Modal Pushover Analysis for High-rise Buildings

by

Ming Zheng

B.S. Civil Engineering
Southwest Jiaotong University, 2011

SUBMITTED TO THE DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ENGINEERING
IN CIVIL AND ENVIRONMENTAL ENGINEERING

AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

JUNE 2013

Signature of Author: ______________________________
Department of Civil and Environmental Engineering
May 10, 2013

Certified by: ______________________________
Jerome J. Connor
Professor of Civil and Environmental Engineering
Thesis Supervisor

Accepted by: ______________________________
Heidi M. Nepf
Chair, Departmental Committee for Graduate Students

Modal Pushover Analysis for High-rise Buildings

by

Ming Zheng

Submitted to the Department of Civil and Environmental Engineering
on May 10, 2013 in Partial Fulfillment of the
Requirements for the Degree of Master of Engineering in
Civil Engineering

ABSTRACT

Pushover analysis is a nonlinear static analysis tool widely used in practice to predict and evaluate seismic performance of structures. Since only the fundamental mode is considered and the inelastic theorem is imperfect for the conventional pushover analysis, a modified Modal Pushover Analysis (MPA) is proposed by researchers. In this thesis, the theories of dynamics for single-degree-of-freedom (SDOF) and multiple-degree-of-freedom (MDOF) are introduced, including elastic analysis and inelastic analysis. The procedures and equations for time history analysis, modal analysis, pushover analysis and modal pushover analysis are discussed in detail. Then an 8-story height model and a 16-story height model are established for analysis. The pushover analysis is conducted for each equivalent SDOF system, and by combination of the distribution of 1 mode, 2 modes and 3 modes, the responses of modal pushover analysis are obtained. The results of pushover analysis and modal pushover analysis are compared with those of time history analysis. The results of the analysis show that the conventional pushover analysis is mostly limited to low- and medium-rise structures in which only the first mode is considered and where the mode shape is constant. The modal pushover analysis is shown to have a superior accuracy in evaluation of seismic demands for higher buildings, especially for story drift ratios and column shears. With this in mind, some design recommendations and areas of future work are proposed in the conclusion.

Thesis Supervisor: Jerome J. Connor
Title: Professor of Civil and Environmental Engineering

Table of Contents

List of Figures

List of Tables

1. INTRODUCTION

1.1. Performance Based Design

The concept of performance based design was first put forward in 1976. In the following two decades, the American and Japanese scholars led the research on this area. Today, performance based design is widely learned and discussed around the world. It is the modern approach to earthquake resistant design. Rather than being based on prescriptive mostly empirical code formulations, performance based design is an attempt to predict buildings with predictable seismic performance (Naeim, Bhatia, & Lobo).

Performance based design is the subset of activities of performance based engineering that focus on the design process. Therefore, it includes identification of seismic hazards, selection of the performance levels and performance design objectives, determination of site suitability, conceptual design, numerical preliminary design, final design, acceptability checks during design, design review, specification of quality assurance during the construction and of monitoring of the maintenance and occupancy (function) during the life of the building (Bertero & Bertero, 2002). As opposed to the traditional strength based design, the performance based design is a new engineering technology and concept focusing on the object of a building asset, in order to prescribe results instead of the procedures to design structures. Based on the importance and usage of buildings, different levels of seismic design are proposed according to their target performance, such that buildings can reach their anticipated functions and largely decrease the damage when subjected to earthquakes (http://en.wikipedia.org/wiki/Performance-based_building_design).

What makes performance based design better is that different performance targets will be determined based on various seismic levels and structural systems, resulting in different construction materials, sequences and structural design methods. By governing these parameters and process, the least economic cost in earthquakes can be reached. In addition, contractors, owners, and designers may put forward their own design requirements. Performance based design is a more flexible design concept to meet the requirements according to individuals and society.

1.2. Time History Analysis

Time history analysis is a rigorous numerical method by integrating differential equation of motion directly. The dynamic responses of displacement, velocity, and acceleration can be determined by the time history analysis, thus the structural internal forces are obtained in each time step. For a rarely met earthquake and deformation demands for a structure, the nonlinear time history is necessary to analyze the structural performance and weak part for seismic design. The current numerical procedure includes Newmark Method, Wilson Method, Collocation Method, Hiber-Hugher-Taylor Method and Chung and Hulbert Method. Based on structural systems, force distributions, computer performance and required accuracy, three types of nonlinear model are selected for the time history analysis: the floor model, the bar model and the finite element model.

However, the nonlinear time history analysis is a complicated process requiring a high performance computing facility and a long computation time. There are also issues when it comes to the input of the earthquake waves and the selection of restoring forces. The current research could only be conducted for 2-D models of critical structures, and the 3-D model is far from mature, so this method is not widely applied. Based on the situation, the scholars in earthquake engineering switched their focus to an easily applicable and approximate manner for seismic performance estimation in practice, the equivalent nonlinear static analysis, or pushover analysis.

1.3. Pushover Analysis and Modal Pushover Analysis

Static pushover analysis is becoming a popular performance based design tool for seismic performance evaluation of existing and new structures. It is expected that pushover analysis will provide adequate information on seismic demands induced by the design ground motion on the structural systems and its components. The purpose of pushover analysis is to estimate the expected performance of a structural system by evaluating its strength and deformation demands under seismic loads by means of a static nonlinear analysis, and comparing these demands to available capacities at the targeted performance levels.

The evaluation is based on an assessment of important performance parameters, including floor displacements, inter-story drift ratios, column shears, inelastic element deformations (either absolute or normalized with respect to a yield value), deformations between elements, and element and connection forces. The inelastic static pushover analysis is regarded as an effective method for predicting seismic forces and deformation demands, which approximately accounts for the redistribution of internal forces that occurs when the structure is subjected to inertia forces that can no longer be resisted within the elastic range of structural behavior (Krawinkler & Seneviratna, 1998).

For the elastic analysis, a multiple-degree-of-freedom (MDOF) system can be decomposed to several singe-degree-of-freedom (SDOF) systems, each of which corresponds to one mode of the MDOF system. The earthquake force distribution is expanded as a summation of modal inertia force distributions, and each modal force component excites the responses of its corresponding mode. The total response can be superimposed by the contribution of each mode (Chopra A. K., Dynamics of Sturctures: Theory and Application to Earthquake Engineering, 2001). Typically the total response is dominated by the fundamental mode. For simplicity, the conventional pushover analysis focuses on the first mode and assumes that the mode shape does not change after the structure yields. It is a powerful tool for its nonlinear analysis, but it has little rigorous theoretical background (Krawinkler & Seneviratna, 1998). In reality, each mode contributes to the total structural response, and the mode shapes will not be constant throughout the inelastic stage. In addition, with the yielding of the structure and the increase of the structural height, the contributions of the higher modes cannot be ignored.

Based on the dynamic theories, A.K Chopra with his research group came up with a new Modal Pushover Analysis (MPA) considering the effect of higher modes on the structural performance. It is an improved pushover analysis by the combination of the responses of each mode with a constant lateral load pattern. The total response is determined from the response of each mode by a certain rule (e.g., SRSS, CQC) (Chopra & Goel, A Modal pushover analysis procedure to estimate seismic demands for buildings: theory and preliminary evaluation, 2001). Since the higher modes are taken into consideration, the modal pushover analysis has a superior accuracy and fits the actual solution better. The response spectrum analysis (RSA) is also introduced in this thesis which is shown to be equivalent to the modal pushover analysis for elastic systems (Chopra A. K., Earthquake Dynamics of Structures, 2005). The advantage of modal pushover analysis lies in its accuracy and simplicity for nonlinear analysis. Nevertheless, the lateral load patterns for MPA are assumed to be constant after yielding, an approximation similar to the pushover analysis, which induces issues that must be solved in the future (Mao, Xie, & Zhai, 2006).

2. SINGLE DEGREE OF FREEDOM SYSTEMS

The equations, figures and some comments in part 2 and part 3 are cited from the following materials: (Chopra A. K., Dynamics of Sturctures: Theory and Application to Earthquake Engineering, 2001), (Chopra & Goel, A Modal pushover analysis procedure to estimate seismic demands for buildings: theory and preliminary evaluation, 2001), (Chopra A. K., Earthquake Dynamics of Structures, 2005)

We start the fundamental theory by introducing simple structures, the single degree of free systems. Based on damping ratios, two different categories of structures are divided, the elastic systems and the inelastic systems.

2.1. Elastic Systems

2.1.1. Equation of Motion

The following idealized one-story structure is shown in Figure 2.1, consisting of a lumped mass at the top, a massless frame providing lateral stiffness k to the system, and a linear viscous damper with its damping coefficient c.

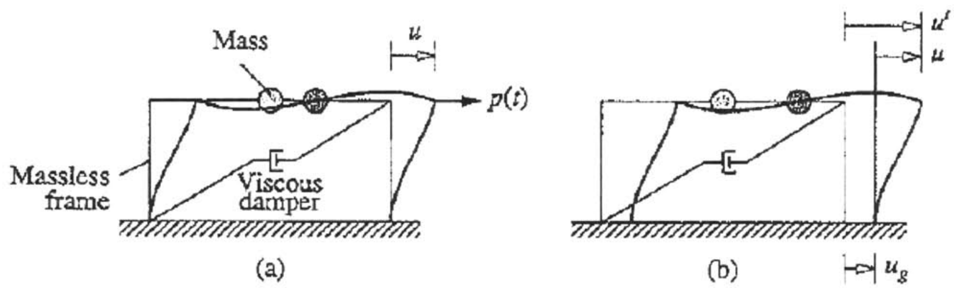


Figure2.1 Single-degree-of-freedom system: (a) applied force p(t); (b) earthquake induced ground motion.

In earthquake areas, the primary issue on dynamics that structural engineers concern about is the structural response under seismic loads. Here the ground displacement is denoted with u_g, the total mass displacement is denoted with u^t, the relative displacement between mass and ground is denoted with u. At each moment the following equation holds:

$$u^t(t) = u(t) + u_g(t) \quad 2.1$$

Figure 2.2 shows the response of an idealized one story system under earthquake excitation, where f_I is the inertia force, f_D the damping resisting force and f_S the elastic resisting force, yielding to the dynamic equation of equilibrium:

$$f_I + f_D + f_S = 0 \quad 2.2$$

Where

$$f_I = m\ddot{u}^t, f_D = c\dot{u}, f_S = ku \qquad 2.3$$

Substituting 2.1.2, 2.1.3 into 2.1.1 yields:

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t) \qquad 2.4$$

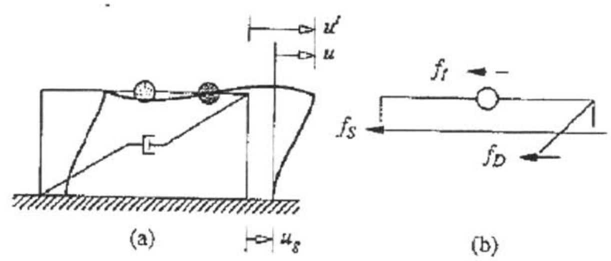


Figure2.2 Forces on a SDOF system

Dividing equation 2.1.3 by m gives the equation of motion in terms of two-system parameters:

$$\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = -\ddot{u}_g(t) \qquad 2.5$$

Where

$$\omega_n = \sqrt{\frac{k}{m}}, \xi = \frac{c}{2m\omega_n} \qquad 2.6$$

2.1.2. Response History

Ground motion varies irregularly so that structures respond irregularly during the earthquake. For a given ground motion $\ddot{u}_g(t)$, the displacement u(t) of a single-degree-of-freedom system relies on the natural period and damping ratio. Figure 2.3 shows the deformation response of three different systems to earthquake excitation at EI Centro.

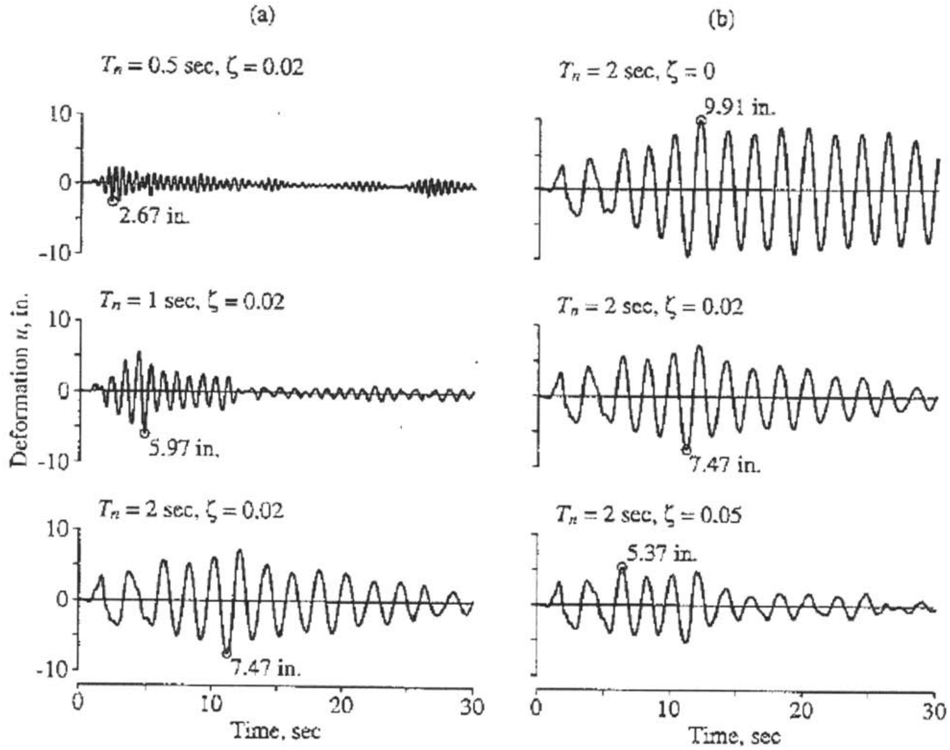


Figure2.3 Deformation response of SDF system to EI Centro ground motion

The deformation response history u(t) could be calculated by numerical methods illustrated in the following part, thus the internal forces of the structure can be figured out by static analysis, which forms a theoretical foundation for pushover analysis and modal pushover analysis. In earthquake engineering, the concept of equivalent static force f_S (Figure 2.4) is proposed.

