A Framework to Account for Flexibility in Modeling the Value of On-Orbit Servicing for Space Systems

by

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Ingénieur diplômée de l'Ecole Polytechnique (1999)

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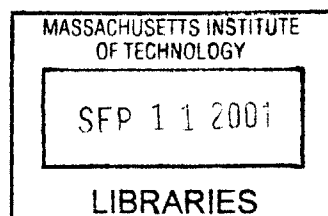
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Abstract

The traditional method for maintenance of space systems consists in building reliable satellites through redundancy and replacing them in case of failure, or whenever an upgrade is necessary. On-orbit servicing could change this paradigm. What would be the missions for which servicing would be most interesting, and what price would they be willing to pay for the capability to be serviced? The answer to these questions would provide valuable guidelines as to which servicing technologies to develop, and at what cost.

Assuming that the technologies enabling automated servicing are available, traditional metrics and models are first proposed to systematically evaluate servicing cost-effectiveness within a representative trade space of serviceable missions and servicing infrastructures. It is shown that though it can capture some elements of cost-effectiveness, the traditional approach tends to underestimate the value of servicing and demonstrate cost advantages smaller than the cost uncertainty.

This issue is solved by then proposing a new approach to on-orbit servicing. First, the intrinsic value of servicing is studied separately from its cost. Furthermore, a first framework to evaluate the flexibility provided by on-orbit servicing to space systems is developed. This framework is used to define models of the value of servicing for two families of space systems faced with different types of uncertainty: commercial systems with uncertain market and military missions with dynamic requirements.

For commercial missions with uncertain market, modeling servicing as an option on life extension shows that space systems should not systematically be designed for the longest possible lifetime. Instead, the optimal design life decreases with increasing uncertainty. The maximum servicing price that would make servicing economically interesting is evaluated as a function of uncertainty and the value of flexibility is illustrated on two current examples.

For military missions, a small number of satellites with the option to maneuver is considered as an alternative to global coverage for flexibility with respect to contingency location. It is shown that while this alternative has little value in the case of a low Earth orbit radar constellation, it has interesting potential for geostationary communication satellites.

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I am also grateful to other students in Prof Hastings' group. Annalisa Weigel and Myles Walton, Jason Andringa, Joseph Saleh and Steve Panetta showed me what true constructive criticism is and I only wish I had been able to be as helpful to them as they were to me.

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I also thank my 'american' family Fabrice and Maria for helping me put matters into perspective by having me spend joyful evenings playing with their kids, and Matthias and Julien for reminding me of the ever-lasting joy of discovery.

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Biographical Note

Elisabeth Lamassoure was born and raised in Paris, France. In July 1996 she was accepted to the French school of engineering *Ecole Polytechnique*, from where she graduated in July 1999 with a Physics major. From April to July 1999 she worked as a research intern at the *Service d'Aéronomie du CNRS*, Verrières-le-Buisson, France, were she studied the monitoring of the active regions on the far side of the Sun using the instrument *Swan* (Solar Wind Anisotropies) on board the Solar and Heliospheric Observatory (SOHO) satellite. In September 1999 she started her Master program at MIT, where she focused her coursework on satellite design and space systems engineering. This thesis is the product of her work as a research assistant in the *Space Systems Laboratory*.

Contents

Chapter 1

Introduction

Space systems are still the only complex engineering systems without routine maintenance, repair and upgrade infrastructure. The *Shuttle* can access and maintain high value assets such as the *International Space Station* (ISS) or the *Hubble Space Telescope* (HST). But for the average space systems, maintenance means are limited to launching spacecraft. Replacement is the only repairing scheme, so that a spacecraft can be lost even if the majority of its components are still operational. One-of-a-kind, reliable and expensive spacecraft have been the natural result of this lack of space logistics, as schematized on figure 1-1. To amortize the high cost of spacecraft, their design lifetimes are made longer, which in turn makes them more expensive.

The space industry as well as the United States governmental agencies recognize today the need for a new paradigm of space systems design. Space technologies are mature enough that pushing the limits of reliability further is becoming extremely expensive. In addition,

$A \xrightarrow{+} B$ *means "A drives B up"*
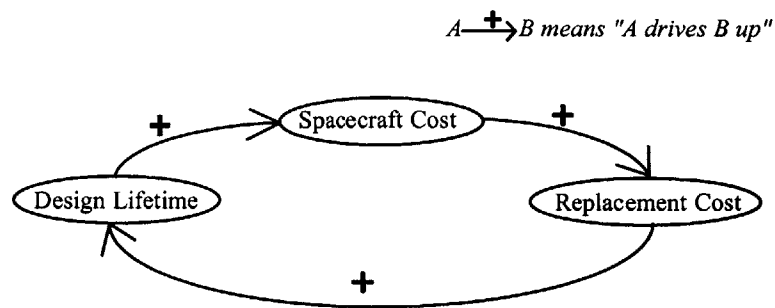
Figure 1-1: Vicious Circle of Traditional Design Methods Leading to Longest Possible Lifetime
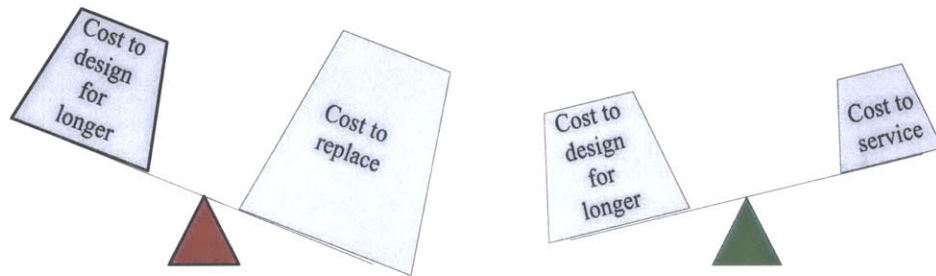
Figure 1-2: On-Orbit Servicing as Breaking the Vicious Circle: Towards Shorter Design Lifetimes?

there is concern about the growing gap between space systems, which are often designed to live more than a decade, and the shorter life cycles of market demand and technology development. The result of this gap is a considerable risk that a spacecraft will become technologically obsolete or will stop addressing any actual market before the end of its design lifetime.

On-orbit servicing, defined as the ability to repair, refuel, replenish, and upgrade satellites on orbit, has long been recognized as having the potential to change the way business is carried out in space. As a cheap alternative to replacement, on-orbit servicing would make possible a new trade between design margins and maintenance costs, as schematized on figure 1-2. This trade is likely leading to less redundant, cheaper spacecraft. As a means of life extension and upgrade, it would foster shorter design lifetimes, thus enabling spacecraft to follow the market and technology dynamics more closely. It would also offer the potential for designing new types of space systems, such as maneuverable spacecraft.

However, the implementation of on-orbit servicing requires a whole new way of designing and managing space systems. In addition, decision makers perceive it as a significant source of technological risk. For investments in on-orbit servicing to be actually deemed worthwhile, considerable advantages in terms of cost-effectiveness must be proven. Many studies have qualitatively explored the potential advantages of autonomous on-orbit servicing. Several projects developed bottom-line architectures for on-orbit servicing of specific space missions and demonstrated potential improvements in terms of cost or cost-effectiveness. However, no advantages have yet been proven that outweigh the perceived risk and cost uncertainty.

Before the decision can be taken to make on-orbit servicing the new paradigm for space systems maintenance, the conditions under which this would be cost-effective still remain to be explored in detail. The objective of this thesis is to propose new tools to participate in answering this question.

The typical research path followed by previous work has been to develop a design tailored to a specific space system and simulate its cost-effectiveness over the mission life. While this approach has been very successful at demonstrating the feasibility of automated on-orbit servicing and at proposing realistic designs, it has not yet proved able to yield any general conclusion about the cost-effectiveness of on-orbit servicing.

One of the reasons may be that such a process overlooks the intrinsic value of servicing for space missions. This value, defined as the price a space mission would be willing to pay for the capability to be serviced, should exist independently of any servicing architecture design. Its systematic study would help identify the space missions that are most likely to become customers of a servicing infrastructure. It would give valuable directions for servicing design as to what space missions to target, and at what cost.

In addition, by using traditional valuation tools such as net present value calculations, previous work has been underestimating an important component of servicing value. The price that a space mission would be willing to pay for servicing is not limited to the cost savings it would make by designing spacecraft for shorter lifetimes and smaller reliability. Servicing would also provide space missions with *options* in the future to adapt to uncertain parameters such as random failures, market dynamics, technological development or changing requirements. This *flexibility* is a significant advantage of servicing, and it is important to take its value into account.

In order to address the holes in previous research, this thesis proposes to step aside from technology development and design, and assume that an infrastructure for on-orbit servicing is available. It can then focus on the main research question:

> *Is there a general way to estimate the* value of servicing for space systems, *defined as the* maximum price *under which servicing improves mission value, taking into account the* options *that servicing provides to decision makers?*

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As a basis for constructing an answer to this question, we will first define its terms and implications in slightly more details.

1.1 Definitions and Motivations for On-Orbit Servicing

1.1.1 A Few Definitions

Waltz [Wal93] gives a definition of on-orbit servicing:

> On-orbit servicing is work in space. The work, performed by men, machines, or a blend of both, relates to space assembly, maintenance, and servicing (SAMS) tasks to enhance the operational life and capability of satellites, platforms, space station attached modules, and space vehicles. In the broadest context of its definition, satellite servicing also includes the in-space launching, reboosting, and retrieval of space systems. Growth versions of some servicing functions involve space debris capture or containment and emergency operations for crew rescue and return to an in-space safe haven or to the Earth.

He divides the set of functions performed by on-orbit servicing into three categories:

Assembly is the fitting together of manufactured parts into a structure; it occurs *before* a space system is operational.

Maintenance is the upkeep of facilities or equipment; it can be scheduled or on-demand; it is performed *after* a system has become operational and includes any on-orbit activity performed for the purpose of extending the operational life of a space system, except replenishment of consumables.

Servicing is the broader term encompassing the replacement of expended consumables and the logistics required to strategically locate supplies; it can be performed before or after a system becomes operational.

In the framework of this thesis, we will refer to on-orbit servicing as any on-orbit activity, including refueling, performed *after* a system has become operational, for the purpose of extending the operational life of the system, or modifying some of its components. The tasks included in this definition are indicated by a cross (X) in table 1.1. We will often use the word *servicing* alone to refer to on-orbit servicing.

Traditional classifications of servicing functions, such as the one proposed by [Wal93] and summarized by table 1.1, have been considering on-orbit servicing from the point of

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Table 1.1: Servicing Functions Classification from [Wal93]

Servicing Tasks	Task Functions	In this thesis
Assembly	Space station assembly	
	Space Station upgrade / modification	X
	Large spacecraft assembly	
	Deployment of appendages	
Orbit Transfer	Delivery to final orbit	X
	Retrieval from orbit	
	Earth return	
Resupply	Fluids	X
	Materials	X
	Film / Tape	X
Maintenance and Repair	Module changeout / replacement	X
	Refurbishment / retrofit	X
	Modification	X
	Decontamination	X
	Cleaning / resurfacing	X
	Test and checkout	X
	Unplanned repair	X
Special	Space debris control	
	Emergency operations	X

view of designing a servicing architecture. For example, resupply and repair are two very different tasks from a servicing mission point of view; while the former is a one-way mass transfer, the latter can require taking out an old module before inserting the replacement module, which is a two-way mass transfer and presents different technological challenges.

From the serviced mission point of view, the relevant distinctions are different. For example, refueling or replacing batteries both aim at extending the lifetime of the existing spacecraft design, while upgrading a module leads to a spacecraft with new or enhanced capabilities. Taking the point of view of the serviceable missions, we will therefore group the servicing functions into only three main categories: life extension, upgrade, and modification.

Life extension includes any on-orbit servicing operation aimed at extending the operational life of the system in its original design; this involves refueling, refurbishing and repairing.

Upgrade includes any on-orbit servicing operation aimed at improving the performance of the operational system in meeting its original mission goal; it involves insertion of

more recent technology into the design, through adding or replacing components.

Modification includes any on-orbit servicing operation aimed at meeting *new* mission goals; examples include design changes through payload replacement, as well as refuel to maneuver into a new constellation configuration.

A last important class of distinction concerns the timing nature of the servicing operations. They can occur either on an on-demand or an a scheduled basis.

On-demand on-orbit servicing is performed as needed; this is well suited for example to repairing after random failures. It requires all servicing material to be constantly available.

Scheduled on-orbit servicing involves setting in advance future servicing times; this is well suited for example for life extension at the end of a design lifetime. Although the time of servicing is set, the components to be delivered can be chosen at the time of service; the decision can also be made not to service at that time after all.

1.1.2 Motivations for On-Orbit Servicing: Traditional Views

On-orbit servicing can enhance the design process by extending the possibilities available to mission designers and the opportunities for trade-offs. It has been recognized to offer many ways of increasing the achievable mission cost-effectiveness, in particular:

Enable Missions Certain missions are simply not viable without servicing because their baseline lifetime is short and the cost of replacement is too high. This is the case for very high value assets such as the *International Space Station* (ISS) or the *Hubble Space Telescope* (HST), and all spacecraft that must be assembled on-orbit.

Reduce Initial Mission Cost The ability to refuel and repair satellites offers an alternative to replacement for trading initial spacecraft costs with mission lifetime costs. For example, long-term consumables and redundant parts make up a mass on satellites that is not immediately useful. The need for corresponding additional structures and fuel increases this mass penalty. Satellites designed for servicing could therefore end up being much smaller than their traditional counterparts.

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Improve Lifetime Performance/ Reduce Risk On-orbit servicing can increase mission lifetime performance by offering cheap and timely ways of mitigating risk [BP91]. This applies to risk in the launch phase (a satellite launched into the wrong orbit could be refueled to maneuver to its desired orbit), risk in the physical life of components (repairing for random failures instead of replacing the whole satellite) and risk in the technological life of components (upgrading a component instead of designing a new satellite).

Most previous work on servicing cost-effectiveness has been considering on-orbit servicing as an alternative to replacement for maintenance of a space system. The typical approach adopted by such studies can be summarized as being made up of the following steps:

1. Choose a specific space mission and analyze its serviceability,

2. Identify one or a trade space of, servicing architecture designs for this mission,

3. Simulate maintenance events over the lifetime of the mission, both for the serviceable case and for a baseline case in which satellites are replaced,

4. Compare lifetime costs and some measure of lifetime performance (such as a utility function, or constellation availability) for the serviceable and the baseline cases,

5. Draw conclusions on the percentage cost advantage, and possibly on the performance advantage, of the chosen on-orbit servicing method.

1.2 A New Approach to On-Orbit Servicing

1.2.1 Defining the Value of Servicing for a Space System

When the approach described above is undertaken, whether on-orbit servicing proves more interesting than traditional methods is actually the result of a trade-off between two main effects: the cost savings from servicing on the one hand, and the price the space mission is going to pay for servicing on the other hand. The cost savings from servicing depend mainly on the satellite design and the elements to be serviced, while the price to pay for servicing depends not only on the cargo to be delivered to the satellites, but also and

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principally on the assumptions, design choices and cost models for the servicing architecture. Therefore the conclusions yielded by such a method are valid only for the specific mission and servicing infrastructure considered.

A first possible approach to solve this issue consists in defining a general trade space of space missions and servicing architectures, and systematically exploring the trade space using general metrics for cost-effectiveness.

Another interesting consideration is that since the infrastructure for on-orbit servicing does not yet exist, results which would give some guidance as to what types of technologies to develop, what space missions to target, and what cost cap not to exceed, would be very valuable. From a theoretical as well as from a conceptual point of view, it is therefore interesting to study the value of servicing for space missions separately from its cost.

Both approaches will be considered in turn in this thesis.

1.2.2 Flexibility through Servicing: Turning Uncertainty into an Asset

Serviceable missions have options But the value of servicing is not limited to the potential cost savings incurred when designing a system for a shorter design life. The capability to be serviced in the future is also a great source of *flexibility* for space missions. A serviceable mission would have *options* to react to the future resolution of parameters that are uncertain at the time of launch. Examples include the option to refuel or repair for life extension, the option to upgrade to avoid technological obsolescence, or the option to modify to meet new requirements.

On uncertainty and risk By not taking flexibility into account, traditional decision making often confuses uncertainty and risk. There is *uncertainty* in a mission if one or several future mission parameters cannot be predicted exactly; uncertain parameters are typically modeled as having a probability density function, and the standard deviation of this distribution is a measure of uncertainty. There is *risk* in a mission if there is uncertainty, and if the results of this uncertainty can have negative outcomes; a typical measure of risk would be the expected negative outcome. For a mission that has no way to react to the resolution of uncertainty, there is often a one-to-one relationship between uncertainty and risk.

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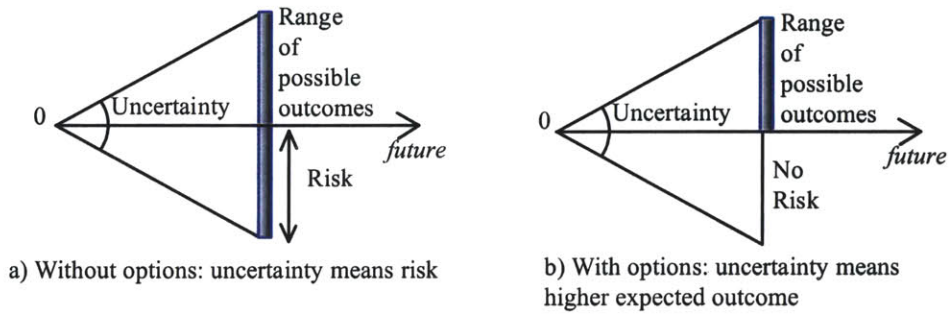
Figure 1-3: The Cone of Uncertainty - Inspired from [AK98]

The cone of uncertainty Options de-couple risk and uncertainty. A good way to conceptually capture this effect is the notion of *cone of uncertainty* proposed by real options theory [AK98] and illustrated on figure 1-3. Decision makers consider the future as seen from the apex of the cone (present). As they look further and further into the future, there is more and more uncertainty associated with their forecast. This is what the cone represents, its angle being a measure of the level of uncertainty. If no option is available, then an increasing uncertainty translates into an increasing probability of a negative outcome; thus uncertainty means risk. But if options are available to react to uncertainty, then negative outcomes can be avoided and a higher uncertainty translates into a higher expected outcome. Thus, *for flexible missions, uncertainty is not a source of risk any more, but a source of value.*

Giving options to space missions is a significant advantage of on-orbit servicing. It is important the capture the value of this flexibility.

1.3 Thesis Outline

This thesis proposes to develop a new framework to yield general results for the value of servicing for space missions, taking flexibility into account. Three main steps are necessary to accomplish this goal: first, analyze the traditional approach and identify its limitations; then based on these limitations, develop a general theoretical framework that fills some holes in previous research; finally, validate the framework on practical examples. These steps are organized into the following chapters:

Chapter 2 summarizes a few results from previous research that are particularly relevant

to this study. Previous work from three complementary areas is presented: the design and evaluation of servicing infrastructures, the impact of servicing of spacecraft design and cost, and the state-of-the-art in valuation methods.

Chapter 3 extends on previous work by defining general metrics to estimate cost-effectiveness on a wide trade space of space missions and servicing infrastructures. It shows how the cost uncertainty yielded by traditional approaches generally outweighs the advantages of servicing in terms of cost-effectiveness.

Recognizing the need to study the full value of servicing independently from its cost, chapter 4 proposes a new approach to on-orbit servicing. Building and expanding on decision tree analysis and real options theory, it defines a framework to embed the value of flexibility into the valuation of space missions faced with uncertainty. The framework relies on the definition of a few *building blocks*, the most important being a model of the uncertainty, a set of reachable operational modes, a sequence of decision points, and a definition of mission value.

Chapter 5 uses this framework to develop a general model for the valuation of commercial space missions with uncertain revenues. The linearity of mission value makes this simple case very similar to real options valuation. The model is first used to estimate the value of the option to abandon, which is available to all space mission but has never yet been accounted for. The option to service for life extension is then considered. The optimal design life is studied as a function of market uncertainty. The *maximum servicing price* under which a serviceable design is optimal is mapped into a market level/market uncertainty space and illustrated on two current examples from the satellite communications world: the Iridium and Globalstar constellations.

Chapter 6 considers the more complex case of military missions with dynamic theater locations. It shows how valuation models can be developed from the same framework in spite of the continuity of the decision points and the non-linearity of the value function, which make both real options theory and decision tree analysis impractical. The value of refueling to make a constellation of satellites maneuverable is studied for two cases: a low-Earth orbit radar constellation taken on the example of the *Discoverer-II* project, and a geostationary fleet of communication satellites taken on the example of the Defense Satellite Communications System (DSCS).

Chapter 7 concludes on the contributions and limitations of this work.

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Chapter 2

Relevant Previous Work

This chapter summarizes and discusses a subjective selection of previous research efforts that have been found particularly relevant to the question of evaluating the value of on-orbit servicing for space missions. There are three main elements in this question: *value, on-orbit servicing* and *space missions*. Our discussion of previous work is accordingly divided into three main areas of research. Section 2.1 summarizes some of the research about on-orbit servicing architectures, which includes historical on-orbit servicing, technology development, and cost-effectiveness studies. Section 2.2 deals with several aspects of the satellite design changes in the presence of on-orbit servicing: the cost savings from designing for a shorter design life on the one hand, and the penalty to design for serviceability on the other hand. Finally, section 2.3 presents and compares three important ways of estimating value for decision making: net present value (NPV), decision tree analysis (DTA) and real options valuation.

2.1 Previous Work on On-Orbit Servicing

2.1.1 History Highlights

Waltz [Wal93] makes the point that although on-orbit servicing is sometimes perceived as revolutionary, it has had an evolving history; maintenance considerations have always been part of spacecraft design and systems engineering.

Space maintenance has been practiced since the beginnings of spaceflight in 1961, when few missions were completed without crew intervention to correct malfunctions. But before

1980, the demonstration of servicing in space had been limited to manned spacecraft such as *Skylab* or *Apollo* in the United States, and the space station program in the USSR.

The Skylab missions (1973-1974) included scheduled maintenance activities [Wal93], but also experienced, from the very launch, failures that required major maintenance efforts: release of a solar array, deployment of a parasol and a twin-pole sun shield, installation of a rate gyro package, servicing of the coolant system, and repair of a microwave antenna. Almost all of the 53 *Skylab* experiments experienced various degrees of maintenance activity during the mission. This maintenance was systematically performed by astronauts.

The Russian space stations program * started in the same years and lasted until the death of the *MIR* space station. An extensive history of on-orbit servicing started with the *Salyut* 6 space station, launched in 1977. *Salyut* 6 had two docking ports: the *Soyuz* spacecraft docked to one port, leaving the other port available for visiting crews or *Progress* resupply vehicles. A total of 12 *Progress* spacecraft, each of length 7 *m* and weight 7 *tons*, delivered more than 20 tons of equipment, supplies and fuel during the station's lifetime; the docking and fuel transfer were performed automatically. The latest version, *Progress-M*, performed more than 40 servicing operations of the *MIR* space station. Its autonomous docking system failed only three times at the first attempt: two *Progress* missions docked at the second attempt, while one (the *Progress M-24*) crashed into the station and had to be maneuvered by hand. In spite of this accident, the program has been a great demonstration of the feasibility of routine autonomous docking and refueling.

The Solar Maximum Mission (*SMM*) spacecraft was the first unmanned spacecraft to be serviced [Wal93]. The spacecraft underwent failure of three of its momentum wheels and of its coronograph/polarimeter instrument ten months after it began collecting spectacular data about the solar activity. After a year-long test program of high fidelity simulation on the ground, astronauts on board the *Shuttle Challenger* were able to retrieve the spacecraft, replace its attitude control module and repair its coronograph electronic box. Amounting to a total estimated cost of $60 million, this repair mission proved less expensive than a $230 million replacement. Some people believe that it proved the usefulness of the *Shuttle*

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*http://www.nauts.com

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and ended the era of the throw-away spacecraft. Since then, there have been numerous examples of unscheduled maintenance on *Shuttle* missions.

The Hubble Space Telescope (HST)[†] is probably the most striking example, as it has been the most extensively serviced unmanned spacecraft is history. Immediately after deployment of the telescope, scientists realized that its primary mirror had a major flaw: a spherical aberration resulting in fuzzy images. The telescope would never have given its revolutionary images of the Universe without the first servicing mission, HST 1 in 1993. Although the overall servicing cost was as high as \$500 million, it proved cheaper than manufacturing a new \$1 billion spacecraft. *Hubble* is also a good example of the power of upgrading. The second servicing mission, in 1997, installed new instruments, multiplying by 30 the spectral resolution and by 500 the spatial resolution of the imaging spectrographs, and allowing the infrared camera to detect even more distant objects. At the same time, new solid state recorders made possible the storage of ten times more data. The next servicing mission, scheduled for November 2001, is expected to provide a tenfold improvement in the *Hubble*'s survey capability.

On-orbit servicing by the *Shuttle* is so expensive that it makes sense only for very high value assets such as *Hubble*. Even in this case, the cost of the three servicing missions have already outweighed the cost of the spacecraft itself. Therefore, recent efforts have been focusing on developing technologies for servicing of the average spacecraft. Only unmanned, autonomous on-orbit servicing can be cheap enough to make this a viable option.

The SAMS project (Space Assembly, Maintenance and Servicing) [Wal93] is a good example of such an effort. It was a joint study between the Department of Defense (DoD), the strategic Defense Initiative Office (SDIO), and NASA. Its primary objective was to define, where cost-effective, SAMS capabilities to meet requirements for improving space systems capability, flexibility, and affordability. The 7-year program consisted of three phases: (1) study, under contract with *TRW* and *Lockheed Martin*, (2) concept development and (3) implementation. The goal was to lead to a national SAMS capability in 2010.

The study identified five orbital regimes and constructed generic design reference mis-

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[†]http://hubble.gsfc.nasa.gov/

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sions (DRMs) to create servicing scenarios for various types of satellites in each regime. It studied the cost-effectiveness of servicing compared to replacing. One of its most interesting results is the identification of *breakpoints* in the curves of potential cost savings from on-orbit servicing, defined as places in the curve when a significant slope change occurs. These breakpoints indicate that on-orbit servicing is most interesting under the following conditions:

- When the cost of orbital replacement units (ORU) is lower than 50% of the satellite replacement cost,

- When servicing equipment charges are lower than 50% of the total satellite replacement cost,

- When servicing time intervals are shorter than 4 to 5 years,

- When servicing time intervals are shorter than on third of the time required to replace the satellite.

2.1.2 Enabling Technologies for Autonomous On-Orbit Servicing

The two basic requirements for autonomous on-orbit servicing to be possible are the ability to access the spacecraft with a maintenance capability (or the capability of the spacecraft to access a maintenance capability), and the ability of the spacecraft to be maintained. The technological prerequisites to meet these requirements can be divided into the functions described below, where the the time sequence of a servicing operation is followed.

- Orbital access (launch) and orbital transfer from an orbit to another are mature technologies, although some argue that a cheaper access to space would be necessary for the success of on-orbit servicing.

- Proximity operations can be defined as two spacecraft sustaining joint actions within 93 km of each other; they include navigation control, safing, docking, thermal control, observation, deployment, and retrieval. Many of these technologies are currently being developed or demonstrated.

- Orbital assembly, modification and upgrade are being demonstrated on the *International Space Station* in the context of close human supervision. Technologies for routine autonomous operations still need to be developed.

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- Safety monitor, defined as the continuous assessment of critical equipment data, and emergency operations and procedures still need to be defined.

- Jettison, defined as the separation of subsystems from a space vehicle with disposal of the separated element on orbit, is being developed as part of the proximity operations research effort.

- Space debris control is becoming a great concern, and mitigating methods are being researched [AH00]. Although servicing can be perceived as a way of reducing the space debris problem by re-using existing resources, it will also create more space debris from launches, remains of servicer vehicles, and disposal of used spacecraft parts.

One of the technologies currently undergoing the most intense development in the United States is autonomous rendezvous and capture (AR&C). A commonly accepted design for AR&C, which uses flight proven technologies, is well described by Polites [Pol99]. A chase spacecraft with both attitude and translation control capability actively navigates to a target vehicle, which is passive in the rendezvous process but has attitude stabilization. This means that spin- and gravity-gradient-stabilized satellites could not be serviced, but also that components critical to the attitude control system could not be replaced, with today's technology. The minimum payload to carry on the chase spacecraft consists of an integrated GPS and inertial guidance sensor (GPS/INS), a video guidance sensor (VGS), AR&C software, grappling mechanisms and possibly an autonomous formation flying sensor (AFF); in the rest of this thesis we will call this the *active AR&C payload*. The target must at least carry a docking interface equipped with retro-reflectors; we will hereafter call this the *passive AR&C* payload.

Other enabling technologies include procedures for operations and ground override, mechanisms and actuators for capture and line connections, and fuel transfer gauges and procedures. In addition, thermal management and attitude control for the docked configuration remain important issues.

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2.1.3 Highlight on a Current Project: Orbital Express

The space industry as well as the U.S. governmental agencies have been recognizing more and more clearly the need for a change in paradigm for space systems design, and the potentials offered by on-orbit servicing to change the economy of space. As a result, several technology demonstration projects are underway, among which the Defense Advanced Research Agency (DARPA) Orbital Express program is probably the most extensive [‡].

One of the goals of the study is to demonstrate in space an Autonomous Space Transporter and Robotics Orbiter (ASTRO) servicing spacecraft. ASTRO will autonomously conduct operations such as inspection, docking, and satellite pre-planned electronics upgrade, refueling and reconfiguration. The demonstration spacecraft will be launched with a companion satellite that it will service on-orbit. The long-term vision is a servicing spacecraft capable of accessing satellites at all orbital altitudes (LEO-to-GEO-to Lagrangian points) and of performing significant plane changes, using ascent-change plane-descent maneuvers, and/or aero-assisted maneuvers.

Research is also underway on other important concepts such as on-orbit storage spacecraft, methods for large-scale on-orbit storage and handling of liquid and/or gaseous consumables, and required changes to serviced spacecraft operational status while servicing.

2.1.4 Previous Work on Unmanned Servicing Cost-Effectiveness

Although the development of on-orbit servicing enabling technologies is well underway, the cost-effectiveness of the concept remains to be proven before the final development steps can be taken. This has been the subject of several research projects, of which this section summarizes a subjective sample.

The SMARD study

The spacecraft modular architecture design study (SMARD) [DCAJ97] has been an extensive research effort making significant steps in both servicing architecture design and cost-effectiveness assessment.

In the area of design, the work categorized different levels of on-orbit servicing in terms of spacecraft serviceability level. It showed that at least 30% of a spacecraft mass would

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[‡]http://www.darpa.mil/tto/programsfrm.html

be readily serviceable, and that this percentage would increase if spacecraft were designed in a modular fashion, with on-orbit servicing in mind. Taking the example of a specific surveillance constellation, it suggested alternatives to the current spacecraft design to make servicing possible. It finally developed a point design for a rendezvous and docking spacecraft tailored to service the baseline constellation, and a concept of servicing operations. One of the most interesting aspects of this design is that *functional* replacement is preferred over *physical* replacement: failed components are not removed but simply unplugged from the main spacecraft data bus. This simplifies the task of the servicer vehicle significantly.

This design was detailed enough to make a bottoms-up cost estimate possible. Combined with a Monte-Carlo simulation of the performance of the constellation for various servicing scenarios (scheduled / unscheduled), this estimate yielded reliable cost-effectiveness results. It showed that the proposed architecture could be up to 38% less expensive than satellite replacement for the baseline space mission.

Upgrading the GPS constellation

Two interesting companion studies published in 1999 addressed the question of autonomous on-orbit servicing for upgrading the satellites of the global positioning system (GPS): one considered the necessary structural modifications on the satellites themselves [HP99], and will be discussed in section 2.2. The other presented a trade study for the best servicing architecture [LWKM99],[LW99].

The goal of the latter work was to determine if the GPS Joint Program Office (JPO) should view a satellite management system of on-orbit servicing as an alternative to its current system of phased upgrade through replacement. The authors elaborated a large two-dimensional trade space of design choices based on existing technology. The first dimension described possible on-orbit servicing architectures in terms of servicer capacity (delivered mass), capability (number of satellites serviced), design life, and propulsion scheme. The second dimension consisted of maintenance strategies varying in time and space. The authors chose a representative sub-set of thirty alternatives from this trade matrix and compared them in terms of cost and value.

The cost was estimated by basing each servicer on a scaled version of an existing

bottoms-up design and using the NASA/Air Force Cost Model'96 (NAFCOM'96) [§]. The value was expressed in terms of what is usually called a utility metric. It was developed using decision analysis (DA) methods in close relationship with the actual GPS decision makers. The value of each architecture was a linear combination of scores along the main areas of concern for the decision maker: life cycle costs (recurring, non-recurring), performance (availability, flexibility) and program viability (shareability, implementability).

This value does not account for the flexibility during the mission lifetime in a direct way. The score for flexibility is made up of three scores dealing with how important the decision makers deems the cycle time, the upgrade frequency and the mass capacity of an servicing scheme. Such a value model is a good, general way of modeling the perception of flexibility a decision maker has at the start of a mission; however, it does not account directly for the options that will be available to decision makers *after* the system is operational.

The thesis concluded on six best alternatives, all comprising one servicer spacecraft per plane, with high to medium capability and capacity, and long design lives. These would deliver orbital replacement units (ORUs) carrying 150 kg to 30 kg of payload, four times over a period of 15 years; the total cost of these four servicing mission would amount to around \$300 M. Although these alternatives would cost more that the baseline maintenance scheme of staged upgrade, they would also score higher on the chosen value metric, due to their reduced time to upgrade or repair; and they are an order of magnitude cheaper than the "brute force" method of lumped replacement for upgrade.

In addition to research about on-orbit servicing design solutions, another main contribution of this work has been to recognize the advantage of on-orbit servicing for upgrading satellites, as a means of solving the conflict between the trend towards longer lifetimes and the need for flexibility to technology development. Stressing the importance of flexibility, the authors even mention the possibility to have satellite platforms in space: the upgrade capability would "make it possible to market their satellites as platforms for customers other than their traditional ones".

[§]As most existing cost models, this model is based on a historical database of satellite costs.
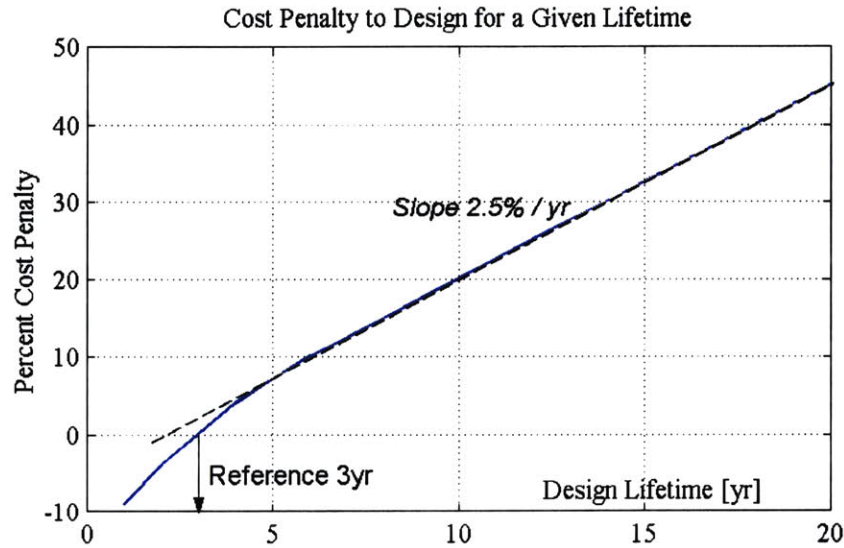
Figure 2-1: Cost Penalty to Design for a Given Lifetime. Adapted from [SHN01]

2.2 Impact of Serviceability on Satellite Design

2.2.1 Cost to Design for a Given Design Lifetime

One of the advantages of on-orbit servicing is to make a new trade between spacecraft design lifetime and maintenance costs possible, as mentioned in introduction and illustrated on figure 1-2. Before being able to quantify this trade, an analysis of the relationship between design lifetime and spacecraft cost is required.

Saleh & al [SHN01] carried out the exploration of this relationship by systematically estimating the impact of the design lifetime requirement on each spacecraft subsystem. The main direct effects of a longer design lifetime requirement are: design margins on solar arrays to outweigh the expected degradation, additional batteries to account for a capacity that decreases with number of cycles, increased electronics redundancy to achieve the same reliability at end of life, and additional fuel mass for station keeping. Indirect impacts on the structures, thermal, and propulsion subsystems only multiply the resulting mass increase. Using standard cost models, this mass increase can be translated into a cost penalty.

For parameters typical of current spacecraft design practices, the final results indicate a cost than increases almost linearly with design lifetime, as illustrated by figure 2-1. For example, designing for 15 years instead of 3 years results in a 35% cost penalty. But the cost is not proportional to lifetime, so that the *cost per operational day* decreases with required

design lifetime. In the absence of any other design driver, this explains and justifies the traditional approach of designing spacecraft for the longest possible time.

However, the authors note that other factors, such as technology obsolescence and market dynamics, should be taken into account in the decision regarding the design lifetime requirement. In addition, the cost-per-operational-day results would change if, instead of implicitly assuming replacement, servicing for life extension was considered. This first estimation of the cost to design for a given lifetime will prove useful in our quantification of the trade illustrated by figure 1-2.

2.2.2 Cost Penalty to Make Satellites Serviceable

The impacts of serviceability are not limited to the positive aspects of designing for a shorter lifetime. Design changes would be required, at least for accessibility of the components.

Few papers have addressed the cost penalty to design a spacecraft for serviceability. An interesting study by the Aerospace Corporation [HP99] concerned the necessary design modifications to make the GPS satellites *upgradable*. Although this investigation was independent of servicer architecture, it had to depend on the interface with the servicing system; the assumption was that systems designed in [LW99] would be used. The study focused on possible satellite upgrades through the addition of new components, which were assumed to consist primarily of electronic boxes. It was therefore assumed that specific upgrade slots would be added to the baseline design and launched empty, ready to receive any additional module. Some thermal mass, data handling capacity and power would be added in the initial design in order to allow for this upgrade.

The authors evaluated several design alternatives. For upgrades only, the best alternative appeared to be the addition of a separate compartment on top of the spacecraft. For a combination of upgrades and repairs, a concept of replaceable panels on the spacecraft sides offered more potential. The corresponding percentage mass penalties on the 1300-kg baseline spacecraft are summarized in table 2.1.

According to this study, the mass penalty to design for upgrade through the addition of new components is of the order of 10% of a spacecraft total mass. Since it would not require any extra spacecraft compartment, designing for repairing and refurbishing should incur an even smaller mass penalty. Furthermore, the penalties described above refer to *modifying an*

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Table 2.1: Average Mass Impact for Serviceability. Adapted from [HP99]

Serviced Mass	Added Power	Mass Penalty
Upgrade 3.5% (50 kg)	125 W	9%
Upgrade 7% (100 kg)	250 W	15%
	0 W	7%
Upgrade+Repair 7%	100 W	15%
	0 W	12%
Upgrade 14%	500 W	25%
	0 W	11%

existing design. If an infrastructure for on-orbit servicing of space systems were available, spacecraft could be designed for serviceability in the first place. In addition, technologies for increased modularity would be developed. The mass penalty for serviceability is likely to be smaller in such a world than any study that uses the current spacecraft design paradigm would estimate.

With the information available so far, it is therefore not unreasonable to assume that the cost penalty to make a design serviceable is negligible compared to the cost penalty to design for a longer lifetime. However, further research into the design of serviceable spacecraft will be required before this assumption can be proven valid.

The previous sections set the *technological* stage for the thesis by depicting the current state of research into technologies, designs and baseline cost impacts of on-orbit servicing. Before exploring the cost-effectiveness of on-orbit servicing for space systems, we still need to set the *economic* stage by investigating the current state of research into valuation methods.

2.3 Estimating Value: Capital Budgeting Methods

This section reviews the three main families of methods to estimate project value: net present value (NPV) calculation, decision tree analysis (DTA), which is becoming popular among space managers, and the relatively new field of real options theory, which, to the author's knowledge has not yet been directly applied to space systems.

In order to fully capture the differences between these three approaches and how they address the valuation problems relevant to space missions, this section will consistently apply them to the simple following example:

A stock has the current value $S = \$200$ and its price after one period is uncertain: it can either go up to $\$400 = uS$ or down to $\$100 = dS$ (where we implicitly assumed $u = 1/d = 2$). Shareholder A ("the seller") holds one stock and gives shareholder B ("the buyer") the option, but not the obligation, to acquire this stock after one period for the set exercise price $E = \$200$. What is the value of this option? In other words, what price for the option are A and B likely to agree upon?

Note that this case is analogous to a *service-or-abandon* real option, where S would be the uncertain expected revenues from the market after the possible date of servicing, and E would an agreed-upon servicing price.

2.3.1 Traditional Method: Net Present Value

The traditional method for capital budgeting has been to calculate the net present value (NPV), which is the sum of future discounted cash flows (expenses and revenues). This method is still the most widely used to evaluate and compare space mission architectures [WL99]. An NPV calculation assumes that a fixed sequence of cash flows will be followed, and accounts for the time value of money by weighting them with a fixed discount factor. If for example, a yearly rate d is used to discount a discrete sequence of cash flows C_i over N periods, the net present value is:

$$NPV(N) = \sum_{i=1}^{N} \frac{C_i}{(1+d)^i} \tag{2.1}$$

For cash flow rates that are continuous in time, this translates into:

$$NPV(T) = \int_0^T C(t)\,e^{-rt}\,dt \qquad (2.2)$$

where the equivalence of the formulas is ensured if $r = \ln(1 + d)$.

Example of net present value calculation   Let us look at our example from an NPV point of view. Traditional valuation would consider that the stock will be bought whatever its value. The NPV of buying the stock would therefore be:

$$NPV = p\,\frac{uS - E}{1 + \hat{r}} + (1 - p)\,\frac{dS - E}{1 + \hat{r}} \qquad (2.3)$$

where p is the probability that the stock price goes up, and $\hat{r}$ is the estimated discount rate over one period. The NPV of not buying the stock would be zero.

Taking for example $p = 1/4$ and $\hat{r} = 0$, we would have:

$$NPV = \tfrac{1}{4} \times 200 + \tfrac{3}{4} \times -100 = \$\ -25$$

From an NPV point of view, deciding to buy the stock is not interesting, and the option would be discarded from the start.

Advantages of Net Present Value

A great advantage of NPV is that it can be easily generalized to non-monetary values.

The goal of a space mission is not always to earn revenues; the case is obvious for scientific as well as military space missions. Instead of being compared to the revenues, the costs of the mission are therefore weighted against what is often called a *mission utility* of a *Function*. Utility describes the metric of performance that is of prime interest to mission decision makers. For an imaging mission for example, this could be the total number of images taken during the mission lifetime that meet a certain resolution requirement.

The approach of NPV calculations, which is to make a best estimate of future benefits and sum them up, can be directly generalized to such value metrics, and has been in the past. We will refer to this type of generalization as *traditional valuation*. A good example is the GINA methodology [Sha99], which proposes a *Cost per Function $CPF = C/F$* metric.

Whenever there is uncertainty about a future cash flow, the expected value of the NPV $E\{NPV\}$ is usually considered. This same approach is easy to generalize for a cost per function metric, where either $E\{C\}/E\{F\}$ or $E\{C/F\}$ are directly available to calculation.

Shortcomings of Net Present Value

If there is little uncertainty about the future, a net present value calculation is a valid method to capture the value of the project. In the presence of uncertainty however, traditional valuation lacks accuracy for two main reasons: its does not account for flexibility, and it is faced with uncertainty in the discount rate.

The main downside of traditional valuation is its failure to account for the flexibility in managerial decisions. In the real world managers do not actually have to set their decisions for years ahead, but can instead adapt future decisions to future conditions. Therefore cash flows are not fixed, but will depend on the resolution of some uncertain parameter(s). Some negative cash flows will be avoided, while some good opportunities will be seized. Net present value calculations underestimate the value of this managerial flexibility, comparatively giving too much importance to less flexible projects.

What about the appropriate discount rate to account for the time value of money? A sum of money earned today can be placed in Treasury bonds, which are guaranteed to offer appreciation at the risk-free interest rate r, so that after T years the initial sum C would be worth $e^{rT}C$. Receiving the money later can be interesting only if it has an *internal rate of return* at least equal to r, in other words its value is growing at a rate faster than r. Symmetrically, paying an amount sooner is interesting only if its internal rate of return exceeds r. The appropriate discount rate, defined as the one that captures the value that people attach to time, is therefore exactly r.

Consider now the opportunity to invest in a project which is risky, i.e. that offers no guarantees on its expected revenues. If the expected return α on the project equals the risk-free interest rate, then it is more interesting to invest directly in Treasury bonds, since they are risk-less. Therefore the project is worthwhile only if is offers a *risk premium* $p = \alpha - r$ that outweighs its risk.

Thus whenever there is uncertainty, the appropriate discount rate $\alpha = r + p$ depends on

the level of project risk. Risk is a function of external uncertainty as well as the internal ways to react to this uncertainty. It is not only difficult to estimate, but also varying with time. Whenever the conclusions drawn from a net present value calculation depend on the discount rate that was used, they should therefore be taken cautiously.

