Analysis of Deployable Strut Roof Structures

By

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Abstract

Deployable structures are structures that can change shape from a compact to an expanded form. Thus, their advantage over conventional structures is adaptability, whether in the sense of adapting to changing environmental conditions or being adapted for repeated transportation and deployment. These features make deployable structure highly desirable for a wide range of applications in the aerospace, military, and architectural fields. However, these structures are often only designed as small scale "products", rather than structures requiring full analysis and design procedures. Much work has focused on the various geometries of the deployment mechanisms without considering practical engineering aspects. If deployable structures are to be designed on the scale of large civil structures, a proper understanding of the flow of forces through the structure is required.

This thesis begins with a brief discussion of deployable structures in general before moving on to geometric constraints of strut-type deployable structures. Then, it details a preliminary analysis of one class of deployable structures, known as angulated element structures. These structures are designed to be operable roofs spanning over sports facilities. During deployment, the center of the structure opens or closes to accommodate changes in weather conditions. Building on the geometry established in other work, the relationships between the basic geometric parameters of angulated element rings and their structural characteristics are determined. SAP2000 analysis results are used to make specific design recommendations. The feasibility of using this type of structure for an operable long span roof is confirmed.

Thesis Supervisor: Jerome J. Connor
Title: Professor of Civil and Environmental Engineering

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1 INTRODUCTION

Traditionally, structures are designed to remain static. Once built, no part of the structure should move or rotate excessively under normal circumstances. However, there are many situations where having moving components allows the structure to change shape according to changing need. For example, the structure of some flowers allows them to open during the day and close at night. Similarly, a stadium roof can remain open during fair weather, but close to keep out precipitation. Alternatively, a structure can be transported in one shape, and then deployed into use once on site. The term 'deployable structure" describes a structure that can change from an initial, compact form to a final, expanded form in which the structure can carry design loads.

The advantages of deployable structures primarily concern adaptability. First, structures can be designed to adapt to multiple service load conditions depending on what structural functions are required. Second, structures can be designed for transport and repeated deployment in multiple locations. The ease and speed of assembly means that deploying a structure is much faster and requires less skilled labor than traditional construction. However, the geometric complexity of these structures means that these advantages come at a higher cost, during both design and fabrication. Additionally, since motion is paramount to the usefulness of these structures, factors such as material fatigue and joint maintenance must be considered in the selection of a deployable design.

Deployable structures are more ubiquitous than one may first imagine. Umbrellas, camping tents, and lawn chairs are common examples of small deployable structures. Structures of this size and scale are commonly viewed as "inventions" rather than "engineering", and thus their structural properties have not been thoroughly analyzed. Recent work [1] [2] has focused on bringing deployable structures to the scale of buildings, complete with the requisite analysis and design methodology. Lightweight roof structures are increasingly popular applications.

Deployable structures are often classified as either surface structures, which consist of 2D modules, or strut structures, which consist of 1D bars [1]. Surface structures have curved surfaces that react and change shape under deployment loading such that they can take design loads in the deployed state. Strut structures have rigid bars that can rotate around pinned joints. When properly linked, articulation of many such joints causes the structure to deploy.

This thesis will describe the current types of deployable structures and their applications with examples. Then, strut-type deployable structures will be discussed in detail. The geometry constraints, deployment mechanisms, and limitations of pantograph structures and angulated element structures will be documented. Emphasis will be placed on deployable roof structures. Finally, analysis of the structural response of angulated element roof structures will be performed using structural analysis software. The geometry of these structures will be varied parametrically to try to establish patterns and determine optimal deployable shapes. These results will be used to make final design recommendations for these types of structures.

2 TYPES OF DEPLOYABLE STRUCTURES AND MECHANISMS

Deployable structures can be broadly categorized into two groups: surface structures and strut structures. Surface structures rely on flat and curved 2D elements that change shape when loaded. Strut structures are assemblies of bars that can be articulated at the joints to change shape. This section will describe further subsections of deployable structures and provide examples of each type, beginning with surface structures.

2.1 Pneumatic Structures

Pneumatic structures, also called inflatables, rely on membranes filled with pressurized air to provide structural stiffness. They can be either high pressure or low pressure systems.

In high pressure structures, chambers in structural components are inflated until the member is stiff enough to support loads. Membranes connect the inflated members to provide enclosure. Inflatable life rafts and bouncy castles are common examples. Rapidly deployed military installations also utilize inflatable "air beams" in order to quickly deploy tents and communication equipment [3]. Companies that specialize in these structures have also developed inflatable bridges and aircraft hangars [4]. The primary advantage of these structures comes from their portability and speed of deployment. Membranes are flat when not pressurized, and can be easily transported and repeatedly erected. However, these advantages diminish as the size of the structure increases since larger structures quickly become cumbersome.

Low pressure inflatables, also called air supported structures, use a pressure differential between the interior and the exterior of the structure to maintain stability. Unlike high pressure structures, inhabitants of the structure experience a slightly higher pressure than atmospheric pressure. To prevent air from rapidly escaping, access to the inside is usually through airlocks consisting of double or revolving doors. However, the structure need not be airtight so long as the pressurization system can quickly replace any air leakage that occurs. The system is commonly used to cover athletic facilities and warehouses, either seasonally or permanently. Figure 1 shows an air supported structure that houses tennis courts at MIT. Since these systems require continuous power to maintain air pressure, they are more commonly used for longer term installations.

Figure 1 - J.B. Carr Tennis Bubble at MIT

For both types of inflatable structures, the shape and material of the membrane in the deflated state strongly influence the final form. Thus, the deployment process is highly nonlinear and requires specialized analysis software to accurately model the membrane stresses during deployment.

2.2 Tape Spring Structures

Tape springs are curved surfaces rolled into a partial cylinder. Each spring can resist some bending in the direction perpendicular to its curvature, giving it structural stability. If the spring is bent perpendicularly to its direction of curvature, it tends to snap back in place. The most common example of a tape spring is its namesake, the measuring tape. In the retracted position, the tape remains flat and coiled. However, when extended, the curvature of the tape allows it to resist bending under its own self weight.

This concept is usually used on deployable components rather than entire deployable structures. It has been used to as a low friction hinge in space structures, as well as a mechanism to deploy structural elements on spacecraft [5]. Since the deployment mechanism relies on nonlinear material deformation, tape springs are currently limited to small scale applications.

2.3 Folding Structures

Folding structures are structures that can expand and contract like an accordion. The structure may be made of a flexible material, or be made of rigid panels attached with hinges at the folds [6]. A simple example is the pop-out panels on campervans and RVs that add to the amount of interior space. Other structures draw their inspiration from Japanese origami. Large solar panel on satellites can be designed to unfold from small storage areas [7]. The method of deployment also varies significantly. Some must be manually deployed by hand, while others use pneumatic or cable actuators. These structures lack stiffness in the deployed state and must therefore be locked in by more durable pieces in order to carry loads.

2.4 Telescoping Structures

Telescoping structures consists of a series of tubular elements that can nest one inside the other. To deploy, the elements slide individually along the common axis such that each element extends out farther than the previous one. Applications include extendable legs on furniture, expandable ladders, telescoping antenna masts, and construction shoring. The structure can be held in place by the actuator, such as a hydraulic piston, or can be locked in place with screws or pins.

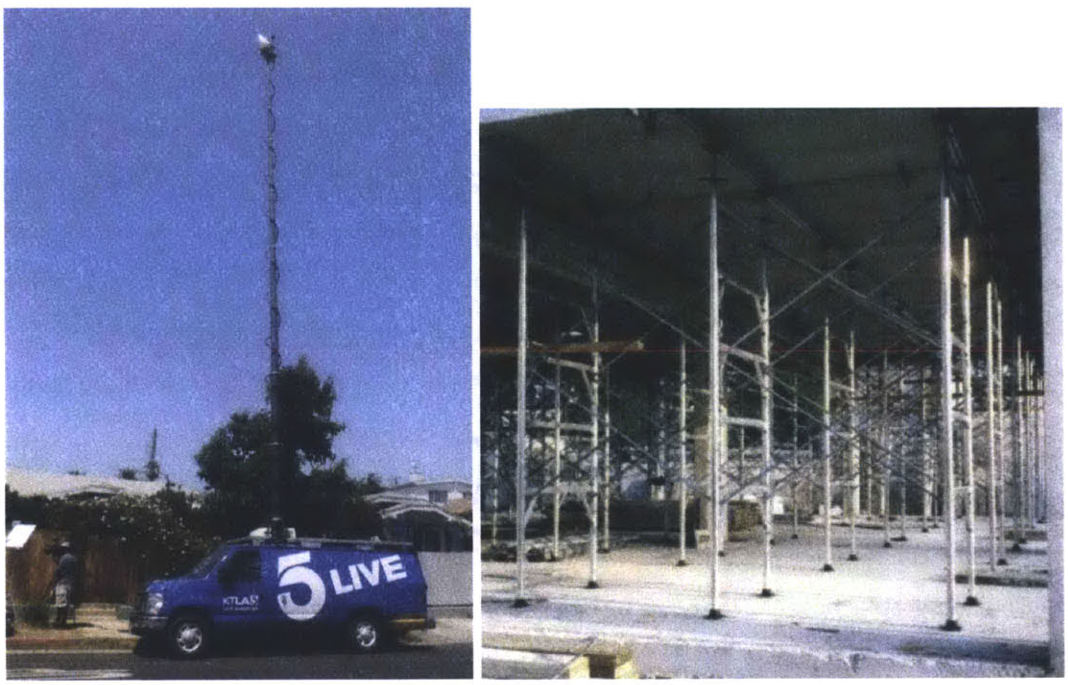

Figure 2 - Left: telescoping antenna on a news van. Right: adjustable height construction shoring

2.5 Pantograph Structures

The pantograph, also called a scissor-like element, consists of two rods that are pinned near their centers while the ends remain free. By rotating the rods about the pin, the structure can be elongated or flattened in the plane of the rods to change shape, as shown in Figure 3. This concept is utilized in many common devices such as scissor lifts and lazy tongs. Connecting multiple pantograph elements together in two dimensions can create flat slabs, beams, or more complex shapes such as arches and domes. In the collapsed state, all members are theoretically collinear, though finite joint and element sizes dictate actual dimensions of the folded structured. Additionally, joint sizes can prevent the structure from fully opening, and therefore their impact on the design must be considered.

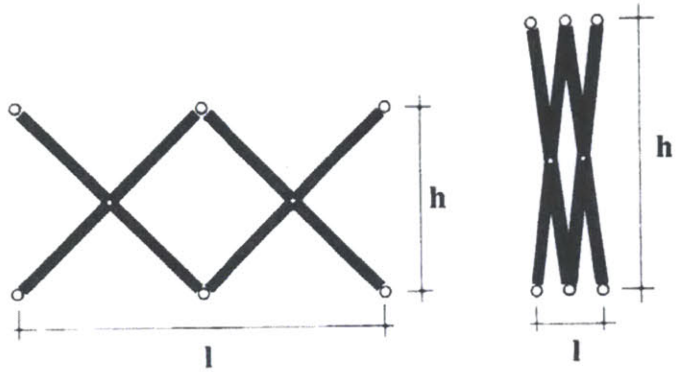

Figure 3 - Basic pantograph mechanism. Borrowed from [6].

The main advantage of these structures is rapid deployability with minimum labor. Since the final shape of the structure is predetermined by the layout and orientation of the struts, merely pulling or pushing in a few locations automatically erects the structure. Some structures have cables built in to enable easy articulation. However, the pantograph on its own is a mechanism, not a stable structure, so care must be taken to ensure that the structure is stable in its final form. This can consist of adding members to lock it in place, attaching the structure to permanent, rigid foundations, or utilizing snap-through behavior. These mechanisms will be discussed further in Section 4.

Pantograph deployable structures are commonly used in tents and temporary shade devices, where portability is a key design factor. Figure 4 depicts a portable shade structure. Sliding doors and security grates also use pantographs because they can be easily adapted to fit any size doorframe.

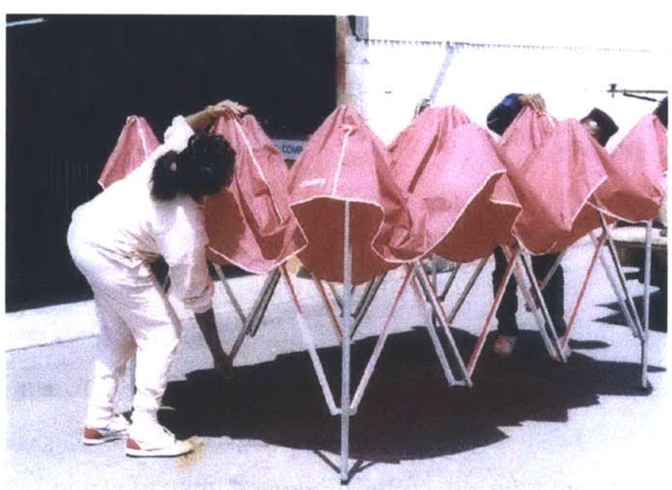

Figure 4 - Pantograph canopy being deployed (eideindusties.com)

3 APPLICATIONS

Deployable structures are well suited to a wide variety of applications, on both small and large scales. Different applications tend to be better suited to particular types of deployable structures, several of which are briefly described in this section.

3.1 Military

To occupy an area, a military must be able to quickly move and set up posts and barracks [3]. The faster an army's facilities can be redistributed, the more effective it can be at coordinating troop movements and organizing support. This requires being able to deploy temporary structures quickly and repeatedly. These structures must be durable and lightweight as well, making deployable tents an ideal solution. Both traditional pantograph tents and inflatable tents are commonly used [3]. Additionally, telescoping structures can be rapidly used to deploy telecommunications masts.

3.2 Disaster Relief

After a major natural disaster, emergency crews must quickly respond with additional critical support facilities to manage an area during a crisis. This may include added capacity for medical treatment, temporary command posts for first responders, or temporary shelter in the event of infrastructure failure. Figure 5 shows a large, rapidly deployable tent developed by Hoberman for this purpose, capable of providing up to 700 sq. ft. for emergency operations.

Figure 5 - Rapidly deployable tent for disaster relief by Johnson Outdoors (hoberman.com)

3.3 Temporary Structures

Many structures, including music stages, exhibition tents and, and construction trailers need to be relocated quickly and frequently, often in the timespan of just a few days or even hours. Deployable structures allow quick assembly and disassembly of these facilities and make them easier to transport from site to site. Construction scaffolding, which must be moved frequently to keep up with construction on a large project, can also employ temporary deployable support systems.

3.4 Aerospace

Since it is very difficult to construct aerospace structures at their target site, nearly all aerospace structures contain some degree of deployability. This may be expanding solar panels on a satellite, deployable radar dishes, or telescoping antennae. The need for structures that can be folded or compacted is compounded by the fact that there is very little room on board a spacecraft in which to store large components. Therefore, the best designed space structures are ones that can be packed tightly during transport and subsequently deployed.

