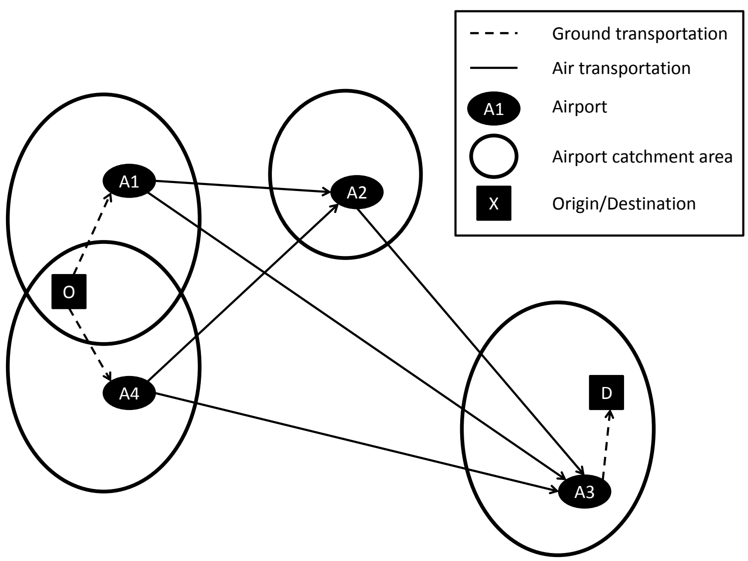

Figure 1.4. Passenger itinerary choice

At the time of booking, passengers also have a preferred time window in which they would like to travel. Passengers have schedule restrictions such that they cannot leave the origin before a given time and they have to be at the destination before a given time. Passengers also have finite financial resources and there exist a threshold value for the fare above which they will decide not to travel. Finally, passengers have different product preferences. For example, they might be interested in booking a refundable ticket in order to keep flexibility in their travel plans. Given all these characteristics, passengers choose the traveling option that maximizes their utility.

1.2.3 Airline decision process

Airlines apply revenue management principles based on their assumptions about passenger behavior and on available historical booking data. Figure 1.5 shows the different data sources and models used in the airlines decision process. In this thesis, we focus on the "optimization model" part of the process.

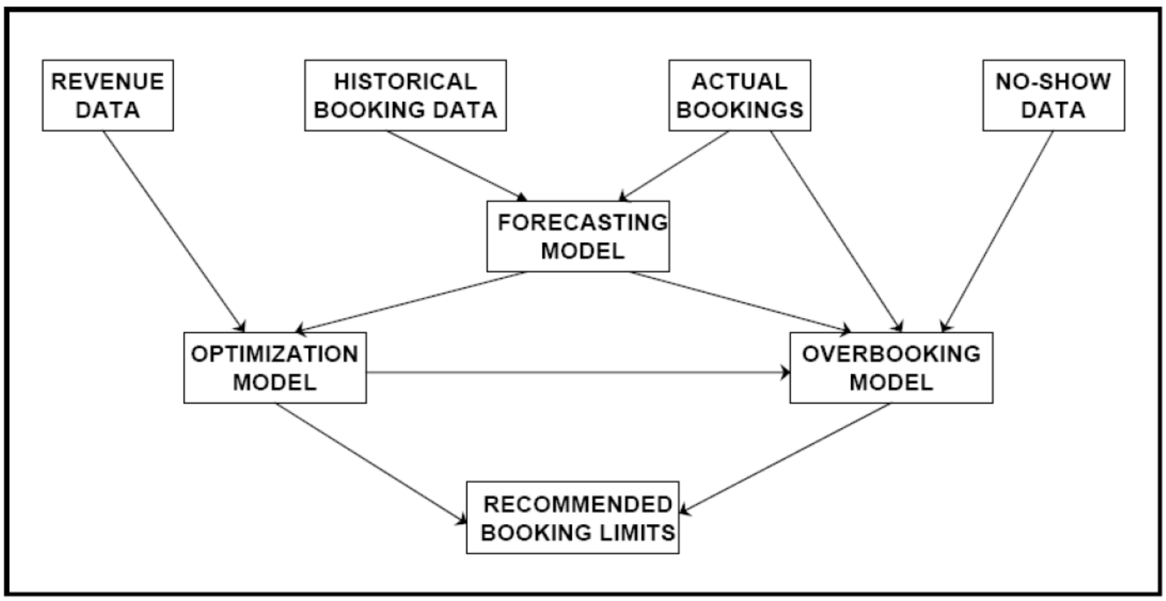

Figure 1.5. Airline information structure

The objective of the airline decision process is to define a decision rule allowing the airline to decide whether to accept or reject incoming booking requests. We explain two different decision rules. First, airlines can use nested booking limits as a decision rule and accept booking requests in a given class as long as the nested booking limit for this class is greater than zero. Second, airlines can use bidprices which are defined as the lowest fare at which the airline is willing to sell its capacity. Airlines accept booking requests in classes for which the fare is greater than or equal to the bidprice. Regardless of the specifics of the airline's selected decision rule, three pieces of information are needed.

1. Average class fares

   In order to make the right decision regarding a booking request, airlines have to know what is the expected financial benefit from this booking and what is the expected revenue from other bookings. Using historical revenue data, airlines can obtain the average fare for a class on a given flight-leg. Since a flight-leg is flown by passengers on different itineraries and fares are different between itineraries, there does not exist a single fare for a given class on a flight-leg and airlines need to find an average fare for each class.

   For example, airlines can allocate the fare of a single passenger to the different flight-legs on his or her itinerary based on the fraction of the total itinerary flying distance traveled on each flight-legs. Averaging these value will give the airline an array of fares, one for each class for each flight-leg.

2. Expected number of bookings to come

   Using historical booking data for similar flight-legs and actual bookings at hand, airlines are able to forecast bookings to come by class. Airlines use this forecast to compute the number of seats to protect for high fare class bookings against low fare class bookings.

3. Number of available units

   Finally, using "no-show" data and the expected number of bookings to come, airlines can decide how many units they make available. It is important to note that the number of available units might be greater than the total number of seats on the plane. Indeed, it is known that some passengers will not show up at the time of departure. Therefore, airlines allow for more bookings than they have seats available in a process called overbooking. However, in this thesis, overbooking is not taken into consideration and, for the sake of simplicity, we will refer to the number of remaining units as the number of remaining seats.

These different sources of information are used to find nested booking limits or bidprices. As stated earlier, these are used together with airline analysts' business knowledge to decide whether to accept or reject booking requests in each classes.

Re-optimization

Since airlines operate a large number of flight-legs in a complex network, it is computationally impossible to continuously optimize to obtain the latest nested booking limits or the latest bidprices. Consequently, airlines divide the booking horizon in a manageable number of time frames and update the decision rule values once per time frame.

1.3 Contribution

The contribution of this work is two-fold. First, we develop different heuristics and extend existing separate cabin revenue management optimization methods to solve the multiple cabin problem. Second, we test these methods in a realistic simulation environment and provide insights on the behavior of the different methods in an environment with realistic passenger choice and competing airlines. More specifically,

- we extend the existing separate cabin dynamic programming formulation by Lautenbacher and Stidham [16] to develop a dynamic programming formulation for the multiple cabin revenue management problem,
- we extend a separate cabin realistic variance approach by Walczak [17] to the multiple cabin revenue management problem,
- we develop four heuristic methods based on the dynamic programming formulation and on the Expected Marginal Seat Revenue (EMSR) heuristic [2],
- we evaluate the performance of the different methods in a simulation environment with realistic passenger choice behavior, and we find that multiple cabin optimization is not systematically leading to a revenue increase.

1.4 Structure of the thesis

First, in Chapter 2 we discuss in detail relevant literature on the revenue management optimization problem and more specifically on the multiple cabin optimization problem. In Chapter 3, we present the different algorithms that on which we focus in this thesis and their underlying assumptions. In Chapter 4, we present the Passenger Origin-destination Simulator (PODS), a realistic passenger choice simulation tool, and we will compare the revenue performance of the different methods in two different competitive environments: first, in a single market competitive environment and, second, in a more realistic network scenario with four airlines competing in multiple markets. Finally, in Chapter 5, we will discuss the different findings and propose future areas for research.

Chapter 2

Literature Review

IN this chapter, we review relevant previous work on the multiple cabin revenue management problem. More specifically, we first look at existing work on the separate cabin revenue management problem. We review a dynamic programming formulation, an extension to this dynamic programming algorithm that allows for higher demand variance, and a static heuristic widely used in the industry. We then briefly review existing work on the multiple cabin revenue management problem. Finally, we review a capacity control mechanism for multiple cabin environments, shared nesting, which allows airlines to keep track of the remaining capacity throughout the booking process and prevents overselling.

■ 2.1 Separate cabin revenue management problem

In this section, we review existing work on the single-leg separate cabin revenue management problem. We present a dynamic programming formulation, an extension to this dynamic programming formulation that allows for higher variance, and the Expected Marginal Seat Revenue (EMSR) static heuristic.

■ 2.1.1 Lautenbacher-Stidham dynamic programming

In their 1999 paper, Lautenbacher and Stidham [16] develop a dynamic programming model for the single leg multiple fare class revenue management problem. Their model divides the booking horizon into time slices for which the probability of observing more than one booking during any one time slice is negligible. At any time slice, the airline decides whether to accept or reject the requested fare. They assume that bookings in a given class i in the set of classes F follow a Poisson process with known rate λ_t^i at time frame t. They then approximate the booking arrival process with a Bernoulli process with $p_t^i = \frac{\lambda_t^i}{N_t}$ as the probability of observing a booking during one of the N_t time slices of time frame t. The probability of observing no bookings is given by $p_t^0 = 1 - \sum_{i \in F} p_t^i$. More formally, the dynamic program is

$$\begin{aligned} U_n(x, r_i) &= \max\{r_i + \mathbb{E}_j(U_{n-1}(x-1, j)), \mathbb{E}_j(U_{n-1}(x, j))\}, \text{ IF } r_i > 0 \\ U_n(x, 0) &= \mathbb{E}_j(U_{n-1}(x, j)), \text{ IF } r_i = 0 \end{aligned}$$

where $U_n(x, r_i)$ is the expected future revenue at time slice n with x seats remaining when observing a booking request of r_i, and $\mathbb{E}_j$ is the expected value over the different fares offered. Or equivalently,

$$U_n(x) = \sum_{i \in F} p_n^i \cdot \max\{r_i + U_{n-1}(x-1), U_{n-1}(x)\} + p_n^0 U_{n-1}(x), \forall x \geq 1; \forall n \geq 1$$

with $U_0(x) = U_n(0) = 0$, $\forall x$ and $\forall n$. In this equation, x is the number of remaining seats, $U_n(x)$ is the expected revenue at time slice n with x seats remaining and r_i is the fare for Class i, and F is the set of fare classes. Airlines compute expected revenues at each time slice for all possible values of x.

As defined previously, a bidprice is the lowest price at which an airline is willing to sell a given seat. Airlines will use bidprices to make their decision regarding a booking request. If the requested fare is higher than the bidprice, the booking request is accepted, and it is rejected otherwise. Bidprice at a given time slice n for a given capacity x ($BP_n(x)$) can be computed using expected revenue computed previously in the following equation:

$$BP_n(x) = U_{n-1}(x) - U_{n-1}(x-1).$$

We now discuss the strengths and weaknesses of the approach proposed by Lautenbacher and Stidham. Their dynamic programming formulation suits the structure of the revenue management problem because it acknowledges the fact that bookings are interspersed among different fare classes over multiple time frames in the booking process. Howerver, since it assumes booking requests arrive according to a Poisson process, the variance of the number of booking requests in Class i at time frame t, $var(B_t^i)$, is equal to the mean number of bookings in Class i at time frame t, $E[B_t^i]$:

$$E[B_t^i] = var(B_t^i) = \lambda_t^i.$$

However, we know from observation in the airline industry that the variance-to-mean ratio for the number of bookings is frequently greater than 1. Consequently, some information about the underlying demand distribution and more specifically the variance in the number of booking requests is not used when applying the Lautenbacher-Stidham approach despite the fact that the airline can estimate the variance of the number of booking requests to come.

As shown by Diwan [11] in his Master's thesis, since the Lautenbacher-Stidham algorithm underestimates the variance, it is overly confident about future high-WTP demand and leads to bidprices that are excessively high. Bidprices that are too high lead to a decrease in revenues because the airline rejects a greater proportion of low fare classes booking requests early in the process while high fare demand predicted by the Poisson process does not materialize later in the booking process.

Finally, computational time can also be an issue with this formulation. As we saw, the Lautenbacher-Stidham formulation requires one calculation for each possible capacity level per time slice. The number of calculations required is so large that the Lautenbacher-Stidham approach is not commonly used in practice. This drawback motivates the need for heuristics that find reasonable sub-optimal solutions in a less computationally intensive way.

■ 2.1.2 Higher demand variance in dynamic programming

In order to address the variance issue raised for the Lautenbacher-Stidham algorithm, Walczak [17] proposes to allow for higher variances by assuming batched booking arrivals. The idea is to allow bookings to arrive in batches of size b_t^i at a rate of rate $\bar{\lambda}_t^i$ such that

$$b_t^i = \left\lfloor \frac{var(B_t^i)}{E[B_t^i]} \right\rceil$$

$$\bar{\lambda}_t^i = \frac{\lambda_t^i}{b_t^i}.$$

By setting b_t^i equal to the variance-to-mean ratio, we keep the expected number of bookings $E[\bar{B}_t^i]$ equal to $E[B_t^i]$ and increase the variance so that $Var[\bar{B}_t^i]$ is b_t^i times greater than $Var[B_t^i]$ where $\bar{B}_t^i$ is the number of Class i booking requests observed during time frame t. We have

$$\begin{aligned} E[\bar{B_t^i}] &= \bar{\lambda_t^i} \cdot b_t^i \\ &= \frac{\lambda_t^i \cdot b_t^i}{b_t^i} \\ &= \lambda_t^i \end{aligned}$$

and,

$$\begin{aligned} Var[\bar{B_t^i}] &= Var[b_t^i \cdot A_t^i] \\ &= \left(b_t^i\right)^2 \cdot Var[A_t^i] \\ &= \left(b_t^i\right)^2 \cdot \bar{\lambda_t^i} \\ &= b_t^i \cdot \lambda_t^i \\ &= b_t^i \cdot Var[B_t^i] \end{aligned}$$

where A_t^i is the number or Class i booking request batches at time frame t. The dynamic program becomes

$$U_n(x) = \sum_{i \in F} \bar{p_n^i} \cdot \max_{j=\{1,\ldots,\min\{b_t^i,x\}\}} \{j * r_i + U_{n-1}(x-j), U_{n-1}(x)\} + \bar{p_n^0} U_{n-1}(x)$$

where $U(x)$, i, F, b_t^i, x, and r_i are defined as previously, j is the number of bookings from the batch that will be accepted, and $\bar{p_n^i}$ is the probability of observing a booking batch arrival in Class i at time n such that:

$$\bar{p_n^i} = \frac{\bar{\lambda_t^i}}{N_t}.$$

Walczak's approach is flexible and addresses one of the criticisms of the Lautenbacher-Stidham formulation. Indeed, the Walczak approach allows for higher variances in the booking arrival process and does not assume equal variance for the different fare classes. That said, computation time is still a major concern with the Walczak approach which reinforces the need for less computationally intensive heuristics.

■ 2.1.3 Expected Marginal Seat Revenue (EMSR)

In his doctoral thesis, Belobaba [2] develops the original version of a the Expected Marginal Seat Revenue static heuristic (EMSRa) to define nested booking limits assuming independent and normally distributed demands for the different classes. Belobaba [3] then further refined the heuristic and presented EMSRb, an improved and more robust version of EMSR. In this thesis, we focus solely on the EMSRb approach.

At a given time and with given demand distribution, the booking limit for the top class (Class 1) is equal to the remaining capacity and the booking limit for subsequent classes is given by

$$BL_i^n \quad = \quad BL_1^n - \left[\mu_{1,\ldots,i-1}^n + \sigma_{1,\ldots,i-1}^n \cdot \Phi^{-1}\left(1 - \frac{r_i^n}{r_{1,\ldots,i-1}}\right)\right].$$

Where BL_i^n is the booking limit for Class i at time n, $\mu_{1,\ldots,i-1}^n$ is the joint mean demand to come for classes 1 to $i-1$, $\sigma_{1,\ldots,i-1}^n$ is the joint standard deviation of demand for classes 1 to $i-1$ and $r_{1,\ldots,i-1}^n$ is the weighted average fare for classes 1 to $i-1$. We have

$$\begin{aligned}
\mu_{1,\ldots,i-1}^n &= \sum_{j=1}^{i-1} \mu_j^n \\
\left(\sigma_{1,\ldots,i-1}^n\right)^2 &= \left(\sigma_{1,\ldots,i-2}^n\right)^2 + \left(\sigma_{i-1}^n\right)^2 \\
r_{1,\ldots,i-1}^n &= \frac{\sum_{j=1}^{i-1} r_j^n \cdot \mu_j^n}{\mu_{1,\ldots,i-1}^n}
\end{aligned}$$

The EMSR heuristic is widely used in the industry and empirical results show that it performs well in practice [3]. We can also use EMSR to obtain an approximation of the expected marginal revenue of each seat at a given time which can be used as a bidprice. Once the airline finds the expected marginal revenue for a given seat, it can then compare it with a booking request and decide whether to accept or reject it. Define the expected marginal revenue from each seat x at time n as $EMSR_n(x)$. We have

$$EMSR_n(x) = \max_{i \in F} \left[\min \left(r_i^n, r_{1,\ldots,i}^n \cdot \left(1 - \Phi \left(\frac{x - \mu_{1,\ldots,i}^n}{\sigma_{1,\ldots,i}^n} \right) \right) \right) \right]$$

Although EMSR is a static heuristic, it is possible to recalculate the $EMSR_n(x)$ value multiple times throughout the booking process, i.e. at each time frame. This implementation is quite common in practice and we will use it as a baseline scenario when comparing the different algorithms.

The main advantage of the EMSRb heuristic is that the computation time required is significantly smaller than the computation time required for the Lautenbacher-Stidham and Walczak approaches.

2.2 Multiple cabin revenue management problem

In this section, we briefly review existing work on the multiple cabin revenue management problem. We first look at two papers dealing with multiple products, each offered at a single price point, and then look at a capacity sharing mechanism, shared nesting, used to manage remaining availability.

2.2.1 Multiple product upgrades

We identify three papers as relevant to our work. First, in their 2009 paper, Shumsky and Zhang [13], provide the optimal dynamic programming solution to the multiperiod capacity allocation. Second, in their working paper, Gallego and Stefanescu [12] introduce two dynamic programming formulations for the revenue management problem with upgrades. Their formulations differ by the time at which the upgrade decision has to be made.

In our terms, they focus on the multiple cabin revenue management problem with a single class in each cabin. They were able to show that using upgrades helps airlines in balancing supply and demand by using excess premium cabin capacity for economy cabin demand. They were also able to show that "fairness" is easy to ensure and does not affect the optimality of the solution. An upgrade mechanism is considered fair if the upgrade priority goes to the passengers who bought higher-end products. In the airline context, this means that a passenger booked in a fully unrestricted and more expensive economy class will be upgraded to the premium cabin before any passenger that booked in a cheaper economy class.