2.3.2 Accounting for Managerial Flexibility: Decision Tree Analysis
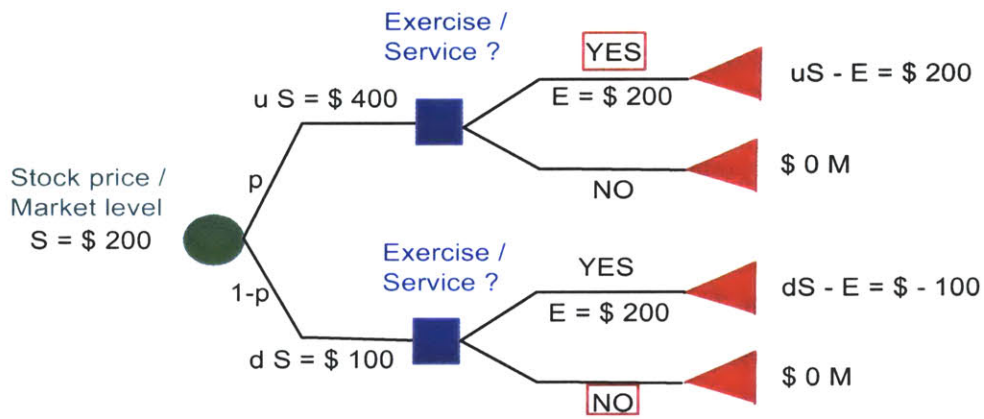
Figure 2-2: Example for Comparison of Net Present Value (NPV), Decision Tree Analysis (DTA) and Real Options Theory

Decision tree analysis (DTA) is a tool to describe a sequence of decisions that are not set from the start but can depend on the resolution of some uncertain parameter(s). Figure 2-2 is the simplest possible example of a decision tree. DTA takes flexibility into account by using the following concepts:

Event nodes are used to represent future events that have an uncertain outcome. In the lifetime of a mission, there can be several such events. In our example, the event is the evolution of the stock price and the event node is represented by a circle.

States of nature represent all the possible outcomes of an event. In our example, each potential future value of the stock is a state of nature. They are represented by branches shooting up from the event node. Decision tree analysis attributes a probability to each possible state of nature after each event node.

Decision nodes represent the times in the lifetime of the mission when a decision can be

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taken. Alternative decisions shoot up as branches from the decision node. In our example, the two possible decisions are to buy or not buy the stock and the decision node is represented by a square.

Decision analysis tree is the term used to describe the structure built up from decision and event nodes and all their associated branches. Time flows in the tree from left to right.

Decision path is the term used to define one particular sequence of decisions and states of nature going from the origin of the tree to one of its possible ends. Summing the cash flows (or the utility function) along one decision path determines the *outcome* of this path. In our example, a decision path could be: the stock goes down and B buys the stock (outcome +$100-$200 = -$100); there are four decision paths in this tree.

Backwards valuation The valuation starts from the outcome of each path and moves backwards into the tree. Combining the outcome with the probability of the states of natures gives the expected value of each decision at the last decision nodes. Only the decision with the highest value is considered at each node, and taken as the new outcome to continue moving backwards into the tree up to the initial decision or event. The value thus yielded is often called the *expanded net present value* (eNPV).

Example of DTA calculation DTA takes the flexibility of decision maker B into account by adding the cash flows that actually correspond to the *optimal* decision for every possible evolution of the stock price, as illustrated on figure 2-2. The value of the option under these conditions becomes:

$$V_{DTA} = p \; \frac{\max(uS - E, 0)}{1 + \hat{r}} + (1 - p) \; \frac{\max(dS - E, 0)}{1 + \hat{r}} \qquad (2.4)$$

which under the same numerical assumptions as above ($\hat{r} = 0$, $p = 1/4$) gives:

$$V_{DTA} = \tfrac{1}{4} \times 200 + \tfrac{3}{4} \times 0 = \$ \; 50$$

This shows that a flexible decision maker would actually find the option so interesting that she would be ready to pay as much as $V_{DTA} = \$50$ for it.

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The difference between NPV and V_{DTA} can be interpreted as the value F_{DTA} of the flexibility in having the right, but no obligation to exercise the option:

$$F_{DTA} = \tfrac{1}{4} \times (200 - 200) + \tfrac{3}{4} \times (0 - (-100)) = \$ 75$$

Advantages of Decision Tree Analysis

Similarly to net present value, decision tree analysis is easily generalizable to situations in which value is not monetary. DTA is actually a part of the *Decision Analysis* framework, which is also active in developing Utility function approaches. This makes the method particularly suited for space missions.

But unlike NPV, DTA considers only the optimal decision after each possible state of nature. Thus, it takes into account the possibility to adapt future decisions to the unfolding of uncertain parameters, which solves the main shortcoming of traditional valuation.

Shortcomings of Decision Tree Analysis

However, this valuation of flexibility is limited. For DTA to remain practical, there must be a finite number of decision nodes, occurring at set decision times. Thus DTA cannot account for continuous flexibility such as on-demand servicing. Similarly, there must be a finite number of possible states of nature after each event node. Thus, DTA cannot account for uncertain parameters that can take values in a continuous interval, such as market demand.

In addition, DTA does not solve the problem of the discount rate faced by NPV. On our example, table 2.2 shows that both NPV and DTA are a strong function of the probability p that the stock goes up, and therefore a strong function of the investment risk. This effect should however be counteracted by the use of the appropriate discount rate. The more likely the stock price is to go up (which increases the option value), the riskier it is to by an option instead of buying the stock today, and therefore the higher the appropriate discount rate (which decreases the option value).

Table 2.2: Results of Net Present Value, Decision Tree Analysis and Arbitrage Pricing Theory on a Simple Numerical Example

Case *bigtrisngleright* Method ▽	$r = 0$ $p = 1/4$	$r = 0$ $p = 1/2$	$r = 10\%$ $p = 1/4$	$r = 10\%$ $p = 1/2$
NPV	$ − 25	$ 50	$ − 22.73	$ 45.45
DTA	$ 50	$ 100	$ 45.45	$ 90.91
APT	$ 66.67	$ 66.67	$ 72.73	$ 72.73

2.3.3 A Leap Forward in Valuing Active Management under Uncertainty: Real Options Theory

Real options theory is the only method that solves the problem of the discount rate. Its principle is to extend the results from financial options theory to capital budgeting for real assets. Options pricing has been building on an initial seminal paper by Black & Scholes [BS73] about the exact situation we describe in our example. The first sentence of their abstract lays down the fundamental principle of option pricing:

> *'If options are correctly priced in the market, it should not be possible to make sure profits by creating portfolios of long and short positions in options and their underlying stocks.'*

Example of option valuation

Options theory cannot be summarized in one paragraph, but some insight into options pricing can be gained by directly applying the above principle to our example.

First consider the decision tree of figure 2-2, but with $p = 1/2$. Decision tree analysis gives $V_{DTA} = \$100$. Assume A offers B the option for this price. Before accepting the offer, B wants to make sure that there is no other investment she could make with the same amount of money, that would have a greater pay-off. For example, she considers buying $N = 2/3$ stocks of the underlying asset at $S = \$200$ and borrowing from a bank the missing money $B = NS - V_{DTA} = \$33.33$. At the end of the period, she could sell the stocks for either $N.uS$ (if the stock price goes up) or $N.dS$ (if the stock price goes down) and reimburse the amount $(1 + r)B$ to the bank, where r is now the risk-free interest rate. For

$r = 0$, the pay-off would be:

$$
\begin{cases}
N.uS - (1+r)(NS - V_{DTA}) = \$\ 233.33 & \text{if stock goes up} \\
N.dS - (1+r)(NS - V_{DTA}) = \$\ 66.67 & \text{if stock goes down}
\end{cases}
$$

By buying the option instead of the stock, the pay-off would have been:

$$
\begin{cases}
max(uS - E, 0) = \$\ 200 & \text{if stock goes up} \\
max(dS - E, 0) = \$\ 0 & \text{if stock goes down}
\end{cases}
$$

Therefore, spending V_{DTA} to buy the option on the stock is actually less interesting than investing in the stock directly, no matter what the future state of nature. Thus, V_{DTA} does not accurately represent the money that B would be willing to pay for the option. The actual value, V_{APT}, should be smaller.

Similarly, now consider $p = 1/4$, and assume that B offers to buy the option from A for the price $V_{DTA} = \$\ 50$. Instead, A could sell $N = 2/3$ stocks, and place the money difference $B = V_{DTA} - N.S$ in a bank. After one period, A would buy the stocks and retrieve from the bank the amount $(1 + r)B$ so that his pay-off would be:

$$
\begin{cases}
-N.uS + (1+r)(NS - V_{DTA}) = \$\ -183.33 & \text{if stock goes up} \\
-N.dS + (1+r)(NS - V_{DTA}) = \$\ 16.67 & \text{if stock goes down}
\end{cases}
$$

By selling the option, A would have had to pay to B:

$$
\begin{cases}
-max(uS - E, 0) = \$\ -200 & \text{if stock goes up} \\
-max(dS - E, 0) = \$\ 0 & \text{if stock goes down}
\end{cases}
$$

Therefore, getting the amount V_{DTA} for selling an option on the stock is in this case less interesting than investing in the stocks directly, no matter what the future state of nature. Thus, V_{DTA} does not accurately represent the money that A would be willing to receive for the option. The actual value, V_{APT}, should be higher.

From these two examples, we can infer that there must exist an equilibrium probability, $\tilde{p}$, and an equilibrium option value, $V_{APT} \neq V_{DTA}$, that make investing in the option

equivalent to investing in the stock directly. In this equilibrium, the portfolio comprised of N stocks and $B = NS - V_{APT}$ in the bank has the same pay-off as the option, no matter what the future state of nature. Therefore by *hedging* the portfolio against the option (i.e, buy the equivalent of one portfolio when you sell one option, and vice-versa), you can create a risk-free situation in which the payoffs are zero no matter what the state of nature . This is what is called an *arbitrage*. The equilibrium value of the option given by this *arbitrage pricing theory* (APT) is obtained by solving for (N, V_{APT}) the system:

$$\begin{cases} N.uS - (1+r)(NS - V_{APT}) = max(uS - E, 0) & \text{if stock goes up} \\ N.dS - (1+r)(NS - V_{APT}) = max(dS - E, 0) & \text{if stock goes down} \end{cases} \tag{2.5}$$

which results in:

$$V_{APT} = \tilde{p}\,\frac{max(uS - E, 0)}{1 + r} + (1 - \tilde{p})\,\frac{max(dS - E, 0)}{1 + r} = \$\ 66.67 \tag{2.6}$$

where

$$\tilde{p} = \frac{(1 + r) - d}{u - d} = 1/3 \tag{2.7}$$

is called the *risk-neutral probability*. It is the probability value for which decision tree analysis and arbitrage pricing yield the same option value.

Advantages of Real Options Theory

Similarly to DTA, real options theory considers only the optimal decision as a function of the outcome of the uncertain parameter(s), thus taking flexibility into account. In addition, options theory was initially developed for stock markets, which are continuous both in time and in possible values. Therefore it developed models that account for continuous probability density functions, as well as for continuous decision making.

Furthermore, real options valuation uses risk-neutral equivalent probabilities. This makes results independent of the risk level p, as shown by the numerical example in table 2.2. This allows to discount money at the risk-free interest rate r, which is usually available to observation and reasonably constant with time. Thus, there is no need for modeling the appropriate discount rate, which can actually be considered as a by-product of the valuation.

Shortcomings of Real Options Theory

Real options theory has never yet been used directly outside of the commercial world. One of its baseline assumptions is that the goal of every company is to increase the wealth of its shareholders. It has therefore never been interested in capturing non-monetary values.

Furthermore, real options theory applies the principles of options pricing to real investment situations. As we saw on the example, option pricing solves the problem of the discount rate by carrying out calculations using risk-neutral probabilities. This relies on the possibility to create risk-free, hedging portfolios of the option and its associated stock. In order to apply options pricing methods to real situations, there must therefore exist what is called a *twin security*, whose behavior on the stock mimics the value of the underlying investment. Thus, real options theory cannot be directly applied to all kinds of investment-making situations, in particular not to most space missions.

Chapter 4 is an attempt at developing a framework that applies decision tree analysis to space systems while capturing some of the advantages of real options theory.

2.3.4 Problem of the Discount Rate

A Measure of Risk-Aversion: The Capital Asset Pricing Method

There is no easy way to determine the discount rate given the risk on a project. But under certain simplifying assumptions, the capital asset pricing model (CAPM) [Mer73] can give an indication. According to the CAPM, if the risk associated with a project is independent on the overall risk of the market, then investors can "diversify away" this risk by investing in many different projects. Investors will only ask a risk premium for the part of the risk that cannot be diversified away, which is the part that is correlated to the overall market. The risk premium p is therefore given by:

$$p = (E\{r_m\} - r)\beta \qquad (2.8)$$

where $E(r_m)$ is the expected return on the market, and r the risk-free interest rate. The "beta" β of the project is a metric commonly used by economists. It represents the sensitivity of the project's returns to the market's returns and can usually be evaluated by

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performing least-squares regression on historical data.

$$\beta = \frac{cov(\alpha, r_m)}{\sigma_m} \qquad (2.9)$$

where α is the rate of return on the project and σ_m the variance of the market returns. This risk premium can also be written:

$$p = \lambda \, cov(\alpha, r_m) \qquad (2.10)$$

where the *market price of risk* λ is a function of the global market conditions only.

Generalizing the Real Options Theory Approach

As in decision analysis, in this thesis we want to determine the optimum decision path as a function of certain uncertain outcomes, and calculate a modified net present value given the probability of each outcome. But we also want to capitalize on the notion of continuous probability distribution for the uncertain parameter and solve as far as possible the discount rate problem. The assumption of close link to the stock exchange is not valid for most space systems, so that we cannot use the results of real options theory directly. Furthermore, we would like to define non-monetary values. The framework we will propose in chapter 4 can be seen as a generalization of the real options theory approach for space systems. It is in essence equivalent to a decision tree analysis with an infinite number of branches and two distinct treatments of the discount rate as described below.

Risk-neutral investors are investors that do not require a risk premium, such as governmental agencies. All costs for government projects can therefore be discounted at the risk-free interest rate r.

Risk-averse investors are investors that do require a risk premium, such as private investors. For commercial projects, we will therefore use two discount rates.

- Costs and revenues that are certain can still be discounted at the risk-free interest rate r.

- Costs and revenues that are uncertain require a risk-premium assumed to verify the CAPM given by equation 2.8. This requires to make an estimate of the β of

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the project on the one hand, and of the global trends of the market $E(r_m) - r$ on the other hand. This also assumes that these values are constant over the time frame of the mission; though clearly a bad approximation, this is at least an improvement over discounting everything at the rate r. We will describe uncertain costs and revenues by their total *rate of return* $\alpha = \alpha_p - p$, where α_p is the expected rate of appreciation (for example, the expected market growth rate). This artifact embeds the risk premium into the growth rate of the parameter. It can then be discounted at the risk-free interest rate r, so that some results of real options theory can remain valid.

Chapter 3

Cost-Effectiveness of On-Orbit Servicing: A Traditional Approach

Few studies have quantitatively addressed the costs and benefits of on-orbit servicing. Savings of up to 40% have been asserted. This was insufficient to outweigh the perceived cost and performance uncertainty for on-orbit servicing, which is often considered a risky new technology. Furthermore, most previous work has been focused on very specific case studies, so that no general conclusions about the cost-effectiveness of on-orbit servicing have yet been drawn.

A solution to this problem would be to systematically explore a wide trade space of servicing infrastructures and space missions, using cost models as a relative tool combined with risk assessment, in search of servicing cost-effectiveness. In a limited time such an approach could not reach the level of detail achieved by previous work. Only first order-of-magnitude approximations would be made possible. But this could help identify the conditions under which servicing makes the most sense, and the directions that would be most interesting for future research to investigate. This chapter proposes to develop a minimal model to make such an approach practical.

To this goal, section 3.1 sets up a general trade space of space missions and servicing infrastructures, defines the minimum parameters necessary to describe a generic on-orbit servicing situation, and proposes metrics for on-orbit servicing cost-effectiveness. Section 3.2 describes a minimal model to estimate these metrics on the trade space. Finally, section 3.3 illustrates the typical results that can be obtained by such an approach and concludes

on its usefulness.

3.1 The Trade Space

The trade space is made up of mission types on the one hand, and maintenance types on the other hand. It should be as large as possible so as to yield general results, but also detailed enough to capture the main drivers for the optimal maintenance type. The ultimate goal is to determine under what conditions the optimal servicing scheme is significantly more cost-effective than a traditional maintenance type. Thus, we need a first-order-of-magnitude model of the serviced and servicing missions that captures the most meaningful cost and performance trends.

3.1.1 Missions

Possible Missions The potential customers for on-orbit servicing can be divided into five main types of missions, which are summarized on figure 3-1:

A. A high value asset: servicing is the only alternative for maintenance of the *International Space Station* (ISS). For such a high value asset, manned servicing with the *Shuttle* is clearly cheaper than replacement. It has also been the option chosen for the *Hubble Space Telescope*, even though the total cost for the so-far three servicing missions exceeded the initial spacecraft cost. These examples suggest that there may be a minimum spacecraft cost over which on-orbit servicing is the optimal solution.

B. New missions: on-orbit servicing would make new types of space missions possible. For example, refueling would enable satellites to become truly maneuverable; this could help reduce the number of satellites in a radar constellation, as will be further discussed in section 6.1. It is also a vital technology for replenishment of reactants in a *Space-Based Laser* (SBL) system*.

C. A low-Earth orbit (LEO) constellation: for systems such as the *Big-LEO* commercial communication systems, the high number of satellites can help amortize the non-recurring cost of servicing.

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*http://www.fas.org/spp/starwars/program/sbl.htm
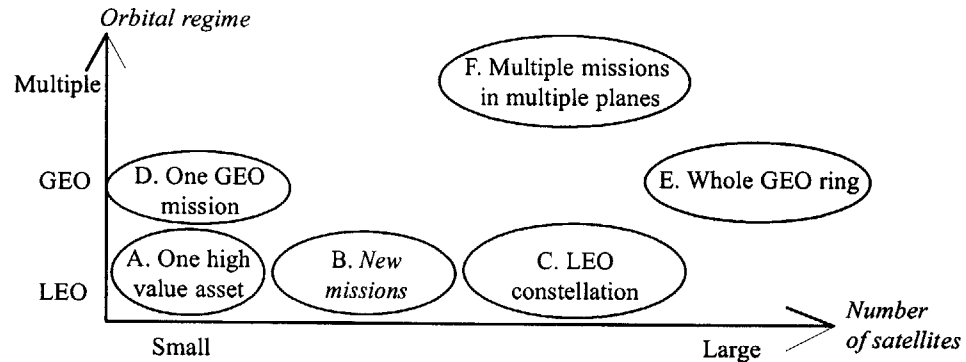
Figure 3-1: Trade Space Part I: Types of Missions

D. One geostationary orbit (GEO) mission: for GEO systems, not only is replacement very expensive, but the required satellite availability is often very high. For example, operators of GEO communication satellites are concerned about the potential loss of market share after any down-time. Servicing would offer a new alternative to trade availability versus cost.

E. The whole ring of GEO satellites: geostationary satellites are very numerous into the same orbital plane and at the same altitude. Thus, not only could their number help amortize the non-recurring servicing costs as for the LEO constellation. The incremental velocity to maneuver between all of them would also be relatively low, which would make servicer vehicles even cheaper.

F. Several missions in several orbital planes: the cost to develop an infrastructure for on-orbit servicing is likely to be much more expensive that what a single space mission could afford. It may be that on-orbit servicing becomes cost-effective only on a big scale, when each mission pays only the marginal cost of servicing. This situation has been compared to the national highway system in the United States, which could never have been developed by a single citizen or company [HLWS01].

Model of a Mission The wider the trade space, the more limited the level of detail that can be achieved in describing each mission. A minimum set of parameters is however necessary to yield meaningful results about the cost-effectiveness of servicing; at least the baseline cost and performance of the mission must be evaluated.

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Station capabilities

Depot/Station in space	3. Satellite roundtrips	4. Disposable servicer making roundtrips	5. Refuelable servicer making roundtrips
No station	1. Satellite replacement (traditional)	2. Disposable servicer launched with all cargo	✕
	No servicer	Non-serviceable servicer	Refuelable servicer

Servicer capabilities
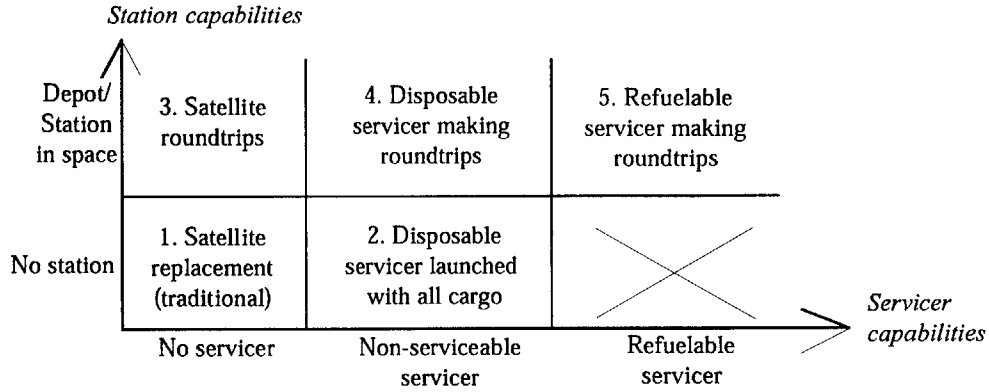
Figure 3-2: Trade Space Part II: Types of Maintenance Infrastructures

Important factors for cost at the constellation level include the number of orbital planes (N_p), the number of satellites per plane (N_{spp}), the orbital altitude (a_0) and inclination (i_0), plus the development (C_D) and yearly operations (o_p) costs. At the satellite level, the serviceable part (subscript C for *cargo*) must be distinguished from the non-serviceable part (subscript N). For each part we consider the failure rate (λ_C, λ_N) or the mean time to failure $(T_C = 1/\lambda_C$, $T_N = 1/\lambda_N)$, the mass (M_C, M_N), and the production cost (C_C, C_N).

Mission performance depends on the probability that the minimum mission requirements are met. In order to evaluate it, at least the required number of operational satellites per plane (N_{nspp}) must be known.

Finally, the impact of serviceability must be captured. In the rest of this chapter, we assume that all satellites are attitude-stabilized and carry either passive or active autonomous rendezvous and capture (AR&C) equipment. Whenever existing satellites are considered, this equipment must be added to their payload cost and mass.

3.1.2 Infrastructures

Changing orbital planes in Earth orbit requires a high incremental velocity. The majority of the reasonable servicing infrastructure designs can therefore be captured by restricting the trade space to launching servicing material separately into each serviceable orbital plane.

Two types of parameters are necessary to describe a maintenance (replacing or servicing) infrastructure. The first type consists of parameters that are uncertain today, but would be set if the technologies for on-orbit servicing were available. It includes production and

launch rates, maximum time allowed for a maneuver (ΔT_{max}), probability to crash when attempting an AR&C maneuver (P_C), and minimum servicer dry mass (M_0, corresponding to the AR&C payload with the power and structures to support it). Parameters of the second type describe the design choices for the maintenance infrastructure and are therefore subject to optimization. Possible maintenance choices can be classified into five families as summarized on figure 3-2:

1. **Without on-orbit servicing,** which is considered as the baseline, satellites are replaced if they fail, run out of fuel, or need an upgrade. The number of spares kept on the ground or on orbit, and the level of redundancy carried on-board, are subject to optimization. The number of spares increases mission cost but also mission availability. The level of redundancy increases initial cost but reduces replacement costs.

2. **Disposable servicer carrying all cargo** corresponds to the minimal servicing capability. The orbital replacement units (ORUs) to be delivered are launched on board a servicer vehicle with active AR&C capability. The servicer is disposed of after it has delivered all its cargo. The potentials for optimization include the number of satellites to visit with one servicer, the level of redundancy on the satellites, and the timing of launches.

The last three families correspond to cases when ORUs can be stored in orbit. They can be stored floating freely on their own, each carrying passive AR&C payload (depot); or they can be attached on an orbiting structure (station). In both cases the depot/station altitude (a_1) is subject to optimization.

3. **Satellites traveling to depot/station** If equipped with active AR&C and maneuver payload, the satellites can travel to the depot or station orbit to get serviced. Each ORU must then be attitude-stabilized and carry passive AR&C payload, which can represent a significant increase in mass and cost. This option has two further disadvantages. First, the down-time in mission availability while the satellites maneuver to the station can be unacceptable. Second, capability for full maneuver and AR&C on the satellites may not be available if they have failed or ran out of fuel.

4. **Servicer roundtrips** corresponds to the case when servicer spacecraft can be re-used

by going to the depot/station and loading new cargo. They perform roundtrips from the depot/station to the satellites orbits until they run out of fuel. Parameters for trade include the total number of satellites to visit per servicer (N_{sps}), the number of satellites to visit during each roundtrip (N), and the number of ORUs to keep on orbit.

5. **Refuelable servicer** is the same as 4 except that the servicer vehicles can be refueled at the station. This implies at least launching fuel tanks with passive AR&C payload. A new parameter for trade is the number of roundtrips that each servicer performs before being refueled (L).

Though they amortize the cost of a servicer over a longer lifetime, options 3-5 are also more risky. They indeed increase the number of autonomous rendezvous and capture maneuvers that are performed by each servicer. In practice, there will be many possible failure modes for a servicing operation, with various degrees of severity. For example, several AR&C attempts may be necessary before successful docking; this could push the servicing time over the required time, but would have no catastrophic consequence. But a failure could also consist of a crash between a servicer vehicle and a satellite, potentially leading to a complete loss of both spacecraft. Whatever the exact risk of the various servicing tasks, increasing the number of AR&C performed by each servicer increases the probability that *something* can go wrong, and thus decreases the servicer availability.

In the rest of this thesis, servicing risk will be described by only one failure mode: *crash with total loss of the two spacecraft*, having a constant probability P_C to occur at each AR&C attempt. This is the most that can be done before more technological data becomes available.

On-Demand versus Scheduled   Each of the schemes 1 to 5 can be carried out either on an on-demand or on a scheduled basis. In the scheduled case, a maintenance period (T_S) is defined. This method is well suited to service components with quasi-deterministic time to failure, such as fuel tanks for station keeping. In the on-demand case, components are serviced as they fail. This is well suited to components with a probabilistic time to failure.

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	A. High value asset	B. New Missions	C. LEO constellation	D. GEO constellation	E. Whole GEO ring	F. Multiple planes
1. Without on-orbit servicing						
2. Disposable servicer						
3. Satellites roudtrip to depot						
4. Servicer roudtrips to depot						
5. Refuelable servicer						

Figure 3-3: Trade Space Matrix Summary

3.1.3 Metrics for Cost-Effectiveness

The Generalized Information Network Analysis [Sha99]

The Generalized Information Network Analysis (GINA) methodology was proposed by Shaw & al [SMH01] as meaningful tool to evaluate space mission cost-effectiveness. It relies on the premise that most satellite systems can be represented as information transfer networks. Their quality of service is measured by four *capability metrics*:

Isolation characterizes the system's ability to isolate and identify the information signals from different sources within the field or regard.

Information rate measures the rate at which the system transfers information between the sources and the sinks. This is most familiarly associated with the data rate for communication systems. The revisit rate is the corresponding parameter for imaging systems.

Integrity characterizes the probability of making an error in the interpretation of an information symbol based on noisy observations. For communications, the integrity is measured by the bit error rate. The integrity of a surveillance radar system is a combination of the probability of a missed detection and the probability of a false alarm.

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Availability is the instantaneous probability that information symbols are being transferred through the network between known and identified origin-destination (O-D) pairs at a given rate and integrity. It is a measure of the mean and variance of the other capability parameters, not a statement about component reliabilities.

The Cost-per-Function Alternative architectures designed to meet the same mission requirements are compared by means of a Cost-per-Function (CPF) metric. The CPF amortizes the total lifetime cost over all satisfied users of the system during its life. The total lifetime costs include costs to initial operating capability (IOC) as well as operation costs and expected failure compensation costs. The Function is the expected total number of times the system will meet the minimum user requirements, expressed in terms of the four capability metrics. The CPF is therefore a meaningful quantitative measure of cost-effectiveness.

GINA has been successfully used to analyze the personal communication systems of *Iridium*, *Globalstar* and *ICO*; the broadband systems of *Spaceway*, *Astrolink*, *Cyberstar* and *Teledesic*; the Air Force *TechSat 21* space based radar experiment and the NASA *Territorial Planet Finder* (TPF) system based around interferometry from separated spacecraft [JMS00].

Servicing Cost-effectiveness

A servicing mission is not an information transfer network in itself. It is rather a mass transfer network. However, the final goal of the mass delivery is to enhance the capabilities of the serviced mission. Therefore on-orbit servicing of an information transfer network is defined as cost-effective if it reduces its Cost-per-Function compared to a traditional approach. The major effects that servicing can have on the CPF are:

- A decrease in the initial cost because less redundancy (or smaller fuel tanks) can be built into each satellite,

- A change in the failure compensation costs, which will not only include routine replacements and servicing, but also satellite and servicer replacement after potential AR&C crashes,

- A change in mission performance, because the probability to meet the requirements depends on the failure compensation scheme. Through failure compensation costs and mission performance, risk assessment is embedded in the mission Cost-per-Function.

Hypothesis

Given this definition of cost-effectiveness, a few hypotheses can serve as guides in the exploration of the design matrix summarized on figure 3-3:

H1. There is a minimum required incremental velocity per unit time over which it is cost-effective to refuel a spacecraft.

H2. There is a minimum cost to initial operating capability over which is it cost-effective to service a high-value asset.

H3. Limits can be drawn in a (serviceable part cost/ serviceable part mass) space as to whether redundancy, replacement, or servicing is most cost-effective.

3.2   A Model to Evaluate Cost-Effectiveness on the Trade Space

Testing these hypothesis requires a model to estimate the impacts of servicing on the various components of mission cost and mission performance. This section proposes a general model to capture the relevant first-order-of-magnitude effects.

3.2.1   Serviceable Spacecraft

Cost Savings from Repairing

One of the main advantages of a repairing capability is to decrease the initial spacecraft cost by designing space systems for a shorter design life. The cost savings incurred by shortening the required design life have been studied by Saleh & al [SHN01], and summarized in section 2.2.1. A linear fit to their final result gives the cost to initial operating capability C_{IOC} as a function of the design life T_D in the form:

$$\frac{C_{IOC}}{C_3} = 1 + \kappa\,(T_D - 3\,yr) \tag{3.1}$$
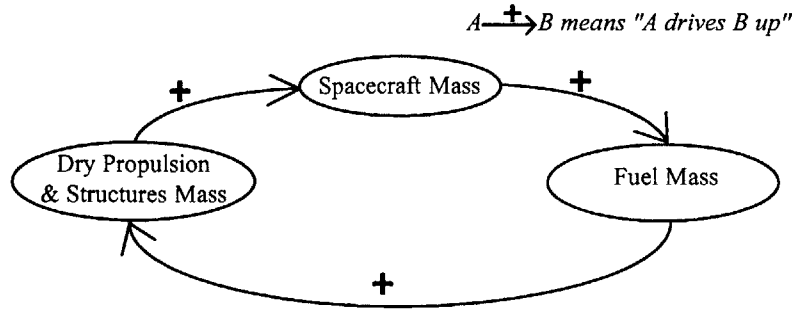
$A \xrightarrow{+} B$ means "A drives B up"

Figure 3-4: Feedback Effect Concerning Fuel Mass

where C_3 is the cost to design for an arbitrary reference of 3 years and $\kappa \approx 2.75\%/yr$.

Cost Savings from Refueling

If a spacecraft can be refueled, it does not have to be designed for the total velocity increment expected over its lifetime, but rather for the maximum total velocity increment between two refueling operations. The corresponding cost savings may be greater than what is suggested by [SHN01] because of a feedback effect between the fuel mass and the propulsion system dry mass. As illustrated on figure 3-4, the fuel mass is not proportional to the spacecraft baseline dry mass as usually assumed in the literature. As fuel mass increases, the propulsion system dry mass and the structures mass to carry it increase too. This increase in total dry mass in turn increases the fuel mass.

This feedback loop is usually taken into account numerically on a case-by-case basis by spacecraft designers. A simple mathematical description would not only be insightful, but also very useful for the implementation of a general model. Let us consider what this formula would be by making first-order-of-magnitude assumptions.

Mass budget as a function of design-ΔV Let us call *design-ΔV* and note ΔV_d the maximum total incremental velocity a spacecraft's propulsion system is designed to provide between two servicing operations.

Recalling the rocket equation (see for example [WL99]) the fuel mass a spacecraft has to carry increases exponentially with its design-ΔV :

$$M_{fuel} = M_{dry}^{tot} \left(e^{\frac{\Delta V_d}{I_{sp} g}} - 1 \right) \tag{3.2}$$

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where M_{dry}^{tot} is the total spacecraft dry mass and I_{sp} the specific impulse of its propulsion system.

Increasing the fuel mass also increases the propulsion system dry mass. For a given type of spacecraft design (a given type of fuel, a given shape of the fuel tank), the dry mass of the propulsion system is roughly proportional to its fuel mass:

$$M_{dry}^{prop} = f_p \, M_{fuel} \tag{3.3}$$

where f_p can be called the *propulsion dry mass factor.*

Similarly, the structures mass can be taken to be roughly proportional to the total spacecraft mass at launch: let us call *structures mass factor* and note f_{st} the constant of proportionality. Finally, if servicing is available, the spacecraft does not have to be launched with all its fuel: let ϵ be the fraction of the total fuel mass that is carried on the spacecraft at launch. Then:

$$M_{st} = f_{st}(M_{dry}^{tot} + \epsilon \, M_{fuel}) \tag{3.4}$$

Combining equations 3.2, 3.3 and 3.4 gives a linear equation for the total spacecraft dry mass:

$$
\begin{aligned}
M_{dry}^{tot} &= M_{dry}^{base} + f_p \, M_{fuel} + f_{st} \, f_p \, M_{fuel} + f_{st} \, \epsilon \, M_{fuel} \\
\Longleftrightarrow \quad M_{dry}^{tot} &= M_{dry}^{base} + (1 + f_p + f_{st} \, f_p + \epsilon \, f_{st}) \left(e^{\frac{\Delta V_d}{I_{sp} \, g}} - 1 \right) M_{dry}^{tot}
\end{aligned}
$$

where M_{dry}^{base} is the baseline spacecraft dry mass without its propulsion system and the structures to support it during launch.

Solving this equation shows that the total spacecraft dry and wet masses behave as the following functions of the design-ΔV :

$$M_{dry}^{tot} = \frac{M_{dry}^{base}}{1 - (f_p + f_{st}f_p + \epsilon \, f_{st}) \left(e^{\frac{\Delta V_d}{I_{sp} \, g}} - 1 \right)} \tag{3.5}$$

$$M_{launch} = M_{dry}^{tot} \left[1 + \epsilon \left(e^{\frac{\Delta V_d}{I_{sp} \, g}} - 1 \right) \right] \tag{3.6}$$
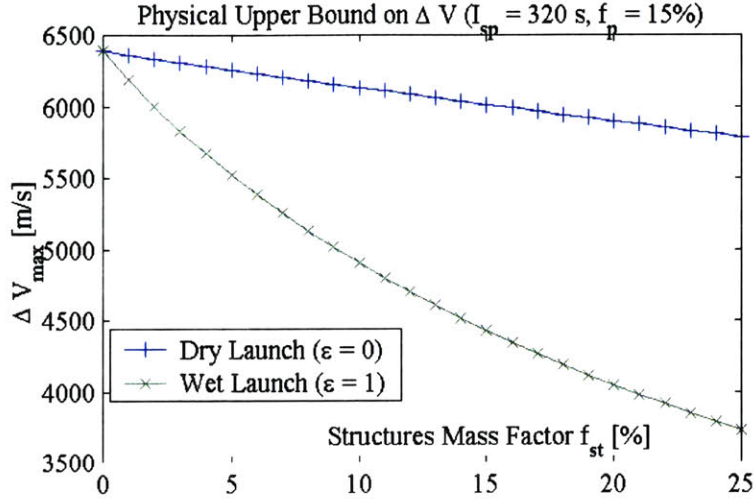
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Figure 3-5: Physical Upper Bound on Design-Incremental Velocity for $f_p = 15\%$, $I_{sp} = 320\,s$

A linear analysis would have given:

$$M_{dry}^{tot} = M_{dry}^{base} \left[1 + (f_p + f_{st}f_p + \epsilon\, f_{st}) \left(e^{\frac{\Delta V_d}{I_{sp}\,g}} - 1 \right) \right]$$

Comparison with equation 3.5 shows that this is a valid approximation only when the dry mass factors and the incremental velocity are very small: $f_{st}, f_p << 1$ and $\Delta V_d << g\, I_{sp}$.

Upper bound on design- ΔV   Equation 3.5 places a physical upper limit on the satellite maneuverability:

$$\boxed{\Delta V_{max} < I_{sp}\, g \, \ln \left[1 + (\epsilon f_{st} + f_p + f_p f_{st})^{-1} \right]} \tag{3.7}$$

Figure 3-5 plots this upper bound as a function the structures mass factor f_{st} in the special case $f_p = 15\%$ and $I_{sp} = 320\,s$ (chemical propulsion). Wertz & Larson [WL99] show that current design practices typically lead to $f_{st} \approx 20\%$. State-of-the-art technologies could give $f_{st} \approx 10\%$. In either case, figure 3-5 shows that launching the satellites dry ($\epsilon = 0$), and refueling them after launch, increases their maximum maneuverability ΔV_{max} significantly. This is due to the fact that any mass present at launch increases the structures mass required to protect the spacecraft from launch stresses.

Design-ΔV as a function of design life   Let us further explore how the above mass budget changes when a spacecraft is made refuelable. A typical spacecraft ΔV-budget is
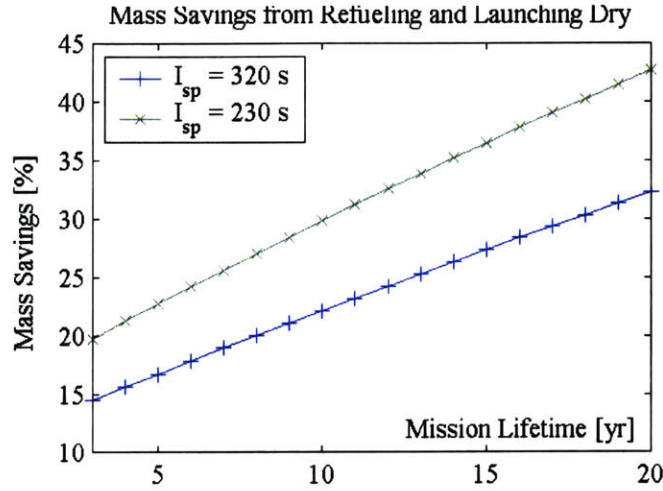
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Figure 3-6: Mass Savings from Refueling and Launching Dry for $\tau = 3\,yr$, $f_p = f_{st} = 15\%$, $\Delta V_{ins} = 100\,m/s$, $\Delta V_{stk} = 30\,m/s/yr$, and $\Delta V_D = 200\,m/s$

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made up of three components:

1. An incremental velocity for initial orbit insertion ΔV_{ins},

2. An incremental velocity for regular station keeping, which can be described as a yearly required ΔV_{stk},

3. And an incremental velocity for de-orbiting at end of life ΔV_D.

If a spacecraft is designed for a lifetime T_H without refueling, then its design-ΔV must be $\Delta V_d = \Delta V_{ins} + \Delta V_{stk}\,T_H + \Delta V_D$ and it must launched carrying all its fuel, so that $\epsilon = 1$. If on the other hand a spacecraft can be refueled every τ years, its design-ΔV is reduced to $\Delta V_d = \max\,(\Delta V_{ins}\,,\,\Delta V_{stk}\,\tau\,,\,\Delta V_D)$. In addition, it could be launched without its fuel and refueled right before final orbit insertion, so that $\epsilon = 0$.

Figure 3-6 shows the corresponding mass savings for numerical assumptions typical of a satellite in low-Earth orbit (LEO). As the design lifetime increases, the fuel for station keeping makes up a larger and larger fraction of the satellite mass, which makes refueling more and more interesting. Even for design lives as short as 3 years, up to 20% of a spacecraft mass can be saved by making it refuelable.

3.2.2 Maneuver Modeling

Maneuver modeling is key to sizing the spacecraft that moves in the servicing process, be it the serviceable satellite itself or a servicing spacecraft. Servicing maneuvers are of at least four kinds as described below:

Changes in inclination Even if the servicing material is launched into the satellites orbital plane, a slight difference in inclination (Δi) may have to be compensated for. The corresponding incremental velocity for an impulsive burn is:

$$\Delta V_{inc} = 2\,V_0\,\sin\left(\frac{\Delta i}{2}\right) \tag{3.8}$$

where V_0 is the orbital velocity at the satellite's altitude a_0.

Transfers from an orbital altitude (a_0) to another (a_1) In order not to impinge on the mission, the servicing material will be stored at a slightly different altitude. In the case of impulsive burns, a simple Hohmann transfer can be used to maneuver between two coplanar circular orbits[†]. Defining $\alpha = a_1/a_0$:

$$\frac{\Delta V_H}{V_0} = \frac{1}{\sqrt{\alpha}}\left|\sqrt{\frac{2}{1+\alpha}} - 1\right| + \left|\sqrt{\frac{2\alpha}{1+\alpha}} - 1\right| \tag{3.9}$$

A Hohmann transfer is also used for de-orbiting any spacecraft at end of life, requiring an incremental velocity ΔV_D. In the case of a LEO satellite or servicer, the transfer brings the spacecraft into an altitude at which it will quickly burn into the atmosphere (typically 150 km). A GEO satellite is instead boosted into a higher altitude, at which it will not impinge on any future mission.

Phasing maneuvers within one orbital plane Circular phasing is useful for a servicer to go from a satellite to the next. It can be performed by slightly raising the apogee and waiting for the difference in period to cancel the difference in phase. The impulsive incremental velocity for changing the phase by an angle ϕ in less than a maximum allowed

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[†]More details are provided in appendix B.1.1

time ΔT_{max} is[‡]:

$$\frac{\Delta V_{ph}}{V_0} = 2 \left| \sqrt{2 - \left(\frac{l+\epsilon}{l-\phi/2\pi}\right)^{\frac{2}{3}}} - 1 \right| \qquad (3.10)$$

$$\text{where} \quad l = \text{Integer part of} \left(\frac{\Delta T_{max}}{T_0} + \frac{\phi}{2\pi}\right)$$

$$\text{and} \quad \epsilon = \begin{cases} -1 & \text{if } \phi > 0 \text{ and apocenter is raised} \\ 1 & \text{if } \phi < 0 \text{ and pericenter is lowered} \\ 0 & \text{otherwise} \end{cases}$$

When the time allowed for the maneuver is much larger than an orbital period, then this incremental velocity becomes inversely proportional to ΔT_{max}:

$$\frac{\Delta V_{ph}}{V_0} \approx \frac{|\phi|}{2\pi} \frac{T_0}{\Delta T_{max}} \quad \text{when} \quad T_0 << \Delta T_{max} \qquad (3.11)$$

Fine maneuvers for the AR&C proximity and final phases   These maneuvers are made up of very small velocity increments, whose sum (ΔV_f) depends on the AR&C control algorithms. The higher the required AR&C reliability, the higher the necessary ΔV_f. As a first-order approximation, let us use a conservative $\Delta V_f = 150 \, m/s$ and assume a high AR&C reliability.

3.2.3   Servicing Infrastructure

Consider a servicer that has to visit N_{sps} satellites at an altitude a_0 and has the capability to load more cargo and to be refueled at an altitude a_1. This is the most general case as defined in section 3.1. Instead of looking for the optimal maneuver scheme by numerical simulation of all possible scenarios, this section proposes a mathematical representation of the servicer mass budget as a function of its maneuver scheme. This will fasten the exploration of the servicing trade space.

Servicer Maneuver Scheme   For all infrastructure types but type 3, the servicer's maneuver scheme can be described in terms of three integers (N, L, K).

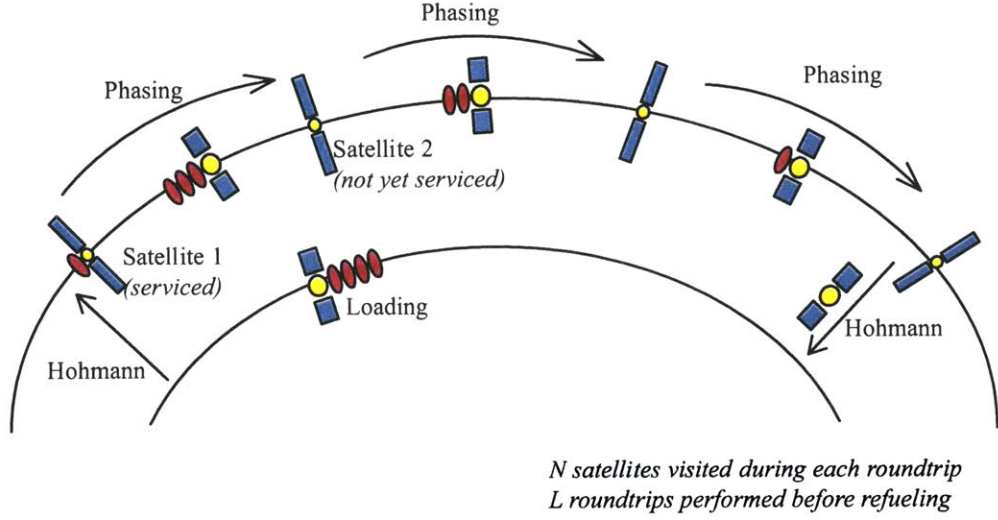[‡]More details are provided in appendix B.1.2

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Figure 3-7: Schematic of Servicer Maneuver Scheme

- N is the number of satellites that a servicer visits before loading more cargo. In the most typical case, these satellites are equally spaced within the same orbital plane, so that the maneuver from a satellite to the next always involves the same incremental velocity: $\Delta V_P = \Delta V_{ph} + \Delta V_f$. We call v_P the normalized velocity increment required to go from one satellite to the next:

$$v_P = \frac{\Delta V_{ph} + \Delta V_f}{I_{sp}\, g} \tag{3.12}$$

After visiting N satellites, the servicer returns to the station/depot altitude where it loads more cargo, then maneuvers back to the constellation altitude to visit the next N satellites. We call v_H the normalized velocity increment required to go from the servicer orbit to docking with one satellite. This is typically the same as the incremental velocity to go from the satellites up to docking with cargo, and is performed using a Hohmann transfer:

$$v_H = \frac{\Delta V_H + \Delta V_f}{I_{sp}\, g} \tag{3.13}$$

- L is the number of sets of N satellites a servicer visits before running out of fuel.

- K is the number of times a servicer is refueled to service NL more satellites. Thus the total number of satellites per servicer is $N_{sps} = NLK$.

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Finally, v_D is the normalized velocity increment to de-orbit:

$$v_D = \frac{\Delta V_D}{I_{sp}\, g} \tag{3.14}$$

This maneuver scheme is reminded by figure 3-7.

