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Punching of column footings - comparison of experimental and calculation results

Authors:

Assist.Prof. **Zoran Bonić**, PhD. CE
University of Niš
Faculty of Civil Engineering and Architecture
zokibon@yahoo.com

Prof. **Radomir Folić**, PhD. CE
University of Novi Sad
Faculty of Technical Sciences
Department of Civil Engineering
folic@uns.ac.rs

Subject review

Zoran Bonić, Radomir Folić

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The column punching through the slab is most often related to floor structures, and less often to foundation slabs under columns. Calculation models for these two problems are often not separated. The deficit of experimental results for column footings results in the use of floor slab models. A review of the theoretical and experimental research references, including Codes/Regulations for punching calculation of column footings, is presented in the paper. The experimental research program and its implementation is described. The results obtained are compared with calculation results based on various regulations. Recommendations and directions for future research are outlined.

Key words:

punching, shear stresses, foundations, codes, experiments, comparative analysis

Pregledni rad

Zoran Bonić, Radomir Folić

Proboj temelja samaca - usporedba proračuna i eksperimenta

Proboj stupa kroz ploču najčešće se vezuje za međukatne konstrukcije, rjeđe i za ploče temelja samaca ispod stupova. Proračunski modeli za ova dva problema često nisu razdvojeni. Zbog nedostatka eksperimentalnih rezultata na temeljima, primjenjuju se modeli za međukatne ploče. U radu je prikazan pregled literature o teorijskim i eksperimentalnim istraživanjima i propisi za proračun proboja stupa kroz ploču. Opisan je program eksperimentalnih istraživanja i njegova realizacija. Dobiveni rezultati uspoređeni su s rezultatima proračuna prema različitim propisima. Dane su preporuke i naznačene smjernice budućih istraživanja.

Ključne riječi:

proboj, posmična naprezanja, temelji, propisi, eksperimenti, usporedna analiza

Übersichtsarbeit

Zoran Bonić, Radomir Folić

Durchstanzen von Einzelfundamenten - Vergleich von Berechnung und Experiment

Das Durchstanzen von Platten bezieht sich vorwiegend auf Betondecken, kann aber auch bei Platten von Einzelfundamenten unter Stützenträgern vorkommen. Berechnungsmodelle werden oftmals für beide Problemstellungen gemeinsam betrachtet. Durch den Mangel an Ergebnissen experimenteller Versuche an Fundierungen, werden häufig Modelle für Deckenplatten verwendet. In der vorliegenden Arbeit ist eine Literaturübersicht hinsichtlich theoretischer und experimenteller Untersuchungen, sowie der sich auf das Durchstanzen von Platten beziehenden Berechnungsrichtlinien gegeben. Die Planung und Realisierung eines Versuchsvorhabens ist beschrieben und die erhaltenen Resultate sind mit den Berechnungsergebnissen nach verschiedenen Verordnungen verglichen.

Schlüsselwörter:

Durchstanzen, Scherspannungen, Fundierungen, Regelwerke, Experimente, Vergleichsanalyse

1. Introduction

Foundations connect and coordinate the work of the structure with the subgrade, whose characteristics in relation to the structure are very different, particularly in terms of deformability. Although foundations have a significant impact on the behaviour of the structure and the surrounding soil, their calculation has not been sufficiently considered, neither in literature nor in technical regulations (standards, norms, requirements). Approximate calculations are most often used in the design of shallow foundations, particularly column footings, which are considered as rigid foundations, and a linear distribution of contact pressures is adopted. In this calculation, a considerable significance is given to the punching shear control, i.e. to the control of column punching through the floor slab with or without shear reinforcement for the assumption of shear in the column area. This control, as well as the punching control of floor slabs, are usually based on experimental research. Due to a small number of experiments aimed at studying the puncture strength of foundation slabs, their analysis is often based on theoretical and/or empirical equations such as those applied to the problem of column punching through floor slabs, which is based on experimental research [1-6].

A broader historical review of solutions to column punching through the slabs without shear reinforcement is given in [7], but here the focus is mostly on punching through floor slabs. At that, it is noted that first recommendations of the American Concrete Institute (ACI) on flat plates, given in 1925, were based on experimental results presented byTablot [8], who investigated column footings, as cited in [7]. However, the difference between the punching mechanism of foundation plates and floor slabs, which is due to their considerable difference in height ("slenderness"), has generally been neglected in technical regulations (hereinafter referred to as "regulations") [9]. This is due to the fact that experimental research relating to foundations has so far been quite scarce, primarily because of the complicated organization of such experiments, higher material costs, and a number of significant parameters that need to be taken into account. Here, It should also be noted that investigations in real soil are rare and expensive, and so in many experiments real conditions are simulated by steel springs or BY a set of small hydraulic jacks connected in parallel [3].

With regard to the punching control of foundations plates, technical regulations allow subtracting the load from the superstructure for a part of soil reaction, but differences between some regulations/standards are quite noticeable [9]. This is the why 17 column footings are studied in [9] (five actually resting on sand to enable analysis of the soil-structure interaction). The varied parameter was the shear span (b) (leg b marked in Figure 4) to effective depth (d) ratio, with values between 1.25 and 2.0, while the concrete strength was between 20 and 40 MPa. It was established that the b/d ratio (shear slenderness) significantly affects the bearing capacity to punching-shear. In addition, the ACI and EN 1992 provisions are critically analysed and their improvements are suggested. It is also shown in [10] and [11] that the failure mechanism substantially depends on the "slenderness" of the floor slab/foundation plate.

According to [11], the most important parameters that should be included in the finite element method (FEM) are: geometry, material properties, and flexural reinforcement. The most important parameters that influence punching are: the effective or total plate depth, and the size effect, which is expressed as the b/d ratio. The flexural reinforcement ratio, plate slenderness, and compressive strength of concrete, also exert a significant influence. The paper includes presentation of the most important fracture-mechanics parameters that are used to describe the ductility or brittleness.

The specificity of column footings is that they have a small b/d ratio. It has been experimentally shown that cracks due to punching are more inclined in case of column footings with greater b/d ratio, than in case of column footings with a smaller b/d ratio. This has been confirmed by theoretical and numerical analyses. The punching test for two circular columns passing through the footing has been simulated numerically by means of the FEM [10]. It was established that failure mechanisms of slender slabs are significantly different from the mechanism related to footings. It is stated that the angle of shear cracks at foundation plates is between 50 - 60 ^{0}, which is significantly higher than the angle for slender slabs (30 - 40 ^{0}). The b/d ratio for foundation plates is much smaller compared to the ratio applied for floor slabs (most regulations are based on them). It has been established by parametric analyses that the compressive strength of concrete has a greater impact on the punching bearing capacity of column footings, compared to "slender" plates that are typical to floor slabs.