Lastly, the paper by Steinhardt and Göensch [15] is closely related to the work presented in this thesis. They address the upgrade and the capacity control problems simultaneously in the revenue management context. They introduce a decomposition heuristic using deterministic linear programming, and develop a dynamic programming algorithm that solves the upgrade and capacity control problems simultaneously.

2.2.2 Shared nesting

In his 2010 AGIFORS presentation, Walczak [18] introduces an availability control mechanism for multiple cabin optimization. We refer to this control mechanism as shared nesting. If we assume that, for a given flight, a premium cabin capacity of CAP_P, an economy cabin capacity of CAP_E, the number of seats available, AVL_P and AVL_E for classes in the premium and in the economy cabin respectively, the number of accepted bookings, BKG_P and BKG_E for the premium and economy cabins respectively, and the number of premium cabin seats available for economy booking upgrades UPG which is assumed to be given at this stage, and to be less than or equal to AVL_P. Shared nesting works as follows:

if $BKG_E < CAP_E$,

$$AVL_P = CAP_P - BKG_P$$

$$UPG_a = \min(UPG, CAP_P - BKG_P)$$

$$AVL_E = CAP_E + UPG_a - BKG_E$$

if $BKG_E \geq CAP_E$,

$$AVL_P = CAP_P - BKG_P - (BKG_E - CAP_E)$$

$$UPG_a = \min(UPG, CAP_P - BKG_P)$$

$$AVL_E = CAP_E + UPG_a - BKG_E$$

This approach assures that the airline is not overselling its capacity.

2.3 Chapter summary

In this chapter, we reviewed relevant literature on the separate cabin revenue management problem. We reviewed dynamic programming approaches and a static heuristic. We also identified relevant existing work on the multiple cabin revenue management problem and reviewed shared nesting, a useful capacity control mechanism. In the next chapter, we will present the different multiple cabin optimization methods we developed in collaboration with MIT PODS consortium airline members.

Chapter 3

Dynamic Formulations and Heuristics

IN this chapter, we present the different algorithms we developed in collaboration with the MIT PODS consortium airline members and on which we focus in this thesis. We first present an extension of the Lautenbacher-Stidham separate cabin model and then present different EMSR-based heuristics and a dynamic programming decomposition heuristic that can be used to solve the multiple cabin problem.

■ 3.1 Multiple cabin dynamic programming (DP) formulations

In this section, we present two different dynamic programming algorithms for the multiple cabin problem. They are extensions of Lautenbacher-Stidham and Walczak's algorithms for the separate cabin revenue management problem presented in Chapter 2.

■ 3.1.1 Multiple cabin DP

We first propose a multiple cabin dynamic programming algorithm which we refer to as "Multiple cabin DP". Similar to the separate cabin Lautenbacher-Stidham formulation, it assumes a single flight with multiple fare classes and known independent demands. It also assumes that booking requests arrive following a Poisson process. We have the following equations:

$$
\begin{aligned}
U_n(x_p, x_e) \;=\; & \sum_{i \in F_p} p_n^i \cdot \max\{r_i + U_{n-1}(x_p - 1, x_e), U_{n-1}(x_p, x_e)\} \\
& + \sum_{j \in F_e} p_n^i \cdot \max\{r_i + U_{n-1}(x_p - 1, x_e), r_i + U_{n-1}(x_p, x_e - 1), U_{n-1}(x_p, x_e)\} \\
& + p_n^0 U_{n-1}(x_p, x_e), \forall x \geq 1; \forall n \geq 1
\end{aligned}
$$

with $U_0(x_p, x_e) = U_n(0, 0) = 0$, $\forall x$ and $\forall n$. In this equation, x_p and x_e are the number of remaining seats in the premium and economy cabin respectively, $U_n(x_p, x_e)$ is the expected revenue at time slice n with x_p and x_e seats remaining in each cabin, r_i is the fare for Class i, F_p and F_e are the sets of premium and economy classes, respectively, and p_n^i is the probability of observing a booking in Class i during time slot n.

As in the separate cabin case, airlines compare requested fares with bidprices. The difference here is that the airline compute a different bidprice for each cabin and compare the requested fare to the appropriate bidprice. Specifically, premium booking requests can be compared with the premium cabin bidprice and economy booking requests are compared with both premium and economy bidprices. A given economy booking request is first compared to the economy cabin bidprice, if it is greater than the economy bidprice, the booking request is accepted in the economy cabin. Otherwise, the booking request is compared to the premium cabin bidprice, if it is greater than the premium bidprice, it is accepted and upgraded to the premium cabin. Otherwise, the booking request is rejected. Bidprices are computed using the following formulas:

$$
\begin{aligned}
BP_n^P(x_p, x_e) &= U_{n-1}(x_p, x_e) - U_{n-1}(x_p - 1, x_e) \\
BP_n^E(x_p, x_e) &= U_{n-1}(x_p, x_e) - U_{n-1}(x_p, x_e - 1)
\end{aligned}
$$

where $BP_n^P(x_p, x_e)$ and $BP_n^E(x_p, x_e)$ are bidprices for premium and economy cabins respectively at time slice n with x_p seats remaining in the premium cabin and x_e seats remaining in the economy cabin. As in the separate cabin case presented in Chapter 2, there is a mismatch between the assumed variance of the "Multiple cabin DP" algorithm and the observed variance. Indeed, the formulation presented above assumes that the variance-to-mean ratio for demand to come in each class is equal to 1, and observations from airline data show that this ratio is generally higher than 1.

■ 3.1.2 Multiple Cabin DP with Variance

Based on the Walczak modification to the separate cabin Lautenbacher-Stidham algorithm, the "Multiple cabin DP with Variance" algorithm allows different variance levels for demand to come in each class. The formulation is very similar to the "Multiple cabin DP" algorithm, but it assumes higher variance-to-mean ratios by allowing booking requests in the different classes to arrive in batches. Following Walczak's approach, bookings arrive in batches of size b_t^i with a rate $\bar{\lambda_t^i}$ such that

$$b_t^i = \left\lfloor \frac{var(B_t^i)}{E[B_t^i]} \right\rceil$$

$$\bar{\lambda_t^i} = \frac{\lambda_t^i}{b_t^i}$$

Where λ_t^i is the arrival rate of booking requests in Class i at time t and B_t^i is a random variable for the number of bookings observed in Class i at time t. Setting b_t^i equal to the variance-to-mean ratio for Class i at time frame t ensures that the expected number of booking requests stays the same while the assumed variance for bookings to come increases to the desired level. The dynamic program is

$$\begin{aligned} U_n(x_p, x_e) \quad = \quad & \sum_{i \in F_p} \bar{p_n^i} \max_{j \in \{1,\dots,\min\{b_t^i, x_p\}\}} [j \cdot r_i + U_{n-1}(x_p - j, x_e), U_{n-1}(x_p, x_e)] + \\ & \sum_{i \in F_e} \bar{p_n^i} \max_{j \in \{1,\dots,\min\{b_t^i, x_p + x_e\}\}} \left[j \cdot r_i + \max_{k \in \{\max(0, j - x_e), \min(x_p, j)\}} \left(U_{n-1}(x_p - k, x_e - j + k) \right), \right. \\ & U_{n-1}(x_p, x_e)] \\ & + \bar{p_n^0} U_{n-1}(x_p, x_e), \forall x \geq 1. \forall n \geq 1. \end{aligned}$$

With $U_0(x_p, x_e) = U_n(0,0) = 0$, $\forall x$ and $\forall n$. As in the "Multiple cabin DP" formulation, x_p and x_e are the number of remaining seats in the premium and economy cabin respectively, $U_n(x_p, x_e)$ is the expected revenue at time slot n with x_p and x_n seats remaining in each cabin, r_i is the fare for Class i, F_p and F_e are the sets of premium and economy classes respectively, $\bar{p_n^i}$ is the probability of observing a batch of b_t^i Class i bookings at time slice n within time frame t, j is the number of bookings requests accepted, and k is the number of economy bookings using premium capacity. The decision rule is the same as in the "Multiple cabin DP".

3.2 Heuristics

In this section, we review different heuristics that we developed to solve the multiple cabin revenue management problem with the help of the MIT PODS consortium airline members. We first cover two different EMSR-based heuristic which we refer to as "Shared nesting with EMSR" and "Shared nesting EMSRc bidprice control", and then present a dynamic programming decomposition which we call "Multiple cabin DP heuristic."

3.2.1 Shared nesting with EMSR

In Chapter 2, we presented shared nesting, which is a control mechanism that keeps track of the number of seats remaining when an airline is using multiple cabin optimization. Suppose that, for a given flight, we have a premium cabin capacity of CAP_P, an economy cabin capacity of CAP_E, the number of seats available, AVL_P and AVL_E for classes in the premium and in the economy cabin respectively, the number of accepted bookings, BKG_P and BKG_E for the premium and economy cabins respectively, and the number of premium cabin seats available for economy booking upgrades UPG which is assumed to be given at this stage, and to be less than or equal to AVL_P. Shared nesting works as follows:

if $BKG_E < CAP_E$,

$$AVL_P = CAP_P - BKG_P$$

$$UPG_a = \min(UPG, CAP_P - BKG_P)$$

$$AVL_E = CAP_E + UPG_a - BKG_E$$

if $BKG_E \geq CAP_E$,

$$AVL_P = CAP_P - BKG_P - (BKG_E - CAP_E)$$

$$UPG_a = \min(UPG, CAP_P - BKG_P)$$

$$AVL_E = CAP_E + UPG_a - BKG_E$$

Table 3.1. Calculation example for "Shared nesting Full EMSR"

Cabin	Class	Avg. fare	Demand	Std. dev.	Protection	Total BL	Mod. BL
Premium	F	$ 800	10	4	7	150	20
	U	$ 600	12	8	12	143	13
Economy	Y	$600	13	7	32	138	138
	M	$ 400	15	9	46	118	118
	B	$ 350	22	10	77	104	104
	L	$ 200	27	9	107	73	73
	Z	$ 149	34	13	147	43	43
	Q	$ 99	42	11	-	3	3

One can think of the shared nesting as a two-step approach. The airline defines the number of premium seats it is willing to share with economy passengers in one step and the airline applies the standard EMSR approach in each cabin in a second step. Both steps are done independently and can be done in any order, as we will see in the following section. We developed three different ways of applying shared nesting with EMSR.

Shared nesting full EMSR

The "Shared nesting full EMSR" approach applies the EMSRb algorithm to the entire capacity and then modifies the booking limit for premium classes.

For example, we have a flight with 20 seats in the premium cabin and 130 seats in the economy cabin. Table 3.1 shows, for each class in the different cabins, the average fare, the mean expected demand, the standard deviation associated with this demand, the protection level computed using the EMSR approach, total booking limits that would be applied if the entire capacity was treated as a separate cabin, and modified booking limits for the "Shared nesting full EMSR" approach. We can see that the difference between total booking limits and modified booking limits is only observed in premium classes. Indeed, we obtain the "Shared Nesting Full EMSR" premium classes' booking limit by subtracting the remaining economy cabin capacity to the total booking limits.

This method is very simple to implement for airlines already using the EMSR heuristic. On the other hand, an important drawback of this method is that is assumes premium demand uses economy capacity when calculating the booking limits. Indeed, the EMSR heuristic protects seats for top classes against lower classes. Therefore, by not changing the joint protect calculation, "Shared nesting Full EMSR" is making an invalid assumption about capacity utilization.

Table 3.2. Calculation example for "Shared nesting Economy EMSR"

Cabin	Class	Avg. fare	Mean demand	Std. dev.	Protection	Modified BL
Premium	F	$ 800	10	4	7	20
	U	$ 600	12	8	12	13
Economy	Y	$600	13	7	10	138
	M	$ 400	15	9	22	128
	B	$ 350	22	10	51	116
	L	$ 200	27	9	80	87
	Z	$ 149	34	13	120	58
	Q	$ 99	42	11	-	18

Shared nesting economy EMSR

"Shared nesting Economy EMSR" is very similar to "Shared nesting Full EMSR", but it fixes the capacity utilization issue identified above. We will look at the same example and apply "Shared nesting Economy EMSR" to compare the booking limits between the two methods.

In "Shared Nesting Economy EMSR", we first apply the EMSRb algorithm to the premium cabin and define the number of seats that must be protected against the highest economy class. In our example, it is determined that 12 seats have to be protected. Therefore, 8 seats can be shared with the economy classes. The second step is to apply the EMSRb algorithm to the economy classes and to find the booking limits using the total economy capacity plus the number of seats shared. We can see that the modified booking limit in premium classes with "Shared Nesting Economy EMSR" are identical the the ones obtained with "Shared Nesting Full EMSR."

The difference between "Shared nesting full EMSR" and "Shared nesting economy EMSR" is at the economy class booking limit level. More precisely, we can see by comparing tables 3.1 and 3.2, that the booking limits for classes M, B, L, Z, and Q are higher with "Shared nesting economy EMSR" when compared to "Shared nesting full EMSR." This is due to two factors: first, the EMSR heuristic protects seats for top classes against lower classes, and second, in "Shared nesting full EMSR" the EMSRb heuristic is applied on the entire set of classes while it is applied independently within each cabin in "Shared nesting economy EMSR." Because premium classes are not taken into consideration when calculating economy classes' booking limit in "Shared nesting economy EMSR", there are fewer valuable classes for which to protect seats against cheaper classes.

Shared nesting EMSRc bidprice control

In "Shared nesting EMSRc bidprice control" algorithm we first define the number of seats to be shared and then apply the EMSRb algorithm within each cabin. As booking requests come, capacity is then managed using the shared nesting control mechanism.

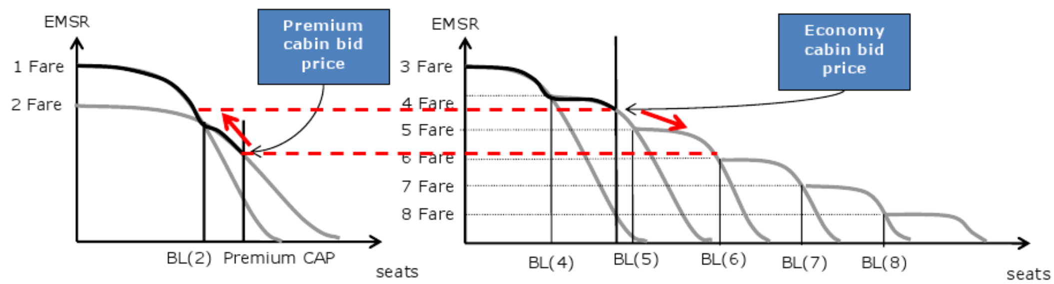

Figure 3.1. Cabin EMSR values

The idea is to use the EMSRb algorithm to find cabin bidprices. We set each cabin bidprice to be equal to the Expected Marginal Seat Revenue of the last seat available in each cabin using the following equation:

$$EMSR_n^h(x) \quad = \quad \max_{i \in F^h} \left[\min \left(r_i^n, r_{j,\ldots,i}^n \cdot \left(1 - \Phi \left(\frac{x - \mu_{j,\ldots,i}^n}{\sigma_{j,\ldots,i}^n} \right) \right) \right) \right]$$

where $EMSR_n^h(x)$ is the Expected Marginal Seat Revenue of the x^{th} seat in cabin h at time n, F^h is the set of classes in cabin h, r_i^n is the fare for class i at time n, $r_{j,\ldots,i}^n$ is the weighted average fare with respect to expected demand for classes j to i with j being the highest Class in cabin h, $\Phi(\cdot)$ is the normal distribution, $\mu_{j,\ldots,i}^n$ is the total expected demand for classes j to i at time n, and $\sigma_{j,\ldots,i}^n$ is the standard deviation for the total expected demand for classes j to i at time n. We share the largest number of premium seats for which the economy cabin bidprice is greater than or equal to the premium cabin bidprice. Figure 3.1 shows a visual representation of the calculation of the number of seats to share. We then apply the EMSRb algorithm in both cabins as we did in the Shared Nesting Economy EMSR case.

Taking the same example as in tables 3.1 and 3.2, we compute the value for different capacities in each cabin and the results are shown in Table 3.3.

We first look at the hypothetical case where, at this specific time, there are 20

Table 3.3. Expected Marginal Seat Revenue for different cabin capacities

Capacities	Premium	Economy
1	790.22	574.06
2	781.80	565.18
...		
5	715.48	524.07
6	673.08	504.81
7	618.70	482.59
8	600.00	457.48
9	600.00	429.69
10	600.00	400.00
11	600.00	400.00
...		
18	464.73	399.10
19	436.20	386.92
20	406.58	373.86
...		

Table 3.4. Example for "Shared nesting EMSRc bidprice control" (0 seat shared)

Cabin	Class	Avg. fare	Mean demand	Std. dev.	Protection	Modified BL
Premium	F	$ 800	10	4	7	20
	U	$ 600	12	8	12	13
Economy	Y	$600	13	7	10	10
	M	$ 400	15	9	22	0
	B	$ 350	22	10	51	0
	L	$ 200	27	9	80	0
	Z	$ 149	34	13	120	0
	Q	$ 99	42	11	-	0

and 10 seats remaining respectively in the premium and economy cabins. We can see that the EMSR for the 20^{th} premium cabin unit is is greater than the EMSR for the 11^{th} economy cabin seat. This implies that the expected revenue from premium cabin passengers for the last seat available in the premium cabin is greater than the expected revenue from an additional unit in the economy cabin. Therefore, in this case, we decide not to share any premium seats. Applying the EMSR heuristic yields the booking limits presented in Table 3.4.

We now look at the hypothetical case where there are 20 and 5 units remaining respectively in the premium and economy cabins. Here, we can see that the ESMR for the 6^{th} economy cabin seat ($504.81) is greater than the EMSR for the 20^{th} premium

Table 3.5. Example for "Shared nesting EMSRc bidprice control" (2 seats shared)

Cabin	Class	Avg. fare	Mean demand	Std. dev.	Protection	Modified BL
Premium	F	$ 800	10	4	7	20
	U	$ 600	12	8	12	13
Economy	Y	$600	13	7	10	7
	M	$ 400	15	9	22	0
	B	$ 350	22	10	51	0
	L	$ 200	27	9	80	0
	Z	$ 149	34	13	120	0
	Q	$ 99	42	11	-	0

cabin seat ($406.58). This implies that the expected revenue from premium cabin passengers for the last seat available in the premium cabin is smaller than the expected revenue from an additional seat in the economy cabin, and we decide to share the last premium seat with economy passengers. We then compare the EMSR of the 7^{th} economy cabin seat ($482.59) and we can see that it is also greater than the EMSR for the 19^{th} premium cabin seat ($436.20). Therefore, we decide to also share the second to last seat in the premium cabin with economy passengers. Using the same logic, we can see that it does not make sense to share a third premium seat with economy passengers. We conclude that the airline should share two seats with economy passengers. Finally, we apply the EMSRb heuristic independently in each cabin which yields the booking limits presented in Table 3.5.