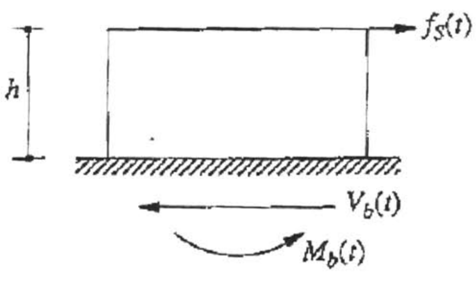


Figure2.4 Equivalent static force

f_s is defined by:

$$f_S(t) = ku(t) = m\omega_n^2 u(t) = mA(t) \tag{2.7}$$

where

$$A(t) = \omega_n^2 u(t) \tag{2.8}$$

Noted that A(t) is the pseudo acceleration, not the absolute acceleration $\ddot{u}^t$(t). A(t) can be determined by displacement u(t) easily. Then the base shear V_b(t) and base overturning moment M_b(t) are obtained:

$$V_b(t) = f_S(t) = mA(t) \qquad\qquad M_b(t) = hf_S(t) = hV_b(t) \tag{2.9}$$

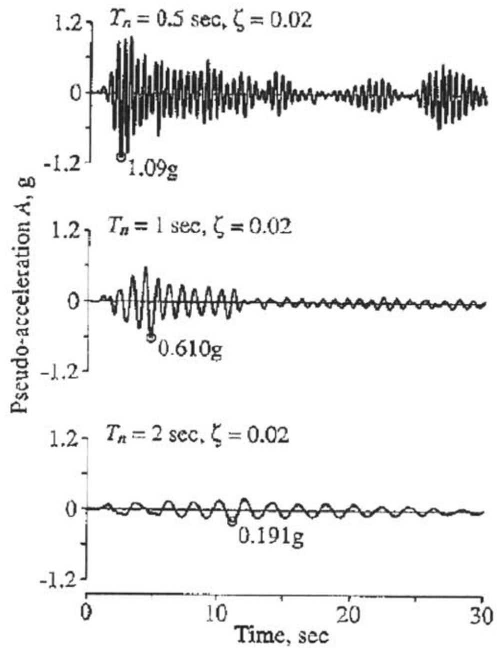


Figure2.5 Pseudo-acceleration response of SDF system to EI Centro ground motion

2.2. Inelastic Systems

2.2.1. Parameters of Inelastic Systems

The normalized yield strength $\overline{f_y}$ for elastoplastic systems is defined as

$$\overline{f_y} = \frac{f_y}{f_0} = \frac{u_y}{u_0} \tag{2.10}$$

where f_0 and u_0 are earthquake induced peak values of resisting force and deformation for corresponding elastic systems.

The yield strength reduction factor R_y is defined as

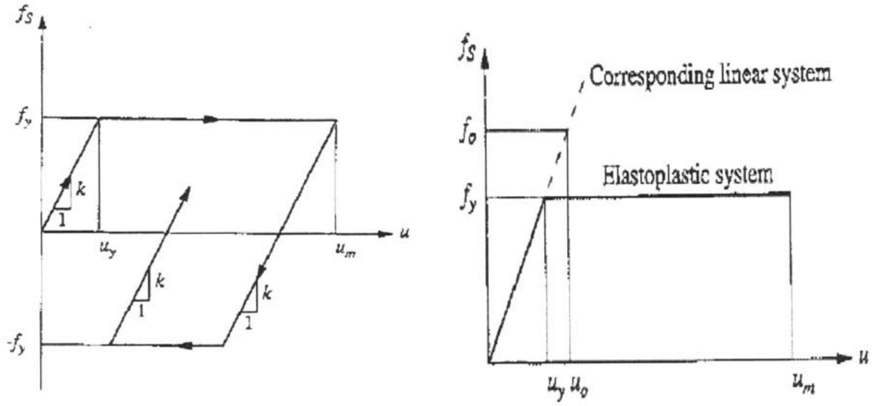
$$R_y = \frac{f_0}{f_y} = \frac{u_0}{u_y} \tag{2.11}$$

The ground motion induced peak value of deformation for an elastoplastic system is denoted by u_m, then and yield deformation is normalized as a dimensionless ratio called ductility factor

$$\mu = \frac{u_m}{u_y} \tag{2.12}$$

Combining 2.10~2.12 yields to

$$\frac{u_m}{u_0} = \mu \bar{f}_y = \frac{\mu}{R_y} \tag{2.13}$$



(a) (b)

Figure2.6 (a) Elastoplastic force-deformation relation; (b)Elastoplastic system and its corresponding linear system

2.2.2. Equation of Motion

The governing equation 2.4 for inelastic system is repeated here with

$$m\ddot{u} + c\dot{u} + f_S(u,\dot{u}) = -m\ddot{u}_g(t) \tag{2.14}$$

where $f_S(u,\dot{u})$ is the resisting force for inelastic systems

For given $\ddot{u}_g(t)$, u(t) depends on three parameters ω_n, ξ and u_y of the system and the force-deformation relation. Diving equation 2.14 by m yields to

$$\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u_y \tilde{f}_S(u,\dot{u}) = -\ddot{u}_g(t) \tag{2.15}$$

where

$$\omega_n = \sqrt{\frac{k}{m}}, \xi = \frac{c}{2m\omega_n}, \tilde{f}_S(u,\dot{u}) = \frac{f_S(u,\dot{u})}{f_y} \tag{2.16}$$

The above equation can be solved by the numerical methods illustrated in part 2.3, special attention should be given in determining of time instants for numerical procedures to ensure enough accuracy. The models analyzed in this thesis apply a time step $\Delta t = 0.01\text{s}$.

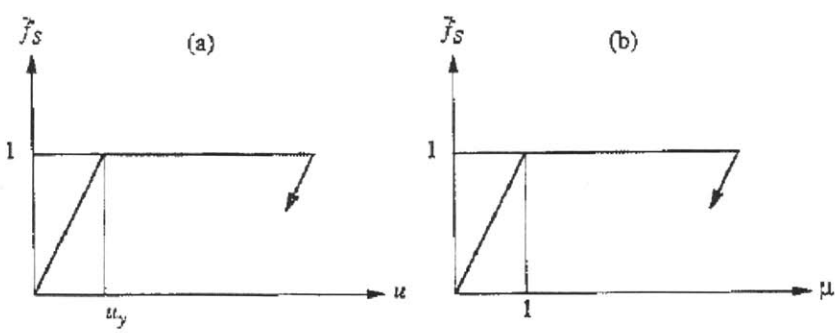


Figure2.7 Force-deformation relations in normalized form

2.3. Introduction to Numerical Methods for Dynamic Response

Under the arbitrary loaded force p(t) or ground acceleration $\ddot{u}_g(t)$, or for a nonlinear system, the analytical solution for a SDOF system is typically not possible to obtain. Numerical time-stepping methods for integration of differential equations are employed to solve these problems. In the subject of mathematics, there exist a lot of different numerical methods among which the most discussed were their accuracy, convergence, stability properties and computer implementation. In this thesis, only a few of them which are widely used in dynamic analysis for SDOF are introduced, since it is adequate for application and research.

2.3.1. Time-Stepping Methods

For an inelastic system, the equation of motion for numerical methods is

$$m\ddot{u} + c\dot{u} + f_S(u,\dot{u}) = p(t) \quad \text{or} \quad -m\ddot{u}_g(t) \tag{2.17}$$

Subject to the initial conditions

$$u_0 = u(0) \qquad \dot{u}_0 = \dot{u}(0)$$

The linear viscous damping is assumed for the system, the external force p(t) will be given by a set of discrete values: $p_i=p(t_i)$, i=0 to N (Figure 2.8). The time interval

$$\Delta t_i = t_{i+1} - t_i \tag{2.18}$$

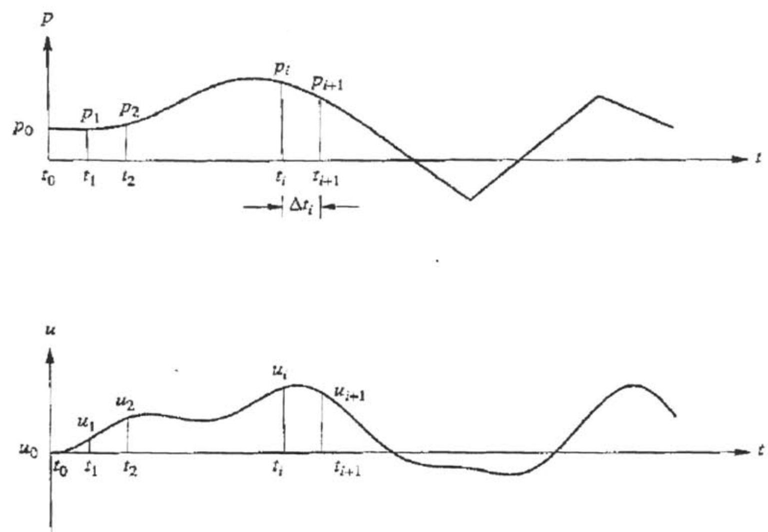


Figure2.8 Notation for time-stepping methods

is usually taken as a constant, although not necessary. In every discrete moment t_i, the structural response is determined for a SDOF, the displacement, velocity and acceleration are denoted as u_i, $\dot{u}_I$ and $\ddot{u}_i$. Assuming that these values are given, the following equation holds at time i:

$$m\ddot{u}_i + c\dot{u}_i + (f_S)_i = p_i \qquad 2.19$$

where $(f_S)_I$ is the resisting force at time i for linear systems, $(f_s)_i = ku_i$, but depends on the prior displacement and velocity history at time i, if the system is nonlinear. The numerical methods to be presented will enable us to determine the response values u_{i+1}, $\dot{u}_{i+1}$, and $\ddot{u}_{i+1}$, i.e., at time i,

$$m\ddot{u}_{i+1} + c\dot{u}_{i+1} + (f_S)_{i+1} = p_{i+1} \qquad 2.20$$

For i = 0, 1, 2, 3..., by applying time-stepping method successively, the response at any moment when i = 0, 1, 2, 3... will be obtained. The given initial conditions provide the necessary information to start the procedure.

The time-stepping for i to i+1 is not an accuracy process, so the three vital requirements for the numerical methods are the convergence, stability and accuracy. For the time-stepping method, we typically have three types: methods based on interpolation of the excitation function, methods based on finite difference expressions of velocity and acceleration and methods based on assumed variation of acceleration. Their detail will not be discussed in this thesis.

3. MULTIPLE DEGREE OF FREEDOM SYSTEMS

3.1. Elastic Systems

3.1.1. Equation of Motion

Under the earthquake induced loading, the governing equation for MDOF system is as follows

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}\boldsymbol{\iota}\ddot{u}_g(t) \tag{3.1}$$

Where **u** is the vector of N lateral floor displacement relative to ground, **m**, **c**, **k** are the mass, classical damping, and lateral stiffness matrices of the system. ι is the influence factor. The right side of the equation can be interpreted as effective earthquake forces:

$$\mathbf{P}_{\mathbf{eff}}(t) = -\mathbf{m}\boldsymbol{\iota}\ddot{u}_g(t) \tag{3.2}$$

The spatial distribution of the effective earthquake forces $\mathbf{P}_{\mathbf{eff}}(t)$ is determined by $\mathbf{s} = \mathbf{m}\boldsymbol{\iota}$, which can be expanded as a summation of modal inertia force distributions $\mathbf{s}_\mathbf{n}$:

$$\mathbf{m}\boldsymbol{\iota} = \sum_{n=1}^{N} \mathbf{s}_\mathbf{n} = \sum_{n=1}^{N} \Gamma_n \mathbf{m}\boldsymbol{\Phi}_\mathbf{n} \tag{3.3}$$

where

$$\Gamma_n = \frac{L_n}{M_n}, L_n = \boldsymbol{\Phi}_\mathbf{n}^\mathrm{T}\mathbf{m}\boldsymbol{\iota}, M_n = \boldsymbol{\Phi}_\mathbf{n}^\mathrm{T}\mathbf{m}\boldsymbol{\Phi}_\mathbf{n} \tag{3.4}$$

3.1.2. Modal Response History Analysis

The preceding equations provide a theoretical basis for the classical modal response history analysis to calculate structural response with respect to time history function. Structural response due to individual excitation terms $P_{eff}(t)$ is determined first for each n, and these N modal responses are combined algebraically at each time instant to obtain the total response.

In equation 3.2, the displacement **u** for an N degree of freedom can be expressed by the superposition of each mode:

$$\mathbf{u}(t) = \sum_{n=1}^{N} \boldsymbol{\Phi}_\mathbf{n} q_n(t) \tag{3.5}$$

where the modal coordinate $q_n(t)$ is governed by

$$\ddot{q}_n + 2\xi_n\omega_n\dot{q}_n + \omega_n^2 q_n = -\Gamma_n \ddot{u}_g(t) \tag{3.6}$$

By comparing equation 3.6 with the equation for nth order SDOF system, it is easy to get the solution for $q_n(t)$, in which ω_n is the natural vibration frequency and ξ_n is the damping ratio for the nth mode. The equation above can be replaced with

$$\ddot{D}_n + 2\xi_n\omega_n\dot{D}_n + \omega_n^2 D_n = -\ddot{u}_g(t) \tag{3.7}$$

where

$$q_n(t) = \Gamma_n D_n(t) \tag{3.8}$$

The solution for $D_n(t)$ can be determined by the time-stepping procedure introduced in part 2.3, then the contribution of nth mode to joint displacement **u**(t) equals to

$$\mathbf{u}_n(t) = \mathbf{\Phi}_\mathbf{n} q_n(t) = \Gamma_n \mathbf{\Phi}_\mathbf{n} D_n(t) \tag{3.9}$$

Based on $\mathbf{u}_n(t)$, the internal forces of structural members can be determined by the equivalent static method. The equivalent static force corresponding to the nth mode is

$$\mathbf{f}_n(t) = \mathbf{k}\mathbf{u}_n(t) = \mathbf{s}_n A_n(t) \tag{3.10}$$

where

$$A_n(t) = \omega_n^2 D_n(t) \tag{3.11}$$

Then any response quantity r(t) --- floor displacement, story drifts, internal element force, etc. --- can be expressed by static analysis under external force $\mathbf{f}_n(t)$. if r^{st}_n denotes the modal static response, namely the static value of r due to external forces $\mathbf{s}_n$, then

$$r_n(t) = r_n^{st} A_n(t) \tag{3.12}$$

Combining the contribution of each mode leads to the total response under ground motion, the nodal displacement is

$$\mathbf{u}(t) = \sum_{n=1}^{N} \mathbf{u}_n(t) = \sum_{n=1}^{N} \Gamma_n \mathbf{\Phi}_\mathbf{n} D_n(t) \tag{3.13}$$

$$r(t) = \sum_{n=1}^{N} r_n(t) = \sum_{n=1}^{N} r_n^{st} A_n(t) \tag{3.14}$$

This is the classical modal response history analysis procedure: equation 3.6 is the standard modal equation governing $q_n(t)$, equations 3.9, 3.12 define the contribution of the nth-mode to the response, equations 3.13 and 3.14 combine the response of all modes to obtain the total response. This is the "Modal" method for time history analysis in SAP2000 which will be applied as a benchmark. Modal expansion of the spatial distribution of the effective earthquake forces will also be introduced providing a conceptual basis for modal pushover analysis.