3.5 Roof Structures

Light weight deployable roofs have a number of applications, usually as weather cover. For example, sports facilities are frequently designed with retractable roofs so that they can remain open during favorable weather and closed during poor weather. Both outdoor and indoor sports can take place at the same facility. Quickly constructed roofs can also be used to seasonally cover athletic fields or pools so that they can remain in use year round [8]. A wide variety of deployment mechanisms have been used including inflatable structures, pantograph structures, and folding structures. The most common type of retractable roof, such as the one on the Arizona Cardinals Stadium, uses panels that slide over one another on a rail system [9]. Other schemes, like the one shown in Figure 6, use a system of cables to fold a flexible membrane roof. Sections 5, 6, and 7 address the idea of using angulated element structures as operable long-span roofs on stadiums or other large facilities.

Figure 6 - Deployable roof on Montreal's Olympic Stadium, designed to be retracted with cables

4 PANTOGRAPH UNIT STRUCTURES

This section will explore pantograph unit structures, the first type of strut type structures, in detail. Geometry of the pantograph unit will be used to show how multiple units can be tessellated to make complex shapes such as domes and arches. The units can be designed to exhibit nonlinear snap-through behavior such that they are stable in both the compact and deployed configurations.

4.1 Pantograph Geometry

As mentioned previously, this type of structure derives its mobility from the scissor-like element found in the pantograph. When these deployable elements are carefully combined to form long chains and units of pantographs, the entire structure remains deployable. Thus the key to the overall geometry of the structure is proper design of the constituent deployable modules.

When the rods in the pantograph are pinned at their centers, the overall shape remains flat. Loads applied at the joints are transferred to the joints through bending of the rods. However, if the rods are connected off center, the pantograph begins to curve, creating an arch (Figure 7). In this way, more complex shapes can be formed that carry forces in both axial and bending action.

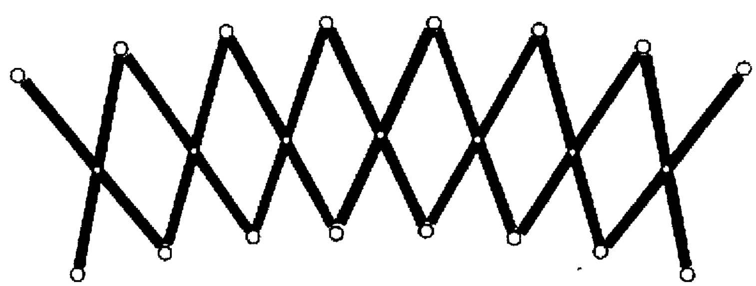

Figure 7 - Arched pantograph created by off-center hinge points. Borrowed from [6].

4.2 Deployable Units

Connecting multiple pantographs can form a deployable unit. Each unit is self-contained, but can be joined with adjacent units to form deployable surfaces. To ensure that there are no gaps in the surface, each deployable unit has a perimeter plan shape that can be tessellated (i.e. a hexagon, a square or a triangle). Pantographs also connect the diagonals of the unit at the center. The three basic units are shown in Figure 8. The angle beta differentiates each unit type and specifies the angles between the unit diagonals.

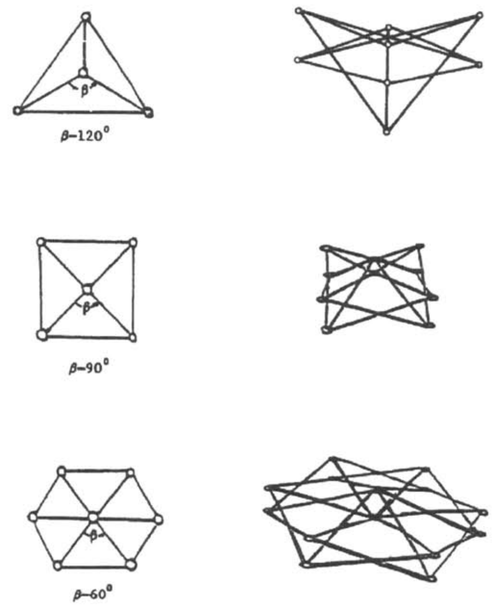

Figure 8 - Deployable units made of pantographs. Borrowed from [1].

To form a flat slab, any of these units can be tessellated in a plane, resulting in a slab with a top and bottom layer connected by vertical pantographs. The distance between the two layers decreases as the structure is expanded in the plane. If the pantographs in each unit are not vertical, but instead are angled toward a common point as shown in Figure 9, the unit begins to curve. Connecting multiple curved units results in an arch or dome shape. In theory, each structure can collapse down to a closed form where all members are collinear, though finite joint and member sizes prevent this from actually occurring.

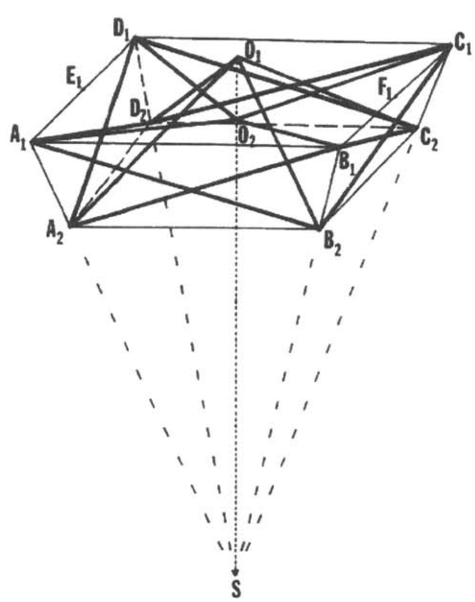

Figure 9 - Deployable unit for curved surface. Borrowed from [1].

4.3 Locking Mechanisms

Since the scissor-like element is a mechanism, care must be taken to ensure that pantograph structures are stable once deployed. Various methods for securing the structures have been implemented. Spanish architect Pinero was the first to create assemblages of pantograph elements [1]. His reticular theatre, made of rigid bars, could be deployed by pulling on a system of cables. Once deployed, the cables were fixed to maintain stability of the structure. Later, Theodore Zeigler expanded on the principle to create expandable domes. He recognized that the members in a geodesic dome could be replaced by scissor-like elements, creating a series of expandable modules. Structural stability was achieved by keeping some members in the dome stressed in the deployed configuration. Escrig built full scale models of pantograph roof structures, and was one of the first to analyze these structures in the deployed condition using finite elements [10]. His structures achieved stability by adding locking members in the deployed state [8].

However, these solutions are not ideal because they require additional time and labor to install locking members. An elegant solution, presented by Gantes [1], is to design the deployable units such that they exhibit snap-through behavior, snapping between open and closed (stress-free) states. The mechanism for the snap-through comes from a geometric incompatibility between the exterior and interior (diagonal) scissor-like elements in a deployable unit. The geometry of the snap-through deployable unit in the expanded configuration is shown in Figure 10.

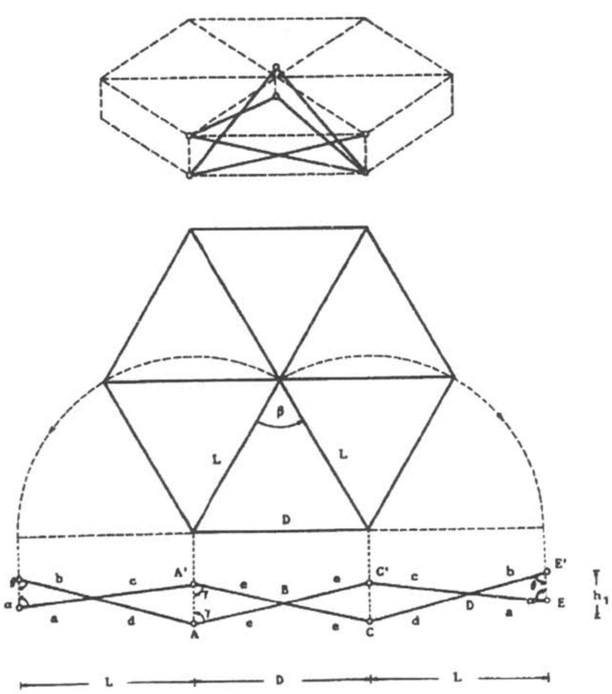

Figure 10 - Snap-through deployable unit geometry. Borrowed from [1].

Here, the pivot in the diagonal pantograph is located off-center, making lengths a and b unequal. Increasing the difference between the two lengths increases the structures resistance to deployment and makes the snap-through effect stronger. If a = b, then the structure will not display snap-through effects and will behave as a mechanism. During deployment, the diagonal scissor-like elements are put under compression while the perimeter pantographs are in tension [11]. The nonlinear response of the structure during deployment is shown below in Figure 11, which was obtained through finite element modeling in ADINA [1]. The structure is stable where the slope of the graph is positive.

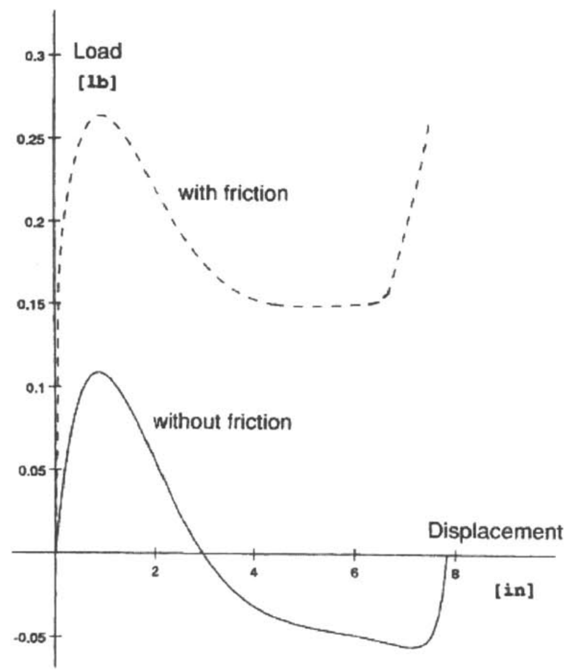

Figure 11 - Force vs. deformation curve for snap-through pantograph structure. Borrowed from [1].

The graph also shows that joint friction plays a large role in the structural response of deployable unit. In this case, the friction is large enough to prevent a load reversal during displacement. It is anticipated that joint friction will be a significant problem for other types of strut structures as well.

5 ANGULATED ELEMENT STRUCTURES

Angulated element structures are a special case of pantograph structures in which the rods used in each pantograph are kinked rather than straight. This kink allows radial expansion to occur in the plane of the pantographs such that the element subtends a constant angle. When several of these angulated elements are connected in series, they maintain constant curvature as they open and close, and can thus create expandable circles and spheres. The element was discovered by Hoberman in 1989 [12], and employed in his Hoberman sphere (Figure 12). Much of Hoberman's kinetic architecture and sculpture relies on this radially expanding mechanism.

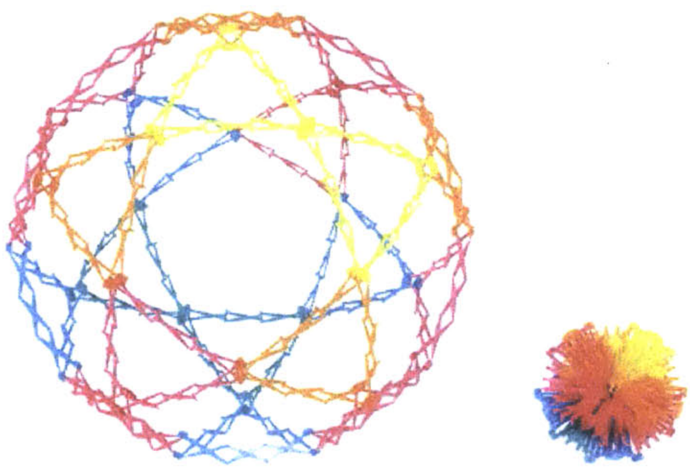

Figure 12 - Hoberman sphere in expanded and contracted form (hoberman.com)

5.1 Angulated Element Geometry

If the hinge in a pantograph is offset from the center of the rods, the pantograph structure takes on a curved shape. However, the curvature is not constant during deployment. Therefore, if the ends of the curved pantograph are connected to form a full circle, it will no longer be deployable. To prove this, consider the single scissor-like element shown in Figure 13, made of two identical straight rods. To expand radially, the element must subtend a constant angle (α), and expand only along the lines OP and OR.

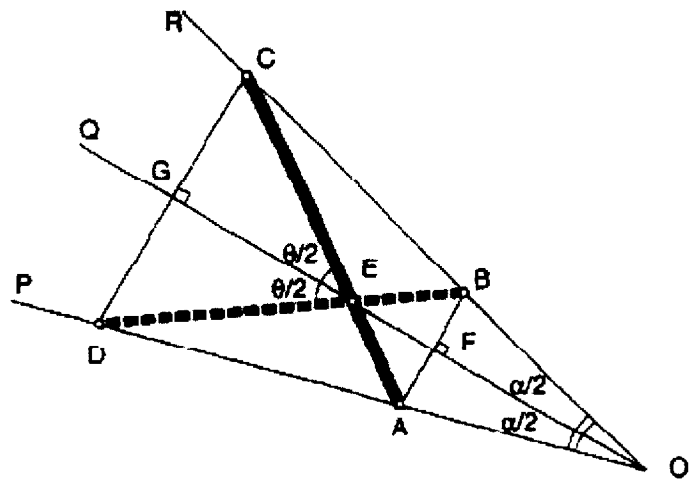

Figure 13 - Off center pantograph made of straight, identical rods. Borrowed from [13].

Note that CG and BF are proportional, and the difference between the two is:

$$CG - BF = FG \tan(\alpha/2)$$

Expressing CG, BF and FG in terms of the deployment angle, θ:

$$CG = CE \sin(\theta/2)$$

$$BF = BE \sin(\theta/2) = AE \sin(\theta/2)$$

$$FG = AC \cos(\theta/2)$$

and substituting yields and expression for α in terms of θ [13]:

$$\tan\left(\frac{\alpha}{2}\right) = \frac{CE-AE}{AC} \tan\left(\frac{\theta}{2}\right)$$

Thus, alpha varies with the deployment angle and cannot remain constant during folding [13]. This means that a ring structure of straight pantographs cannot be deployed. To enable movement, a kink the pantograph rods is introduced such that the center pivot is moved to a new point, as shown in Figure 14. The new equation relating alpha and theta is [13]:

$$\tan\left(\frac{\alpha}{2}\right) = \frac{CE-AE}{AC} \tan\left(\frac{\theta}{2}\right) + 2\frac{EF}{AC}$$

Therefore, if CE = AE, alpha is independent of theta and the structure can deploy as a mechanism. It follows [13] [14] that the angle of kink (also called the angle of embrace) in the angulated rod is 180° – α.

