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DATA AND MEASUREMENT IN YEAR 4 OF THE NATIONAL CURRICULUM: MATHEMATICS

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The activities introduced here were used in association with a research project in four Year 4 classrooms and are suggested as a motivating way to address several criteria for Measurement and Data in the *Australian Curriculum: Mathematics*. The activities involve measuring the arm span of one student in a class many times and then of all students once.

Introduction

The *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment, and Reporting Authority [ACARA], 2013a) has had a mixed reception from mathematics educators (e.g., Atweh, Goos, Jorgensen, & Siemon, 2012) and the debate will lead to continuing improvement and relevance over time. There are places in the curriculum, however, whether planned or not, where valuable links can be made that offer great opportunity for classroom activities. One of these appears in Year 4 in the strands of Measurement and Data. The relevant descriptors are presented in Table 1.

Table 1. Year 4 content descriptors, *Australian Curriculum: Mathematics* (ACARA, 2013a).

Measurement and Geometry
Using units of measurement
• Use scaled instruments to measure and compare lengths, masses, capacities and temperatures (ACMMG084)
Statistics and Probability
Data representation and interpretation
• Select and trial methods for data collection, including survey questions and recording sheets (ACMSP095)
• Construct suitable data displays, with and without the use of digital technologies, from given or collected data. Include tables, column graphs and picture graphs where one picture can represent many data values (ACMSP096)
• Evaluate the effectiveness of different displays in illustrating data features including variability (ACMSP097)

The final word in Table 1 is the key connecting concept across these descriptors. Variation is the concept underlying all of statistics (e.g., Moore, 1990; Watson, 2006) and research has suggested that students develop an appreciation for variation before an appreciation of expectation, where expectation is related to "expected values" such as probabilities or averages (Watson, 2005). These two concepts form the foundation of informal inference, which although not explicitly stated in the curriculum, must be the aim of the "interpretation" part of "Data representation and interpretation." Makar and Rubin (2009) define informal inference in terms of using data as evidence, generalising beyond those data, and acknowledging uncertainty in the generalisation. It is precisely the variation in the data that leads to the uncertainty in the expectation expressed in the generalisation.

Returning to the descriptors in Table 1, in measuring length students may have an expectation that their own measurement is precise and accurate, but if several students measure the same object, they will quickly see that there is variation in the measurements. Variation will also occur if different students use different tools to measure the length of an object, which links to the second descriptor in Table 1 about methods of data collection. Variation is inherent in both the different methods of data collection and in the data collected. As intimated in the fourth descriptor, the suitable data displays from the third descriptor will have different qualities in displaying the variation in the data collected.

The activities described here, based on measuring arm span length (Watson & Wright, 2008), were the basis of a research project that will involve students in Years 4 to 6 over three years. Students were given workbooks (the questions from which are condensed in the Appendix) and worked in groups of two or three. The teachers were provided with extensive teaching notes, including instructions for carrying out and recording the measurements on a whiteboard in a list and with yellow stickies in a stacked "dot" plot. Students produced hand-drawn plots and later used the *TinkerPlots* software (Konold & Miller, 2011) for creating other plots of the data. The researchers were present observing and occasionally answering questions. The students were sometimes in their classroom and sometimes in a computer laboratory; the researchers fit in and were not considered an intrusion.

The lessons and student outcomes

The first lesson began with a discussion on how accurately the class could measure length, reviewing the units and tools they could use. The question was asked. "If we all measure the same object will we get the same answer for its length? Why or why not?" The scene was then set for measuring the arm span of one student, posing the question of everyone getting the same value, and being able to make a "best guess" of the person's arm span. Students were given a choice of measuring tools, including rulers, tape measures, string, and metre sticks. The instructions in the workbook assisted students in recording the data on the whiteboard and in their workbooks; students answered questions Q3, Q4, and Q5 in the workbook (see Appendix) while the measurements were being made and recorded. One of the stacked dot plots of class data is presented in Figure 1.