Servicer mass budget The effective dry mass of a servicer decreases over time as it delivers a mass M_C to each satellite. Appendix B.2 shows that its fuel mass is a function of its dry mass $M_{dry}^{servicer}$, the delivered mass M_C, and the maneuver scheme (N, L) as follows:

$$M_{fuel}^{servicer} = A\, M_{dry}^{servicer} + B\, M_C \tag{3.15}$$

$$\text{where} \quad A = \exp\left[L(N-1)\,v_P + 2L\,v_H + v_D\right] - 1$$

$$\text{and} \quad B = \frac{\exp\left[L(N-1)\,v_P + 2L\,v_H\right] - 1}{\exp\left[(N-1)\,v_P + 2\,v_H\right] - 1} \left(\frac{e^{N\,v_P} - 1}{e^{v_P} - 1}\, e^{v_H} - N\right)$$

Adding to the servicer minimum dry mass (M_0) the propulsion system dry mass and the structural mass to carry the propulsion system and the cargo yields:

$$M_{dry}^{servicer} = M_0 + (f_p + \epsilon f_{st} + f_p\, f_{st})\, M_{fuel}^{servicer} + f_{stC}\, N\, M_C \tag{3.16}$$

where f_{stC} is a structures mass factor corresponding to cargo mass $(f_{stC} > f_{st}$ because the cargo has to be moved for delivery). Solving the linear system given by the above two equations gives the servicer dry mass as a function of its maneuvering scheme and its specific impulse:

$$M_{dry}^{servicer} = \frac{M_0 + f_{stC}\, N\, M_C + (f_p + \epsilon f_{st} + f_{st}f_p)\, B\, M_C}{1 - A\,(f_p + \epsilon f_{st} + f_{st}f_p)} \tag{3.17}$$

We note that this places a physical upper limit on the extent of the servicer maneuvers in the form:

$$L\,(N-1)\,\Delta V_P + 2L\,\Delta V_H + \Delta V_D < I_{sp}\, g\, \ln\left(1 + \frac{1}{f_p + \epsilon f_{st} + f_{st}f_p}\right)$$

Optimizing the servicing infrastructure The total number of servicer vehicles and the dry mass of each servicer are the two main drivers of the total servicer cost. For a given number of satellites per servicer N_{sps}, equation 3.17 makes it possible to find the servicer

maneuver scheme that minimizes servicer dry mass. A cost model will be the tool to trade this dry mass against the number of servicers $N_{serv} = N_{spp}/N_{sps} = N_{spp}/NLK$.

3.2.4 Markov Model

Random Failures Satellite failures that are not deterministic are usually modeled by a failure rate $\lambda(t)$ such that probability P_f to be failed at $t + dt$ is only a function of the probability to be failed at t: $P_f(t + dt) = P_f(t) + [1 - P_f(t)]\lambda(t)dt$. This is a Markov process. Constellation performance, defined as the probability to meet the requirements as a function of time, is therefore modeled in the GINA framework via a Markov matrix A_M. A_M contains failure rates such that the vector of state probabilities (P) is described by:

$$\dot{P} = A_M P \qquad (3.18)$$

On-Demand Maintenance On-demand maintenance can be modeled by a repair rate μ such that the probability for a failed device to be repaired/replaced during t and $t + dt$ is μdt. If the failure rates and the repair rates are all constant, the system remains a Markov process; the Markov matrix now contains both λ's and μ's.

Deterministic Failures Satellite failures that are quasi-deterministic, such as fuel consumption for station keeping, cannot be modeled by a failure rate. We take these failures into account by incorporating a change in the Markov model initial conditions at the expected date of failure.

Scheduled Maintenance A simple change in the initial conditions is not adequate for scheduled maintenance because of two kinds of risks. First, the time of maintenance is not perfectly deterministic; a repair rate is a simple way of modeling this uncertainty. Second, there is a certain probability of failure (for example AR&C crash). A way to take both these effects into account while keeping a Markov process is to multiply the number of states in the model. If N_{sched} is the number of scheduled events, each state is divided into $(1+N_{sched})$ sub-states, going from *never maintained* until maintained N_{sched} times. At each scheduled time, the Markov matrix is changed to incorporate new repair rates for transition to the next sub-state.

Table 3.1: Markov Model Cases Summary

	Probabilistic Occurrence	Deterministic Occurrence
Failure	Failure rate λ appears in matrix	Change in the conditions at time of failure T_F
Maintenance	Repair rate μ appears in matrix	Repair rate μ appears at time of repair T_{sched}

Number of Failures The expected number of times that a system has to be "repaired" (serviced or replaced) from a state k is simply:

$$N_{rep/serv,k} = \int_0^{T_H} \mu_k \, P_k(t) \, dt \tag{3.19}$$

where $P_k(t)$ is the probability to be in the failed state k (result of the Markov model), and μ_k is the repair rate from this state. This relation is valid for the expected number of satellite replacements as well as for the expected number of servicing operations over the mission lifetime T_H.

Number of Satisfied Users The mission's Function is the result of the Markov model on the one side and a market model for the information transfer network on the other side. Let $\mathcal{M}_j(t)$ be the instantaneous number of *satisfied users* per unit time in the operational state j. Then the Function is:

$$Fn = \sum_j \int_0^{T_H} P_j(t) \, \mathcal{M}_j(t) \, dt \tag{3.20}$$

3.2.5 Cost Modeling

Three spacecraft cost models are publicly available: the unmanned spacecraft cost model (USCM7), the small satellite cost model (SSCM8) and a rule-of-thumb industry model. These models are based on cost-estimating relationships (CERs), which are equations relating cost to given design parameters. These relationships were derived from historical data. Their validity is therefore limited to a range of application, and is not exact but associated with a standard deviation.

- The CERs for the unmanned spacecraft cost model 7th edition (USCM7) are reproduced in [WL99] (tables 20-4 and 20-5, p 795-796). For each spacecraft subsystem, this model provides CERs for the theoretical first unit (TFU) cost, which represents the recurring part, and for the research, development, test and evaluation (RDT&E) cost, which represents the non-recurring part. The applicability ranges correspond to the traditional way of designing satellites.

- The CERs for the small satellite cost model (SSCM8) are reproduced in [WL96]. This model is more suitable to small, lightweight spacecraft designed with an aggressive cost-reducing approach. For convenience of use, each CER gives the total (recurring plus non-recurring) costs as a function of only one or two design parameters; when more parameters are known, the CERs can be combined to improve the accuracy of the cost estimate.

- The rule-of-thumb (ROT) industry model is convenient when very little information is available about the design. It relies on the premise that spacecraft dry mass is the main cost driver. The theoretical first unit cost C_{TFU} is simply proportional to dry mass and the development costs scale with C_{TFU} by a technological factor F. A typical numerical scale is $C_{TFU}/M_{dry} = 77,000\,FY00\$/kg$ [GVH$^+$97].

In the case of a servicer vehicle, the different subsystems fall into the applicability ranges for different cost models. While the payload, communication, and power subsystems correspond to SSCM8-type satellites, the propulsion and attitude control systems require the use of USCM7. This is due to the fact that cost models are not valid for such a spacecraft because it differs too much from the historical data that was used to derive them. Figure 3-8 illustrates this problem. Since a bottoms-up cost estimate is not possible when exploring a large trade space, the most that can be done to mitigate the problem is to use a combination of the three cost models. The total standard deviation σ is then obtained from the individual standard deviations according to:

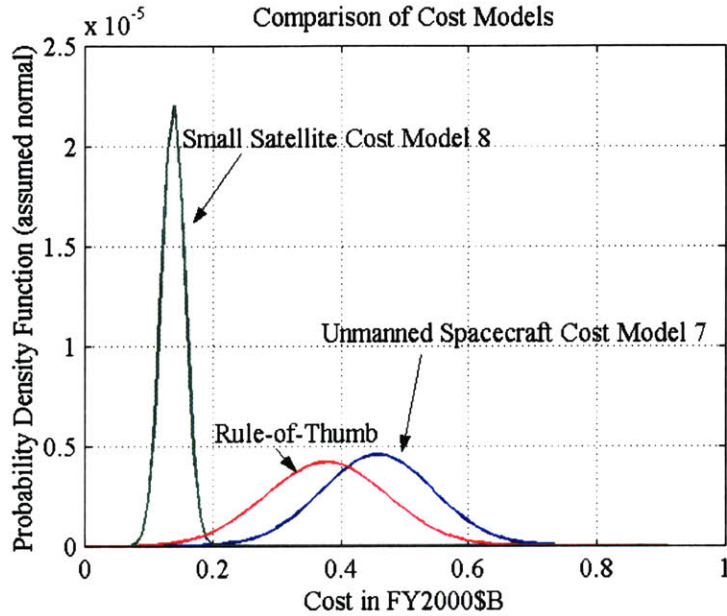$$\frac{1}{\sigma^2} = \sum_{i=1}^{3} \frac{1}{\sigma_i^2} \qquad (3.21)$$

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Figure 3-8: Comparison of 3 Cost Models for 4 Typical Servicer ($M_C = 200\,kg$)

An uncertainty $\sigma_{TRL} = 25\%$ linked to the low technology readiness level (TRL) of on-orbit servicing must be added to the servicing cost uncertainty obtained from the cost models:

$$\sigma_S = \sqrt{\sigma^2 + \sigma_{TRL}^2} \tag{3.22}$$

The total production costs are obtained from the TFU cost by applying a standard learning curve factor:

$$
\begin{aligned}
C_P &= N^B\, C_{TFU} \\
\text{with} \quad B &= 1 - \frac{\ln{(100\%/S)}}{2}
\end{aligned}
\tag{3.23}
$$

where as recommended in [WL99], $S = 95\%$ when less than 10 units are produced, $S = 90\%$ when the number of units produced is $N \in [10; 50]$, and $S = 85\%$ for more than 50 units.

Launch costs are minimized by a look-up-table method as a function of spacecraft mass, number of spacecraft, number of orbital planes, and launching nation; [WL99], table 20-18 p 812 is used as reference.

The same first-order model of operations costs [BL96] is used for the serviced mission and the servicing infrastructure. Operations for scheduled servicing are considered to occur
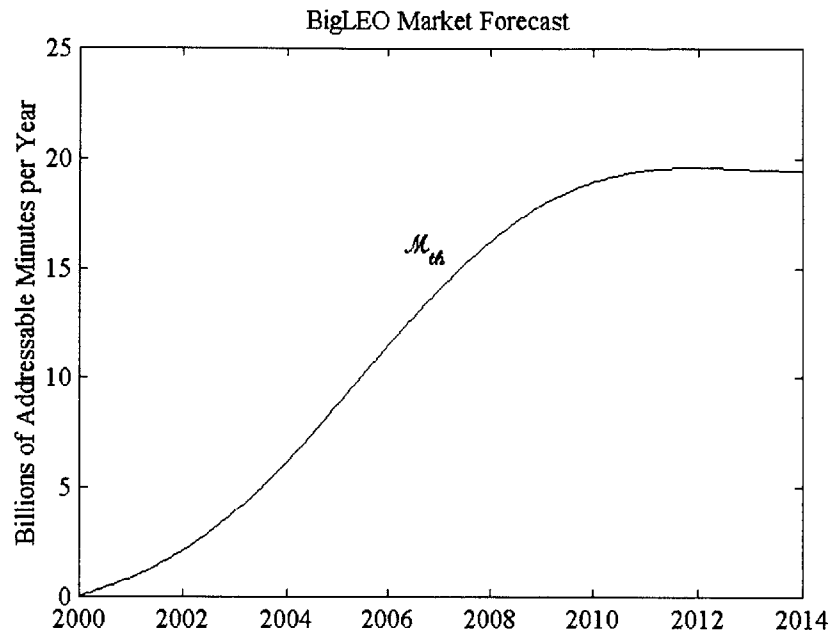
Figure 3-9: LEO 66 Market Forecast as Adapted from [GVH⁺97] (1997)

within the year of the event only. For on-demand servicing, a minimal operations team is considered to be constantly available.

Initial costs are spread to yield a funding profile ([WL99] equation 20-3), which is added to yearly operations, replacement, and servicing costs. Total costs are finally discounted to account for the time value of money and calculate the net present value of the total expenses.

The model is now ready to be applied throughout the trade space matrix (figure 3-3). Although several applications were considered, only one result will be analyzed here, chosen for its representative features.

3.3 A Typical Result: LEO Constellation of Communication Satellites

Problem Statement

Let us consider refueling a LEO communication mission, taking mission parameters on the example of the Iridium constellation [FRGT98], hereafter called LEO66. As seen by decision makers before the launch of the mission, the market was expected to grow quasi-linearly as described in [GVH⁺97] and illustrated on figure 3-9. If the actual market had followed this forecast, then it would have been interesting to extend the mission lifetime beyond its initial requirement of 8 years.

Three design alternatives The goal of this case-study is to compare three design alternatives, all aimed at achieving an effective lifetime of 16 years:

1. Keep the baseline required lifetime of 8 years and *replace* the satellites after 8 years,

2. Keep the baseline required lifetime of 8 years and *refuel* the satellites after 8 years. The non-recurring costs of building an on-orbit station are too high for only one mission. The mission cannot afford to move the satellites to a depot because mission availability is critical for market capture. The only choice is therefore infrastructure type 2, where servicers are launched with their cargo into each orbital plane. The refueling can be scheduled because fuel consumption is quasi-deterministic. The main servicing design parameter is the number of satellites to be refueled by each servicer.

3. Design the spacecraft propulsion system for *16 years of operation*. This increases the cost to initial operating capability, but requires no scheduled maintenance.

Tables 3.2 and 3.3 summarize the numerical assumptions.

Results in the Baseline Case

In the baseline case, the satellites propulsion requirements are dominated by the fuel to de-orbit, which accounts for almost a third of their total mass. Servicing makes it possible to do away with this weight by refueling at end of life.

The Cost-per-Function is here the lifetime cost per billable minute as defined in [GVH⁺97]. Figure 3-10 compares several servicing schemes with the two traditional alternatives; total

Table 3.2: LEO66 Constellation Assumptions

Parameter Description	Name	Value	Source
Constellation altitude	a_0	$780\,km$	[FRGT98]
Satellite launch altitude	a_{00}	$650\,km$	[FRGT98]
Number of orbital planes	N_p	6	[FRGT98]
Number of satellites per plane	N_{spp}	12	[FRGT98]
Sats per plane to meet the reqs	N_{nspp}	11	[FRGT98]
Satellite specific impulse	I_{spS}	$320\,s$	Chemical
Sat mean time to failure (MTTF)	$1/\lambda$	$9.18\,yr$	[FRGT98]
Time to replace one satellite	$1/\mu$	$3\,months$	Spare available
Satellite development cost factor	F_S	4	State-of-the-art
Discount factor	d	7%	Observed
Ballistic coefficient	BC	$50\,kg/m^2$	Assumed

Table 3.3: LEO66 Servicing Assumptions

| Parameter Description	Name	Value	Source
Probability to crash at AR&C	P_C	0.1%	Parameter
Servicer specific impulse	I_{spO}	$320\,s$	Chemical
Servicer launch altitude	a_1	$700\,km$	Assumed
Error in servicer inclination	Δi	$1°$	Conservative
Time allowed for approach	ΔT_{max}	$2\,days$	Typical
Final approach time	ΔT_f	$1\,day$	Typical
Docked time	ΔT_{docked}	$1\,day$	Typical
Final approach ΔV	ΔV_f	$120\,m/s$	Conservative
Attitude control ΔV	ΔV_{yr}	$30\,m/s/yr$	Typical LEO
Servicer development cost factor	F_0	6	New technology
Structures mass factor	f_{st}	0.1	Optimistic
Propulsion dry mass factor	f_p	0.1	Optimistic
Structures mass factor for cargo	f_{stC}	0.2	Estimated
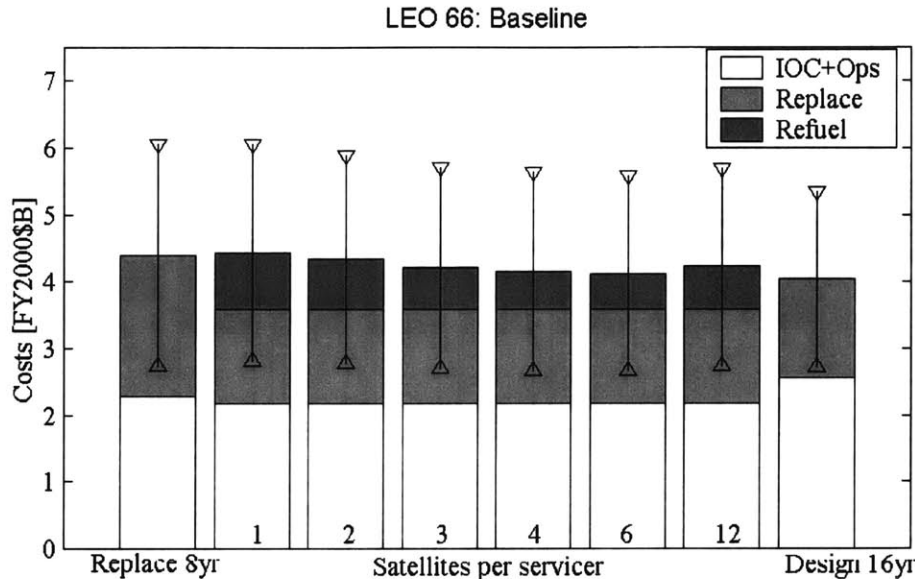
Figure 3-10: LEO66: Baseline Case Costs

costs are indicated in place of Cost-per-Function because the Function adds up to almost the same amount for all cases. The most striking result is the extent of the uncertainty bars, which outweigh the cost differences; we will return to this issue. Without considering uncertainty bars, the optimal servicing scheme appears to be 6 satellites per servicer, which corresponds to two servicers per orbital plane). First suppose that we want to improve flexibility, defined here as the ability to extend the mission life as a response to unexpected market growth; then refueling after 8 years is hardly more cost-effective than replacing the whole constellation. If 16 years was the initial desired lifetime, refueling is even less interesting. The satellite's ΔV requirements are low enough that they can carry fuel for 16 years without significantly increasing lifetime costs.

Allowing the parameters to vary and studying the response of the Cost-per-Function indicates what are the key parameters for servicing cost effectiveness.

Most Relevant Sensitivities

Sensitivity to Nominal Failure Rate In the baseline case, the satellites failure rate is so high that even with refueling, 75% of the constellation has to be replaced before end of life. At lower failure rates, empty fuel tanks become the dominating reason for satellite failure, increasing the relative advantage of refueling. Figure 3-11 illustrates the results for
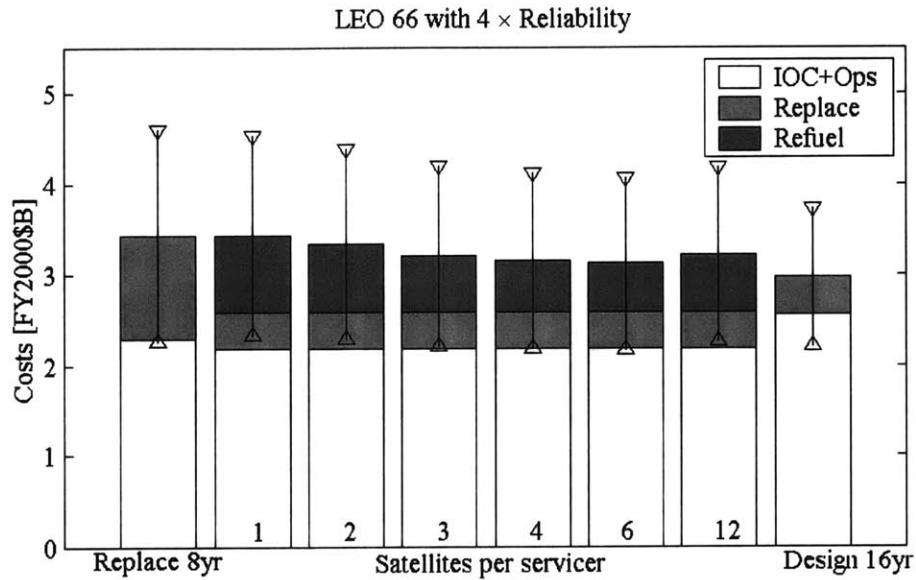
Figure 3-11: LEO66: Costs for a Higher Reliability

four times the nominal satellites mean time to failure.

Sensitivity to altitude   The lower the constellation altitude, the higher the ΔV_{yr} needed to compensate for atmospheric drag.   Under an altitude of around $400\,km$, it becomes prohibitive to carry fuel for 16 years. The advantage of refueling thus increases as altitude decreases, even though shorter refueling periods or bigger servicers become necessary. Figure 3-12 illustrates the results at an altitude of $400\,km$.

Sensitivity to AR&C Risk   The cost-effectiveness of refueling is sensitive to the probability to crash when attempting AR&C. Increasing P_C both decreases the mission performance and increases its failure compensation costs. Figure 3-13 shows how for P_C higher than one percent, the mission performance, which is the probability that the minimum number of satellites necessary to meet the requirements are available, drops below 70% just after each refueling event. This is an unacceptable risk when market capture is at sake.

Conclusion

This example is typical of all the results that can be obtained with the model we proposed, and that have been obtained by previous work on other special cases. Although situations
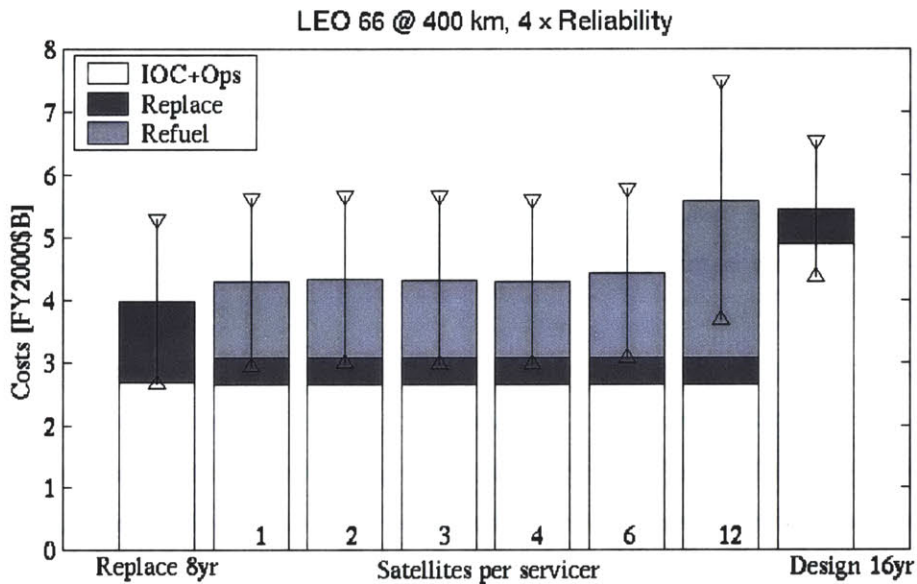
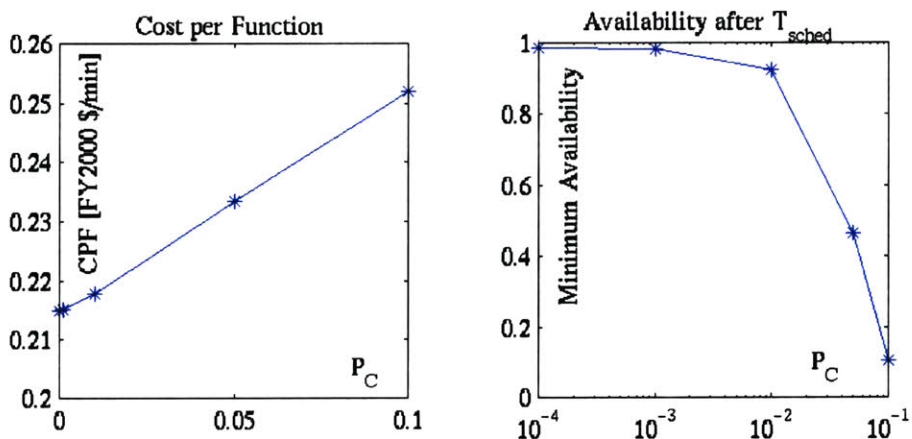Figure 3-12: LEO66: Costs at $400\,km$ Altitude

Figure 3-13: LEO66: Sensitivity to P_C: Performance and Cost per Billable Minute

can be found for which servicing proves cost-effective, the cost advantage always remains smaller than the cost uncertainty. This uncertainty is made up of two parts: the uncertainty in the constellation cost and the uncertainty in the servicing cost.

The uncertainty in the constellation cost arises from the standard deviation in the CERs used. CERs are designed from historical data to model the way costs depend on various design parameters. Their standard deviation reflects an uncertainty in *absolute* cost, but the *relative* cost difference between various designs are usually well captured. Thus cost uncertainty is only a minor limiting factor for the conclusions in terms of relative constellation costs.

However, we saw that a servicer spacecraft would be very different from historical satellites, so that cost models would not be directly applicable. Therefore, the uncertainty in the servicing price is here an absolute limiting factor, which makes any definitive conclusion about the cost-effectiveness of servicing impossible. This has also been one of the major problems faced by previous work on on-orbit servicing.

It is however interesting to note that at least one conclusion can be drawn by considering figure 3-10: if servicing were free, the lifetime costs for a serviceable design would be 10% lower than for a 16-year design. Whatever the price of servicing, the potential cost savings for this case cannot exceed this limit. This is an indication of the intrinsic *value of servicing*. It suggests a new perspective on on-orbit servicing, which we will now explore in more detail.

Chapter 4

A General Framework for the Value of Servicing Under Uncertainty

This chapter proposes a general framework that represents a new perspective on on-orbit servicing for space systems.

Based on the issues faced by the traditional methods to study on-orbit servicing , section 4.1 identifies the need for two major changes of approach as suggested by Saleh & al [SLH01]. First, the value of servicing should be evaluated separately from its cost. Second, a framework should be developed to account for the value of the flexibility provided by on-orbit servicing.

Section 4.2 constitutes a first attempt at proposing such a framework. It lays out fundamental principles for the valuation of flexibility, which will be used in the next chapters to develop models for the value of servicing in various special cases.

4.1 Motivation and Approach

4.1.1 Model Motivation: Inadequacies of Previous Methods

Separating Servicing Value from Servicing Cost

In the common approach used by previous work and expanded in chapter 3, whether on-orbit servicing proves more interesting that traditional methods is the result of a trade-off

between two main effects that we may summarize as: the cost savings from servicing versus the price the space mission is going to pay for servicing.

The cost savings from servicing depend mainly on the satellite design and the elements to be serviced. For any given space mission, they can be estimated with reasonably good accuracy. Using typical design assumptions and cost models based on historical data, their order of magnitude can even be estimated on a large trade space of missions. For example, we saw how Saleh & al [SHN01] estimated the cost penalty to design a typical spacecraft for a given lifetime.

On the other hand, the price that a space mission will have to pay for servicing depends on two factors that are both still very uncertain:

1. The *cost* of servicing depends not only on the cargo to be delivered to the satellites, but also on the technologies, design choices and cost models assumed for the servicing architecture. Servicing mass depends a lot on orbital dynamics and therefore on the specifics of the constellation to service on the one hand, and on the specifics of the servicing propulsion scheme on the other hand. Any conclusion can only apply to the type of infrastructure that was assumed, not to on-orbit servicing in general. Moreover, chapter 3 showed that cost models are not adequate for such an infrastructure. Therefore even these limited conclusions bear a very high uncertainty.

2. Furthermore, the *price* of servicing is not necessarily equal to its *cost*. The price also depends on the development policy for the infrastructure: the cost of the whole architecture could be paid by one space mission, or shared among several missions. Even better, an infrastructure could be developed by a governmental organization, so that only the marginal cost of servicing would be charged to individual space missions [HLWS01].

Estimating the cost savings from servicing separately from the servicing price would therefore present several major advantages. From the conceptual viewpoint, it could serve as a good indicator of the maximum price that a space mission would be willing to pay for servicing. This would correspond to the intrinsic value of servicing, independently of the servicing architecture. Missions for which servicing could make sense would be the ones for which servicing value is significant compared to total mission value. Furthermore, it would significantly reduce the uncertainty in the conclusions drawn.

Since no infrastructure for on-orbit servicing exists yet, results which would give some guidance as to what types of technologies to develop, what space missions to target as potential customers, and what cost cap not to exceed, would be very valuable. This is the type of results that can be obtained when studying the value of servicing separately from its cost.

Accounting for the Flexibility Provided by On-Orbit Servicing

One of the main advantages of on-orbit servicing is that it gives decision makers *options* to react to sources of uncertainty in the future.

The most obvious source of uncertainty consists of "random" failures; for example, a satellite launched into the wrong orbit could be refueled to maneuver to its design-orbit; or electronic components victim of single event upsets could be replaced. But another major source of uncertainty is becoming more and more problematic. Spacecraft design lifetimes are typically much longer than the time scales on which markets and technologies evolve. By the time a spacecraft is launched, there is a fair probability that it will be obsolete or not respond to any actual market. With on-orbit servicing, spacecraft could be designed for shorter lifetimes with an option on life extension if market demand is high, and/or with an upgrade option to avoid technological obsolescence. More generally, on-orbit servicing provides decision makers with the option to *modify* their mission to respond to changes in its requirements after the system has been fielded. This is a perfect example of *flexibility* as defined by Saleh & al [SH00].

Thus the price that a space mission would be willing to pay for servicing is greater than the potential cost savings incurred if choosing servicing instead of replacing. It may even be that a serviceable mission is more expensive in a strict sense, but provides the mission with so much flexibility that it is more interesting than a traditional design. The value of servicing should therefore account for the *value of flexibility*. This important advantage of on-orbit servicing over traditional methods has never been quantified by previous work, which relied on traditional valuation.

Building on decision tree analysis and real options theory, this chapter proposes a framework to account for the value of flexibility in modeling the value of space missions faced with external sources of uncertainty.

Table 4.1: Examples of Options Available to Space Missions

Source of Uncertainty	Option for Traditional mission	Additional Option for Serviceable mission
Operations cost	Abandon	-
Random failures	Replace	Refuel, Repair
Market demand	Abandon-or-replace	Abandon-or-service
Technology	Upload software Replace	Upgrade (e.g. computer)
New requirements	New mission	Modify

4.1.2 Example of Options Available to Space Missions

All space missions are at least flexible to some extent: the course of action for future operations and maintenance is never perfectly set on the day the mission it launched. In a way, the value of space missions has always been underestimated by not taking this flexibility into account.

The option to abandon if operational costs turn out to be to high compared to mission benefits is an example of option available to all space missions and that has been exercised in the past. Recent examples include the *Iridium* constellation, which was abandoned because its market turned out to be too low; or the *NEAR* (*Near Earth Asteroid Rendezvous*) spacecraft, which was abandoned though operational in order to save money. Section 5.2 studies the value of this option for the case of commercial missions.

The option to replace for life extension or after a random failure is another type of option available to all space missions. This expensive option has been exercised for very successful missions only, such as key scientific and military missions, or in-the-money geostationary communication satellites. For missions with high uncertainty, the value of the option to replace for life extension should be traded against the cost to design for a long lifetime. This trade has rarely been performed in the past, because of the high cost of replacing, but also the lack of a way to quantify flexibility.

The option to service for life extension would only be available to serviceable missions. Traditionally seen as an alternative to replacement, this option also offers an interesting potential to trade spacecraft design life against flexibility. We will see in section 5.3 how this trade leads to short optimal design lives for commercial missions with highly

uncertain markets.

The option to upgrade would be available to serviceable spacecraft to avoid technological obsolescence and improve their performance with respect to their baseline requirements. The same type of option is also often exercised by traditional systems when they upload new software to their satellite. Another example is the periodic upgrade of the *Hubble Space Telescope*'s components.

The option to modify would make it possible for serviceable spacecraft to respond to changes in their requirements. For a commercial mission, this could mean addressing new markets, such as data communication instead of voice. For a military mission, an example of an uncertain requirement is the location of the main theater of action; chapter 6 explores this option.

4.2 Servicing as Providing Options for Space Missions under Uncertainty

The next chapters will propose models to estimate the value of on-orbit servicing for different types of missions (commercial, military), having different types of options (abandon, replace, service, upgrade, modify) to react to different sources of uncertainty (market, requirements, technology, random failures). These models all deal with the same abstract goal: estimate the value of having options to react to the future resolution of uncertainty.

This section lays out fundamental principles that are common to any option valuation. These underlying principles, which apply to real options theory as well as decision tree analysis, can be seen as a general framework to embed the value of flexibility into the estimation of space mission value.

4.2.1 Basic Elements of the Framework

This subsection defines the basic elements required to fully describe a situation where options are available to adapt to the resolution of uncertainty. These elements are the building blocks of the framework. Their particular values and behaviors must be defined for each case under study as the first step in the valuation process.

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Uncertain parameter X In a world of certainty, options have no value. Therefore the cornerstone of any option valuation is modeling the uncertainty in the future states of nature. At least one uncertain parameter is required, which can be modeled as an instantaneous stochastic process X^*. This is a generalization of decision tree analysis to account for continuity in the possible states of nature. If several parameters are uncertain, X can be taken to have several dimensions: $X = (X_1, ..., X_n)$.

The most fundamental assumption on this source of uncertainty is that it should be *external* to the mission and not be affected by decisions taken after the system has been fielded. This is a valid assumption when desiring to capture how the availability of options reduces risk *not* by reducing uncertainty, but rather by affecting the *consequences* that uncertainty has on the mission.

In the following, we will also assume that the uncertain parameter follows a Markov process: the distribution of X at time $t > t_0$ knowing the path $X([0; t_0])$ is only a function of $X(t_0)$; in other words, only the latest information about X is relevant. We will note $p_t(x|x_0)$ the probability density function of X at t knowing $X(t_0) = x_0$. The main reason for this assumption is that it greatly simplifies the valuation process while allowing to capture some aspects of flexibility. It is reasonably valid for sources of uncertainty such as market dynamics, military contingency location, or random failures.

Mission horizon T_H In order to evaluate alternative mission scenarios on a fair basis, their costs and benefits should be compared over the same elapsed time $[0; T_H]$, where T_H will be called the *mission horizon*. For example, satellites can be designed for lives shorter than the mission horizon, in which case they may have the option to be serviced or replaced at the end of their design lifetime. This option must be considered in order to fairly compare them with satellites designed for a lifetime equal to the mission horizon.

The mission horizon must be long enough for the options to have a chance to be exercised. It must also be short enough to represent what is actually of interest to decision makers at $t = 0$.

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*Appendix A has a few more details on stochastic processes

Decision points T_k The valuation of options relies on the existence of decision points, which are times in the future when decision makers will have the option to choose between several alternative decisions. For example in the case of an option on life extension, the first decision point would occur at the end of the design lifetime, then on a periodic basis. In the case of on-demand servicing, decision makers have the option to continuously revise their strategy. This can be captured by modeling decision points as periodic with a very short period. It corresponds to a generalization of a decision tree to account for continuity in the decision points.

The time $T_0 = 0$ can be considered as the first decision point, at which there is a choice between doing nothing or building and launching a space system. The decision points are finally $T_0, ... T_N$ with $0 = T_0 < T_1 < ... < T_N < T_H$. For each T_k let us call *the next period* and note τ_k the time to the next decision point:

$$\tau_k = T_{k+1} - T_k$$

with the convention $\tau_N = T_H - T_N$. For simplicity of notation, for any variable Y we will note Y_k the quantity of Y incurred during the kth period: $Y_k = Y_{[T_k; T_{k+1}]}$.

Modes of operation (m) At each decision point, alternative decisions can be represented as several possible modes of operation as suggested by Trigeorgis [Tri96]. Typical examples of modes of operation that could be available to space missions are: (0) *abandoned,* (1) *operational in its initial design,* or (m) *operational with modification m.* We will mark the value of any variable Y in mode of operation (m) by an upperscript: $Y^{(m)}$; the value of any variable linked to a switch from mode (n) to mode (m) by $Y^{(n \to m)}$; and any variable linked to a history of successive modes of operation $(m_1, m_2, ... m_n)$ by $Y^{(m_1, m_2, ... m_n)}$.

Utility metric U The utility metric is a generalization of the notion of revenues encountered in real options for commercial missions. For $t_1 < t_2$, $U([t_1; t_2])$ is a measure of the aggregated benefits from the mission over the time interval $[t_1; t_2]$. These benefits are not necessarily monetary.

In most non-commercial cases, there can be several choices for the utility metric. For example, the benefits from a space-based radar mission could be the total number of

kilometer squared protected, or the total time a given area has been protected. The right choice should be the one that most describes what is of importance to decision makers. It must be such that, among several architectural alternatives with the same cost, decision makers would choose the one offering the highest utility function.

The utility metric can be a function of the uncertain parameter X; for example, market returns are a function of market demand. In such a case, U is a stochastic process.

Utility rate u For any meaningful model, mission benefits are an increasing function of time. In any given state of the system, there is be an instantaneous utility rate u such that $u = dU/dt$. If this utility rate depends only on the present state of the system, then we can say that *utility is time-additive*, i.e. total utility is the sum of past and future utility.

For example, the utility rate of a commercial mission would be its revenues per unit time. For an information-disseminating network, it would be the number of satisfied users per unit time. In both cases, the utility incurred over two years is the sum of utility incurred over the first year and the utility incurred over the second year.

Matrix of switching costs C The cost metric is the sum of all the expenses associated with the mission and its options. For $t_1 < t_2$, $C([t_1; t_2])$ is the present value of the aggregated costs of the mission over the time interval $[t_1; t_2]$. Costs are always time-additive. Certain cost components may be a function of the uncertain parameter X: in such a case, C is a stochastic process.

Three types of costs are associated with any mission:

1. The initial cost to develop, produce and launch the space system. This cost is commonly called *cost to initial operating capability* (IOC), and will therefore be noted C_{IOC}. It is a function of the mission type, the design life of the system, and the required reliability at end of life.

2. The cost to operate the system, which can often be represented by a constant operations cost per unit time $o_p^{(m)}$. In this case the present value at T_k of the

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cost to operate in mode (m) during the kth period is:

$$O_k^{(m)} = cd(\tau_k, r)o_p^{(m)}$$

(4.1)

where we define the *cumulative discount* function cd by

$$cd(\tau, r) = \begin{cases} \tau & \text{if} \quad r = 0 \\ \left(1 - e^{-r\tau}\right)/r & \text{otherwise.} \end{cases}$$

(4.2)

3. The present value $C_k^{s(n \to m)}$ of the costs to transition from mode (n) to mode (m). These are the cost incurred at a given time when the current mode is (n) and the decision to choose mode of operation (m) is made. Servicing and replacing costs are typical examples. Note that deciding to remain in the same mode of operation can also incur a cost, so that $C_k^{s(n \to n)} \neq 0$. For example, in order to stay operational at the end of its design lifetime, a spacecraft needs to be serviced. The fact that a certain mode (l) may not be accessible from mode (m) is taken into account by setting $C_k^{s(m \to l)} = \infty$; for example, a mission cannot be re-initiated once its spacecraft have been de-orbited.

We define the total *switching cost* $C_k^{(n \to m)}$ from mode (n) to mode (m) at a decision point T_k as the sum of all costs incurred during the kth period after deciding to switch to mode (m). This includes not only servicing costs, but also operations cost during the next period. Thus:

$$C_k^{(n \to m)} = C_k^{s(n \to m)} + O_k^{(m)}$$

The various switching costs from any mode of operation to any other mode of operation make up a *matrix of switching costs* $C_k = \left(C_k^{(m \to n)}\right)_{m,n}$.

Note on discounting costs Two types of discount rates are necessary for the valuation of non-financial options such as space systems, as was discussed in section 2.3:

- Costs and revenues that are not subject to uncertainty, or that mimic the behavior of a twin-security that is traded on the stock exchange, can be discounted at the risk-free interest rate r, but

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Table 4.2: Examples of Possible Value Functions

	Commercial	Military	Scientific
Utility U	Revenues	Utility f^n	GINA Function
Mission value V	$U - C$	U/C	$U/C = 1/CPF$
	$(U - C)/C$	$\max(U; C < cost\,cap)$	$\max(U; C < cost\,cap)$
Source of uncertainty	Revenues	Requirements	Funding

- Costs and revenues that are uncertain and not linked to a twin-security must be described by a rate of return $\alpha = r + \delta$.

Value metric V The value of a mission is a trade-off between its benefits and its costs:

$$V([t_1; t_2]) = f\{U([t_1; t_2]), C([t_1; t_2])\}$$

The value metric should be chosen such that among several alternatives, decision makers would systematically choose the one that maximizes the future expected value.

Examples of possible value metrics are given in table 4.2. For a commercial mission, value is often simply the difference between benefits and costs: $V = U - C$. For information-disseminating missions (which includes communications, scientific and most military missions), Shaw [Sha99] introduced the notion of Cost-per-Function (CPF). Capitalizing on this framework, utility can be taken to be the same as Function, in other words the total number of satisfied users over the mission lifetime. Value can then be the utility per cost: $V = U/C = 1/CPF$.

It is important to note that although costs and utility are time-additive, value is not necessarily. This means in particular that unless f is linear, maximizing future value is not equivalent to maximizing lifetime value[†].

The relevant uncertain parameters and decision modes are the ones that can affect the mission value. Therefore, value will always be a function of the uncertain parameter X. Value is thus a stochastic process.

Decision model The decision model describes how the decision should be taken at a

---

[†]The condition on the value function f for these goals to be equivalent is:

$$f(U_1, C_1) > f(U_2, C_2) \iff f(U + U_1, C + C_1) > f(U + U_2, C + C_2) \tag{4.3}$$

which is not met by $f(U, C) = U/C$

decision point T_k as a function of the current mode of operation and the current state of the uncertain parameter $x_k = X(T_k)$. If the value metric has been correctly defined, the decision will be to chose the mode of operation that maximizes future mission value. Let us call $EV_{\geq k}^{(n \to m)}(x_k)$ the expected value of the mission after T_k knowing the current mode (n) and the current state of the uncertain parameter $x = X(T_k)$ and assuming that mode of operation (m) is chosen:

$$EV_{\geq k}^{(n \to m)}(x) = E\left\{ V^{(n \to m)}([T_k; T_H]) \mid X(T_k) = x \right\}$$

The cornerstone of the valuation process is to consider only the optimal decision at each decision point. The optimal mode of operation $\hat{m}_k(n, x)$ at T_k is given by:

$$\hat{m}_k(n, x) \quad : \quad \max_m \left(EV_{\geq k}^{(n \to m)}(x) \right) \tag{4.4}$$

The total mission value that needs to be evaluated is:

$$V = EV_{\geq 0}^{(0)}(x_0)$$

4.2.2 Valuation Process Illustrated on a Simple Example

Once the building blocks described in the previous section have been properly defined, the expected value of the mission at time $t = 0$ can be deduced from an iterative backwards process. This section describes the simple case when there is only one decision point and two modes of operation, and the value metric is monetary. Building on this case, the next section will describe the valuation process in its most general setting.

The Simple Option on Life Extension

Consider a space mission designed for $T_D = T$ with the option to be serviced at T, thus increasing its lifetime up to T_H. At T decision makers will choose between two modes of operation: (0) *not operational* or (1) *operational*. Choosing (0) would incur the cost $C^{(1 \to 0)} = 0$ and choosing (1) would incur the cost $C^{(1 \to 1)} = C^s + O_p = E$, where C^s is the cost of servicing and O_p is the cost to operate the system from $t = T$ until $t = T_H$. We note this cost E because it is similar to the exercise price of a stock option: whereas a stock
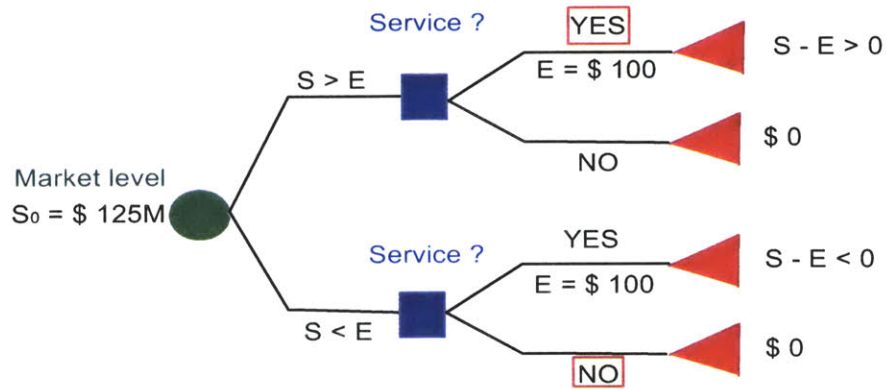
Figure 4-1: The Simple Option on Life Extension for a Commercial Mission

trader can buy an option on a stock, here the decision maker can buy an option on a life extension.

What source of uncertainty would make this option interesting? Let us consider a commercial mission. The revenues after time T are uncertain at the time of launch; we will call these S because they are similar to the stock price for a stock option. S is a stochastic process and at time $t = 0$ its value S_0 is observed. The uncertain parameter can be defined as the ratio $X = S/S_0$. Suppose we can observe the value $x = X(T)$ at time T: then staying operational is interesting only if it incurs more revenues than expenses, i.e. only if $x > E/S_0$. The decision model will thus define the intervals of possible values of x for which each decision should be taken:

$$\text{Abandon} \iff x \in I^{(0)} = [0\,; E/S_0]$$
$$\text{Service} \iff x \in I^{(1)} = [E/S_0\,; +\infty]$$

This situation is illustrated on figure 4-1. It is very similar to the example given in section 2.3 to compare valuation methods. The difference is that the possible values of the uncertain parameter are now the continuous range $[0; +\infty]$.

What is the probability distribution function of these revenues? If the market behaves

as a stock, its rate of change can be described as a diffusion process with volatility $\sigma^{\dagger}$:

$$\frac{dS}{S} = \alpha \, dt + \sigma \, dB \tag{4.5}$$

where

- B is the Brownian process with unit volatility and zero mean.

- α represents the expected rate of return of the revenues. For a risk-neutral financial investment, this is simply the risk-free interest rate r. In a more general setting, this can be written as $\alpha = r + \delta$.