■ 3.2.2 Multiple cabin DP heuristic

The last heuristic that we developed in collaboration with the PODS MIT consortium member airlines is based on the Lautenbacher-Stidham algorithm presented in Chapter 2. The idea behind the heuristic is to decompose the large multiple cabin problem into smaller separate cabin problems and use the dynamic programming solution of these smaller problems depending on the number of seats remaining in each cabin.

More specifically, we apply the separate cabin Lautenbacher-Stidham algorithm to three different single-leg revenue management problems.

1. Premium pax

   Premium classes' demand using premium cabin capacity

2. Economy pax

   Economy classes' demand using economy cabin capacity

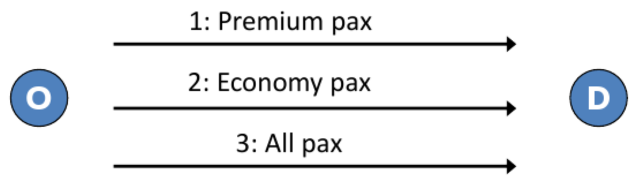

Figure 3.2. Three different separate cabin DP problems

3. All pax

   All classes' demand using total capacity

We then compare the different booking requests with the bidprice of the different solution depending on the capacity remaining. Specifically, when there are seats remaining in the economy cabins, premium booking requests are compared to the "Premium pax" bidprice and economy booking requests are compared to the "Economy pax" bidprice. When there are no seats left in the economy cabin, all booking requests are compared to the "All pax" bidprice.

3.3 Chapter summary

In this chapter, we presented the different algorithms we developed in collaboration with MIT PODS consortium airline members. We presented a dynamic programming formulation for the multiple cabin problem that extends the separate cabin Lautenbacher-Stidham dynamic programming formulation, as well as an extension to the Walczak formulation for the multiple cabin dynamic programming formulation. We also presented different EMSR-based heuristics and a dynamic programming decomposition heuristic. These heuristics can be used to control capacity in a multiple cabin environment. More precisely, we saw three different ways that an airline can calculate the number of units to be shared in a shared nesting scheme: "Shared Nesting Full EMSR," "Shared Nesting Economy EMSR," and "Shared nesting EMSRc bidprice control." Finally, we presented a dynamic programming decomposition that uses the solution from smaller problems at different times to control capacity in the multiple cabin environment. In the next chapter, we will compare the performance of these different approaches in two different simulation environments using the Passenger Origin-Destination Simulator.

Chapter 4

Simulation and Results

IN this chapter, we compare the performance of the different algorithms presented in Chapter 3 based on simulation results in a competitive environment. More specifically, we explain differences in revenue and load factor performance by analyzing class closure rates and fare class mix, and test the sensitivity of these results to different demand levels.

In this chapter, we introduce the Passenger Origin-Destination Simulator (PODS), we explicitly define the performance metrics of interest, and compare the performance of the different algorithms in the realistic passenger choice simulation environment of PODS.

4.1 Simulator description

In this subsection, we describe the simulation tool used in this thesis, the Passenger Origin-Destination Simulator, which was created by Hopperstad at Boeing in the 1990s.

PODS simulates hypothetical airlines competing for virtual demand in a virtual network. Figure 4.1 presents its architecture. It is divided in two parts: (1) the Passenger Choice Model, and (2) the Revenue Management System. In the first part, virtual passengers are created with multiple characteristics such as a preferred schedule, a maximum willingness-to-pay, and different sensitivity to fare class constraints. In the second part, the airlines' logic is defined. Airlines in PODS manage their capacity like real airlines in the industry by forecasting future demand based on recorded historical bookings, and can choose from an array of revenue management methods to make their seat availability decisions.

We limit our description to a relatively high level since PODS has been extensively described in previous work. We refer readers interested in implementation details to Carrier's Master's thesis [8].

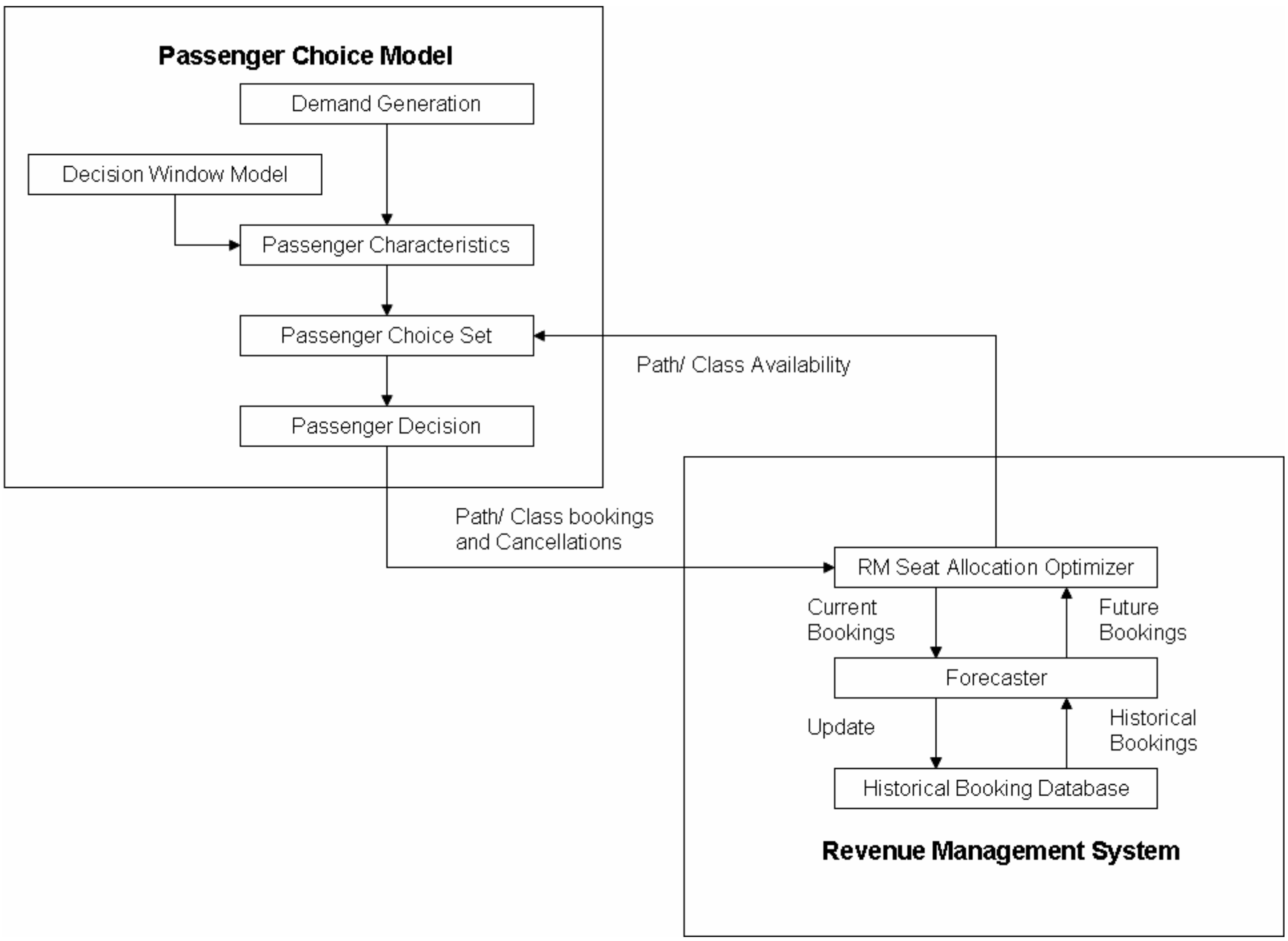

Figure 4.1. PODS Architecture representation [4]

■ 4.1.1 Passenger Choice Model

The Passenger Choice Model generates virtual passengers with a need for transportation between the different cities in the network. The simulator generates two different passenger types, business and leisure passengers. The generation process for both passenger types is identical, the only difference being the value of the different input parameters. Figure 4.2 shows typical passenger arrival curves for the different passenger types. As we can see, business passengers arrive, on average, later in the booking process when compared to leisure passengers. There are three main characteristics that are defined for each passenger:

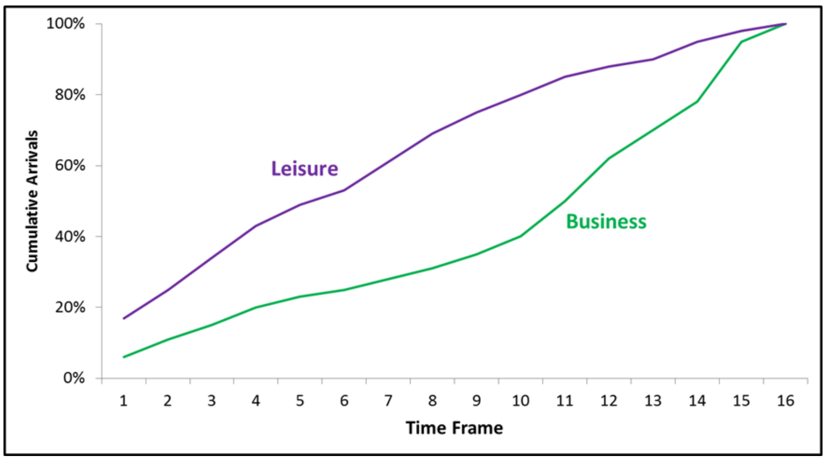

Figure 4.2. Passenger arrival curves

1. Willingness-to-pay

   Passengers are randomly assigned a maximum out-of-pocket willingness-to-pay. As one would expect, business passengers have, on average, a higher maximum willingness-to-pay than leisure passengers.

2. Schedule preference

   Based on the Time Of Day Demand curves [10], passengers are assigned a preferred traveling time window; they are then assigned a rescheduling cost. Passengers prefer itineraries that are within their preferred time window and incur a rescheduling cost if their selection option is outside their preferred window. As one would expect, business passengers are generally more sensitive to schedule changes than leisure passengers.

3. Restriction sensitivity

   Passengers are randomly assigned a sensitivity level to the different restrictions applied by the airlines to fare classes. As explained in Chapter 1, examples of restrictions are, among others, minimum stay at destination, change fee, and cancel fee. The restrictions applied to the different fare classes force passengers to book in higher fare classes. Another restriction introduced in PODS is the "Economy cabin" restriction. It represents the disadvantage of not having a confirmed seat in the premium cabin. As expected, business passengers are, on average, more sensitive to restrictions than leisure passengers.

As shown on Figure 4.1 after defining passenger characteristics, the passenger choice set is defined. As its name indicates, the passenger choice set is the subset of fare classes from which a given passenger makes its final choice. It is created by eliminating fare classes that have a fare greater than the passenger's maximum willingness-to-pay from the set of classes made available by the airline. It is worth noting that "no-go" is an option for passengers with a WTP lower than the lowest fare class in the set of classes made available by the airline.

At the last step in the Passenger Choice Model, passengers select the option from the passenger choice set that minimizes their total cost, where total cost is defined as the sum of the fare and the costs associated with rescheduling and the cost of restrictions. The passenger's final choice is then recorded as a booking by the airline's revenue management system.

4.1.2 Revenue Management System

In this subsection, we describe the Revenue Management System. The Revenue Management System is the airline side of the simulation and does not have any knowledge of the underlying parameters of the Passenger Decision Model. The function of the Revenue Management System is to maximize revenues assuming a given flight schedule and fixed capacity. There are three main components of the Revenue Management System:

1. Historical Booking Database

   Like in the real airline industry, airlines in PODS record historical booking data. The observed booking history information stored in the Historical Booking Database is the only historical information available to the airlines to forecast future demand and make their seat allocation decision.

2. Forecaster

   Using historical booking data and accepted bookings up to the current reoptimization time, airlines forecast bookings-to-come in two steps: (1) Detruncation, and (2) using different statistical methods (e.g. exponential smoothing).

   Detruncation is the process needed to convert observed bookings to an estimate of true or unconstrained demand. Indeed, airlines have no way of knowing real demand for a given class on a specific itinerary, and one classic way to estimate it is to use the Boeing Spill model [9]. Once the airline has an estimate for the unconstrained demand for a past flight, it can use statistical methods to forecast demand for future flights.

3. RM Seat Allocation Optimizer

   Given forecasted bookings-to-come, airlines use different optimization methods such as the ones presented in Chapter 3.

   The RM seat allocation optimizer keeps track of the number of seats remaining and decides, based on a specified revenue management algorithm, which fare classes are available at any given time. The set of available classes is passed to passengers so that they can define their own "passenger choice set". As described earlier, the passenger choice set is then used in the Passenger Choice Model.

PODS represents the state of the art in terms of realistic simulation environment for airline revenue management and is constantly evolving based on the inputs from the PODS consortium airline members. Now that we have a better understanding of how the simulator works, we define the performance indicators that we use to evaluate the performance of the different methods presented in Chapter 3.

■ 4.2 Performance indicators

Before comparing the different methods, we need to define the different performance metrics that we use to evaluate their the performance.

- *Revenue Passenger Miles (RPM)* is a measure of the total capacity used by passengers. It is computed by summing over all the flight-legs operated by an airline the product of the number of passengers flying to the distance flown.

- *Available Seat Miles (ASM)* is a measure of the total capacity offered by a given airline. It is computed by summing over all the flight-legs operated by an airline the product of the number of seats to the distance flown.

- *Revenue* is the total amount of money received by the airline. It is calculated by summing the product of the number of confirmed bookings by the fare in each class across all itineraries. Revenue is generally the first metric we examine since, in revenue management, the objective is to maximize revenues.

- *Load factor* is a measure of utilization of capacity. It is the ratio of used capacity to the amount of available capacity. For a specific leg, it is calculated by dividing the total number of passengers sitting on the plane at departure by the total number of seats on the plane, whereas at the network level, it is calculated by dividing the total number of RPM with the total number of ASM. In this thesis, we look at total load factor, which is computed for the entire capacity, and at cabin load factor, which is computed for each cabin separately.

  Load factor is a critical variable in revenue management. Indeed, one of the variable directly influencing revenue is the total number of passengers flying. However, high load factor does not necessarily lead to high revenues. Indeed, one can think of a case where a flight is filled with many low fare class passengers.

- *Yield* is the average revenue generated by a passenger flying one mile. It is calculated by dividing total revenue by total RPM.

  Yield is the other critical variable in revenue management. Because of the law of supply and demand, high yield is generally associated with lower load factor. Therefore, one can see that there exists a trade-off between yield and load factor when maximizing revenues.

- *Fare class mix* is a measure of the booking spread across the different fare classes. One can observe the relative strengths and weaknesses of different methods by comparing fare class mix.

- *Closure rate* is a measure of availability of the different fare classes. Airline Revenue Management analysts can use closure rates to see what percentage of flights had a specific class available at a given time.

We will use these different metrics to compare the relative performance of the methods, and provide some insights on the practical implications of the results.

Table 4.1. Single market fare structure

Cabin	Class	Fare	AP	R1	R2	R3
Premium	1	\$ 779	0	NO	NO	NO
	2	\$ 702	3	NO	YES	NO
Economy	3	\$ 624	0	NO	NO	NO
	4	\$ 440	3	NO	YES	NO
	5	\$ 309	7	NO	YES	YES
	6	\$ 208	10	YES	YES	YES
	7	\$ 171	14	YES	YES	YES
	8	\$ 135	14	YES	YES	YES

4.3 Results

In this section, we compare the performance of different seat allocation methods. We first present the general characteristics of the simulations, we then analyze each method using the performance metrics defined previously and test the sensitivity of the results to different demand levels.

4.3.1 Network and fare structure

We use two distinct network and fare structures to compare the different algorithms. The first network is a single market case with two airlines offering a single flight each in which we can test computationally intensive methods. The second network is a realistic network with four airlines competing in multiple markets in which we can better assess the practical performance of the different algorithms.

Single market

First, we have a single market competitive network with two airlines offering one flight each in the market. The fare structure in this market is presented in Table 4.1. It shows the array of different classes offered with their respective fare, Advance Purchase requirement (AP), and which of the three product restrictions apply. We note that R1 is the most restrictive of all three restrictions and that R3 is more restrictive than R2. It is important to remember that all economy classes have the additional “Economy cabin” disutility restriction to model the decision process with regards to the different cabins. By more restrictive, we mean that the associated contribution to the total cost used at the passenger decision step in the Passenger Choice Model is higher then the one associated with other restrictions.

Table 4.2. Realistic network restricted fare structure

Cabin	Class	avg. Fare	AP	R1	R2	R3
Premium	1	$ 1666	0	NO	NO	NO
	2	$ 1500	0	NO	YES	NO
	3	$ 1140	7	YES	YES	NO
	4	$ 801	14	YES	YES	YES
Economy	5	$ 1333	0	NO	NO	NO
	6	$ 920	3	NO	YES	NO
	7	$ 725	7	NO	YES	YES
	8	$ 602	10	YES	YES	YES
	9	$ 504	14	YES	YES	YES
	10	$ 416	14	YES	YES	YES

Table 4.3. Realistic network semi-restricted fare structure

Cabin	Class	avg. Fare	AP	R1	R2	R3
Premium	1	$ 683	0	NO	NO	NO
	2	$ 594	0	NO	YES	NO
	3	$ 446	7	NO	YES	NO
	4	$ 343	14	NO	YES	YES
Economy	5	$ 506	0	NO	NO	NO
	6	$ 364	0	NO	YES	NO
	7	$ 280	7	NO	YES	YES
	8	$ 223	7	NO	YES	YES
	9	$ 183	14	NO	YES	YES
	10	$ 153	14	NO	YES	YES

This single market case will be used to assess the performance of the multiple cabin dynamic programming formulation. We compare the dynamic programming methods to the base case and to the "Multiple cabin DP heuristic." Indeed, since the dynamic programming formulations are computationally intensive, we can only test them in a relatively small network.

Realistic network

The second scenario is a more realistic network where four airlines compete in 572 markets. These markets are divided in two market types: (1) higher value restricted markets, and (2) lower value semi-restricted markets. There are two different market types in order to simulate the market diversity observed in the industry.

The fare structures for both markets are presented in table 4.2 and 4.3. The re-

Table 4.4. Distinct EMSR results in single market scenario at low demand

AL	ASM	RPM	LF	Yield	Rev
1	166940	126028	75.5	0.2440	30749
2	166940	125355	75.1	0.2469	30945

Table 4.5. Distinct EMSR results in single market scenario at medium demand

AL	ASM	RPM	LF	Yield	Rev
1	166940	138091	82.7	0.2523	34840
2	166940	136580	81.8	0.2596	35458

stricted fare structure is used in 276 markets while the semi-restricted fare structure is used in the remaining 296 markets. We can see that this network has more classes than the single market case, and that the average fare for the lowest classes in the premium cabin is lower than the average fare for top classes in the economy cabin. These differences make this network more realistic and we will use it to compare the performance of the different heuristics. It is important to remember that all economy classes have the additional "Economy cabin" disutility restriction to model the decision process with regards to the different cabins.

■ 4.3.2 Base case scenarios

In this section, we define base case scenarios at different demand levels in each network. Upgrades are not allowed in the base cases and every airline uses the EMSR heuristic independently in both cabins.