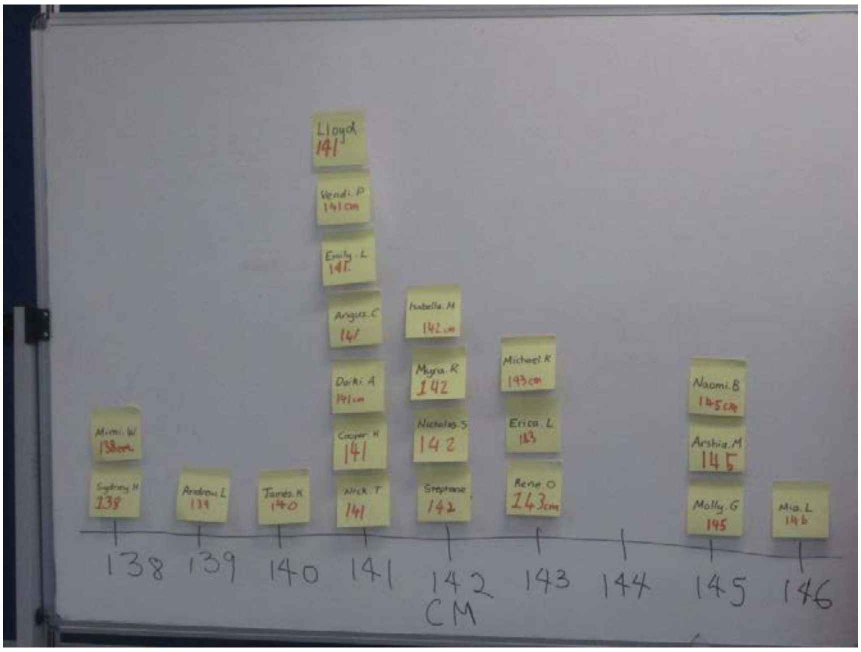

Figure 1. Stacked dot plot of one class's data.

In answering Q3, "Were all of the measurements the same? Why or why not?", students were well aware of the reasons for variation. Most answers focussed on the measuring tools or whether the measuring was carried out accurately.

- No, because the class was using different measuring materials.
- No, because some people didn't start at zero.
- No, because [student] could have moved her arms.
- No, because the units of measurement were different.

Other responses, however, were more vague.

- No, because everybody can get a different answer.

In answering Q4 "Were you surprised at some of the values? Which ones? Why?" there was variation across the classes because of the different data collected but well over half of the students identified an outlier, whereas others noticed the range or lack of variation.

- Yes, [student] did 99 cm and everybody else did over 110 cm.
- Yes, 146 and 138 because they were the biggest and smallest number.
- I wasn't surprised because the numbers are around the same.

Q5 was more comprehensive, asking for a summary of the accuracy of the measurements, a "best guess" of the arm span, and the student's confidence in the guess. Comments on accuracy mirrored some of the reasons given for Q3. Examples presented here hence focus on the reasons for the expectation associated with the "best guess" of the person's arm span, which varied considerably.

- I think it is 141 cm because there are more 141 cm in the measurements.
- I think that our guess was pretty good because we guessed 142 cm and it was in the middle of all of the guesses.
- I think 141 cm ... is the right size for [student].

When asked to create a representation of the data in a graph, picture or plot, the results were generally of two types. Figure 2 shows three representations that kept track of the values recorded for the measurements. On the left is an alphabetical list of the measurers and their measurements. In the centre is a horizontal line plot of unordered measurements with scale from 138 cm and on the right is a vertical value plot with the scale beginning at 10 cm.