With these assumptions, the uncertain parameter at T follows the log-normal probability density function:

$$p(x) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma\sqrt{T}} \frac{1}{x} \exp\left\{ -\frac{\left[\ln(x) - (\alpha - \sigma^2/2)T\right]^2}{2\sigma^2 T} \right\} \tag{4.6}$$

Derivation of the Black-Scholes Equation

For this commercial mission, the utility function U is equal to the revenues and the value is simply $V = E\{U - C\}$. This value being time-additive, the value at $t = 0$ of the option on life extension is simply the expected present value of the potential benefits incurred after time T. For risk-neutral investors, this is:

$$
\begin{aligned}
V &= \int_{I^{(0)}} V_{\geq 0}^{(0,1)}(x)\,p(x)\,dx + \int_{I^{(1)}} V_{\geq 0}^{(0,2)}(x)\,p(x)\,dx \\
&= \int_{0}^{E/S_0} 0 \times p(x)\,dx + \int_{E/S_0}^{\infty} e^{-rT}(x\,S_0 - E)\,p(x)\,dx
\end{aligned}
$$

---

†The definition can be found in appendix A.

Making the changes of variables $w = \frac{\ln(x) - (\alpha - \sigma^2/2)T}{\sigma\sqrt{T}}$ and $y = w - \sigma\sqrt{T}$ and defining $d_2 = -\frac{\ln(E/S_0) - (\alpha - \sigma^2/2)T}{\sigma\sqrt{T}}$:

$$V = \int_{-d_2}^{\infty} e^{(\alpha - r - \sigma^2/2)T} S_0 \frac{1}{\sqrt{2\pi}} e^{\frac{w^2}{2}} \, dw - e^{-rT} E \int_{-d_2}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{w^2}{2}} \, dw$$

$$= e^{(\alpha - r)T} S_0 \int_{-d_2 + \sigma\sqrt{T}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{y^2}{2}} \, dy - e^{-rT} E N(d_2)$$

The option value is finally:

$$\boxed{V = e^{\delta T} S_0 N(d_1) - e^{-rT} E N(d_2)} \tag{4.7}$$

where:

- N is the cumulative normal distribution function $N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{\frac{t^2}{2}} \, dt = 1 - N(-x)$

- $d_1 = \left[\ln(S_0/E) + (\alpha + \sigma^2/2)T\right]/\sigma\sqrt{T}$

- $d_2 = d_1 - \sigma\sqrt{T}$

When $\delta = 0$, equation 4.7 is identical to the *Black-Scholes equation*, which was a key result in the foundation of options pricing theory in 1973 [BS73]. The generalization of this equation for $\delta \neq 0$ is useful for cases when the underlying option does not behave as a financial asset [MS85].

Numerical example

Let us illustrate this result on a typical numerical example. Consider a mission designed for $T = 10\,yr$ with the capability to be serviced to extend its life until $T_H = 20\,yr$ for a service-and-operations price $E = \$100\,M$. Let us assume for this example that the forecast revenues after T are $S_0 = \$125\,M$, and that $r = 5\%/yr$ and $\delta = 0$. Figure 4-2 plots the value of the option to be serviced for life extension and compares it with the net present value after time T, as a function of the volatility σ.

A few general trends, which result from equation 4.7, are worth noting:

- When there is little uncertainty on the revenues, the option value equals the net present value of the project $tV = e^{\delta T} S_0 - e^{-rT} E$. This occurs when the volatility
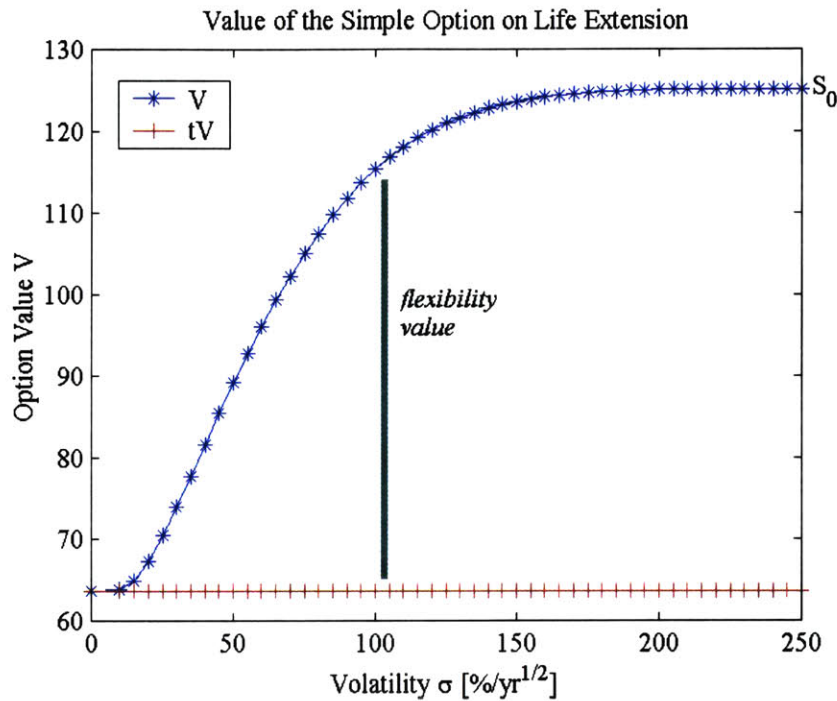
Figure 4-2: Simple Option on Life Extension for a Commercial Mission: Case $T_H = 20\,yr$, $E = \$\,100\,M$, $S_0 = \$\,125\,M$, $r = 5\%/yr$ and $\delta = 0$

is very small ($\sigma \to 0$), or when the valuation is performed very close to the decision time ($T \to 0$). It shows that traditional valuation is valid in a world of certainty.

- The option value increases with uncertainty. This is a direct illustration of the uncertainty being turned into an asset when having options.

- When there is high uncertainty in the forecast, the option value equals the net present value of the revenues alone: $V \to e^{\delta T}\, S_0$. This occurs when the volatility if very high ($\sigma \to \infty$) or when the valuation is performed very long before the decision time ($T \to \infty$). The amount $e^{\delta T}\, S_0$ is the worst possible difference between actual revenues and forecast revenues. It is therefore a statement that the value of an option cannot exceed the value of the potential losses it helps prevent.

Comparison with Net Present Value: Flexibility Value

A net present value calculation would not take into account the existence of an option.

For $e^{\delta T}\, S_0 < e^{-rT}\, E$, it would consider that the spacecraft will never be serviced, and therefore that there is no value in the servicing option. This corresponds to neglecting the

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probability that the mission be more successful than expected, thus being over-pessimistic.

For $e^{\delta T} S_0 > e^{-rT} E$, it would assume that the spacecraft will be serviced whatever the market level S. Therefore it would be over-optimistic, and overestimate the future expenses; the net present value incurred after time T would be:

$$NPV = tV = \int_0^{\infty} e^{-rT}(S - E)\,p(S)\,dS$$

which corresponds to underestimating the value of serviceability by an amount that can be defined as the *flexibility* value:

$$F = V - tV = \int_0^{E} e^{-rT}(E - S)\,p(S)\,dS > 0 \qquad (4.8)$$

Figure 4-3 is general plot of the option value and the flexibility value as a function of the cumulative volatility $\sigma\sqrt{T}$, for various values of the revenues forecast S_0 and the servicing price E. Two additional general trends are worth noting on this plot:

- As could be expected, the option value increases with decreasing servicing price or increasing forecast revenues.

- On the other hand, the *relative flexibility* value F/V decreases with increasing forecast revenues. This is due to the fact that the higher the forecast revenues, the lower the probability that the actual revenues will drop below the threshold level E, and therefore the lower the losses that can be prevented by having an option.

This example presented a situation that is realistic while simple enough to be solved analytically. Is showed general trends that will remain valid for almost all option valuation situations. The rest of this chapter proposes to build on this example to construct an options valuation process valid in the most general setting.

4.2.3 Valuation Process for a General Compound Option

Now consider the general case when there can be several decision points in the mission lifetime, at times $0 < T_1 < T_2 < ... < T_N < T_H$, several modes of operation $(0), (1), ...(N_M)$, and a non-linear value metric $V = f(U, C)$.

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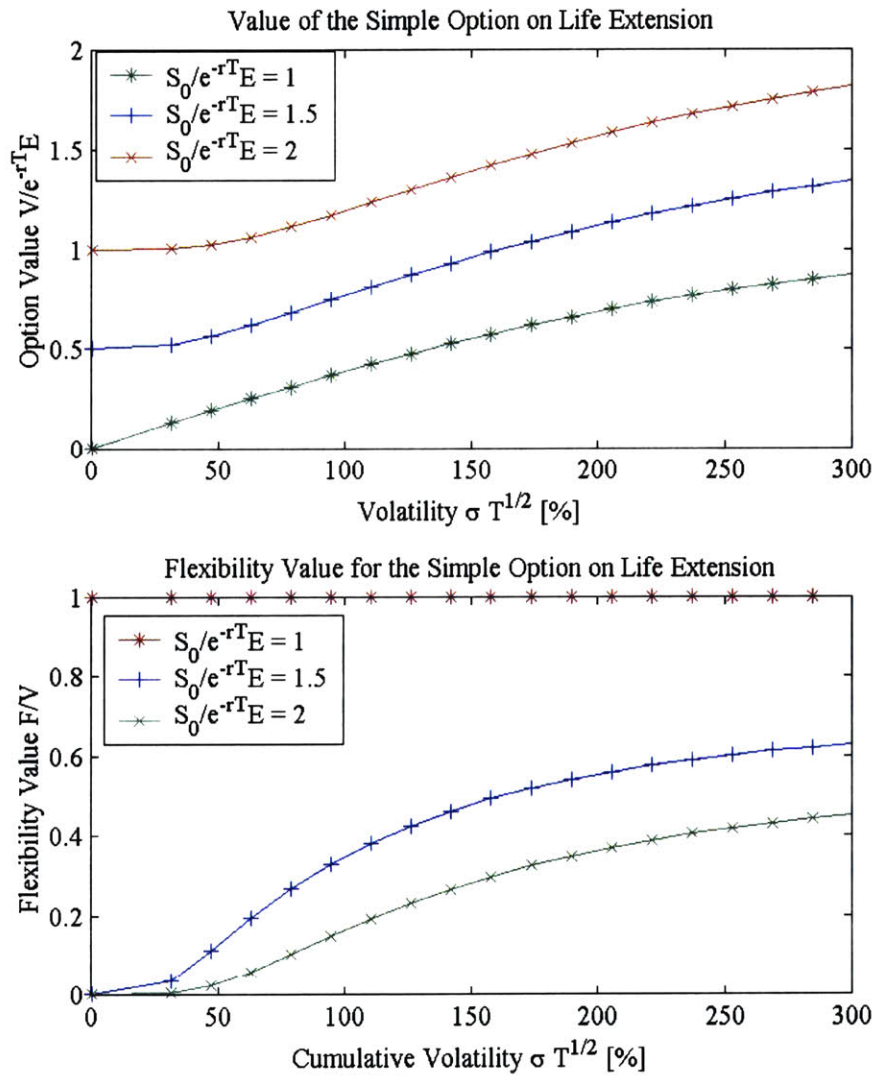
Figure 4-3: Simple Option on Life Extension: Value from Black-Scholes Equation. Case $T_H = 20\,yr$, $r = 5\%/yr$ and $\delta = 0$
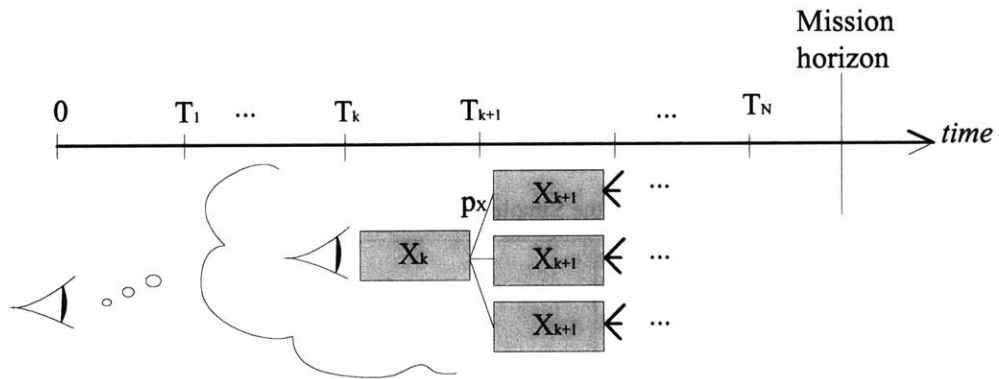
Figure 4-4: Schematic of Decision Situation at Node k

Choosing a certain mode of operation at time T_k gives the option to choose other modes of operation at time T_{k+1}. For example, a spacecraft that has been serviced once has bought not only a life extension, but also the option to be serviced once again. This corresponds to an option on an option, which is called a *compound option*. The value of this future option must be taken into account in estimating the value of the first option. The valuation process therefore starts off with the last option and proceeds backwards in time. It is very similar to working backwards in a decision tree, except that the finite number of branches is now replaced by a continuous density probability function.

Figure 4-4 is a schematic representation of this valuation process. The valuation is performed at $t = 0$, when only the initial value X_0 of the uncertain parameter is known. For each step T_k in the backwards process, the decision maker at $t = 0$ imagines:

> *"When I observe the uncertain parameter at T_k and look into the future, what mode of operation will I choose if I see $X(T_k) = x$?"*

Last decision

The last decision gives the "initial" condition of the induction process. At $t = 0$, both the uncertain parameter $X(T_N)$ and the mode of operation (n) in which the mission will be at T_N are unknown. The decision model must therefore determine for each possible entering mode (n) the sets $I_N^{(n \to m)}$ of values of the uncertain parameter at T_N for which a switch to

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mode (m) will maximize future value:

$$x \in I_N^{(n \to m)} \iff \max_l \left\{ EV_N^{(l)}(x) \right\} = EV_N^{(m)}(x) = f(U^{(m)}(x), C^{(n \to m)}(x))$$

Thus, while traditional valuation assumes only one possible mode of operation for the period $[T_N; T_H]$, with the new process the decision maker at $t = 0$ must think:

"If the previous mode of operation was (n) and $X(T_N) = x \in I_N^{(n \to m)}$ occurs, then I will choose to switch to mode (m)."

Induction Relation

Now consider decision point T_{N-1}. A future mode must be chosen on the basis of the previous mode of operation (l) and the observed value of the uncertain parameter $x = X(T_{N-1})$. In making the decision to switch to a mode (n), two things must be traded off:

- The cost $C_{N-1}^{(l \to n)}(x)$ and the utility $U_{N-1}^{(n)}(x)$ that will be incurred during $[T_{N-1}, T_N]$ as a result of choosing (n), and

- The cost and the utility that will be incurred after T_N, given that the decision point T_N will be entered in mode (n) and that the uncertain parameter $y = X(T_N)$ will follow the density probability function $p_{T_{N-1}}(y|x)$.

The choice of mode of operation that will be made at T_N is known as a function of (n) and y through the sets $I_N^{(n \to m)}$ determined at the previous step. Thus the value to maximize is:

$$EV_{\geq N-1}^{(l \to n)}(x) = \sum_{m=1}^{N_M} \int_{I_N^{(m)}} f\left(U_{N-1}^{(n)}(x) + U_N^{(m)}(y), \, C_{N-1}^{(l \to n)}(x) + C_N^{(n \to m)}(y) \right) p_{T_{N-1}}(y|x) \, dy$$

This determines the sets $I_{N-1}^{(l \to n)}$ of values of the uncertain parameter at T_{N-1} for which a switch to mode (l) maximizes future value.
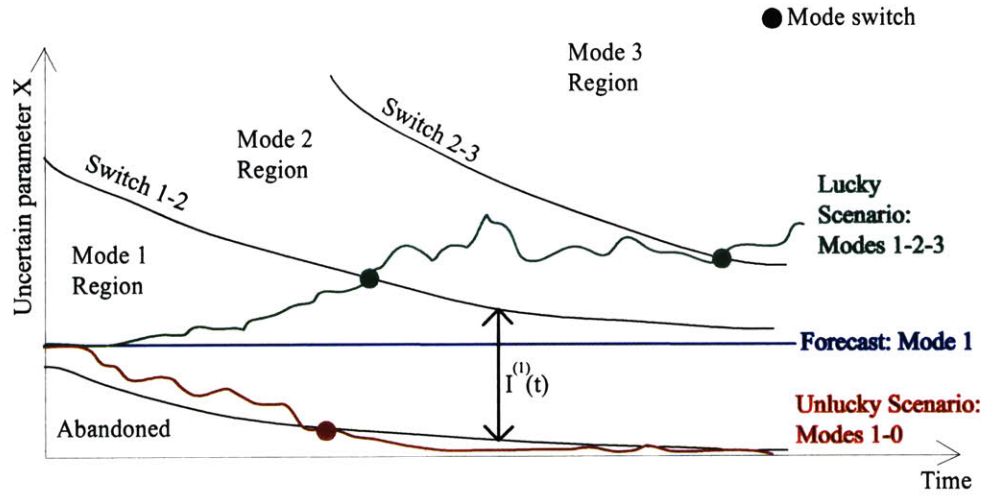
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Figure 4-5: Decision Model

At decision point T_{N-2}, the future value in turn writes:

$$EV_{\geq N-2}^{(k \to l)}(x) \sum_{n,m=1}^{N_M} \int_{I_{N-1}^{(n)}} p_{\tau_{N-2}}(y|x)\, dy \int_{I_N^{(m)}} p_N(z|y)\, dz \dots$$
$$f\left(U_{N-2}^{(l)}(x) + U_{N-1}^{(n)}(y) + U_N^{(m)}(z),\, C_{N-2}^{(k \to l)}(x) + C_{N-1}^{(l \to n)}(y) + C_N^{(n \to m)}(z)\right)$$

$$(4.9)$$

The same principle can be applied according to a backwards iterative process up to $T_0 = 0$ where it gives the total mission value as seen from the initial point.

Thus, the decision process maps the (time / uncertain parameter value) space into regions corresponding to different optimal modes. A mode switch will occur at each decision point when a boundary in this space has been crossed. Figure 4-5 is a conceptual representation of this mapping. Traditional valuation would assume that the forecast sequence of events would occur (straight line), overlooking the possibility of unlucky scenarios for which the mission would have to be abandoned, or lucky scenarios for which (for example) the spacecraft would undergo several upgrades. The proposed valuation process on the other hand recognizes the ability of decision makers to adapt to the resolution of uncertainty and make optimal decisions in the future. It therefore considers the optimal sequence of events for each possible evolution of the uncertain parameter with time.

4.2.4 Determination of a *Maximum Servicing Price*

In this chapter, we saw the different types of options available to space missions and proposed a general framework to embed this flexibility into the valuation of space mission architectures.

This framework can now be used to derive three types of information about the market base for servicing:

- The cost penalty that a space mission would be willing to pay to design for serviceability is directly given by the value of the option on life extension; an example is the Black-Scholes equation for the simple option on life extension seen in section 4.2.2.

- The value of a serviceable mission is a function of the price of servicing. Comparing this function with the value of a non-serviceable mission will give the *maximum servicing price* Υ that would make a space mission choose a serviceable design:

$$\text{Serviceable} \quad > \quad \text{Traditional} \quad \Longleftrightarrow \quad C_{service} < \Upsilon$$

- Determining the flexibility value relative to traditional value:

$$\frac{F}{tV} = \frac{V - tV}{tV}$$

will indicate by how much traditional valuation underestimates mission value, and therefore to what extent this framework is interesting.

The rest of this thesis applies this framework to find the value of on-orbit servicing and the maximum servicing price for two types of space missions. Chapter 5 builds a general model to deal with commercial missions faced with uncertainty in their future revenues. As examples of non-commercial missions, chapter 6 considers military space missions faced with uncertainty in the occurrence and location of contingencies.

But before jumping into the applications, let us consider the limitations to keep in mind.

4.3 Limitations of the Framework

The framework we proposed in the previous sections is only a first attempt at defining a general framework for the valuation of options for space missions faced with uncertainty.

Although it can account for many practical cases of space options, some simplifying assumptions had to be made, which limit its generality as described below.

4.3.1 Non-Fundamental Limitation

Discrete decision times For the sake of clarity, the previous sections implicitly assumed a finite number of set decision points. By setting the period to be infinitesimal, the framework can easily be generalized to continuous decision points. However, it is clear that an iterative backwards process such as described by equation 4.9 becomes impossible as the number of decision points tends to infinity. A solution to this problem is to alter the definition of value, defining

$$V' = f\left(E\{U\}, E\{C\}\right)$$

instead of

$$V = E\left\{f(U, C)\right\}$$

This makes no difference for linear valuation functions as the one used for commercial missions in chapter 5. It does for utility-per-cost metrics, and chapter 6 gives an example of a continuous-time model using $V = E\{U\}/E\{C\}$.

Finite number of modes of operation For the sake of clarity, the previous sections described the possible modes of operation as a *finite* set. This is not a fundamental assumption and the same framework can directly be generalized to the case of a continuous range of possible modes, such as a whole interval of possible orbital altitudes. The implementation would however become more complex.

4.3.2 Fundamental Limitations

Exogenous uncertainty The framework applies for cases when the uncertainty is exogenous, i.e. the source of uncertainty is external to the mission and cannot be affected by decisions taken after the system has been fielded. The presence of options reduces risk by affecting the *consequences* that uncertainty has on the mission.

This assumption can be an adequate description of the option to service to react to random failures, market and technology dynamics, or changing requirements. It can not be used to describe the interactions between the source of uncertainty and the mission's

decisions, such as the dynamics of competitive markets.

"Describing" the uncertainty   The proposed valuation process relies on the availability of the information necessary to define the *building blocks* of the framework. For many practical cases, this information is not observable. In particular, the probability density function $p(x|x_0)$ of the uncertain parameter in the future, which describes the uncertainty in the parameter's forecast, is usually very uncertain itself. Assumptions have to be made, and the sensitivity of the results to the assumed distribution must be estimated. As we will see in the following chapters, there is usually a *threshold uncertainty* over which the conclusions change. No conclusion can be drawn in situations where the uncertainty is estimated to lie close to this limit.

Forms of flexibility   This framework describes flexibility as a *known* set of possible modes of operation available to decision makers. It can therefore not account for the most general form of flexibility, which lies in the ability to define new, unpredictable modes of operation to respond to unknown sources of uncertainty.

Chapter 5

Value of Servicing for Commercial Missions with Uncertain Revenues

One of the most typical situations in which a significant source of uncertainty affects a space system is the case of a commercial space mission with uncertain revenues. In the light of the framework developed in the previous chapter, this chapter proposes to study the value of on-orbit servicing as a way to provide options to react to this source of uncertainty.

Section 5.1 describes the form taken by the building blocks and develops a general valuation model for the commercial case. It shows how a monetary definition of value simplifies the valuation process by making the model linear and very similar to the situations encountered in real options theory. The section concludes with a convenient method for numerical implementation of the model.

Section 5.2 uses this model to isolate and study the value of the compound option to abandon, which the rest of the thesis will consider available to all space missions.

This serves as a preliminary study for the crux of the chapter, section 5.3, which considers the value of servicing as a option on life extension to be traded against the cost to design for a certain lifetime. It shows that the framework can prove a powerful tool to define a new decision making approach regarding the choice of a design lifetime requirement when servicing is available. After identifying the general conditions under which servicing has the highest value, the section illustrates the value of flexibility on two case-studies with very high market uncertainty, inspired from the constellations *Iridium* and *Globalstar*.

5.1 General Model for Commercial Missions with Uncertain Revenues

The general valuation process presented in chapter 4 becomes relatively simple when considering the case of a commercial mission with uncertain revenues, for which the linear models and methods developed for real options theory apply almost directly. Section 5.1.1 sets up the model by defining the particular form taken in this case by the main basic framework elements. Using these baseline assumptions, section 5.1.2 lays out the valuation process for commercial missions with uncertain revenues is its general mathematical form. Section 5.1.3 finally describes a convenient numerical method to put this valuation process into practice.

5.1.1 Basic Elements of the Model

Section 4.2.1 defined the basic elements necessary to describe any option situation. For the case of a commercial mission with uncertain market, a few of these building blocks take a particularly simple form.

Uncertain Parameter X: Market Forecast

Definition Most commercial missions start off with a theoretical forecast for their expected market demand. Let us call $\mathcal{M}_{th}^{(m)}(t)$ the revenues per unit time that would be incurred based on this forecast if choosing mode of operation (m). Two typical examples are: a constant expected market $\mathcal{M}_{th}(t) = \mathcal{M}_0$; or a market base linearly increasing with time $\mathcal{M}_{th}(t) = \mathcal{M}_0 + a\,t$.

Market demand cannot be predicted perfectly: it is an uncertain parameter. The uncertainty in market demand translates into uncertainty in mission revenues, which is the parameter of direct interest to decision makers. We will take the ratio of the actual potential revenues in each mode of operation (m) over the theoretical market as our uncertain parameter(s):

$$X^{(m)} = \frac{\mathcal{M}^{(m)}(t)}{\mathcal{M}_{th}^{(m)}(t)}$$

With this definition, there can be as many uncertain parameters as there are modes of operation. This is necessary to take into account possible spacecraft modifications to address

new markets.

The exogenous uncertainty assumption In the real world, managers can make decisions that affect the revenues. All such decisions should be modeled as possible modes of operation, so that each uncertain parameter $X^{(m)}$ remains an external source of uncertainty. For example, $X^{(m)}$ can represent the number of people who would be interested in the service provided by the space system if it were upgraded to a certain mode (m). This number, which corresponds to *potential* revenues, varies as the result of external sources of uncertainty, such as global economical growth and the action of competitors. Only the *actual* revenues depend on the choice made by decision makers as to whether to upgrade the constellation or not. Thus, as soon as the modes of operation have been properly chosen, it is a fair assumption to assume that the uncertain parameter is an *exogenous* stochastic process, i.e an external source of uncertainty.

The geometric random walk assumption In order to describe the uncertainty, an assumption on its probability density function must be made. A convenient assumption used by real options theory is the geometric random walk process with drift α_m and volatility σ_m*, which is a good description of the behavior of stocks values. Under this assumption, if $X^{(m)}(t)$ is known then $x = X^{(m)}(t+\tau)/X^{(m)}(t)$ has a log-normal probability density function with mean $e^{\alpha_m \tau}$ and variance $\sigma_m \sqrt{\tau}$:

$$p_\tau^{(m)}(x) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_m \sqrt{\tau}} \frac{1}{x} \exp\left\{ -\frac{\left(\ln(x) - (\alpha_m - \sigma_m^2/2)\tau\right)^2}{2\,\sigma_m^2\,\tau} \right\} \tag{5.1}$$

The drift α_m is typically used to account for the time value of money.

This assumption presents two advantages in addition to its simplicity. First, the normal distribution (here used to describe the rate of deviation from expectation) is a usual assumption when the form of uncertainty is unknown. It is a valid approximation when the observed uncertainty is the sum of many independent uncertain parameters, which is often the case for the market dynamics that make revenues vary. Second, the variation of the standard deviation as $\sqrt{t}$ is a good description of the increase in uncertainty as one makes predictions further away in time. Examples of practical *cones of uncertainty* obtained form
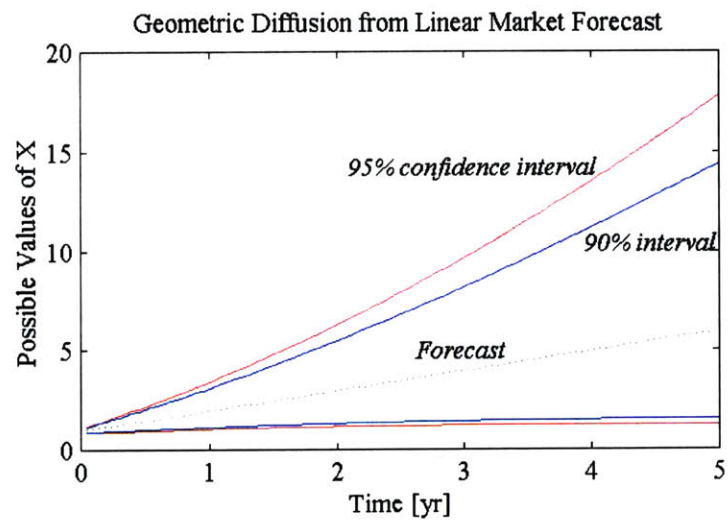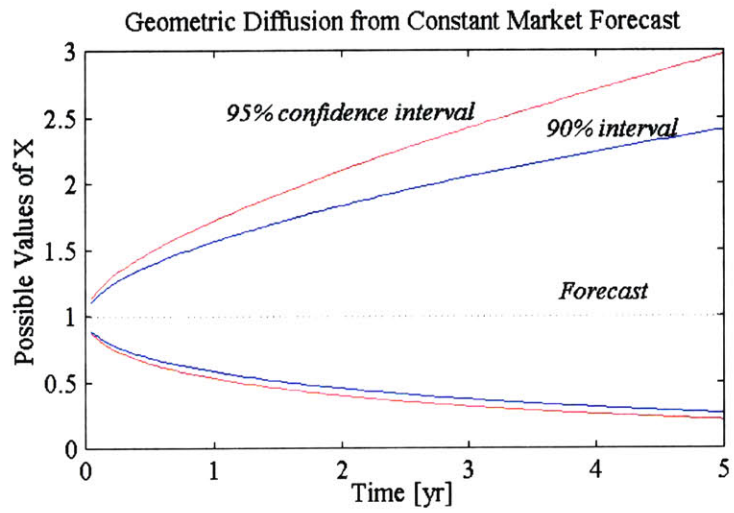
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*Defined in appendix A.

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Figure 5-1: Practical *Cone of Uncertainty*: 90% and 95%-confidence Intervals for the Log-Normal Probability around two Market Forecast as a Function of Time ($\sigma = 30\%.yr^{-\frac{1}{2}}$)

this assumption are given on figure 5-1.

The greatest shortcoming of the random walk assumption is the symmetry in the uncertainty, which gives equal probability to revenues that exceed expectations as to revenues that are lower than expectation. Though this would be a reasonable assumption for an ideal, unbiased forecast, in the real world space mission revenues rarely explode significantly over their forecast.

Cross-correlation In order to keep the model simple without sacrificing the main effects of uncertainty, we will further assume that we are in one of the two following extreme cases:

1. The various components of the uncertain parameter $(X^{(m)})$ are proportional. This corresponds to only one effective uncertain parameter $X = X^{(1)}$ with $X^{(m)} = \eta^{(m)} X$. This is a good description of a situation where potential modifications increase the level of performance for the same market. Or,

2. The various components of the uncertain parameter $(X^{(m)})$ are completely independent. This is a good description for a situation where potential modifications address different markets.

Utility Metric

Utility rate u For a commercial mission, the utility rate simply corresponds to the potential mission revenues per unit time. It is therefore equal to the uncertain parameter X times the revenues $\mathcal{M}_{th}$ predicted by the market forecast, and can depend on the current mode of operation (m):

$$u^{(m)} = \mathcal{M}^{(m)}(t) = \mathcal{M}_{th}^{(m)}(t) \, X^{(m)}(t) \tag{5.2}$$

We will note $x_k^{(m)}$ the value of the uncertain parameter *observed* at decision points T_k: $x_k^{(m)} = X^{(m)}(T_k)$.

Expected utility U The utility is simply the present value of the aggregated revenues. If its internal rate of return α has been defined as described in the introductory financial

section (2.3), it can be discounted at the risk-free interest rate:

$$U([t_1; t_2]) = \int_{t_1}^{t_2} e^{-rt} u\, dt \tag{5.3}$$

At a decision point T_k, the potential revenues rate $u_k^{(m)} = x_k^{(m)}\, \mathcal{M}_{th}^{(m)}(T_k)$ will be observed for each mode (m). According to the probability density function 5.1, the revenues rate expected at any latter time $t' = T_k + t$ will then be $E\{u^{(m)}(t')\} = e^{\alpha_m t}\, x_k^{(m)}\, \mathcal{M}_{th}^{(m)}(t')$. The value at T_k of the expected utility $EU_k^{(m)}(u_k)$ over the next period if choosing mode (m) will therefore be:

$$
\begin{aligned}
EU_k^{(m)}(x_k) &= E\left\{ \int_0^{\tau_k} e^{-rt}\, \mathcal{M}_{th}^{(m)}(T_k + t)\, X^{(m)}\, dt \right\} \\
&= x_k^{(m)} \int_0^{\tau_k} \mathcal{M}_{th}^{(m)}(T_k + t)\, e^{(\alpha_m - r)t}\, dt
\end{aligned}
$$

Define the forecast incremental revenues:

$$\boxed{ R_k^{(m)} = \int_0^{\tau_k} \mathcal{M}_{th}^{(m)}(T_k + t)\, e^{(\alpha_m - r)t}\, dt } \tag{5.4}$$

$R_k^{(m)}$ is a deterministic parameter. It is a function of the market forecast and the sequence of decision points only. It corresponds to the value at T_k of the revenues that would be incurred in mode (m) between times T_k and T_{k+1}, if the market followed its initial forecast. The utility expected if choosing mode (m) in the kth period is the corrected prediction once the uncertain parameter x_k is observed:

$$\boxed{ EU_k^{(m)}(x_k) = x_k^{(m)}\, R_k^{(m)} } \tag{5.5}$$

Value Metric V

For a commercial mission, value is simply the difference between benefits and costs: $V = U - C$. This linearity is extremely satisfying from a conceptual as well as from a practical point of view. For example, total mission value is the sum of past and future value. Similarly,

the expected future value is the sum of the value incurred in each of the future periods. It also ensures that:

- Maximizing future value is consistent with maximizing total mission value, and that

- The obtained value V is greater than the net present value of the mission: $V > tV$. This is a necessary condition to define a *flexibility value* $F = V - tV$.

5.1.2 *Expanded Net Present Value*

Last Decision point   Now consider the decision model at the last decision point T_N. The expected future value if switching from mode (n) to mode (l) given $X^{(l)}(T_N) = x^{(l)}$ is:

$$EV_N^{(n \to l)}(x) = EU_N^{(l)}(x^{(l)}) - C_N^{(n \to l)} = x_k^{(l)} R_N^{(l)} - C_N^{(n \to l)}$$

Maximizing $EV_N^{(n \to l)}(x)$ gives the sets $I_N^{(n \to m)}$ of values of $x = (x^{(0)}, ... x^{(N_M)})$ for which the optimal mode switch is (m) if the previous mode was (n). For example, if choosing between an abandoned state with $X^{(0)} = 0$ and a serviced state with $X^{(1)} = X$, then there is a minimum value $\Pi_N^{(1)} = C_N^{(1)}/R_N^{(1)}$ to choose mode (1), so $I_N^{(1)} = [\Pi_N^{(1)}; +\infty]$ and $I_N^{(0)} = [0; \Pi_N^{(1)}]$. This is exactly the situation encountered in 4.2.2 with $\Pi_N^{(1)} = E/S_0$.

The expected future value if entering the decision point T_N is mode (n) is then given by:

$$\forall x \in I_N^{(n \to m)} \quad EV_N^{(n)}(x) = x^{(m)} R_N^{(m)} - C_N^{(n \to m)} \tag{5.6}$$

Induction Relation   Now consider any decision point T_k entered in mode (l) observing the uncertain parameter $x = (x_k^{(m)})_m$. The future value $EV_{\geq k+1}^{(n)}(x)$ of the mission after the next decision point is known for each possible entering mode (n). The values of the uncertain parameters at $t = T_{k+1}$ have the joint probability density function $p_{T_k}(y|x)$. The expected future mission value if switching from the current mode (l) to mode (n) will therefore be:

$$\boxed{EV_{\geq k}^{(l \to n)}(x) = x^{(n)} R_k^{(n)} - C_k^{(l \to n)} + e^{-r\tau_k} \sum_{m=1}^{N_M} \int_{I_{k+1}^{(n \to m)}} EV_{\geq k+1}^{(n \to m)}(y) \, p_{T_k}(y|x) \, dy} \tag{5.7}$$

Maximizing $EV_{\leq k}^{(l \to n)}(x)$ in turn gives the sets $I_{k+1}^{(l \to n)}$. Thus, knowing the functions $EV_{\geq k+1}(x)$, equation 5.7 gives the functions $EV_{\geq k+1}(x)$. This relation is much simpler than the general

case described the previous chapter, for which a cascade of integrals on all future periods was necessary (equation 4.9).

The backwards iterative process finally gives the expected value of the mission as seen from $t = 0$:

$$\boxed{V = EV_{\geq 0}^0(1)} \tag{5.8}$$

Since it is the difference between the present value of the expected revenues and the present value of the expected costs, V is similar to a net present value (NPV). But unlike what is traditionally done when calculating an NPV, the calculation of V takes into account the optimal choices at each decision point as a function of the resolution of the uncertainty in X. For this reason, we may call this value *expanded net present value*, and abbreviate it eNPV.

Flexibility Value   The flexibility value is here simply the difference between the expanded net present value and the net present value:

$$F = V - tV = eNPV - NPV \tag{5.9}$$

An NPV calculation would assume a set sequence of modes of operation $(m_0, ..., m_N)$ according to the market forecast at $t = 0$, which simply gives:

$$tV = NPV = \sum_{k=0}^{N} e^{-rT_k} \left[x_0^{(m_k)} R_k^{(m_k)} - C^{(m_{k-1} \to m_k)} \right] \tag{5.10}$$

where by convention $m_{-1} = 0$ (*not operational*).

We can note that NPV and eNPV are equivalent when there is no uncertainty, in other words when the volatility of the market is null:

$$\boxed{tV = V\left\{\sigma^{(m)} = 0\right\}} \tag{5.11}$$

5.1.3 Calculating *Expanded Net Present Values*: Numerical Analysis Method

The previous section defined the expanded net present value (eNPV) for commercial missions faced with a geometric Brownian source of uncertainty in their revenues, and gave the backwards induction relation to calculate this value (equation 5.7).

This section presents the numerical method we chose to implement this equation. This convenient method is an application with only slight modifications of the *log-transformed binomial lattice method* proposed by Trigeorgis for the valuation of real options [Tri96].

For simplicity, we will here assume that there is only one uncertain parameter X, which follows a geometric Brownian motion process with expected drift α and volatility σ.

Log-transformation

The log-transformation consists in defining the intermediate variable $Y = \ln X$. Then $dX/X = \alpha\,dt + \sigma\,dB$ translates into[†]:

$$dY = \left(\alpha - \frac{\sigma^2}{2}\right) dt + \sigma\,dB \tag{5.12}$$

which means that Y is a Brownian motion with mean drift $\alpha - \sigma^2/2$ and volatility σ. Thus X, which has an exponential drift, is replaced by Y, which has a linear drift. This ensures the stability of the numerical method.

Discretized Brownian Motion: the Random Walk

The Brownian motion is the limit of a random walk as the size of the steps in the walk tend to zero. Since numerical methods require discretization, it is natural to discretize Y "*back*" into a random walk process. Let δt be the time step of the simulation. During each time step, Y will change by a probabilistic amount δY. The discretization reduces the continuous probability density function of δY to a probability P that $\delta Y = +\Delta Y$ and a probability $1 - P$ that $\delta Y = -\Delta Y$. The appropriate values of ΔY and P are the ones that

---

[†]dY cannot be obtained through standard calculus because B is not differentiable. The demonstration of this formula can be found in appendix A

Y_j

P

Y_0

$1-P$

δt

$j\delta t$

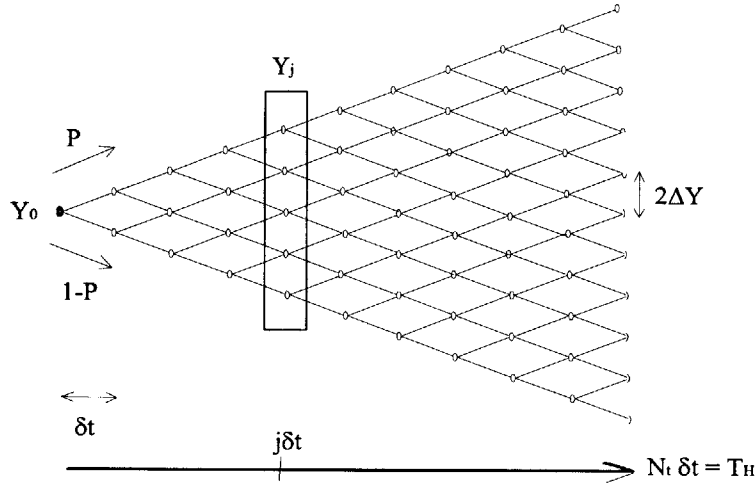$2\Delta Y$

$N_t\,\delta t = T_H$

Figure 5-2: The Log-Transformed Binomial Lattice

conserve the mean drift and volatility of the process. They must therefore verify:

$$E\{\delta Y\} = \left(\alpha - \frac{\sigma^2}{2}\right)\delta t = P\,\Delta Y + (1-P)(-\Delta Y)$$

$$Var\{\delta Y\} = \sigma^2\,\delta t = \left(P\,\Delta Y^2 + (1-P)(-\Delta Y)^2\right) - E\{\delta Y\}^2$$

which requires:

$$\Delta Y = \sqrt{\sigma^2\delta t + \left(\alpha - \frac{\sigma^2}{2}\right)^2 \delta t^2} \qquad (5.13)$$

$$P = \frac{1}{2}\left(1 + \frac{(\alpha - \sigma^2/2)\delta t}{\Delta Y}\right) \qquad (5.14)$$

The lattice is the two-dimensional grid representing the time flowing in N_t increments of the time steps δt, versus the possible values of the log-transformed uncertain parameter Y in increments of ΔY. The values of Y that are reachable through the random walk fill up only half of a rectangular grid (in other words, a triangle) as illustrated on figure 5-2.

Backward Iterative Process

The last time step corresponds to $T_N = N_t\,\delta t$. At that time Y is a vector taking values in $[Y_0 - N_t\,\Delta Y; Y_0 + N_t\,\Delta Y]$ with increment $2\Delta Y$; its length is $N_Y = 1 + N_t$. The values of the

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(1+j+k) uncertain parameter values (i) at (j+k).δt

mode switches (l)

$C_j^{(n \to l)}$

modes of operation (n)

$A_{j<(j+k)}$

modes of operation (n)

EV$_j$

modes of operation (n)

(1+ j) uncertain parameter values (i) at j.δt

(1+ j+k) uncertain parameter values (i) at (j+k).δt

$X_iR^{(l)}$

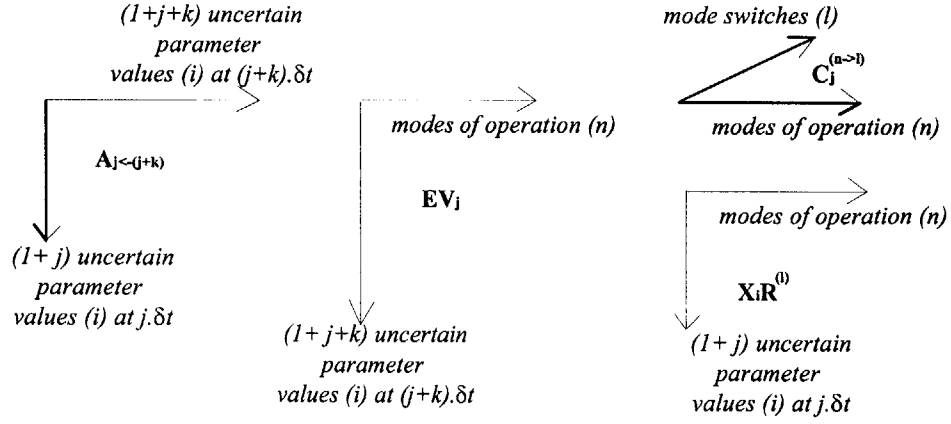*(1+ j) uncertain parameter values (i) at j.δt*

]

Figure 5-3: Main Matrices Used in the Numerical Simulation

uncertain parameter are given by $X_i = e^{Y_i}$. To each of these values corresponds the matrix of size $N_M \times N_M$ containing future mission values if switching from mode (n) to mode (l):

$$EV_{N_t}^{(n \to l)}(X_i) = X_i^{(l)} R_N^{(l)} - C_N^{(n \to l)} \tag{5.15}$$

Determining the optimal mode switch (l) for each value of X and each entering mode (n) gives the value after step N_t as a matrix $(EV_{N_t})_{i,n}$ of size $N_Y \times N_M$.

Now consider any two times $j\delta t$ and $(j + 1)\delta t$ which are not decision points, in other words at which the mode of operation cannot be changed. For each value $i\Delta Y$ of Y at $j\delta t$, Y can move either up or down by ΔY. Thus the present expected value at $j\delta t$ is determined from the present expected future value at $(j + 1)\delta t$ by:

$$(EV_j)_{i,n} = e^{-r\delta t} \left[P \ (EV_{j+1})_{i+1,n} + (1 - P) \ (EV_{j+1})_{i-1,n} \right]$$

This can be applied on each of the modes of operation (n), i.e. on each of the rows of EV_{j+1}.

Trigeorgis [Tri96] recommends an iterative backward process in j. But with computer languages that allow for efficient matrix manipulation (such as MATLAB) it can be useful to note that k time steps can be reduced to one operation with the following consideration.

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First note that EV_j is a matrix of size $(1 + j) \times N_M$ that verifies:

$$EV_j = A_j\, EV_{j+1} \qquad (5.16)$$

where the matrix A_j has size $(1 + j) \times (1 + j + 1)$ and is such that

$$(A_j)_{i,i} = e^{-r\delta t}\, P$$
$$(A_j)_{i,i+1} = e^{-r\delta t}\, (1 - P)$$

Using this induction relation, it can be proven that:

$$EV_j = A_{j\leftarrow(j+k)}\, EV_{j+k} \quad \text{where} \qquad (5.17)$$

$$[A_{j\leftarrow(j+k)}]_{i,i+n} = e^{-r\, k\delta t} \binom{k}{n} P^n\, (1 - P)^{k-n} \quad \forall i \in \{1, ..., 1+j\} \quad \forall n \in \{0, ..., k\}$$

Adjusting Value at Decision Points

Whenever j reaches a decision point $T_j = j\delta t$, the value must be adjusted to account for the optimal mode switch. If the next decision point is at $(j + k)\delta t$, then:

$$EV_j^{(n\to l)}(x) = x^{(l)}\, R_j^{(l)} - C_j^{(n\to l)} + \int EV_{j+k}^{(l)}(y)\, p_j(y|x)\, dy$$

translates into:

$$(EV_j)_{n,i,l} = X_i\, R_j^{(l)} - C_j^{(n\to l)} + A_{j\leftarrow(j+k)}\, (EV_{j+k}) \qquad (5.18)$$

Figure 5-3 illustrates these various matrix elements. Choosing the optimal mode switch (l) for each value i of Y and each entering mode (n) gives the expected future value $(EV_j)_{i,n}$. This is particularly convenient with computer languages allowing manipulation of three-dimensional arrays.