Passengers have the flexibility to choose a seat in both cabins depending on their inherent characteristics. That said, passengers selecting a premium cabin fare class have a confirmed seat in the premium cabin and passengers selecting an economy cabin fare class have a confirmed seat in the economy cabin and it is impossible for them to be upgraded to a seat in the premium cabin.

Single market

For the single market case, tables 4.4, 4.5, and 4.6 show ASM, RPM, total load factor, yield, and revenue for the two airlines in the market at a low, medium and high demand respectively.

The results presented here are sample level averages over five simulation trials of 600 samples where each sample simulates the entire booking process for all flights in the

Table 4.6. Distinct EMSR results in single market scenario at high demand

AL	ASM	RPM	LF	Yield	Rev
1	166940	143464	85.9	0.2610	37437
2	166940	141796	84.9	0.2689	38131

Table 4.7. Distinct EMSR fare class mix in single market scenario at low demand

Class	AL 1	AL 2
1	8.4	8.4
2	0.9	1.0
Premium total	9.3	9.4
3	1.5	1.7
4	15.3	15.8
5	10.5	10.8
6	2.6	2.8
7	1.0	1.1
8	88.0	86.2
Economy total	119.0	118.3
Grand total	128.3	127.7

network. In other words, it is the average flight revenue performance of each airline. We can see that both airlines have the exact same number of ASMs at all demand levels. This is due to the fact that they both offer a single 982-mile flight with two cabins, economy and premium, having 150 and 20 seats respectively. Both airlines are using the exact same forecasting and revenue management method, and the fact that Airline 2 has a slight revenue advantage over Airline 1 at all demand levels is due to random variation in the simulation. It is interesting to note the trade-off between load factor and yield. Indeed, although Airline 2's load factor is 0.9 percentage point higher than Airline 1's, which could be interpreted as advantageous for Airline 1 by an untrained analyst, Airline 2's yield is higher which explains why Airline 2's revenues are higher than Airline 1's.

Tables 4.7, 4.8, and 4.9 show the average number of bookings in each class over the different trials and the different samples at three different demand levels. We can see that the large majority of bookings observed are Class 8 bookings and that Class 1 is the fare class for which we observe the most bookings in the premium cabin. We can also see that Class 8 bookings represent a smaller fraction of bookings as demand increases. This indicated that the revenue management system is able to select the most valuable passengers and reject lower fare class when demand increases. Consistent with

Table 4.8. Distinct EMSR fare class mix in single market scenario at medium demand

Class	AL 1	AL 2
1	10.1	9.9
2	1.3	1.2
Premium total	11.4	11.1
3	1.4	1.8
4	17.1	18.8
5	13.4	14.3
6	4.1	5.2
7	2.8	3.6
8	90.5	84.3
Economy total	129.3	127.95
Grand total	140.6	139.1

Table 4.9. Distinct EMSR fare class mix in single market scenario at high demand

Class	AL 1	AL 2
1	10.8	10.6
2	1.4	1.3
Premium total	12.1	12.0
3	1.4	1.8
4	18.8	20.7
5	15.9	17.0
6	7.2	8.5
7	7.3	7.7
8	83.3	76.8
Economy total	134.0	132.4
Grand total	146.1	144.4

results in tables 4.4, 4.5, and 4.6, Airline 1 accepted more bookings than Airline 2, but the mix is such that the average fare per booking or, equivalently, the yield is higher for Airline 2. It is also interesting to note that, on average, a little over 8 seats out of the 20 available premium seats at medium demand are left empty. Successful multiple cabin optimization methods will accept additional economy bookings and allow these passengers to use available premium cabin capacity.

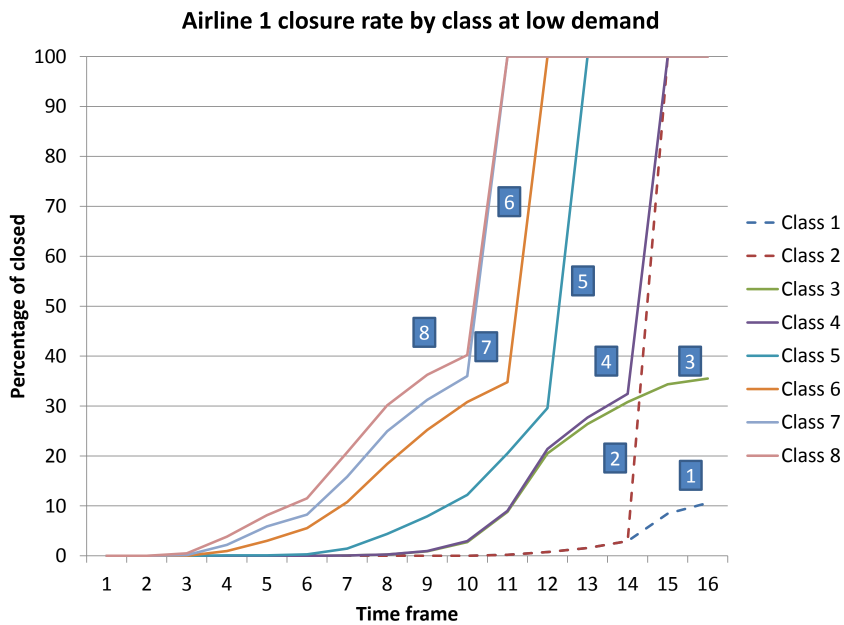

Figure 4.3. Airline 1 class closure rates in single market scenario at low demand

Figures 4.3, 4.4, and 4.5 show Airline 1's percentage of samples for which the specified class is closed as a function of time before departure, which we refer to as closure rates. The booking horizon is divided in sixteen "time frames" and departure is at the end of time frame 16. As one would expect, lower classes close earlier than in the booking process than higher classes, and closure rates are generally increasing as demand is increasing.

Realistic network

Tables 4.10, 4.11, and 4.12 show average sample results over 2 trials of 600 samples. One can see by comparing ASMs that this scenario is much bigger than the single market scenario. In this network, Airline 1 is the dominant carrier in terms of ASMs, RPMs,

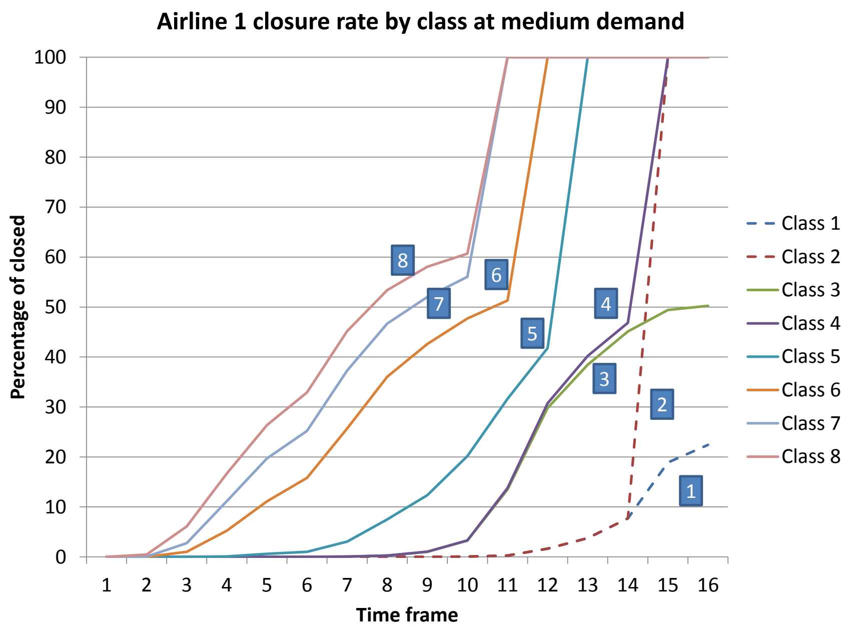

Figure 4.4. Airline 1 class closure rates in single market scenario at medium demand

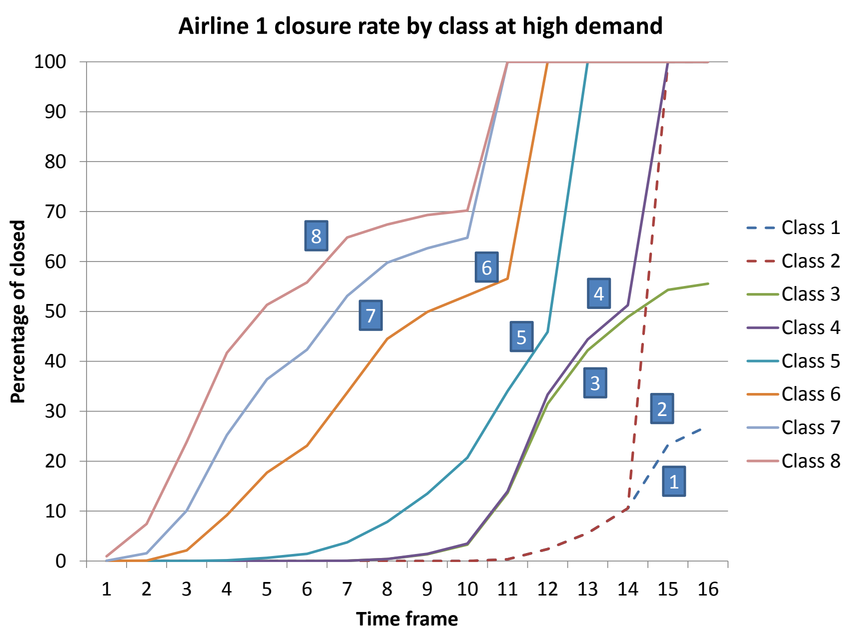

Figure 4.5. Airline 1 class closure rates in single market scenario at high demand

Table 4.10. Distinct EMSR results in realistic network scenario at low demand

AL	ASM	RPM	LF	Yield	Rev
1	28,743,562	20,821,930	72.4	0.1585	3,301,295
2	23,584,496	17,401,240	73.8	0.1500	2,609,477
3	10,311,962	7,115,968	69.0	0.1485	1,056,680
4	17,870,366	12,413,867	69.5	0.1540	1,911,148

Table 4.11. Distinct EMSR results in realistic network scenario at medium demand

AL	ASM	RPM	LF	Yield	Rev
1	28,743,562	22,928,808	79.8	0.1590	3,645,111
2	23,584,496	19,059,300	80.8	0.1500	2,859,374
3	10,311,962	7,839,079	76.0	0.1485	1,163,760
4	17,870,366	13,849,365	77.5	0.1534	2,123,841

yield and revenues. Airlines 1, 2 and 4 are providing service in a mix of restricted and semi-restricted markets. Airline 3 has less capacity than other airlines, as exposed by its significantly smaller number of ASMs, and is offering service in semi-restricted markets only. We can see that total load factor is increasing as demand increases.

Tables 4.13, 4.14, and 4.15 show the average number of bookings per sample on the entire network for each class and for each airline at different demand levels. We can see that the majority of bookings are observed in Class 10. This is due to the fact that Class 10 has the lowest fare in all markets and that the restrictions in place are such that a majority of the bookings are observed in this class. For similar reasons, Class 3 is the fare class for which we observe the most bookings in the premium cabin.

Figures 4.6, 4.7, and 4.8 show Airline 1's rate across samples and trials of flights for which the specified class is closed as a function of time in the booking process, which we refer to as closure rates. We can see that high class closure rates are lower than low class closure rate in both cabins and that closure rates increase for all classes when demand increases.

In this subsection, we presented the base line performance at different demand levels

Table 4.12. Distinct EMSR results in realistic network scenario at high demand

AL	ASM	RPM	LF	Yield	Rev
1	28,743,562	24,722,054	86.0	0.1600	3,954,362
2	23,584,496	20,372,184	86.4	0.1516	3,089,123
3	10,311,962	8,497,807	82.4	0.1496	1,271,283
4	17,870,366	15,113,955	84.6	0.1538	2,324,899

Table 4.13. Distinct EMSR fare class mix in realistic network scenario at low demand

Class	AL 1	AL 2	AL 3	AL 4
1	323.03	245.66	46.86	156.98
2	49.82	39.95	14.57	36.34
3	548.28	526.65	547.89	454.34
4	36.84	71.55	34.36	34.95
Premium total	957.97	883.81	643.68	682.61
5	534.93	429.09	338.59	302.87
6	1076.15	980.85	312.15	576.83
7	327.94	261.78	38.84	158.04
8	139.1	150.18	115.42	119.07
9	11.09	7.95	7.72	3.78
10	4954.94	4478.53	3312.53	3698.96
Economy total	7044.15	6308.38	4125.25	4859.55
Grand total	8002.12	7192.19	4768.93	5542.16

Table 4.14. Distinct EMSR fare class mix in realistic network scenario at medium demand

Class	AL 1	AL 2	AL 3	AL 4
1	346.97	257.93	46.24	168.66
2	54.56	40.64	13.5	38.63
3	564.08	552.08	572.2	474.17
4	35.23	69.7	34.32	34.84
Premium total	1000.84	920.35	666.26	716.3
5	624.05	496.53	400.63	355.76
6	1218.24	1107.85	365.96	652.31
7	379.89	300.65	43.32	182.27
8	157.02	170.48	138.33	135.48
9	58.17	49	38.33	35.94
10	5421.71	4865.65	3585.75	4088.21
Economy total	7859.08	6990.16	4572.32	5449.97
Grand total	8859.92	7910.51	5238.58	6166.27

Table 4.15. Distinct EMSR fare class mix in realistic network scenario at high demand

Class	AL 1	AL 2	AL 3	AL 4
1	373.27	273.28	45.08	183.34
2	60.27	44.41	13.19	42.24
3	576.23	568.94	598.08	488.75
4	35.84	67.74	35.27	34.43
Premium total	1045.61	954.37	691.62	748.76
5	689.9	558.89	468.28	394.66
6	1374.02	1250.69	430.77	740.39
7	446.28	353.42	52.5	216.9
8	188.5	212.29	185.58	168.28
9	213.5	183.93	133.25	136.83
10	5652.62	5019.01	3707.52	4333.51
Economy total	8564.82	7578.23	4977.9	5990.57
Grand total	9610.43	8532.6	5669.52	6739.33

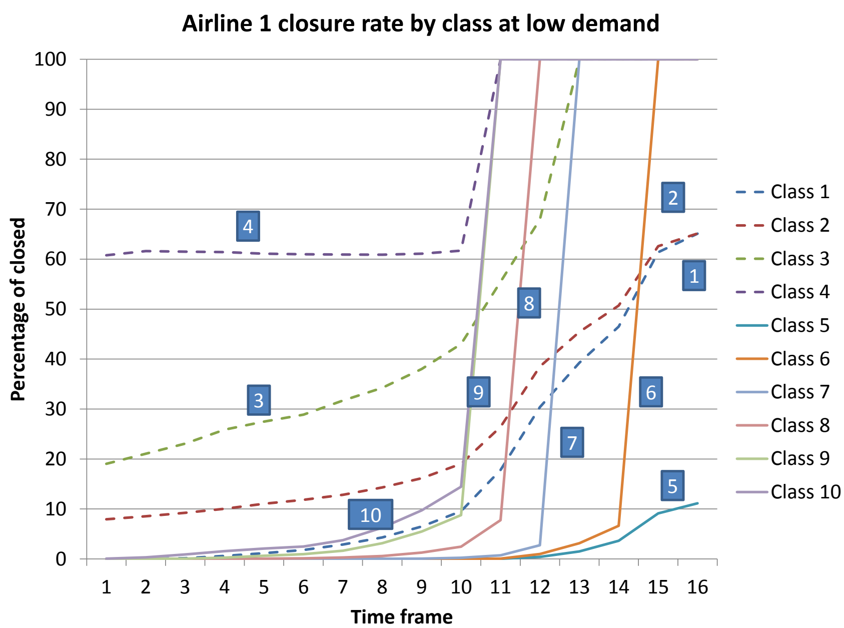

Figure 4.6. Airline 1 class closure rates in realistic network scenario at low demand

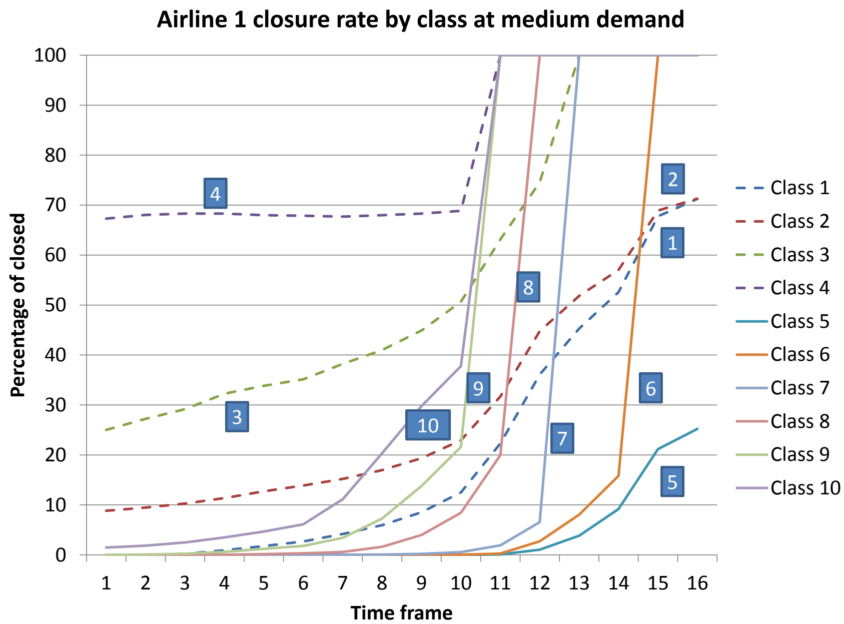

Figure 4.7. Airline 1 class closure rates in realistic network scenario at medium demand

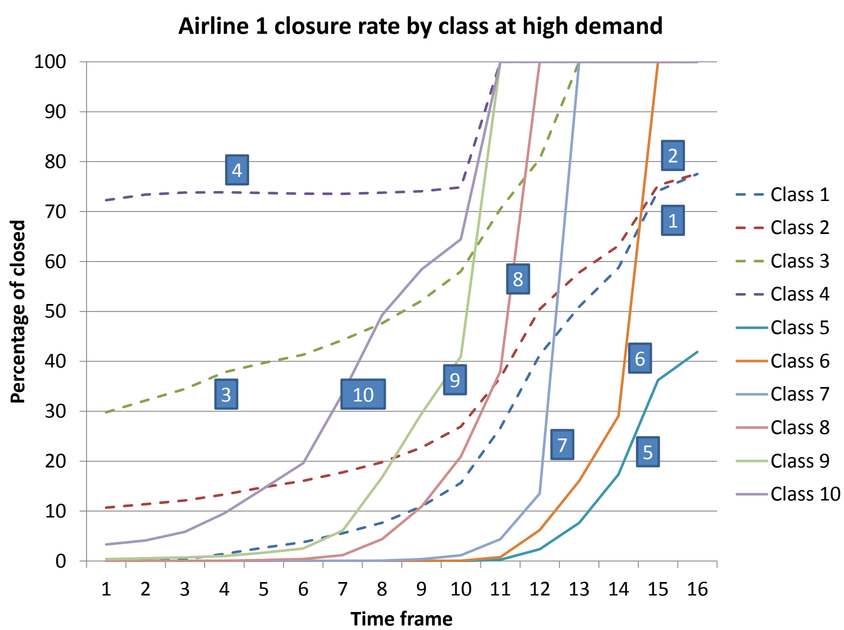

Figure 4.8. Airline 1 class closure rates in realistic network scenario at high demand

in the two scenarios of interest. We saw that, in the single market case, both airlines perform very similarly whereas there are clear differences between the airlines in the realistic network scenario.