It should be noted that the punching failure through a footing is brittle, and so the failure of one column/support can lead to the progressive and unexpected failure of neighbouring supports, as cracks and deformations remain small until just prior to the actual failure. The use of shear reinforcement increases the punching capacity significantly, while also contributing to the increase in ductility and the possibility of redistribution of forces [12]. It is emphasized in the same paper that most models described in regulations and literature are related to empirical connections, and that they do not contribute to the understanding of phenomena, which is why their introduction into the analysis is more difficult. The theory of plasticity, fracture mechanics, and nonlinear finite element method, are increasingly used in theoretical analyses.

The absence of properly developed theories that would explain behaviour of reinforced concrete slabs has lead to practical use of models contained in various technical regulations. In Switzerland, the shear reinforcement is calculated on the basis of the theory of plasticity, according to SIA 262. At that, rough approximations are made, and the contribution of concrete to punching capacity is neglected, which leads to conservative calculation results for shear reinforcement. Generally, technical regulations do not distinguish between punching through floor slabs and punching through foundation slabs. The comprehensive monograph *fib* 12 [13] also places a high emphasis on the problem of column

Table 1. Overview of previous column-footing experiments

Author	Year	Type of support	Number of tested footings	Geometry of footing		
				Shape	Dimension [mm]	Effective depth [mm]
Hegger and Ricker	2005.	Sand in the box	5	square	900	150 to 250
Timm	2003.	Line	10	square	760 to 1080	172 to 246
Hallgren	1998.	Line/Surface	14	square and circular	850 to 960	273 to 278
Dieterle and Rostasy	1987.	Surface	13	square	1500 to 3000	320 to 800
Kordina and Nölting	1981.	Surface	11	rectangular	1500 to 1800	193 to 343
Dieterle and Steinle	1981.	Surface	6	square	1800 to 3000	700 to 740
Rivkin	1967.	Surface /clay and sand	6	square	650 and 1000	120
Richart	1948.	Spring	149	square and circular	610 to 3000	200 to 740
Talbot	1913.	Spring	20 (punching)	square	1520	250

punching through floor slabs. A comparative analysis is made for experimental results and the results obtained according to regulations. The data bank contains tests for slabs with and without shear reinforcement. Without a broader discussion, it is stressed that foundations slabs are specific in that they have a smaller shear span to height ratio, and that the foundation slab failure mechanism is different when compared to slender slabs.

The theoretical explanation of the plate punching phenomenon, based on the critical shear crack opening, is given by Muttoni in [14] for the reinforced concrete slab without transverse reinforcement, and in [15] for slabs with transverse reinforcement. This is a new formulation of punching criteria based on plate rotation. It is called the critical shear crack theory (CSCT). The application of this theory to the punching of slabs with transverse reinforcement is presented in [16]. In the draft of the new *fib* Model Code 2010 [17], provisions for the punching-related slab design point to the above-mentioned CSCT formulated by A. Muttoni, and also to papers [14] and [16]. This is regulated in the draft [17] by provisions contained in section 7.3.5.

In order to test some regulations and their applicability to the punching resistance calculation of column footings, an experimental program was conducted in the scope of paper [18]. The program of experimental investigations was made with goals that have also been set by some other researchers [3]. The column footing punching behaviour was studied so as to obtain answers to the following questions: how does the soil pressure distribution at the base of the footing affect the punching bearing capacity, and what is the difference in behaviour between floor slabs and foundation slabs in terms of column punching through the slab? During the tests, punching mechanisms have also been registered, especially the angles at which shear cracks are formed.

The above overview points to the need for comparative analysis of experimental results and provisions contained in appropriate regulations. A brief overview of some experimental results obtained on column footings, and provisions from different regulations, are presented in this paper to enable their comparative analysis. The comparative analysis comprises the following documents: Byelaw BAB 87 (*Byelaw on technical standards for concrete and reinforced concrete*) which was until recently used in a number of countries of our region, and which was implemented on a large number of projects; American ACI 318-02 [22]; British BS 110-1-1997 [20]; German DIN 1045-1 [21]; European EN 1992 EC-2 [22], and Russian regulations SNiP-84 [23]. A modified model based on the Model Code (*fib*) [24] and EN 1992-1: 2004, and also on the European Concrete Platform from 2010 [25], is proposed in this paper.

The above regulations were compared based on the results obtained by experimental testing of the column footings punching on a gravel bedding. The punching failure of the column footing supported by gravel was recorded by varying the slab height: 10; 12.5; 15.0; 17.5; 20.0 and 25.0 cm.

2. Comparative review of some experiments

According to published data, a relatively small number of punching tests for column footings resting on a real bedding has been realized since the start of the twentieth century [18]. In most experiments conducted so far, the natural bedding was simulated by steel springs, Richart [3] and Talbot [8], by a battery of small hydraulic jacks, or by line loads which produced the same effect as the load exerted by a uniformly reactive soil, Timm [26] and Hallgren [6, 10]. The tests carried out by Dieterle and Rostasy, Kordina and Nölting, and Dieterle and Steinle, cited in [3], are also quite significant. The punching test on real soil was practically implemented only by Hegger and others [3-4] - in a box of sand, and by Rivkin [27] - on the clay and sand *in situ*. The data about geometric characteristics, the number of tested footings, and supports used, are clearly arranged and presented in Table 1.

Expressions from some regulations, based on the slab punching tests database, are analysed and evaluated in paper [28]. The regulations/standards analysed in this paper comprise ACI (1983), BS (1985), CEB-FIP (1990), prEN (1991), and two Japanese regulations.

The first of the two Japanese regulations is the one issued by the Architectural Institute of Japan (AIJ) and the other one is by the Japan Society of CE (JSCE). Thus, punching capacity parameters currently used in Japan have also been considered. It is emphasized that the shear strength of concrete is used in most expressions. The analysis focused on more than 300 tests of reinforced concrete slabs. The results show that the expressions recommended in ACI, prEC 2 and CEB-FIP are rather conservative. Much more realistic results are obtained if Japanese regulations are applied.

A brief overview of the column-footing punching calculations, as given in some current technical regulations, is provided below. The methods described in this way are then used in original footing-punching experimental tests [18].

3. Overview of calculation methods according to technical regulations

The fact that none of the most frequently mentioned slab and footing punching models (Kinnunen and Nylander, Menetrey, Shehat and Regan, Broms, etc.) has generally been accepted so far, has led to the considerable disparity of recommendations contained in actual international and national codes. In most of them the semi-empirical method of critical cross-section is used for calculating the punching resistance of slabs or foundations. This method is based on the assumption that the plate is punched when a vertical fracture cross-section is established along the entire perimeter of the penetration body that is formed near the column. This cross-section is called a critical or control section, with the perimeter length *u*. The slab punching occurs when the shear stress in the critical cross-section attains the shear strength of concrete. Based on this, the punching calculation is reduced to the control of shear stress in the critical section, i.e. the calculated shear stress in critical cross section τ_{cal}, at some distance from column edges, is compared with the punching shear stress v. If the requirement $\tau_{cal}<v$, is fulfilled there is no danger of punching. Otherwise, the shear reinforcement should be introduced to prevent punching.