This backwards process can then be iterated up to $j = 0$ where it gives the value EV_n for each possible initial mode of operation.

5.2 Value of the Compound Option to Abandon

The option most immediately available to any space system is the option to abandon the mission if the operations cost is considered too high. For the case of a commercial mission with uncertain market, this will occur as soon as the operations cost exceeds the expected revenues.

The value of this option has never been taken into account in the literature. Thus, the value of space missions has been underestimated. The simplicity of the situation makes it a good candidate for a first application of the model.

5.2.1 Building Blocks

The decision points for the *periodic* option to abandon with period τ_a, are $T_k = k\tau_a$. At each of these decision points, the mission must choose between two modes of operation: (0) *not operational* and (1) *operational*. The revenues rate in mode (0) is $X^{(0)} = 0$ so that there is actually only one uncertain parameter $X^{(1)} = X$.

In order to model the effect of the abandoning option alone, we will here assume a constant market forecast $\mathcal{M}_{th}(t) = \mathcal{M}_0$. The expected revenues during period k if $X(T_k) = x$ is observed are then simply:

$$
\begin{aligned}
EU_k^{(0)}(x) &= 0 \\
EU_k^{(1)}(x) &= x\,e^{(\alpha-r)T_k}\,cd(\tau_a, r - \alpha)\,\mathcal{M}_0
\end{aligned}
\tag{5.19}
$$

Assume it costs nothing (or a negligible amount of money) to abandon the mission and let o_p be the operations costs per unit time in the operating mode. The matrix of switching costs will be the same at each decision node:

$$
C_k = \begin{pmatrix} 0 & o_p\,.cd(\tau_a, r) + C_{re-init} \\ 0 & o_p\,.cd(\tau_a, r) \end{pmatrix}
\tag{5.20}
$$

where $C_{re-init}$ is the cost to re-initiate the mission once it has been abandoned. Two bounds on the value of this cost are easy to model:

- For an *irrevocable* option to abandon, $C_{re-init} = \infty$. The value of this option corresponds to a lower bound on the general option to abandon.

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- For a perfectly *temporary* option to abandon, $C_{re-init} = 0$. The value of this option corresponds to an upper bound on the general option abandon.

The rest of this section proposes to study these two bounds.

5.2.2 Modeling the Option Value

Value of the Irrevocable Option to Abandon

First consider the case when abandoning is irrevocable. This is typical of a space system in low-Earth orbit, where abandoning the mission means de-orbiting the satellites. Let's consider that the mission also loses its licenses, so that even launching new satellites is not possible.

Consider decision point $T_k = k\,\tau_a$. If the system is operational, then there is a decision to make between abandoning, which will produce no future value, and staying operational, which will produce the utility $EU_k^{(1)}$, incur the cost $o_p\,.cd(\tau_a, r)$, and give a further option at T_{k+1}. Thus the decision model will be of the form:

$$
\begin{cases}
EV_{\geq k}^{(1 \to 1)}(x) & = x.cd(\tau_a, r - \alpha) - o_p\,.cd(\tau_a, r) + e^{-r\tau_a} \int EV_{\geq k+1}^{(1)}(y)\,p(y|x)\,dy \\
\text{Abandon} & \Longleftrightarrow EV_{\geq k}^{(1)\to(1)}(x) \leq 0
\end{cases}
$$

The future value is always an increasing function of the observed market level x. Thus the decision model will yield a market threshold Π_k over which it is interesting to pursue the mission after time T_k. The total option value is then determined from the backwards iterative process:

$$
\begin{cases}
EV_{\geq k}(x) & = x.cd(\tau_a, r - \alpha) - o_p\,.cd(\tau_a, r) + e^{-r\tau_a} \int\limits_{\Pi_{k+1}}^{\infty} EV_{\geq k+1}(y)\,p(y|x)\,dy \\
EV_{\geq k}(\Pi_k) & = 0 \quad \text{defines} \quad \Pi_k \\
V & = EV_{\geq 0}(1)
\end{cases}
\tag{5.21}
$$

Value of the Temporary Option to Abandon

Now consider the case when abandonment is temporary and the mission can be re-initiated at no cost. This is an approximation of the case of a system in geostationary orbit, for which the satellites need not be de-orbited but simply boosted to a slightly higher orbit.

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Table 5.1: Irrevocable Option to Abandon: Numerical Assumptions

Parameter	Notation	Baseline Value
Mission horizon	T_H	$10\,yr$
Risk-free interest rate	r	$5\%/yr$
Appreciation rate	α	$0\%/yr$
Operations costs	o_p	$\$\,5\,M/yr$
Market forecast	$\mathcal{M}_0$	$\$\,8\,M/yr$
Volatility	σ	$30\%.yr^{-\frac{1}{2}}$
Number of abandon options	T_H/τ_a	10

In practice, there will be a cost to maneuver the satellites back and possibly regain the licenses; the results of this section can therefore only serve as an upper bound on the value of the option to abandon.

When $C_{re-init} = 0$, the decision whether to operate in the next period is independent on the entering mode of operation:

$$
\begin{cases}
EV_{\geq k}^{(1)}(x) = & x.cd(\tau_a, r - \alpha) - o_p.cd(\tau_a, r) + ... \\
& +e^{-r\tau_a} \int_0^{\Pi_{k+1}} EV_{\geq k+1}^{(0)}(y)\,p(y|x)\,dy + e^{-r\tau_a} \int_{\Pi_{k+1}}^{\infty} EV_{\geq k+1}^{(1)}(y)\,p(y|x)\,dy \\
EV_{\geq k}^{(0)}(x) = & e^{-r\tau_a} \int_0^{\Pi_{k+1}} EV_{\geq k+1}^{(0)}(y)\,p(y|x)\,dy + e^{-r\tau_a} \int_{\Pi_{k+1}}^{\infty} EV_{\geq k+1}^{(1)}(y)\,p(y|x)\,dy \\
EV_{\geq k}^{(1)}(\Pi_k) = & EV_{\geq k}^{(0)}(\Pi_k) \quad \text{defines} \quad \Pi_k
\end{cases}
$$

5.2.3 Study of the Option Value

Numerical Example

Figure 5-4 plots the resulting value and compares it with a net present value for the numerical assumptions summarized in table 5.1. These numbers include all costs *after* initial operating capability (IOC); they do not include the cost to produce and launch the system. The plot shows that mission value increases almost linearly with uncertainty. For very high uncertainty, the value incurred after IOC becomes twice what would have been estimated from a net present value calculation.

General Case

Figures 5-5 and 5-6 illustrate the results in the general case. The plots use non-dimensional parameters so as to be readily applicable to any special case. As for the Black-Scholes
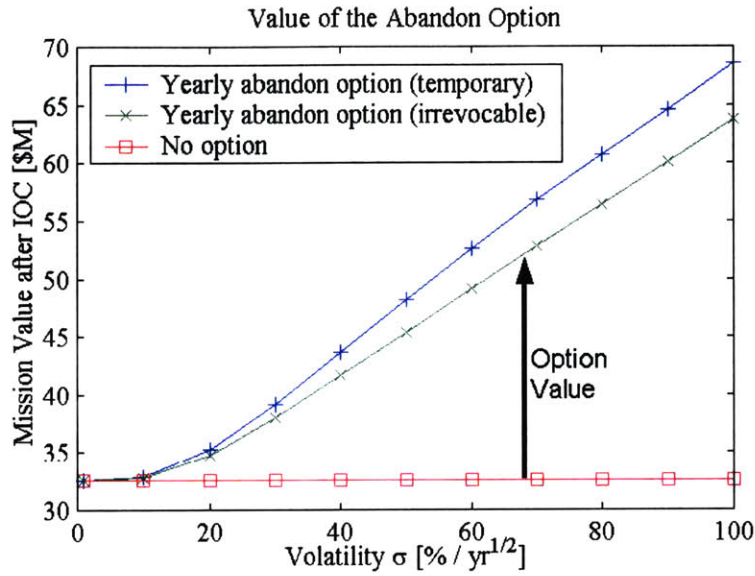
Figure 5-4: Value of the Compound Abandon Option for the numerical assumptions in table 5.1

equation (4.7), the results depend on costs in a relative fashion only, and depend on time only through the products $\sigma\sqrt{t}$, αt and rt. The option value V can therefore be written as a function of only five variables:

$$\frac{V}{tV} = f^n\left(\frac{\mathcal{M}_0}{o_p}, \frac{T_H}{\tau_a}, \sigma\sqrt{T_H}, r\,T_H, \alpha\,T_H\right) \tag{5.22}$$

Value as a Function of Decision Period   Figure 5-5 shows that the value of the option to abandon increases with the number of decision points available over the lifetime of the mission. In other words, it increases with the frequency at which a decision to abandon can be taken. This result is intuitive since more frequent decisions represent an enhanced flexibility. However, the losses that are prevented by increasing the decision frequency, decrease as the abandon period decreases; this explains why the option value quickly reaches an upper bound.

The upper bound is reached as soon as at least 6 options are available over the mission lifetime. In order to take a continuous option to abandon into account, it is therefore sufficient to assume a yearly abandon option for a mission horizon longer than 6 years, or a quarterly option to abandon for a mission horizon of a few years.
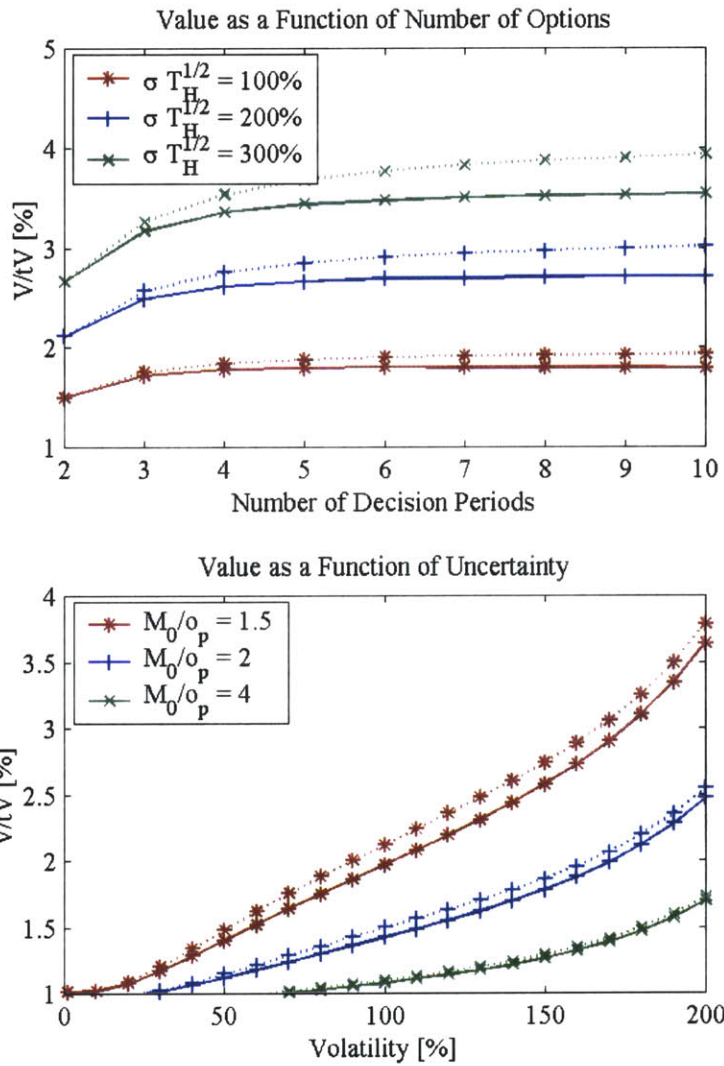
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Figure 5-5: Value of the Compound Abandon Option $r = 5\%/yr$ and $\alpha = 0$: (a) As a function of number of decision points for several values of volatility and for $\mathcal{M}_0/o_p = 1.2$ (b) As a function of uncertainty for several market levels and for $\tau_a = 1\,yr$.

Value as a function of market level The bottom part of figure 5-5 plots mission value as a function of uncertainty for different market levels $\mathcal{M}_0$. It shows that as the expected revenues increase, the relative flexibility value decreases. This effect was already noticed in section 4.2.2 for the simple option on life extension. It is due to the fact that the higher the expected revenues, the lower the probability that the mission will be abandoned, and therefore the less interesting the ability to abandon.

Sensitivity to interest and appreciation rates It is interesting to test on this simple example the importance of using the appropriate discount rates. Figure 5-6 shows that the relative abandon option value is sensitive both to the risk-free interest rate r and to the difference δ between the internal rate of return on the mission and the discount rate r.

The risk-free interest rate r captures the concept of time-value of money. As it increases, what happens later in the mission is deemed less and less important. This reduces the potential losses incurred if not being able to abandon, which in turn reduces the value of the abandon option.

The difference δ is the effective internal rate of return of the market relative to the risk-free interest rate. $\delta > 0$ represents an exponential market increase; since it corresponds to a higher market, it reduces the relative option value. On the other hand, $\delta < 0$ represents a risk-premium, corresponding to discounting the risky revenues at a higher rate than the (riskless) operations costs; it is equivalent to a exponential market decrease, which increases the relative option value.

The sensitivity of the option value to these rates becomes more serious as longer mission horizons are considered. A careful estimate of r and δ is required for an accurate option valuation.

Irrevocable versus Temporary

On figures 5-5 and 5-6, the value of the temporary option to abandon is indicated in dotted lines. As could be expected, the temporary option has a more value than the irrevocable option, since it provides more flexibility. However, the difference is of only of a few percent. Since these two values can be seen as a lower and an upper bound on the value of realistic options to abandon, there is not need to model the abandon option further: the estimate can be deemed precise enough.
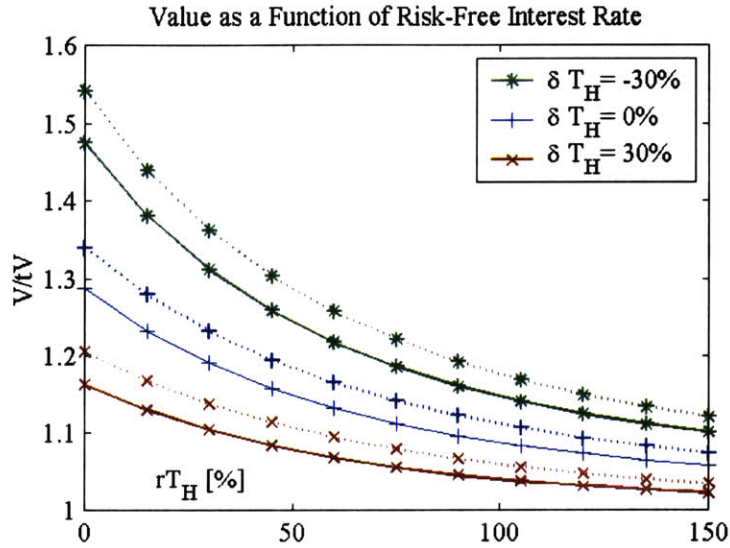
114

Figure 5-6: Sensitivity of the Abandon Option Value to r and α in the case $M_0/o_p = 2$, $\sigma \sqrt{T_H} = 200\%$

In the rest of this chapter, we will be interested in comparing systems with various design lifetimes. A system designed for a shorter life always has the option to abandon at the end of this lifetime. In order to compare it on a fair basis with a system designed for a longer lifetime, we will assume that all missions have a periodic abandon option with the same period. An appropriate choice for this period is 1 year. This compound abandon option will represent additional decision points.

5.2.4 Conclusions

These results show that traditional valuation methods have been significantly underestimating the value of all missions with uncertain revenues, creating a bias in favor of conservative projects. By recognizing the flexibility of decision makers to shut off an unsuccessful mission, the proposed valuation framework shows that some projects that would be deemed uninteresting by traditional valuation can actually have significant value.

Thus, the application of the proposed framework proves useful even in the simple case of the option to abandon a space mission, which is a limited form of flexibility. It should prove even more interesting in studying the value of on-orbit servicing for space missions, which is the focus of the rest of this thesis.

5.3 Optimal Design Life under Market Uncertainty

This section considers servicing as an option on life extension for commercial satellites with uncertain market.

Satellites are typically designed for the longest possible lifetime. This not only leads to high mass and cost due to the requirement for large design margins, especially for power, propulsion and redundancy [SHN01]; it also corresponds to a design lifetime that is often longer than the characteristic market and technological life cycles. Thus there is a fair risk that satellites will be obsolete and/or not respond to any actual market before their end of life. If on-orbit servicing were available, satellites could be designed for a period of time closer to the market dynamics, with the option to extend their life, abandon the mission or upgrade their payload according to the market and technological conditions at a later time.

Is there sufficient value in this added flexibility to make servicing a commercial space mission interesting? The framework we developed and its application to commercial missions are ideal to address this question.

Section 5.3.1 explains how to use this model for two types of study. For a given servicing price and servicing interval, it yields the optimal design lifetime as a function of market level and market volatility. Then for a given uncertainty level, it identifies the maximum price that a space mission would be willing to pay for each on-orbit servicing operation, by considering the conditions under which the optimal design life is shorter than the mission horizon.

Section 5.3.4 applies these results to two examples of commercial missions with highly uncertain market: the *Iridium* and *Globalstar* low-Earth orbit constellations of communication satellites.

5.3.1 Towards a New Decision Making Approach

Traditional Decision Making

With the traditional way of designing satellite systems, there is a strong incentive to design for the longest possible lifetime. In the absence of uncertainty, a longer lifetime indeed means a smaller cost per operational day [SHN01]. The typical traditional decision making process is illustrated on figure 5-7. The net present value of the project is estimated without taking into account the uncertainty in the revenues and the possible options to react to it.
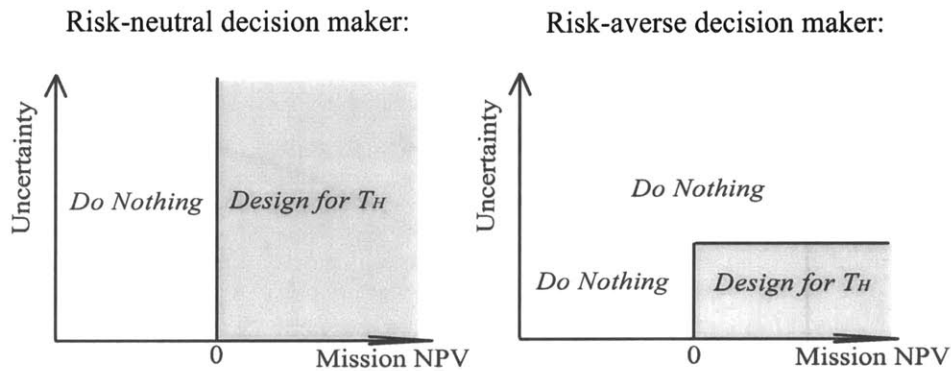
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Figure 5-7: Traditional Decision Making

Two attitudes are then possible depending on the level of awareness of the decision makers regarding uncertainty:

- Risk-neutral decision makers would simply not take uncertainty into account. They would approve the project if its net present value is above a given threshold (it should at least be positive), and design it for the longest possible lifetime T_H to minimize its cost-per-operational-day.

- Being concerned about uncertainty but lacking a way to quantify its effects, risk-averse decision makers would simply reject any project whose uncertainty is higher than a subjective threshold. This greatly limits the space of possible projects.

Towards a New Decision Making Approach

The framework proposed is chapter 4 can represent a useful tool to define a new decision making process as illustrated on figure 5-8. By providing a means of quantifying the effects of uncertainty and the value of managerial flexibility, it can make it possible to draw two new decision boundaries:

Minimum market level: effect of the abandon option As seen in section 5.2, for missions with a compound option to abandon and a low market level, mission value increases significantly as uncertainty increases. Even if the net present value is negative, the actual value can be positive because there is some probability that the revenues will rise unexpectedly. Therefore, the minimum market level over which the mission is worthwhile
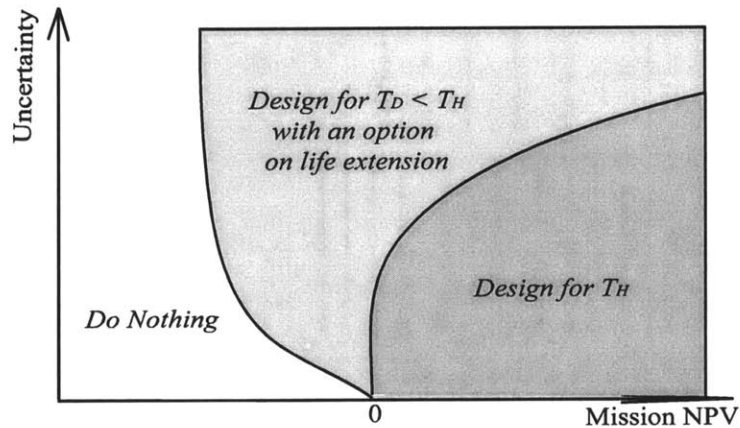
117

Figure 5-8: Decision Making with Options

decreases with increasing uncertainty. This explains the shape of the left boundary on figure 5-8.

Minimum uncertainty level: effect of the servicing option   We saw that the cost to produce and launch a space system increases almost linearly with increasing design lifetime requirement. This leads to a decreasing cost-per-operational day if replacement is the only solution for life extension. But life extension through servicing has the potential to be much cheaper than life extension through replacement. Therefore on-orbit servicing would make possible a trade-off between the cost to design for a given lifetime and the price to service for life extension. This trade-off could lead to an optimal design lifetime T_D shorter than the mission horizon T_H. As uncertainty increases, the value of the option on life extension increases as seen in section 4.2.2. Thus we can expect the existence of a minimum uncertainty level over which the optimal design is serviceable. This explains the shape of the right boundary on figure 5-8.

Through this new decision making approach, the proposed framework would provide decision makers with the tools to fully understand the effects of external sources of uncertainty and take their future options into account when deciding on a design-and-maintenance strategy.

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The next sections will show how to use the framework to quantify the boundaries in the above decision diagram, and to determine the maximum servicing price that a commercial space mission would be willing to pay for each servicing operation.

5.3.2   Modeling the Irrevocable *Service or Abandon* Option

The option to service for life extension corresponds to a *service-or-abandon* option. Section 5.2 showed that there is little difference between the values of the irrevocable and of the temporary options to abandon. When considering the option to service for life extension, it can therefore be assumed, without loosing much generality, that abandoning is irrevocable. This will at least give a lower bound on the option value.

Building Blocks

Consider a space system designed for a time T_D with the option to be serviced in increments of the service interval τ up to the mission horizon T_H: the decision points are $T_k = T_D + (k - 1)\tau$. At each decision point there is a choice between two modes of operation: (0) *abandoned* and (1) *operational in the initial design*. The utility in mode (0) is zero, so that there is only one uncertain parameter $X^{(1)} = X = \mathcal{M}(t)/\mathcal{M}_{th}(t)$.

The value at T_k of the expected revenues during the kth period if $X(T_k) = x$ is observed is:

$$\begin{cases} EU_k^{(0)}(x) & = 0 \\ EU_k^{(1)}(x) & = x\, R_k \end{cases} \tag{5.23}$$

Let $C_{IOC}(T_D)$ be the cost to initial operating capability as a function of the design life T_D; this function is described in sections 2.2.1 and 3.2.1. For the purpose of this study, it is sufficient to use a linear cost model and a linear fit to the mass penalty as given by 3.1:

$$\frac{C_{IOC}}{C_3} = 1 + \kappa\,(T_D - 3\,yr) \tag{5.24}$$

where C_3 is the cost to initial operating capability for a arbitrary reference design lifetime requirement of 3 years. This corresponds to defining a percentage cost penalty κ per unit time of design life.

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The initial matrix of switching costs at $T_0 = 0$ is then:

$$C_0 = \begin{pmatrix} 0 & C_{IOC}(T_D) + o_p \,.cd(T_D, r) \\ \infty & \infty \end{pmatrix}$$

At each following decision point, the situation it is very similar to the irrevocable option to abandon studied in the previous section. The only difference is that at each potential service time $T_{k+1} = T_D + k\tau$, the cost to stay operational is now the sum of the operations cost and the servicing price. Let C_S be the price of each servicing operation. The matrix of switching costs is then:

$$C_j = \begin{cases} \begin{pmatrix} 0 & \infty \\ 0 & o_p \,.cd(\tau, r) \end{pmatrix} & \text{for the yearly option to abandon, at } T_j \neq T_k, \\[2em] \begin{pmatrix} 0 & \infty \\ 0 & C_S + o_p \,.cd(\tau, r) \end{pmatrix} & \text{for the service-or-abandon option, at } T_j = T_k \end{cases}$$

Decision model

The total option value is determined from the following backwards iterative process:

$$\begin{cases} EV_{\geq k}(x) & = x\,R_k - C_S - o_p\,.cd(\tau, r) + e^{-r\tau} \int\limits_{\Pi_{k+1}}^{\infty} EV_{\geq k+1}(y)\,p(y|x)\,dy \\[1.5em] EV_{\geq k}(\Pi_k) & = 0 \quad \text{defines} \quad \Pi_k \\[1em] V & = EV_{\geq 0}(1) \end{cases} \tag{5.25}$$

It is convenient here to define all costs as percentages of the cost to design for 3 years, $C_{IOC}(3) = C_3$. The present value of the mission at time $T_0 = 0$ is finally a function of the design life T_D, the cost penalty per unit design time κ, the servicing price C_S, the service interval τ, the market forecast $\mathcal{M}_{th}(t)$, the market volatility σ, the operations cost o_p, the mission horizon T_H, the risk-free interest rate r and the rate of return α:

$$\frac{V}{C_3} = f^n\left(T_D, \frac{C_S}{C_3}, \tau\,;\, \sigma, \frac{\mathcal{M}_{th}}{C_3}\,;\, \frac{o_p}{C_3}, \kappa, T_H, r, \alpha\right) \tag{5.26}$$

Optimal design life

For a given servicing price and service interval, the optimal design life $\widehat{T_D}$ is the design life that gives the maximum expected mission value $\widehat{V_S}$.

$$\widehat{T_D} = f^n \left(\frac{C_S}{C_3}, \tau \; ; \; \sigma, \; \frac{\mathcal{M}_{th}}{C_3} \; ; \; \frac{o_p}{C_3}, \; \kappa, \; T_H, \; r, \; \alpha \right) \qquad (5.27)$$

In particular, for a given mission and market forecast, the valuation process will yield the optimal design life as a function of the market uncertainty and the servicing price. A serviceable design will be chosen only if the optimal design life is shorter that the mission horizon T_H:

$$\text{Serviceable} \iff \widehat{T_D} < T_H$$

Comparison with the option to replace The *replace-or-abandon* option can be studied in exactly the same way as the *service-or-abandon* option, simply using $C_S \approx C_{IOC}$. An optimal design life $\widehat{T_{D,R}}$ giving the maximum value $\widehat{V_R}$ can also be determined for the replacement case. This adds an additional condition on the choice for serviceability:

$$\boxed{\text{Serviceable} \iff \widehat{T_D} < T_H \quad \text{and} \quad \widehat{V_S} > \widehat{V_R}}$$

Maximum Servicing Price

For a given service interval, the maximum servicing price Υ can finally be defined as the price under which the optimal design life is shorter than the mission horizon:

$$\widehat{T_D} \left(\frac{C_S}{C_3}, \tau, ... \right) < T_H \iff \frac{C_S}{C_3} < \Upsilon$$

$$\Upsilon = f^n \left(\tau \; ; \; \sigma, \; \frac{\mathcal{M}_{th}}{C_3} \; ; \; \frac{o_p}{C_3}, \; \kappa, \; T_H, \; r, \; \alpha \right) \qquad (5.28)$$

Lower Bound on the Servicing Price

Whatever the choice of servicing infrastructure, it will be necessary to launch the mass that has to be delivered to the serviceable spacecraft. Therefore the cost to produce and launch this mass can be considered as a lower bound on the marginal cost of servicing, and hence

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Table 5.2: Irrevocable Service-or-Abandon Option: Constant Assumptions

Parameter	Notation	Baseline Value	Source
Mission horizon	T_H	$15\,yr$	Typical GEO
Risk-free interest rate	r	$7.9\,\%/yr$	Adapted from [HJJK00]
Rate of return	α	$4.2\,\%/yr$	Adapted from [HJJK00]
Penalty rate	κ_{LEO}	$2.75\,\%/yr$	Adapted from [SHN01]
Operations cost	o_p	$5\,\%C_3\,/yr$	Typical [WL99]

a lower bound on the servicing price. The cost relationship 5.24 suggests that this lower bound be approximated by:

$$\boxed{\frac{C_S}{C_3} \geq \kappa\,\tau = \Upsilon_{min}} \tag{5.29}$$

5.3.3   Results in the General Case

Three of the eight parameters that set the value of the maximum servicing price are particularly interesting to study:

- The service interval τ, which is a free variable. In particular, the variation of the maximum servicing price Υ as a function of τ can be compared to the linear variation of the lower bound on the servicing price $\Upsilon_{min} = \kappa\,\tau$,

- The market forecast $\mathcal{M}_{th}/C_3$ and its volatility, which are the two parameters that can vary widely among space missions. For this very general study, we will consider only constant market forecasts: $\mathcal{M}_{th}(t) = \mathcal{M}_0$.

The five remaining parameters will be held constant as indicated in table 5.2. The value for the interest and return rates in this table were obtained by considering current industry data. We remember from section 2.3 that under the capital asset pricing method (CAPM) assumptions, the rate of return on an asset should be of the form:

$$\alpha = r + p = r + E\{r_m - r\}\,\beta \tag{5.30}$$

Harbison & al [HJJK00] plot the observed value of β and α for a variety of major industries, including the aerospace industry. A fit to the equation 5.30 gives the risk-free interest rate $r \approx 7.9\%$ and the overall market trend $E\{r_m - r\} \approx 8.3\%$. The Aerospace & Defense
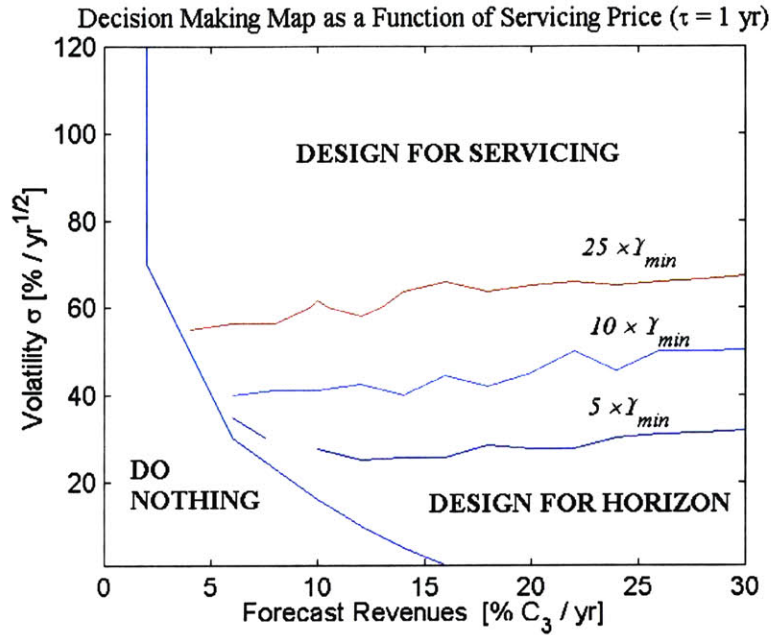
122

Figure 5-9: Maximum Servicing Price for the Option on Life Extension: a Map in a Market Level / Uncertainty Space as a New Tool to Decision Making. Case $\tau = 1\,yr$.

industry as a whole (A&D) has $\alpha_p \approx 10\%$ and $\beta \approx 0.7$, which corresponds to :

$$\alpha_{A\&D} = \alpha_p - p = \alpha_p - E\{r_m - r\}\beta \approx 4.2\%$$

$$\delta_{A\&D} = \alpha_{A\&D} - r \approx -3.7\%$$

Quantifying the Boundaries in the New Decision Making Map

Expressing the resulting maximum servicing price Υ in units of its lower bound $\Upsilon_{min} = \kappa\tau$ makes the results almost independent of the service interval τ. This results from the fact that the cost savings to design for a shorter lifetime are also proportional to $\kappa\tau$. Figures 5-9 and 5-10 show that even the two extreme cases $\tau = 1\,yr$ and $\tau = 7\,yr$ present indeed very similar features.

These plots provide an estimate of the numerical values for the boundaries in the decision making map envisioned in the previous section, in the most general case possible. They constitute therefore a very valuable, directly applicable tool for decision making regarding the design of serviceable spacecraft. The position of the left boundary is determined by the option to abandon; it is a function of the operations cost only. The position of the

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Decision Making Map as a Function of Servicing Price ($\tau = 7$ yr)

DESIGN FOR SERVICING

$25 \times \Upsilon_{min}$

$10 \times \Upsilon_{min}$

$5 \times \Upsilon_{min}$

DO NOTHING

DESIGN FOR HORIZON

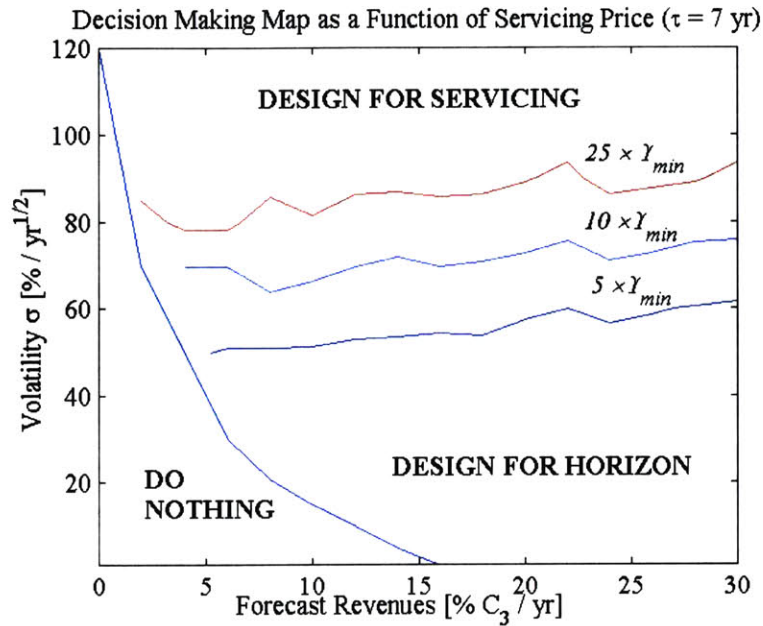Volatility σ [% / yr$^{1/2}$]

Forecast Revenues [% C_3 / yr]

Figure 5-10: Maximum Servicing Price for the Option on Life Extension: a Map in a Market Level / Uncertainty Space as a New Tool to Decision Making. Case $\tau = 7\,yr$

right boundary, which determines when a design should be made serviceable, depends on the price of on-orbit servicing. The lower the servicing price, the more interesting it is to design for servicing for a given uncertainty.

These maps confirm that in the absence of uncertainty ($\sigma = 0$), it is always optimal to design for the longest possible lifetime. Thus, traditional valuation underestimates the value of on-orbit servicing for commercial space missions by not taking into account the effects of uncertainty. Only by taking the value of flexibility into account can the trade between the cost to design for a given lifetime requirement and the price to service for life extension be captured.

Furthermore, figure 5-9 shows that as soon as there is significant uncertainty ($\sigma > 40\%.yr^{-\frac{1}{2}}$), the maximum servicing price is an order of magnitude greater than the cost to produce and launch the serviceable mass. Thus, the value of on-orbit servicing for commercial missions with high uncertainty is likely to be significant compared to the marginal cost of servicing.
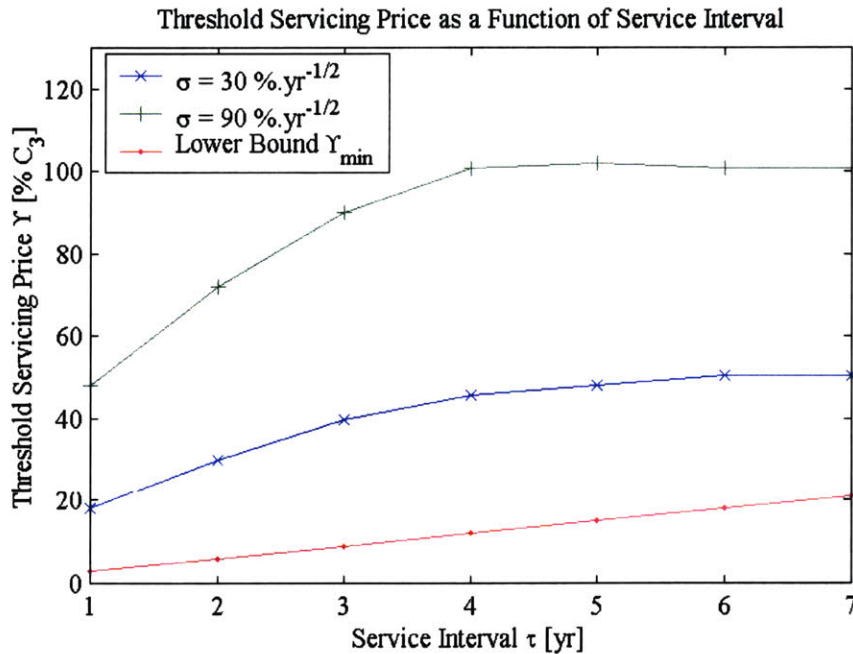
124

Figure 5-11: Maximum Servicing Price as a Function of Servicing Interval: General Case with $\mathcal{M}_0/C_3 = 10\%/yr$

Maximum Servicing Price as a Function of Service Interval

Figure 5-11 shows how the maximum servicing price Υ varies with the servicing interval τ and compares it with the minimum servicing price Υ_{min}, for two values of the volatility and for the example $\mathcal{M}_0/C_3 = 10\%/yr$. This type of plot could prove useful as a guide for future technology development as to what servicing price a space mission can be charged: the price should be higher than the cost Υ_{min} to produce and launch the serviceable mass, but lower than the maximum servicing price Υ that the mission is willing to afford. This figure shows that the lower the service interval, the greater the range of possible servicing prices. This result is in agreement with [Wal93], which also identified shorter service intervals as more interesting.

Maximum Flexibility Value as a Function of Volatility

Figure 5-12 plots the ratio of the expanded net present value (eNPV) over the traditional NPV, and the relative value of the flexibility provided by on-orbit servicing as a function of volatility. This plot corresponds to the case of free servicing: $C_S = 0$; it is therefore the *maximum* flexibility value. This value not only increases as expected with increasing
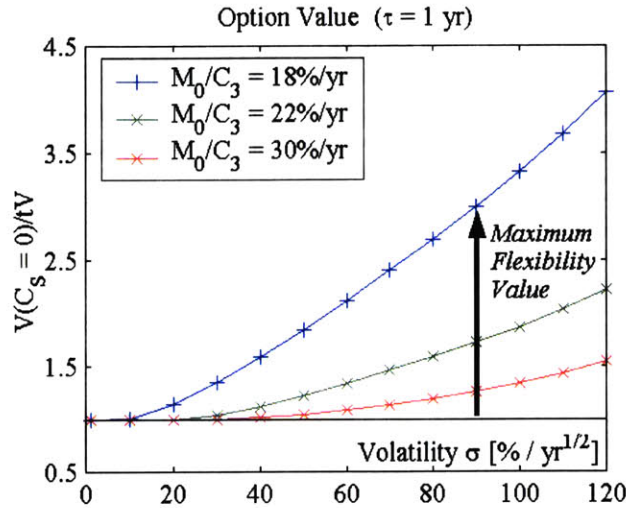
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Figure 5-12: Expanded Net Present Value and Maximum Flexibility Value as a Function of Volatility: General Case with $\tau = 1\,yr$.

volatility, but is also much greater than in the case of the option to abandon. Even for high market forecasts, the value of flexibility can make up as much as 50% of the total eNPV of the mission. This amount represents by how much traditional valuation underestimates the value of a serviceable mission.

The above results accomplish the goal of the section: they quantify the boundaries on the decision making diagram envisioned on figure 5-8, show that on-orbit servicing is of great value to commercial missions with highly uncertain markets, and estimate the corresponding flexibility value. In doing so, they demonstrate the significance and many uses of the proposed framework.

An additional piece of information would however be interesting. On the maps 5-9 and 5-10, the broad area labeled *design for servicing* corresponds to any design life shorter than the mission horizon. *How does the optimal design life actually behave as uncertainty increases?* Let us further explore this question by considering a realistic numerical example.

5.3.4 Application to Two LEO Communications Missions

Iridium and *Globalstar*

Iridium and *Globalstar* were two of the *big LEO* constellations of satellites conceived in the early 1990's to address the great potential market of mobile telephony. By the time the

systems were launched, the market they had been expecting had shrunk significantly due to the advent and rapid evolution of cellular telephony networks. As a result, *Iridium* soon filed for bankruptcy, and the same end may be awaiting *Globalstar*. The great uncertainty in their market make these two constellation a perfect case-study for our valuation model.

Table 5.3: Approximate Parameters for *Iridium* and *Globalstar*

Parameter	Name	*Iridium*	*Globalstar*
Cost to IOC	C_{IOC}	$3 B	$2 B
Design life	T_D	5 yr	7.5 yr
Operations cost	o_p	$245 M/qt	$125 M/qt
Operational satellites	N_{sats}	66	52

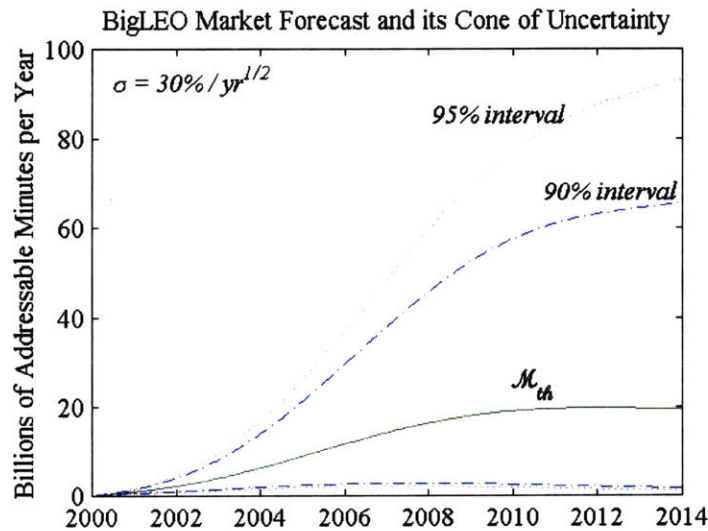**A Highly Uncertain Market**

Figure 5-13: LEO66 Market Forecast [GVH$^+$97] with a Reference Cone of Uncertainty $\sigma = 30\%.yr^{-\frac{1}{2}}$

Market forecast   The expected market for these missions as forecast in 1997 is the same as used in chapter 3 when studying an *Iridium*-like case from a traditional point of view. The new approach requires to study not only the magnitude, but also the volatility of this market. The market magnitude can be found in [GVH$^+$97] in terms of number of billable minutes per year for various assumptions of market penetration. This market is reproduced on figure 5-13, where a *cone of uncertainty* corresponding to $\sigma = 30\,\%\,yr^{-\frac{1}{2}}$ is represented for reference. The average price per minute around was $3/*min* for *Iridium* and $1/*min*
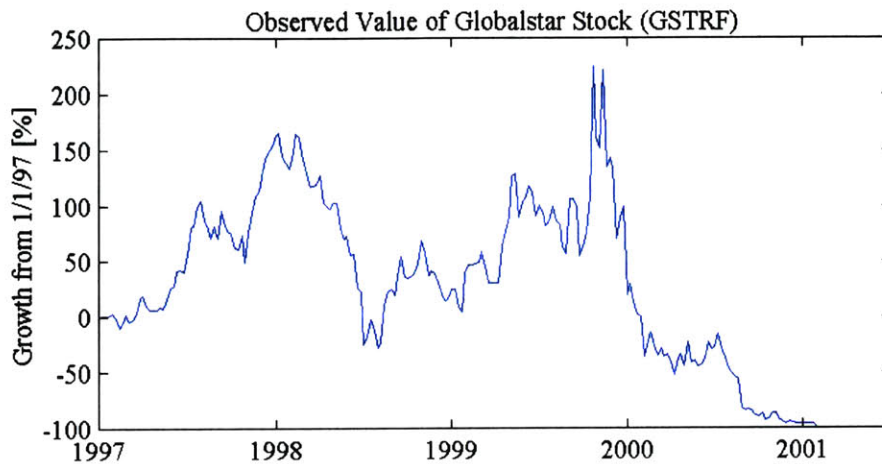
Figure 5-14: Time Variation of the *Globalstar* Stock (GSTRF) (Source http://finance.yahoo.com/)

for *Globalstar*. Combining these numbers with a typical market penetration of 10% gives the market forecast in terms of revenues: $\mathcal{M}_{th}(t)$.

Market volatility A major practical difficulty with any option valuation is the estimation of the volatility of the market. While a market forecast is a necessary part of a business plan, the uncertainty in the forecast is by definition unknown. The situation is made easier when as in this case, historical data about the projects is available. We will therefore perform an *a posteriori* volatility estimate.

A first, very crude, method of estimation consists in considering the actual number of users. Six months after it started operating, *Iridium* had only 10000 customers instead of the 52000 expected. This corresponds to an approximate volatility of $\sigma \approx 1.2 \, yr^{-1/2}$.