4.3.3 Performance evaluation

In this subsection, we compare the relative effect on Airline 1's performance of different multiple cabin optimization methods. For each method, we focus on relative revenue performance, load factor changes, fare class mix variation, and closure rates differences; we test the sensitivity of the results to different demand levels; and we also discuss the practical implications of the results.

Single market

We first compare all algorithms presented in Chapter 3 in the single market case. We are interested in the competitive implication of the methods, so we test the variation when only Airline 1 uses the different optimization methods while Airline 2 uses the EMSR heuristic independently in each cabin (Distinct EMSR).

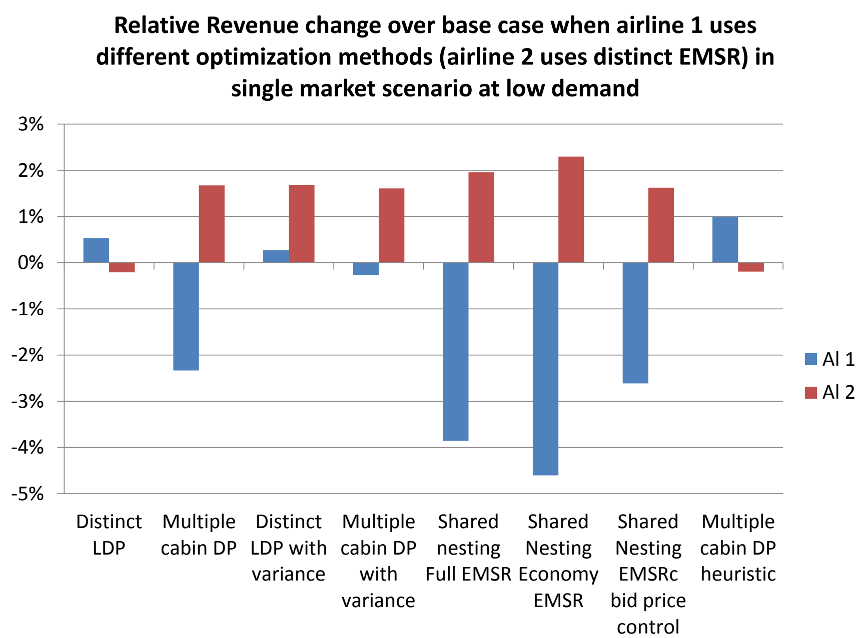

Figure 4.9. Relative revenue change over base case in single market scenario at low demand

Figures 4.9, 4.10, and 4.11 show the revenue percentage change for both airlines when

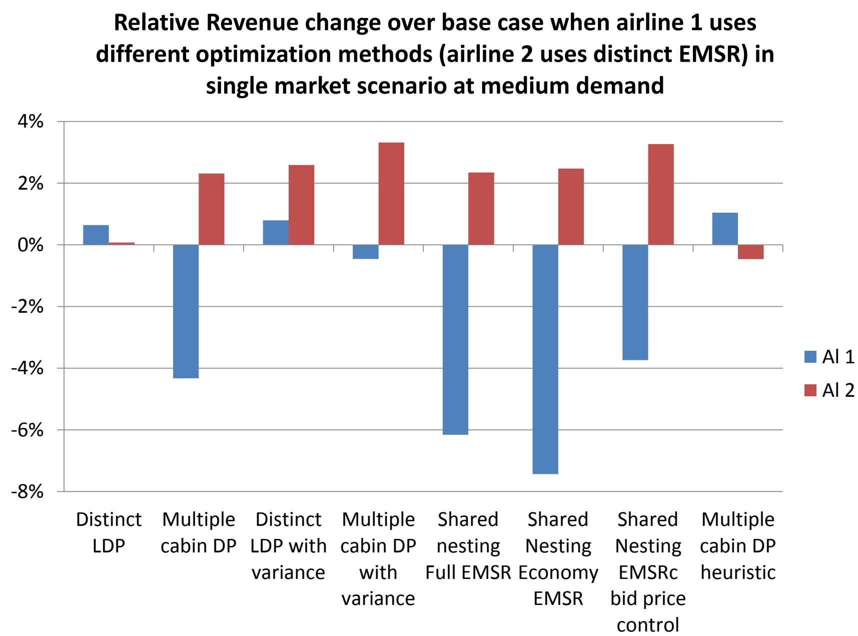

Figure 4.10. Relative revenue change over base case in single market scenario at medium demand

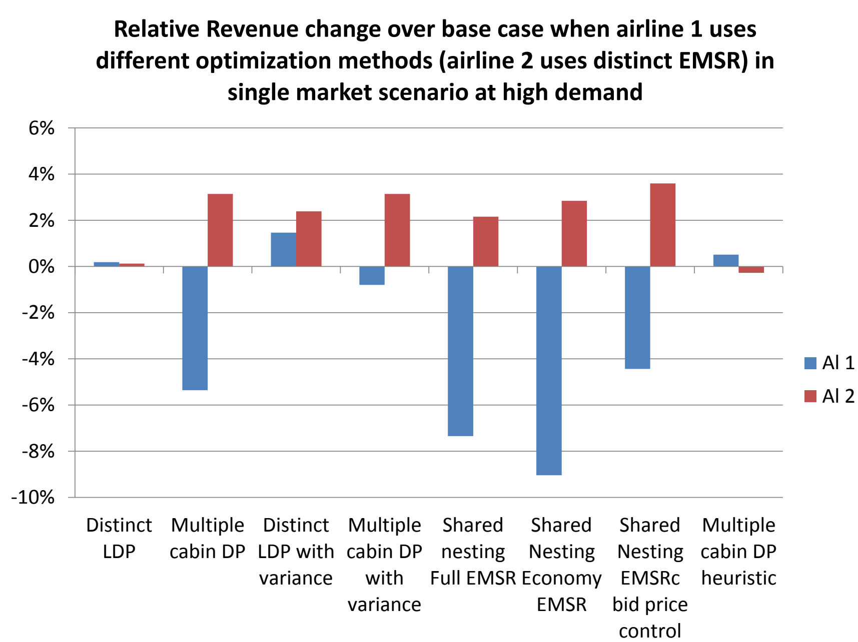

Figure 4.11. Relative revenue change over base case in single market scenario at high demand

Airline 1 is using the specified method. It is important to note that "Distinct LDP" and "Distinct LDP with variance" are not multiple cabin optimization methods. These methods were presented in Chapter 2 and they were introduced in the analysis to allow us to isolate the effects of multiple cabin optimization from dynamic programming. We can see that most of the multiple cabin optimization methods lead to a revenue decrease for Airline 1. In fact, only the "Multiple cabin DP heuristic" leads to a 1% increase in the low and medium demand levels and to a 0.5% increase in the high demand case. We can see, by comparing "Multiple cabin DP" with "Multiple cabin DP with Variance" that accounting for variance improves the performance of multiple cabin optimization. Finally, in the single market case, none of the EMSR-based multiple cabin heuristics improve revenues over the distinct cabin base case.

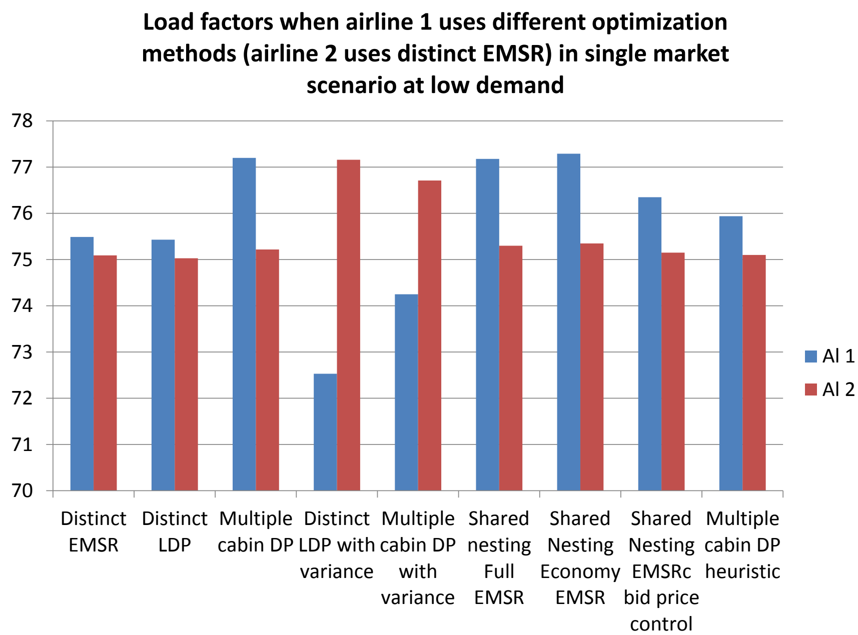

Figure 4.12. Load factors in single market scenario at low demand

Figures 4.12, 4.13, and 4.14 show total load factor changes for both airlines when Airline 1 is using different methods. We observe that allowing additional economy bookings to use available premium capacity by using a multiple cabin optimization method leads to a total load factor increase for Airline 1. Indeed, comparing EMSR-based heuristics with the "Distinct EMSR" base case, and the Multiple cabin DP methods with their respective distinct case, we can see that total load factor increases when

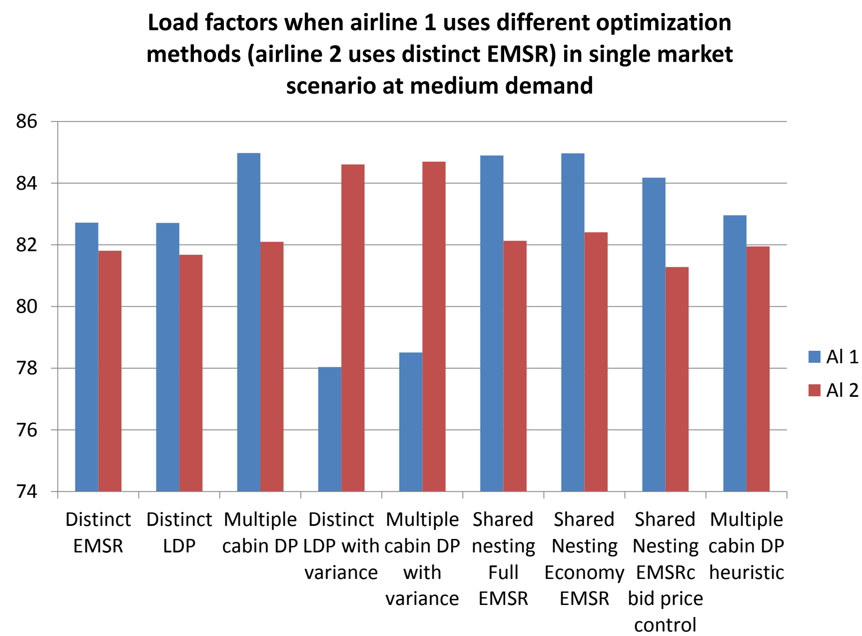

Figure 4.13. Load factors in single market scenario at medium demand

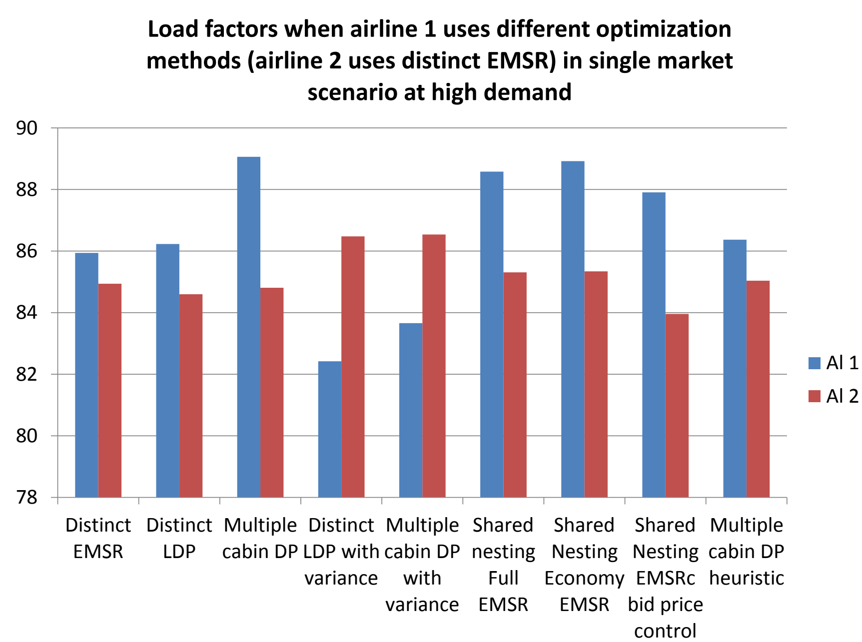

Figure 4.14. Load factors in single market scenario at high demand

using a multiple cabin optimization method.

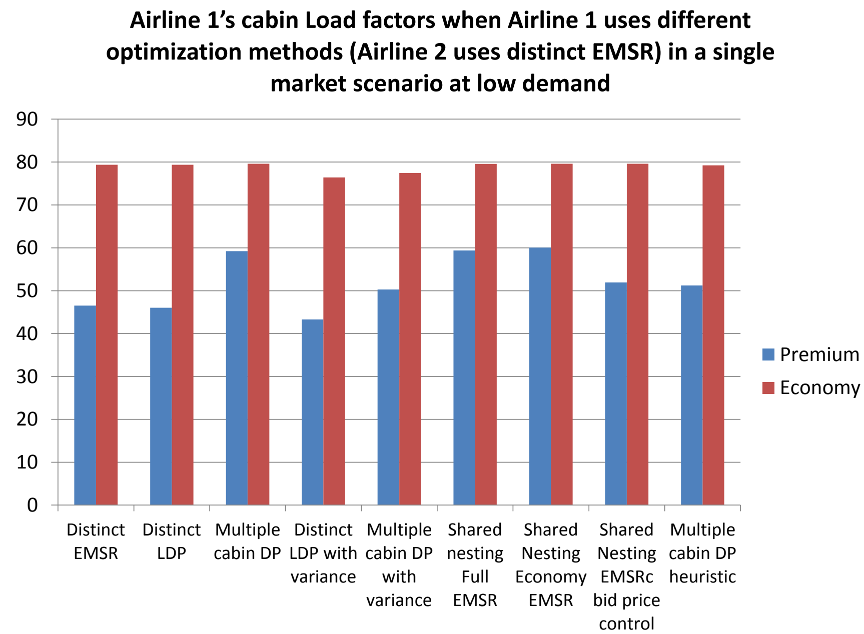

Figure 4.15. Cabin load factors in single market scenario at low demand

Figures 4.15, 4.16, and 4.17 show Airline 1's cabin load factor when Airline 1 uses the specified revenue management algorithm at different demand levels. Because multiple cabin methods allow additional economy booking requests to use available premium capacity, we observe that all multiple cabin methods lead to an increase in premium cabin load factor and that the economy cabin load factor is not changing significantly when compared to the base case scenario.

Figures 4.18, 4.19, and 4.20 show the average number of bookings per sample in each class for Airline 1 with exact numbers shown for classes 1 and 8. We note that most multiple cabin methods lead to an increase in Class 8 bookings and in a decrease in Class 1 bookings. This means that there is a trade-off between the additional economy bookings accepted by the airline and displaced high value premium bookings. The difference in revenue performance is explained by two things. First, the magnitude of the displacement effect of premium passengers by economy passengers and, second, the average fare paid by additional economy cabin bookings. Figures 4.9, 4.10, and 4.11 show that "Multiple cabin DP heuristic" is the only multiple cabin optimization method that leads to a revenue increase. Looking at the fare class mix, we can see that the