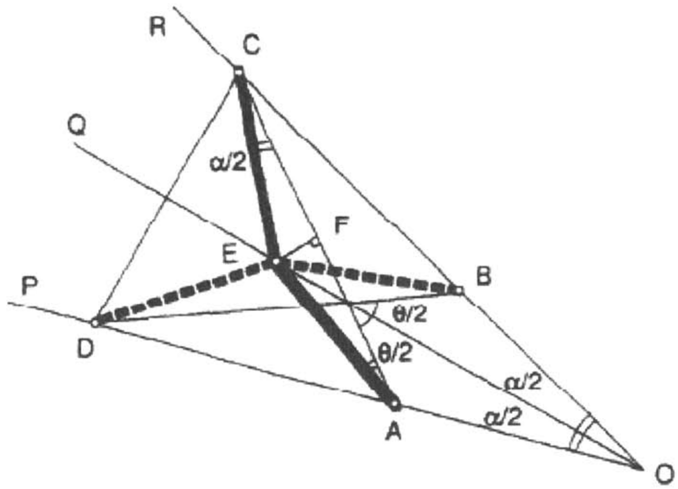

Figure 14 - Kinked pantograph, or angulated element. Borrowed from [13].

You and Pelligrino generalize this proof to angulated elements where the rods need not be identical. They show [13] that angulated elements of a general type can be created as long as the triangles formed by ADE and BCE are either isosceles (Type I) or similar (Type II). In either case, the key to deployability is that the two kinked rods share the same coupler curve at the center hinge (point E) [15]. Using a variety of these general angulated elements (GAEs), any arbitrary plan shape can be designed as a deployable linkage. An elliptical plan shape is shown in Figure 15.

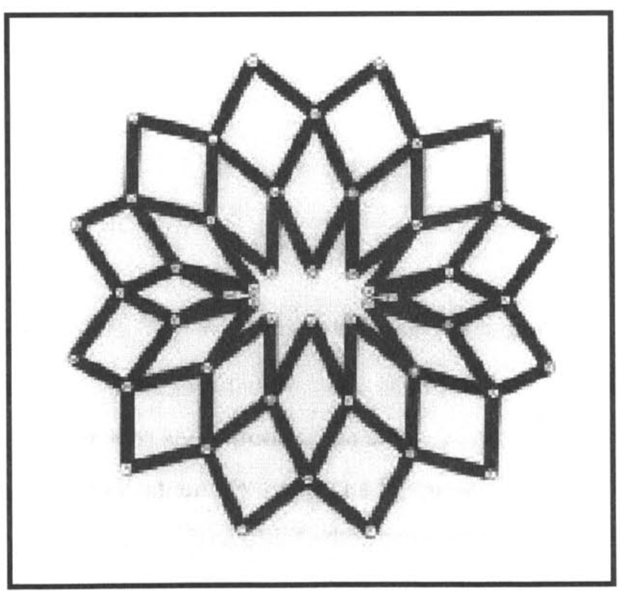

Figure 15 - Deployable mechanism with elliptical plan shape. Borrowed from [2].

For any shape, the maximum and minimum deployment states are limited by hinges meeting at the center or perimeter. Circles are useful plan shapes because all of the hinges in a circular structure will meet simultaneously, and the mechanism can close completely. In other shapes, certain hinges will intersect before others, preventing full contraction or expansion. In Figure 15, note how the hinges at the ends meet before those near the center. In addition, using a circular plan means that all angulated rods can be identical, and alpha is the same for each individual angulated element. Thus alpha can be determined by dividing 360° by the number of angulated elements in the circle (hereafter referred to as the number of divisions, n). Increasing the number of divisions causes a decrease in alpha, and creates "straighter" angulated elements.

5.2 Multi-angulated Elements

When connected, a series of angulated elements forms a ring of rhombuses. Any ring of angulated elements can be augmented by adding an additional row of kinks such that they form another layer of rhombuses, as demonstrated in Figure 16.

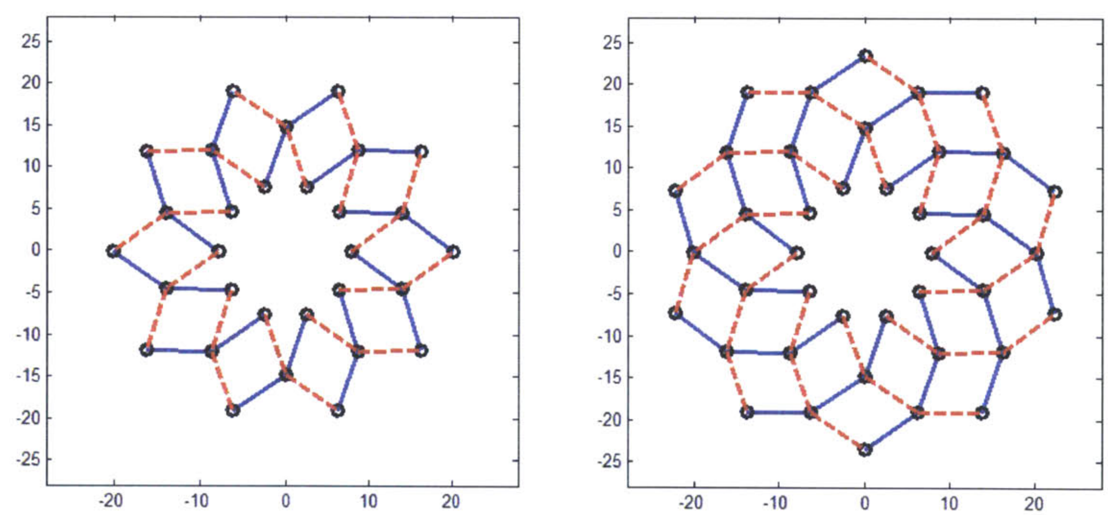

Figure 16 - Adding kinks to a ring of angulated elements (n = 10). Left: two kinks. Right: three kinks.

It can be shown [13] that the kink angle remains constant between inner and outer rings of rhombuses. In other words, the angulated elements can be made up of continuous rods with any number of kinks, called multi-angulated elements, rather than a series of angulated elements. This is significant because it greatly reduces the number of individual parts that need to be fabricated to construct an angulated element mechanism. It is helpful to visualize the mechanism as a series of overlapping rods, oriented clockwise (shown in solid blue) and counter clockwise (shown in dashed red). The number of kinks in a multi-angulated element is denoted by the variable k. Since the layers are separate and only connected at the hinge points, the elements could also be replaced by rigid plates [16].

5.3 Deployment Process of Circular Angulated Element Rings

During deployment, all hinges on the ring expand radially toward or away from the center of the ring, ending in one of two extreme positions. The closed configuration occurs when the hinges meet in the center, closest to the origin. The open configuration occurs when the exterior hinges meet at the perimeter, furthest from the center. Between these positions, the hinges travel on straight lines to expand and contract. However, the motion of a single layer of the structure can also be described by the circular motion of individual angulated elements. Kassabian showed that the hinge points on a multi-angulated element lie on a circle, and deployment can be described as the rotation about the center of this circle [2]. The radius of this sub-circle is defined as r*, as shown in Figure 17.

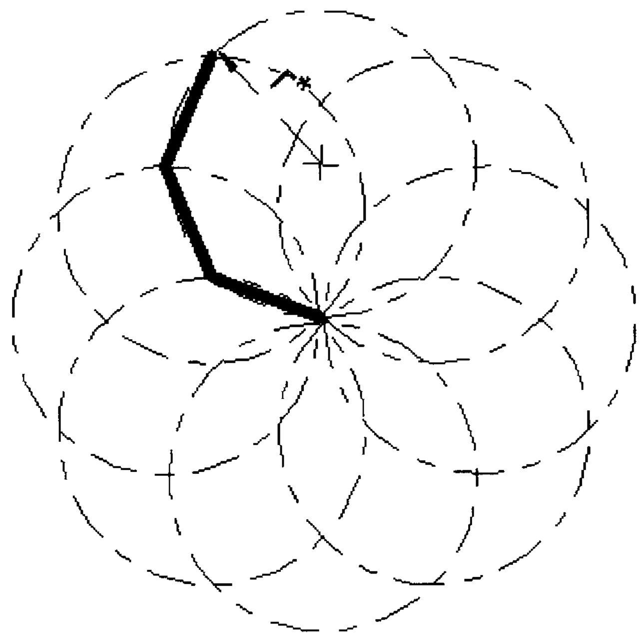

Figure 17 - Rotation circle of individual angulated element.

Using r*, the maximum and minimum radii of the structure can be derived. Uppercase R denotes the outer radius of the ring, while lowercase r denotes the radius of the central opening. The equations for the various radii are [17]:

$$R_{max} = 2r^*$$

$$R_{min} = 2r^* \sin\left(k\frac{\alpha}{2}\right)$$

$$r_{max} = 2r^* \cos\left((k-1)\frac{\alpha}{2}\right)$$

$$r_{min} = 0$$

The equations for R_{max} and r_{min} are for the theoretical limits; in reality the size of the hinges prevents the structure from fully opening to R_{max} or fully closing to r_{min}. Jensen has modified these equations to account for this discrepancy by the means of a reduction angle [17].

Note that angulated element rings are mechanisms, and are therefore not stable on their own. They must be locked in place by other means before they can properly resist loads. This could be done by locking joints at the desired stage of deployment, or by attaching the structure to rigid supports once deployed.

5.4 Angulated Element Structures as Deployable Roofs

Angulated elements structures are well suited for use as deployable roofs. Unlike pantograph structures, angulated element rings do not collapse into a small bundle. Rather, they oscillate between a closed and open ring state, each with a fairly large perimeter. This limits their usefulness as temporary, portable structures, but increases their appeal as operable coverings for permanent facilities. This could include swimming pool covers, tennis court arenas, or large scale domed stadiums. The roof would remain closed during poor weather, but could be quickly opened during favorable weather. Retractable roofs have been built successfully into many sports facilities using a variety of opening mechanisms [9]. The adaptability of these structures increases their utility, and can make the facilities more profitable for owners.

Since large changes in the size of the opening are possible without corresponding changes in the outer ring, it is convenient to support these roof structures at their perimeter, where supports will not need to move excessively during deployment. Architecturally, the structure should maximize the area that can be covered in the closed position while minimizing the area that is covered in the open position. Simultaneously, the movement of the outer radius of the structure should be small so that support movement during deployment will be minimized. In other words, the ratio of R_{min} to R_{max} should be close to one. This ratio can be expressed as:

$$\frac{R_{min}}{R_{max}} = \sin\left(k\frac{\alpha}{2}\right)$$

This shows that increasing k has the effect of decreasing the difference between R_{min} and R_{max} as desired. However, increasing k also decreases the size of the central opening:

$$r_{max} = 2r^*\cos\left((k-1)\frac{\alpha}{2}\right)$$

Therefore, a balance must be found between these two conflicting design objectives. A ring with the minimum value of k, two kinks, will have a large central opening, but the outer radius will also expand and contract a great deal. Conversely, a ring with many kinks in its elements will have a very stable outer radius, but will not be able to open very far. Rings with three to four kinks seem to be a suitable compromise. The maximum value for k corresponds with $r_{max} = 0$:

$$0 = 2r^*\cos\left((k-1)\frac{\alpha}{2}\right)$$

$$90 = (k-1)\frac{\alpha}{2}$$

$$\frac{180}{\alpha} = (k-1)$$

$$k = \frac{n}{2} + 1$$

Another useful parameter for investigating the geometry of the deployable roof is the opening ratio, OR, as defined by Jensen [17]. The opening ratio is defined as the radius of the central opening in the open position, r_{max}, divided by the outer radius in the closed position, R_{min}.

$$OR = \frac{r_{max}}{R_{min}} = \frac{\cos\left((k-1)\frac{\alpha}{2}\right)}{\sin\left(k\frac{\alpha}{2}\right)}$$

Larger opening ratios are desirable, since they correspond to larger central openings. If OR = 1, then the closed structure will fit perfectly inside the opening of the open structure. Keeping OR less than or equal to one ensures that there will be some overlap between the open and closed positions, which can be useful in designing the perimeter supports. Figure 18 shows the relationship between the opening ratio and the number of divisions, n, for different values of k. Note that increasing k quickly decreases the opening ratio.

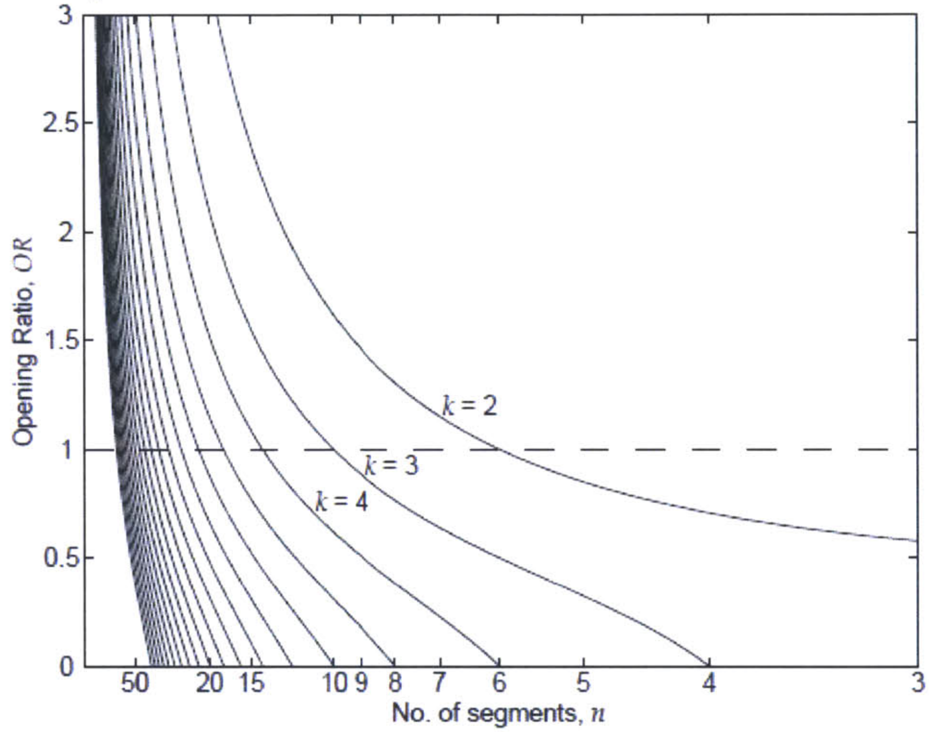

Figure 18 - Opening ratio as a function of n and k. Borrowed from [17].

This section has detailed the geometry parameters of angulated element rings. An appealing application of these shapes is in deployable roofs. With this in mind, the architectural effects of various angulated element geometries were considered. The next section will explore the structure response of these roofs under design loads.

6 ANALYSIS OF ANGULATED ELEMENT STRUCTURES

Previous work has extensively analyzed pantograph strut structures during deployment and operation [1]. However, similar analysis of other types of deployable strut structures has not been completed. This may be due to framing the field as a geometric problem rather than an engineering one. In *Motion Structures,* You notes that the book considers all members to be rigid bars, and thus their motion is not influenced by practical engineering constraints such as elastic deflections, tolerances, and fabrication techniques [14]. This is understandable, given that many deployable structures are small-scale "inventions," and conventional structural analysis techniques are not typically necessary. However, these issues must be considered to expand deployable structures to the scale of large civil structures. Engineers must understand both the strut geometry and how forces are carried through them.