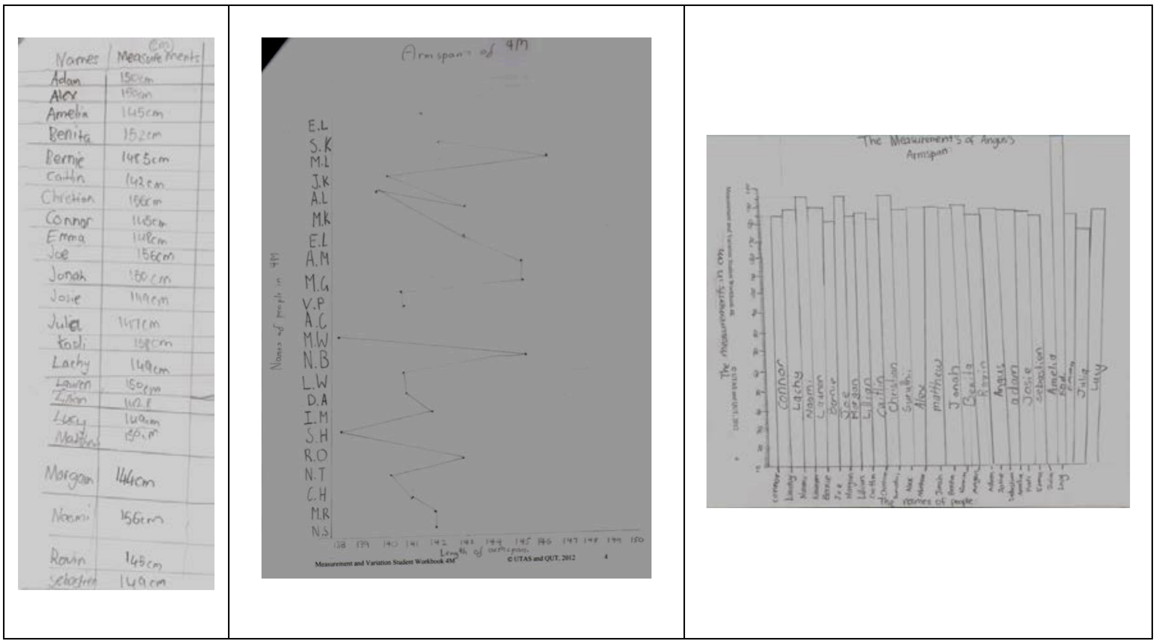

Figure 2. Three examples of representations of the values measured.

Figure 3 contains three representations of the frequency type. On the left is an unordered tally list; in the centre, an unordered frequency bar chart; and on the right, a frequency pictograph. The representations in Figures 2 and 3 illustrate the tremendous variation in tables and plots that are created by students when they are given a blank page rather than a labelled grid to fill in.

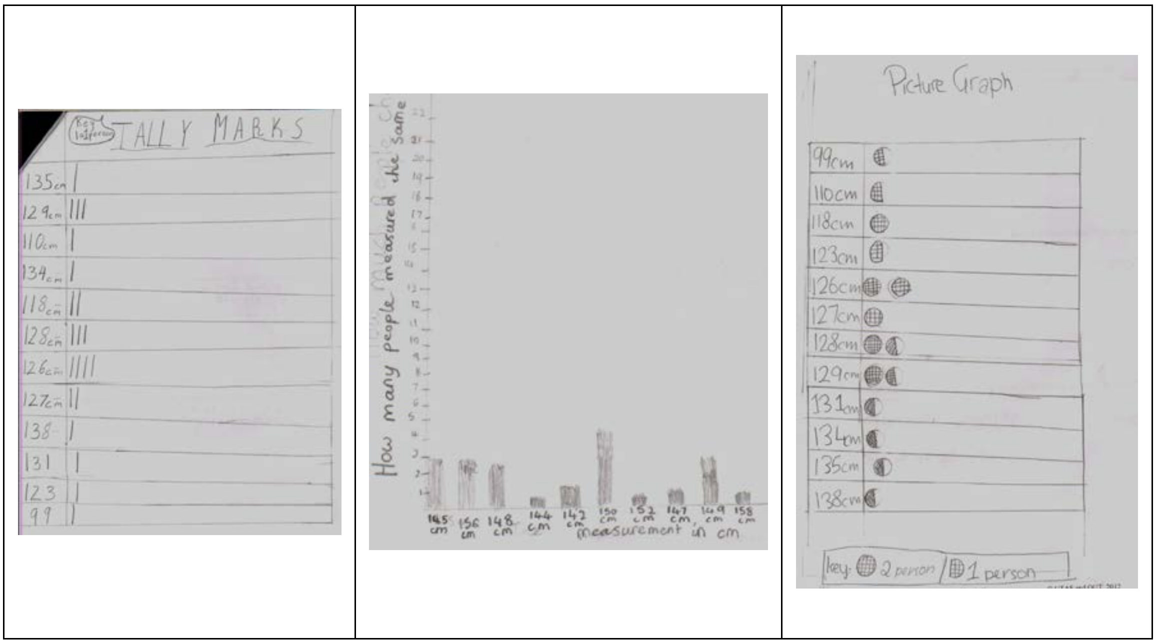

Figure 3. Three examples of representations of the frequencies of values measured.