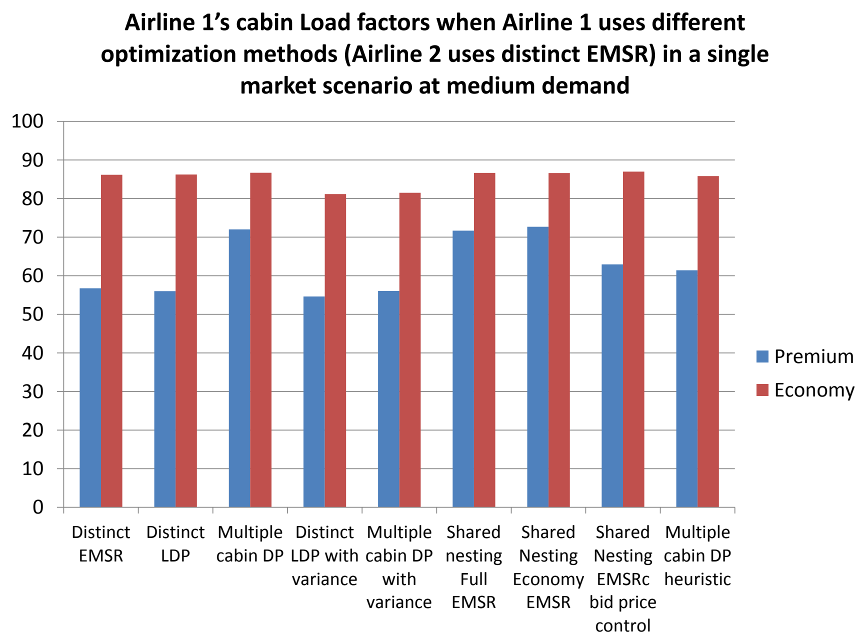

Figure 4.16. Cabin load factors in single market scenario at medium demand

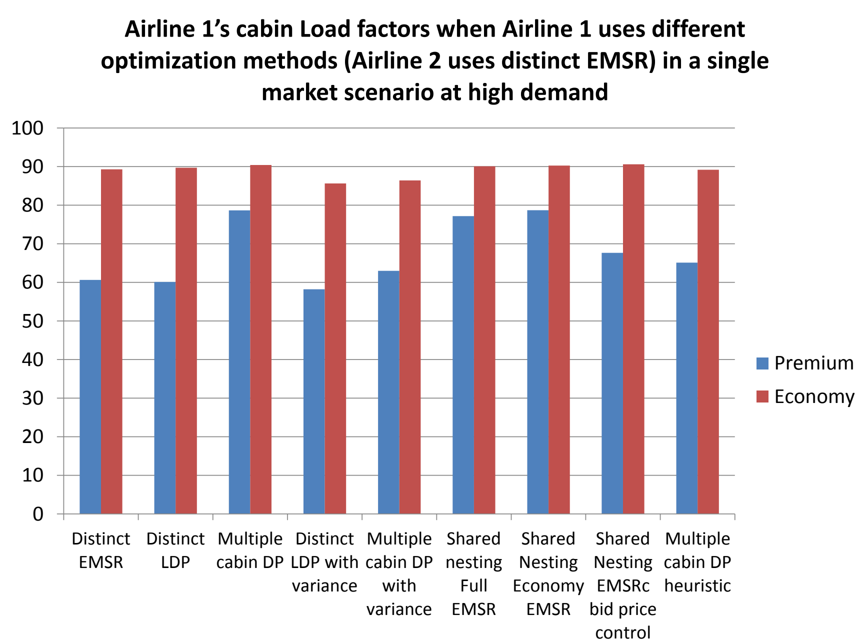

Figure 4.17. Cabin load factors in single market scenario at high demand

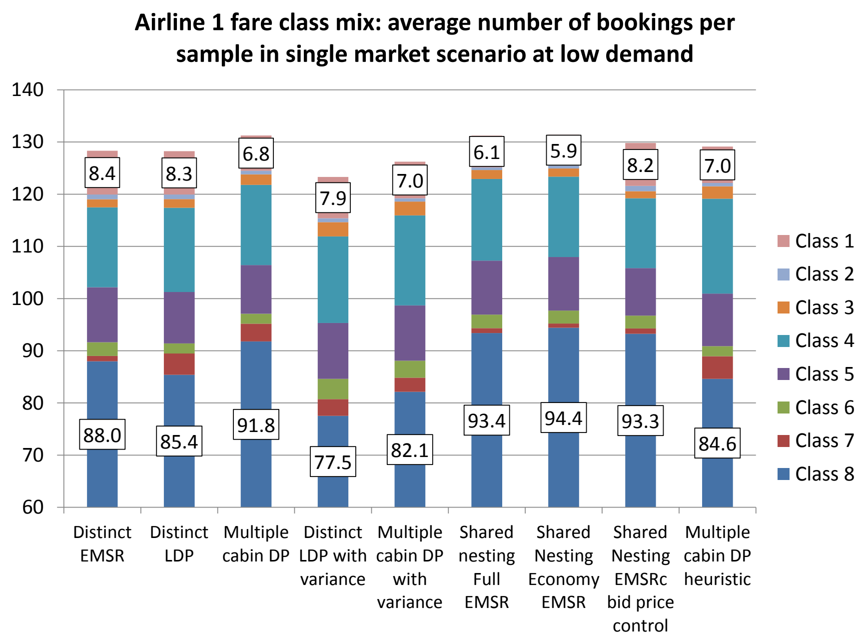

Figure 4.18. Fare class mix in single market scenario at low demand

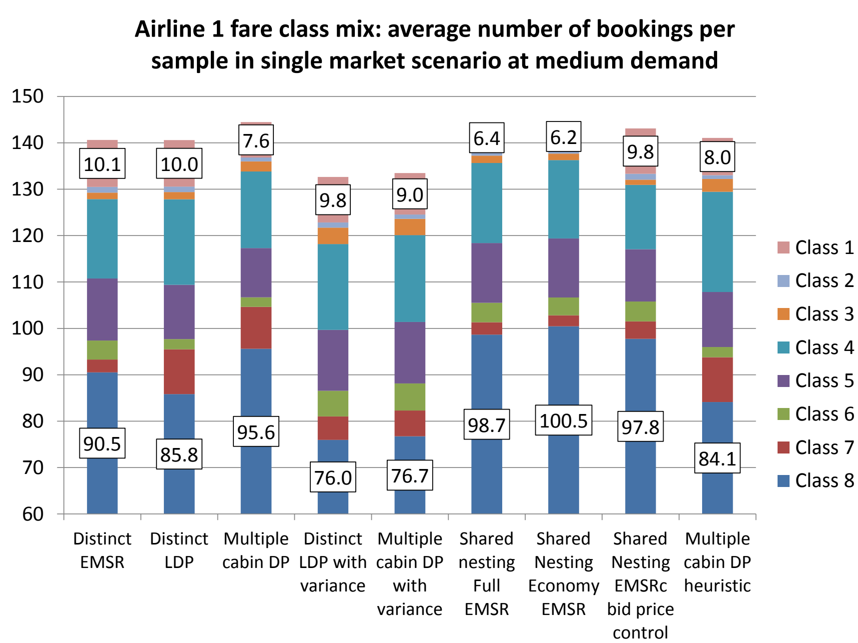

Figure 4.19. Fare class mix in single market scenario at medium demand

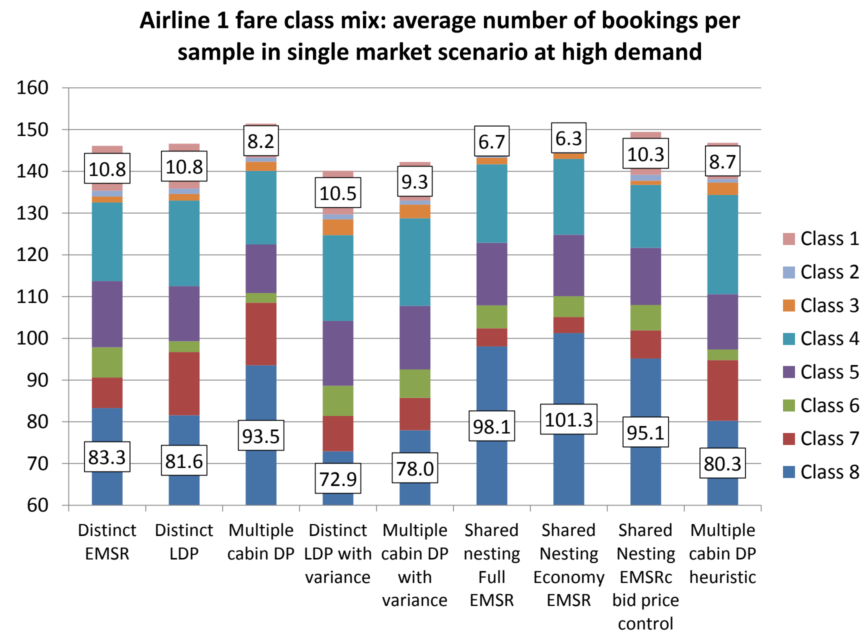

Figure 4.20. Fare class mix in single market scenario at high demand

"Multiple cabin DP heuristic" is the only one for which the number of Class 8 bookings increase over its relevant distinct case (Distinct LDP). Furthermore, we can see that the number of Class 3 bookings increases over the distinct case. This indicates that the "Multiple cabin DP heuristic" forces additional economy bookings into more valuable economy classes, which leads to a revenue increase.

Figures 4.21, 4.22, and 4.23 show the average percentage of samples for which a specified class is closed as a function of time in the booking process for different optimization methods, which we refer to as closure rates. First, we observe that, in general, closure rates have very similar patterns from one method to the other and that there are a few methods that have significantly different closure rates. We focus on "Multiple cabin DP with variance" at medium demand, and we can see from figure 4.22(D) that closure rates for the lowest economy cabin fare class are relatively high early in the booking process, decrease midway to a level below the average closure rates of other methods and finally closes late in the booking process. We can also see from figure 4.22(A) and figure 4.22(B) that closure rates for premium classes are slightly higher early in the booking process, and generally flatter later in the booking process. The overall trend is not affected by different demand levels.

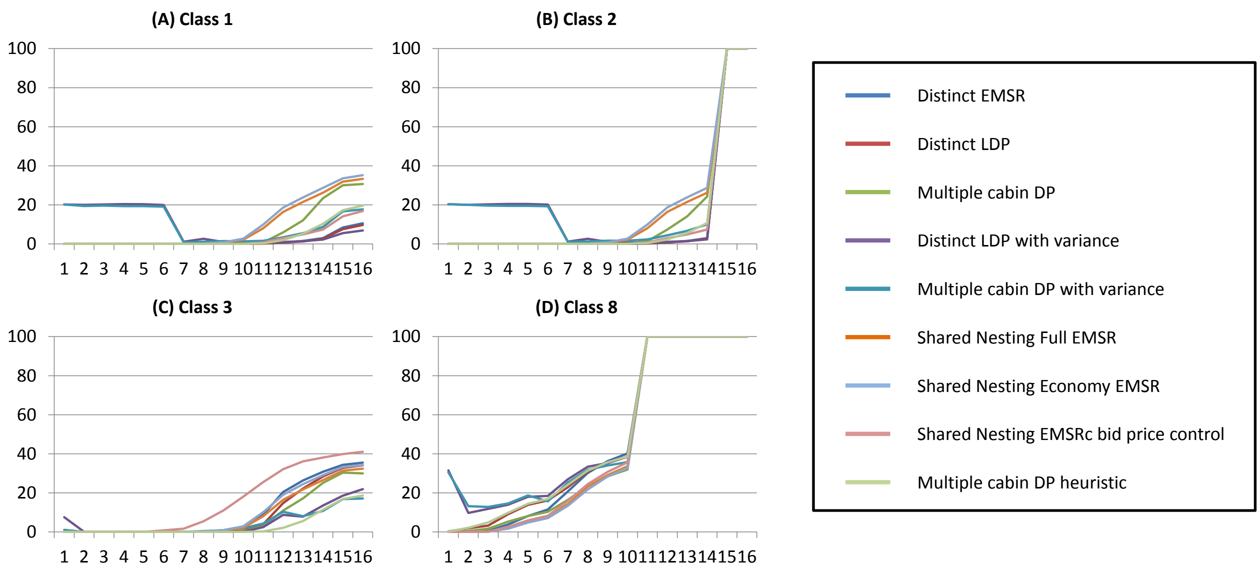

Figure 4.21. Closure rates in single market scenario at low demand

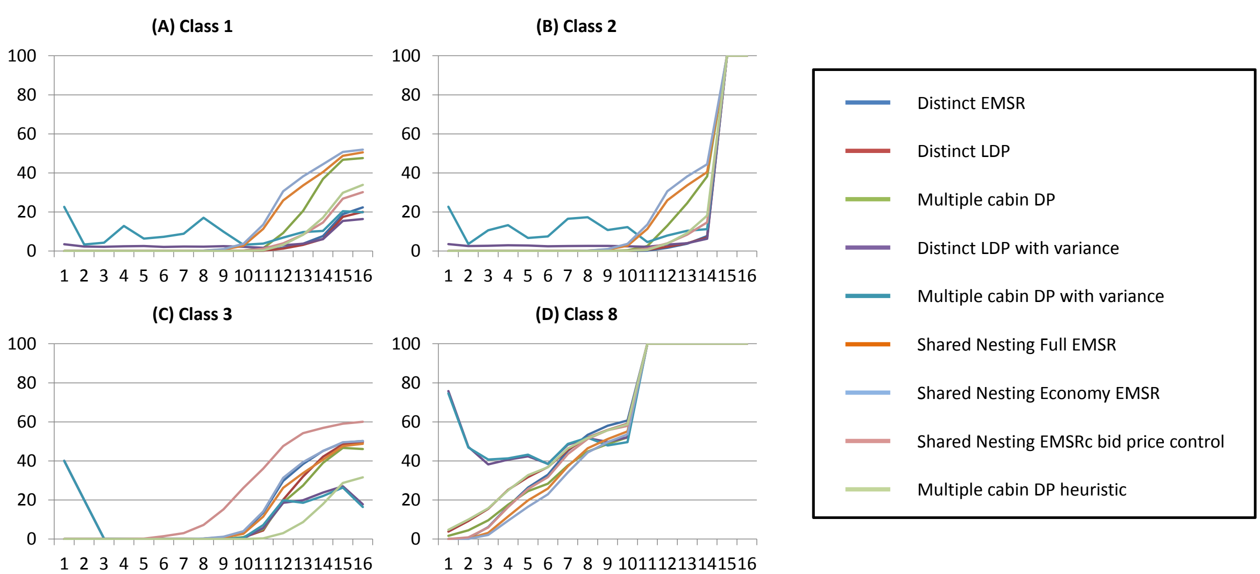

Figure 4.22. Closure rates in single market scenario at medium demand

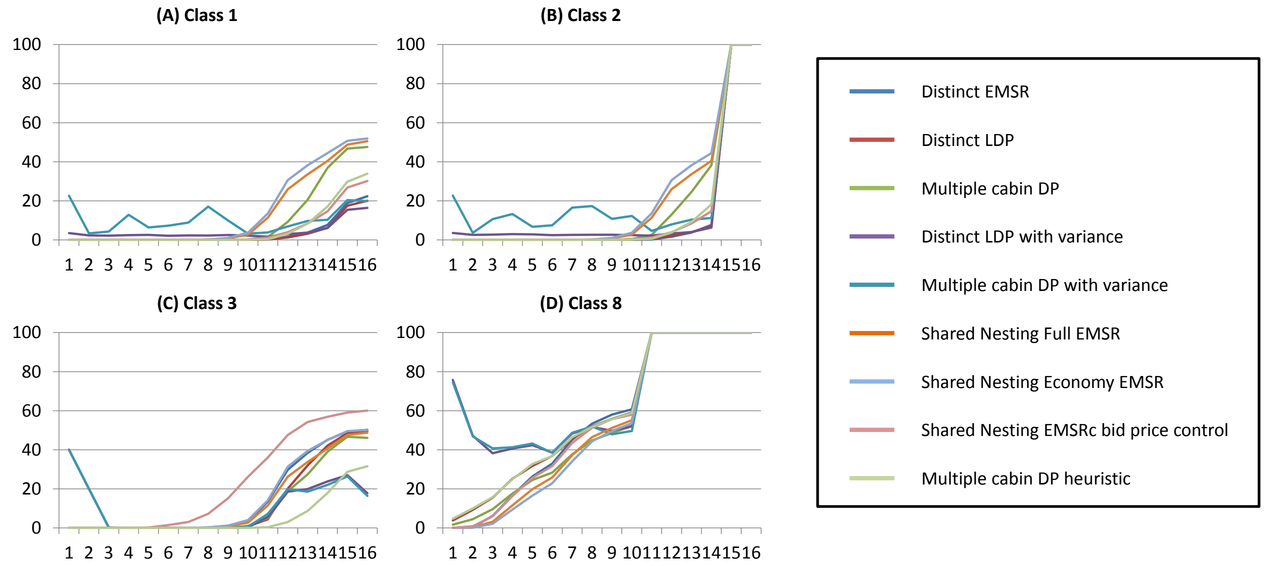

Figure 4.23. Closure rates in single market scenario at high demand

The relatively poor performance of most of the multiple cabin optimization methods presented in this section is due to the fact that revenues gained from additional economy bookings are smaller than revenues lost from displaced premium passengers. The "Multiple cabin DP heuristic" leads to a revenue increase because additional economy bookings are in higher classes. Benefits from multiple cabin optimization methods are driven by (1) the number of additional bookings, (2) the average fare paid by these extra economy passengers, (3) the number of displaced premium passengers, and (4) the average fare paid by displaced premium cabin passengers. In the single market case, the vast majority of premium bookings are Class 1 bookings; this implies that most of the displaced premium passengers are high-value Class 1 passengers. Since all premium fares are higher than economy fares, additional economy passengers have to outnumber displaced premium passengers. Since the number of extra economy bookings is limited by the available capacity in premium, only methods that force extra economy bookings in higher classes lead to a revenue increase. Another interesting observation, as we noted when analyzing Figure 4.13, is that multiple cabin optimization methods all lead to a total load factor increase. This is viewed as highly positive by airline management teams since it sends the signal that their company is more efficient and better uses its capacity. Finally, the multiple cabin results were not significantly different from one demand level from the other.

When dynamic programming methods were introduced in chapters 2 and 3 we presented them as being well suited for the revenue management problem. Therefore, it is surprising that "Multiple cabin DP" and "Multiple cabin DP with variance" do not perform better than the "Multiple cabin DP heuristic." This is due to the mismatch between the underlying assumptions of multiple cabin dynamic programming and the reality of the booking and passenger choice process. We identified the variance assumption implied by the Poisson arrival process as the reason why "Multiple cabin DP" underperforms compared to the base case scenario. As exposed by Diwan [11], this variance assumption causes the algorithm to be overconfident regarding late high-WTP booking requests, which in turn leads to decreasing closure rates during the booking process as shown in figures 4.21, 4.22, and 4.23. Consequently, high-WTP passengers arriving later in the booking process are able to book in lower classes; this effect explains the change in fare class mix and the reduction in revenues for "Multiple cabin DP."

"Multiple cabin DP with Variance" addresses the variance issue, but still underperforms compared to the base case scenario and compared to the "Multiple cabin DP heuristic." It is also interesting to note that "Multiple cabin DP with variance" also suffers from the declining closure rates effect as shown on figures 4.21, 4.22, and 4.23. This indicates that the variance assumption is not the only reason explaining dynamic programming methods' underperformance. Indeed, the dynamic programming formulations also assume that the airline can reoptimize and change the bidprice value after every single booking. We saw in Chapter 1 that airlines in practice reoptimize at a relatively small number of times throughout the booking process; this causes dynamic programming to respond more slowly to changes in the booking behavior and leads to the observed revenue performance and closure rate patterns.

Also, dynamic programming methods, like the other multiple cabin algorithms tested, do not take competitive interactions or passenger choice into consideration. In other words, they do not take into account competitors' strategy and they assume that demand is independent between the different fare classes. Our results suggest that the dynamic programming methods tested are more sensitive to the assumption misalignment than the heuristics we developed.

Realistic network

We now look at the more realistic network scenario. Since this is a more complex and bigger scenario, it is not practical to run "Multiple cabin DP" and "Multiple cabin DP with variance". Therefore, we compare the performance of different heuristics.

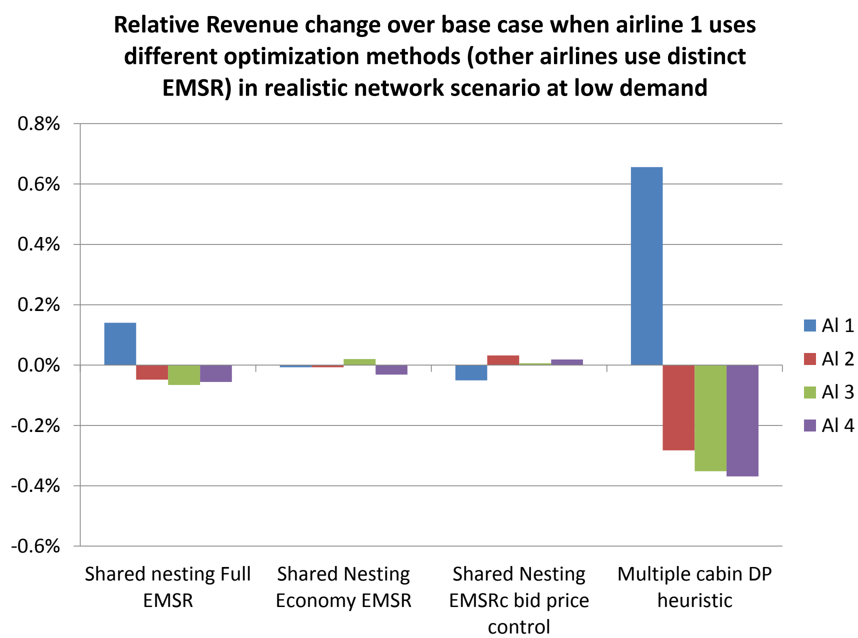

Figure 4.24. Relative revenue change over base case in realistic network scenario at low demand

Figures 4.24, 4.25, and 4.26 show relative revenue change for all four airlines when Airline 1 uses different optimization methods. We can see that the "Multiple cabin DP heuristic" leads to a 0.6%, 1.2%, and 2.4% increase in revenues at low, medium, and high demand, respectively. This result is in line with what we observed in the single market scenario. As in the single market case, most of the EMSR-based heuristics lead to a revenue decrease. However, we can see that "Shared Nesting Full EMSR" leads to a revenue increase for Airline 1 ranging from 0.1% to 0.4% over the base case depending on the demand level.