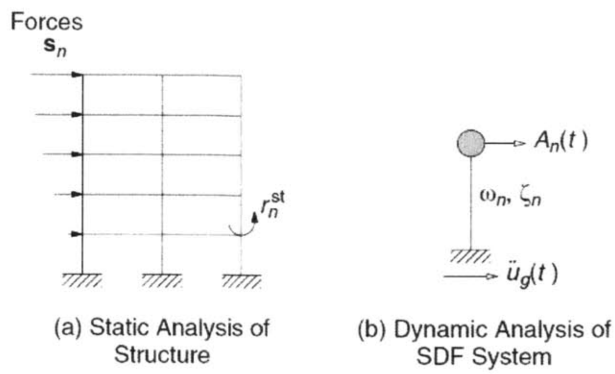


Figure3.1 Conceptual explanation of modal response history analysis of elastic MDF systems

The first step for this dynamic analysis method is to calculate the vibration properties of the structures (natural vibration frequency and modes), and then expand the force distribution vector $\mathbf{m}\boldsymbol{\iota}$ to modal components $\mathbf{s}_n$. The contribution of nth mode to the dynamic response can be obtained by multiplying the following two parts: (1) the static analysis under forces $\mathbf{s}_n$; (2) the dynamic analysis for the nth mode SDOF system under $\ddot{u}_g(t)$. By combining the static response of these N sets of forces $\mathbf{s}_n$ and dynamics response of these N different SDOF system, the total seismic response will be obtained

3.1.3. Multistory Buildings with Symmetric Plan

Assuming that a multistory building has two orthogonal axes of symmetry, and the ground motion is along one of them. The equation of motion now is repeated as:

$$\mathbf{m\ddot{u}} + \mathbf{c\dot{u}} + \mathbf{ku} = -\mathbf{m1}\ddot{u}_g(t) \tag{3.15}$$

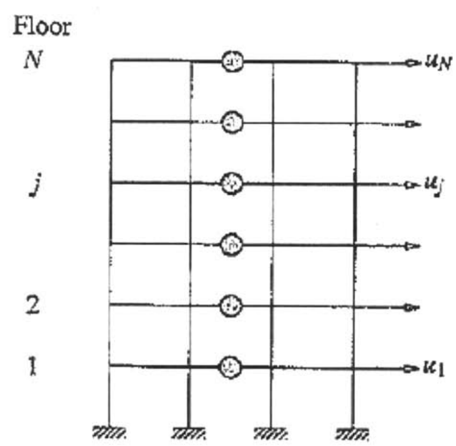


Figure3.2 Dynamic degrees of freedom of a multistory frame: Internal displacements relative to the ground

wherein each element of $\mathbf{1}$ is unity, substituting $\boldsymbol{\iota} = 1$ into equations 3.3, 3.4 yields to the modal expansion of the spatial distribution of effective earthquake forces:

$$\mathbf{m}\boldsymbol{\iota} = \sum_{n=1}^{N} \mathbf{s}_n = \sum_{n=1}^{N} \Gamma_n \mathbf{m} \boldsymbol{\Phi}_n \tag{3.16}$$

where

$$\Gamma_n = \frac{L_n^h}{M_n}, L_n^h = \sum_{j=1}^{N} m_j \Phi_{jn}, M_n = \sum_{j=1}^{N} m_j \Phi_{jn}^2 \tag{3.17}$$

Specifically, the lateral displacement of the jth floor is

$$u_{jn}(t) = \Gamma_n \Phi_{jn} D_n(t) \tag{3.18}$$

The modal static response r^{st}_n is determined by static analysis of under external forces $\mathbf{s}_n$ (Figure 3.3)

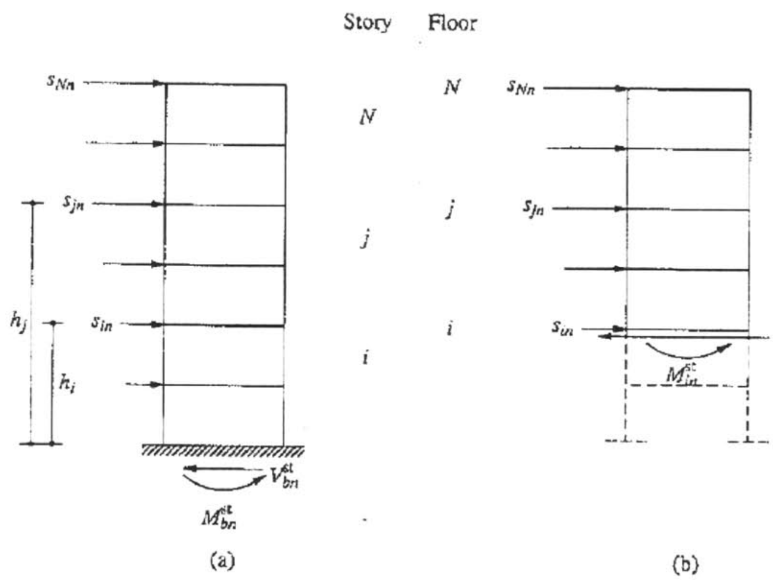


Figure3.3 Computation of modal static response of story forces from force vector $\mathbf{s}_n$: (a) base shear and base overturning moment; (b) i th story shear i th floor overturning moment

Table 3.1 gives the six response quantities for modal static response: the i th floor shear V_i, the i th floor overturning moment M_i, the base shear V_b, the base overturning moment M_b, story displacement u_j, and story drifts Δ_j

Response r	Modal static response r_n^{st}	Response r	Modal static response r_n^{st}
V_i	$V_{in}^{st} = \sum_{j=i}^{N} s_{jn}$	M_b	$M_{bn}^{st} = \sum_{j=i}^{N} h_j s_{jn} = \Gamma_n L_n^{\theta} \equiv h_n^* M_n^*$
M_i	$M_{in}^{st} = \sum_{j=i}^{N} (h_j - h_i) s_{jn}$	u_j	$u_{jn}^{st} = (\Gamma_n / \omega_n^2)\Phi_{jn}$
V_b	$V_{bn}^{st} = \sum_{j=i}^{N} s_{jn} = \Gamma_n L_n^h \equiv M_n^*$	Δ_j	$\Delta_{jn}^{st} = (\Gamma_n / \omega_n^2)(\Phi_{jn} - \Phi_{j-1,n})$

Table3.1 Modal static responses

3.1.4. Response Spectrum Analysis

The structural response r(t) obtained from response history analysis is the function of time, but it takes longer time and is expensive for the program to run it. For the response spectrum analysis, only the peak response values are obtained from the response spectrum without carrying out the time-costing response history analysis. For a SDOF system, there is no difference for the outcome of RSA and RHA; for a MDOF system, RSA usually cannot lead to accurate results, compared with RHA. But the estimate is exact enough for engineering application.

In such a response spectrum analysis, the peak value r_{no} of the nth mode contribution $r_n(t)$ to response r(t) is determined from

$$r_{no} = r_n^{st} A_n \qquad 3.19$$

where A_n is the ordinate of the pseudo-acceleration response (or design) spectrum for the nth mode SDOF system. There are two commonly used modal combination rules, the Complete Quadratic Combination (CQC) or the Square-Root-of-Sum-of-Squares (SRSS) rules. The SRSS rule, which is valid for structures with well-separated natural frequencies such as multistory buildings with symmetric plan, provides an estimate of the peak value of the total response:

$$r_o = (\sum_{n=1}^{N} r_{no}^2)^{1/2} \qquad 3.20$$

In the following analysis for the models, the SRSS rule will be applied.

3.1.5. Modal Pushover Analysis for Elastic Systems

For an elastic system, the modal pushover analysis is consistent with RSA, since the static analysis of the structure subjected to lateral forces

$$\mathbf{f}_{n0} = \Gamma_n \mathbf{m} \mathbf{\Phi}_n A_n \qquad 3.21$$

provides the same values of r_{no}, the peak nth mode response in equation 3.19. Alternatively, this response value can be obtained by static analysis of the structure subjected to the lateral forces distributed over the building height according to

$$\mathbf{s}_n^* = \mathbf{m} \mathbf{\Phi}_n \qquad 3.22$$

and the structure is pushed to the roof displacement, u_{rno}, the peak value of the roof displacement due to the nth mode, which from equation 3.9 is

$$u_{rno} = \Gamma_n \Phi_{rn} D_n \qquad 3.23$$

where $D_n = A_n/\omega_n^2$. D_n and A_n are available from the response (or design) spectrum.

The peak modal responses, r_{no}, each determined by one pushover analysis, can be combined according to SRSS rule to obtain an estimate of the peak value of r_o of the total response. This modal pushover analysis (MPA) for linear elastic systems is equivalent to the well-known RSA procedure.

3.1.6. Summary

The response history of an N-story building with two orthogonal symmetric plan under ground motion along x or y direction can be computed as the following steps:

1. Define the ground acceleration $\ddot{u}_g(t)$ numerically at every time step Δt.
2. Define the structural properties:
   a. Determine the mass and lateral stiffness matrices **m** and **k**.
   b. Estimate the modal damping ratio ξ_n
3. Determine the natural vibration frequency ω_n and natural modes of vibration Φ_n.
4. Determine the modal components s_n of the effective force distribution.
5. Compute the response contribution of the nth mode by following steps, which are repeated for all modes, n=1,2,3…N:
   a. Perform static analysis of the building subjected to lateral forces s_n to determine r_n^{st}, the modal static response for each desired response quantity r from table 3.1
   b. Compute the pseudo acceleration $A_n(t)$ for nth mode SDOF system under ground motion by applying time-stepping method introduced in part 2.3
   c. Determine $r_n(t)$ by Eq. 3.12
6. Combine the modal contributions $r_n(t)$ to determine the total response using Eq. 3.20.

Typically only the lower order modes contribute significantly to the total response, so apply steps 3, 4, 5 mainly to these steps.

3.2. Inelastic Systems

3.2.1. Equation of Motion

The modal pushover analysis becomes more attractive when it comes to the inelastic analysis. As is discussed before, the nonlinear static analysis, or the pushover analysis is an effective tool to predict seismic demands and estimate structural performance. The safety requirements allows for a ductility deformation of the structure without collapse.

The relationship between lateral forces $\mathbf{f}_S$ at the N floor levels and the ultimate lateral floor displacements $\mathbf{u}$ is no longer linear, but depends on the history of displacements, thus,

$$\mathbf{f}_S = \mathbf{f}_S(\mathbf{u}, sign\dot{\mathbf{u}}) \qquad 3.24$$

substituting 3.21 into equation 3.1 yields to

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{f}_S(\mathbf{u}, sign\dot{\mathbf{u}}) = -\mathbf{m}\boldsymbol{\iota}\ddot{\mathbf{u}}_g(\mathrm{t}) \qquad 3.25$$

This matrix equation contains N nonlinear differential equations for the N floor displacement $u_j(t)$, j=1,2,3…N. The solution for these coupled equations is the exact nonlinear response history analysis.

The classical modal analysis does not hold for inelastic analysis, because the theoretical basis for the modal analysis is that the nth mode component of the effective earthquake forces induces structural response only in its nth mode of vibration, but not any other modes. Still, it is useful to expand the displacement of the inelastic system in terms of the natural vibration modes of the corresponding linear system as follows:

$$\mathbf{u}(t) = \sum_{n=1}^{N} \mathbf{\Phi}_n q_n(t) \qquad 3.26$$

Substituting equation 3.26 into equation 3.25, premultiplying by Φ_n^T, and using mass and classical damping orthogonality property of modes gives

$$\ddot{q}_n + 2\xi_n \omega_n \dot{q}_n + \frac{F_{sn}}{M_n} = -\Gamma_n \ddot{u}_g(t) \qquad 3.27$$

The resisting force

$$F_{sn} = F_{sn}(\mathbf{q}_n, sign\dot{\mathbf{q}}_n) = \mathbf{\Phi}_n^T \mathbf{f}_s(\mathbf{u}_n, sign\dot{\mathbf{u}}_n) \qquad 3.28$$

depends on all modal coordinates $q_n(t)$, implying coupling of modal coordinates because of yielding of the structure. Equation 3.27 represent a set of coupled equations for inelastic systems, neglecting the coupling of the N equations in modal coordinates in equation 3.27 leads to uncoupled modal response history analysis procedure. Expanding the spatial distribution **s** of the effective earthquake forces into the modal contributions $\mathbf{s_n}$ according to equation 3.3, where ϕ_n are now the modes of the corresponding linear system. The equations governing the response of the inelastic system is

$$\mathbf{m\ddot{u}} + \mathbf{c\dot{u}} + \mathbf{f}_S(\mathbf{u}, sign\mathbf{\dot{u}}) = -\mathbf{s}_n \ddot{u}_g(t) \tag{3.29}$$

The solution of equation 3.29 for inelastic systems will no longer be described by equation 3.5 because $q_r(t)$ will generally be nonzero for modes other than the nth mode, implying that other modes will also contribute to the solution. For linear systems, however, $q_r(t)=0$ for all modes other than the nth mode; therefore, it is reasonable to expect that the nth mode should be dominant even for inelastic systems. The governing equation for the nth-mode inelastic SDOF system is

$$\ddot{D}_n + 2\xi_n\omega_n\dot{D}_n + \frac{F_{sn}}{L_n} = -\ddot{u}_g(t) \tag{3.30}$$

and

$$F_{sn} = F_{sn}(D_n, sign\dot{D}_n) = \mathbf{\Phi}_n^T \mathbf{f}_s(D_n, sign\dot{D}_n) \tag{3.31}$$

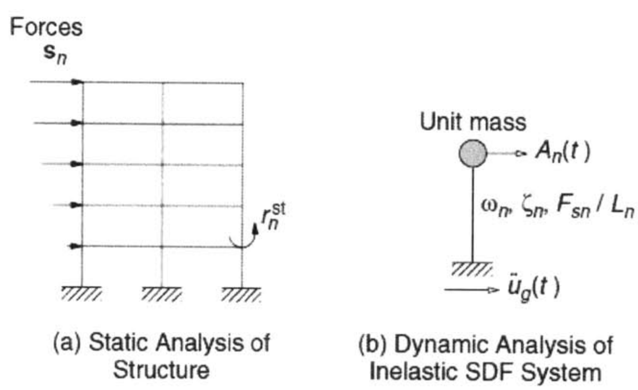


Figure3.4 Conceptual explanation of uncoupled modal response history analysis of inelastic MDOF systems

3.2.2. Inelastic Modal Pushover Analysis

Summarized below are a series of steps used to estimate the peak inelastic response of a symmetric-plan, multistory building about two orthogonal axes to earthquake ground motion along an axis of symmetry using the MPA procedure developed by Chopra and Goel:

1. Compute the natural frequencies, ω_n and modes Φ_n, for linearly elastic vibration of the building.

2. For the nth-mode, develop the base shear-roof displacement, V_{bn} - u_{rn}, pushover curve for force Distribution

$$\mathbf{s}_n^* = \mathbf{m}\mathbf{\Phi}_n$$

For the first mode, gravity loads, including those present on the interior (gravity) frames, were applied prior to the pushover analysis. The resulting P-delta effects lead to negative post-yielding stiffness of the pushover curve. The gravity loads were not included in the higher mode pushover curves, which generally do not exhibit negative post-yielding stiffness.