For column footing, the shear stress at punching is calculated in the critical section τ_{cal} according to

$$\tau_{cal} = \frac{P_{u,red}}{O_{kp} \times d} \tag{1}$$

where:

O_{kp} - perimeter of critical (control) section

d - effective cross section depth (mean value of two perpendicular directions).

According to regulations, the normal ultimate force in column P_u can be reduced for the part of the soil reaction beneath the punching body based on the following expression

$$P_{u,red} = P_u - A_0 \cdot \sigma_n \tag{2}$$

where:

σ_n - net reactive soil pressure at the contact surface (without dead weight of the footing),

A_0 - surface area of the punching body basis in the reinforcement plane.

Table 2. Ultimate shear stress calculation methods according to various regulations

Eurocode 2 [22]	**ACI 318-02 [19]**	**BS 8110-1: 1997 [20]**
$v = C_{Rd,c} \cdot k \cdot (100 \cdot \rho_t \cdot f_c)^{1/3} \frac{2d}{a_{EC2}} \geq v_{\min} \frac{2d}{a_{EC2}}$ (3) $C_{Rd,c} = 0{,}18/\gamma_c$ γ_c - material resistance factor for concrete $k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$ ρ_t - flexural reinforcement ratio a_{EC2} - distance from loaded area to control perimeter	$v = \min \begin{cases} 0.083(2 + \frac{4}{\beta_c})\sqrt{f_c} \\ 0.083(2 + \frac{\alpha_s \cdot d}{b_0})\sqrt{f_c} \\ 0.332\sqrt{f_c} \end{cases}$ (4) α_s = 40 for interior column α_s = 30 for edge column α_s = 20 for corner column	$v = \frac{0.79}{\gamma_m}(100\rho_s)^{\frac{1}{3}}(\frac{400}{d})^{\frac{1}{4}}(\frac{f_{cu}}{25})^{\frac{1}{3}}$ (5) γ_m - partial safety factor ρ_s - flexural reinforcement ratio f_{cu} - characteristic concrete cube compressive strength
SNIP-84 [23]	**BAB 87**	**DIN 1045-1 [21]**
$P_{u,red} = \alpha \cdot R_{bl} \cdot u_m \cdot d$ (6) α - coefficient of concrete characteristics (1.0 for normal weight concrete) R_{bl} - design strength of concrete subjected to axial tension u_m - mean circumference of the upper and lower bases of the punching pyramid/cone $u_m = 2(b_c+l_c+2d)$ b_c, l_c - column dimensions at the basis	$v \leq \frac{2}{3}\gamma_1 \cdot \tau_a$ (7) $\gamma_1 = 1.3 \cdot \alpha_a \cdot \sqrt{\mu}$ μ - mean value of flexural reinforcement ratio of two perpendicular directions α_a - coefficient depending on the type of reinforcement τ_a - limits of permissible main tensile stresses	$v_{Rd,ct} = \left(\frac{0.21}{\gamma_c}\eta_l \cdot k(100\rho_l \cdot f_{ck})^{1/3}\right)$ (8) γ_c - material resistance factor for concrete (1.50) η_1 - for normal weight concrete (1.0) ρ_t - flexural reinforcement ratio
d – effective depth of footing; f_c - characteristic compressive strength of concrete cylinder; b - perimeter of critical section		

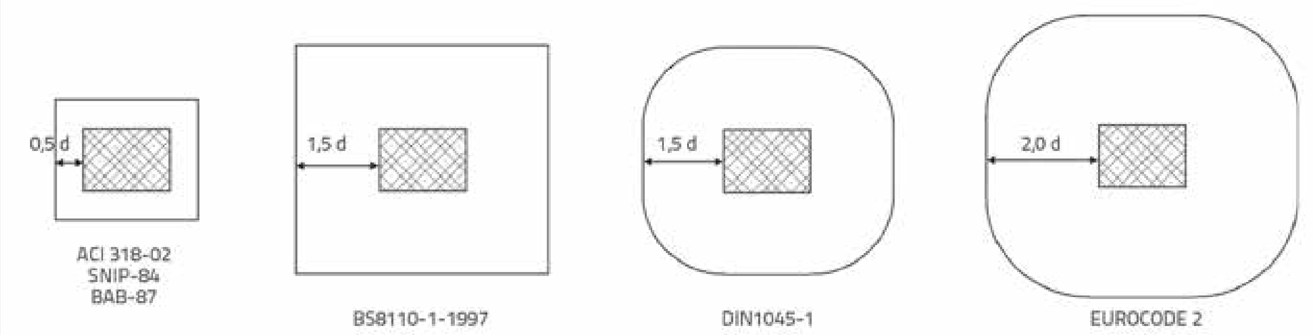

Figure1. The control section in some regulations, as dependent on the effective footing depth d

Regulations for calculating ultimate shear stress at punching in the control section are very varied, as illustrated in Table 1. In addition, the regulations greatly differ from one another when the position and shape of the control section is determined, cf. Figure 1. Control sections from different regulations are used for comparative analysis.

Equations (3) through (8) are included in Table 2. As already mentioned, the comparison of these regulations includes experimental test results for the punching of column footings resting on sub-grade made of cohesionless materials.

4. Experimental studies

The schematic of the experimental setup consisting of the test frame, test specimen – column footing, hydraulic jack, and prepared sub-grade is shown in Figure 2. Experiments were conducted in 2009.