Figures 4.27, 4.28, and 4.29 show total load factors for all four airlines when Airline 1 is using different optimization methods. We can see that total load factor changes are smaller in magnitude compared to the single market case. It is important to note that total load factor increases from the base case when using all multiple cabin heuristics apart from "Shared Nesting Full EMSR."

Figures 4.30, 4.31, and 4.32 show Airline 1's cabin load factors when it is using the specified revenue management method. We observe a similar pattern to the one observed in the single market scenario, with an increase in premium cabin load factor

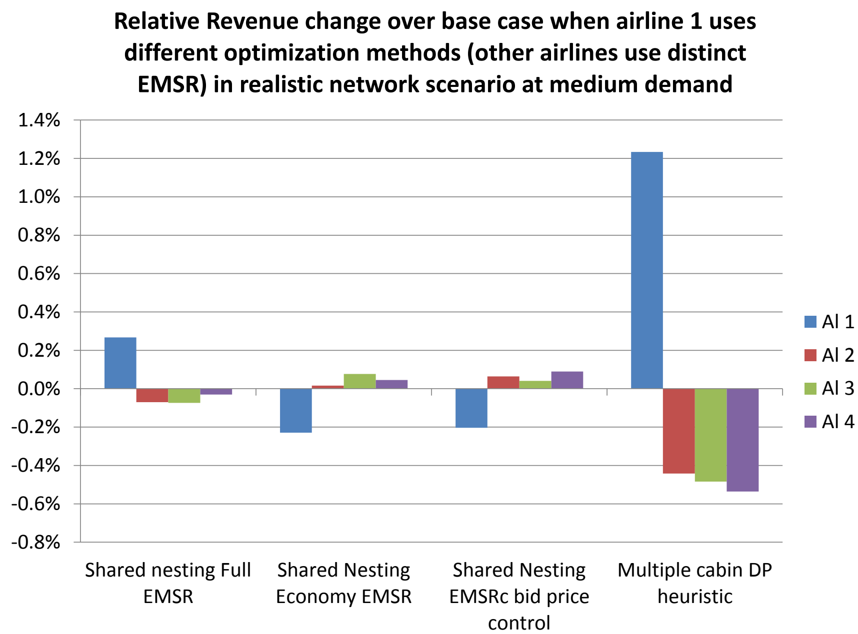

Figure 4.25. Relative revenue change over base case in realistic network scenario at medium demand

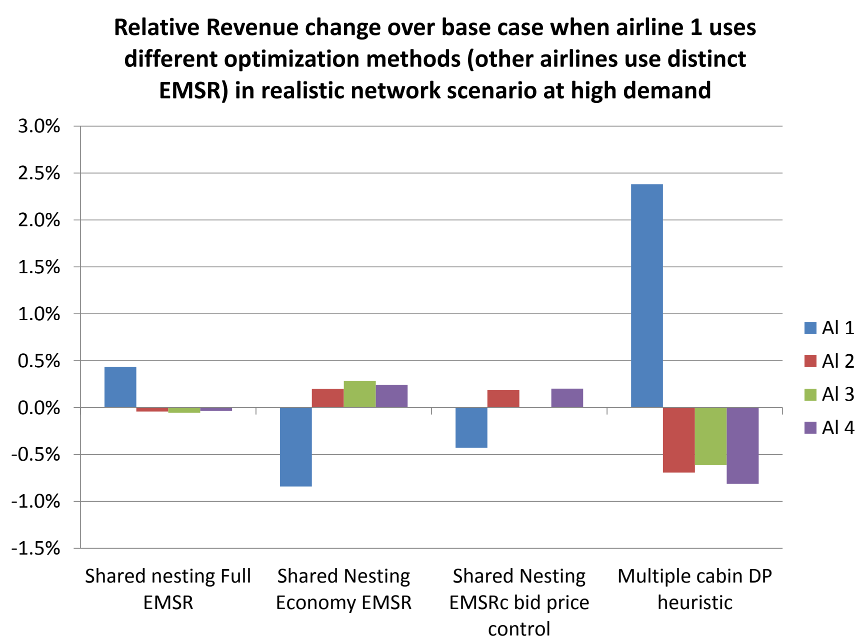

Figure 4.26. Relative revenue change over base case in realistic network scenario at high demand

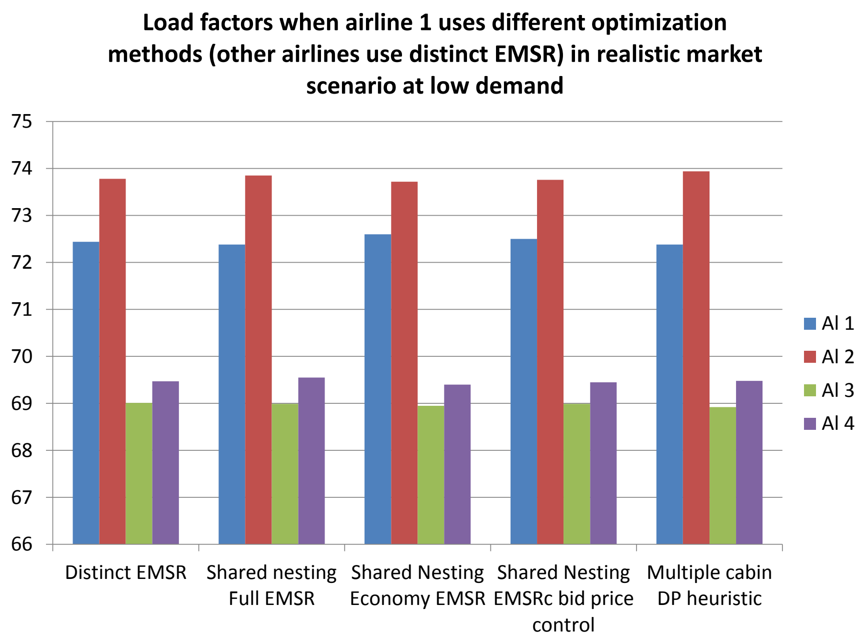

Figure 4.27. Load factor in realistic network scenario at low demand

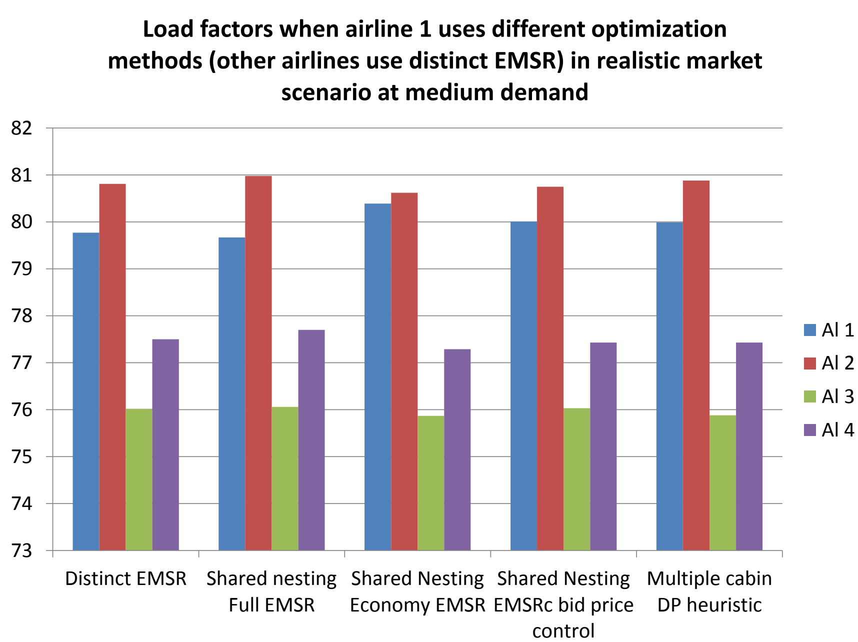

Figure 4.28. Load factor in realistic network scenario at medium demand

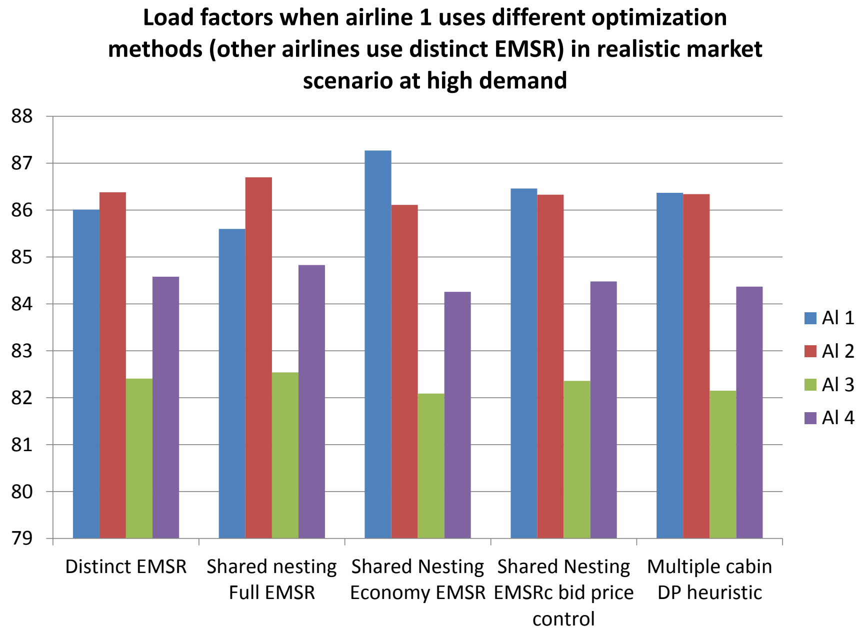

Figure 4.29. Load factor in realistic network scenario at high demand

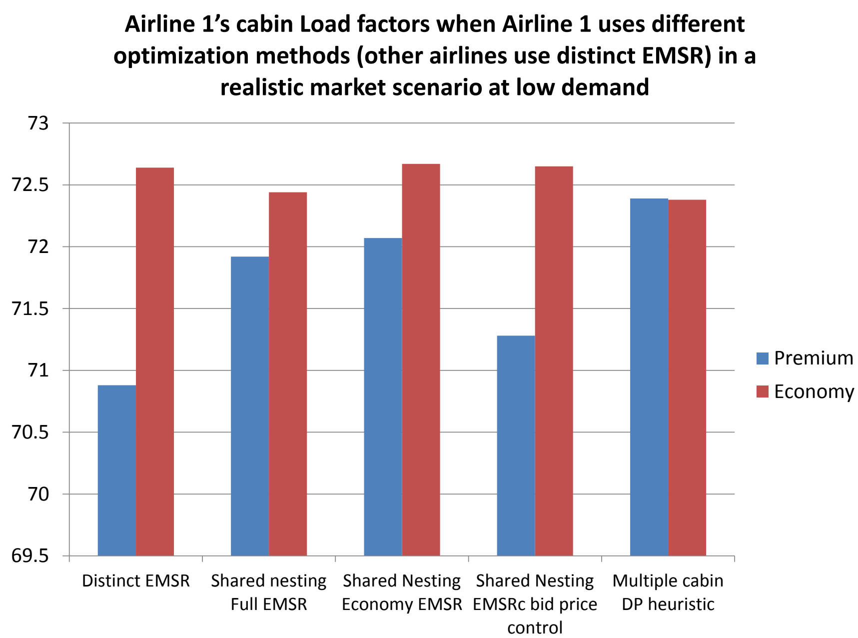

Figure 4.30. Cabin load factor in realistic network scenario at low demand

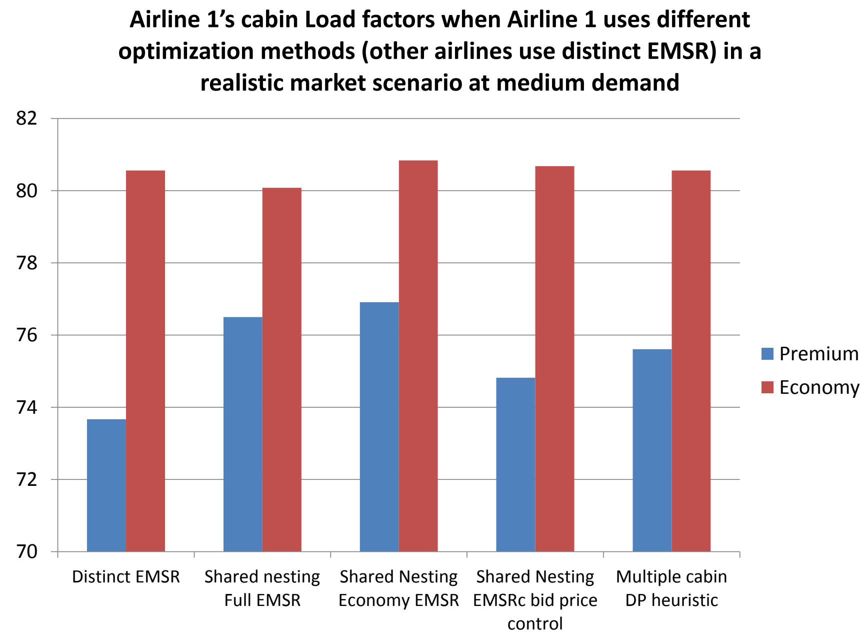

Figure 4.31. Cabin load factor in realistic network scenario at medium demand

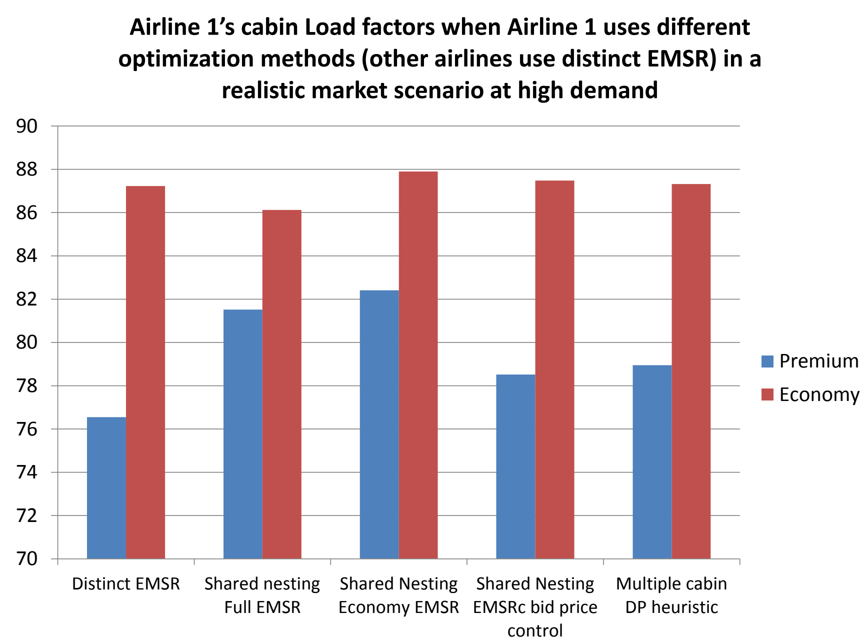

Figure 4.32. Cabin load factor in realistic network scenario at high demand

and slight variations in economy cabin load factor when using multiple cabin methods.

Figure 4.33. Fare class mix in realistic network scenario at low demand

Figures 4.33, 4.34, and 4.35 show the average number of bookings across samples in the different fare classes for multiple optimization methods. We first see that the vast majority of bookings are observed in Class 10 for all methods. We can also see that "Shared Nesting Full EMSR" is the only method that has a smaller number of Class 10 bookings when compared to the base case. It is also important to note that only "Shared Nesting Full EMSR" and "Multiple cabin DP heuristic" are leading to an increase in Class 5 (i.e. top economy) bookings.

Figures 4.36, 4.37, and 4.38 show the average percentage of Airline 1 flights for which the specified class is closed over the simulation samples, known as closure rates, for the highest and lowest class in both cabin. All the EMSR-based heuristics have very similar closure rate behavior, but the "Multiple cabin DP heuristic" has the lowest closure rate for Class 10 midway in the booking process and the lowest Class 5 closure rate late in the booking process. This explains why this method is able to keep a high number of Class 10 bookings while accepting more Class 5 bookings late in the booking process.