3. Idealize the pushover curve as a bilinear curve. If the pushover curve exhibits negative post-yielding stiffness, idealize the pushover curve as elastic-perfectly-plastic.

4. Convert the idealized pushover curve to the force-displacement, F_{sn}/L_n - D_n , relation for the nth -"mode" inelastic SDF system by utilizing

$$\frac{F_{sn}}{L_n} = \frac{V_{bny}}{M_n^*} \qquad D_{ny} = \frac{u_{rny}}{\Gamma_n \phi_{rn}}$$

in which M_n* is the effective modal mass, Φ_{rn} is the value of Φ_n at the roof.

5. Compute peak deformation D_n of the nth-"mode" inelastic SDF system defined by the force-deformation relation of and damping ratio ξ_n. The elastic vibration period of the system is

$$T_n = 2\pi\left(\frac{L_n D_{ny}}{F_{sny}}\right)^{1/2}$$

For an SDF system with known T_n and z_n, D_n can be computed by nonlinear response history analysis (RHA) or from the inelastic design spectrum.

6. Calculate peak roof displacement urn associated with the nth-"mode" inelastic SDF system from

$$u_{rn} = \Gamma_n \phi_{rn} D_n$$

7. From the pushover database, extract values of desired responses rn: floor displacements, story drifts, plastic hinge rotations, etc.

8. Repeat Steps 3-7 for as many modes as required for sufficient accuracy. Typically, the first two or three "modes" will suffice.

9. Determine the total response (demand) by combining the peak "modal" responses using the SRSS rule:

$$r \approx \left(\sum_n r_n^2\right)^{1/2}$$

3.2.3. Nonlinear Time History Analysis

Time-history analysis provides for linear or nonlinear evaluation of dynamic structural response under loading which may vary according to the specified time function. Dynamic equilibrium equations, given by

$$Ku(t) + C\frac{du(t)}{dt} + M\frac{d^2u(t)}{dt^2} = r(t)$$

are solved using either modal or direct-integration methods. Initial conditions may be set by continuing the structural state from the end of the previous analysis. Additional notes include:

- Step Size – Direct-integration methods are sensitive to time-step size, which should be decreased until results are not affected.
- HHT Value – A slightly negative Hilber-Hughes-Taylor alpha value is also advised to damp out higher frequency modes, and to encourage convergence of nonlinear direct-integration solutions.
- Nonlinearity – Material and geometric nonlinearity, including P-delta and large-displacement effects, may be simulated during nonlinear direct-integration time-history analysis.
- Links – Link objects capture nonlinear behavior during modal (FNA) applications.

In the project, the implicit Hilber – Hughes – Taylor Method is adopted. The elastic forces are taken here between t_n and t_{n+1}.

$$\boldsymbol{U}_{n+1} = \boldsymbol{U}_n + \mathrm{h}\dot{\boldsymbol{U}}_n + h^2(1/2 - \beta)\ddot{\boldsymbol{U}}_n + (h^2)\beta\ddot{\boldsymbol{U}}_{n+1}$$

$$\dot{\boldsymbol{U}}_{n+1} = \dot{\boldsymbol{U}}_n + h(1-\gamma)\ddot{\boldsymbol{U}}_n + h\gamma\ddot{\boldsymbol{U}}_{n+1}$$

$$\boldsymbol{M}\ddot{\boldsymbol{U}}_{n+1} + (1+\alpha_H)\boldsymbol{K}\boldsymbol{U}_{n+1} - \alpha_H\boldsymbol{K}\boldsymbol{U}_n = \boldsymbol{F}_{n+1}$$

The authors of this method do not give the range of application, mutual relation between parameters αH, β and γ and their influence on the stability condition. Numerical tests performed by the author of the present paper proved that the change of the parameters should be done with attention. The method can be considered as the alternative to the Bossak method. However, since it contributes potential forces not clearly definite, applications to nonlinear problems should be investigated.

4. THE APPLICATION OF MODAL PUSHOVER ANALYSIS IN HIGH-RISE BUILDINGS

4.1. Basic Information of the Models

With the assistant of computer program, the seismic analysis is more convenient to perform. The University of California, Berkeley was an early base for computer-based seismic analysis of structures, led by Professor Ray Clough (who coined the term finite element). Students included Ed Wilson, who went on to write the program SAP in 1970, an early "Finite Element Analysis" program. Today SAP2000 is a powerful tool in structural analysis.

For simplicity, two 2-D frame models are established for the analysis (Fig. 4.1) in SAP2000; one is 8 stories and the other 16 stories. The two frames have the same materials made of steel, and the same W sections. Both of the buildings are two bay frames with the same story height 3m and bay width 6m. The basements are restrained in all directions, and the models are set as plane frames, that is to say, the external forces will be loaded in X direction and the structures also move along this axis.

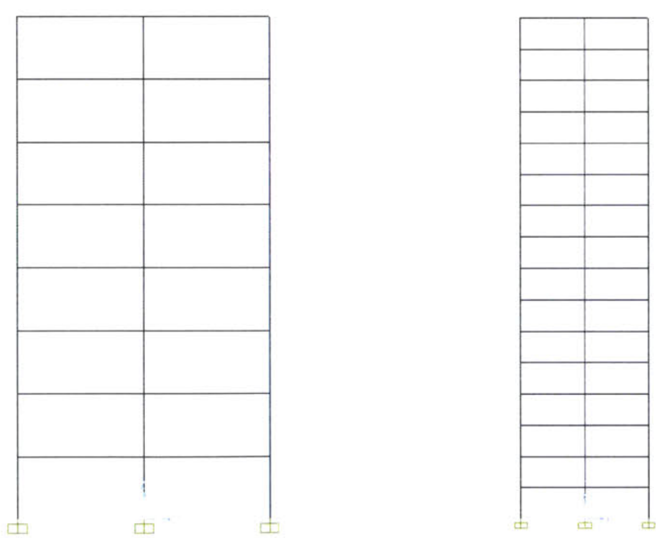


Figure4.1 Eight-story building and sixteen-story building models

In this thesis, we only consider the elastic stage of the structures, so a week ground motion was selected: the S_MONICA-1 time history function in X direction scaled down by a factor of 0.5.

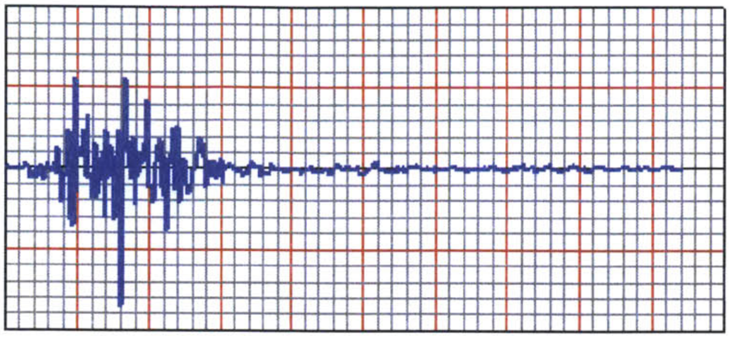


Figure4.2 S_MONICA-1 time history function

Next a new time history load case is defined. In the "Time History Type" option, the "Modal" is the modal response history analysis, which is a linear method; while the "Direct Integration" is the nonlinear procedure with a superior accuracy but spends more time for computing. Here we check the former option "Modal". The "Output Time Step Size" is the stepping time introduced in part 2.3, we set it 0.01s, output time steps is 500. Lastly, the modal damping ratio is set constant at 0.05.

4.2. Structural Properties

The first three vibration modes and their properties are listed in the table below:

8-floor	**StepNum**	**Period**	**Frequency**	**CircFreq**	**Eigenvalue**
Text	Unitless	Sec	Cyc/sec	rad/sec	rad2/sec2
Mode	1	1.242376	0.80491	5.0574	25.577
Mode	2	0.378316	2.6433	16.608	275.84
Mode	3	0.196117	5.099	32.038	1026.4
16-floor	**StepNum**	**Period**	**Frequency**	**CircFreq**	**Eigenvalue**
Text	Unitless	Sec	Cyc/sec	rad/sec	rad2/sec2
Mode	1	2.650	0.377	2.371	5.622
Mode	2	0.851	1.175	7.382	54.489
Mode	3	0.476	2.102	13.206	174.390

Table4.1 Structural properties for the two buildings

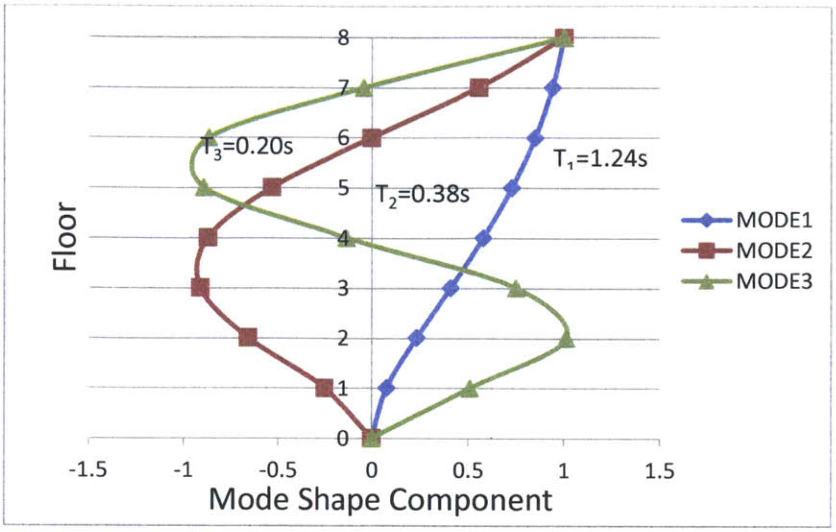


Figure4.3 First three natural-vibration periods and modes of the 8-story building

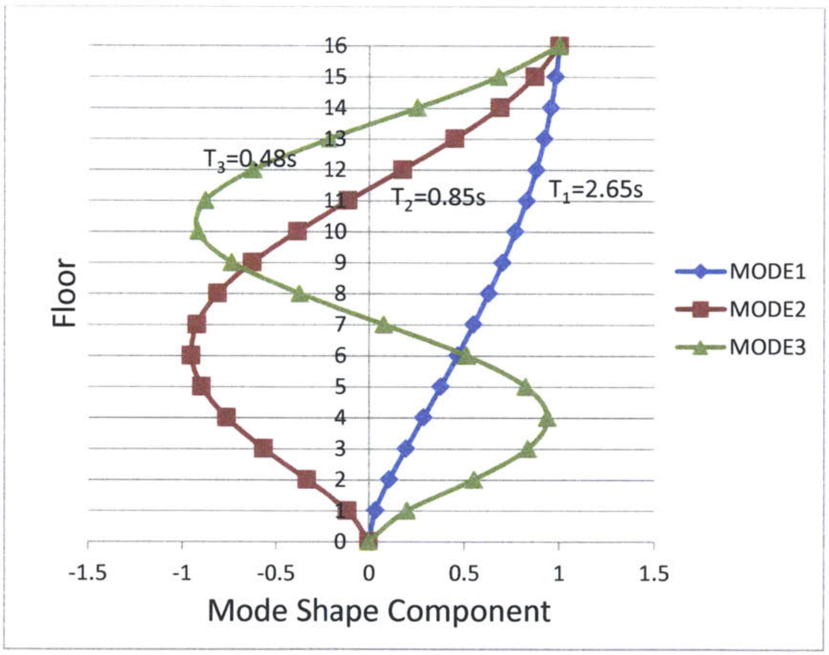


Figure4.4 First three natural-vibration periods and modes of the 16-story building

The point displacements representing the mode shapes can be obtained directly from SAP2000, but they are only the original values. Dividing by the value of roof displacement, the normalized mode shapes are obtained. By applying Eqs. 3.17, we can determine the force distributions. This matrix calculation process is performed by a set of MATLAB codes attached in APPENDIX.

Floor	$\emptyset_1$	$\emptyset_2$	$\emptyset_3$	s1	s2	s3
0	0	0	0	0	0	0
1	0.0777	-0.2519	0.5110	0.101	0.114	0.156
2	0.2334	-0.6573	1.0151	0.304	0.298	0.310
3	0.4094	-0.9103	0.7523	0.533	0.413	0.230
4	0.5793	-0.8697	-0.1318	0.754	0.395	-0.040
5	0.7292	-0.5311	-0.8908	0.949	0.241	-0.272
6	0.8508	-0.0011	-0.8641	1.107	0.001	-0.264
7	0.9402	0.5548	-0.0424	1.224	-0.252	-0.013
8	1	1	1	1.302	-0.454	0.305

Table4.2 Normalized mode shape values of the 8-story building

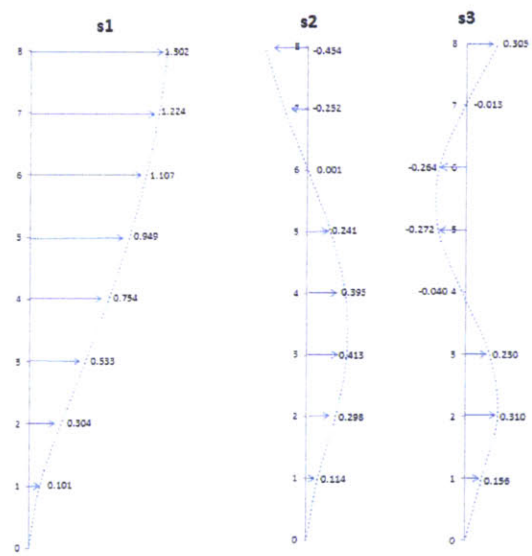


Figure4.5 Force distributions $\mathbf{s}_n$ of the 8-story building

Floor	$\emptyset_1$	$\emptyset_2$	$\emptyset_3$	s1	s2	s3
0	0	0	0	0	0	0
1	0.0357	-0.1137	0.2009	0.046	0.052	0.061
2	0.1089	-0.3338	0.5516	0.141	0.152	0.166
3	0.1957	-0.5658	0.8356	0.254	0.258	0.252
4	0.2864	-0.7623	0.9401	0.372	0.347	0.283
5	0.3769	-0.8959	0.8251	0.490	0.408	0.248
6	0.4649	-0.9513	0.5133	0.604	0.433	0.155

7	0.5490	-0.9223	0.0787	0.713	0.420	0.024
8	0.6282	-0.8109	-0.3739	0.816	0.369	-0.113
9	0.7015	-0.6270	-0.7350	0.911	0.285	-0.221
10	0.7682	-0.3866	-0.9171	0.998	0.176	-0.276
11	0.8275	-0.1110	-0.8759	1.075	0.051	-0.264
12	0.8787	0.1756	-0.6209	1.141	-0.080	-0.187
13	0.9216	0.4485	-0.2125	1.197	-0.204	-0.064
14	0.9558	0.6852	0.2548	1.241	-0.312	0.077
15	0.9815	0.8698	0.6803	1.275	-0.396	0.205
16	1	1	1	1.299	-0.455	0.301