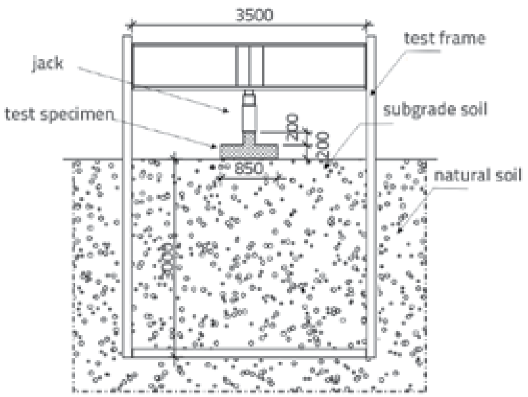

Figure 2. Section of the experimental setup

Figure 3. Experimental testing *in situ*

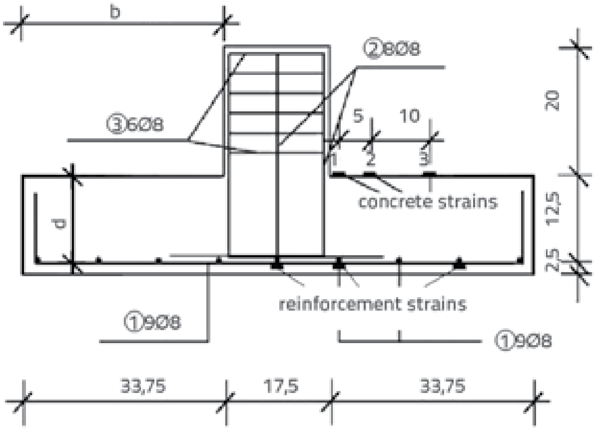

Figure 4. Dimensions and reinforcement of some footings

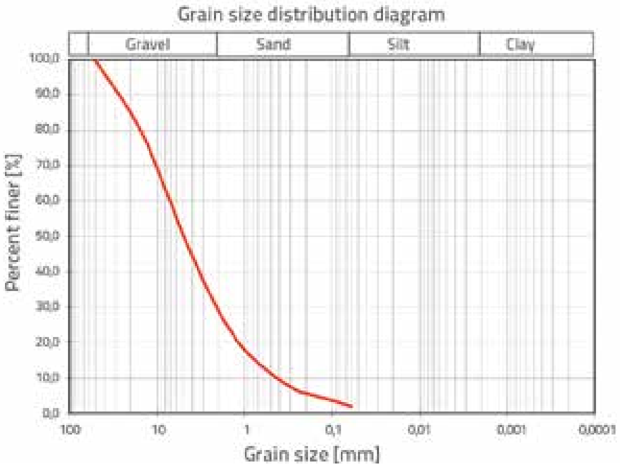

Figure 5. Particle size distribution of gravel

The test frame is placed at the bottom of the prepared footing pit measuring 4,0 x 5,0 m in plan and 3,0 m in depth. Gravel layers, each 30 cm in thickness, are placed at the bottom of the frame measuring 3,5 x 3,5 m, and then each of the layers is compacted by plate vibrator to the required modulus of compressibility. The compaction level is controlled for each layer by the circular plate test. The measured mean values of the modulus of compressibility of these layers ranged from 43,3 to 66,7 MPa, which corresponds to the normal

Table 3. Characteristics of tested footings (test specimens)

Designation the footing	Footing depth h [cm]	Effective depth d [cm]	Concrete strength fc, cube [MPa]	Bar size [mm]	Reinforcement ratio [%]	Failure load [kN]
TI	20	17.5	38.37	8	0.40	1001/906*
TII	15	12.5	38.37	8	0.40	1050
TIX	12.5	10.0	21.25	8	0.40	430
TX	17.5	15.0	21.25	8	0.40	656
TXI	15	12.5	19.29	8	0.40	451
TXII	15	12.5	10.0	8	0.40	440

* During the first test. the column failed at 1001 kN. After a new column was fabricated. the footing was punched at the load of 906 kN

** Foundations TIII-TVIII did not fail as their bearing capacity was higher than the capacity of the equipment (1000 kN).

compaction of subsoil. The compaction of sub-grade was controlled before each footing was tested. It ranged between 39,5 MPa and 76,7 MPa.

The truss structure of the frame, as well as its dimensions, enable an undisturbed formation of sliding surfaces in the soil beneath the foundations in case the soil failure should precede the punching shear in the course of the loading. This setup allows footings testing under completely realistic boundary conditions in terms of soil, as well as proper comparison and verification of earlier laboratory testing results with *in situ* tests.

The adopted footing dimensions are 85x85cm in plan, and they correspond to experiments made by Kinnunen and Hegger et al. [4], with regard to result comparison possibilities, and also according to the capacity of the available measuring equipment (approximately 1000 kN). The dimensions and characteristics of the footing are given in Table 3. and shown in Figure 4.

The compressive strength of concrete at the time of testing was obtained based on 15 cm cube specimens and standard cylinders. Average and calculated values of the 15 cm cube specimens are given in Table 3.

Steel bars 8 mm in diameter were used for the reinforcement of the footings, and the percentage of reinforcement amounted to approximately 0.4 % for all footings. Reinforcing steel characteristics were determined on three samples of the reinforcement used in the testing. The obtained mean values were: tensile strength f_{su} = 653 MPa, yield point f_{sy} = 570 MPa, and the corresponding yield strain $\varepsilon \approx 2{,}7$ ‰.

The experimental testing was conducted by placing the footing on the soil surface and by loading it with a vertical centric force, which was applied by a hydraulic jack positioned between the steel cross-beam and the footings (Figure 3). A hydraulic jack 1000 kN in capacity was used for applying load to the footing, and the load was applied in 50 kN load increments. The load was kept constant at every load increment until the total consolidation of soil at that load. The consolidation was registered by observing vertical displacements of points at the footing corners and at the column of the footings. During the experiment, the following parameters were measured after each second: strains in the reinforcement and in the concrete of the footings, vertical displacements of points at the footing corners and at the column of the footings, the intensity of loading force, and contact pressures beneath the footings. A more detailed data and test results are given in [18].

5. Use of regulations in the calculation of test specimens

The ultimate axial force in the column for the footings from Table 3 was calculated for the purpose of mutual comparison, and to compare results obtained by applying the regulations with the results of experimental investigations.

The ultimate axial force in column was calculated based on actual characteristics of the material, and it was assumed that the safety factors relating to the material (concrete) are equal to one.

For the tested footings, the ultimate axial force in column after transformation based on equation (2) can be determined according to the formula:

$$P_u = P_{u,red}\frac{A}{A-A_0} = \frac{P_{u,red}}{1-\frac{A_0}{A}} \tag{9}$$

where A is the area of the footing base.

All regulations define the shape and position of the control section, which is the boundary of the surface A_0, and so the capacity of the control section can be calculated as follows:

$$P_{u,red} = v \cdot O_{kp} \cdot d \tag{10}$$

where:

v – ultimate shear stress at punching of the control section according to the adopted byelaw,

O_{kp} – perimeter of the control section according to adopted regulations,

d – effective depth of footing.

In this way, the ultimate axial force in column was calculated for tested footings in accordance with the rules and regulations mentioned in introduction.

5.1. Results obtained using Eurocode 2

According to Eurocode (EC) 2, the shear bearing capacity at the slab punching is checked along the column perimeter, and in the basic control section at the distance of a_{EC2} = 2,0 from the edge of the column (Figure 1), where d is the effective depth of the section. This document does not prescribe specific calculation for the column footing punching, and it does not explicitly prescribe which control section should be adopted as a reference value at footings (distance a_{EC2} in equation (3)). It is however advisable to check the section at a smaller distance of 2.0 d from the column edge. Thus, the German standard DIN 1045-1 from 2008 (based on EC-2) defines the control section at 1.5d from the column edge. In comments to the EC 2, in the European Concrete Platform - ECP [25], instructions are given for the determination of the control section position, and the use of a special diagram – based on experimental results - is recommended. These instructions for the ratio of the footing length to the column length *l / c*, and the ratio of the column length to the effective depth of the footing *c / d*, give the ratio a_{crit} / d (according to diagram on Figure 6b. Also, the value of the column punching force can be determined based on similar ratios with the diagram on Figure 6c. In most cases a_{crit} value is less than 2*d*, which means that the slope of the footing punching body is much steeper than that of the slab punching body.