The method most widely used by financial experts however, is historical regression on the value of a *twin security* traded on the stock exchange. This type of estimation is possible for the case of the *Globalstar* mission, whose stock (GSTRF) is still traded. Figure 5-14 shows the five-year evolution of the *Globalstar* stock. It is obvious from this plot that the volatility of GSTRF has not been constant as models always assume. Rather, figure 5-15 plots its evolution with time over the last four years: fairly high before 2000, the volatility exploded as the stock price shrank after *Iridium*'s failure. Thus, while $\sigma = 30 \, \%.yr^{-\frac{1}{2}}$ could have been estimated at the start of the mission, a more realistic value given recent history
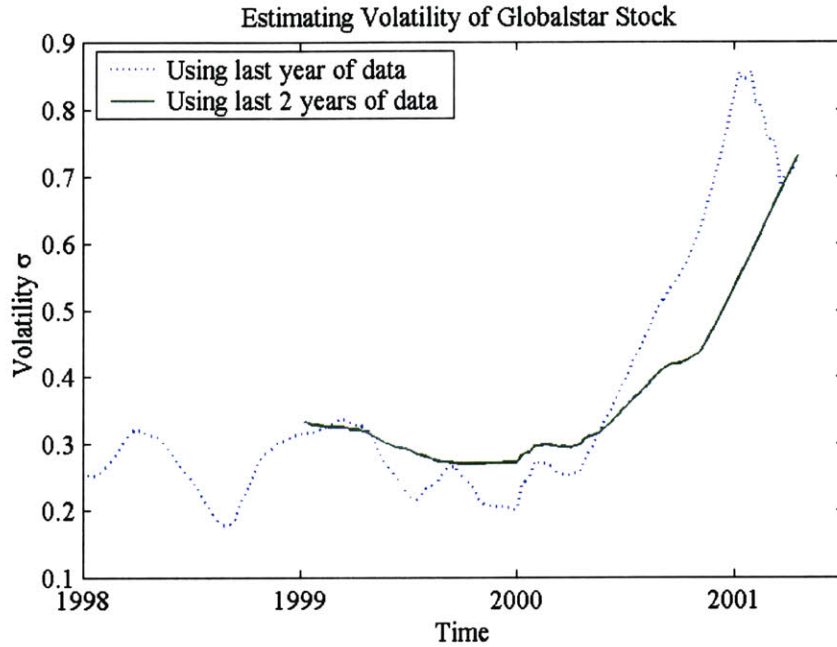
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Figure 5-15: Estimation of the Volatility of the *Globalstar* Stock (GSTRF)

would be $\sigma = 90\,\%.yr^{-\frac{1}{2}\ddagger}$.

Given the uncertainty on the volatility, we shall keep it as a parameter as long as possible.

Risk premium In the absence of any more information, a reasonable approximation is to assume the same discount rate and internal rate of return as for the Aerospace and Defense industry as a whole: $r = 7.9\%$ and $\delta = -3.7\%$.

Optimal Design Life

Figure 5-16 maps the regions of different optimal design lifetimes for the parameters of the *Globalstar* case, in a servicing price/volatility space. Expressing the servicing price in units of its lower bound $\kappa\,\tau\,C_3$ makes the optimal design lifetime approximately the same for the two missions. As expected, the optimal design life decreases with decreasing servicing price and increasing uncertainty. This map reveals the existence of two very different situations:

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‡Although from this plot the shrinking of the stock value seems to show a deterministic trend, in the first quarter of 2001 the *Globalstar* customer base has been growing significantly. The assumption of symmetric uncertainty is thus not that bad.
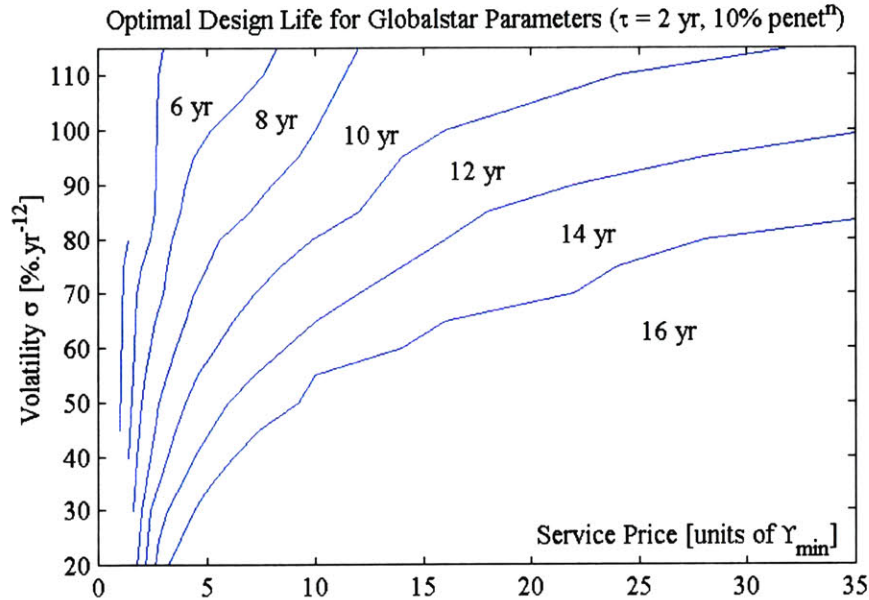
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Volatility σ [%.yr$^{-1/2}$]

110
100
90
80
70
60
50
40
30
20

6 yr   8 yr   10 yr

12 yr

14 yr

16 yr

Service Price [units of Υ_{min}]

0   5   10   15   20   25   30   35

Figure 5-16: *Globalstar*: Optimal Design Life as a Function of Servicing Price for $\tau = 2\,yr$

- If servicing is very cheap (below two times the lower bound), then the servicing price is the main driver for the optimal design lifetime, whatever the uncertainty. But the optimal design lifetime is so sensitive to the servicing price that it is not a good guide for decision making.

- When the servicing price is significant on the other hand, uncertainty becomes the main driver for the optimal design lifetime. The regions of different optimal design lifetimes become large enough to represent a significant tool for decision making. However, an accurate estimation of the market uncertainty would be required to make the right decision. For example, the *Globalstar* volatility as estimated at the start of the mission ($\sigma = 30\,\%.yr^{-\frac{1}{2}}$) would recommend the longest possible lifetime. However, the actual stock volatility as observed after five years ($\sigma = 90\,\%.yr^{-\frac{1}{2}}$) corresponds on the map to a shorter design lifetime with the option to service.

Flexibility Value

Figure 5-17 plots the relative value of flexibility for the two missions as a function of volatility. The results are very similar to those obtained in the general case. The relative flexibility value of *Globalstar* is higher for the same volatility because its cost per minute is lower,
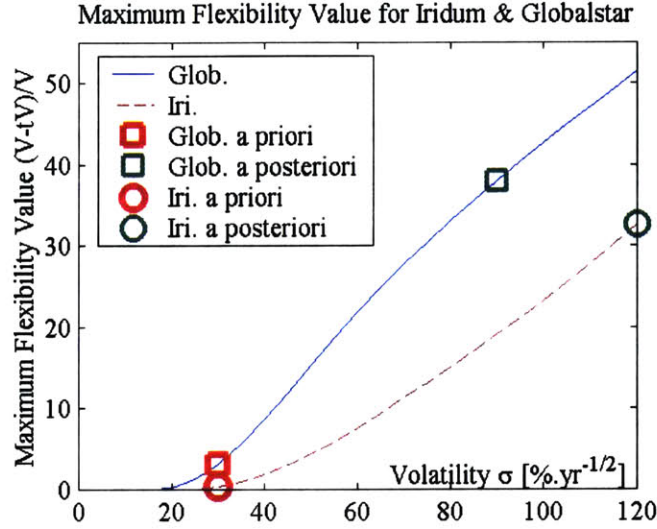
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Figure 5-17: *Iridium* and *Globalstar*: Flexibility Value as a Function of Volatility

making its overall market lower for the same penetration. If $\sigma = 90\,\%.yr^{-\frac{1}{2}}$ is assumed for *Globalstar*, then the flexibility value is 40% of the NPV, which represents \$1.5B. If $\sigma = 120\,\%.yr^{-\frac{1}{2}}$ is assumed for *Iridium*, then the relative flexibility is 30%, which corresponds to almost \$6B. These cases thus confirm once more the importance of accounting for flexibility when evaluating the advantages of on-orbit servicing.

5.3.5 Conclusions

This section proposed a new approach to decision making regarding the choice of a design lifetime requirement and a maintenance strategy. The option valuation framework can provide decision makers with two types of tools to strategically manage the uncertainty in their revenues:

- If the price of on-orbit servicing is known, then the optimal design choice can be plotted in a market level/volatility map such as figure 5-8. Provided that the uncertainty is not close to the decision boundary, then broad estimations of market forecast and volatility suffice to decide between designing for the longest possible lifetime and designing for servicing.

- If accurate estimates of the market forecast and market volatility are developed, then the optimal design life can be plotted in a servicing price/volatility map such as 5-16.

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Then only a broad estimation of the servicing price suffices to decide on a design lifetime requirement.

Using these tools on the most general case possible, we showed that the maximum servicing price that commercial space missions should be willing to pay for on-orbit servicing is an order of magnitude higher than the cost to produce and launch the serviceable mass, as soon as the volatility in their revenues is higher than about $40\%.yr^{-\frac{1}{2}}$. This proves not only that on-orbit servicing has significant value for commercial space missions, but also that this conclusion can be reached only by accounting for the value of the flexibility it provides them with.

Finally, theses results show how the valuation framework can be used as a guide for future on-orbit servicing technology development, by defining the range of possible servicing prices that a space mission can be charged (figure 5-11).

Note The above results suggest that the optimal design lifetime can be as long as 16 years, whereas the lifetime actually chosen by both missions was shorter than 8 years. This is due to the simplifying assumptions made here that the cost to IOC is a linear function of the design lifetime. In practice, there are technological limits to the design lifetime.

Chapter 6

Towards a Value of Servicing for Military Missions

For a significant proportion of space missions, value is not monetary and the linear valuation model described in the previous chapter is not valid. This chapter proposes to tackle the problem of non-commercial on-orbit servicing valuation by choosing the example of military space missions.

The decision process for a military mission differs greatly from what happens in the commercial world. First, mission value is not a measure of revenues minus cost, but rather takes the form of a complex utility function divided by cost. Thus, value does not have the same linear properties as for the cases studied by real options theory. But even more importantly, there exists two possible decision processes. When designing a space mission during a peaceful time, the optimal design is taken to be the one the maximizes utility per cost under certain constraints. But when making a decision about an operational space mission involved in contingencies, the cost factor becomes much less critical than the performance. The alternative that maximizes utility is generally chosen.

This chapter proposes a way to adapt the options valuation framework to the special case of military missions faced with uncertainty in the location of contingencies over the world. The value of refueling as a way to make satellites maneuverable will be explored in two cases. Section 6.1 considers the potential for reducing the number of satellites in a LEO radar constellation. Section 6.2 studies the potential improvement in capacity when optimizing the distribution in longitude of a GEO fleet of communication satellites.

6.1 A *Thin Radar* Constellation

This section is interested in the potential offered by on-orbit servicing to enable new ways of designing space systems. It addresses the example of maneuverable satellites, which would be made possible by a refueling capability. In particular, using maneuverable spacecraft could help reduce the number of satellites in a low Earth orbit (LEO) radar constellation.

6.1.1 Problem Statement

Discoverer II   The Discoverer II (DII) program[*] was an Air Force, Defense Advanced Research Projects Agency (DARPA), and National Reconnaissance Office (NRO) joint initiative to develop and demonstrate revolutionary capabilities for space-based radar. The program was based on DARPA's work on a new lightweight satellite called *STARLITE.* Its goal was to develop, design, fabricate and launch two research and development satellites capable of detecting and tracking moving targets on the Earth's surface, producing high-resolution imagery and collecting high-resolution, digital terrain mapping data. If full funding had been approved, deployment of additional 22 satellites was projected by DARPA for 2003-2005. The resulting *Discoverer II* constellation would consists of 24 low cost satellites, placed at 770 km altitude in 8 orbital planes in a Walker Delta-pattern [†] with a phase value of 4 and an orbital inclination of 53°; this constellation was designed to meet a commander's requirement for an imaging operation within 15 min after receiving tasking, 90% of the time, averaged across 65° north and south latitude. The *Discoverer II* program also intended to show how individual satellite costs could be cut to less than $100M$, reducing the 20-year life-cycle cost of a large operational system to less than $10B$. Despite these efforts, the project was judged too expensive and was canceled.

The Idea of a Thin Constellation   Space-base radar requirements are usually focused around a few critical theaters. However, the location of these theaters cannot be predicted at the time of mission design and can be expected to change several times over a mission lifetime of more than a decade. Therefore, space-based radar missions need to be designed with the flexibility to adapt to any possible theater location. Traditionally, this flexibility is build up in the system by designing the constellation for global coverage over the range of

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[*]http://www.fas.org/spp/military/program/imint/starlight.htm
[†]See next section for a definition

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possible theater latitudes, as was the case for the *Discoverer II* constellation. Space-based radar systems need to be implemented in low-Earth orbit (LEO) in order to meet their resolution requirement. Therefore global coverage requires many satellites, much more than would be necessary for coverage of the current critical theater only. Radar constellations are thus over-designed for instantaneous requirements.

This over-design for flexibility leads to high costs. The existence of a refueling capability in space could offer an alternative to global coverage for flexibility to theater location. Making satellites refuelable increases their maneuver capabilities. A constellation designed for coverage over one location could therefore maneuver to optimize its orbital characteristics for coverage over any new theater. Since it would be designed for instantaneous local coverage, such a constellation would require fewer satellites, hence the name *thin constellation*. A thin constellation offers the potential to reduce the lifetime cost of a radar constellation. Maneuvering could also enable the constellation to focus its coverage over one theater, improving its utility for the same cost. Thus there are at least two ways in which a refueling capability could improve a mission's cost-effectiveness. But this maneuver-and-refuel capability will of course come at an additional cost.

Goal of the Study The goal of this section is to evaluate the value of servicing to reduce the number of satellites in a space-based radar constellation. For this purpose two definitions of *value* are interesting:

- Value can be the lifetime utility per cost, which we will define in the next section,

- Or it can refer to mission's lifetime cost only, because systems with high utility per cost are unrealistic if their cost is higher than what decision makers are willing to spend.

Mission Value

Four Capability Metrics The mission requirements for a radar constellation can be expressed in terms of four GINA-type capability metrics as follows [Sha99]:

Isolation Isolation measures the ability to identify a target from the ground clutter. It is expressed in terms of three numbers: ground resolution (cell size; 200 m), minimum detectable velocity (MDV; 1m/s), and velocity precision.

Integrity Integrity measures the probability of an error in the information transferred. It is here expressed in terms of the probability of false alarm and the probability of detection.

Rate There are two important rates for a radar mission.

1. The first is the area search rate of a ground cell within a theater, omitting times when the theater is not being searched. It measures how much territory can be surveyed per unit time during the times in view.

2. The second is linked to the gap times between coverage of an area by successive satellites. *Discoverer II* requirements are expressed in terms of what we will called a *reaction time*. Reaction time is defined as the time it takes from a request to observe an area to the actual collection of data over this area.

Availability In the GINA framework, availability is the probability to meet the mission's requirements for isolation, rate, and integrity. It can be measured as a function of any of these variables.

This study focuses on the trade between number of satellites and their maneuverability. For this trade to remain unbiased, the satellites altitude and payload design will be held constant. Under these conditions, isolation and integrity will be approximately the same for all designs, because they are only function of the radar signal link budget. The theater area search rate will also be constant at constant payload design.

In the framework of this study, availability is therefore a function of reaction time. The requirements must specify a minimum availability for a minimum reaction time:

$$\mathcal{R} = (Av, Rxn)$$

The baseline requirements specified for *Discoverer II* are $\mathcal{R}_b = (90\%, 15\,min)$.

Function, Cost per Function; Utility and Value In the initial work by Shaw [Sha99], Function was defined for a radar constellation as the total number of km^2 protected over the lifetime of the mission. This definition holds when the constellation is designed for global coverage and the actual target locations are unknown. In that case, coverage is

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indeed approximately uniform and the *total* number of km^2 protected can be considered to be proportional to the number of *critical* km^2 protected.

But at any instant in time, a user can actually be satisfied only by observations of the current critical theater. Therefore, we will define utility as the total time in view of the instantaneous theater of interest. This theater has a probabilistic time and space distribution, which must be taken into account.

6.1.2 Number of Satellites versus Maneuverability

Instead of designing a constellation for global coverage within a latitude band, we thus propose to design a thin constellation using a distinct orbital configuration for each theater location; the satellites would maneuver between each orbital configuration as the theater location changes. This corresponds to carrying out a trade-off between the number of satellites and their maneuverability. The more satellites can maneuver, the better they can optimize their coverage over the current critical theater, and therefore the fewer the required number of satellites for a given coverage requirement. On the other hand, there is a cost to design satellites for maneuverability. This section proposes a simple maneuverability model to help quantify this trade-off.

Reminder: Walker Delta patterns [Wal71] are a family of configurations for constellations of satellites in circular orbits at a given altitude and inclination. They are defined by three numbers $\mathcal{T}/\mathcal{P}/\mathcal{F}$:

- $\mathcal{T}$ is the total number of satellites

- $\mathcal{P}$ is the number of orbital planes; $\mathcal{P}$ can be any divisor of $\mathcal{T}$

- $\mathcal{F}$ is the angle past its ascending node at which a satellite is when the satellite in the neighboring plane to its East reaches its node; it has units of $2\pi/\mathcal{T}$ and can take any integer value in $\{0, 1, ...\mathcal{T} - 1\}$

A Simplified Model of Maneuverability

Maneuverability Δi_{max} A good metric for maneuverability is the maximum incremental velocity that a satellite is designed to perform before it needs to be refueled. With this incremental velocity ΔV_{max}, each satellite could *a priori* change any or all of its six orbital

elements. In order to keep isolation and integrity constant without changing the payload design, we will consider a constant altitude and therefore constant eccentricity. The longitude of the ascending node has little impact on the average coverage for circular low-Earth orbits. Therefore inclination and phase between satellites in adjacent planes are the two orbital elements that are interesting to change. In order to gain some insight about maneuverability while keeping the study simple, let's further limit ourselves to Walker Delta-patterns with a constant number of satellites placed into a constant number of planes. Then ΔV_{max} can be used only for uniform inclination change, and change of Walker phase number.

The incremental velocity required for a change in phase, which is to first order inversely proportional to the time allowed (equation 3.10), can safely be considered negligible compared to the incremental velocity required for an inclination change. Therefore in this simplified model, the maneuverability ΔV_{max} can simply be expressed in terms of a maximum inclination change before running out of fuel: Δi_{max}.

As more satellites are available, mission requirements can be met at orbital inclinations that are further and further away from the target's latitude, and at more diverse Walker phases. Therefore increasing the number of satellites decreases their required inclination change capability.

Viewing angle conditions   Three viewing angle conditions must be met for a target to be considered in view of the satellite, as illustrated by figure 6-1:

- A minimum grazing angle ε_{min}; the is the angle from the user's horizon to the target-satellite line-of-sight.

- A minimum nadir hole angle η_{min}; this is the angle from the sub-satellite radial vector to the satellite-target line-of-sight.

- A minimum cone angle β_{min}; this is the angle from the satellite velocity vector to the target-sub-satellite-point vector.

Meeting the requirements over a target   Due to the rotation of the Earth below satellites in LEO, the availability of a constellation over a location is a function of the location latitude only. We will say that a constellation configuration meets the requirements
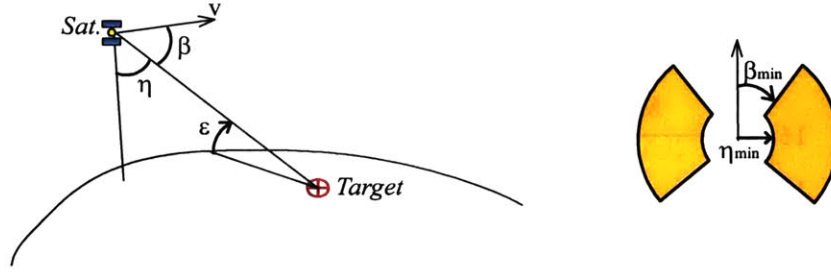
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Figure 6-1: Definition of Viewing Angles: Satellite / Target Configuration and Satellite Footprint

$\mathcal{R} = (Av, Rxn)$ over a target at latitude θ if its reaction time for observing *this* target is smaller than Rxn, $Av\%$ of the time.

Minimum number of satellites for a maneuverability For a constellation with a fixed number of satellites $\mathcal{T}$ and a fixed number of orbital planes $\mathcal{P}$, an orbital configuration $\mathcal{C}$ is defined by two numbers, namely its inclination and its Walker phase number: $\mathcal{C} = (i, \mathcal{F})$.

Consider a target at a latitude θ. The orbital inclination i is *possible* if there exists a phase $\mathcal{F}$ for which the configuration $(i, \mathcal{F})$ meets the requirements over the target. Let $\mathcal{I}(\theta) = [i_{min}(\theta), i_{max}(\theta)]$ be the range of possible orbital inclinations. In order to be able to meet the requirements over any hot spot, it must have the maneuverability:

$$\Delta i_{req}(\mathcal{T}, \mathcal{P}) = \max\left(0, \max_n(i_{min}) - \min_n(i_{max})\right)$$

The minimum number of satellites $\mathcal{T}_{min}$ for a maneuverability Δi_{max} is the minimum number of satellites $\mathcal{T}$ for which there exists a number of planes $\mathcal{P}$ such that $\Delta i_{req}(\mathcal{T}, \mathcal{P}) \leq \Delta i_{max}$. The function $\mathcal{T}_{min}(\Delta i_{max})$ is illustrated on figure 6-2 for various requirements. This estimate was obtained by numerically simulating all possible Walker Delta-patterns as Keplerian orbits perturbed only by the oblateness of the Earth (J_2-effect).

Ratio of time in view The ratio of time in view $\zeta(\theta, \mathcal{C})$ of a given target latitude θ is the average percentage time that the spot is viewed by a satellite in the constellation $\mathcal{C}$ with the required viewing angle conditions. By convention, a constellation that does not meet the (Av, Rxn) requirement for a target will be defined as having a null ratio of time
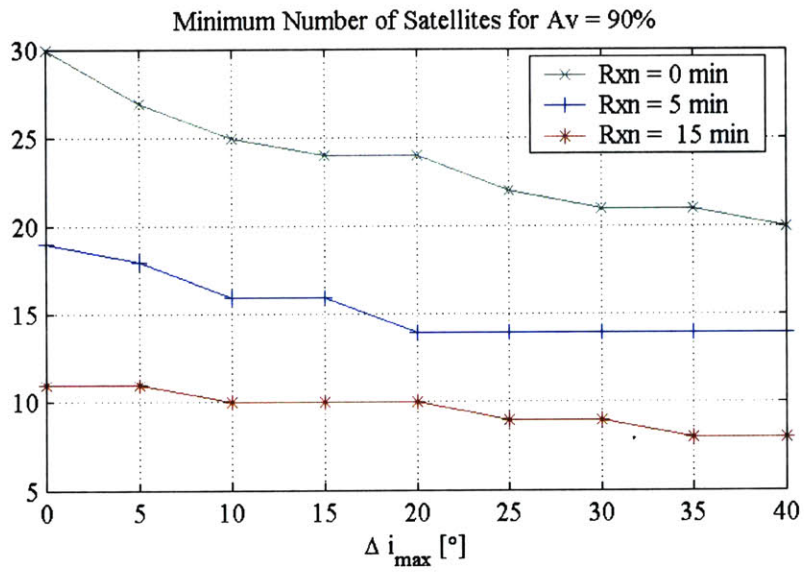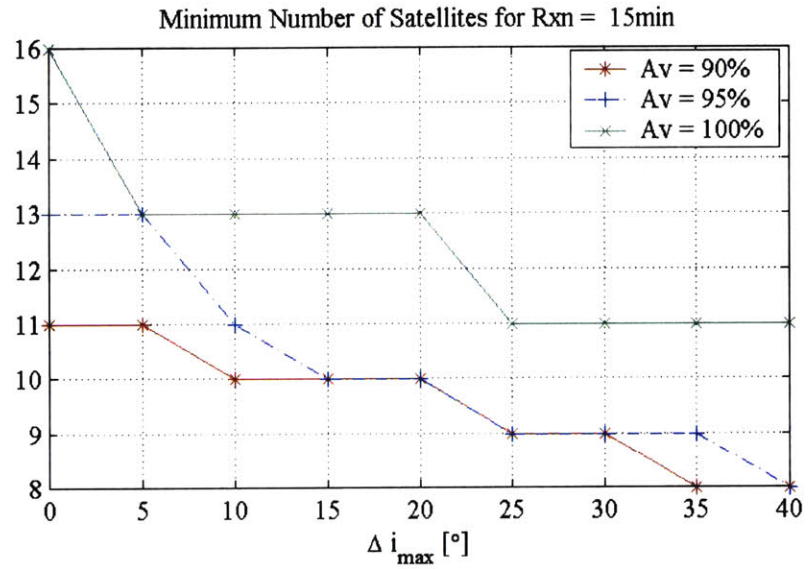
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Figure 6-2: Minimum Number of Satellites as a Function of Maneuverability. Ground moving target indication (GMTI) viewing angle conditions ($\varepsilon_{min} = 6°$, $\eta_{min} = 20°$ and $\beta_{min} = 0°$) and 10 possible targets between $\theta = 0°$ and $\theta = 48.5°$ are assumed.

in view over this location. The optimal orbital configuration $\widehat{\mathcal{C}}(\theta) = \left(\widehat{\imath}(\theta), \widehat{\mathcal{F}}(\theta)\right)$ is the configuration giving the maximum ratio $\widehat{\zeta}$.

Optimal number of planes Suppose we have a finite number of possible theaters and let P_{Hn} be the probability that the critical theater be number n. The expected utility rate for a maneuverable constellation with $\mathcal{T}$ satellites in $\mathcal{P}$ planes is then:

$$\overline{u}(\mathcal{T}, \mathcal{P}) = \sum_{n=1}^{N_{HS}} P_{Hn} \, \zeta \left(\theta_n, \widehat{\mathcal{C}}(\theta_n) \right)$$

Among the possible number of orbital planes $\mathcal{P}$ for a given number of satellites $N_{sat} = \mathcal{T} \geq \mathcal{T}_{min}(\Delta i)$, the optimal number of orbital planes $\widehat{\mathcal{P}}(\mathcal{T}, \Delta i)$ is the one that yield the maximum expected utility rate $\widehat{u(\mathcal{T})} = \overline{u}(\mathcal{T}, \widehat{\mathcal{P}})$. If costs do not depend on the number of planes, then this is also the number of planes that maximizes value.

Capturing the main trade With the above definitions, the number of orbital planes and the optimal orbital configurations over each possible target are set once the number of satellites and their maneuverability are chosen. Thus the cost as well as the utility rate of a constellation depend only on two parameters: its number of satellites $\mathcal{T}$ and its maneuverability Δi_{max}. This is the level of simplification that we wanted to reach.

6.1.3 Maneuverable Satellite Propulsion System

Modeling the effect of maneuverability on the satellite cost is key to capturing the trade between the number of satellites and their maneuverability. This effect is mainly dependent on the design of a propulsion system.

This section considers the effect of choosing between two very different types of propulsion systems: chemical propulsion, which is very fuel consuming but allows for quasi-instantaneous maneuvers; and electric propulsion, which requires long maneuvering times but leads to much lighter spacecraft.

Chemical Propulsion

An example of a baseline chemical propulsion system that was designed for refueling is the Gamma Ray Observatory (GRO) Propulsion Subsystem [WCH88]. It could carry up to

Table 6.1: Selection of Representative Electric Propulsion Systems from [WL99]

Concept	$I_{sp}[s]$	$P_{in}[kW]$	$T_P[mN/kW]$	$M_P[kg/kW]$	Propellant
Resistojet	299	0.9	905	1	N_2H_4
Arcjet	580	2.17	113	2.5	N_2H_4
Pulsed Plasma Thruster (PPT)	1200	0.02	16.1	85	Teflon
Hall Effect Thruster (HET)	2042	4.5	54.3	6	Xenon
Ion Thruster	3400	0.6	25.6	23.7	Xenon

1800 kg of hydrazine.

If the maneuverable spacecraft use several times the same tank as for the GRO, or tanks designed with the same sizing proportions, then the ratio of the total tank mass over the usable fuel mass remains constant. It is then a fair approximation to define as in chapter 3 a constant propulsion subsystem mass factor f_p such that:

$$M_{dry}^{propulsion} = f_p \, M_{fuel}$$

For the GRO, this factor is found to be $f_p \approx 15\%$.

Chemical maneuvers are well modeled as impulsive burns, which means that the change in inclination can be considered to be instantaneous. The incremental velocity and time required to perform an inclination change Δi_{max} are therefore:

$$\begin{cases} \Delta V_{max}^{chem} &= 2\,V_0\,\sin\left(\frac{\Delta i_{max}}{2}\right) \\ \Delta T_{max}^{chem} &= 0 \end{cases} \tag{6.1}$$

where V_0 is the orbital velocity at the satellite's altitude a_0.

Electric Propulsion

Unlike chemical impulses, electric propulsion maneuvers consist of low-thrust, continuous burns over long periods of time.

[WL99] gives the input power P_{in}, the thrust/power ratio T_P, the specific impulse I_{sp} and the specific mass M_P for various existing electric propulsion systems. A representative selection of these systems is reproduced in table 6.1. The thrust available from N_T thrusters is:

$$F = N_T\,T_P\,P_{in} = T_P\,P_{input} \tag{6.2}$$

where $P_{input} = N_T\, P_{in}$ is the total input power to the propulsion system.
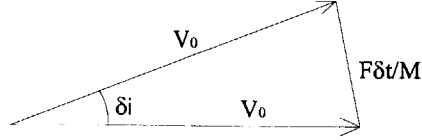

Figure 6-3: Low-Thrust Inclination Change

Since the maneuver is not instantaneous, the formula for inclination change $\delta V = 2\,V_0\, sin(\delta i/2)$ is valid for short time scales only. Making the approximation that the orbit remains circular at all points during the low-thrust maneuver, this gives:

$$\frac{di}{dt} = \frac{1}{V_0}\frac{dV}{dt} \tag{6.3}$$

If the thrust F and the exhaust velocity $g\,I_{sp}$ are constant, then the mass flow rate $\dot{M} = F/g\,I_{sp}$ is constant and the total time for the maneuver can be estimated from the rocket equation:

$$\frac{V_0\,\Delta i}{g\,I_{sp}} = \ln\left(\frac{M_{init}}{M_{init} - F\Delta T/g\,I_{sp}}\right)$$

which finally yields:

$$\begin{cases} \Delta V_{max}^{elec} & = V_0\,|\Delta i_{max}| \\[2mm] \Delta T_{max}^{elec} & = \frac{g\,I_{sp}}{T_P}\left(1 - e^{-\frac{V_0\,|\Delta i_{max}|}{g\,I_{sp}}}\right)\frac{M_{init}}{P_{input}} \end{cases} \tag{6.4}$$

Thus the time to maneuver is proportional to the spacecraft wet mass M_{init} and inversely proportional to the power P_{input} available for propulsion.

Spacecraft Mass Budget

The spacecraft mass budget is then the same as described in chapter 3. Recalling equation 3.5:

$$\begin{cases} M_{dry,tot} & = \dfrac{M_{dry}^{base}}{1-(f_p+f_{st}f_p+\epsilon\,f_{st})\left(e^{\frac{\Delta V_d}{g\,I_{sp}}}-1\right)} \\[4mm] M_{launch} & = M_{dry,tot}\left(1 + \epsilon\left(e^{\frac{\Delta V_d}{g\,I_{sp}}} - 1\right)\right) \end{cases}$$

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Figure 6-4 compares the mass increase, and the time for a given inclination change per kg/kW of spacecraft, for the electric propulsion systems given in table 6.1 and for chemical propulsion.

Figure 6-5 plots the total spacecraft dry mass versus the time to maneuver for a spacecraft similar to the *Discoverer II* baseline satellites, which had $M_{dry}^{base} = 4,400kg$ and a payload radiating $4\,kW$ of power, hence $P_{input} \approx 10\,kW$. A spacecraft with chemical propulsion would accomplish a $\Delta i_{max} = 20°$ change in less than a day and have a dry mass of $6000\,kg$; a spacecraft with arcjets would maneuver in 195 *days* and have a dry mass of $5000\,kg$; and a spacecraft with ion thrusters would maneuver in 571 *days* and have a dry mass of $4470\,kg$. Thus without any increase in available power, the mass savings allowed by electric propulsion come at the expense of very long maneuver times.

6.1.4 Modeling Utility per Cost

Building Blocks

The basic elements of the flexibility valuation take a special form here for two reasons: on-demand refueling requires continuous decision points, and the decision model is to maximize *current utility* instead of maximizing *future value*.

Uncertain parameter: theater dynamics The uncertain parameter is the latitude $X = \theta$ of the current critical theater of action.

Bonds & al show that the historical occurrence of world conflicts has been similar to a Poisson process, i.e. the probability that a contingency appears between the times t and $t + dt$ is of the form $\nu_c\,dt$. Here we are interested in a radar constellation that must constantly cover the *most critical* theater of action. A reasonable assumption is that any new contingency has a certain probability to become the new theater of action, so that the probability that the theater changes between t and $t + dt$ is of the form $\nu\,dt$ where we will can ν the *hot spot frequency*.

A new theater is relevant for the radar constellation only if it appears at a significantly different latitude. Thus it is sufficient to define a finite number N_H of *hot spots*, each spot n representing a different region around the latitude θ_n, and having the probability P_{Hn} to contain the most critical theater. For the purpose of this study, we will assume that the P_{Hn} are constant. If the theater is in spot n at t, then at $t + dt$
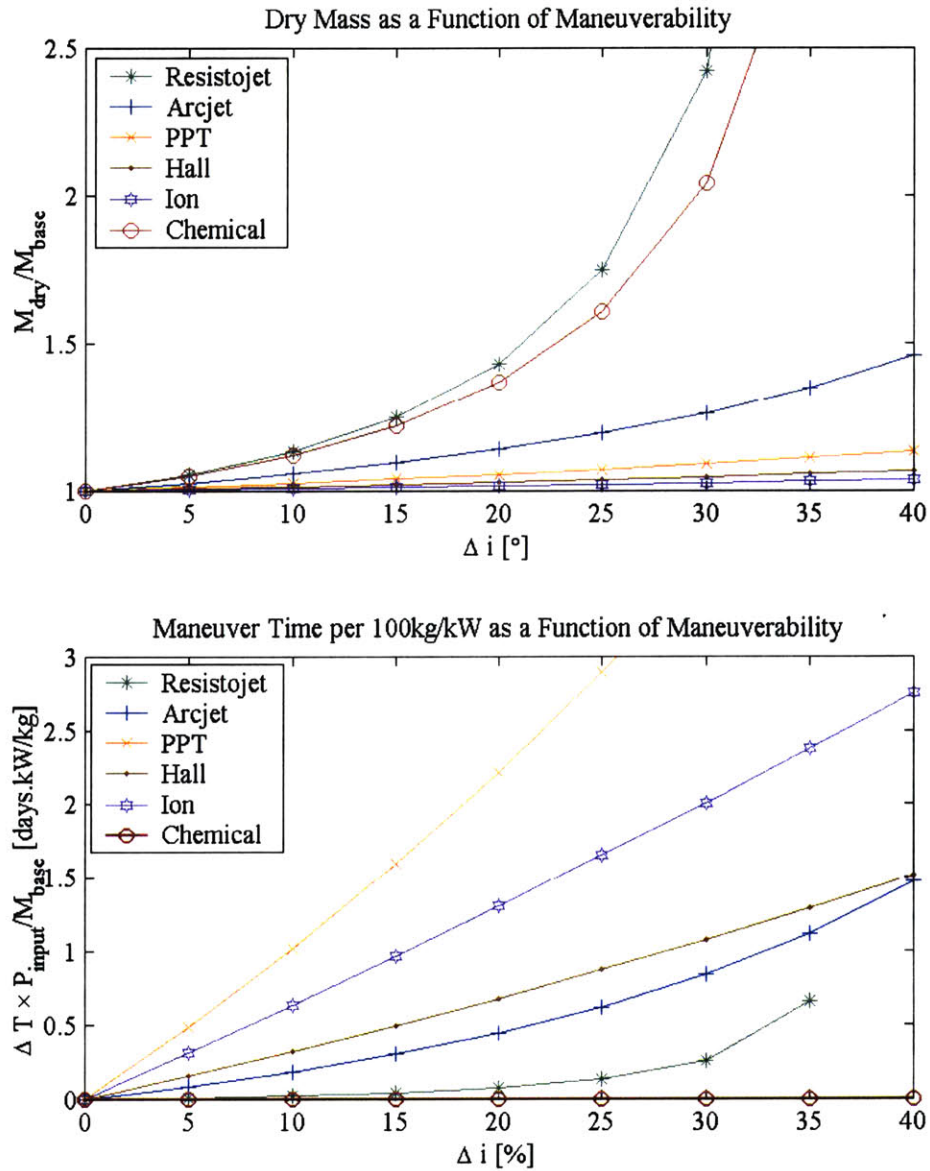
Figure 6-4: Spacecraft Mass and Time for Inclination Change for Selected Electric Propulsion Systems. To know the number of days for a given Δi_{max}, multiply the value read on the plot by the total mass of the spacecraft in kg and divide it by the power available for propulsion in kW.
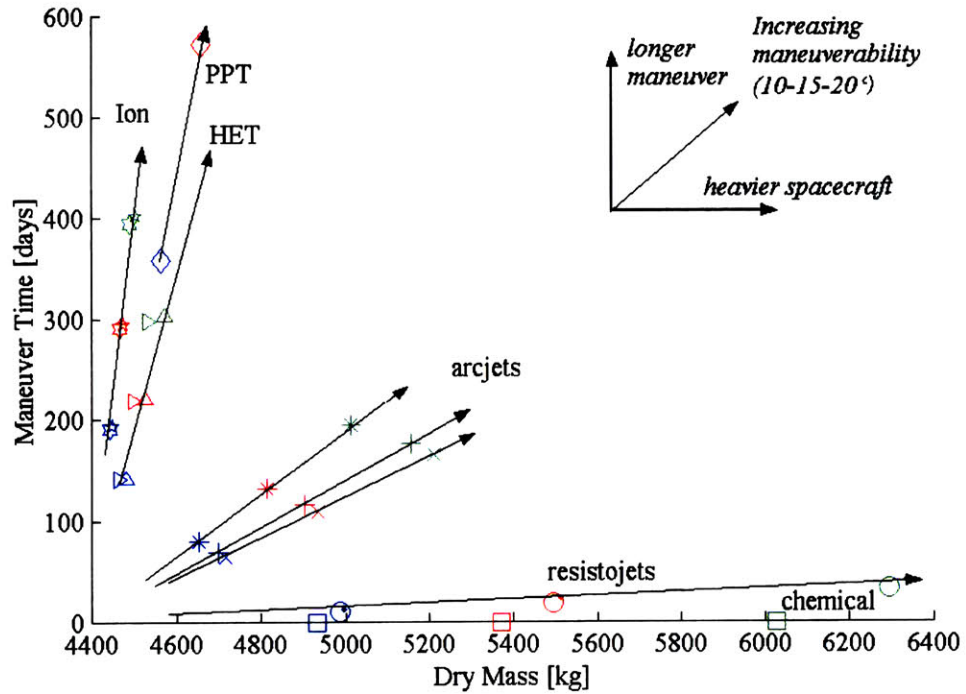
Figure 6-5: Spacecraft Mass versus Time for Inclination Change for Selected Electric Propulsion Systems in the Case $M_{dry}^{base} = 4400\,kg$ and $P_{input} = 10\,kW$.

it has the probability $(1 - \nu\,dt)$ to still be in n, and the probability $\nu\,dt\,P_l/(1 - P_{Hn})$ to be in any other spot l.

Modes of operation The possible modes of operation are the N_H constellation configurations $\widehat{C}_n = (\widehat{i}_n, \widehat{F}_n)$ optimized over each hot spot n.

Utility rate The utility rate is the ratio of time in view of the current configuration (n) over the current theater latitude X:

$$u^{(n)}(X) = \zeta\left(X, \widehat{C}_n\right)$$

Decision points The decision points are here continuous: at each time t, the decision can be made whether to maneuver to a new configuration (n). If the decision is to not maneuver, then the next decision point occurs at $t + dt$. However, if the decision is to maneuver and the time to perform a maneuver T_M is finite, then there won't be any other decision point until $t + T_M$. This makes the system dependent on history, which

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is contrary to one of the framework assumptions made in chapter 4 (Markov process assumption). This contradiction can be leveraged, and the system made dependent on the current state of nature only, by defining a *maneuver rate* $\mu_M = 1/T_M$: the probability to finish the maneuver between t and $t + dt$ verifies $dP = P\mu_M\, dt$.

Matrix of switching costs We saw that the main impact of maneuverability was mass. Since the added mass is primarily made up a fuel and structures to support the fuel, a reasonable approximation is to assume that the development and production cost do not depend on maneuverability. The cost to initial operating capability is a function of the number of satellites in the constellation and their maneuverability: $C_{IOC} = f^n(N_{sat}, \Delta i_{max})$. The dependence of the satellite mass on maneuverability is given by equation 3.5. The dependence on number of satellites is made up of three terms: constant development costs, launch costs proportional to the number of satellites, and production costs subject to a learning curve factor B. So that finally:

$$C_{IOC} = C_D + N_{sat}^B C_P + \frac{1 + \epsilon \left(e^{\frac{\Delta V_d}{g\, I_{sp}}} - 1 \right)}{1 - (\epsilon f_{st} + f_p + f_p f_{st}) \left(e^{\frac{\Delta V_d}{g\, I_{sp}}} - 1 \right)} N_{sat}\, M_{dry}^{base}\, C_L \qquad (6.5)$$

where C_L is the launch cost per unit mass.

Switching modes of operation requires refueling. The servicing price is likely to depend on the mass to be delivered, which is a function of the inclination change to be performed. Let us assume that there is a constant servicing price C_{Sm} per unit mass delivered. Then any result on servicing price per unit mass will be directly comparable to launch prices per unit mass. The incremental velocity to maneuver between configurations (m) and (n) is a function of their difference in inclination:

$$\Delta V^{(m \leftrightarrow n)} = \begin{cases} 2\, V_0 \sin \left(|\widehat{\imath}_m - \widehat{\imath}_n| / 2 \right) & \text{with chemical propulsion}, \\ V_0\, |\widehat{\imath}_m - \widehat{\imath}_n| & \text{with electric propulsion}. \end{cases}$$

The mass of fuel to be delivered is then $M_{fuel}^{(m \rightarrow n)} = M_{dry} \left(e^{\Delta V / I_{sp} g} - 1 \right)$, so that the

servicing price to switch modes is of the form:

$$C^{(m \mapsto n)} = \frac{\exp\left(\frac{\Delta V^{(m \mapsto n)}}{I_{sp}\,g}\right) - 1}{1 - (\epsilon f_{st} + f_p + f_p\,f_{st})\left(e^{\frac{\Delta V_{max}}{g\,I_{sp}}} - 1\right)}\, M_{dry}^{base}\, C_{Sm} \qquad (6.6)$$

Decision model It makes sense that the initial decision between different designs be made on a utility per cost basis. However, in a military framework it also makes sense that as a theater location changes, the decision be to maximize the performance achievable with the current system. The decision model therefore dictates to maneuver on-demand to the new optimal configuration $\widehat{C} = (\widehat{\imath}, \widehat{\mathcal{F}})$.

Value model

In this study the decision model thus takes of new form, which is to continuously optimize performance instead of periodically maximize future value. Thus the decision depends on the uncertain parameter and the current mode of operation, but not on the decision time. The best method to estimate value in this case is therefore not a backwards iterative process. A Markov model is more appropriate. Such a model presents the further advantage of facilitating the introduction of a servicing rate μ_S and a probability of a crash P_C.

At least five states are necessary to describe the constellation behavior:

1. The satellites have enough fuel for a maneuver and the constellation is optimized over the current theater.

2. The satellites do not have enough fuel for a maneuver but the constellation is optimized over the current theater.

3. The satellites have enough fuel for a maneuver and the constellation is not optimized over the current theater.

4. The satellite do not have enough fuel for a maneuver and the constellation is not optimized over the current theater.

5. Some satellites are failed, so that the constellation cannot meet the requirements.

The Markov process is illustrated on figure 6-6: the constellation stops being optimized at a rate ν, at which point it must maneuver with a rate μ_M, then be refueled with a rate
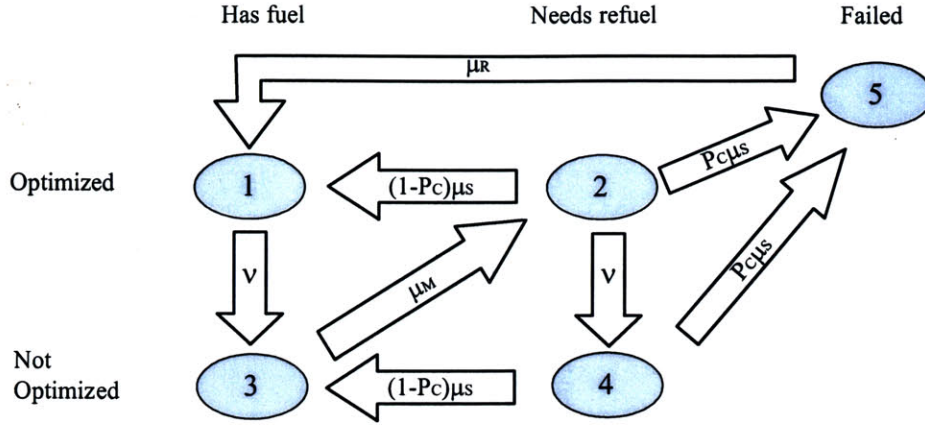
Figure 6-6: Markov Model for a Maneuverable Radar Constellation

$(1 - P_C)\,\mu_S$, which carries the risk of a failure with rate $P_C\,\mu_S$. Finally, failed satellites are replaced at a rate μ_R. If desired, it is also easy at this point to model the **aging of satellites** by a failure rate λ; however this captures no fundamental difference between a maneuverable constellation and a baseline constellations. The resulting Markov matrix is:

$$
A_M = \begin{pmatrix}
-\nu\,(-\lambda) & (1 - P_C)\mu_S & 0 & (1 - P_C)\mu_S & \mu_R \\
0 & -\nu - \mu_S\,(-\lambda) & \mu_M & 0 & 0 \\
\nu & 0 & -\mu_M\,(-\lambda) & 0 & 0 \\
0 & \nu & 0 & \mu_S\,(-\lambda) & 0 \\
0\,(+\lambda) & P_C\,\mu_S\,(+\lambda) & 0\,(+\lambda) & P_C\,\mu_S\,(+\lambda) & -\mu_R
\end{pmatrix} \tag{6.7}
$$

The vector of probabilities $(P_i)_i$ to be in each state i is given by solving the differential equation:

$$
\begin{cases}
P(0) & = [1 \quad 0 \quad 0 \quad 0 \quad 0]; \\
\dot{P}(t) & = A_M\,P(t)
\end{cases} \tag{6.8}
$$

The final calculation of the mission value is summarized on table 6.2.