For question Q6, students were asked to write a summary of their representations, thinking about variation. Only a few students missed the point and talked about colour or people. The display of appreciation of variation itself revealed variation, from little recognition to quite meaningful suggestions.

- I used a bar chart to show [student's] arm span.
- My graph is counting in centimetres. It has the names of everyone on the horizontal axis and the measurement on the vertical axis.
- It tells you from a number to a number e.g. 126 to 129 and how many people chose between the two numbers.
- Bar graph and 141 is most likely to be the correct arm span.
- I chose this graph because it shows the different variations of cm for one person's arm span. It also shows how many people got the same measurement.
- I did a range of measurements to see which one is most popular.

Students then entered the data into *TinkerPlots* to consider other possible representations of the measurements. It was their first experience with data in *TinkerPlots* except for an introductory session in the computer laboratory a few weeks earlier. Students were keen to identify the people with the values they had measured and in this first exposure to the software unconventional representations were sometimes created. Two of the *TinkerPlots* graphs are shown in Figure 4, again representing measured values or frequencies of measured values.

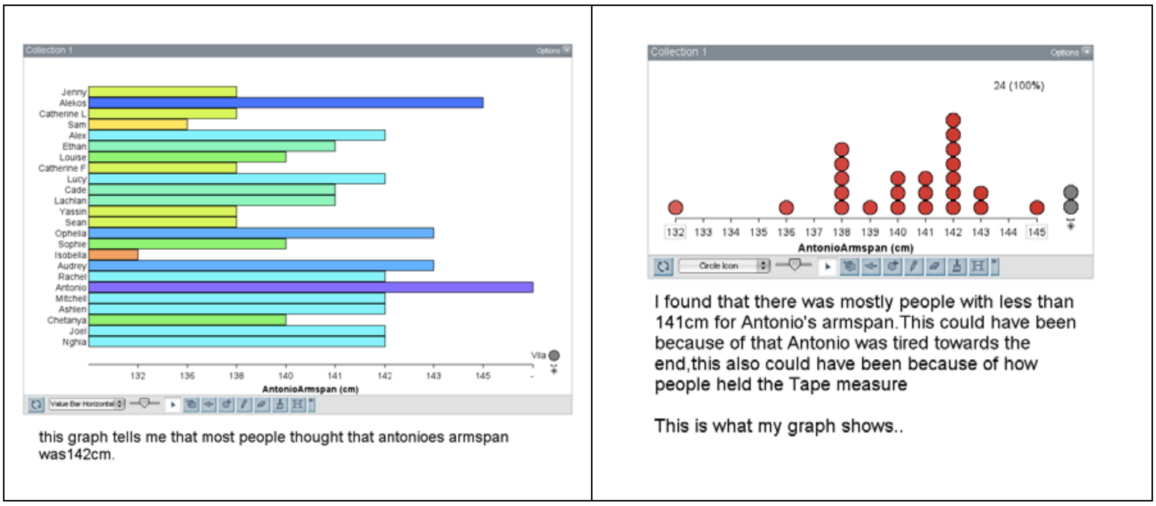

Figure 4. Two examples of *TinkerPlots* representations of measurements for one student.

Moving to the second lesson where students measured the arm spans of all members of their class once, the questions involved the accuracy of measurement and the class data were entered directly into *TinkerPlots* rather than the students creating their own representations first on paper. Following class discussion it was expected by the teachers and researchers that the issue of having many measurements to make a "best guess" of arm span, as in the first lesson, versus having only one measurement in the second lesson, would motivate students to question the accuracy of the single values for each student. In fact the circumstances of the lessons, where to speed the process, in most classes the researchers helped measure the arm spans of students along a tape measure attached to a wall, meant other salient features captured the attention of students. When answering Q4 on accuracy, most suggestions were of a practical rather than theoretical nature.

- More accurate because we were putting the measuring tape on the wall.
- They will be [more accurate] because the adults did it.
- Yes. I think they will be because now we have had practice at measuring we might be more accurate than last time.

Only one response appeared to recognise the issue of repeated measurements.

- I think [student's] arm span measurements were more accurate than ours because we were all measuring the same person.

For the final part of the activity students were asked to display the data from measuring the arm spans of the entire class. Because the data were entered into the same file that contained the measurements on the single student, students could be asked to describe differences between the plots for the two data sets. Two of the final plots and explanations are shown in Figure 5.