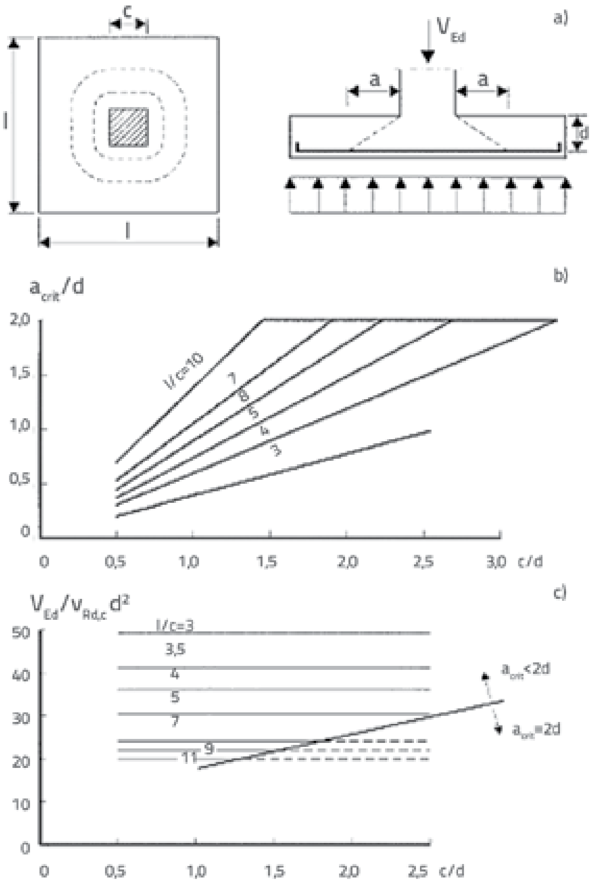

Figure 6. Calculation of punching force in column, after [25]

If the main control section at the distance 2,0 *d* from the edge of the column is adopted as the control section, which corresponds to the punching body slope of only 26,6°, then it would cover a major part of the footing base for most footings. Since the column force is reduced for the part of the soil reaction within the control section, the relevant force in punching control would be very small. Moreover, there might be cases in which the control section is outside the footing base, which would lead to the absurd fact that the relevant force in punching control is negative.

To emphasize significance of the selection of control section, a comparative calculation of experimental footing punching was made, where the control section is at the distance of a_{crit} = 2,0 *d* from the edge of the column, and at the distance obtained using the diagram from Figure 6a.

Expressions (9) and (10) are used in the calculation. Here, the concrete punching shear capacity in an appropriate cross-section is calculated from the expression (3), and the concrete safety factor γ_c = 1,0 is adopted in the calculation. The calculation results are given in Table 4.

5.2. Results obtained according to DIN 1045-1 (2008)

The punching control according to DIN 1045-1 (2008) is implemented analogously to EN 1992, but the control section is located at the distance of 1,5 d from the edge of the column (Figure 1). The relevant reduced ultimate normal force in column $P_{u,red}$ is calculated by reducing the column ultimate normal force for the net average reactive soil pressure σ_n at the surface of 0,5 A_0. herefore, the expression (9) is modified as follows:

$$P_u = \frac{P_{u,red}}{1-\frac{0.5A_0}{A}} \qquad (11)$$

5.3.Results obtained according to BS 8110-1:1997

According to British standards, shear stresses are calculated in a rectangular section that is located at the distance of 1.5d from the boundary of the surface subjected to load (Figure 1). The punching shear capacity of concrete is calculated in this section according to equation (5), using the partial safety factor of γ_m = 1,0. Calculation results are given in Table 4.

5.4. Results obtained according to ACI 318-02

This Regulation takes into account the critical section at *d*/2 from the column edge and its shape corresponds to the shape of the column (Figure 1). The research conducted by Hegger [4], reveals that these results are in better accord with experimental results if it is adopted that the control section is at the distance of 1,0 *d*, and so the calculation of the ultimate normal force in column is conducted in accordance with both proposals. The concrete punching shear capacity in the control section is calculated from equation (4), using the reduction factor of $\Phi = 1{,}0$ (in this standard, the concrete shear capacity is usually reduced by the coefficient $\Phi = 0{,}75$). Calculation results are shown in Table 4.

5.5. Results obtained according to СНиП 2.04.01 – 84

According to current Russian regulations, which were also in effect in the Soviet Union, the punching body slopes at 450 with respect to the horizontal, and it is replaced in punching control with a parallelepiped whose sides are at the distance of *d*/2 from the edges of the column. Unlike previous regulations in which the characteristic compressive strength of concrete was used for determining the punching shear capacity in the control section, the tensile strength of concrete is used in current Russian regulations. Considering that the actual concrete strength (with the safety factor for material equalling to one) was used in calculations according to regulations considered until now, and as the ultimate punching force is calculated in expression (6) based on the design concrete strength at axial tension (R_{bl}), the tabulated values for the calculation of the concrete axial tension strength have to be increased by adding an average safety coefficient of the material, which amounts to 1,35 in this particular case. Calculation results obtained in this way are given in Table 4.

5.6. Results obtained according to BAB 87

The regulation BAB 87, used until recently in former Yugoslavia for the design and construction of structures, is mostly based on the former German DIN 1045. Unlike the above mentioned standards, here the punching control is conducted based on allowable stress. This value is used to obtain bearing capacity of the control section in the exploitation/service phase, and so the resulting value has to be multiplied by the minimum safety factor (1,75) as required by DIN 1045. This must be done to enable comparison of BAB 87 results with the above mentioned results based on regulations in which the theory of limit states is used. Since BAB 87 contains no provisions on the punching of foundations, the reduction in the normal force of the column for a part of soil reaction under punching body is not accounted for. In our technical literature, the punching control of reinforced concrete column footings is usually conducted based on Leonhardt's recommendations. Otherwise, it is specified in BAB 87 that the punching control of slabs be performed at the control section that is located at the distance of d/2 from the edge of the column, which was adopted for column footings as well. Calculation results are shown in Table 4.