Table4.3 Normalized mode shape values of the 16-story building

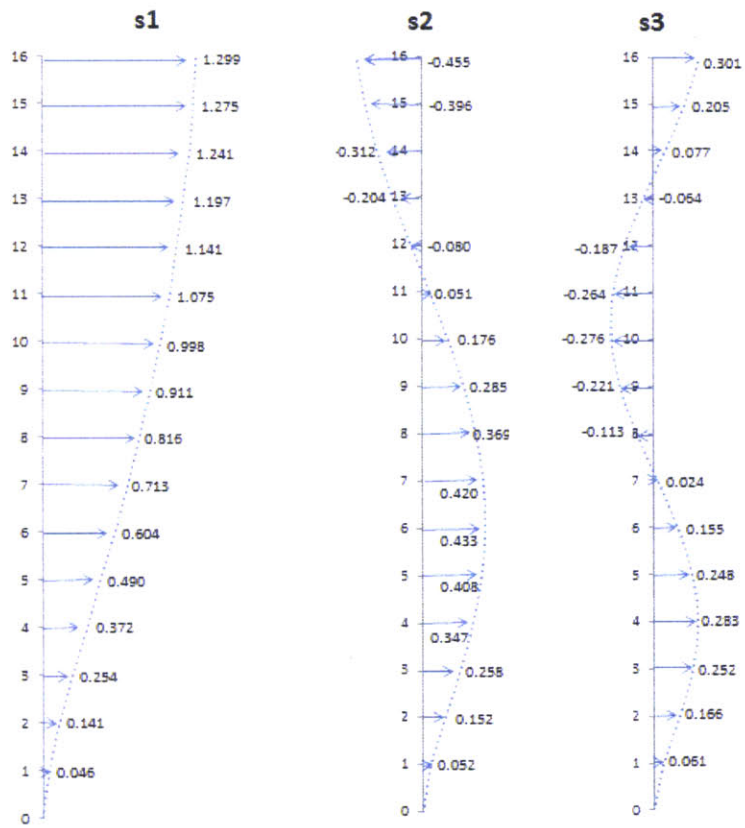


Figure4.6 Force distributions s_n of the 16-story building

4.3. Elastic Analysis

4.3.1. Modal Response History Analysis

The modal response history analysis is firstly performed as a benchmark, and its basis theory has been discussed in part 3.1.2. In SAP2000, there are two different procedures for the time history analysis, "Modal" and "Direct Integration", in which the modal method is actually the linear analysis.

The time history function is applied to the whole building first as ground acceleration parallel to X direction. The roof displacements and base shear forces including their peak values for the two buildings are shown in Fig. 4.7 and Fig. 4.8. These figures contain the time history responses for the top floor and the bottom floor, so the responses of the other floors can be multiplied with the force distribution $\mathbf{s}_n$ proportionally, since they are elastic systems.

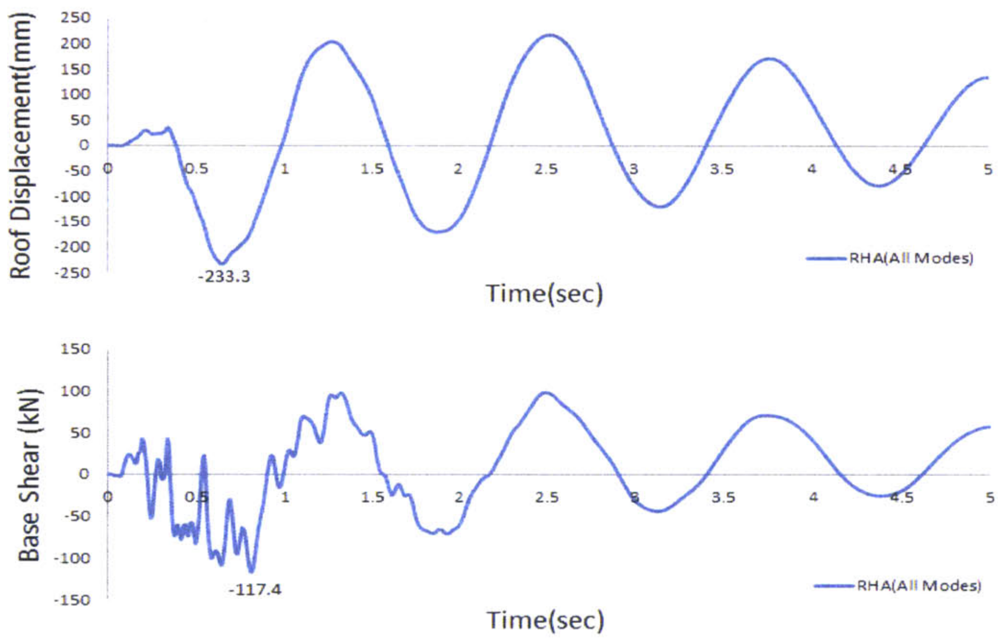
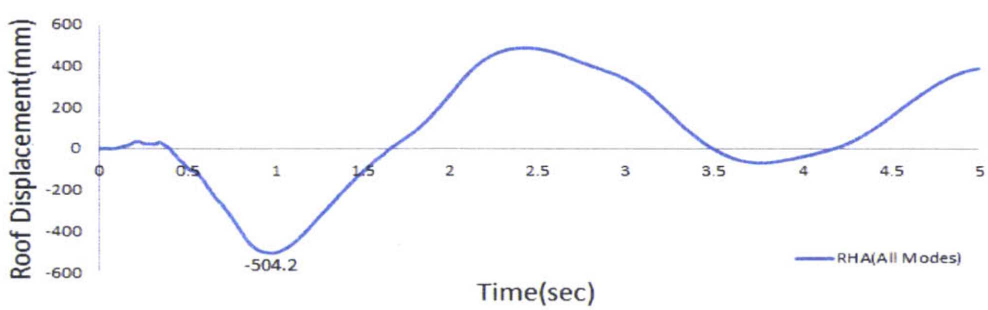


Figure4.7 Roof displacement and base shear history (all modes) for the 8-story building



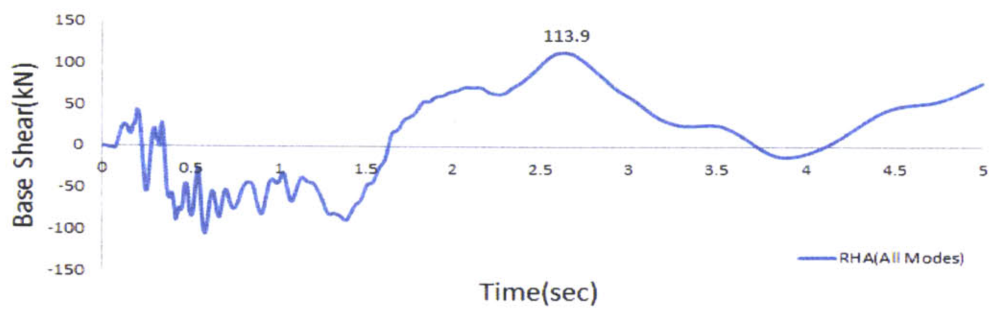


Figure4.8 Roof displacement and base shear history (all modes) for the 16-story building

The next stage will be the most critical analysis, since the responses mentioned above are the combinations of all modes, but we still need to know the response of each mode under the same ground motion excitation. In practical application, the peak values can be determined directly for the response spectrum for an individual mode. In this thesis, the MDOF model is switched to the equivalent SDOF systems for the first three modes. Based on Fig. 3.1 and the formulas in Table 3.1, the models have the following corresponding relation:

8-Floor Building	Mode 1	Mode 2	Mode 3
M_n^*	6.273m	0.759m	0.412m
h_n^*	5.689h	-0.867h	1.731h
Γ_n	1.3016	-0.4541	0.3055
16-Floor Building	Mode 1	Mode 2	Mode 3
M_n^*	12.574m	1.503m	0.646m
h_n^*	10.873h	-2.177h	3.018h
Γ_n	1.2989	-0.4551	0.3012

Table4.4 Effective modal mass and effective modal height of the two buildings

where m is the equivalent lumped mass for each floor, and h is the story height. Γ_n is the multiplying factor for the floor displacement in Eq. 3.13

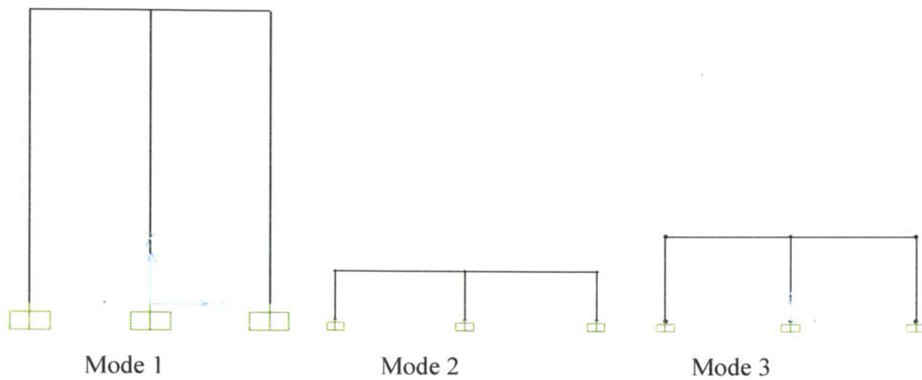
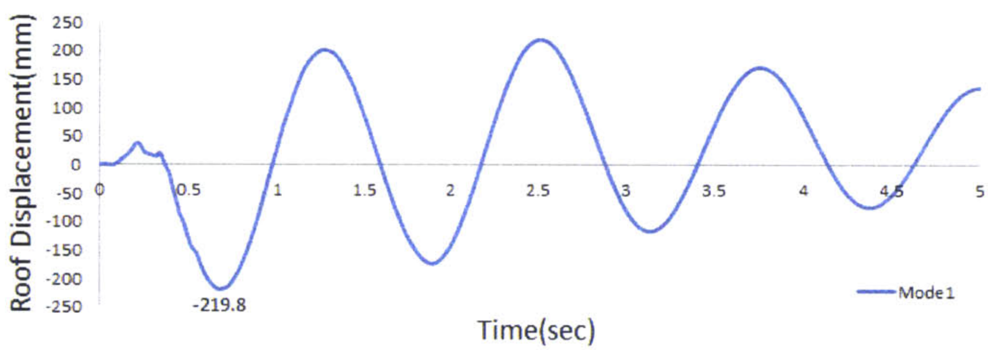
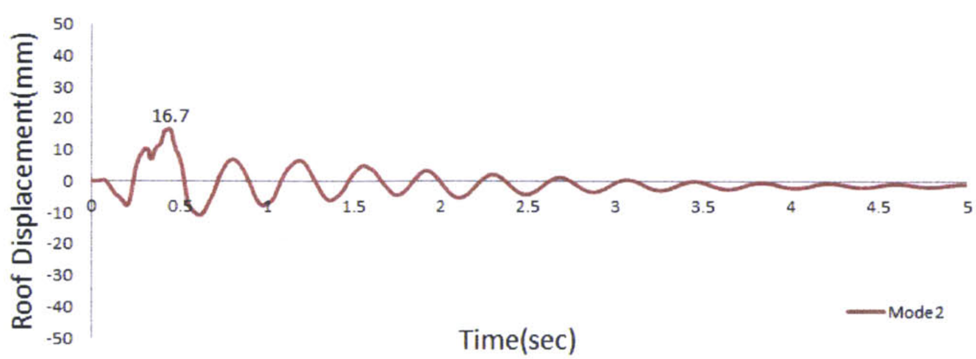


Figure4.9 Equivalent SDOF systems for the first three modes of the two buildings

In the new equivalent SDOF systems, the height is from Table 4.4 directly, and the lumped mass is defined by the "mass source" option in SAP2000 to make it equal to the values in Table 4.4. The material properties, cross sections and the time history functions are all the same in SDOF systems with those in MDOF systems. The figures below show the roof displacements of each mode under the same ground motion:





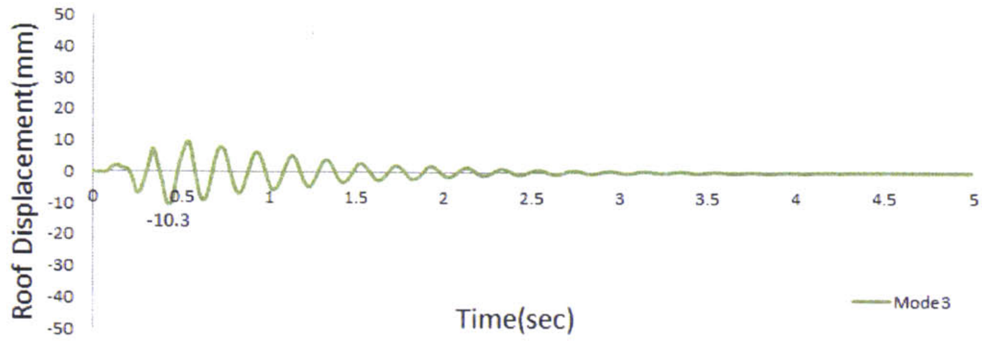


Figure4.10 Roof displacements due to the first three modes of the 8-story building