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Table 6.2: Summary of Thin Constellation Valuation Process

$$V = \frac{E\{U\}}{E\{C\}}$$

$$E\{U\} = \overline{T_o}\,\overline{u_o} + \overline{T_{no}}\,\overline{u_o}$$

$$E\{C\} = C_{IOC}(N_{sat} + N_R, \Delta i_{max}) + N_S\,\overline{C} \quad \text{where:}$$

$$N_R = \int_0^{T_H} \mu_R\, e^{-rt}\, P_5(t)\, dt \qquad \text{Discounted number of replacements}$$

$$N_S = \int_0^{T_H} \mu_S\, e^{-rt}\, (P_2(t) + P_4(t))\, dt \qquad \text{Discounted number of servicings}$$

$$\overline{C} = \sum_{n=1}^{N_H} \sum_{n \neq m} \frac{P_{Hn} P_{Hm}}{1 - P_{Hn}} C^{(n \leftrightarrow m)} \qquad \text{Average cost to change configuration}$$

$$\overline{u_o} = \sum_{n=1}^{N_H} \widehat{\eta}(\theta_n) \qquad \text{Average utility rate if optimized configuration}$$

$$\overline{u_{no}} = \sum_{n=1}^{N_H} \sum_{m \neq n} \eta(\theta_n, \widehat{C_m}) \qquad \text{Average utility rate if non-optimized configuration}$$

$$\overline{T_o} = \int_0^{T_H} (P_1(t) + P_2(t))\, dt \qquad \text{Time spent with optimized configuration}$$

$$\overline{T_{no}} = \int_0^{T_H} (P_3(t) + P_4(t))\, dt \qquad \text{Time spent with non-optimized configuration}$$

6.1.5 Results for the Baseline Case

Numerical Assumptions

Distribution of Potential Hot Spots   Let us assume that there are ten hot spots around
the world, as shown in table 6.3. These hot spots are representative of any set of locations
within $0°$ and $48°$ latitude: the limitation to ten spots thus simplifies the study without
sacrificing any generality.

Table 6.3: Latitude of Hot Spots

Hot Spot Number n	1	2	3	4	5	6	7	8	9	10
Latitude θ_n	48.51°	45°	42°	40.5°	33.5°	32°	31.9°	30°	13.8°	0°
Probability P_n	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Table 6.4: Baseline Assumptions

Parameter	Symbol	Nominal Value	Source
Mission horizon	T_H	$20\,yr$	*Discoverer II*
Required reaction time	Rxn	$15\,min$	*Discoverer II*
Required availability	Av	90%	*Discoverer II*
Altitude	h_0	$770\,km$	*Discoverer II*
Min. grazing angle	ε_{min}	12°(SAR)/ 6°(GMTI)	*Discoverer II*
Min. nadir angle	η_{min}	20°	*Discoverer II*
Min. cone angle	β_{min}	45°(SAR)/ 0°(GMTI)	*Discoverer II*
Hot spot frequency	ν	$1yr^{-1}$	Adapted from [BMH$^+$00]
Mean time to refuel	μ_S	$1\,week$	Estimate from lit.
Mean time to maneuver	m	$[1\,day; 2\,months]$	Parameter
Satellite's specific impulse	I_{spC}	$320\,s$	Chemical [WL99]
	I_{spE}	$299 - 3400\,s$	Electric [WL99]
Structures mass factor	f_{st}	0.2	Robust design
Propulsion dry mass factor	f_p	0.15	GRO [WCH88]
Learning curve factor	B	0.926	95% slope

The satellites are launched with fuel for orbit insertion, station-keeping and de-orbiting
as if they where not to maneuver. Fuel for maneuver (maximum load) is filled right after
orbit insertion. After each maneuver the amount of fuel that has been used is refilled.

Chemical Propulsion: Instant Maneuver but Prohibitive Mass

If the location of the main theater of action change, it is critical that the radar satellite
be able to observe the new theater as fast as possible. Chemical propulsion offers the great
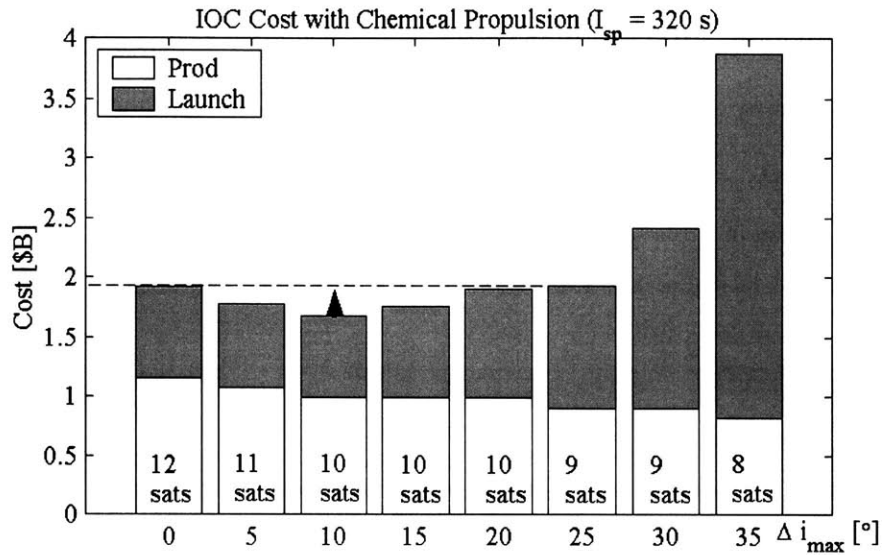
Figure 6-7: Chemical Propulsion: IOC Cost as Function of Maneuverability

advantage of minimizing the time for an inclination change. This however comes at the expense of an exponential mass increase.

Figure 6-7 shows the minimum cost of a constellation to IOC as a function of its maneuverability, assuming that the optimal number of satellites is chosen. This optimal happens to be the minimum number of satellites required for each maneuverability, so that the production costs are as expected a decreasing function of maneuverability. But while production costs decrease linearly, launch costs increase exponentially with maneuverability as a results of the exponential mass increase.

A very slight minimum of cost and maximum of utility per cost (see figure 6-8) is observed for a maneuverability $\widehat{\Delta i_{max}} = 10°$. But the difference with zero maneuverability is so slight that it allows no room for servicing price: whatever the price of servicing, chemical maneuverability cannot be interesting for this radar constellation.

It is interesting to note that in this case, there is no value to refueling whatever the design of a servicing infrastructure. This proves the interest of studying the value of servicing before even attempting to model its cost.
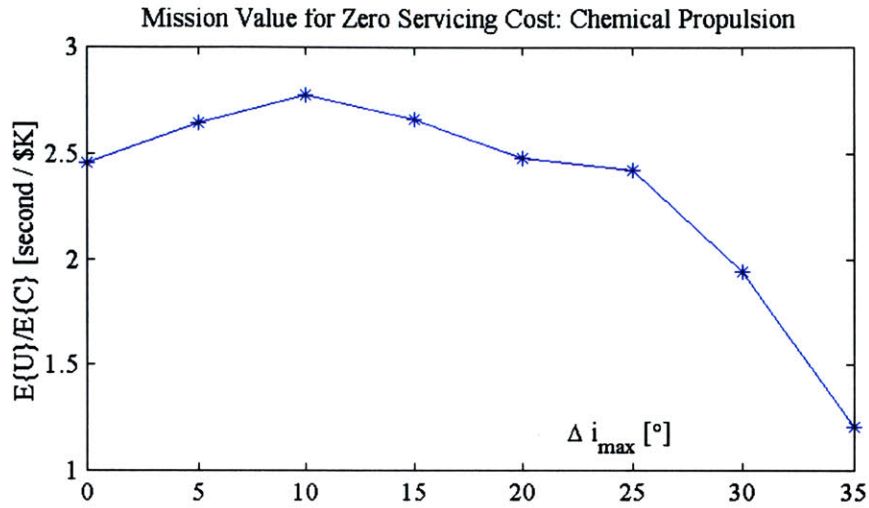
Figure 6-8: Chemical Propulsion: Mission Value for Null Servicing Price

Electric Propulsion: High Maneuverability and but Very Long Maneuver

The exponential mass increase observed in the previous section is the direct effect of the low specific impulse of chemical propulsion. With electric propulsion, the propulsion system mass can stay within reasonable bounds, as we saw on figure 6-4. This cost advantage however comes at the expense of a very long time to maneuver. Though a long maneuver time is unacceptable in the context of war, it may be acceptable in the context of peacetime surveillance. An upper bound on the value of servicing can be found by assuming that there is no limit on the allowed time to maneuver and considering the various propulsion schemes given in table 6.1.

Figure 6-9 shows that for a sufficiently high specific impulse, the impact of maneuverability on spacecraft mass becomes negligible, so that the sum of production and launch costs decreases with increasing maneuverability. In the case of free servicing, the optimal design would be the highest possible maneuverability, $\Delta i_{max} = 40°$. However, the higher the maneuverability, the higher the total mass that must be delivered to the constellation over the lifetime of the mission. As soon as servicing has a price, the optimal maneuverability is thus lower. Considering the difference in cost and in utility between the maneuverable cases and the non-maneuverable case, one can determine the *maximum servicing price per unit mass* under which the optimal design is maneuverable.

This maximum is illustrated on figure 6.1.5 as a function of time allowed to maneuver,
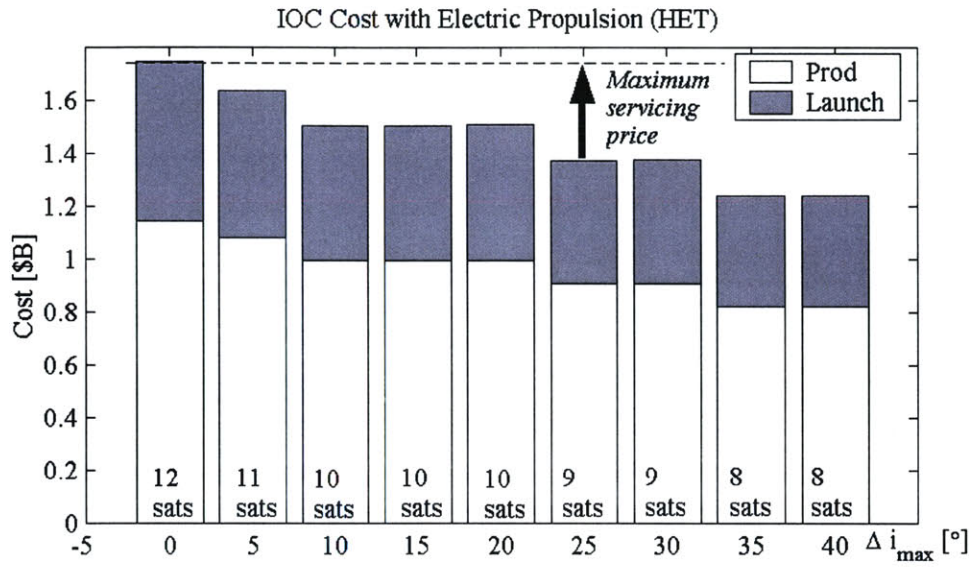
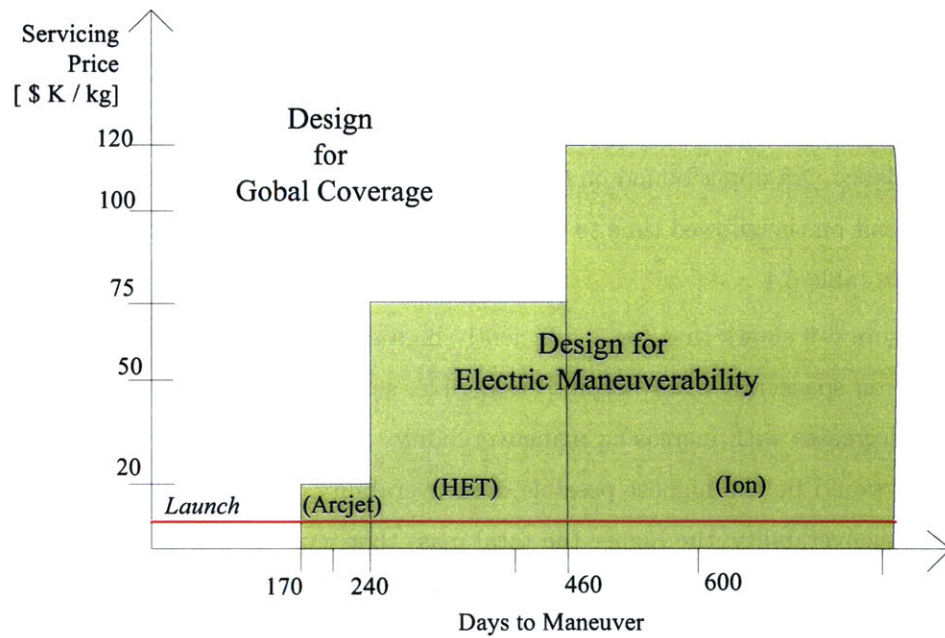Figure 6-9: Electric Propulsion: IOC Cost for Null Servicing Price

Figure 6-10: Electric Propulsion: Threshold Servicing Price per Unit Mass versus Time Allowed to Maneuver

taking into account the different types of electric propulsion systems. Approximately the same results are obtained whether considering utility per cost or cost alone. The maximum servicing price is greater than five times the cost to launch to LEO (which is approximately $10\,K/kg$) for any electric propulsion system but arcjets and resistojets. Thus, if the price of servicing is kept close to the marginal servicing cost, an electric thin constellation is more cost-effective than a global coverage, non-maneuverable constellation. In the case of ion thrusters, the maximum servicing price is even an order of magnitude higher than typical launch costs. However, this must be traded against maneuver time. The maneuver time that makes electric propulsion optimal is of the order of a year for the baseline satellite's mass and power. Given that no power is available for the payload during a maneuver, such a long time is unacceptable in the critical context of radar coverage of a military theater of action.

6.1.6 Conclusions

By introducing a Markov model of the dynamics of contingencies, this section successfully expanded the framework to account for continuous decision times in the special case when the decision is to always optimize performance. The trade of number of satellites versus maneuverability was explored for a radar constellation for two types of propulsion systems. While chemical propulsion offers the fastest maneuverability, its exponential mass increase outweighs the advantages of servicing. Refueling proves much more promising for electric propulsion systems, for which the maximum servicing price per unit mass can be as high as an order of magnitude greater than launch costs to LEO. However, electric propulsion systems require unacceptable times for a change in inclination.

These results suggest that refueling would have no value for a LEO radar constellation unless revolutionary propulsion technologies, offering fast maneuver *and* high specific impulse, were developed. Before drawing definitive conclusions, other options should of course be explored, such as other types of maneuverability (not limited to inclination change of a Walker Delta-pattern) or step-wise chemical inclination change (refueling after each step). These studies are however outside the bounds of this research effort.

Instead, let us use this generalization of the framework to study another military case, for which the incremental velocity for maneuvering is less problematic.

6.2 Military Communications under Uncertain Contingency Locations

The uncertainty in the occurrence and location of contingencies does not affect only radar satellites. As the military deploy troops, their communications needs over the longitude of the theater increase significantly. An answer to these needs is to lease capacity on available commercial geostationary satellites [BMH+00]. However, commercial satellites can be used only for information that requires a low level of security.

There is therefore a need for flexibility in the distribution of the capacity provided by military satellites. This flexibility could be achieved by making geostationary military satellites maneuverable, so that their distribution in longitude as a function of the distribution of contingencies. Changes in longitude require only small incremental velocities even for a short allowed maneuvering time, thus solving the problem faced in the previous section. *Is this enough to make refueling of significant value ?* This section proposes a basic, first-order-of-magnitude model to estimate the maximum servicing price that would make on-orbit servicing interesting for this case.

6.2.1 Satellite Design: Design-ΔV

Incremental Velocity for Phasing Maneuvers

Changing longitude by an angle $\Delta\Phi$ corresponds to a phasing maneuver, which can be accomplished by altering the apogee of the orbit so that the slightly different period cancels out the difference in phase. The required incremental velocity depends on the time ΔT_{max} allowed to perform the maneuver and was given in chapter 3 [‡]:

$$\frac{\Delta V_{ph}\left(\Delta\Phi, \Delta T_{max}\right)}{V_0} \;=\; 2\left|\sqrt{2 - \left(\frac{l}{l - \Delta\Phi/2\pi}\right)^{\frac{2}{3}}} - 1\right|$$

$$\text{where} \quad l \;=\; \text{Integer part of} \left(\frac{\Delta T_{max}}{T_0} + \frac{\Delta\Phi}{2\pi}\right)$$

Figure 6-11 illustrates this function. It shows in particular that the velocity increment is proportional to $\Delta\Phi/\Delta T_{max}$ for $\Delta T_{max} \geq 7\,days$.
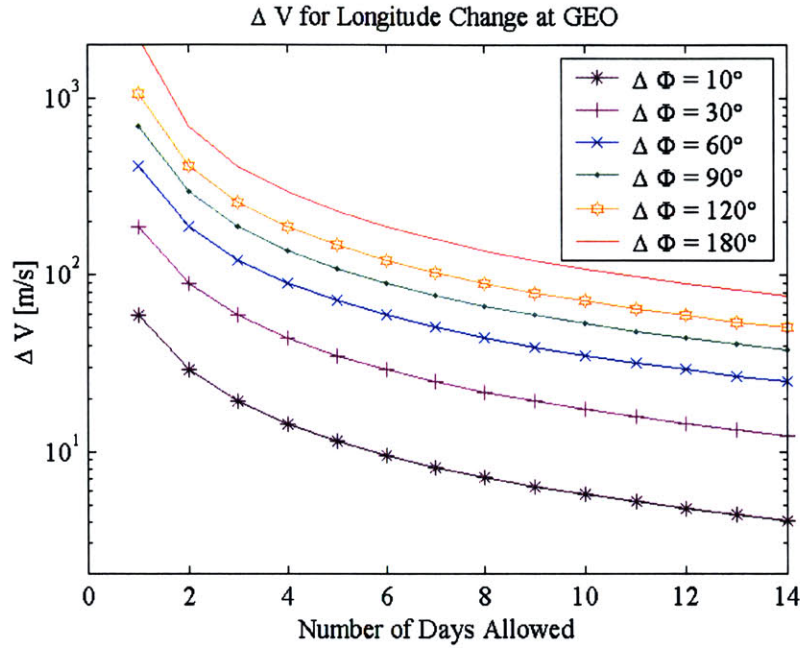

[‡]And more details are provided in appendix B.1.2

Figure 6-11: ΔV to Change the Longitude of a GEO Satellite as a Function of Time Allowed to Maneuver

Fleet Maneuver Scheme

Suppose that capacity is required to be moved between two regions separated in longitude by an angle $n\Delta\Phi$. There are two ways to maneuver the fleet of satellites, as illustrated on figure 16:

1. Move one satellite by an angle $n\Delta\Phi$ so as to minimize the number of satellites to be refueled, or

2. Move each satellite to the nearest slot in the direction of the capacity move; the satellites replace each other in a row so that each of them maneuvers the least far possible. If satellites are placed every $\Delta\Phi$, this corresponds to moving n satellites by an angle $\Delta\Phi$.

Previous results suggest that it is important to minimize spacecraft design-ΔV. However, the incremental velocity required for station keeping of a GEO satellite with a design life or 10 years is of the order of $500\,m/s$. Figure 6-11 shows that as soon as two days are allowed for the maneuver, the incremental velocity for any longitude change is smaller than $700\,m/s$. It is therefore reasonable to design the satellites for $\Delta\Phi_{max} = \pi$ and choose
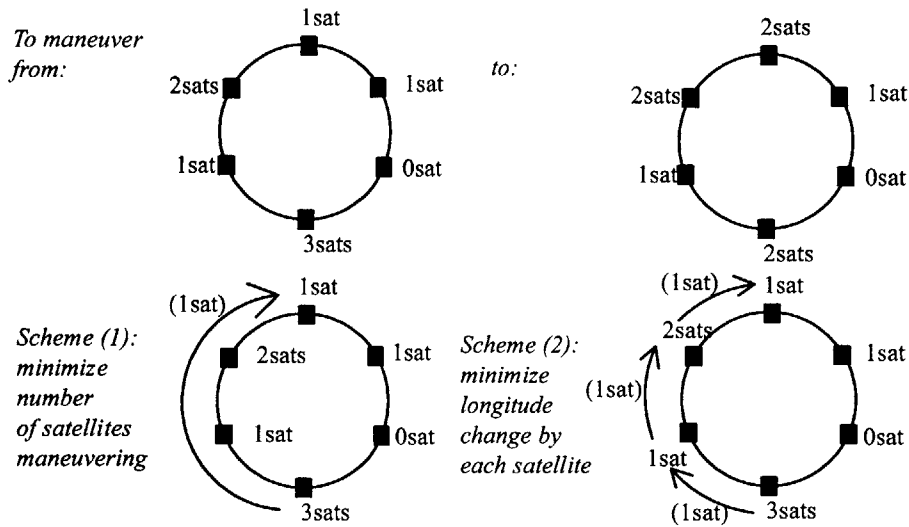
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Figure 6-12: Possible Maneuver Schemes for a Military Communications GEO Fleet

maneuver scheme (1), which maximizes the communications availability during the time of the maneuver.

The satellites can be assumed to be refueled after each maneuver, as well as every time they have performed ΔV_{max} for station keeping. Their design-ΔV is therefore:

$$\Delta V_d = \max \{\Delta V_{ins}, \ \Delta V (\pi, \Delta T_{max})\}$$

where ΔV_{ins} is the incremental velocity required for orbit insertion if the satellites are launched into geostationary transfer orbit. The mass budget is then the same as for the chemical radar constellation considered in section 6.1.

6.2.2 Modeling Value

Building Blocks

The building blocks for the valuation process are very similar to the previous case. In particular, on-demand maneuvering requires continuous decision points, and the decision model is to maximize current performance instead of maximizing future value; the same baseline principles lead to the same valuation method.

Uncertain parameter Assume that the total number of contingencies to be covered

around the world is a constant N_C. The uncertainty lies in the distribution of these contingencies as a function of longitude. It is practical to divide the globe in N_R regions and define the *uncertain parameter* as the vector $X = (X_1, ..., X_{N_R})$ describing the number of contingencies that lie in each region. We note that $\sum_{k=1}^{N_R} X_k = N_C$; there are therefore $N_R - 1$ independent random variables X_k. If at time t contingencies have the same probability $p = 1/N_R$ to occur in each region, then the probability that the distribution of contingencies be $\mathcal{C} = (X_1, X_2..., X_{N_R})$ is:

$$P\left\{(X_1, X_2..., X_{N_R})\right\} = p^a (1-p)^b \frac{N_C!}{\prod\limits_{i=1}^{N_R} X_i!} \tag{6.9}$$

$$\text{where} quad = \sum_{k=1}^{N_R - 1} X_i$$

$$\text{and} \quad b = (N_R - 1) N_C - \sum_{k=1}^{N_r - 1} k X_{N_R - k}$$

Modes of operation The fleet is made up of N_{sat} military GEO satellites whose only degree on freedom is their longitude. The possible *modes of operation* are the distributions of the fleet over the N_R regions: $\mathcal{C} = (n_1, n_2..., n_{N_R})$ where n_k is the number of satellites attributed to region k.

Decision model As for the thin constellation case, the decision model is independent of time: it says to maximize the achievable performance of the current system. This is achieved by maneuvering into the optimal distribution of satellites $n_k = I(X_k N_{sat}/N_C)$ as soon as demand moves, under the constraint $\sum_{k=1}^{N_R} n_k = N_{sat}$.

Decision times The *decision process* is continuous. This requires the definition of a maneuver rate $\mu_M = 1/\Delta T_{max}$; and a *contingency frequency* ν_c as the rate at which contingencies move; in other words, between t and $t + dt$ there is a probability $\nu_c dt$ that a contingency will move to a different region.

Utility The demand is expressed in terms of total capacity (data rate) required over the world. The part of the demand that is not met by the maneuverable fleet will be met by commercial satellite leases or use of other military systems. Therefore, the *utility* of the system can be defined as the percentage of data transfer required that has been

met over the lifetime of the mission. The *utility rate* at t is thus the percentage of the data rate $\mathcal{R}(t)$ required that is provided at t.

Let R be the data rate provided by each satellite, and assume that several satellites above the same region can be used at full capacity. The required capacity over a region k is $\mathcal{R}(t)\,X_k\,/N_C$. Therefore the utility rate of configuration $\mathcal{C}$ is:

$$u(X,t) = \frac{1}{\mathcal{R}(t)} \sum_{k=1}^{N_R} \min\left(n_k\,R,\,\frac{\mathcal{R}(t)\,X_k}{N_C}\right) \tag{6.10}$$

For a maneuverable constellation, $n_k = I\,(X_k\,N_{sat}/N_C)$ as soon as the maneuver is performed, so that:

$$\overline{u}(X,t) = \overline{u_o}(t)\,P_o(t) + \overline{u_{no}}(t)\,P_{no}(t) \tag{6.11}$$

where $P_o(t)$ is the probability to be in an optimized configuration, which at t has the expected utility rate:

$$\overline{u_o} = \sum_{\substack{X_k \geq 0 \\ \sum X_k = N_C}} P\left\{(X_1, \ldots X_{N_R})\right\} \sum_{k=1}^{N_R} \min\left(I\left(\frac{X_k\,N_{sat}}{N_C}\right)\frac{R}{\mathcal{R}(t)},\,\frac{X_k}{N_C}\right) \tag{6.12}$$

and $P_{no}(t)$ is the probability to be in a non-optimized configuration, which corresponds to missing one satellite over one region:

$$\overline{u_{no}}(t) = \overline{u_o}(t) - \frac{R}{\mathcal{R}(t)} \tag{6.13}$$

Switching costs Is is convenient to assume as previously that the price to service is proportional to the mass to be delivered, and look for a maximum in terms service price per serviced kg. In this case the price to refuel after a change of longitude $\Delta\Phi$ is:

$$C(\Delta\Phi) = \frac{\exp\left(\frac{\Delta V(\Delta\Phi, \Delta T_{max})}{I_{sp}\,g}\right) - 1}{1 - (\epsilon f_{st} + f_p + f_p f_{st})\left(e^{\frac{\Delta V_d}{g\,I_{sp}}} - 1\right)}\,M_{dry}^{base}\,C_{Sm} = x(\Delta\Phi)\,M_{dry}^{base}\,C_{Sm} \tag{6.14}$$

where C_{Sm} is the servicing price per unit mass.

Contingencies that require a satellite in a region k to maneuver have equal probability
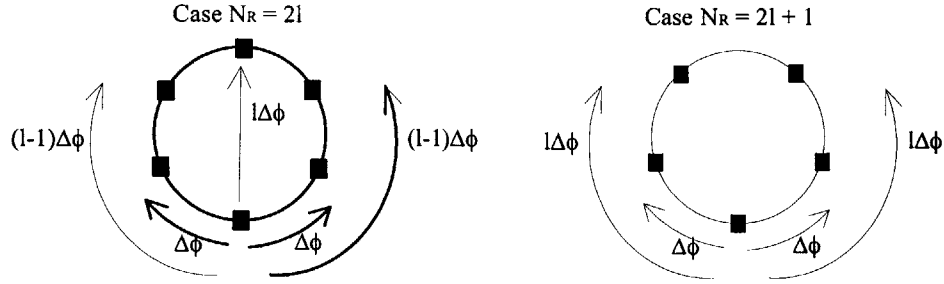
Figure 6-13: Possible and Equally Likely Maneuvers

$1/(N_R - 1)$ to appear in any of the $N_R - 1$ other regions. A satellite will always take the shortest past to maneuver, so that the possible maneuvers are illustrated on figure 6-13. Given the choice of maneuver scheme, this means that the average fuel cost for a maneuver is:

$$\overline{C}_S = \overline{x}_S M_{dry}^{base} C_{mS} \tag{6.15}$$

$$\overline{x}_s = \begin{cases} \frac{1}{l} \sum\limits_{k=1}^{l} x(k\Delta\Phi) & \text{if} \quad N_R = 2l + 1 \\ \frac{2}{2l-1} \sum\limits_{k=1}^{l} x(k\Delta\Phi) + \frac{1}{2l-1} x(l\Delta\Phi) & \text{if} \quad N_R = 2l \end{cases}$$

Markov model

The continuous decision process can be described by a Markov process very similar to the one proposed in section 6.1. Thanks to the symmetry of the regions, the performance of the fleet can be described by two states: (o) *optimized configuration* and (no) *non-optimized configuration*. On average, a satellite will need to maneuver only every N_{sat}/N_C changes in a contingency location. Therefore transitions from (o) to (no) occur with rate $\nu = N_{sat} \nu_c/N_C$, while satellites maneuver with rate μ_M from state (no) to state (o).

Satellites will maneuver one at a time. In order to take into account the effect of the servicing rate μ_S, each state must be divided into $(N_{sat} + 1)$ sub-states, where sub-state k means k *satellites need to be refueled*. Finally, the failed state (f) and the replacement rate μ_R must be considered to account for a non-zero probability of catastrophic event P_C.

The transitions in the Markov model are illustrated on figure 6-14, where $N_{sat} = 2$ for
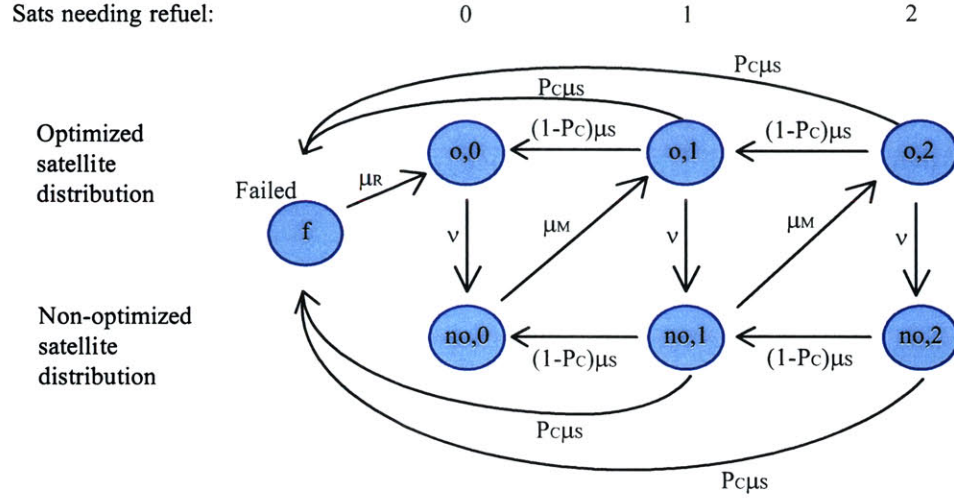
Figure 6-14: Markov Model for Maneuverable Milcom GEO fleet

the sake of clarity. The corresponding Markov matrix is finally (with $\mu'_S = (1 - P_C)\mu_S$):

$$
A_M = \begin{pmatrix}
-\nu & \mu'_S & 0 & 0 & 0 & 0 & \mu_R \\
0 & -\nu - \mu_S & \mu'_S & \mu_M & 0 & 0 & 0 \\
0 & 0 & -\nu - \mu_S & 0 & \mu_M & 0 & 0 \\
\nu & 0 & 0 & -\mu_M & \mu'_S & 0 & 0 \\
0 & \nu & 0 & 0 & -\mu_S - \mu_M & \mu'_S & 0 \\
0 & 0 & \nu & 0 & 0 & -\mu_S & 0 \\
0 & P_C\,\mu_S & P_C\,\mu_S & 0 & P_C\,\mu_S & P_C\,\mu_S & -\mu_R
\end{pmatrix} \tag{6.16}
$$

The discounted number of attempted servicings is:

$$
N_{serv} = \mu_S \int_0^{T_H} e^{-r\,t}\,dt \sum_{k=1}^{N_{sat}} \left(P_{o,k}(t) + P_{no,k}(t) \right) \tag{6.17}
$$

The discounted number of satellite replacements due to servicing catastrophic events is:

$$
N_{rep} = \mu_R \int_0^{T_H} e^{-r\,t}\,P_f(t)\,dt \tag{6.18}
$$

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Mission value The final value is given by:

$$\begin{cases} E\{U\} & = \int_0^{TH} P_o(t)\,\overline{u_o}(t)\,dt + \int_0^{TH} P_{no}(t)\,\overline{u_{no}}(t)\,dt \\ E\{C\} & = C_{IOC}(N_{sat} + N_{rep}, \Delta V_d) + N_{serv}\,\overline{C_S} \\ V & = \frac{E\{U\}}{E\{C\}} \end{cases} \qquad (6.19)$$

Comparison with the Baseline Fleet

For a baseline constellation, which cannot maneuver, $n_k = N_{sat}/N_R$ satellites are attributed to each region permanently, so that the utility rate for a contingency distribution X is:

$$u_b(X,t) = \frac{1}{\mathcal{R}(t)} \sum_{k=1}^{N_R} \min\left(\frac{N_{sat}\,R}{N_R}, \frac{X_k\,\mathcal{R}(t)}{N_C} \right)$$

The expected utility rate at t is:

$$\overline{u_b}(t) = \sum_{\substack{X_k \geq 0 \\ \sum X_k = N_C}} P\{(X_1, ... X_{N_R})\} \sum_{k=1}^{N_R} \min\left(\frac{N_{sat}\,R}{N_R\,\mathcal{R}(t)}, \frac{X_k}{N_C} \right) \qquad (6.20)$$

The value of this baseline mission is finally

$$\begin{cases} E\{U_b\} & = \int_0^{TH} \overline{u_b}(t)\,dt \\ E\{C_b\} & = C_{IOC}(N_{sat}, \Delta V_{d,b}) \\ V_b & = \frac{E\{U_b\}}{E\{C_b\}} \end{cases} \qquad (6.21)$$

Maximum Servicing Price

The maximum servicing price per unit mass Υ is the servicing price under which the value of the maneuverable fleet is greater than the value of the baseline fleet:

$$V > V_b \iff E\{C\} < \frac{E\{U\}}{E\{U_b\}} E\{C_b\}$$

$$\iff C_{IOC}(N_{sat} + N_{rep}, \Delta V_d) + N_{serv}\,\overline{x}_S\,M_{dry}^{base}\,C_{mS} < \frac{E\{U\}}{E\{U_b\}} E\{C_b\}$$

hence:

$$\boxed{\Upsilon = \frac{E\{U\}\,E\{C_b\}/E\{U_b\} - C_{IOC}(N_{sat} + N_{rep}, \Delta V_d)}{N_{serv}\,\overline{x}_S\,M_{dry}^{base}}} \qquad (6.22)$$

6.2.3 Results

Numerical Assumptions

Baseline fleet of satellites Let us consider as our baseline satellite fleet the Defense Satellite Communication System (DSCS) satellites [§]. The Air Force Space Command (AFSC) currently operates $N_{sat} = 9$ Phase III DSCS satellites in geostationary orbit, over $N_R = 5$ main areas of coverage. The system is used for high priority command and control communications such as the exchange of wartime information between defense officials and battlefield commanders. Each satellite is designed for 10 years, has a lift-off weight $M_{launch} = 2615\,kg$ and an on-orbit weight $M_{base} = 1170\,kg$, from which we can infer that an upper stage is used for orbit insertion, so that the design-ΔV of the baseline satellites is to provide station keeping and end-of-life disposal only; for baseline satellites this is approximately $\Delta V_{d,b} = 600\,m/s$ with a 20% margin. The cost of each unit is approximately $C_u = \$\,200\,M$, which we can approximate as being equal to:

$$C_u = \frac{C_P}{1 - (\epsilon f_{st} + f_p + f_p f_{st}) \left(e^{\frac{\Delta V_{d,b}}{g\, I_{sp}}} - 1 \right)}$$

Baseline demand The overall demand is increasing with time at an approximate rate $1\,Gbps/yr$ starting from a current value of 3 $Gbps$ [BMH$^+$00]. Given the definition of utility, it is sufficient to define the demand in terms of a percentage compared to its present value, so that:

$$\mathcal{R}(t) = 1 + a_{\mathcal{R}}\, t$$

with $a_{\mathcal{R}} = 1/3\,yr^{-1}$. We will further assume that the demand is exactly met by the current satellite fleet, so that $N_{sat}\,R = 1$.

Dynamics of Contingencies Bonds & al [BMH$^+$00] analyze the communications needs of the military. They note on historical data that on average, 6.5 contingencies are occurring at any time over the world, among which 2.9 have small communications needs (weight 1), 2.7 have medium communications needs (weight 4) and 0.9 have large communications needs (weight 10). This corresponds to $N_C \approx 10$ concurrent small contingencies at any time.

[§]http://www.fas.org/spp/military/program/com/dscs_3.htm

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Table 6.5: Fleet of GEO Military Communication Satellites: Numerical Assumptions

Parameter	Name	Value	Source
Number of contingencies	N_C	10	Adapted from [BMH$^+$00]
Number of regions	N_R	5	DSCS satellites coverage
Number of satellites	N_{sat}	9	DSCS satelllites
Contingency frequency	ν_c	$3/yr$	Adapted from [BMH$^+$00]
Max time to maneuver	ΔT_{max}	$2\,days$	Priority estimate
Mission horizon	T_H	$10\,yr$	DSCS design lifetime
Orbit insertion ΔV	ΔV_{ins}	$0\,m/s$	Use upper stage
Specific impulse	I_{sp}	$320\,s$	Chemical propulsion
Fuel fraction at launch	ε	1	Not too heavy
Structures mass factor	f_{st}	0.2	Robust design
Propulsion dry mass factor	f_p	0.15	GRO [WCH88]
Baseline unit cost	C_u	$\$\,200\,M$	DSCS (web)
Cost of launch	C_L	$\$\,30K/kg$	DSCS launchers
Risk-free interest rate	r	7%	Typical
Baseline satellite mass	M_{base}	$1170\,kg$	DSCS
Servicing rate	μ_S	$1/(7\,days)$	Estimate
Probability of crash	P_C	0.001	Estimate
Replacement rate	μ_M	$3/yr$	Estimate

They model contingencies as a Poisson process. Each contingency has a probability $p = 0.6$ to end after three months (which would correspond to $\nu_0 = 4$) and a probability $1 - p = 0.4$ to be extended for three additional months. This would correspond to an expected contingency frequency:

$$\nu_c = \frac{p\,\nu_0}{1-p} \sum_{k=1}^{\infty} \frac{(1-p)^k}{k} = -\frac{p\,\nu_0\,\ln(p)}{1-p} \approx 3/yr$$

Table 6.5 summarizes the other baseline numerical assumptions.

Improvement in Utility

With a maneuverable constellation, the capacity of the fleet is almost fully exploited at any time. With a baseline fleet on the other hand, some satellites are wasted over areas with small number of contingencies, while others would be needed where contingencies concentrate. Therefore the utility of the baseline fleet is smaller than the utility of the maneuverable fleet.

Figures 6-15 and 6-16 illustrate the function U_b/U as a function of number of satellites, number of simultaneous contingencies, and contingency frequency. For the baseline case,
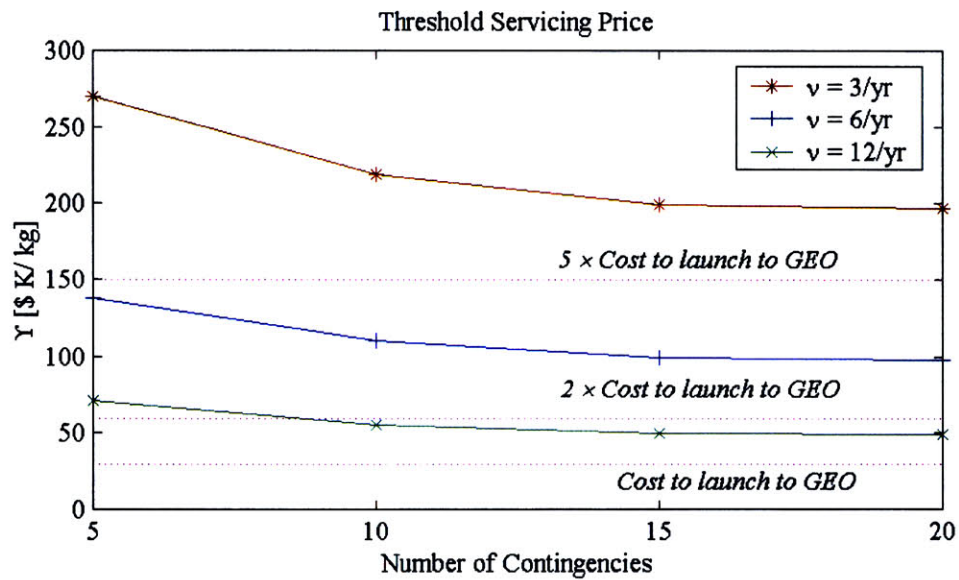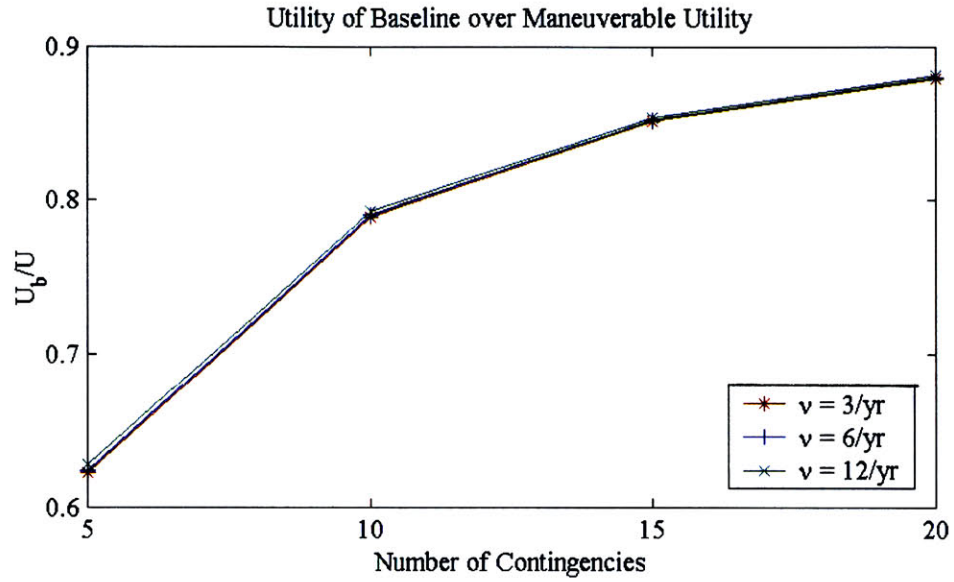
Figure 6-15: Utility Improvement and Maximum Servicing Price as a Function of Number of Simultaneous Contingencies

maneuverability enables a 20% improvement in utility. This number increases as the number of satellites increases, due to the improved flexibility of the fleet. It decreases as contingency frequency increases, due to finite time necessary to react to a contingency move. However, these two effects are almost negligible. Much stronger is the sensitivity of the baseline utility to the number of contingencies: the more numerous the contingencies requiring the same overall capacity, the smaller the relative deviations from a uniform distribution of contingencies over longitude, and therefore the smaller the capacity wasted by the baseline fleet.

Maximum Servicing Price

Figures 6-15 and 6-16 also plot the maximum servicing price as a function of number of satellites, number of simultaneous contingencies and contingency frequency.

For the baseline values, the maximum servicing price if $ 220 K/kg, which is more than seven times the cost to launch to GEO. This shows that servicing would be significantly interesting for this case as soon as the fleet of satellites pay only the marginal cost of servicing.

The maximum servicing price Υ decreases at higher contingency frequencies because of the increasing number of maneuvers necessary per year. For $\nu_c = 12/yr$, the maximum servicing price is of the order of the cost to launch to GEO: following the contingencies becomes too expensive compared to the increase in utility.

Υ also decreases with increasing number of contingencies, as a result of the increasing utility of the baseline constellation. As the distribution of contingencies becomes homogeneous, the need for maneuverability decreases. For the baseline $\nu_c = 3/yr$, the maximum servicing price remains however always an order of magnitude higher than the launch cost.