5.7. Calculation according to the proposed procedure

In the preceding calculations, the ultimate normal force in the column P_u, is calculated using the equation (9), which is derived from the expression (2) where the reactive net soil pressure, σ_n, is mentioned. In all the regulations, it is normally assumed that σ_n is calculated as an average pressure exerted on the entire contact surface of the footing. However, based on the conducted experiment, and according to previous research by other authors [4], it is clear that a high concentration of contact pressures in the area of the column occurs during loading of the column footing on granular soil. Keeping this fact in mind, the punching calculation must be modified by correcting the net reactive soil pressure using the contact pressure concentration factor F_c. Taking this into account, the expression (9) for the ultimate normal force in column can be written as:

$$P_u = \frac{P_{u,red}}{1 - \frac{A_0}{A} F_c} \tag{12}$$

By using the equation (12), an additional calculation of the limit normal force in the column is made for all tested footings. The concentration factor F_c is taken as the ratio of average pressure under the punching body, to the average pressure under the entire footing. This ratio is calculated for each tested footing,

Table 4. Ultimate normal force in column for some regulations, in kN

Designation the footing / Regulation	TI-3	TII	TIX	TX	TXI	TXII
EC2 $a_{crit} = 2{,}0$ d	5013	539 (1,95)	195 (2,21)	914 (0,72)	429 (1,05)	345 (1,28)
EC2 based on ECP [8]	811	570 (1,84)	368 (1,17)	531 (1,24)	453 (0,99)	260 (1,7)
DIN $a_{crit} = 1{,}5$ d	584	284 (3,70)	153 (2,81)	317 (2,07)	226 (1,99)	181 (2,44)
BS $a_{crit} = 1{,}5$ d	1200	401 (2,62)	230 (1,87)	583 (1,13)	376 (1,20)	376 (1,18)
ACI $a_{crit} = 0{,}5$ d	540	314 (3,34)	140 (3,07)	311 (2,11)	222 (2,03)	160 (2,76)
ACI $a_{crit} = 1{,}0$ d	1088	519 (2,02)	254 (1,69)	564 (1,16)	368 (1,22)	264 (1,67)
СНиП $a_{crit} = 0{,}5$ d	1010	487 (2,16)	247 (1,74)	514 (1,28)	338 (1,33)	249 (1,78)
BAB 87 $a_{crit} = 0{,}5$ d	482	239 (4,39)	104 (4,13)	199 (3,30)	146 (3,08)	101 (4,38)
The proposed procedure	1486	743 (1,41)	345 (1,25)	659 (0,99)	470 (0,96)	464 (0,95)
Measured	-	1050	430	656	450	442

Note: Values in brackets represent the ratio of experimental and calculation results and, their arithmetic mean (in this case amounting to 1.4) is adopted for the final value, which is applied in the calculation. At that, the angle of α=45^0, has been adopted, based on our own results and previous research results, as the angle of the punching body slope, while the following expression is proposed for the punching ultimate shear stress in the control section defined in this way:

$$v = \frac{0.25}{\gamma_c}\left(\frac{0.4}{d}\right)^{\frac{1}{3}} f_{ck}^{\frac{2}{3}}\, \rho^{\frac{1}{3}} \tag{13}$$

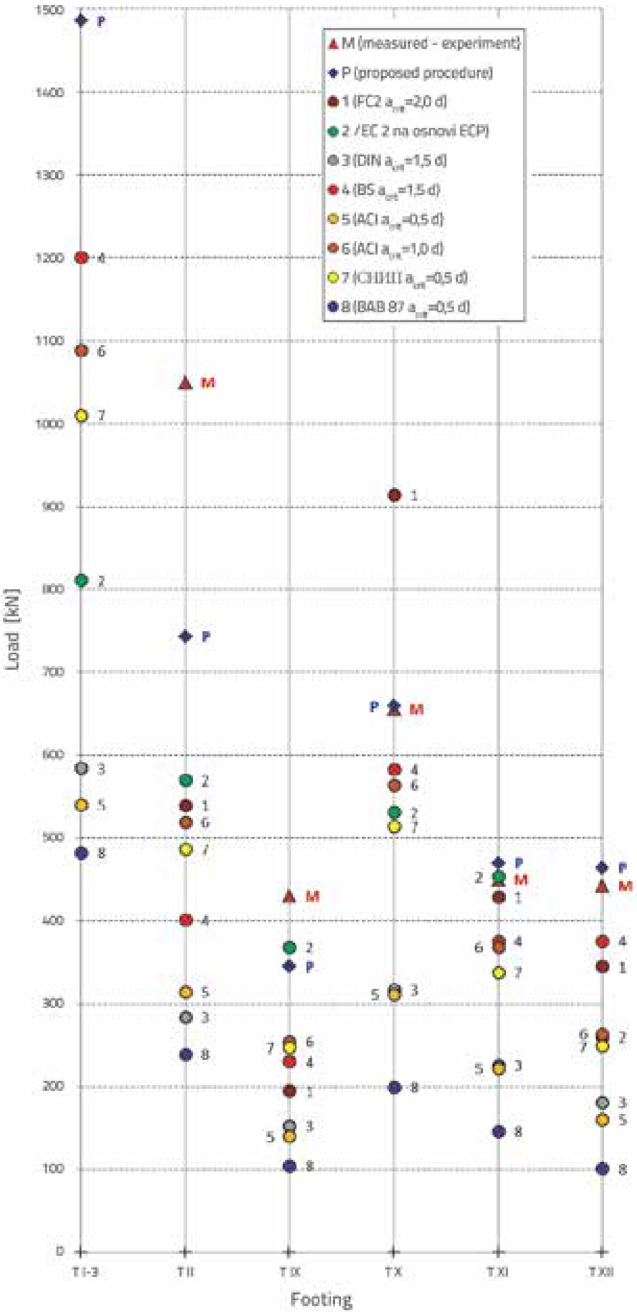

Figure 7. Comparative diagrams of ultimate normal force based on experiments and current regulations

where:

γ_c - resistance factor for concrete (1,5, here calculated as γ_c = 1,0),
d - effective depth of footing [m],
f_{ck} - characteristic compressive strength of concrete cylinder, which to be adopted as min 15,0 MPa,
ρ - flexural reinforcement ratio [%].

The results obtained by this procedure, and the results based on the mentioned regulations, are given in Table 4. The values from Table 4 are clearly shown in Figure 7.

6. Discussion of results

The values in parentheses given in Table 4 are safety coefficients, i. e. the ratio of experimentally obtained failure load to calculated failure load (with real material properties) $F_s = P_{um}/P_{ur}$. When the value of this ratio is greater than 1.0, the regulations are conservative and underestimate the bearing capacity of footings, while the bearing capacity of footings is overestimated in regulations if the value is less than 1.0.