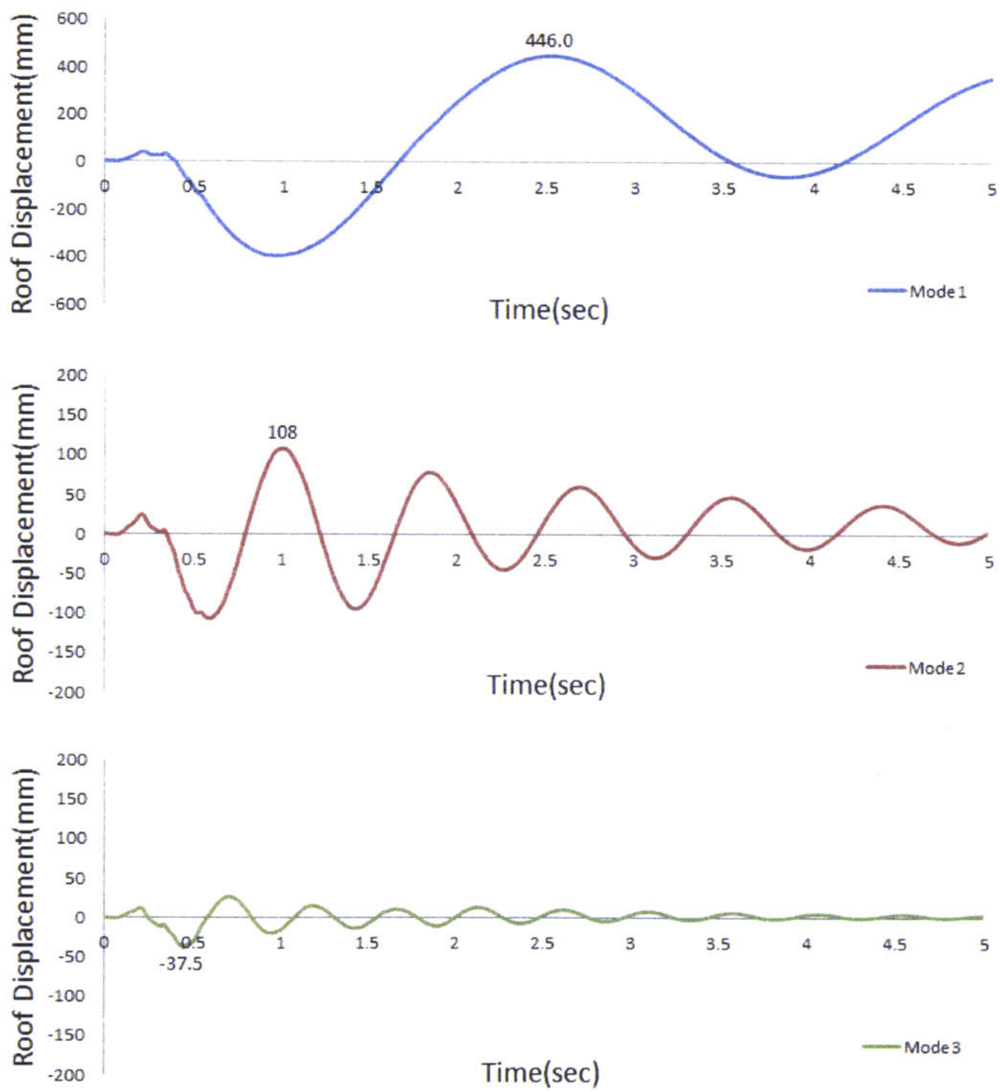


Figure4.11 Roof displacements due to the first three modes of the 16-story building

4.3.2. Elastic Modal Pushover Analysis

The modal pushover analysis is based on the superposition of the contribution of each mode, so the pushover analysis is firstly conducted for the individual mode. The models are pushed at the roof points to the target displacement determined from time history analysis, i.e., pushing the structure to 219.8mm, 16.7mm and 10.3mm for the 8-story building, and 446mm, 108mm and 37.5mm for the 16-story building, to the roof displacements. The pushover curve should be a line with constant slope since the structure does not yield. The displacements of other floors are strictly proportioned to the mode shapes in elastic systems. The shape vectors $\emptyset_n$ are also taken as load patterns for modal pushover analysis, which has a superior accuracy compared with the uniform load pattern or monotonic increased load pattern. For each pushover analysis, the column shear of every story is also recorded. Once the floor displacements, story drifts and column shears for each mode are obtained, we may conduct the modal pushover analysis by utilizing the SRSS modal superposition rule to determine the combined modal responses for 1 mode, 2 modes and three modes. The story drift ratios are the peak values obtained from time history records. The results of the modal pushover analysis are compared with the elastic time history analysis shown in Table 4.5~4.10 and Fig.4.12~4.17.

Floor Displacement(mm)

Floor	Pushover Analysis			Modal Pushover Analysis			Response History Analysis
	Mode 1	Mode 2	Mode 3	1 Mode	2 Modes	3 Modes	
0	0	0	0	0	0	0	0
1	-17.1	-4.2	-5.3	17.1	17.6	18.4	19.2
2	-51.3	-11.0	-10.5	51.3	52.5	53.5	55.2
3	-90.0	-15.2	-7.7	90.0	91.3	91.6	94.8
4	-127.3	-14.5	1.4	127.3	128.2	128.2	131.5
5	-160.3	-8.9	9.2	160.3	160.5	160.8	166.7
6	-187.0	0.0	8.9	187.0	187.0	187.2	194.2
7	-206.6	9.3	0.4	206.6	206.9	206.9	216.7
8	-219.8	16.7	-10.3	219.8	220.4	220.7	233.3

Table4.5 Peak values of floor displacements for the 8-story building (elastic analysis)

Drift Ratio(%)

Floor	Pushover Analysis			Modal Pushover Analysis			Response History Analysis
	Mode 1	Mode 2	Mode 3	1 Mode	2 Modes	3 Modes	
0	0	0	0	0	0	0	0
1	-0.569	-0.140	-0.175	0.569	0.586	0.612	0.640
2	-1.140	-0.226	-0.173	1.140	1.162	1.175	1.200
3	-1.289	-0.141	0.090	1.289	1.297	1.300	1.320
4	-1.245	0.023	0.304	1.245	1.246	1.282	1.223
5	-1.098	0.188	0.261	1.098	1.114	1.144	1.173
6	-0.891	0.295	-0.009	0.891	0.938	0.938	0.917
7	-0.655	0.309	-0.282	0.655	0.724	0.777	0.750
8	-0.438	0.248	-0.358	0.438	0.504	0.618	0.553

Table4.6 Peak values of story drift ratios for the 8-story building (elastic analysis)

Floor	Column Shear (kN)						
	Pushover Analysis			Modal Pushover Analysis			Response History Analysis
	Mode 1	Mode 2	Mode 3	1 Mode	2 Modes	3 Modes	
1	87.1	-26.3	42.5	87.1	91.0	100.4	117.4
2	100.6	-25.5	27.5	100.6	103.8	107.4	112.4
3	99.9	-14.6	-10.2	99.9	101.0	101.5	105.6
4	92.0	1.3	-38.4	92.0	92.0	99.7	102.7
5	79.0	16.7	-33.3	79.0	80.7	87.3	91.4
6	62.6	26.2	0.5	62.6	67.9	67.9	70.9
7	42.8	26.1	32.5	42.8	50.1	59.7	60.0
8	27.1	19.8	39.6	27.1	33.6	51.9	53.2

Table4.7 Peak values of column shears for the 8-story building (elastic analysis)

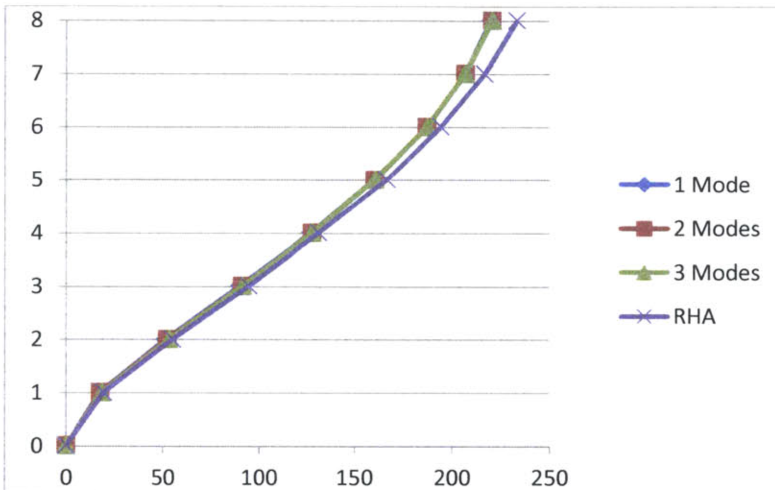


Figure4.12 Heightwise variation of floor displacements for the 8-story building (elastic analysis)

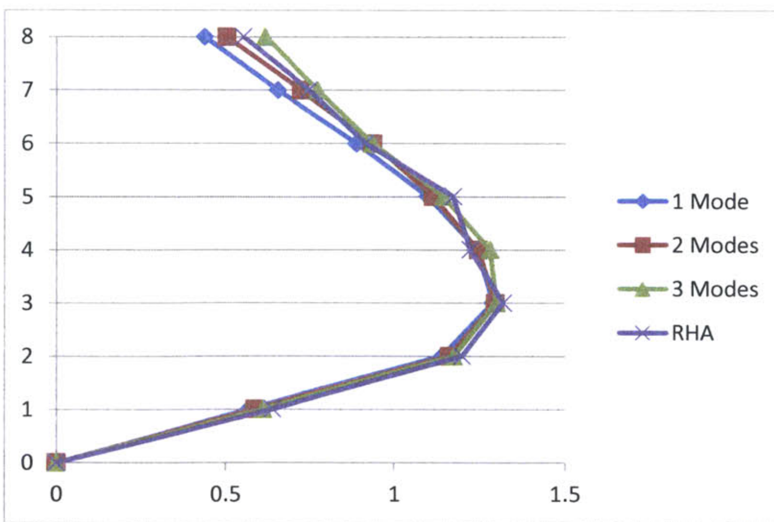


Figure4.13 Heightwise variation of story drift ratios for the 8-story building (elastic analysis)

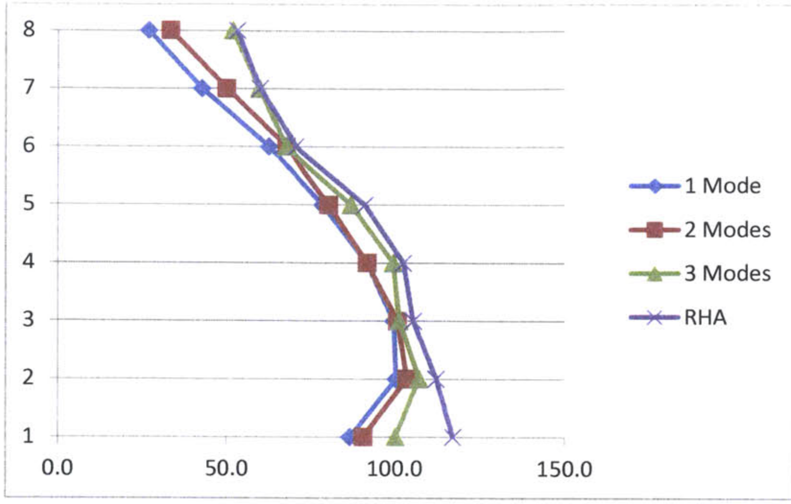


Figure4.14 Heightwise variation of column shears for the 8-story building (elastic analysis)

Floor	Floor Displacement(mm)						
	Pushover Analysis			Modal Pushover Analysis			Response History Analysis
	Mode 1	Mode 2	Mode 3	1 Mode	2 Modes	3 Modes	
0	0	0	0	0	0	0	0
1	15.9	-12.3	-7.5	15.9	20.1	21.5	25.5
2	48.6	-36.0	-20.7	48.6	60.5	63.9	67.7
3	87.3	-61.1	-31.3	87.3	106.5	111.0	115.6
4	127.7	-82.3	-35.3	127.7	152.0	156.0	159.0
5	168.1	-96.8	-30.9	168.1	194.0	196.4	199.2
6	207.3	-102.7	-19.2	207.3	231.4	232.2	236.5
7	244.9	-99.6	-3.0	244.9	264.4	264.4	270.5
8	280.2	-87.6	14.0	280.2	293.6	293.9	298.6
9	312.9	-67.7	27.6	312.9	320.1	321.3	324.4
10	342.6	-41.7	34.4	342.6	345.1	346.8	353.8
11	369.0	-12.0	32.8	369.0	369.2	370.7	374.3
12	391.9	19.0	23.3	391.9	392.4	393.1	401.8
13	411.0	48.4	8.0	411.0	413.9	414.0	420.6
14	426.3	74.0	-9.6	426.3	432.6	432.8	441.8
15	437.7	93.9	-25.5	437.7	447.7	448.4	456.1
16	446.0	108.0	-37.5	446.0	458.9	460.4	470.5

Table4.8 Peak values of floor displacements for the 16-story building (elastic analysis)

Floor	Drift Ratio(%) Pushover Analysis Mode 1	Mode 2	Mode 3	Modal Pushover Analysis 1 Mode	2 Modes	3 Modes	Response History Analysis
0	0	0	0	0	0	0	0
1	0.530	-0.409	-0.251	0.530	0.670	0.715	0.724
2	1.088	-0.792	-0.438	1.088	1.346	1.416	1.432
3	1.290	-0.835	-0.355	1.290	1.537	1.578	1.597
4	1.349	-0.707	-0.131	1.349	1.523	1.529	1.525
5	1.345	-0.481	0.144	1.345	1.429	1.436	1.452
6	1.308	-0.199	0.390	1.308	1.323	1.380	1.441
7	1.251	0.104	0.543	1.251	1.255	1.368	1.433
8	1.177	0.401	0.566	1.177	1.243	1.366	1.393
9	1.090	0.662	0.451	1.090	1.275	1.353	1.346
10	0.991	0.865	0.228	0.991	1.316	1.335	1.352
11	0.881	0.992	-0.051	0.881	1.327	1.328	1.365
12	0.763	1.032	-0.319	0.763	1.283	1.322	1.388
13	0.637	0.982	-0.511	0.637	1.171	1.277	1.292
14	0.508	0.852	-0.584	0.508	0.992	1.151	1.210
15	0.382	0.664	-0.532	0.382	0.766	0.933	1.025
16	0.276	0.469	-0.400	0.276	0.544	0.675	0.712

Table4.9 Peak values of story drift ratios for the 16-story building (elastic analysis)

Floor	Column Shear(kN) Pushover Analysis Mode 1	Mode 2	Mode 3	Modal Pushover Analysis 1 Mode	2 Modes	3 Modes	Response History Analysis
1	78.1	69.2	45.8	78.1	104.3	114.0	113.9
2	93.2	79.0	47.8	93.2	122.2	131.2	135.5
3	97.2	74.3	34.7	97.2	122.3	127.2	133.9
4	96.9	61.0	12.5	96.9	114.5	115.2	125.9
5	94.3	41.7	-12.9	94.3	103.1	103.9	114.8
6	90.2	18.7	-35.2	90.2	92.1	98.6	108.0
7	85.0	-5.6	-49.0	85.0	85.2	98.3	106.2
8	79.0	-29.1	-50.9	79.0	84.2	98.4	105.7
9	72.1	-49.5	-40.7	72.1	87.5	96.5	108.6
10	64.5	-65.1	-20.9	64.5	91.6	94.0	109.7
11	56.3	-74.6	3.6	56.3	93.5	93.5	107.1
12	47.6	-77.1	26.8	47.6	90.6	94.5	104.2
13	38.5	-72.5	43.2	38.5	82.1	92.8	98.8
14	29.1	-61.5	49.0	29.1	68.0	83.8	82.3
15	19.3	-44.2	42.0	19.3	48.2	64.0	71.9
16	12.1	-28.8	29.8	12.1	31.2	43.2	58.6

Table4.10 Peak values of column shears for the 16-story building (elastic analysis)