The results observed in the realistic network are in line with what was observed

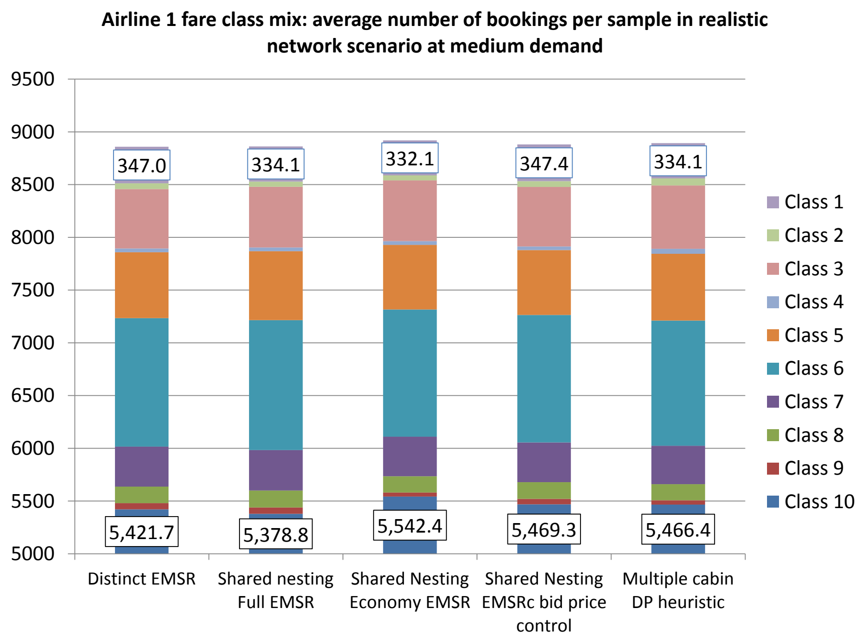

Figure 4.34. Fare class mix in realistic network scenario at medium demand

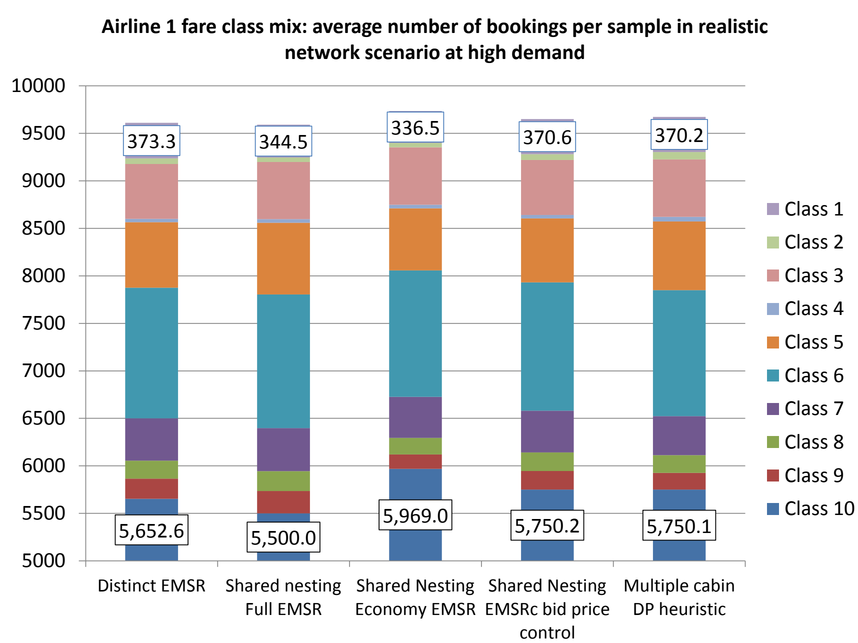

Figure 4.35. Fare class mix in realistic network scenario at high demand

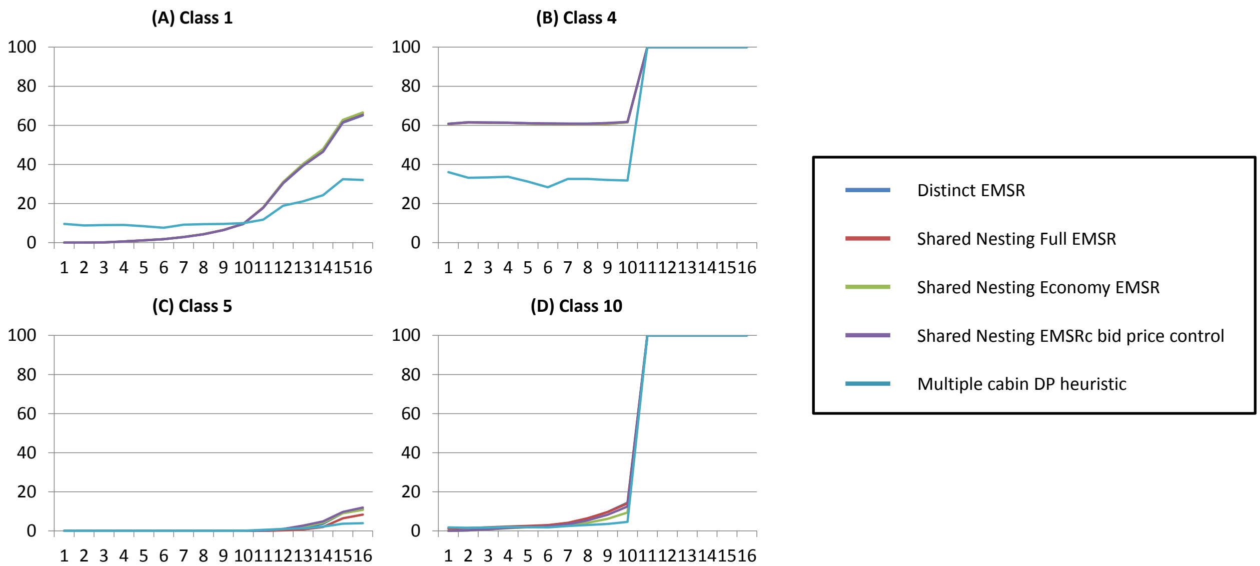

Figure 4.36. Closure rates in realistic network scenario at low demand

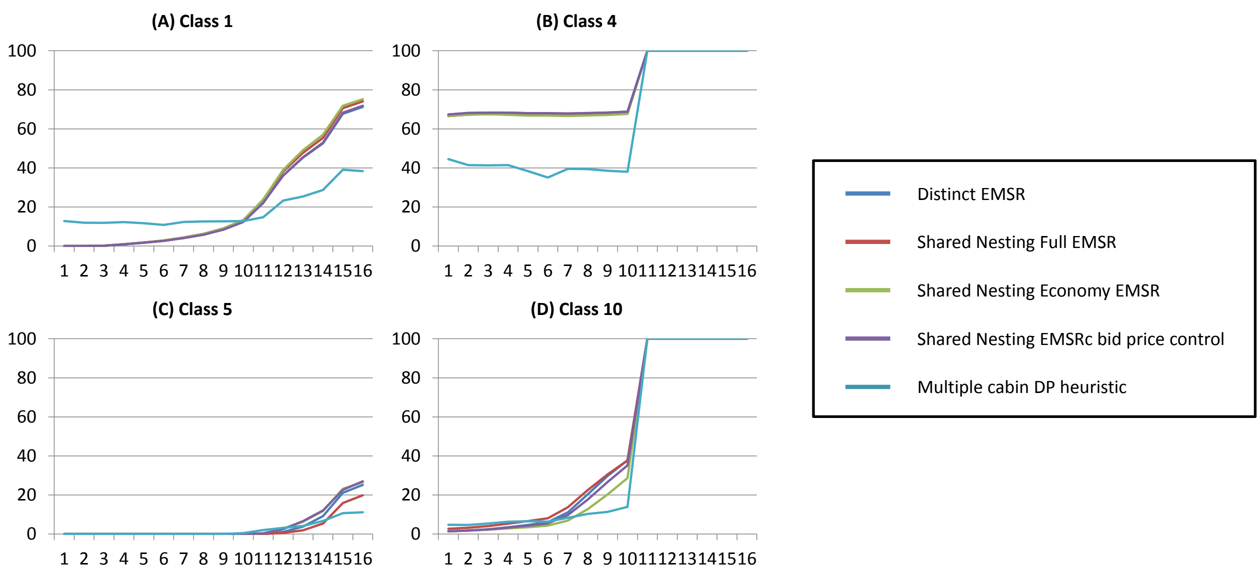

Figure 4.37. Closure rates in realistic network scenario at medium demand

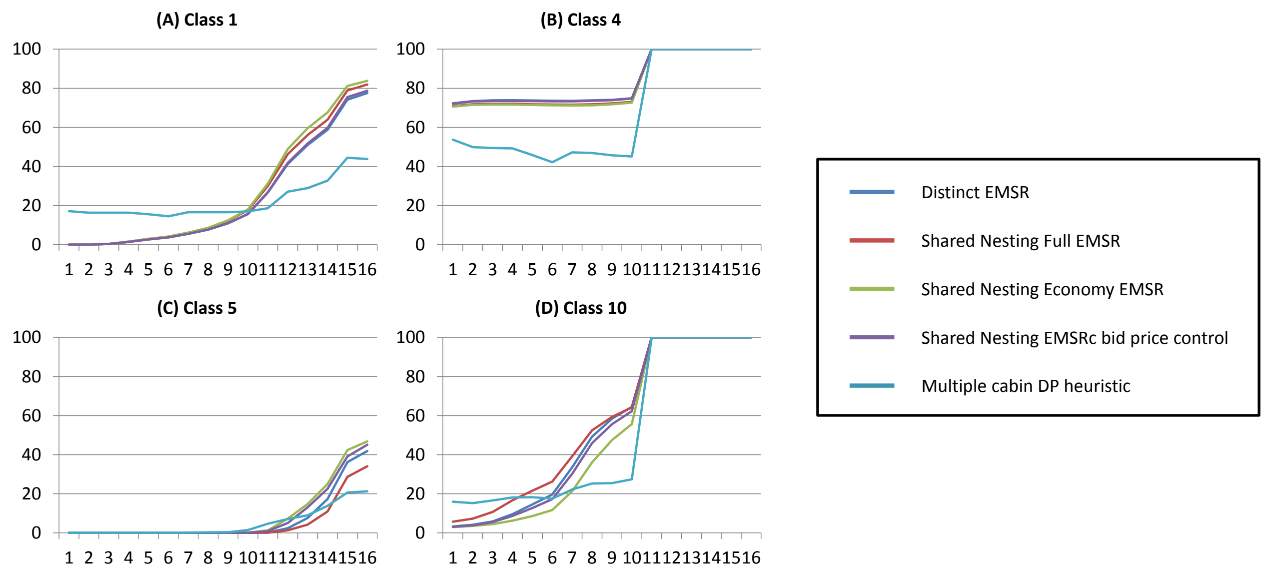

Figure 4.38. Closure rates in realistic network scenario at high demand

in the single market scenario. Indeed, the multiple cabin optimization methods that leads to a revenue increase forces additional economy bookings in top economy classes. Interestingly, "Shared Nesting Full EMSR" leads to a total load factor decrease in this scenario. This is a counterintuitive result since one would expect that allowing for upgrades would systematically lead to a total load factor increase. Although the premium cabin load factor increases, the decrease in total load factor for "Shared Nesting Full EMSR" is driven by a decrease in economy cabin load factor due to an increase in the protection levels against low economy classes. Higher protection levels against low economy classes means that passengers are forced to book in high economy classes which, in turn, increases the forecasted demand for these high economy classes, which further increases the protection levels against low economy classes. In fact, this effect is the exact opposite of the "spiral down" effect in revenue management. The fact that we observe a total load factor decrease for this method is an indication that this "spiral up" effect is stronger than the effect of adding available seats in the economy cabin.

We also note that since the fare of the top economy classes are higher than the fare for low premium classes, some of the displaced premium passengers are replaced by more valuable economy passengers. Therefore, there is a greater potential for revenue increase for multiple cabin optimization methods. This realistic network scenario also highlights the competitive interactions. Indeed, we can see that airlines 2, 3, and 4 experience a revenue decrease when Airline 1 experiences a revenue increase and vice versa. This indicates that competitors are affected by Airline 1's decision and they accept premium passengers rejected by Airline 1, while Airline 1 takes economy passengers from its competitors. Finally, as in the single market scenario, multiple cabin optimization performance was not significantly affected by the change in demand levels.

4.4 Chapter summary

In this chapter, we compared the relative performance of the different multiple cabin optimization methods explained in Chapter 3. We presented the Passenger Origin-Destination Simulator used to test the algorithms. Finally, we discussed the performance of the different optimization methods in two different simulation scenarios.

We saw that multiple cabin optimization can lead to a revenue increase. We observed that the "Multiple cabin DP heuristic" leads to an increase in revenues ranging from 0.5% to 2.5% depending on the network and the demand level. "Shared nesting Full EMSR" leads to a revenue increase ranging from 0.1% at low demand to 0.4% at high demand in the realistic network scenario. Other methods led to a revenue decrease ranging from 9.6% for "Shared nesting Economy EMSR" at high demand in the single market scenario to 0.05% for "Shared nesting EMSRc bid price control" at low demand in the realistic network scenario. The performance of the different optimization methods is explained by the trade-off between revenues gained from additional economy cabin bookings and revenues lost from rejected premium cabin bookings, and successful multiple cabin methods were all increasing the gain from additional economy cabin bookings by forcing passengers to sell up to top economy classes.

We also discussed the mismatch between the underlying assumptions of the dynamic programming approaches and the reality of the booking process. Although dynamic programming is theoretically appealing for the revenue management problem, our results show that it is not performing as well in a competitive environment as some of the heuristics we developed. Diwan [11] suggested that dynamic programming underperforms because of the mismatch between the Poisson variance assumption and the higher observed variance in the airline industry. This assertion is supported by the results obtained with "Multiple cabin DP". Although taking variance into account improves dynamic programming's performance as exposed by the "Multiple cabin DP with Variance" performance, we observe that the "Multiple cabin DP heuristic" still performs better than "Multiple cabin DP with Variance". This is due to the fact that dynamic programming methods assume that airlines reoptimize and can change the decision rule after every booking request whereas airlines do not have the ability to do so in practice. This implies that airlines applying dynamic programming have to average their bidprices, which reduces the ability of the dynamic programming algorithm to respond to changes in the booking behavior. Also, our results suggests that dynamic programming methods, “Multiple cabin DP” and “Multiple cabin DP with Variance”, are less robust to the fact that other underlying assumptions do not match the competitive reality of the booking and passenger choice process when compared to the proposed heuristics.

Chapter 5

Conclusion

IN this thesis, we defined the multiple cabin single-leg revenue management problem and compared the performance of different optimization methods and heuristics in a competitive environment. In this chapter, we review our contribution, summarize our findings, and discuss future research opportunities.

5.1 Contribution and findings

The contribution of this work is two fold. First, we developed multiple optimization methods to solve the multiple cabin problem. We extended existing separate cabin single-leg dynamic programming formulations to the multiple cabin problem. Furthermore, we developed several heuristics based on existing dynamic programming formulations or on approaches widely used in the industry. Second, we compared and discussed the performance of the different methods developed using simulation results from the Passenger Origin-Destination Simulator (PODS).

The objective of multiple cabin revenue management is to better use the airline's capacity by allowing economy bookings to use available premium capacity. One would think that this can only lead to a revenue increase since airlines allow additional economy bookings to use available premium capacity. Interestingly, we found that most methods tested did not lead to a revenue increase when simulated in a competitive environment with passenger choice. As expected, we observed an increase in premium cabin load factor with all the methods tested, and an increase in total load factor when using most of the multiple cabin optimization methods, "Shared nesting Full EMSR" in the realistic network scenario being the only exception. However, these additional economy passengers were using the premium capacity, and because of the stochastic nature of the observed demand, some premium booking requests were rejected. Therefore, instead of a systematic revenue increase driven by additional bookings in economy classes, we observed a trade-off between revenue gains from additional economy bookings and revenue losses from displaced premium passengers.

Only two multiple cabin optimization methods led to a revenue increase over the base case: “Shared nesting Full EMSR” led to an increase ranging from 0.1% to 0.4% in the realistic network scenario depending on the demand level, and the “Multiple cabin DP heuristic” led to increases ranging from 0.5% to 2.4% depending on the demand level and on the scenario. These methods were successful because they forced additional economy bookings in top economy classes. Since top economy classes are more valuable than low economy classes, the revenue benefit from these additional economy bookings is greater. This effect is in fact more important than the impact of increased load factor from better capacity usage. This is illustrated by the fact that “Shared nesting Full EMSR” led to a revenue increase despite a slight total load factor decrease in the realistic network scenario.

Although theoretically appealing, dynamic programming methods have not performed well compared to some of the heuristics we developed. This is due to existing differences between the underlying assumptions of the dynamic programming formulations and the characteristics of the booking process faced in practice by the airlines. More precisely, the combination of at least two effects is leading to the observed result. First, as shown by Diwan [11], the assumed variance by the Poisson process behind dynamic programming makes the method overconfident about late high-WTP booking requests, and the fact that airlines cannot reoptimize after each booking request reduces the ability of dynamic programming to correct the bidprice when it realizes that late high-WTP booking requests do not materialize or when it identifies changes in the booking behavior. Moreover, our results also suggest that the dynamic programming methods tested are less robust to underlying assumption mismatch when compared to the proposed heuristics.

5.2 Directions for further research

We focused on the multiple-cabin revenue management problem. While providing solutions to this problem, we made some simplifying assumptions. Most importantly, we focused on the single-leg revenue management problem and ignored the fact that airlines are offering multiple interdependent flight-legs in a complex network. Future work should address the network multiple-cabin revenue management problem. In the single-leg multiple-cabin revenue management problem we are making the simplifying assumption that all the demand on a given flight-leg is flying from the flight-leg's origin to the flight-leg's destination. Therefore, future work should take into account the trade-off between accepting a booking request from a passenger using multiple flight-legs and a booking request from a passenger using a single flight-leg. There exist network formulations for the separate cabin revenue management problem. Extending the existing virtual nesting approach [14] to the multiple cabin problem using one of the EMSR-based heuristics proposed in this thesis seems to be a reasonable practical starting point.

Computational time is a major issue with the dynamic programming algorithms we proposed in this work. Therefore, developing a robust heuristic that reduces computation time while providing a solution of quality can still be explored further. In this thesis, we proposed many EMSR-based heuristics, and a dynamic programming decomposition, but future work could incorporate dynamic programming roll-outs as Bertsekas [6] described in his book. Indeed, using the EMSRb heuristic, a relatively fast heuristic, to estimate expected future revenue at a given time, one can use dynamic programming over a reduced number of time slices to improve on the EMSR estimate and, in theory, find a better solution.

Bibliography

[1] Peter P. Belobaba; Amedeo Odoni; Cynthia Barnhart. *The Global Airline Industry*. Wiley, 2009.

[2] Peter P. Belobaba. *Air Travel Demand and Airline Seat Inventory Management*. PhD thesis, Massachusetts Institute of Technology, 1987.

[3] Peter P. Belobaba. Optimal vs. heuristic mehtods for nested seat allocation. In *AGIFORS Reservations Control Study Group Meeting*, 1992.

[4] Peter P. Belobaba. Passenger origin-destination simulator (pods) summary of processes and models. Working document, September 2010.

[5] Peter P. Belobaba. Route planning and schedule development. Class presentation, October 2012.

[6] Dimitri P. Bertsekas. *Dynamic Programming and Optimal Control*, volume 1. Athena Scientific, 2012.

[7] Theodore Charles Botimer. *Airline Pricing and Fare Product Differentiation*. PhD thesis, Massachisetts Institute of Technology, 1994.

[8] Emmanuel Carrier. Modeling airline passenger choice: passenger preference for schedule in the passenger origin-destination simulator (pods). Master's thesis, Massachusetts Institute of Technology, 2003.

[9] Boeing Commercial Airplane Company. Load factor analysis, market share modeling and profit profiles. Seattle, WA, 1978.

[10] Boeing Commercial Airplane Company. Decision window path preference methodology time mode. Seattle, WA, November 1993.

[11] Sarvee Diwan. Performance of Dynamic Programming Methods in Airline Revenue Management. Master's thesis, Massachusetts Institute of Technology, June 2010.

[12] Guillermo Gallego and Catalina Stefanescu. Upgrades, upsells and pricing in revenue management. Working paper.

[13] Robert A. Shumsky and Fuqiang Zhang. Dynamic capacity management with substitution. *Operations Research*, 57, No. 3:671–684, May-June 2009.

[14] Penn C.W. Smith B.C. Analysis of alternative origin-destination control strategies. In *AGIFORS*, 1988.

[15] Claudius Steinhardt and Jochen Goensch. Integrated revenue management approaches for capacity control with planned upgrades. *European Journal of Operational Research*, 223:380–391, 2012. URL http://ssrn.com/abstract=1515865.

[16] Conrad J. Lautenbacher Shaler Stidham. The underlying markov decision process in the single-leg airline yield management problem. *Transportation Science*, 33: 136–146, 1999.

[17] Darius Walczak. Modeling high demand variance in dynamic programming. *Journal of Revenue and Pricing Management*, 5:94–101, 2006.

[18] Darius Walczak. Capacity sharing. In *AGIFORS*, 2010.