In case of EC2 (a_{crit} = 2,0 *d*), the Fs ranges from 0,72 to 2,21, where the minimum value of 0,72 is obtained for the footing TX, which is 15,0 cm in effective depth. The maximum value of 2,21 is obtained for footing TIX, whose effective depth is 10,0 cm. It can therefore be concluded that the bearing capacity of footings with larger effective depth is overestimated in EC2, with basic control (critical) cross-section at 2,0 *d* from the edge of the footing column. This could have been expected as this position of the control section is primarily related to floor slabs, which are characterized by smaller depth. This is even more pronounced in footing TI-3 whose bearing capacity reaches up to 5013 kN according to EC2, which is definitely quite excessive, although the punching strength of the footing has not been reached, i.e. no measured value has been obtained. The reason for such design punching force is certainly the effective depth of this footing, which amounts to 17,5 cm.

Using the EC 2 with the control-section positioned according to ECP [25], the following F_s values are obtained: from 0,99 for footing TXI to 1,84 for footing TII. The dissipation value for Fs is now much smaller, and the punching force for footings of larger effective depth is not overestimated. It can even be argued that this proposal somewhat underestimates the bearing capacity for footings of larger depth, as the bearing capacity of 811 kN is defined for footing TI-3, and this footing was tested, without punching failure, until the force of 1001 kN in the column. It can be concluded that, combined with the proposed ECP, this regulation is much more suitable for application in footings.

In case of DIN the values of F_s range from 1,99 for the footing TXI to 3,70 for the footing TII. Since the footings TXI and TII have the same static height (12.5 cm), and as their other characteristics are similar, with the exception of the concrete compressive strength (for footing TII it was 30,39 MPa and for footing TXI it was 15,28 MPa), it can be concluded that this regulation reflects with insufficient accuracy the increase of concrete compressive strength with the increase in bearing capacity of footing.

In general terms, the calculation according to British Standards (BS) provides quite a good estimate of bearing capacity of footings because the values of F_s are in the range of 1,13 to 2,62, although it underestimates the bearing capacity of footings with the large compressive strength of concrete. This is due to the fact that the bearing capacity of footings TII, TXI and TXII (the same height with a very different compressive strength of concrete - 30,39 MPa, 15,28 MPa and 7,92 MPa, respectively) increases very slowly with an increase in the compressive strength of concrete. The quality of the results is enhanced by the fact that this document equates the impact of compressive concrete strengths of less than 25 MPa.

The ACI 318 regulation, with the control section proposed at the distance of 1.0d from the edge of the column, gives much better results than the standard ACI regulation. In fact, with this proposal, the results offered by ACI 318 are generally quite good (Fs are in the range of 1,16 to 2,02), although it may broadly be stated that it underestimate the influence of the compressive strength of concrete.

Russian regulations СНиП – 84 provide very good results, with the F_s ranging from 1,28 to 2,16, and so it can be concluded that they are on an equal footing with much more recent regulations. It takes properly into account the increase in punching capacity with an increase in depth of the footings, while it underestimates the influence of the concrete tensile strength for higher class types of concrete.

Calculations according to BAB 87 provide results that are characterized by high values of the coefficient $F_{s,}$ and by considerable scattering of results, from 3,08 to 4,39.

The proposed/modified design procedure gives the best results because the coefficient Fs values are closest to one, and dispersion values of F_s are relatively low (0,95 – 1,41). Here we must remember that the experiment on the basis of which this procedure is proposed, was conducted on footings that had almost the same percentage of reinforcement (0,4 %), and so the influence of this parameter was adopted to be similar to the EC 2 and BS. This fact calls for additional experimental analysis of that parameter with respect to the footing punching capacity.

All regulations analysed in this text (except BAB 87) use partial safety factors for loads and materials, which significantly differ from one another. A review of partial safety factors used in various regulations for dead and live loads, and also for materials, is given in Table 5.

Considering the above mentioned differences, and the fact that some regulations use additional safety factors in relation to the type of stress state or working conditions (BS, ACI, СНиП), it is possible to make an analysis that would invalidate or annul the mentioned differences. It is based on mutual comparison of forces in service that are obtained by dividing design failure forces with the total safety factors that include safety coefficients for the load, material, type of stress state, and working conditions, i.e:

$$F_{su} = \gamma_o \cdot \gamma_m \cdot \gamma_{n.s} \cdot \gamma_{u.r.} \tag{14}$$

where:

F_{su} - total safety factor,
γ_o - safety factor for load,
γ_m - safety factor for material,
$\gamma_{n.s.}$ - safety factor for the corresponding stress state,
$\gamma_{u.r.}$ - safety factor for working conditions.

Usually, the participation of live load in relation to the dead load in buildings amounts to approximately 15 %, and so the global safety factor for load can be calculated for all regulations as follows:

$$\gamma_{opć.} = 0{,}85\,\gamma_g + 0{,}15\,\gamma_p \tag{15}$$

Thus obtained values are given in the column "Global" in Table 5.

In EC 2, DIN 1045-1 and BS 8110, the safety factor for material is explicitly expressed by the value of 1.5 (for concrete, while reinforcement characteristics are not included in the calculation

Table 5. Partial safety factors used in some regulations

Regulation	Load - γ_0			Material - γ_m		Stress state $\gamma_{n,s,}$	Working conditions $\gamma_{u,r,}$
	Dead γ_g	Live γ_p	Global $\gamma_{opć}$	Concrete	Reinforcement steel		
EC 2	1,35	1,50	1,37	1,50	1,15	-	-
DIN 1045-1	1,35	1,50	1,37	1,50	1,15	-	-
BS 8110	1,40	1,60	1,43	1,50	1,05	1,25	-
ACI 318-02	1,20	1,60	1,26	1,30 (average) [19]	1,11 [1]	1,33	-
СНиП 2,04,01–84	1,10	1,10 (average)	1,10	1,35 (average)	-	-	1,18

Table 6. Total safety factors at punching for some regulations

Regulation / Factor	EC 2	DIN 1045-1	BS 8110	ACI 318-02	СНиП 2,04,01–84	Proposed procedure
F_{su}	2,06	2,06	2,68	2,18	1,75	2,06