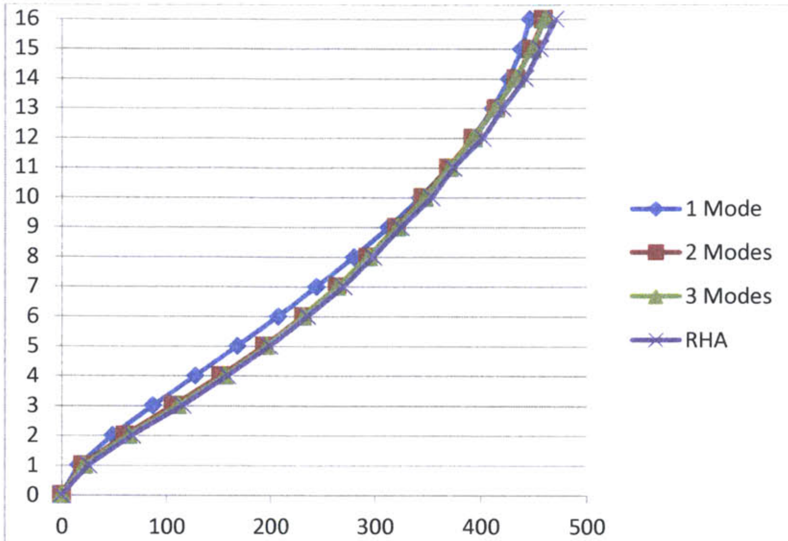


Figure4.15 Heightwise variation of floor displacements for the 16-story building (elastic analysis)

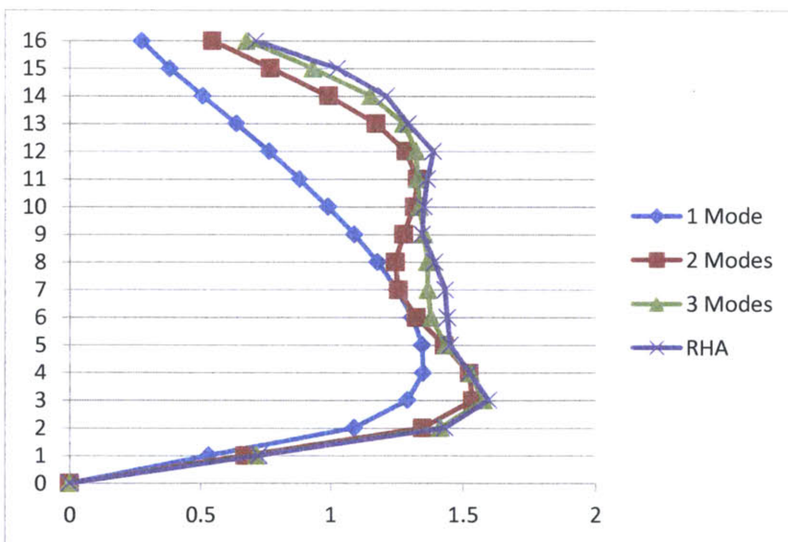


Figure4.16 Heightwise variation of story drift ratios for the 16-story building (elastic analysis)

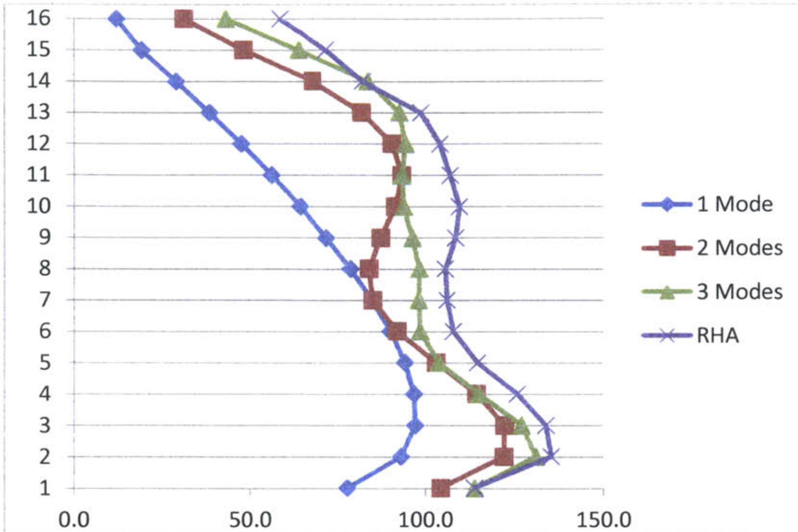


Figure4.17 Heightwise variation of column shears for the 16-story building (elastic analysis)

The peak values mentioned above can also be determined from a response spectrum, which is more popularly used in practice. The elastic modal time history analysis is proofed to be equivalent with the response spectrum analysis. From the analysis we find that for median-rise elastic buildings, the fundamental mode dominates the structural responses, there is no apparent difference between the responses of 1 mode, 2 modes, 3 modes pushover analysis and the time history analysis.

With the increase of the structural height, there is an increasingly participation ratio of the higher modes to the structural response. This difference is not displayed significantly in the floor displacements, but it is obvious in story drift ratios and column shears. We see that for high-rise buildings, first mode is usually inadequate for predicting structural demands, especially in story drift ratios and column shears, while 2 modes or 3 modes typically suffice. Notice that the invariance of the story drift ratios in the medium stories, the combination of higher modes could be more conservative though the response of 1 mode is very close to that of time history analysis.

4.4. Inelastic Analysis

The numerical algorithm for time history analysis is now changed to the nonlinear "Direct Integration", which is the rigorous method. The time step is the same 0.01 with a total output of 500. The time history function is now multiplied by a factor of 2, four times as large as that for elastic analysis. The plastic hinges are assigned at two ends of the columns and beams.

The equivalent SDOF models are again employed for the analysis, and the structures are pushed to the target displacements similarly as before. Each floor is subjected to the lateral forces distributed over the building height according to Eq 3.22. Lastly, the responses of each mode are combined by SRSS and then compared with the nonlinear time history analysis. However, for elastic systems, this procedure lacks theoretical foundation since the force distribution s_n may not be invariant in the inelastic range. The inelastic analysis is a complex process with a number of influential factors and parameters, the methods introduced for modal pushover analysis in this thesis is an approximate way turned out to be accurate enough. For simplicity, only the final results and figures are shown here.

Floor	Floor Displacement(mm)						
	Pushover Analysis			Modal Pushover Analysis			Response History Analysis
	Mode 1	Mode 2	Mode 3	1 Mode	2 Modes	3 Modes	
0	0	0	0	0	0	0	0.0
1	-60.5	35.9	-24.4	60.5	70.4	74.5	93.1
2	-181.7	93.7	-48.5	181.7	204.4	210.1	235.2
3	-318.7	129.7	-35.9	318.7	344.1	346.0	370.2
4	-451.1	123.9	6.3	451.1	467.8	467.8	490.8
5	-567.8	75.7	42.5	567.8	572.8	574.4	602.4
6	-662.4	0.2	41.3	662.4	662.4	663.7	693.7
7	-732.0	-79.1	2.0	732.0	736.3	736.3	768.3
8	-778.6	-142.5	-47.8	778.6	791.5	793.0	833.6

Table4.11 Peak values of floor displacements for the 8-story building (inelastic analysis)

Floor	Drift Ratio(%)						
	Pushover Analysis			Modal Pushover Analysis			Response History Analysis
	Mode 1	Mode 2	Mode 3	1 Mode	2 Modes	3 Modes	
0	0	0	0	0	0	0	0
1	-2.017	1.197	-0.813	2.017	2.345	2.482	2.582
2	-4.040	1.926	-0.802	4.040	4.475	4.546	4.623
3	-4.567	1.202	0.418	4.567	4.723	4.741	4.781
4	-4.412	-0.193	1.407	4.412	4.416	4.635	4.724
5	-3.890	-1.608	1.208	3.890	4.210	4.380	4.336
6	-3.155	-2.517	-0.043	3.155	4.036	4.036	4.145
7	-2.320	-2.641	-1.308	2.320	3.515	3.750	3.828
8	-1.553	-2.115	-1.659	1.553	2.624	3.104	3.205

Table4.12 Peak values of story drift ratios for the 8-story building (inelastic analysis)

Floor	Column Shear (kN)						
	Pushover Analysis			Modal Pushover Analysis			Response History Analysis
	Mode 1	Mode 2	Mode 3	1 Mode	2 Modes	3 Modes	
1	183.7	166.5	-236.2	183.7	247.9	342.4	310.7
2	214.9	161.2	-180.2	214.9	268.6	323.5	332.5
3	202.8	122.2	80.2	202.8	236.8	250.0	312.7
4	190.7	-30.1	218.3	190.7	193.1	291.4	282.4
5	171.6	-148.8	188.6	171.6	227.1	295.2	287.4
6	150.2	-164.1	-7.3	150.2	222.5	222.6	230.3
7	117.4	-160.3	-136.2	117.4	198.7	240.9	231.5
8	86.5	-130.1	-154.7	86.5	156.2	219.9	212.3

Table4.13 Peak values of column shears for the 8-story building (inelastic analysis)

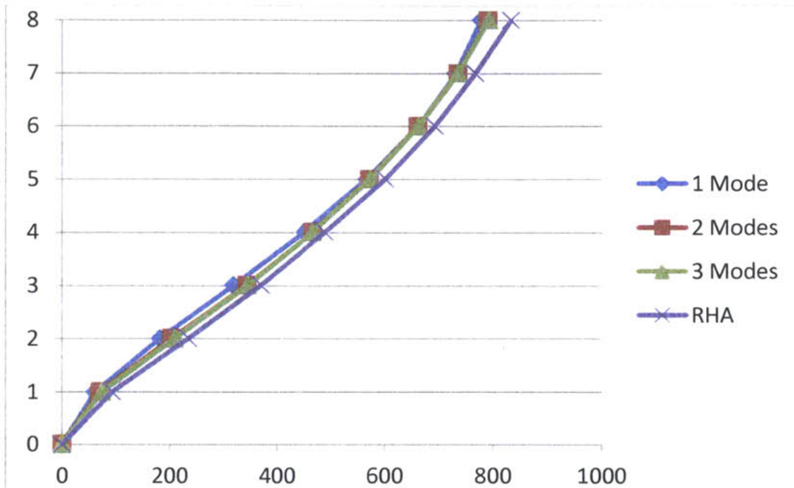


Figure4.18 Heightwise variation of floor displacements for the 8-story building (inelastic analysis)

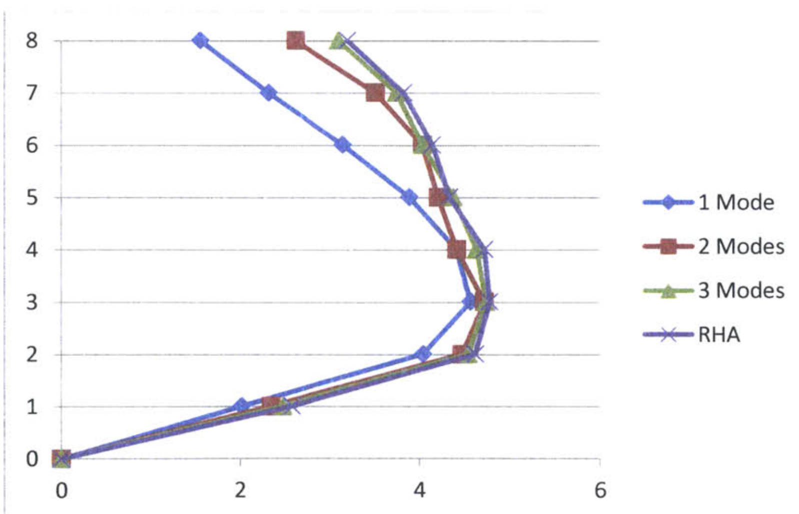


Figure4.19 Heightwise variation of story drift ratios for the 8-story building (inelastic analysis)

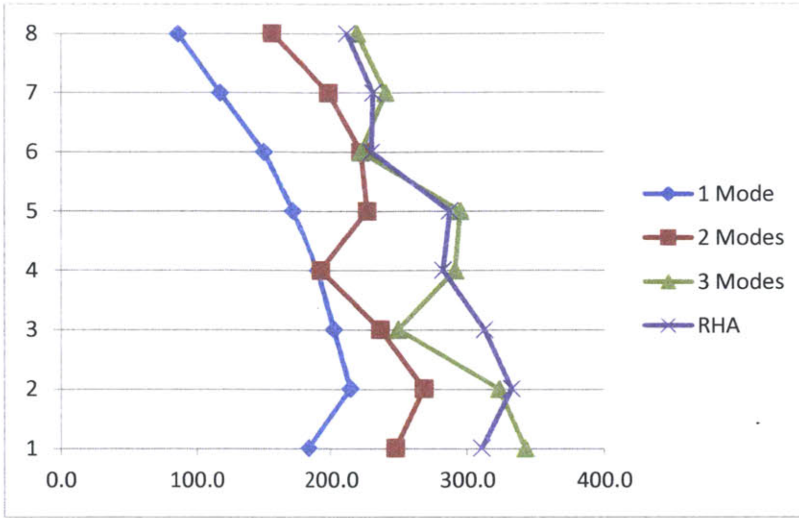


Figure4.20 Heightwise variation of column shears for the 8-story building (inelastic analysis)

Floor	Floor Displacement(mm)						
	Pushover Analysis			Modal Pushover Analysis			Response History Analysis
	Mode 1	Mode 2	Mode 3	1 Mode	2 Modes	3 Modes	
0	0	0	0	0	0	0	0
1	45.6	50.8	-24.2	45.6	68.3	72.5	91.9
2	139.3	149.3	-66.5	139.3	204.2	214.7	239.7
3	250.3	253.0	-100.8	250.3	355.9	369.9	387.8
4	366.4	340.9	-113.4	366.4	500.5	513.1	529.6
5	482.2	400.7	-99.5	482.2	626.9	634.7	673.3
6	594.7	425.4	-61.9	594.7	731.2	733.9	788.8
7	702.4	412.5	-9.5	702.4	814.5	814.6	870.2
8	803.7	362.7	45.1	803.7	881.7	882.9	940.5
9	897.5	280.4	88.6	897.5	940.2	944.4	1003.3
10	982.7	172.9	110.6	982.7	997.8	1003.9	1068.6
11	1058.6	49.6	105.6	1058.6	1059.7	1065.0	1130.2
12	1124.2	-78.5	74.9	1124.2	1126.9	1129.4	1207.4
13	1179.0	-200.6	25.6	1179.0	1195.9	1196.2	1275.6
14	1222.7	-306.4	-30.7	1222.7	1260.5	1260.9	1355.4
15	1255.6	-389.0	-82.0	1255.6	1314.5	1317.0	1418.8
16	1279.3	-447.2	-120.6	1279.3	1355.2	1360.6	1470.8

Table4.14 Peak values of floor displacements for the 16-story building (inelastic analysis)