In order to further investigate the analysis of deployable strut structures, an analysis study of various angulated element structures was completed for this thesis. Using the geometry constraints detailed in Section 5, a series of models was generated in MATLAB and analyzed in the structural analysis program SAP2000. For simplicity, the analysis was restricted to circular structures so that the response was symmetrical. Bending stresses and deflections are calculated for each model. In the following section, the results of this analysis are used to make recommendations on the engineering considerations of such structures.

6.1 Methodology

6.1.1 MATLAB Program

In order to parametrically control the shape of the angulated element rings, a MATLAB program was written to find the coordinates of the ring hinges in various stages of deployment. The program makes circular rings in which all elements are the same size. The program inputs are the number of divisions (n), the number of kinks in each element (k), the outer radius of the ring (R_{min}), and the distance from the origin to the closest hinge. Using these values, the program calculates the length of each segment in the element, the size of alpha, and the parameters R_{max}, r_{max}, and r* as defined in Section 5. Then, given the distance (d_1) to the first hinge, the location of the second hinge (d_2) can be found using the law of sines, as shown in Figure 19.

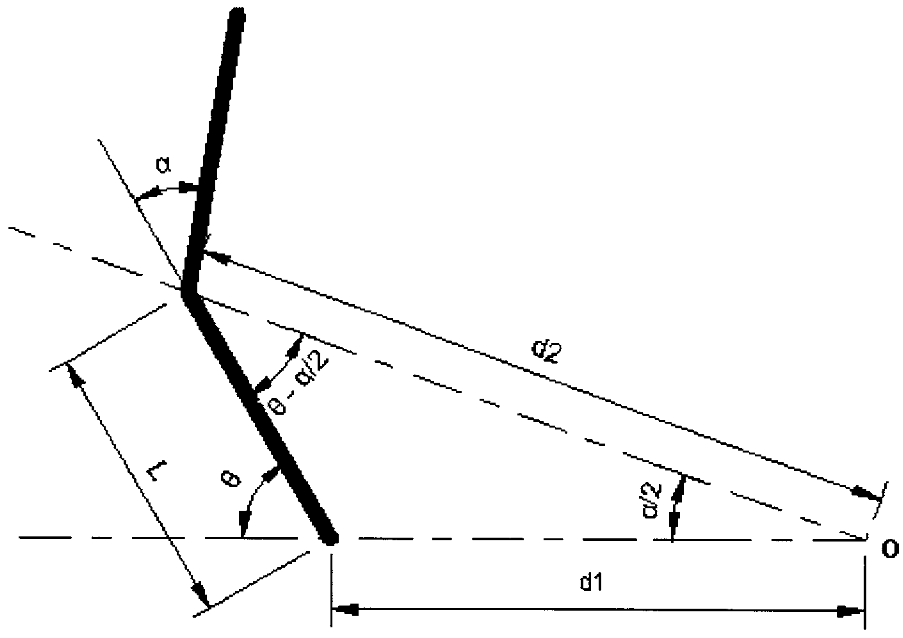

Figure 19 - Mathematical definition of hinge locations in an angulated element

$$\frac{\sin(\alpha/2)}{L} = \frac{\sin(180 - \theta)}{d_2} = \frac{\sin(\theta - \alpha/2)}{d_1}$$

From here, theta can be calculated from the given distance d_1:

$$\theta = \arcsin\left(\frac{d_1 \, sin(\alpha/2)}{L}\right)$$

The distance to the second hinge, d_2, is given by:

$$d_2 = \frac{L\sin(180 - \theta)}{\sin(\alpha/2)}$$

The program loops through for each kink to find the location of all the hinges on one angulated element and stores the coordinates in an array. The coordinates of the next angulated element hinges are found by using a rotation matrix on the first array. By looping through the number of divisions, all of the hinge locations are calculated. The MATLAB program code is included in the Appendix: MATLAB Code.

With the geometry of the structure calculated, the program returns the information in three formats. First, the plot command is used to draw the structures in a MATLAB figure. This gives an initial view of the structure before it is imported into analysis software. Second, an Excel file containing the joint coordinates is created. Third, an AutoCAD Drawing Exchange Format file (.dxf) is created with a unique filename. These other file types are used to interface with SAP2000. Code for writing to a .dfx file from MATLAB was written by Gzregorz Kwiatek, and is freely available from the Mathworks file exchange website at http://www.mathworks.com/matlabcentral/fileexchange/33884.

6.1.2 SAP2000

To create the model, the structure geometry was imported from the .dxf created by MATLAB (though unfortunately it was necessary to open and re-export the .dxf file in a third program, SketchUp). As usual, frame sections, joint supports, and joint loads were applied. Special care was taken to ensure that the hinges in the model would behave like a pantograph. The clockwise and counterclockwise elements do not lie in the same plane; instead, they are separated by a one foot gap. This means that the elements on either side of a kink can be modeled as continuous without interfering with the element members in the intersecting angulated element. To vertically tie the two elements, the joints are connected with a link member that allows free rotation about its own axis but is fixed in all other degrees of freedom. This setup is shown in Figure 20. Just like the actual hinge, this link transfers horizontal shear and moments, but allows rotation between upper and lower elements in the plane of the roof. Since all the loading will be applied perpendicular to the plane of the angulated elements, the one foot separation between elements will not affect the analysis results.

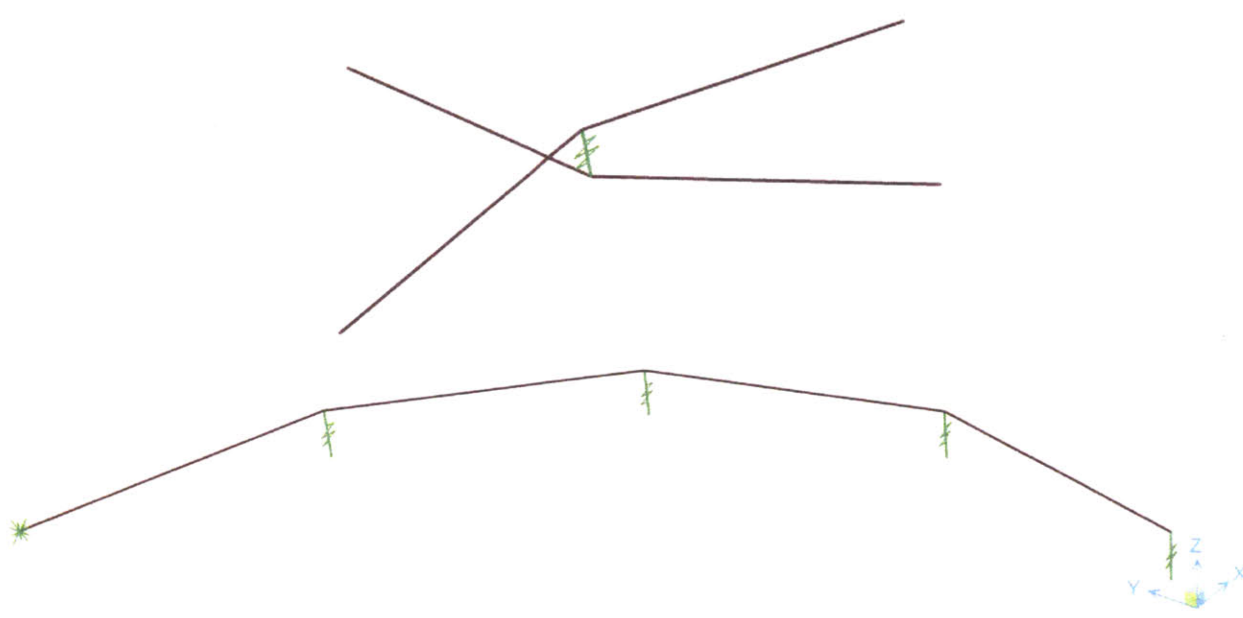

Figure 20 - Top: Single hinge link between upper and lower elements. Bottom: Hinges at each kink

To save time, the Excel file of joint coordinates was used to make some models from ones previously completed. Instead of importing a new .dxf file and reapplying the links and loads, the joint connectivity table was edited directly using SAP's Interactive Database Editing mode. By copying joint coordinates from the Excel table, the structural model could be modified quickly and easily. Of course, this method only works when the number of divisions and number of kinks does not change from one model to the next.

6.2 Loading

Since angulated element rings are best suited for use as operable roofs, the analysis models are all loaded transversely, perpendicularly to the plane of the elements. The structures must support their own weight in addition to snow, rain, and maintenance loads. These imposed loads must be transferred to the angulated elements by a covering structure. One possibility is to use a flexible fabric as a membrane that folds easily to open the roof. However, such membranes have durability issues due to fatigue loading. A more enticing option is to design cover plates that allow the roof to open and close without overlapping [18]. One cover plate would be attached to each angulated element at the hinge points and allowed to rotate with the element below. Alternatively, cover plates that slide over one another can be attached at the hinges as implemented on Hoberman's Iris Dome exhibition at the MoMA [19]. The precise geometry of cover plates has been explored in other work [16] [18], but all cover plates transfer a distributed roof load to a concentrated load at the hinge points. Therefore, the roof loading for this thesis was approximated as a point load applied to every hinge in the model. All point loads are of equal magnitude.

To determine the magnitude of the load, the total area of the roof in the closed position is multiplied by sixty pounds per square foot. This conservatively accounts for a factored live load and snow load that would be applied to a typical roof. Since the covered area of the roof is maximized when the roof is closed, this estimate will be even more conservative for when the roof is in the open position. To design a specific structure, this number should be revised to reflect local conditions and building codes.

6.3 Analysis Results

6.3.1 General

Due to the curved nature of the angulated elements, the resulting force distribution in each member is a combination of moment about both axes and torsion. These forces vary in magnitude depending on the number of kinks, the number of divisions, the radius of the ring, and the deployment state.

Support conditions at the ring perimeter strongly influence the deflected shape and forces present in the members. Two different support conditions were considered: pinned and fixed. The pin connections allow rotation about any axis without translation, while the fixed connections only allow rotation in the plane of the hinge. Practically, either of these conditions is reasonable, and the choice of one or the other would depend on the actuation method and the desired performance. Figure 21 shows the different moment distributions due to each type of support. Moment diagrams are drawn on the compression side, with negative moment in red and positive moment in blue.

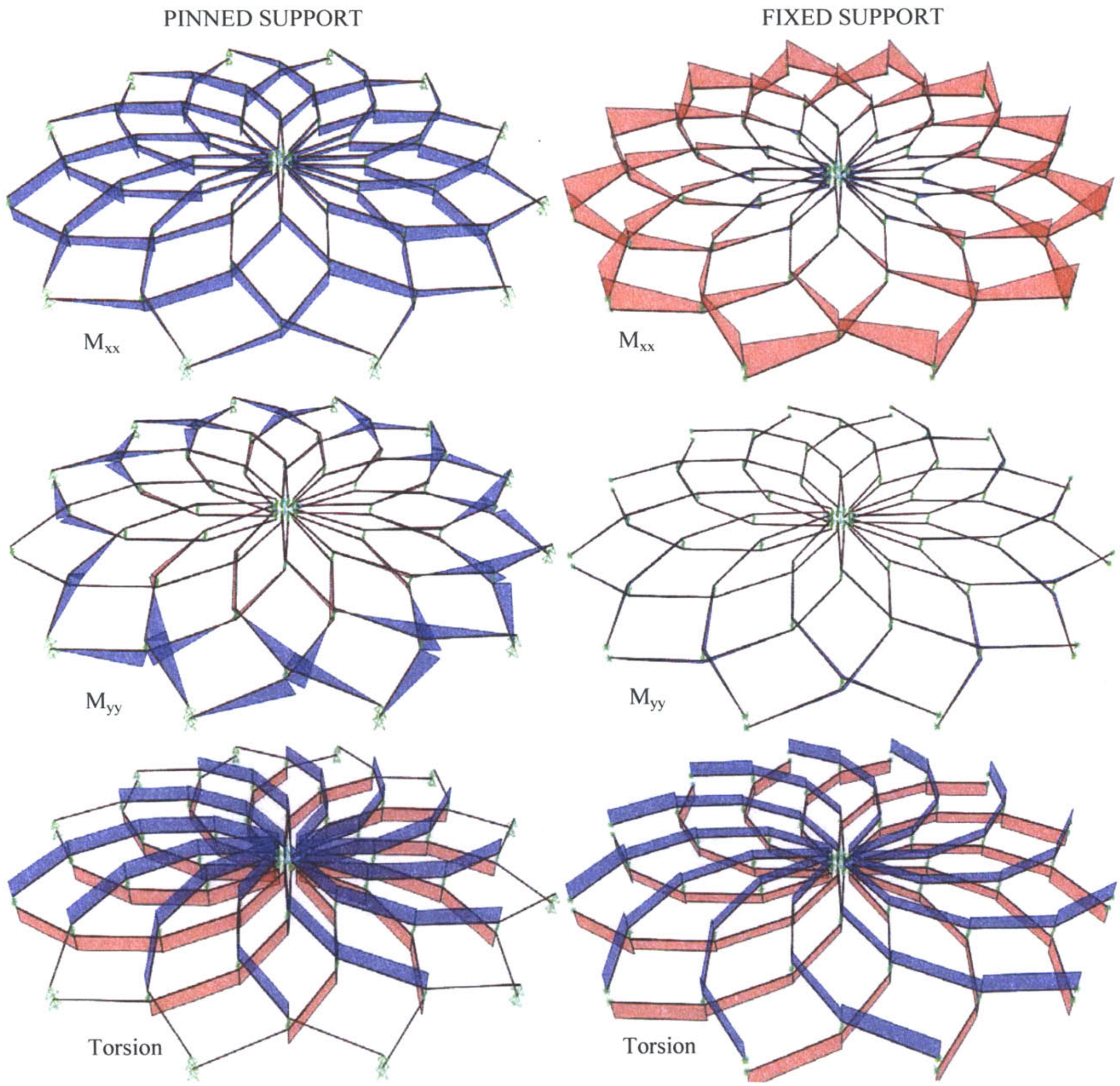

Figure 21 - Moment and torsion diagrams for each support type. n = 12, k = 4

Looking at the M_{xx} diagram, one interpretation of these results is to relate the pin-supported structure to a simply supported beam and the fixed-supported structure to a series of fixed-end cantilevers; the first is under primarily positive moment, while the second is under negative moment. The deflected shape of each angulated element (Figure 22) also reflects this conclusion. However, this is overly simplistic because analysis of other configurations shows that the moment in each angulated element can change signs. In these configurations, the structure behaves more like a reciprocal frame, in which long spans are bridged by interlocking beams atop one another. In these frames, the bending moment can change sign between intermediate beam supports. Figure 23 shows the moment variation for a deployable ring with pinned supports, twenty-four divisions and four kinks. Note that there is a negative bending moment on the middle kink in each element, while there is a positive moment elsewhere.