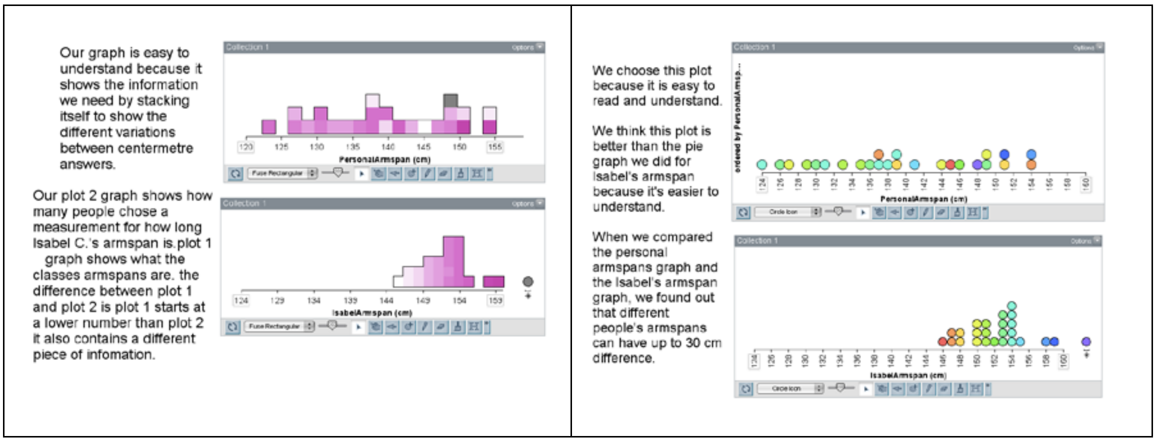

Figure 5. Two examples of summaries from the measurement activities.

For students who had completed their analyses, they were encouraged to consider other questions that might arise from the data collected, such as the possibility of gender difference in the arm span lengths. Figure 6 shows two examples. On the left is a plot showing the gender difference in one class and on the right is a plot showing the variation in measurements of a single student for the different tools used in another class.

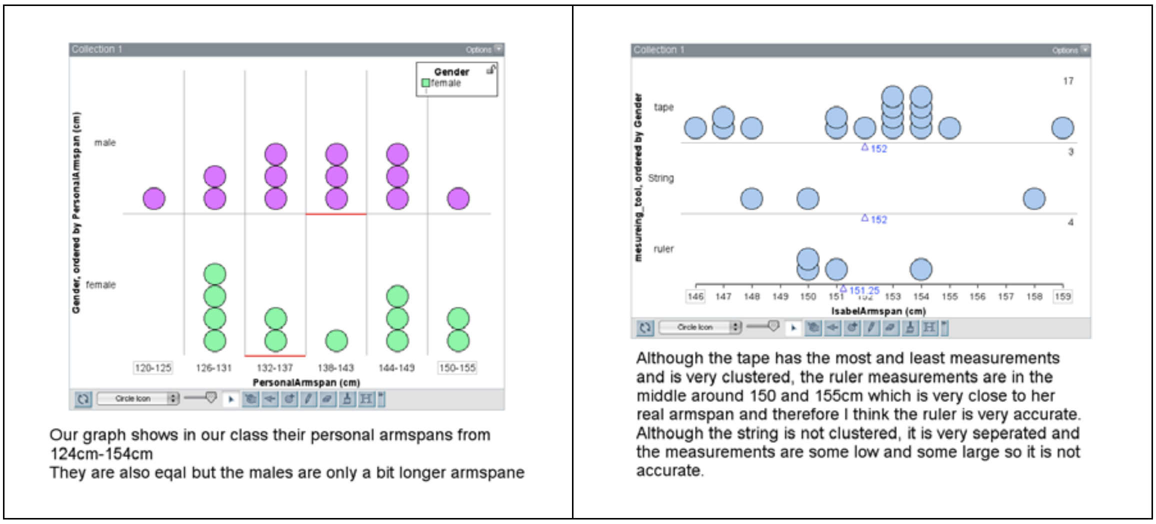

Figure 6. Two examples of extensions to the measurement activity.