Sensitivity Studies

Sensitivity to time allowed to maneuver As more time is allowed to maneuver, the spacecraft design-ΔV decreases and the maneuverable satellites become lighter and therefore less expensive. Figure 6-17 (a) plots the sensitivity of Υ to ΔT_{max}, all other parameters being equal to their baseline value. Although very interesting from this plot, longer times to maneuver may not be acceptable from a military point of view, when security is at stake. This is why $\Delta T_{max} = 2\ days$ was assumed as the baseline requirement.
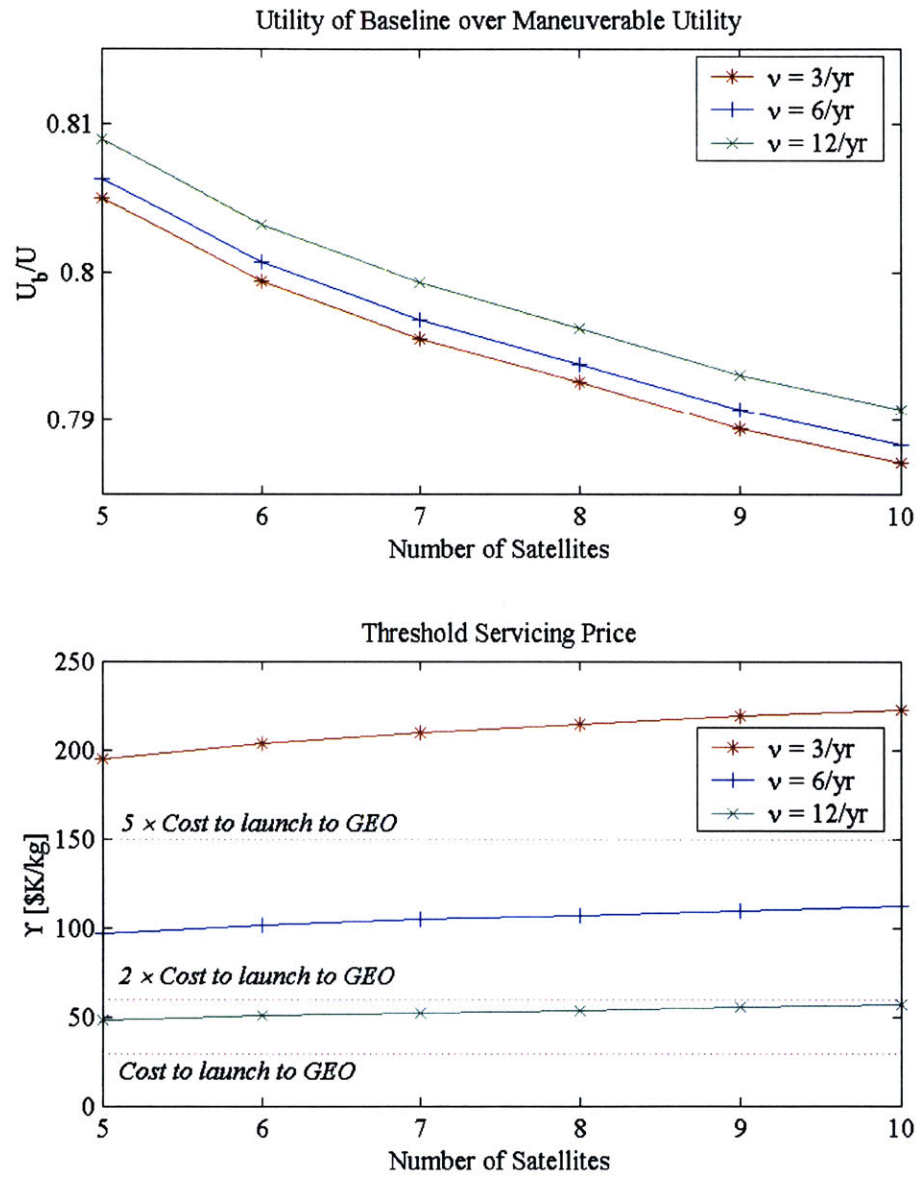
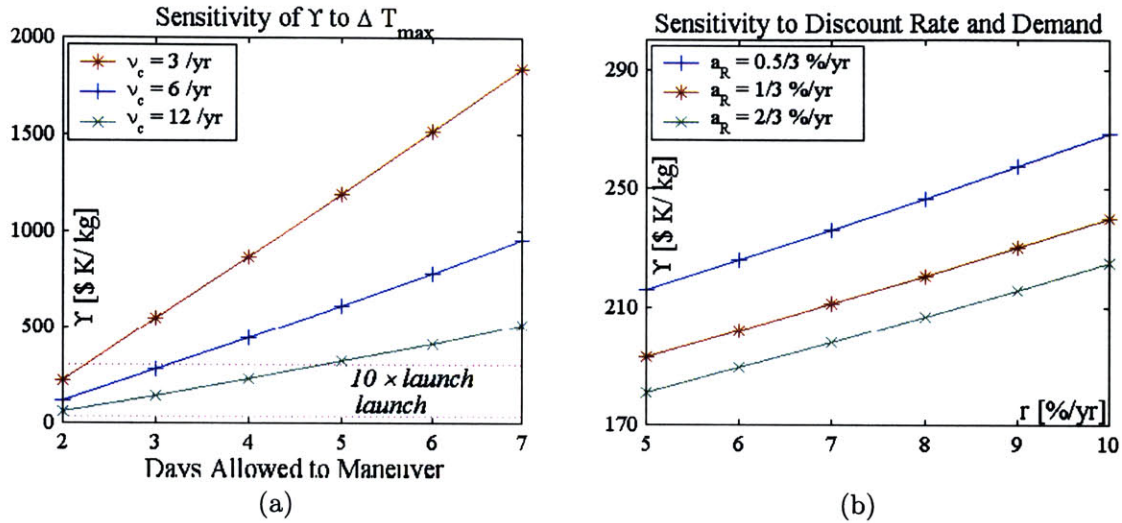Figure 6-16: Utility Improvement and Maximum Servicing Price as a Function of Number of Satellites

Figure 6-17: Sensitivity of Υ to (a) Time Allowed to Maneuver (b) Demand Growth Rate and Discount Factor

Sensitivity to demand growth and discount rate The utilities of both the baseline and the maneuverable fleet are decreasing functions of the overall demand growth rate $a_{\mathcal{R}}$. Since the utility of the maneuverable fleet is higher, its sensitivity to demand growth rate is higher, which results in a maximum servicing price that decreases with increasing demand, as shown by figure 6-17 (b).

This figure also illustrates the effect of the discount rate: as r increases, the cost of the baseline fleet remains unchanged, while the servicing expenses of the maneuverable fleet, which occur later in time, become cheaper. Therefore the maximum servicing price is an increasing function of the discount rate r. However, the order of magnitude of Υ remains the same over the range of reasonable risk-free interest rates.

Sensitivity to probability of a catastrophic event Figure 6-18 (a) shows that the sensitivity of Υ to the probability of a crash, P_C, is very small. This is due to the fact that P_C was taken into account only in the Markov model: the effect of a 1% probability of crash is simply a 1% increase in costs, to account for replacements. The actual impact of P_C, which would be a temporary loss of communications over a region and the military consequences thereof, are much more serious than what this figure suggests, but not easily captured by a simple model.
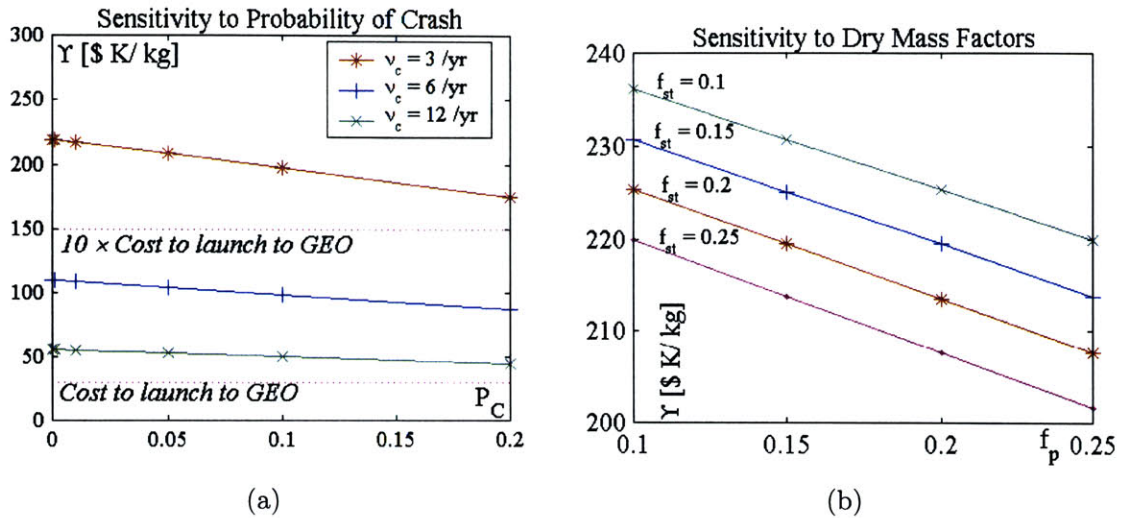
Figure 6-18: Sensitivity of Υ to (a) Probability of a Catastrophic Event (b) Dry Mass Factors

Sensitivity to mass factors Finally, figure 6-18 (b) shows the sensitivity of Υ with respect to the assumed *dry mass factors* f_{st} and f_p. As expected, the cost of maneuverability increases with increasing dry mass factors, which leads to a decreasing maximum servicing price. However, the design-ΔV of the maneuverable satellites is here very close to the design-ΔV of the baseline satellites. As a result, baseline and maneuverable satellites are affected in a similar fashion by the dry mass factors. Thus, the sensitivity of Υ to f_{st} and f_p is linear and not exponential; the maximum servicing price varies by less than 40% over the whole range of realistic mass factors.

6.2.4 Conclusions

This section showed that the maneuverability concept is very promising for a fleet of GEO satellites. The maximum servicing price is an order of magnitude greater than the cost to launch mass into geostationary orbit, which guarantees that servicing can be interesting for realistic marginal infrastructure costs.

Sensitivity studies show that whenever broad assumptions on some parameters had to be made, the same conclusions hold over the whole range of reasonable parameter values.

Only two parameters were shown to make a significant difference in maximum servicing price. The first is the time allowed for a maneuver: doubling this time roughly doubles the maximum servicing price. However, this parameter may not be a subject of trade in the

framework of responding to military contingencies. The second is the *contingency frequency*, which gives a measure of how often a satellite needs to maneuver. As this frequency doubles, the maximum servicing price is roughly divided by two: when contingencies move too fast, the costs of maneuverability outweigh its advantages. However, servicing remains significantly interesting up to twice the baseline estimated contingency frequency.

Chapter 7

Summary and Conclusions

7.1 Summary

The goal of this thesis was to develop of general way to estimate the value of on-orbit servicing for space systems. In the introduction, we defined two possible research directions to yield general results about the cost-effectiveness of on-orbit servicing and set out to explore them both.

Expansion of the traditional approach Chapter 3 explored the first direction. It expanded the traditional approach to on-orbit servicing by defining a method to estimate servicing cost-effectiveness on a wide trade space of space missions and servicing infrastructures. It also developed the first equations to model the optimal maneuver scheme for a fleet a servicer vehicles visiting a constellation of satellites.

A typical application of the model was then presented and used to illustrate the limitations of the traditional approach to on-orbit servicing. This approach tends to underestimate the value of servicing and to demonstrate cost advantages smaller than the cost uncertainty. The analysis of these limitations served as a motivation to define a new research direction.

Definition of a new framework for the value of flexibility provided by on-orbit servicing Chapter 4 laid out the foundations of this new approach. First, the *value of on-orbit servicing for space systems*, defined as the maximum price a space mission is willing to pay for being serviced, should exist independently of the design choices internal to the servicing infrastructure. Second, on-orbit servicing would provide space systems with

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options to react to the resolution of uncertainty, and the value of this *flexibility* should be taken into account.

Building and expanding on decision tree analysis and real options theory, a framework to embed the value of flexibility into the valuation of space missions faced with uncertainty was developed. The framework relies on the definition of a few *building blocks*, the most important being a model of the uncertainty, a set of reachable operational modes, a sequence of decision points, and a definition of mission value. Once these building blocks are set up, the mission value can be obtained from a backwards iterative process similar to a decision tree with infinite number of branches. This value embeds the value of flexibility by taking into account the ability of decision makers to always optimize their decision using the latest available information.

This framework accomplishes the main goal of the thesis: it defines a general method to estimate the value of on-orbit servicing for space missions, taking the value of flexibility into account. It also represents a new perspective to space missions decision making, in which the value of flexibility is estimated as a tool to strategically manage external sources of uncertainty.

The following chapters proved a solid validation of the framework. They also yielded interesting information about the value of on-orbit servicing for two types of space systems.

Value of the compound option to abandon   The framework was first used in section 5.2 to estimate the value of the option to abandon, which is available to all space missions but has never yet been accounted for. The results show that traditional valuation methods have been significantly underestimating the value of all missions with uncertain revenues, creating a bias in favor of conservative projects. By recognizing the flexibility of decision makers to shut off an unsuccessful mission, the proposed model shows that some projects that would be deemed uninteresting by traditional valuation can actually have significant value.

New decision making maps for the choice of a design lifetime requirement Building on this case, section 5.3 proposed a new tool to decision making regarding the choice of a design lifetime requirement when on-orbit servicing is available. It showed how the option valuation framework can be used to produce two types of maps for commercial

decision makers to manage the uncertainty in their revenues. If the price of on-orbit servicing is known, then the optimal design choice can be plotted in a market level/market volatility map. If accurate estimates of the market forecast and market volatility are developed, then the optimal design lifetime can be plotted in a servicing price/market volatility map. The proposed valuation framework can also be used as a guide for future on-orbit servicing technology development, by defining the range of possible servicing prices that a space mission should be charged.

Maximum servicing price for commercial space missions with uncertain revenues
The market level/market volatility map in the general case with constant market forecast showed that the *maximum servicing price* for a commercial space mission is an order of magnitude higher than the cost to produce and launch the serviceable mass, as soon as the volatility in mission revenues is higher than about $40\% .yr^{-\frac{1}{2}}$. This proves not only that on-orbit servicing has significant value for commercial space missions, but also that much of this value resides in the flexibility it provides them with.

The servicing price/market volatility map for two realistic examples (*Globalstar* and *Iridium*) showed that as soon as the servicing price is more than five times the cost to produce and launch the serviceable mass, the main driver for the optimal design lifetime is the market uncertainty. The optimal choice is the longest possible lifetime up to a *minimum uncertainty*, at which serviceable designs become interesting.

Maximum servicing price for military space missions with dynamic distribution of contingencies By introducing a Markov model of the dynamics of contingencies, chapter 6 expanded the framework to military missions in the case when the decision is to continuously optimize performance.

For a low-Earth orbit radar constellation, the trade of number of satellites versus maneuverability was explored for two types of propulsion systems: chemical and electric. While chemical propulsion offers the fastest maneuverability, its exponential mass increase outweighs the advantages of refueling. For electric propulsion systems on the other hand, the maximum servicing price per unit mass can be an order of magnitude greater than the cost to launch to LEO. However, electric propulsion systems require unacceptable times for a change in inclination. Thus, the results suggest that refueling would have little value for a

LEO radar constellation.

On the other hand, the maneuverability concept was shown to be very promising for a fleet of GEO satellites as a means to continuously optimize their longitude over a dynamic distribution of contingencies. The maximum servicing price per unit mass is an order of magnitude greater than the cost to launch into geostationary orbit, which guarantees that servicing can be interesting for realistic marginal infrastructure costs.

7.2 Recommendations for Future Work

In a way, this work has been the first step into new territory. This means that it is still very incomplete. But also that it opens up a wealth of new questions and interesting research directions.

Note on applying the framework This thesis proposed a new valuation *process* that takes building blocks as input. We illustrated the use of this process by making assumptions on the *numerical forms* of the building blocks. Although the valuation process can be used as such, the numerical assumptions represent zeroth-order approximations only and would need refinement. For the sake of generality, linear numerical assumptions were made about the cost to design for a given lifetime, the cost to operate a system, and the relation between mass and cost. In the real world, non-linear effects and discontinuities exist, that depend on the particularities of the design. Using this framework on a real case study, the actual mass and costs functions should first be derived in order to yield more accurate results.

Expansion to overcome the main limitations Several limitations of the proposed framework could be the object of future development.

First, all the results we presented here assumed that uncertainty is *external*, follows a *Markov process*, and is *symmetric*. The generalization to internal sources of uncertainty is far from trivial. The generalization to non-symmetric sources of uncertainty, such as revenues having a relatively higher probability to be lower than their forecast, is the easiest step because it involves only the definition of a different probability density function. More difficult but very interesting would be the application to processes that are not Markovian. This would account for the fact that decision makers do not only rely on the latest available information: they also learn from history.

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Second, while the case of a linear value function has been well defined, there remains interesting work to do in the area of non-commercial missions. We demonstrated how to expand the framework to account for continuous decision making in the case of military space missions which optimize their *instantaneous performance*; this decision scheme made it possible to use a Markov model. For scientific missions, which optimize their *future utility per cost ratio*, a backwards iterative valuation process would be necessary, but made very complex if decision points have to be continuous, or if the expected value of the ratio has to be estimated (instead of the ratio of the expected values). Investigations into new models and/or new numerical methods to solve this problem would be valuable.

Finally, all the applications studied in this work assumed that there was only one uncertain parameter. Real space missions are however usually faced with a combination of uncertain parameters, such as uncertain revenues *and* uncertain servicing price *and* random failures. The generalization to such a case is conceptually easy as soon as theses sources of uncertainty are independent. It however multiplies the complexity of the numerical analysis.

Further applications Future work should apply the proposed models to other types of space systems. Of particular interest would be the study of two other important sources of uncertainty: random satellites failures and technology obsolescence.

Random failures can be modeled by a constant failure rate. They require the definition of a continuous decision process which will depend on the type of mission.

A typical assumption for the study of technological obsolescence could be that the potential utility rate achievable with new technology increases as an exponential law (such as Moore's law) with uncertain rate. This would be a good example of non-symmetric uncertain parameter. The different modes of operation would correspond to technologies made available in different years.

Market base for on-orbit servicing Thus, the application of the proposed framework can be used to estimate the maximum servicing price that various types of space missions would be willing to pay for the capability to be serviced. Combining this information with an approximation of the number of space missions of each type that is expected in the future would yield an estimation of the *market base* for on-orbit servicing.

This market base would represent valuable information for future technology and policy

development regarding on-orbit servicing.

7.3 Conclusion

There is value in the options that on-orbit servicing would provide to space systems. By not accounting for this flexibility, traditional valuation methods have been underestimating the value of servicing.

This work proposed a general framework to take into account the value of the flexibility to react to any source of uncertainty that can be modeled as external. It showed that options can make up a significant fraction of total space mission value as soon as the uncertainty is significant. Since they do not require to estimate the cost of any on-orbit servicing infrastructure, these results are not plagued by the high servicing cost uncertainty.

The systematic application of this framework should prove very useful in identifying the space missions for which on-orbit servicing would offer the most potentials. It should serve as a guide for future on-orbit servicing technology development as to what maximum price can be charged to various types of space missions. More generally, it can be used by space missions as a new tool for decision making, with which the value of flexibility, seen as a means to actively manage external sources of uncertainty, can be quantified.

The author wishes lots of fun to future graduate students carrying out research in this area. She can be reached at elisabeth.lamassoure@polytechnique.org for any question.

Appendix A

The Mathematics of Uncertainty: Elements of Stochastic Calculus

The information in this appendix was directly adapted from [Shr97].

A.1 Definitions

Stochastic process A stochastic process X is a sequence of random variables. It can be discrete: $X_0, X_1, ...X_n$ or continuous: $X(t)$.

Martingale property Let $X(t)$ be a stochastic process et let us denote by $E\{Y|Z\}$ the expectation of any random variable Y knowing the value of the random variable Z. X is a martingale if

$$E\{X(t + \tau)|X([0;t])\} = X(t) \quad \forall\, t, \tau > 0 \tag{A.1}$$

Knowing all the values taken by X during the interval $[0;t]$, the expectation of X at any later time is simply its latest known value $X(t)$. In other words, martingales tend to go neither up nor down.

Markov property In this thesis we assumed that uncertain parameters could be modeled as Markov processes. Here is a reminder of the rigorous definition of a Markov process.

Let X be a stochastic process, h a function, and for $0 \leq t_0 \leq t_1$ let $E^{t_0,x}\{h[X(t_1)]\}$ be the expectation of $h[X(t_1)]$ given that $X(t_0) = x$. The Markov property says that
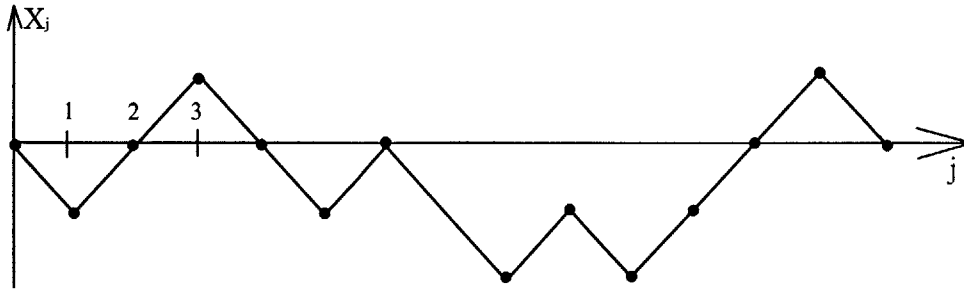
Results of tosses: {t,h,h,t,t,h,t,t,h,t,h,h,h,t,...}

Figure A-1: Example of Random Walk

whatever the initial condition $X(0) = \xi$:

$$E^{0,\xi} \{h\left[X(t_1)\right] | X(t_0) = x\} = E^{t_0,x} \{h\left[X(t_1)\right]\} \tag{A.2}$$

In other words, if you want to estimate a function of X based on the observation of the path of X over the interval $[0; t_0]$, the only relevant information is the last observed value $X(t_0)$. The differences with the martingale property are that the Markov property is valid for *any* function of X, and that it allows X to move up or down.

A.2 Random Walks and the Brownian Process

A.2.1 Symmetric Random Walk

Suppose you toss a coin a infinite number of times and define the random variable X such that:

$$
\begin{aligned}
X_0 &= 0 \\
X_{j+1} &= \begin{cases} X_j + 1 & \text{if the result of the } j^{th} \text{ toss is heads,} \\ X_j - 1 & \text{if the result of the } j^{th} \text{ toss is tails.} \end{cases}
\end{aligned}
\tag{A.3}
$$

Then X_j is called a symmetric random walk process. This notion is illustrated by figure A-1.

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The central limit theorem states that:

$$\frac{1}{\sqrt{k}} X_k \rightarrow \text{standard normal variable as } k \rightarrow \infty \qquad (A.4)$$

A.2.2 Brownian Process

Let n be an integer. For $t \geq 0$ of the form $t = k/n$, define

$$B^{(n)}(t) = \frac{1}{\sqrt{n}} M_{tn} = \frac{1}{\sqrt{n}} M_k$$

and for other values of $t \geq 0$, define $B^{(n)}(t)$ by linear interpolation. The Brownian process is the limit of $B^{(n)}(t)$ as $n \rightarrow \infty$, in other words the limit of a random walk as the step of the walk tends to zero.

More specifically, a random variable $B(t)$ is called a Brownian motion if it satisfies the following properties:

1. $B(0) = 0$

2. $B(t)$ is a continuous function of t

3. B has independent, normally distributed increments; if you define $0 = t_0 < t_1 < ... < t_n$ and $Y_k = B(t_k) - B(t_{k-1})$, then:

   - $Y_1, ... Y_n$ are independent

   - $E\{Y_k\} = 0 \quad \forall \, k$

   - $Var\{Y_k\} = t_k - t_{k-1} \quad \forall \, k$

Two important properties of the Brownian motion are that it is both a martingale and a Markov process.

A.3 Measures of Uncertainty

A.3.1 First Variation

The first variation is a measure of the ups and downs of a function over an interval. It is defined as follows. Let $\Pi = \{t_0, ... t_n\}$ be a partition of the interval $[0; T]$, i.e. $0 = t_0 < t_1 <$

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$.... < t_n = T$. The *mesh* of the partition is defined to be

$$||\Pi|| = \max_k (t_{k+1} - t_k)$$

Then the first variation of a function f over the interval $[0; T]$ is by definition:

$$FV_{[0;T]}(f) = \lim_{||\Pi|| \to 0} \sum_{k=0}^{n-1} |f(t_{k+1}) - f(t_k)| \qquad (A.5)$$

The link of this definition to the amount f varies is even clearer in the case of a differentiable function. If f if differentiable, then the first variation reduces to:

$$FV_{[0;T]}(f) = \int_0^T |f'(t)| \, dt$$

A.3.2 Quadratic Variation

In a similar fashion, the *quadratic variation* of f is defined as

$$\langle f \rangle(T) = \lim_{||\Pi|| \to 0} \sum_{k=0}^{n-1} |f(t_{k+1}) - f(t_k)|^2 \qquad (A.6)$$

It can be shown that for a differentiable function, $\langle f \rangle(T) = 0$. But for a Brownian process:

$$\langle B \rangle(T) = T \qquad (A.7)$$

The quadratic variation of f is a measure of the randomness in f. The above property of the Brownian motion proves in particular that *the paths of a Brownian motion are not differentiable.* In can be re-written informally:

$$dB(t) \, dB(t) = dt$$

A.3.3 Volatility

The *squared sample absolute volatility* of f over the interval $[T_1; T_2]$ partitioned by $\{t_0, ...t_n\}$ is:

$$\frac{1}{T_2 - T_1} \sum_{k=0}^{n-1} [f(t_{k+1} - f(t_k)]^2$$

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In the case of a Brownian motion, this is equal to 1 whatever the choice of the interval $[T_1; T_2]$. This property can be re-written $\langle B \rangle(T) = T = \int_0^T 1 \, dt$. In other words, the volatility of the Brownian motion is the rate at which its quadratic variation accumulates.

A.4   Basics of Itô Calculus

A.4.1   The Itô Integral

Let δ be a function. Consider the integral $I(T) = \int_0^T \delta(t) \, dB(t)$: if B were a differentiable function, then we would simply have $I(T) = \int_0^T \delta(t) \, B'(t) \, dt$. This cannot work if B is a stochastic process, because the paths of a stochastic process are not necessarily differentiable (i.e., $B'(t)$ is not necessarily defined).

The Itô integral is a generalization of the integral $I(T)$ when $B(t)$ is a Brownian motion.

Itô integral of an elementary process   Let $0 = t_0 \leq t_1 \leq \ldots \leq t_n = T$ be a partition of $[0; T]$ and let $\delta(t)$ be a function that is constant on each subinterval $[t_k; t_{k+1}]$; we call $\delta(t)$ an elementary process. The Itô integral of such a process is defined as:

$$I(T) = \sum_{k=0}^{n} \delta(t_k) \, [B(t_{k+1}) - B(t_k)] \tag{A.8}$$

Itô integral of a general integrand   Now let $\delta(t)$ be a process such that $E\left\{\int_0^T \delta^2(t) \, dt\right\} < \infty$. There is a sequence of elementary processes $\{\delta_n\}_{n=1}^{\infty}$ such that:

$$\lim_{n \to \infty} E\left\{ \int_0^T |\delta_n(t) - \delta(t)|^2 \, dt \right\} = 0 \tag{A.9}$$

The Itô integral of the general process $\delta(t)$ is defined by:

$$\int_0^t \delta(t) \, dB(t) = \lim_{n \to \infty} \int_0^t \delta_n(u) \, dB(u) \tag{A.10}$$

Properties of the Itô integral   include:

- Linearity: $I(t)$ is linear with respect to $\delta(t)$

- Continuity: $I(t)$ is a continuous function of t

- Martingale: $I(t)$ is a martingale

- Itô isometry: $E\left\{I^2(t)\right\} = E\left\{\int_0^t \delta^2(u)\,du\right\}$

Quadratic variation of the Itô integral

$$\langle I \rangle(t) = \int\limits_0^t \delta^2(u)\,du$$

Which means that the instantaneous absolute volatility of I is $\delta^2(t)$. Informally, it can be written as:

$$dI(t)\,dI(t) = \delta^2(t)$$

Important Example   The example of the Brownian process illustrates how Itô calculus differs from standard calculus:

$$\int\limits_0^T B(t)\,dB(t) = \frac{1}{2}B^2(T) - \frac{1}{2}T \tag{A.11}$$

A.4.2   Itô's Formula

Let $f(x)$ be a differentiable function. If $B(t)$ were also differentiable, then the ordinary *chain rule* would give:

$$\frac{d}{dt}f\left[B(t)\right] = f'\left[B(t)\right]\,B'(t)$$

which in differential notation corresponds to $df\left(B(t)\right) = f'\left[B(t)\right]\,dB(t)$

Since the Brownian process is not differentiable, the actual formula has an extra term, which gives Itô's formulas in differential form:

$$df\left[B(t)\right] = f'\left[B(t)\right]\,dB(t) + \frac{1}{2}f''\left[B(t)\right] \tag{A.12}$$

and in integral form:

$$f\left[B(t)\right] - f\left[B(0)\right] = \int\limits_0^t t\,f'\left[B(u)\right]\,dB(u) + \frac{1}{2}\int\limits_0^t f''\left[B(u)\,du\right] \tag{A.13}$$

A.5 Geometric Brownian Motion

The typical assumption for stocks, also used in real options and in chapter 5 of this thesis, is to model the uncertain parameter as a geometric Brownian motion.

A geometric Brownian motion with drift α and volatility σ is a process $S(t)$ of the form:

$$S(t) = S(0) \exp\left[\sigma B(t) + \left(\mu + \frac{1}{2}\sigma^2\right) t\right] \tag{A.14}$$

where μ and $\sigma > 0$ are constant.

We can note that $S(t) = f(t, B(t))$ with $f(t, x) = S(0) \exp\left[\sigma x + \left(\mu + \frac{1}{2}\sigma^2\right) t\right]$ so that Itô's formula gives:

$$dS(t) = \mu S(t)\, dt + \sigma S(t)\, dB(t) \tag{A.15}$$

which is equivalent to say that $\ln\left[S(t)\right]$ is a Brownian process with drift μ and volatility σ.

Informally:

$$dS(t)\, dS(t) = \sigma^2 S^2(t)\, dt$$

Appendix B

Calculations of Velocity Budgets

B.1 Incremental Velocities for Various Maneuvers

B.1.1 Hohmann Transfer

A Hohmann transfer minimizes the incremental velocity to maneuver between two coplanar circular orbits at the respective altitudes a_0 and a_1. It consists of two impulsive burns. The first burn transfers the spacecraft to the elliptical orbit with apocenter and pericenter distances a_0 and a_1: the semi-major axis of the transfer orbit is $a_t = (a_0 + a_1)/2$. The second burn circularizes the orbit at a_1. The total incremental velocity is (see also [WL99]):

$$\Delta V_H = \left| \sqrt{\frac{2\mu}{a_0} - \frac{2\mu}{a_0 + a_1}} - \sqrt{\frac{\mu}{a_0}} \right| + \left| \sqrt{\frac{2\mu}{a_1} - \frac{2\mu}{a_0 + a_1}} - \sqrt{\frac{\mu}{a_1}} \right|$$

where $\mu = G\,M_{Earth}$. Defining the ratio of altitude $\alpha = a_0/a_1$ and the baseline orbital velocity $V_0 = \sqrt{\mu/a_0}$, this can be re-written:

$$\Delta V_H = V_0 \left| \sqrt{2 - \frac{2}{1+\alpha}} - 1 \right| + V_0 \left| \sqrt{\frac{2}{\alpha} - \frac{2}{1+\alpha}} - \sqrt{\frac{1}{\alpha}} \right|$$

$$\frac{\Delta V_H}{V_0} = \left| \sqrt{\frac{2\alpha}{1+\alpha}} - 1 \right| + \sqrt{\frac{1}{\alpha}} \left| \sqrt{\frac{2}{1+\alpha}} - 1 \right|$$

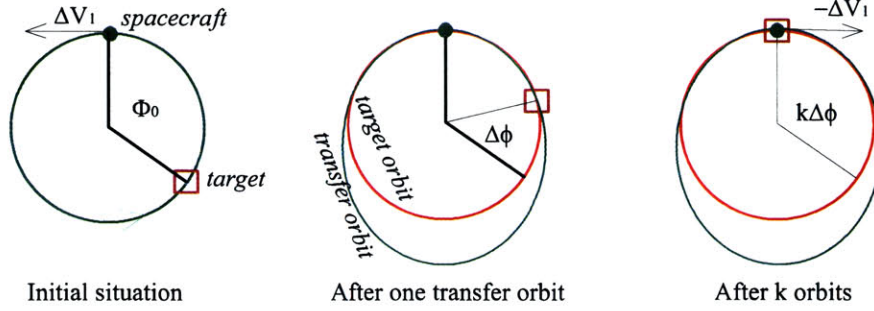This is the final result used in 3.9.

Figure B-1: Phasing Maneuver: Example

B.1.2 Incremental Velocity for Phase Change

Now consider a maneuver to a position in the same orbital plane at the same altitude, but with by a difference in angular phase $\Phi_0 \in [-\pi\,;\,\pi]$. The maneuver must be performed in less than a maximum allowed time ΔT_{max}. This can be achieved by altering the orbital semi-major axis, so that the difference in orbital period cancels out the phase. Four different situations can be imagined, according to the sign of Φ_0 and of the semi-major axis alteration.

Case 1 Let us first consider $\Phi_0 > 0$ and choose to raise the apocenter. Let the number of orbits per unit time be n_0 for the baseline altitude a_0 and n_t for the transfer orbit with semi-major axis a_t. The change in phase after one period in the transfer orbit is $\Delta\phi = 2\pi\,n_0\,T_t - 2\pi = 2\pi\,(n_0/n_t - 1)$. A phase change Φ_0 will therefore be obtained after k orbits if:

$$\frac{n_0}{n_t} = \frac{1}{k}\left(1 - \frac{\Phi_0}{2\pi}\right) + 1 \tag{B.1}$$

The time to perform this maneuver is then $k\,T_t = k\,T_0\,n_0/n_t$ where T_0 is the baseline orbital period. The maximum allowed maneuver time therefore requires:

$$k \le \frac{\Delta T_{max}}{T_0} - 1 + \frac{\Phi_0}{2\pi}$$

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The maneuver is made up of two opposite, equal-magnitude impulsive burns. The total incremental velocity is:

$$\Delta V_{ph} = 2 \left| \sqrt{\frac{2\mu}{a_0} - \frac{\mu}{a_t}} - \sqrt{\frac{\mu}{a_0}} \right|$$

$$\frac{\Delta V_{ph}}{V_0} = 2 \left| \sqrt{2 - \frac{a_0}{a_t}} - 1 \right|$$

But:

$$\frac{a_0}{a_t} = \left(\frac{n_t}{n_0} \right)^{\frac{2}{3}}$$

The minimal incremental velocity is obtained for a_t as close as possible to a_0, i.e. for the maximal possible number of orbits:

$$k = I \left(\frac{\Delta T_{max}}{T_0} + \frac{\Phi_0}{2\pi} \right) - 1$$

where $I(x)$ denotes the integer part of x. This finally gives:

$$\frac{\Delta V_{ph}}{V_0} = 2 \left| \sqrt{2 - \left(\frac{l-1}{l - \Phi_0/2\pi} \right)^{\frac{2}{3}}} - 1 \right|$$

$$l = I \left(\frac{\Delta T_{max}}{T_0} + \frac{\Phi_0}{2\pi} \right)$$

Case 2 If the apocenter is raised to obtain $\Phi_0 < 0$, then k is now given by $k\Delta\phi = |\Phi_0|$ so that equation B.1 becomes:

$$\frac{n_0}{n_t} = \frac{1}{k} \frac{|\Phi_0|}{2\pi} + 1$$

Similar calculations then yield:

$$\frac{\Delta V_{ph}}{V_0} = 2 \left| \sqrt{2 - \left(\frac{l}{l - \Phi_0/2\pi} \right)^{\frac{2}{3}}} - 1 \right|$$

with the same definition of l.

Case 3 If the pericenter is lowered to obtain $\Phi_0 > 0$, then $\Delta\phi = 2\pi - 2\pi\, n_0/n_t$ so that equation B.1 becomes:

$$\frac{n_0}{n_t} = 1 - \frac{\Phi_0}{2\pi}$$

which yields:

$$\frac{\Delta V_{ph}}{V_0} = 2 \left| \sqrt{2 - \left(\frac{l}{l - \Phi_0/2\pi}\right)^{\frac{2}{3}}} - 1 \right|$$

Case 4 Finally, if the pericenter is lowered to obtain $\Phi_0 < 0$, then:

$$\frac{n_0}{n_t} = 1 - \frac{1}{k}\left(1 - \frac{|\Phi_0|}{2\pi}\right)$$

which yields

$$\frac{\Delta V_{ph}}{V_0} = 2 \left| \sqrt{2 - \left(\frac{l+1}{l - \Phi_0/2\pi}\right)^{\frac{2}{3}}} - 1 \right|$$

The results can finally be summarized as follows:

$$\frac{\Delta V_{ph}}{V_0} = 2 \left| \sqrt{2 - \left(\frac{l+\epsilon}{l - \Phi_0/2\pi}\right)^{\frac{2}{3}}} - 1 \right| \tag{B.2}$$

$$l = I\left(\frac{\Delta T_{max}}{T_0} + \frac{\Phi_0}{2\pi}\right) \tag{B.3}$$

$$\epsilon = \begin{cases} -1 & \text{if } a_t > a_0 \text{ and } \Phi_0 > 0 \\ 1 & \text{if } a_t < a_0 \text{ and } \Phi_0 < 0 \\ 0 & \text{otherwise} \end{cases} \tag{B.4}$$

Note on optimal maneuver For a given allowed time ΔT_{max}, the optimal maneuver scheme is the one that minimizes ΔV_{ph}. For $\Phi_0 > 0$, this is obtained by lowering the pericenter, while for $\Phi_0 < 0$, raising the apocenter is more interesting. In a real situation however, the choice between these two options may not exist. For example, a spacecraft in low-Earth orbit can not lower its pericenter too close to the Earth because of atmospheric drag.

B.2 Velocity and Mass Budget for a Servicer Vehicle

The servicer maneuver scheme described in chapter 3 is reminded on figure B-2. We consider a servicer vehicle with total dry mass $M_{dry}^{servicer}$ able to carry a maximum cargo mass $N\,M_C$. The servicer loads a cargo $N\,M_C$ at an altitude a_1, performs a Hohmann transfer to the satellites altitude a_0 to deliver a mass M_C to the first satellite, performs phase changes to

Phasing

Phasing

Phasing

Satellite 2
(not yet serviced)

Satellite 1
(serviced)

Loading

Hohmann

Hohmann

Hohmann

N satellites visited during each roundtrip
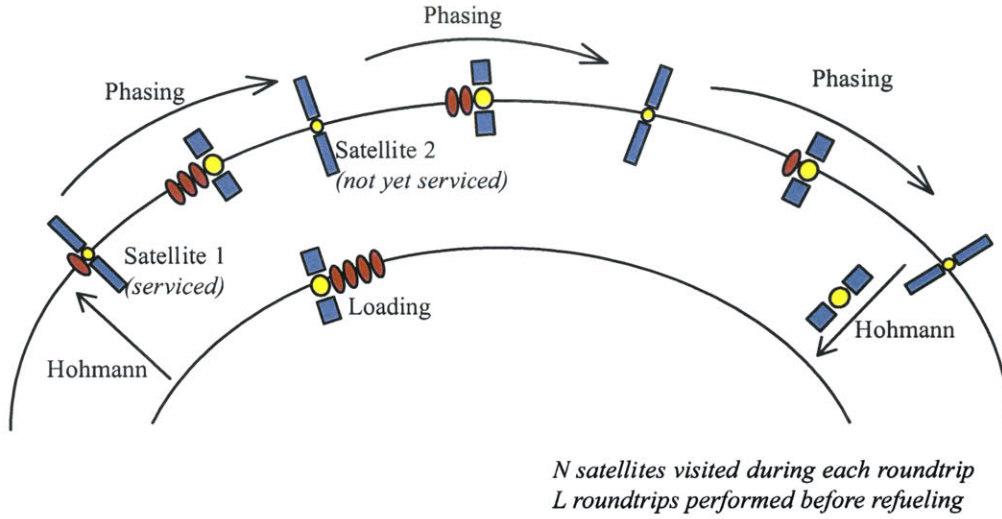*L roundtrips performed before refueling*

Figure B-2: Schematic of Servicer Maneuver Scheme

visit $N - 1$ other satellites in the same orbital plane, then performs a Hohmann transfer back to the depot altitude a_1, where it loads more cargo and repeats the whole process $L - 1$ times. We note h the total velocity increment for the Hohmann transfer and docking, normalized by the exhaust velocity $I_{sp}g$; and p the normalized total velocity increment for the phase change, and docking.

The mass budget for such a maneuver scheme is not a direct application of the rocket equation, because the dry mass of the servicer decreases as it delivers cargo to each satellite. The following shows how to calculate the total required servicer fuel mass on a maneuver-per-maneuver basis, staring from the last maneuver.

B.2.1   One Roundtrip to Visit N Co-Planar Satellites

Last Hohmann transfer   After the very last Hohmann transfer, which requires the normalized velocity increment h, the final mass is $M_{dry}^{servicer}$. Therefore the rocket equation gives the fuel required for this last maneuver in the form:

$$M_0^{fuel} = \left(e^h - 1\right) M_{dry}^{servicer} \tag{B.5}$$

Phasing maneuvers   Now consider the phasing maneuvers. The fuel mass M_{k+1}^{fuel} required to visit $(k + 1)$ satellites is the sum of tow terms:

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1. The fuel mass required to visit the next satellite corresponds to a normalized velocity increment p. The total mass after this maneuver is the sum of the dry mass, the cargo for the k next satellites, and the fuel mass to visit the k next satellites.

2. The fuel mass required to visit the last k satellites.

Therefore the rocket equation gives:

$$M_{k+1}^{fuel} = (e^p - 1) \left(M_{dry}^{servicer} + k\, M_C + M_k^{fuel} \right) + M_k^{fuel}$$

Thus we have the induction relation:

$$M_{k+1}^{fuel} = (e^p - 1)\, M_{dry}^{servicer} + (e^p - 1)\, k\, M_C + e^p\, M_k^{fuel} \qquad (\text{B.6})$$

Define a_k and b_k such that

$$M_k^{fuel} = a_k\, M_{dry}^{servicer} + b_k\, M_C$$

Then equations B.5 and B.6 are equivalent to:

$$\begin{cases} a_{k+1} &= e^p\, a_k + (e^p - 1) \quad \text{and} \quad a_0 = e^h - 1 \\ b_{k+1} &= e^p\, b_k + (k+1)\,(e^p - 1) \quad \text{and} \quad b_0 = 0 \end{cases} \qquad (\text{B.7})$$

A simple demonstration by induction gives:

$$\begin{cases} a_k &= e^{kh+h} - 1 \\ b_k &= (e^p - 1) \sum_{l=1}^{k-1} (k-l)\, e^{lp} \end{cases}$$

b_k can be expressed without a summation by the following considerations. Define

$$f(x) = \sum_{l=0}^{k-1} x^l = \frac{x^k - 1}{x - 1}$$

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Then

$$\sum_{l=1}^{k-1} (k-l)\, x^l \quad = \quad k\, f(x) - x\, f'(x)$$

$$= \quad k\, \frac{x^k - 1}{x - 1} - \frac{(k-1)\, x^{k+1} - k\, x^k + x}{(x-1)^2}$$

which finally gives:

$$\begin{cases} a_k & = e^{kp+h} - 1 \quad \forall k \leq N - 1 \\ b_k & = \frac{e^{kp}-1}{e^p-1}\, e^p - k \quad \forall k \leq N - 1 \end{cases} \tag{B.8}$$

First Hohmann transfer The fuel mass M_N^{fuel} required for one whole roundtrip of N satellites is the sum of tow terms:

1. The fuel mass required to visit the first satellite corresponds to a Hohmann transfer with normalized velocity increment h. The total mass after this maneuver is the sum of the dry mass, the cargo for $N - 1$ satellites, and the fuel mass M_{N-1}^{fuel} to visit the $N - 1$ next satellites.

2. The fuel mass M_{N-1}^{fuel} required to visit the last $N - 1$ satellites.

Therefore the rocket equation gives:

$$M_N^{fuel} = \left(e^h - 1\right)\left(M_{dry}^{servicer} + (N-1)\, M_C + M_{N-1}^{fuel}\right) + M_{N-1}^{fuel}$$

This corresponds to:

$$\begin{cases} a = a_N & = e^h\, a_{N-1} + \left(e^h - 1\right) = e^{(N-1)p+2h} - 1 \\ b = b_N & = e^h\, b_{N-1} + N\left(e^h - 1\right) = \frac{e^{Np}-1}{e^p-1}\, e^h - N \end{cases} \tag{B.9}$$

The total fuel mass for one roundtrip is finally:

$$M_1^{RT} = M_N^{fuel} = a\, M_{dry}^{servicer} + b\, M_C \tag{B.10}$$

B.2.2 L Roundtrips

For a final mass $M_{dry}^{servicer}$, the total fuel mass for the last roundtrip is given by equation B.10. The same relation is valid for any roundtrip, except that the final mass needs to be

adjusted to include the fuel required for later roundtrips. Thus, the total fuel mass M_{k+1}^{RT} required for $(k+1)$ roundtrips is the sum of two terms:

1. The fuel mass required for the next roundtrip, after which the final mass will be the sum of the dry mass and the fuel mass M_k^{RT} required for the last k roundtrips.

2. The fuel mass the fuel mass M_k^{RT} required for the last k roundtrips.

Therefore the rocket equation gives:

$$M_{k+1}^{RT} = \left[a \left(M_{dry}^{servicer} + M_k^{RT} \right) + b\, M_C \right] + M_k^{RT}$$

which corresponds to the induction relation:

$$M_{k+1}^{RT} = a\, M_{dry}^{servicer} + b\, M_C + (a+1)\, M_k^{RT} \tag{B.11}$$

Define c_k and d_k such that $M_k^{RT} = c_k\, M_{dry}^{servicer} + d_k\, M_C$. Then equations B.11 and B.10 are equivalent to:

$$\begin{cases} c_{k+1} & = (a+1)\, c_k + a \quad \text{and} \quad c_1 = a \\ d_{k+1} & = (a+1)\, d_k + b \quad \text{and} \quad d_1 = b \end{cases} \tag{B.12}$$

A simple demonstration by induction gives:

$$\begin{cases} c_k & = (a+1)^k - 1 \\ d_k & = \frac{b}{a} \left[(a+1)^k - 1 \right] \end{cases} \tag{B.13}$$

Using equation B.9 finally gives the total fuel mass $M_L^{RT} = M_{fuel}^{servicer}$ required for L roundtrips:

$$M_{fuel}^{servicer} = A\,.M_{dry}^{servicer} + B\,.M_C \tag{B.14}$$

$$\text{where} \quad A = \exp\left[L(N-1)p + 2Lh \right] - 1 \tag{B.15}$$

$$\text{and} \quad B = \frac{\exp\left[L(N-1)p + 2Lh \right] - 1}{\exp\left[(N-1)p + 2h \right] - 1} \left(\frac{e^{Np} - 1}{e^p - 1}\, e^h - N \right) \tag{B.16}$$

This corresponds to the mass budget mentioned in chapter 3 (equation 3.15).

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