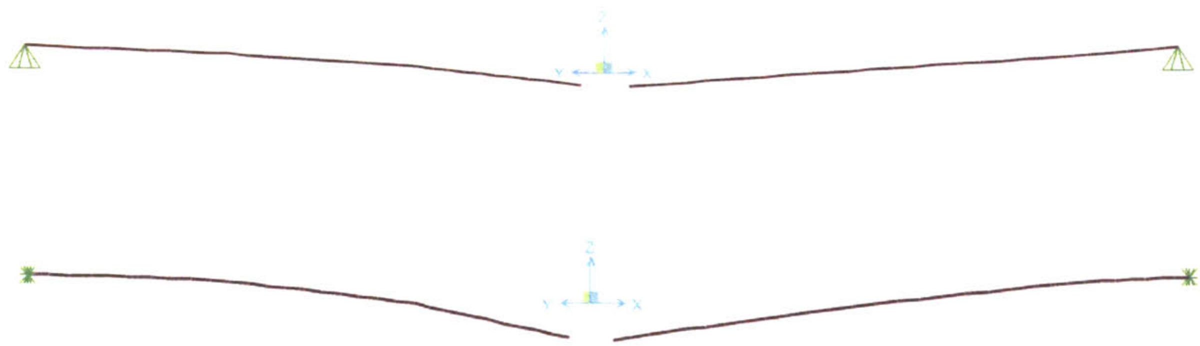

Figure 22 - Deflected shape of angulated elements. Top: pinned support. Bottom: Fixed support

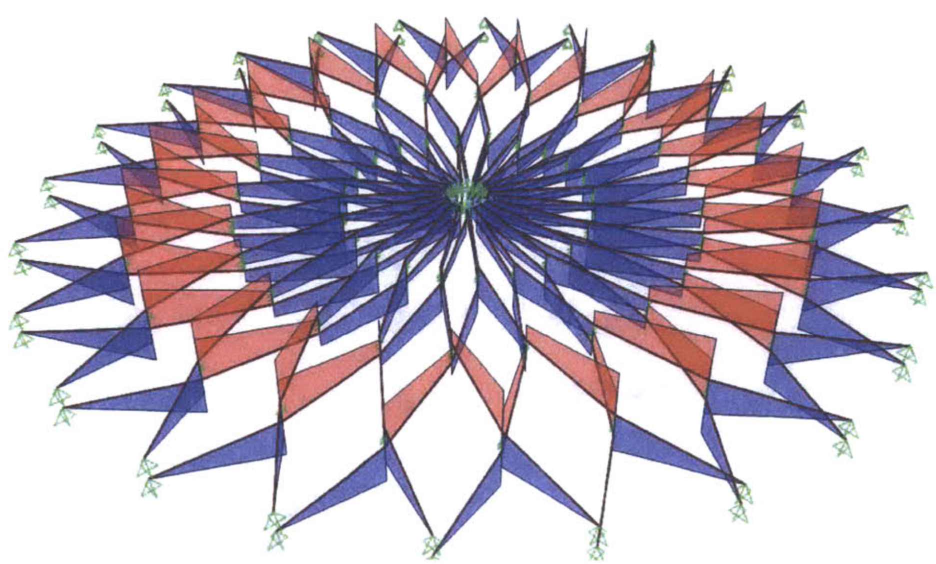

Figure 23 - Moment distribution with changing sign. n= 24, k =4

Despite the irregular moment distributions, the largest negative moment in the fixed supported rings always occurs at the support. This provides a valuable point of comparison between structures with various geometries. A similar benchmark for pinned supported rings could be the largest positive moment in each element, but this moment does not necessarily occur at the same place in every ring. Well-designed structures should minimize the deflection at the center and the moments and torsion in each element. In this analysis, three key response measures were used to compare the structures. For rings with fixed supports, the maximum negative moment was recorded. For pin supported rings, the maximum positive moment was recorded. For both support types, the maximum torsion and the maximum deflection were recorded. The maximum deflection always occurs at the center of the ring, furthest from the supports at the perimeter.

6.3.2 Response During Deployment

In order to properly design a retractable roof, it is important to understand how the structure carries loads when open, closed, and at all positions in between. Using MATLAB, a series of rings in various stages of deployment were created and analyzed in SAP2000. Each ring has three kinks and twelve divisions, and has an outer radius of twenty feet when closed, as shown in Figure 24.

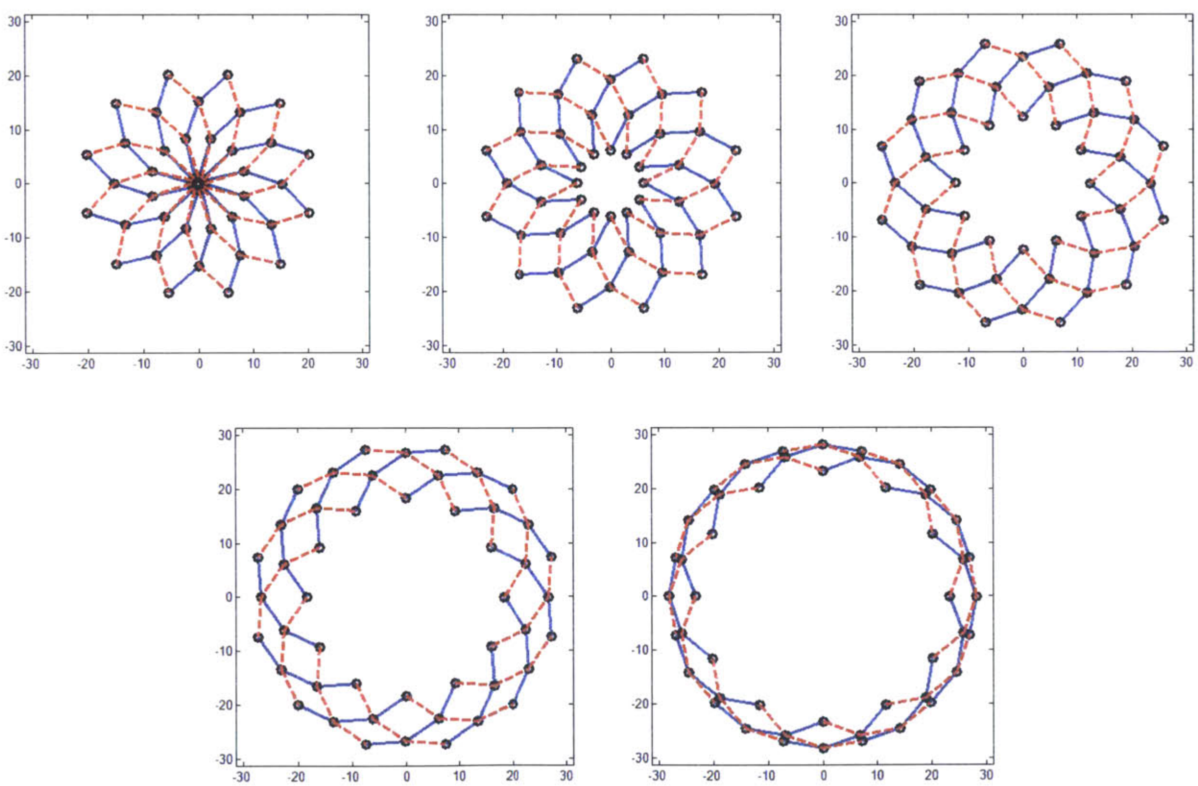

Figure 24 - Opening of ring with n =12, k =3, and Rmin = 20

In order to quantify how open each ring is, the distance to the first hinge is specified in MATLAB as a fraction of the maximum inner radius (r_{max}). To account for finite hinge sizes, the most open position is limited to 95% of the theoretical r_{max}, while the most closed position is limited to 5% of r_{max}. In between these two extremes, values of 25%, 50%, and 75% of r_{max} were chosen to as interim deployment stages to analyze. Each of these five positions is shown in Figure 24.

In SAP2000, each of these rings was analyzed with both fixed and pinned connection types. It was expected that the moment, torsion, and deflection would be maximized when the ring was in the closed position. All members in the structure are HSS20x12x3/8. Figure 25 shows the deflection at the center hinge during various deployment stages. The x-axis has increasing values of r_{max} percentage, meaning that the central opening size increases from left to right. In other words, the graph shows the deflection at the center as the ring retracts from the closed position to the open position.

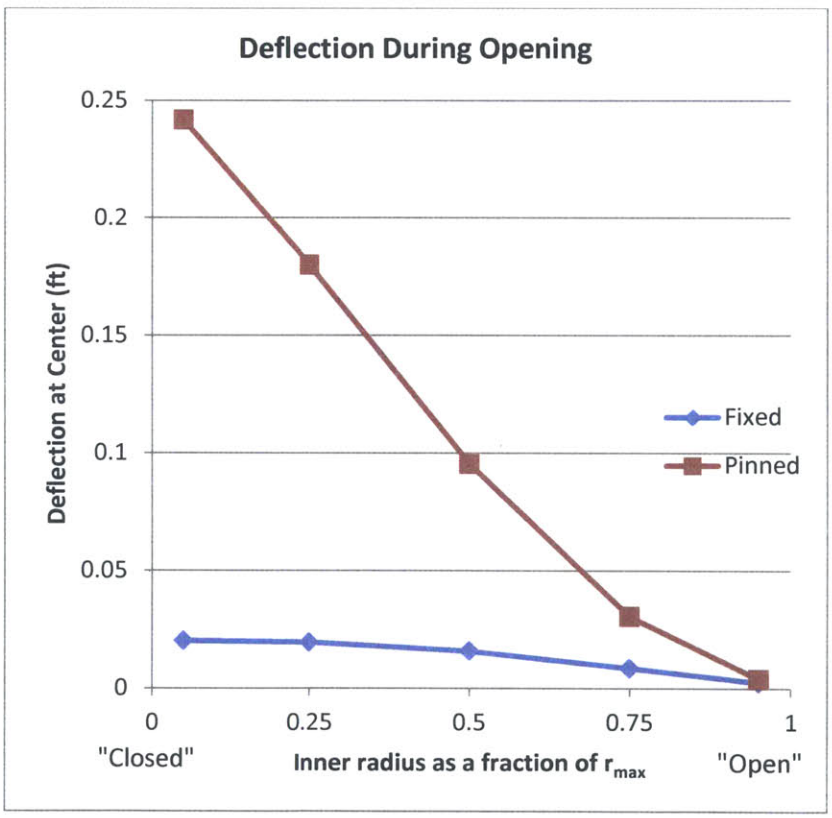

Figure 25 - Deflection at center hinges vs. stage of retraction

As expected, the deflection is maximized when the ring is in the closed position, and approaches zero as the ring fully opens. The graph also demonstrates the superior performance of the fixed supported ring over pinned supported rings, which deflects almost twelve times as much in the closed configuration.

Compared to a conventional roof with a forty foot span, the pinned supported ring would not meet deflection criteria of L/240.

Figure 26 shows the variation of the absolute value of the maximum moment during the deployment process. For the pin supported rings, the moment taken was the maximum positive moment, typically found in the middle kink of each angulated element. For the fixed support rings, the moment taken was the negative moment at the support, even if both positive moment and negative moment were present in the structure. Figure 27 shows the variation of the maximum torsion at any point in the angulated element during retraction.

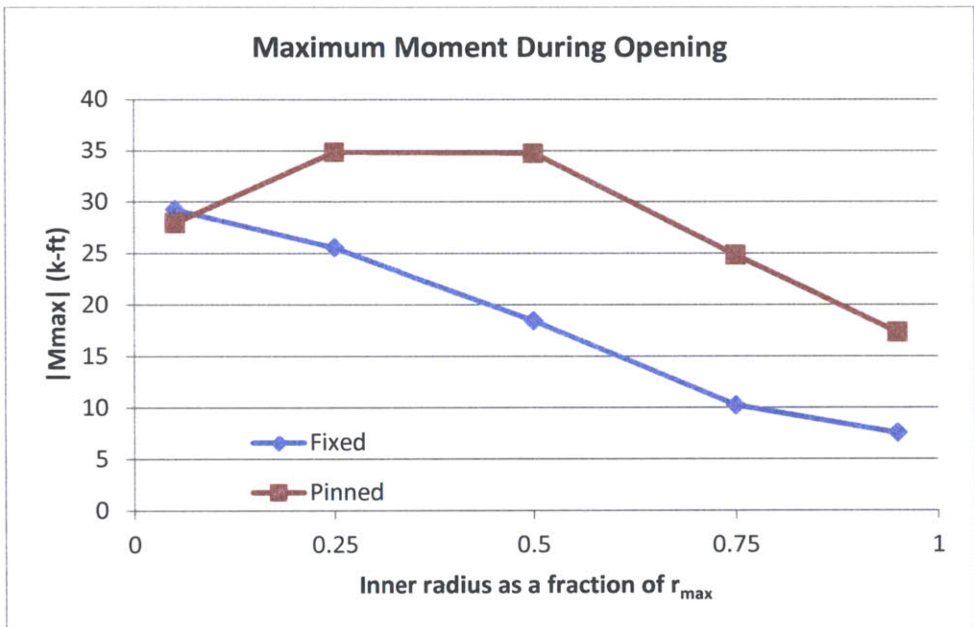

Figure 26 - Variation of Mmax during retraction

The moment in the elements generally decreases as the ring opens, as anticipated. However, the maximum moment in the pinned supported ring reaches a maximum when the ring is 25% open.

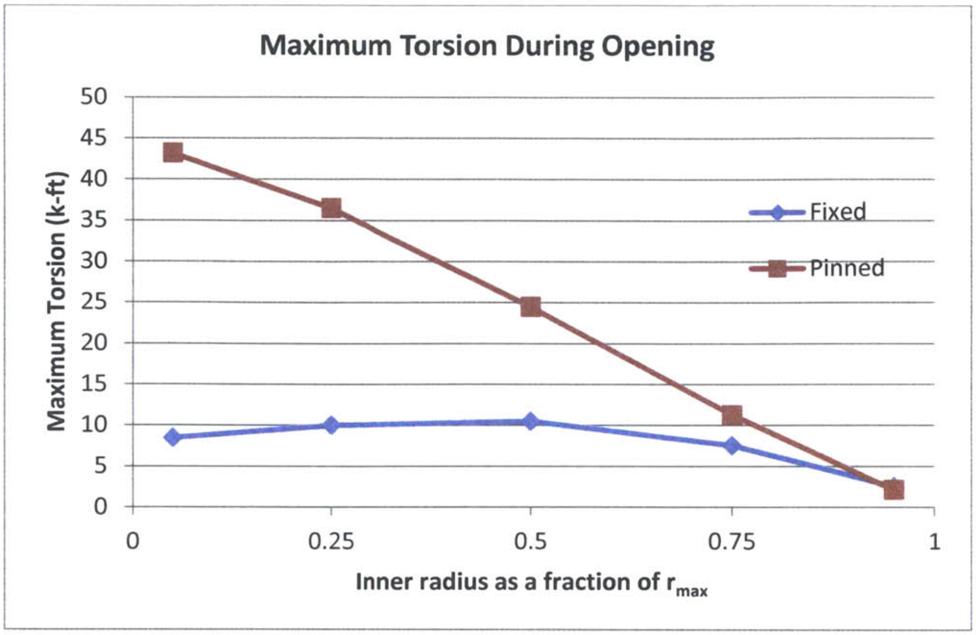

Figure 27 - Variation of maximum torsion during retraction

Similarly, the maximum torsion occurs at mid deployment for the fixed support ring. Minimum torsion for both rings occurs in the open position.