Table 7. Normal exploitation force in column for some regulations, in kN

Designation the footing / Regulation	TI-3	TII	TIX	TX	TXI	TXII
EC2 a_{crit} = 2,0 d	2433	262 (4,01)	95 (4,52)	444 (1,48)	208 (2,16)	167 (2,65)
EC2 based on ECP [25]	394	277 (3,79)	179 (2,40)	258 (2,54)	220 (2,05)	126 (3,51)
DIN a_{crit} = 1,5 d	283	139 (7,55)	74 (5,81)	154 (4,26)	110 (4,09)	88 (5,02)
BS a_{crit} = 1,5 d	448	150 (7,00)	86 (5,00)	218 (3,01)	140 (3,21)	140 (3,16)
ACI a_{crit} = 0,5 d	248	144 (7,29)	64 (6,72)	143 (4,59)	102 (4,41)	73 (6,05)
ACI a_{crit} = 1,0 d	499	238 (4,41)	117 (3,66)	259 (2,53)	169 (2,66)	121 (3,65)
СНиП a_{crit} = 0,5 d	525	253 (4,17)	129 (3,33)	269 (2,44)	175 (2,57)	129 (3,43)
BAB 87 a_{crit} = 0,5 d	275	136 (7,72)	59 (7,29)	114 (5,75)	84 (5,36)	57 (7,75)
Proposed procedure	721	361 (2,91)	167 (2,57)	320 (2,05)	228 (1,97)	225 (1,96)
Measured	-	1050	430	656	450	442

of punching capacity, and therefore the factor of safety for the reinforcing steel is not of interest). In case of ACI 318-02 and СНиП (SNiP) 2.04.01-84, the safety factor for concrete depends on the strength of concrete (it increases with strength) and amounts to approximately 1.30 and 1.35, respectively. However, in order to obtain ultimate punching loads, material safety coefficients equalling to one are used in this text.

The safety factor for the type of stress state exists in BS 8110 and it amounts to 1,25 (for shear). In ACI 318-02, this factor amounts to 1/0,75 = 1,33 (for shear). These coefficients were also assumed to be equal to 1,0 in the calculation of the ultimate punching load.

The factor of safety for working conditions is also included in СНиП 2.04.01-84. There, the value of 1,0 is adopted for weather conditions favourable for concrete hardening (which is usually fulfilled for foundations due to humidity of the environment). However, for concreting in vertical position, an additional safety coefficient must be applied: 1/0,85 = 1,18.

Taking all this into account, total safety factors at punching F_{su}, are calculated for some regulations, as shown in Table 6.

Considering that ultimate punching loads are calculated based on actual rather than calculated characteristics of the material (concrete), the service load can be obtained for some regulations by dividing the calculated ultimate punching load by total safety factors given in Table 6. Thus obtained exploitation forces are given in Table 7, while coefficients F_{ss}, which represent the ratio of registered punching loads to calculated loads, are given in brackets next to them.

To gain a proper insight into the quality of results provided by the studied regulations, the actual safety coefficients given in Table 7. should be compared with total safety coefficients at punching given in Table 6.

The calculation according to EC-2 provides much higher values of actual coefficients compared to the total projected safety factor (F_{su}=2,06). Thus, values F_{ss} range from 1,48 for the footing TX (which is on the side of safety) to 4,52. However, if suggestions given in the ECP are used, this regulation gives much better results, i.e. the range of F_{ss} values is smaller (2,05 - 3,79) and all the results are on the safe side.

The regulation DIN 1045-1, which is based on EC2, provides results that are more conservative compared to EC 2, and so the values ranging from 4,09 to 7,55 are obtained for actual safety coefficients.

The regulation BS 8110 is also very conservative, and actual safety coefficients based on this regulations range from 3,01 – 7,00, which is much more when compared to the total safety coefficient at punching which amounts to 2,68 according to this regulation/standard.

Results provided by ACI 318-02 recommendations are also highly conservative (F_{ss} ranges from 4,41 to 7,29, as related to the F_{su} = 2,18). However, the suggestion that this regulation should place the control section at the distance of 1,0 *d* from the edge of column gives more rational results, and in this case the F_{ss} ranges from 2,53 to 4,41.

Results obtained by СНиП 2.04.01-84 are also conservative and F_{ss} values range from 2,44 to 4,17.

In this comparison, the most conservative results are those that are based on the regulation BAB 87. Here, the F_{ss} values are very high for all footings, i.e. they sometimes attain 7,75. This could have been expected as this is the only regulation that is based on the concept of allowable stresses.

Even in this case, the results provided by the proposed calculation procedure are the best, i.e. the values of actual safety coefficients correspond the most to the total safety coefficient (estimated value: 2,06 and obtained value: 1,96 - 2,91). However, here it should be noted that there is still room for improvement of the proposed method, as the influence of the percentage of reinforcement on the punching bearing capacity has not been considered in the experimental research. The results would be even more accurate it this parameter were introduced.

7. Final remarks and conclusions

Based on the analysis presented in the paper, it can be concluded that all current regulations give conservative results, and that the most rational result is provided by EC 2, when the procedure proposed in the ECP is used [25]. The results clearly point to the need for improvement - corrections to existing regulations - because the footing bearing capacity values based on these regulations are significantly underestimated, and so the actual coefficients F_{ss} go up to 7,55. The impact of various parameters used in the calculation of punching capacity of footings must therefore be reexamined, and the stress concentration factor must be introduced into the calculation. The necessity of introducing this parameter in the punching calculation is further evidenced by the measurement of contact pressures in this and in previous experiments.

The number of footings tested on real soil is too small to formulate general conclusions. Therefore, a more detailed parametric study must be conducted to determine the influence of the stress concentration factor and other parameters that are introduced in calculation of the punching bearing capacity of footings. This opens an ample space for further experimental research in this area.

Following the analysis of tests presented in available literature, it is proposed that an approximate angle of 45° be adopted for punching through the footing. In paper [3] the authors conclude that the density of sand does not affect distribution of contact stresses under the footing, and that the adoption of their uniform distribution provides a sufficient safety against column punching through footing. Also, it was established that EN are less conservative than the ACI 318 which underestimate the influence of "shear slenderness" on calculated values.

A probabilistic approach to punching analysis is adopted in some studies. Thus, the punching probability of reinforced-concrete footings is studied in [29] where the following variables are considered: errors of the theoretical model for strength determination, compressive strength of concrete, reinforcing-steel strength, cross-sectional dimensions, and the applied load. Since in practice the safety factor 3 is used for load, it is recommended that this factor be increased to 4 so that the probability of column punching through the plate can be less than $1,35 \times 10^{-3}$.

The stress concentration factor F_c depends on the effective depth of the footing, and it is higher for lower effective depth values, and vice versa. This factor is also dependent on the intensity of the applied force, and it tends to increase as the punching load is approached. It is also dependent on the type of soil. Although the experiment was conducted on the gravel subgrade, it would also be appropriate to find out how the factor F_c changes when other subgrade types are used: sand, silt and clay. A larger number of experiments should be conducted on gravel subgrade so that the values of this factor could be considered relevant. Therefore, a larger number of experiments and appropriate analyses should be made in order to determine the value of this factor with greater accuracy. It is only then that adequate values of parameters for different soil types and different footing characteristics would be obtained, and these values could then be used in the design process. This parameter is not further analysed in this paper; it is in fact given as an arithmetic mean obtained on the tested footings. Theoretical models should be developed in further research, but with verification based on experimental results. The study of plates with a small shear span ratio is of special interest. The application of high strength concrete in foundations involves examination of impact of the concrete compressive strength at punching [30].

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