Discussion

The activities described here have demonstrated the feasibility of linking Measurement and Data at Year 4. Without a context, statistics are meaningless (Rao, 1975) and measurement provides a realistic and motivating context within the *Australian Curriculum: Mathematics*. The activities also fulfil the detail of constructing "data displays, with and without the use of digital technologies" within Data Representation and Interpretation and reinforce the general capability of ICT across the curriculum (ACARA, 2013b). In particular, Investigating with ICT, Creating with ICT, and Communicating with ICT (p. 53) are illustrated with these activities. The foundation concept of variation was stressed throughout discussions with students and although not all written responses reflected it (43% of students were ESL students), the teachers expressed complete satisfaction with the outcomes.

Turning to the proficiency strands of the mathematics curriculum, these activities clearly contributed to understanding in building "a robust knowledge of adaptable and transferrable mathematical concepts" (ACARA, 2013a, p. 5), particularly in relation to linear measurement and variation as a foundation for statistical investigations. Variation must be one of the most transferable concepts in all of mathematics! Fluency in terms of "choosing appropriate procedures [and] carrying out procedures flexibly, accurately, efficiently and appropriately" (p. 5) was observed to develop during the lessons in the new techniques encountered by the students. Students also were developing "the ability to make choices, interpret, formulate, model and investigate problem situations" (p. 5) as they worked through the activities presented to them. Finally most students displayed progress in reasoning as they used logical thought patterns in the measurement context to infer, justify and generalise, explaining their outcomes (p. 5).

For those who would meet the content, proficiency and capability standards of the new curriculum, these activities provide an excellent starting point, as well as a foundation for informal inference in later years. Although not all students performed at the level displayed in the figures, all students were engaged, discussed difference and variation with members of their groups and the teacher, and could answer casual questions posed by the researchers.

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Appendix: Student Workbook (Condensed)

Measuring a person's armspan

Name: [Other information]

How easy is it to get an accurate measurement of something? Today, we will be discussing methods of measuring the armspan of one person in your class and everyone will have a chance to do the measuring.

Q1. Explain here the method you and your partner have decided to use:

Q2. Each person in your group is to record below their name and the measurement they took.

Name: ______________________________Armspan (cm): ____________
Name: ______________________________Armspan (cm): ____________
Name: ______________________________Armspan (cm): ____________

Once you have your measurement, also record the information on the whiteboard when instructed by your teacher. [Table provided with names of students to record measurements.]

Q3. Were all of the values the same?
Why or why not?
Which was the largest?
Which was the smallest?

Q4. Were you surprised at some of the values? Which ones? Why?

Q5. Write a summary of how accurate you think the measurements in the table are. What is your "best guess" of the armspan of the person the class measured? How confident are you of this value?

Use the next page to create a graph or plot or picture to represent the values in the table. [Blank page provided.]

Q6. Write a summary statement about what your representation shows about the measurements your class made of the armspan of the person you measured. Think about the variation that is seen in your plot or picture.

Q7. After looking at the other representations presented in class, which do you think is the best way to tell the story of the person's armspan? Why?

Q8. What is your new "best guess" for the armspan of the person the class measured? Explain why you chose this value.

Q9. [Instructions given for entering class data into *TinkerPlots*, which students complete and create plots.] Has your "best guess" changed for the person's actual armspan? If so, explain why?

Q10. After looking at the other *TinkerPlots* files created by your classmates, write a summary below of which representation you think shows the variation in the measurements the best.

Lesson 2 – Measuring our armspan

Last lesson we measured the armspan of a single person. This lesson we are going to measure and plot the armspan of all members of the class.

Q1. Describe the method your class have decided to use:

Q2. All people in your group are to record below their names and their armspan measurements. [space provided]

Once you have your measurement, also record the information on the whiteboard when instructed by your teacher.

Q3. Do you think all the values will be the same?
Why/why not?

Q4. How accurate do you expect your results to be compared to our last lesson?

[Table provided with names of students to record measurements.]

Q5. In your groups, enter the data into the *TinkerPlots* file.
You will need to:

- Create Data Cards and enter their data
- Create a plot that best describes the data set and tells the story
- Create a text box and write:
  i. A summary statement about what your plot shows and
  ii. At least two sentences that describe the differences between this plot and the earlier one.

Extension question

Do boys have longer armspans or do girls? How might this be explored? Can you make a plot showing the difference between boys' and girls' armspans?