6.3.3 Number of Divisions

The next series of analyses attempted to explore the relationship between the number of divisions in a deployable ring and its structural response. Using MATLAB, a series of rings with four kinks and a closed radius of twenty feet was created. The number of divisions in each ring varied from eight to twenty-four. The shape of these rings is shown in Figure 28.

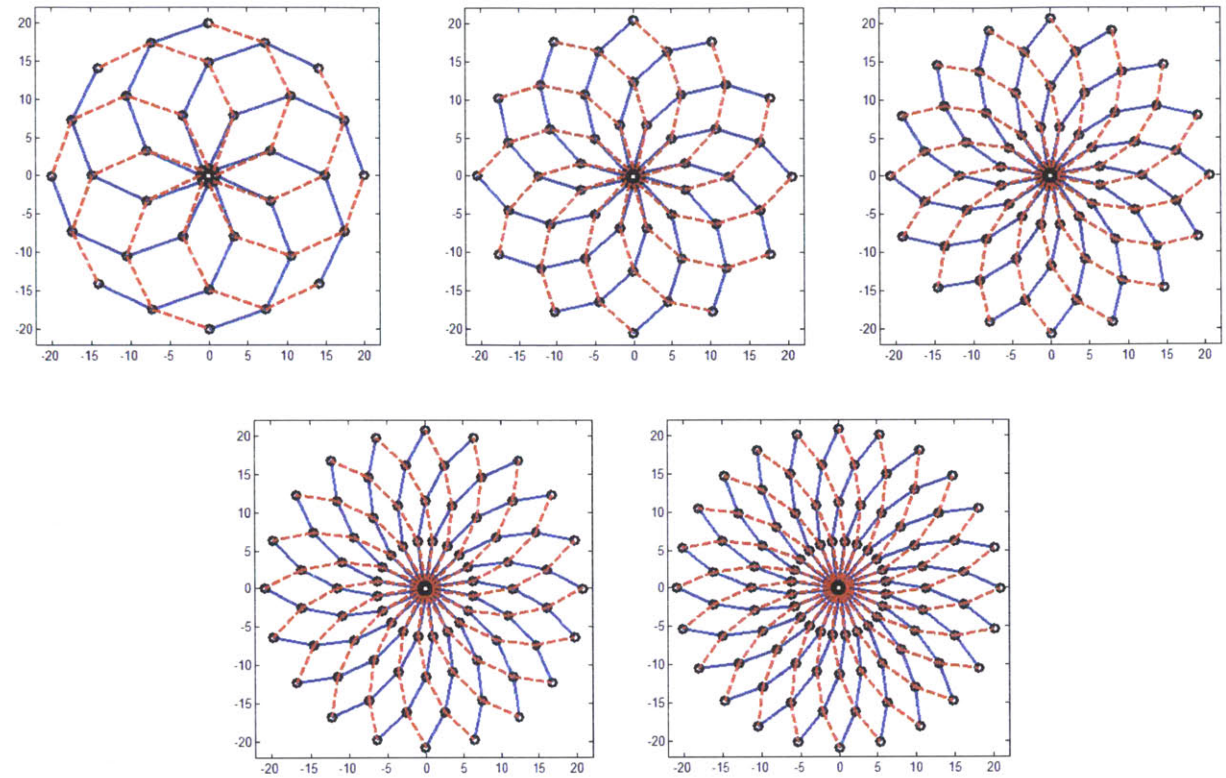

Figure 28 - Closed rings with varying number of divisions. From left to right, n = 8, 12, 16, 20, 24

A constant cross section (HSS20x12x3/8) was used for each member, which expedited the analysis process and allowed meaningful comparisons between the deflections in each model. However, since the member size does not change between models, the self-weight of the structure increases linearly with the number of elements. To make sure the effect of added self-weight did not influence the results, only the applied roof loads at the joints were included in the analysis, while self-weight of the beams was ignored. This loading on each hinge is summarized in Table 1.

Table 1 - Joint loading for each n

Closed Radius (ft)	Covered Area (sq ft)	Roof load (psf)	Total Load (kip)	Number of Kinks	Number of Divisions	Number of Hinges	Load per hinge (kip)
20	1256.6	60	75.4	4	8	40	1.885
20	1256.6	60	75.4	4	12	60	1.257
20	1256.6	60	75.4	4	16	80	0.942
20	1256.6	60	75.4	4	20	100	0.754
20	1256.6	60	75.4	4	24	120	0.628

The deflection of the center hinges of each ring is plotted in Figure 29. Again, it is apparent that fixed supported rings deflect much less than pinned supported rings. For the fixed support rings, the results are as expected: the additional elements provide additional support and stiffen the structure, decreasing the total deflection. However, this pattern does not hold for rings with pinned supports. Instead, the deflection increases with added divisions up to a maximum at sixteen. After sixteen divisions, the deflection begins to decrease.

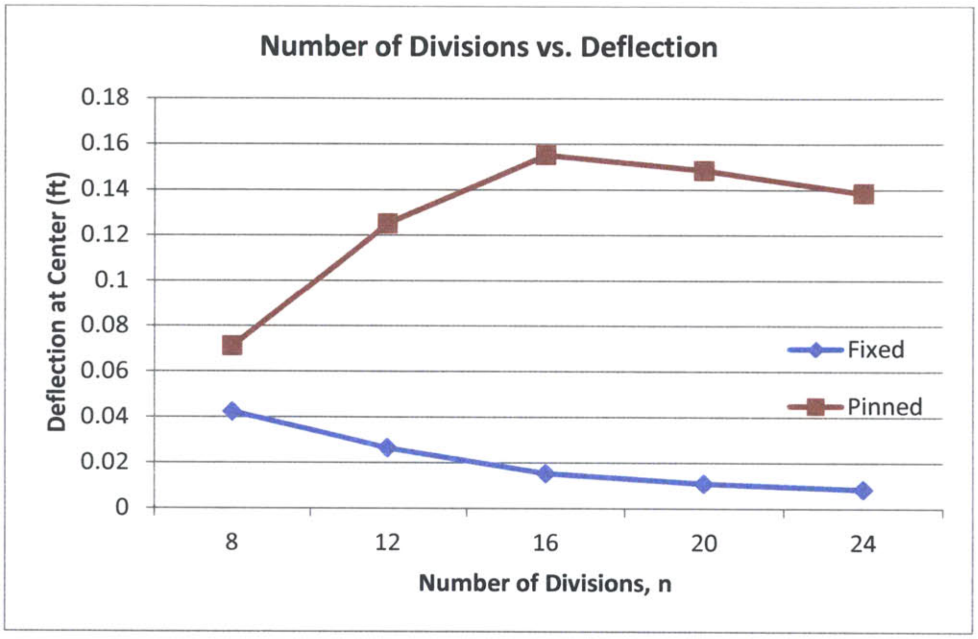

Figure 29 - Number of Divisions vs. deflection (closed position)

Figure 30 shows how the maximum moment varies with the n. Figure 31 shows how the maximum torsion varies with n. Again, each graph shows local maxima, indicating that simply adding more divisions will not always reduce the moment or torsion in the members. Only as the number of divisions increases to twenty or twenty-four will the maximum moment and torsion both decrease.

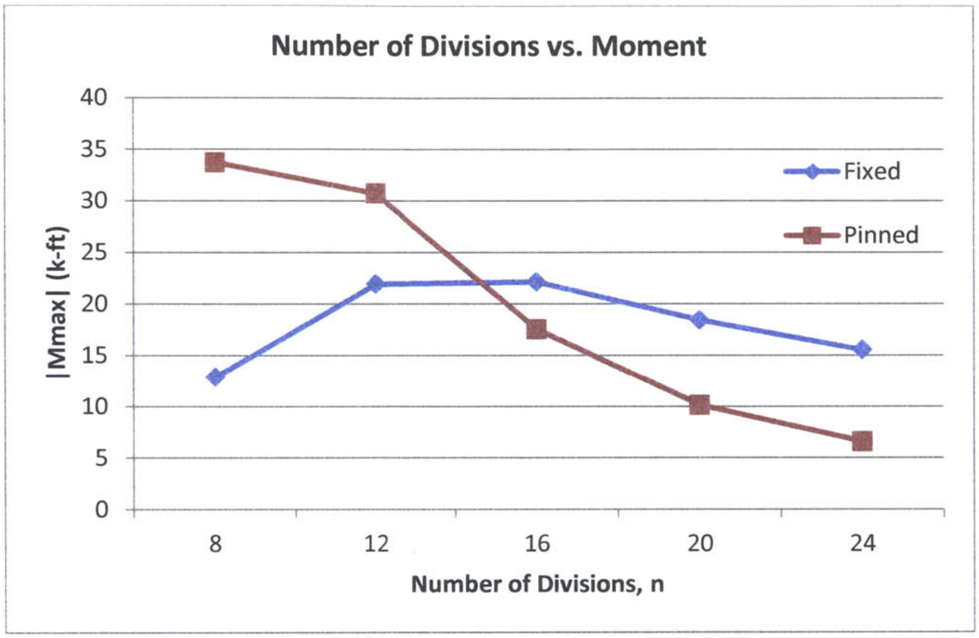

Figure 30 - Number of divisions vs. maximum moment (closed position)

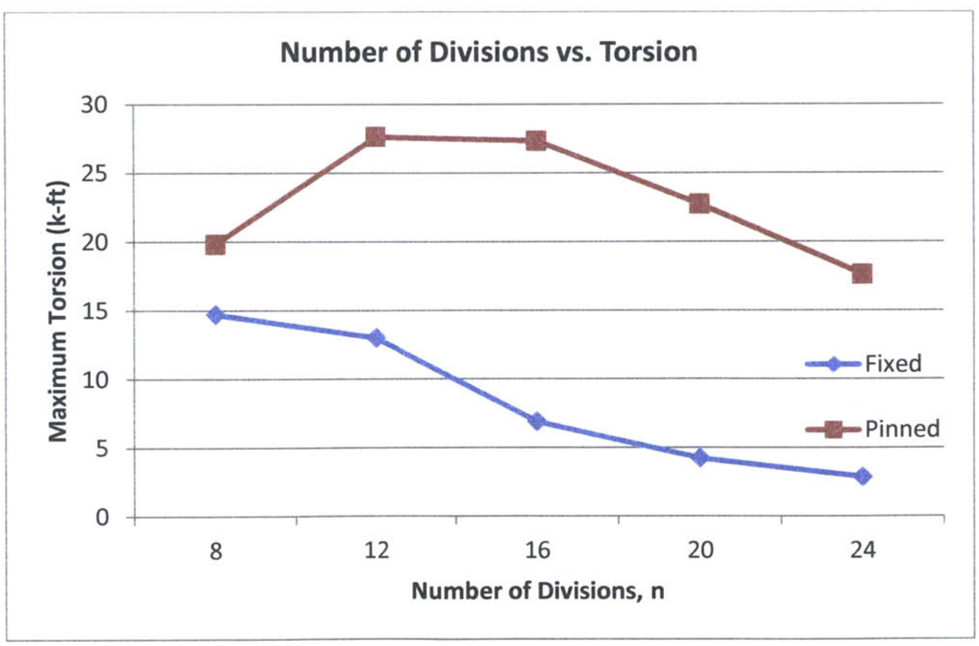

Figure 31 - Number of divisions vs. maximum torsion (closed position)

6.3.4 Roof Span

In order for a deployable roof structure to be implemented on large civil structures such as stadiums, the manner in which the structural response scales with size must be investigated. To explore this, MATLAB was used to create a series of deployable rings with increasing radii, ranging from a diameter of 20 feet to a span of 60 feet. Each ring had three kinks and twelve divisions. The loading on each hinge was calculated with a 60 psf load on the entire roof, divided by the number of hinges. Once again, the self-weight of the beams was ignored. This loading is shown in Table 2.

Table 2 - Joint loading for each ring radius

Closed Radius (ft)	Covered Area (sq ft)	Roof load (psf)	Total Load (kip)	Number of Kinks	Number of Divisions	Number of Hinges	Load per hinge (kip)
10	314.2	60	18.85	3	12	48	0.393
15	706.9	60	42.41	3	12	48	0.884
20	1256.6	60	75.40	3	12	48	1.571
25	1963.5	60	117.81	3	12	48	2.454
30	2827.4	60	169.65	3	12	48	3.534

In theory, both the moment and deflection should increase significantly with larger spans. Not only is there a father distance to span, but there is also more snow load on a larger roof. The deflection at the center hinges of the ring is plotted in Figure 32.

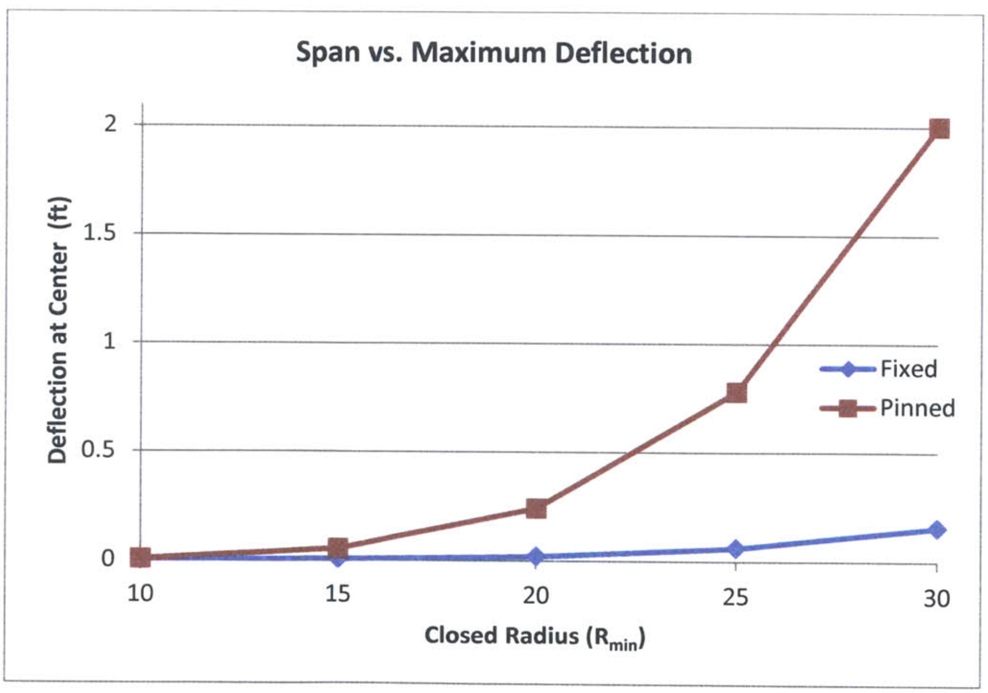

Figure 32 - Roof span vs. maximum deflection

As expected, the deflection curves have an increasing slope. This figure also demonstrates that the rings with fixed supports are about ten to twelve times as stiff as the pin supported rings, similar to Figure 25.