Floor	Drift Ratio(%) Pushover Analysis Mode 1	Mode 2	Mode 3	Modal Pushover Analysis 1 Mode	2 Modes	3 Modes	Response History Analysis
0	0	0	0	0	0	0	0
1	1.521	1.694	-0.808	1.521	2.277	2.416	2.525
2	3.122	3.281	-1.410	3.122	4.529	4.743	5.124
3	3.701	3.459	-1.142	3.701	5.066	5.193	5.440
4	3.870	2.929	-0.420	3.870	4.853	4.871	4.793
5	3.858	1.992	0.463	3.858	4.342	4.367	4.462
6	3.753	0.826	1.253	3.753	3.843	4.042	4.205
7	3.588	-0.433	1.747	3.588	3.614	4.014	4.102
8	3.376	-1.660	1.819	3.376	3.762	4.179	4.410
9	3.126	-2.743	1.452	3.126	4.159	4.405	4.706
10	2.842	-3.583	0.732	2.842	4.574	4.632	5.030
11	2.528	-4.108	-0.166	2.528	4.824	4.826	5.234
12	2.187	-4.272	-1.025	2.187	4.800	4.908	5.148
13	1.827	-4.067	-1.642	1.827	4.459	4.752	4.923
14	1.457	-3.529	-1.878	1.457	3.818	4.255	4.335
15	1.096	-2.751	-1.710	1.096	2.961	3.420	3.540
16	0.790	-1.942	-1.285	0.790	2.096	2.459	2.917

Table4.15 Peak values of story drift ratios s for the 16-story building (inelastic analysis)

Floor	Column Shear(kN) Pushover Analysis Mode 1	Mode 2	Mode 3	Modal Pushover Analysis 1 Mode	2 Modes	3 Modes	Response History Analysis
1	129.5	-147.2	163.2	129.5	196.1	255.1	271.6
2	153.9	-165.3	169.9	153.9	225.9	282.6	295.3
3	153.5	-153.0	122.9	153.5	216.7	249.1	288.9
4	153.9	-129.8	43.3	153.9	201.3	205.9	253.0
5	150.7	-90.0	-52.2	150.7	175.5	183.1	231.9
6	145.0	-39.3	-130.6	145.0	150.2	199.1	241.3
7	137.4	13.9	-169.9	137.4	138.1	218.9	245.3
8	128.4	66.9	-174.4	128.4	144.8	226.7	238.8
9	118.7	111.3	-147.8	118.7	162.7	219.8	229.1
10	108.4	137.6	-81.0	108.4	175.2	193.0	212.6
11	93.7	151.8	12.8	93.7	178.4	178.8	201.3
12	76.4	155.8	100.0	76.4	173.5	200.3	203.3
13	60.6	148.0	153.7	60.6	159.9	221.8	227.8
14	45.6	131.0	167.6	45.6	138.7	217.6	193.7
15	30.1	99.7	146.9	30.1	104.1	180.1	169.9
16	18.8	67.0	114.5	18.8	69.6	134.0	106.6

Table4.16 Peak values of column shears for the 16-story building (inelastic analysis)

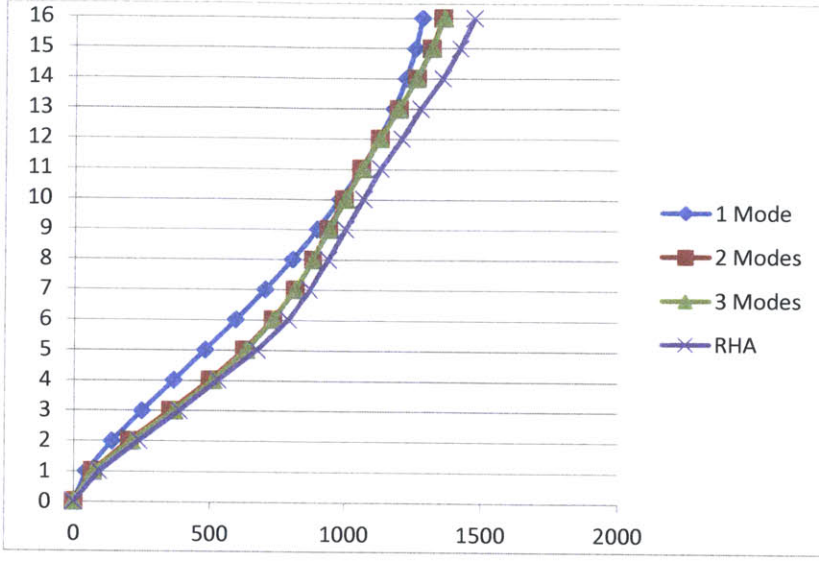


Figure4.21 Heightwise variation of floor displacements for the 16-story building (inelastic analysis)

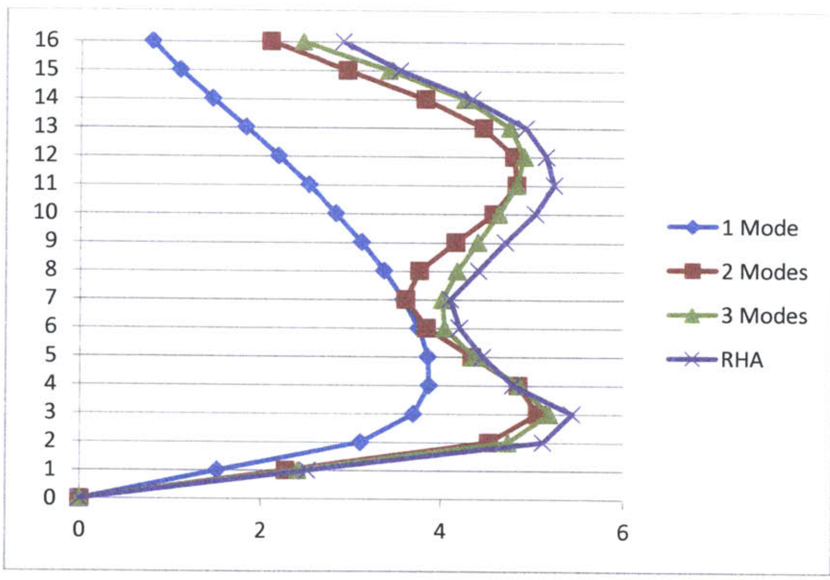


Figure4.22 Heightwise variation of story drift ratios for the 16-story building (inelastic analysis)

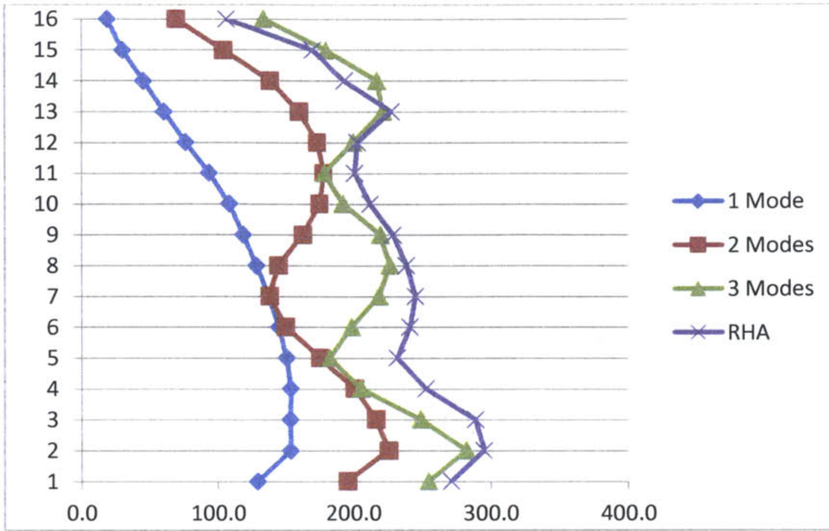


Figure4.23 Heightwise variation of column shears for the 16-story building (inelastic analysis)

By comparing nonlinear static pushover analysis with the nonlinear time history analysis and the elastic analysis, we find that the 1-mode pushover analysis is not adequate for estimating seismic demands, especially for high-rise buildings. The difference between the 1 mode response, 2 modes response, 3 modes response and the time history analysis is larger for inelastic analysis than those for elastic analysis. The errors of inelastic analysis are generally larger than those of elastic analysis, since we take the assumption that the mode shapes keep constant in nonlinear stage. In addition, for inelastic systems, the governing equation is coupled, which means that the effective lateral force $\mathbf{s}_n$ contributes to not only its corresponding mode, but other modes. This contribution is small so that we do not take into consideration.

5. CONCLUSTIONS AND FUTURE WORK

By establishing two models and analyzing their elastic and inelastic behavior under seismic loads, the results of 1 mode, 2 modes and 3 modes modal pushover analysis are compared with the time history analysis, Table 5.1 shows the average errors in each analysis case:

	Floor Displacment			Drift Ratio			Column Shear		
	1 mode	2 modes	3 modes	1 mode	2 modes	3 modes	1 mode	2 modes	3 modes
8-Elastic	5.94%	4.89%	3.97%	8.59%	3.60%	1.26%	28.66%	10.68%	5.09%
16-Elastic	31.53%	9.48%	5.67%	20.49%	4.83%	1.56%	33.39%	11.33%	6.31%
8-Inelastic	16.39%	9.93%	8.47%	33.55%	7.30%	2.18%	73.70%	26.24%	6.71%
16-Inelastic	57.53%	13.66%	10.00%	38.56%	7.39%	3.97%	79.08%	36.83%	16.27%

Table5.1 Average errors in each analysis case

Generally speaking, the higher the building, the more the higher order modes contribute to the total response, and the more modes needed for accuracy. The conventional 1-mode pushover analysis provides perfect seismic demands for low- and medium-rise buildings, especially when structures are basically elastic. For inelastic systems, due to the assumption that the mode shapes do not change during deformation and the other assumptions, at least 3 modes should be employed for modal pushover analysis.

Compared with the 1-mode pushover analysis with single monotonic load pattern, the improved modal pushover analysis combines the contribution of higher order modes, resulting in a superior accuracy in evaluating seismic performance. It is demonstrated that the elastic modal pushover analysis is essentially equivalent to the well-known elastic response spectrum analysis, so the MPA becomes more attractive and effective when applied to inelastic systems and high-rise buildings. Another important application of pushover analysis and modal pushover analysis is to predict the failure mechanism for structures. The results of the comparison between different height structures, pushover analysis and modal pushover analysis are not performed in the thesis

Although the MPA is conceptually clear and applicable, it is ultimately an approximated procedure. It would still be a huge amount of work if applied in application, especially when the structure is complicated. In addition, the assumptions and imperfect inelastic theorems mentioned above should be emphasized and solved in future work.

BIBLIOGRAPHY

Benjamin, L., Ima , M., Irma, J. H., & Hartano, W. (2007). Performance of modal pushover analysis on a first mode dominant moment resisting frame. *The 1st International Conference of European Asian Civil Engineering Forum.* Univerisitas Pelita Harapan, Indonesia.

Bertero, R. D., & Bertero, V. V. (2002). Performance-based seismic engineering: the need for a reliable conceptual comprehensive approach. *Earthquake Engineering and Structural Dynamics, 31*(Johe Wiley & Sons, Ltd.), 627-652.

Chopra, A. K. (2001). *Dynamics of Sturctures: Theory and Application to Earthquake Engineering.* New Jersey: Prentice Hall.

Chopra, A. K. (2005). *Earthquake Dynamics of Structures.* Oakland, CA: Earthquake Engineering Research Institute.

Chopra, A. K., & Geol, R. K. (1999). Capacity-demand-diagram methods based on inelastic design spectrum. *Earthquake Spectra*, 637-656.

Chopra, A. K., & Goel, R. K. (2001). *A Modal pushover analysis procedure to estimate seismic demands for buildings: theory and preliminary evaluation.* Berkeley, CA: Pacific Earthquake Engineering Research Center, University of California Berkeley.

http://en.wikipedia.org/wiki/Performance-based_building_design. (n.d.). *Performance-based Building Design.*

Krawinkler, H., & Seneviratna, G. (1998). Pros and cons of a pushover analysis of seismic performance evaluation. *Engineering Structures, 20*(Elsevier Science Ltd), 452-464.

Mao, J., Xie, L., & Zhai, C. (2006). Studies on the improvments in modal pushover analysis. *Earthquake Engineering and Engineering Vibration, 26*(China Academic Journal Electronic Publishing House).

Mwafy, A. M., & Elnashai, A. S. (2000). Static pushover versus dynamic collapse analysis of RC buildings. *Engineering Structures, 23*(Elsevier Science Ltd), 407-424.

Naeim, F., Bhatia, H., & Lobo, R. M. (n.d.). *Performance-based seismic engineering.* Los Angeles, CA: John A. Martin & Assocates, Inc.

Oguz, S. (2005). *Evaluation of pushover analysis procedures for frame structures, M.S. Thesis.* Middle East Technical University.

Seneviratna, G. (1997). *Evaluation of inelastic MDOF effects for seismic design, PhD Thesis.* Stanford, CA: Stanford University.

APPENDICES

MATLAB codes for 8-story buildings:

```
phi1=[0.077719174 0.233366532 0.409351222 0.57932963 0.729217301 0.850773451
0.940153231 1]';
phi2=[-0.25191314 -0.657303522 -0.910269543 -0.869711036 -0.531091255 -0.001137911
0.554777624 1]';
phi3=[0.51104658 1.015142466 0.752330768 -0.131800945 -0.8908266 -0.864095536
-0.042402044 1]';

L1=sum(phi1);
M1=sum(phi1.^2);
gama1=L1/M1;
s1=gama1*phi1;

L2=sum(phi2);
M2=sum(phi2.^2);
gama2=L2/M2;
s2=gama2*phi2;

L3=sum(phi3);
M3=sum(phi3.^2);
gama3=L3/M3;
s3=gama3*phi3;

s=[s1 s2 s3];

M1=sum(s1);
M2=sum(s2);
M3=sum(s3);
H1=[1 2 3 4 5 6 7 8]*s1/M1;
H2=[1 2 3 4 5 6 7 8]*s2/M2;
H3=[1 2 3 4 5 6 7 8]*s3/M3;
```

MATLAB codes for 16-story buildings:

```
phi1=[0.035677908 0.108882495 0.195665497 0.286416877 0.376895223 0.464901359
0.549030309 0.628208237 0.701524506 0.768175318 0.827451345 0.878747796
0.921598886 0.955768543 0.981465657 1]';
phi2=[-0.113673384 -0.333759519 -0.565824696 -0.762280827 -0.895912847 -0.951319639
-0.922297285 -0.810946914 -0.626961889 -0.386566196 -0.110971593 0.175626497
0.448469615 0.685194937 0.869754767 1]';
phi3=[0.200930814 0.551568238 0.835564866 0.940109811 0.825052628 0.513300101
0.0787401 -0.373857167 -0.734962872 -0.917060507 -0.875868911 -0.620885881
-0.212461866 0.254781671 0.680253455 1]';

L1=sum(phi1);
M1=sum(phi1.^2);
gama1=L1/M1;
s1=gama1*phi1;

L2=sum(phi2);
M2=sum(phi2.^2);
gama2=L2/M2;
s2=gama2*phi2;

L3=sum(phi3);
M3=sum(phi3.^2);
gama3=L3/M3;
s3=gama3*phi3;

s=[s1 s2 s3];

M1=sum(s1);
M2=sum(s2);
M3=sum(s3);
H1=[1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16]*s1/M1;
H2=[1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16]*s2/M2;
H3=[1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16]*s3/M3;
```