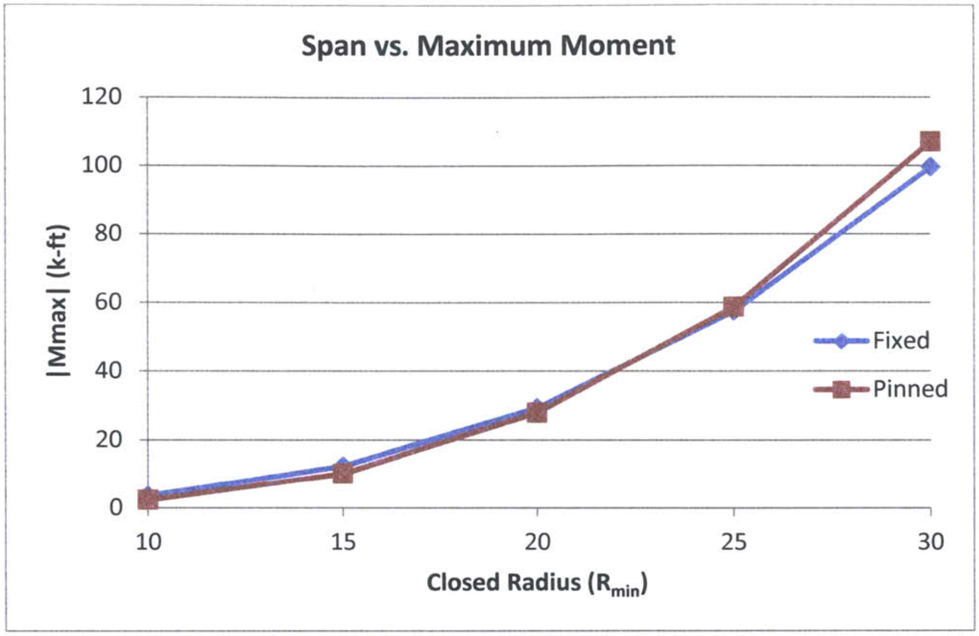

Figure 33 - Roof span vs. maximum moment

The absolute value of the maximum moment scales parabolically with the span distance of the roof (Figure 33). The choice of one support type or another makes almost no difference in the response.

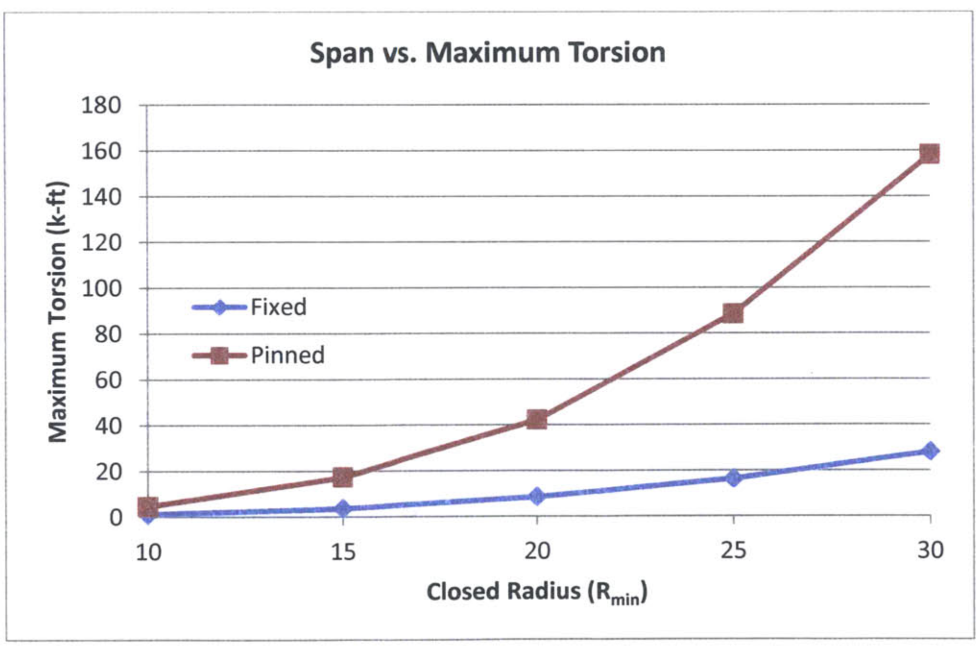

Figure 34 - Roof span vs. maximum torsion

Torsion also scales parabolically with the size of the roof span, but fixed supported rings experience up to 82% less torsion (Figure 34).

6.3.5 Number of Kinks

The last parametric study varied the number of kinks and measured the deflection, moment, and torsion as before. MATLAB was used to create rings with a diameter of twenty feet, twelve divisions, and k ranging from two to six. The shape of this series of rings is shown in Figure 35. Again, self-weight of the structure was ignored, and the only load applied corresponded to the sixty psf factored roof load (see Table 3).

Table 3 - Joint loading for each k

Closed Radius (ft)	Covered Area (sq ft)	Roof load (psf)	Total Load (kip)	Number of Kinks	Number of Divisions	Number of Hinges	Load per hinge (kip)
20	1256.6	60	75.40	2	12	36	2.094
20	1256.6	60	75.40	3	12	48	1.571
20	1256.6	60	75.40	4	12	60	1.257
20	1256.6	60	75.40	5	12	72	1.047
20	1256.6	60	75.40	6	12	84	0.898

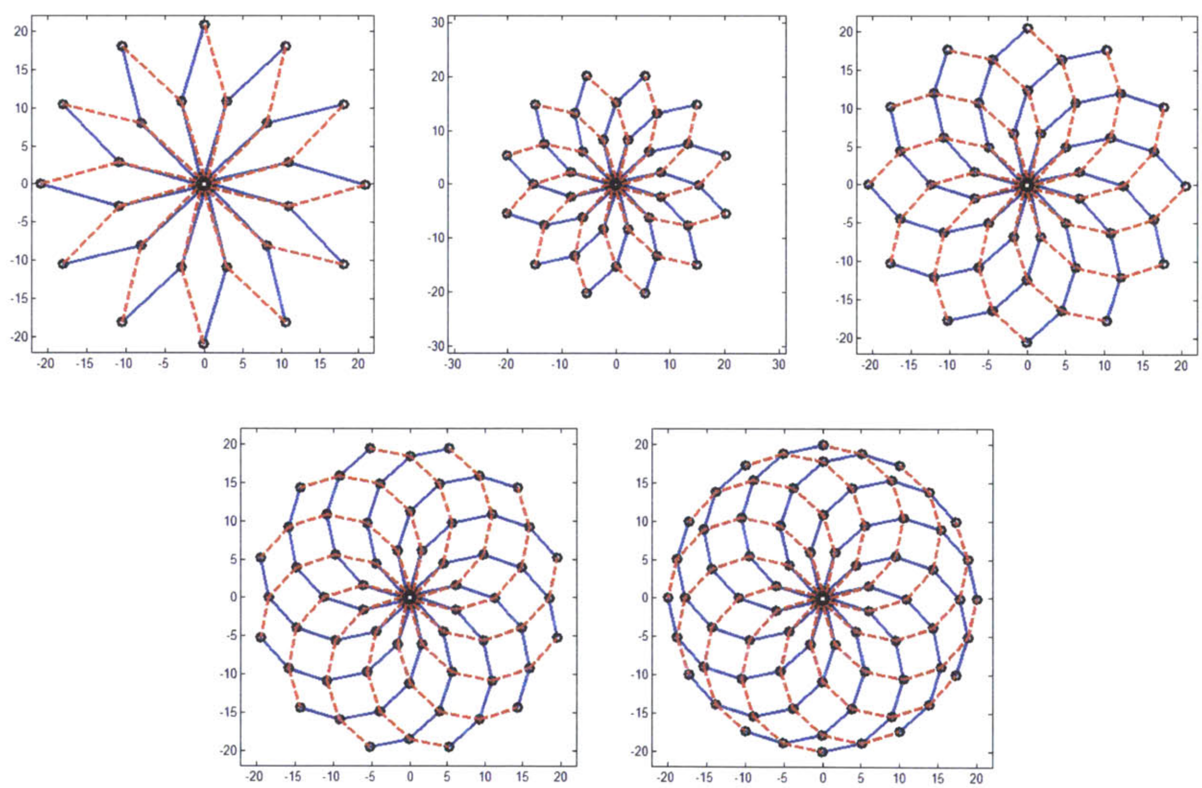

Figure 35 - Rings with varying k. From left to right, k = 2, 3, 4, 5, 6

Once again, the element self-weight was not included. It was expected that adding more kinks would stiffen the structure by adding more points where the elements intersect. Thus, the deflection should decrease as the value of k increases. Figure 36 shows the deflection results. The pinned-supported rings behave as anticipated, but the fixed supported rings did not vary significantly with k. In fact, the deflection increased slightly with higher k.

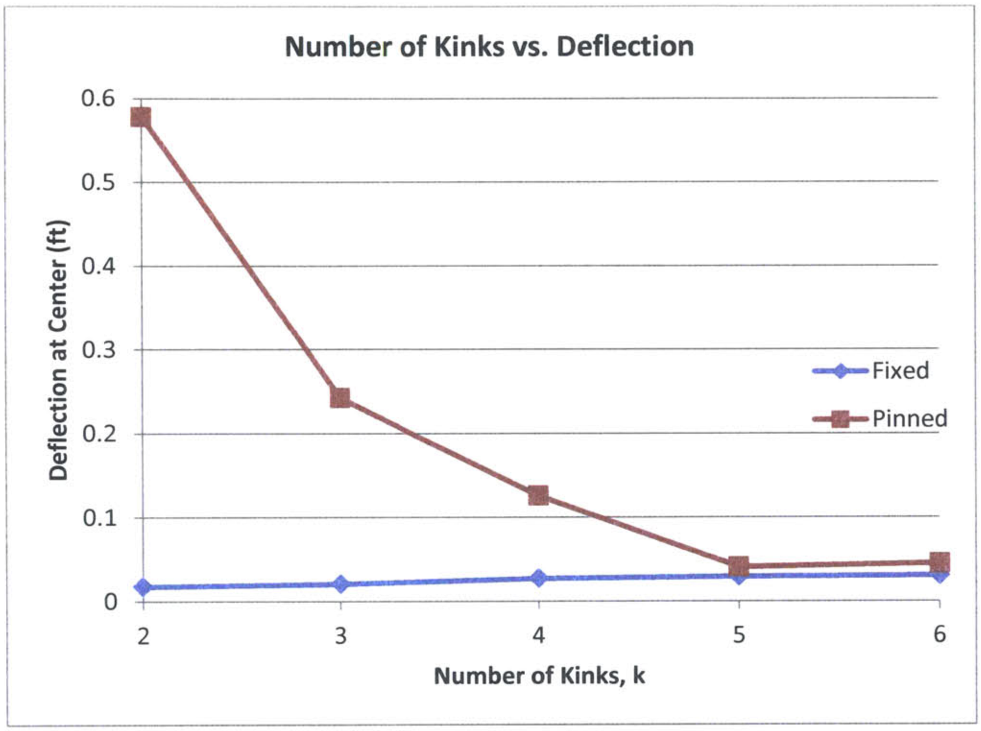

Figure 36 - Number of kinks vs. deflection

The moment in the fixed supported rings decreases as expected with the number of kinks (Figure 37).

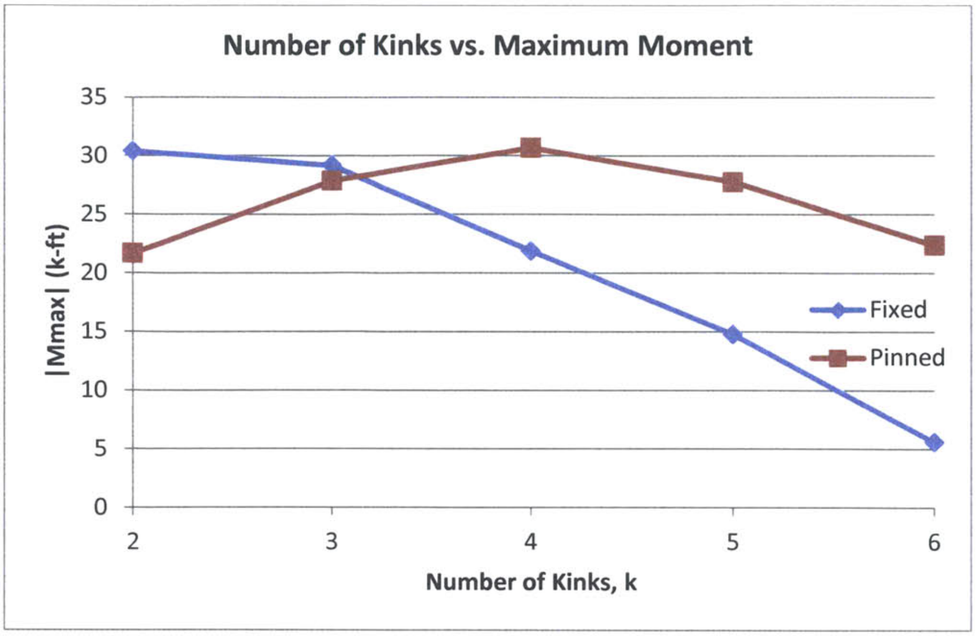

Figure 37 - Number of kinks vs. maximum moment

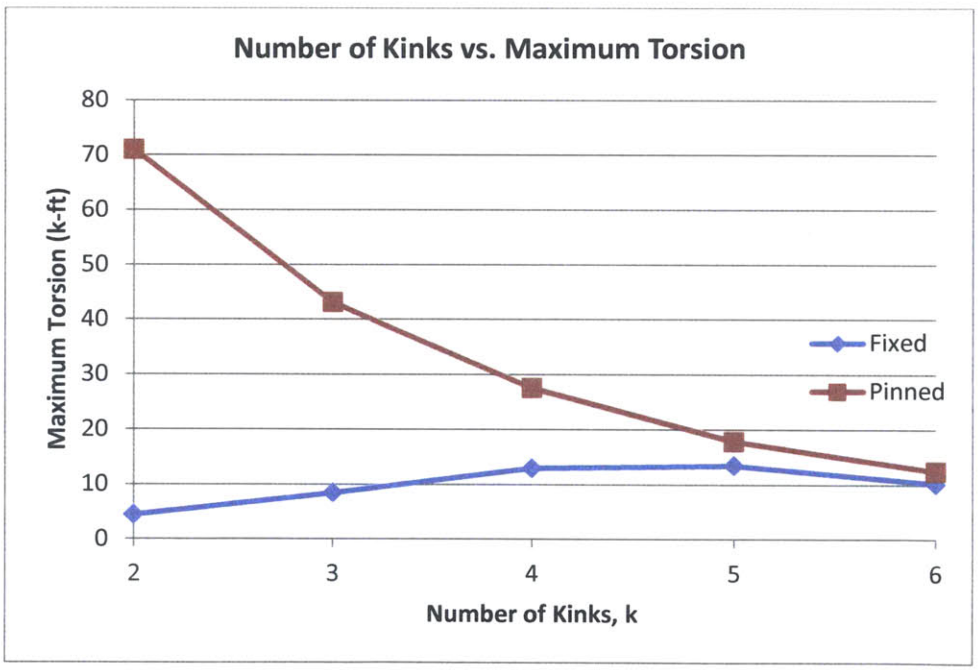

Figure 38 - Number of kinks vs. maximum torsion

The results of this analysis were used to make the design recommendations in Section 7.

7 DESIGN RECOMMENDATIONS

Using the results of the previous analysis, several design recommendations can be made regarding the geometry and structure of deployable roofs made of angulated elements. First, the most critical aspect of the structural response in almost every case was whether fixed or pinned support conditions were used. Looking at the deflection curves, it is apparent that the fixed supported structures are up to twelve times as stiff as the pin supported ones. Therefore, fixed supports should be specified wherever at all possible. Pinned supports should only be used on small spans with limited loading. There are numerous advantages to using fixed supports. First, the angulated elements will be more structurally efficient overall since the beams will not need to be oversized to account for excessive deflection. Increasing the stiffness also increases the fundamental frequency of the roof, causing a significant reduction in vibration due to wind and live loads. As shown in Figure 31 through Figure 38, using fixed supported members generally reduces the maximum moment and torsion on each member. This, in turn will reduce the load on the hinges and make the mechanism easier to actuate. The fixed support rings have more intuitive structural behavior. Finally, since the maximum moment and maximum torsion generally occur at the fixed support, the angulated elements can be tapered towards the center of the ring. This will reduce the beam self-weight and therefore reduce the moment and deflection in the closed position.

No matter what support conditions are used, members must be designed for biaxial bending and torsion at each stage of deployment, not just open and closed. Figure 26 and Figure 27 indicate that the maximum moment and maximum torsion will not necessarily occur when the ring is fully closed. Since the members are under significant torsion, HSS tubes or other torsion resistant sections are recommended. Designers must also look for moment reversals during the opening process. For example, fixed supported rings in the fully open position showed both positive and negative moment (Figure 39), while only negative moment is present in the more closed positions. While these moment reversals will not likely be enough to require resizing the member, the load reversal on the connections could cause fatigue problems.

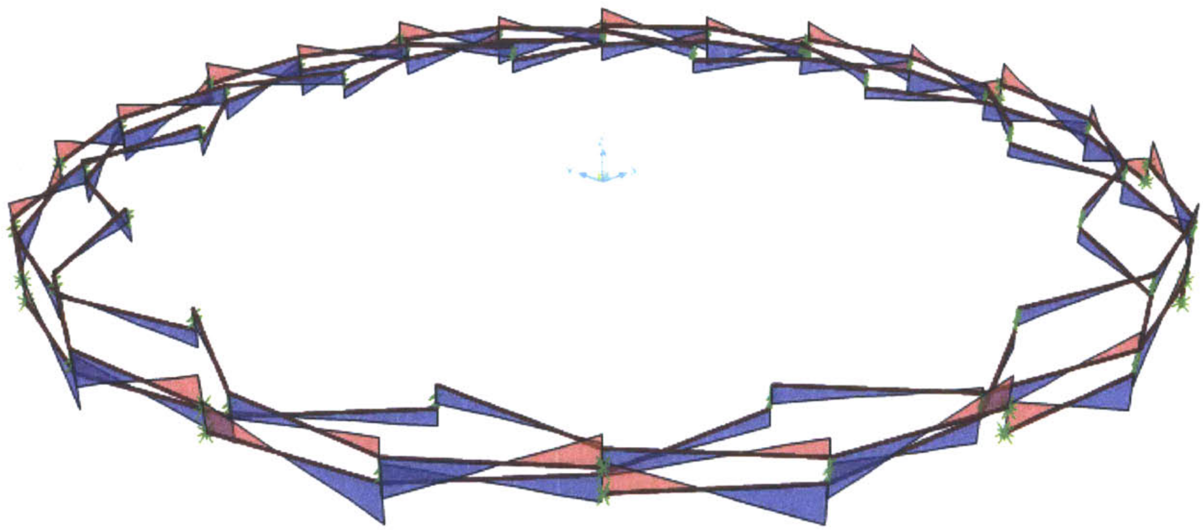

Figure 39 - Moment diagram of fixed supported ring in open position. Note both + and – moments.

The number of divisions (n) in the ring should be based on both structural and architectural (geometry) characteristics. As shown in Section 5.4, using more divisions yields better opening ratios. Structurally, increasing n initially increases the moment in the element, but then begins to decrease the moment after about n = 16 (assuming fixed supports, see Figure 30). This suggests that high values of n would also be suitable structurally. However, adding more divisions would also lead to an increase in the self-weight of the structure. It is unknown whether this increase in self-weight would offset the benefits associated with having more divisions. Future study could use an optimization scheme to determine the best value for the number of divisions.

The number of kinks (k) should also be dependent on both the structural and architectural design goals, though not necessarily equally on each. For fixed supported rings, the deflection is roughly independent of k, while k has a very large effect on the open geometry. Therefore, architectural constraints should play a larger role in selecting a desired number of kinks. However, three to four kinks seems to work well both structurally and architecturally. Self-weight also increases with k, so future investigation should model self-weight as well.

Both increasing the number of divisions and increasing the number of kinks results in more hinge points. Since friction is the only force opposing the motion of the structure during deployment, having more hinges means it may be more difficult to operate a roof with more divisions or more kinks. Proper maintenance of the hinges will be required to ensure smooth opening and closing procedures.

The fixed supported deployable rings also scaled well to larger structures. Even for the largest ring diameter (60 feet), the deflection at the center did not exceed L/380 when using relatively small HSS20x12x3/8 members. This suggests that long span roof stadiums are indeed structurally feasible. The structure could be made even more efficient for long span roofs by devising a three-dimensional dome out of multi-angulated elements. The dome would allow much of the forces to be transferred to the supports axially rather than through bending.

This study looked at each parameter associated with deployable rings individually and attempted to isolate recurring patterns. However, given the clearly defined parameter inputs and outputs, this problem is well suited to an optimization scheme. Future work could develop this scheme to minimize the weight of a structure for a given span, minimize the number of hinges for a given loading, or solve other similar problems.

8 CONCLUSION

Deployable structures are structures that can change shape from a compact to an expanded form. They offer wider versatility than conventional structures because they can be easily assembled and transported, and because they can adapt to changing demands. However, the structures are more difficult to design and construct, and are therefore tailored to specific applications.

These structures can be broadly classified as surface structures, such as inflatables, or strut structures, such as camping tents. There is a wide variety of articulation mechanisms for deployable structures. Some of these include inflating, folding, telescoping, and radially expanding. Different mechanisms are suited for different purposes.

Primary applications of deployable structures include military and disaster relief tents. Such structures must be able to be transported easily and be erected quickly. Other temporary structures, such as stages, exhibition stalls, and construction scaffolding also benefit from this feature of rapidly deployable structures. Aerospace structures utilize expanding structures that can be compacted during launch, but still unfolded into large components such as solar panels and communication equipment. Finally, lightweight deployable roofs are used to cover sports facilities to provide variable weather protection as needed.

Pantograph structures operate on the principle of two rigid bars that are pinned in the center. By changing the endpoints of the rods, this scissor-like-element can expand and contract in the plane of its rods. If many scissor-like-elements are connected into deployable units, complex expandable geometries, such as arches and domes, can be formed. Additionally, if properly designed, the pantograph structures can exhibit snap-through behavior that locks them into their expanded configuration. Since this snap through process is nonlinear, a proper understanding of the strut forces is required to design this behavior.

By adding a kink to the bars of a pantograph, an angulated element is formed. This element subtends a constant angle as it deploys, and therefore a series of these elements forms a radially expanding ring. Since these rings have a central opening that changes size, they can be used as the basis for operable roofs of sporting facilities that open and close depending on the weather.

The geometry of these angulated element structures has been comprehensively detailed in previous work, but no prior attempts to have been made to properly analyze them in analysis software. Using MATLAB and SAP2000, this thesis performed a preliminary analysis of these structures and successfully identified patterns in the structural response. It was determined that the support conditions of these rings have the largest effect on the bending moment and peak deflections in the rings. Also, various trade-offs were identified in terms of reducing forces in the members while retaining functionality of the deployment mechanism. In spite of these trade-offs, angulated element structures were found to be feasible for long span roofs if properly designed. Future work is required to investigate the effects of joint fatigue and optimize the structures further.

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Appendix: MATLAB Code

```
%Deployable ring generator
%Plots rings in MATLAB
%writes a .dxf file of for use in analysis
%creates xcel file of joint coordinates to transfer into SAP connectivity
table
%Maxwell Wolfe
%April 24, 2013

loop = 1; %global loop counter
for param = [8 12 16 20 24]; %global parameter values to vary

%Bar Info
divisions = param;
no_kinks = 3;
R_min = 20; %Outside radius of ring in closed position

alpha = 360/divisions;
no_hinges = no_kinks+1;
bar_length = R_min*(sind(alpha/2)/sind(no_kinks*alpha/2));
r_star = bar_length/(2*sind(alpha/2));
r_max = 2*r_star*cosd((no_kinks-1)*alpha/2); %maximum inner radius
R_max = 2*r_star; %maximum outer radius

%Expansion Info
d1 = 1; % distance from origin to closest hinge, measure of how "open" the
ring is

%*****CLOCKWISE ELEMENTS******%
theta = asind(d1*sind(alpha/2)/bar_length)+alpha/2; %From law of sines
d2 = bar_length*sind(180-theta)/sind(alpha/2); % distance from origin to
second hinge

%local_coord stores x,y,z coord of hinge points, each row is a hinge point
local_coord = zeros(no_hinges,3);
local_coord(1,:) = [-d1 0 0];
local_coord(2,:) = [d2*cosd(180-alpha/2) d2*sind(180-alpha/2) 0];
for n = 3:no_hinges %step through and find coordinates of each hinge in an
elem.
    theta = theta + alpha; %theta (measured from 0) increases with each bar
    local_coord(n,1) = local_coord(n-1,1) + bar_length*cosd(180-theta); %x-
coordinates of next hinge
    local_coord(n,2) = local_coord(n-1,2) + bar_length*sind(180-theta); %y-
coordinates of next hinge
end

%Array single element in circle
Rot = [cosd(alpha) -sind(alpha) 0;sind(alpha) cosd(alpha) 0; 0 0 1];
%Rotation matrix about z axis
clckws_coord = zeros((no_hinges)*divisions,3); %initialize matrix of all
clockwise elem.

array_el = local_coord; %initialize element to array
```

```
rotated_el = zeros(size(local_coord));
i = 1; %clockwise coordinate index
for j=1:divisions %array as many times as needed to complete circle
    clckws_coord(i:i+no_kinks,:)=array_el; %place current elem. coordinates
into matrix
    for k = 1:no_hinges
        rotated_el(k,:) = Rot*array_el(k,:)'; %Find rotated hinge position
    end
    array_el = rotated_el; %make rotated bar the current elem. to be arrayed
    i=i+no_hinges; %move to correct row in storage area
end

% Plot elements
i=1;
figure(loop);
for k = 1:divisions %loop through to draw each angulated elem.
    for n = i:i+no_kinks-1;

plot([clckws_coord(n,1),clckws_coord(n+1,1)],[clckws_coord(n,2),clckws_coord(
n+1,2)],'linewidth',2.5,'marker','o','markeredgecolor','k');
    hold on
    end
    i = i+no_hinges; %move to correct row in storage array
end

%*****COUNTERCLOCKWISE ELEMENTS*****%
%local_coord stores x,y,z coord of hinge points
local_coord = ones(no_hinges,3); %vertically separates counterclockwise
elements by 1
local_coord(1,:) = [-d1 0 1];
local_coord(2,:) = [d2*cosd(180+alpha/2) d2*sind(180+alpha/2) 1];
%Counterclockwise
theta = asind(d1*sind(alpha/2)/bar_length)+alpha/2; %reset value  of theta
for n = 3:no_hinges
    theta = theta + alpha;
    local_coord(n,1) = local_coord(n-1,1) + bar_length*cosd(180+theta); %Uses
180+theta
    local_coord(n,2) = local_coord(n-1,2) + bar_length*sind(180+theta);
end

Rot = [cosd(alpha) -sind(alpha) 0;sind(alpha) cosd(alpha) 0; 0 0 1];
%Rotation matrix
cntrclckws_coord = zeros((no_hinges)*divisions,3);

array_el = local_coord; %initialize element to array
rotated_el = zeros(size(local_coord));
i = 1; %global coordinate index
for j=1:divisions
    cntrclckws_coord(i:i+no_kinks,:)=array_el;
    for k = 1:no_hinges
        rotated_el(k,:) = Rot*array_el(k,:)';
    end
    array_el = rotated_el;
    i=i+no_hinges;
end
% Plot elements
```

```
i=1;
for k = 1:divisions
    for n = i:i+no_kinks-1;

plot([cntrclckws_coord(n,1),cntrclckws_coord(n+1,1)],[cntrclckws_coord(n,2),c
ntrclckws_coord(n+1,2)],'linewidth',2.5,'linestyle','- -','color','r');
    hold on
    end
    i = i+no_hinges;
end
xlim([-1.1*R_min 1.1*R_min]);
ylim([-1.1*R_min 1.1*R_min]);
axis square;

%******Create xcel file with joint coordinates*****
global_coord = [clckws_coord; cntrclckws_coord];
xlsheet = sprintf('%g',param); %label sheet based on parameter being modified
xlswrite('Ring_opening.xls',global_coord,xlsheet); %write joint coordinates
to xcel file

%*****Create .dxf file*****
file = sprintf('xxxringR%gk%gn%g.dxf',R_min,no_kinks,divisions); %create file
name with parameter values
FID = dxf_open(file); %create dxf with correct filename
i=1;
for j = 1:divisions*2

dxf_polyline(FID,global_coord(i:i+no_kinks,1),global_coord(i:i+no_kinks,2),gl
obal_coord(i:i+no_kinks,3));
    i = i + no_hinges;
end
dxf_close(FID);

loop = loop+1; %increment global loop counter
end
```