

# On semantics in dynamic argumentation frameworks

Maria Laura Cobo, Diego C. Martinez, and Guillermo R. Simari

Artificial Intelligence Research and Development Laboratory (LIDIA)  
Department of Computer Science and Engineering, Universidad Nacional del Sur  
Av. Alem 1253 - (8000) Bahía Blanca - Bs. As. - Argentina  
{mlc, dcm, grs}@cs.uns.edu.ar  
<http://www.cs.uns.edu.ar/lidia>

**Abstract.** A Timed Abstract Argumentation Framework is a novel formalism where arguments are only valid for consideration in a given period of time, which is defined for every individual argument. Thus, the attainability of attacks and defenses is related to time, and the outcome of the framework may vary accordingly. In this work we study the notion of stable extensions applied to timed-arguments. The framework is extended to include intermittent arguments, which are available with some repeated interruptions in time.

**Keywords:** Timed Abstract Argumentation, Abstract Argumentation, Timed Information

## 1 Introduction

One of the main concerns in Argumentation Theory is the search for rationally based positions of acceptance in a given scenario of arguments and their relationships. This task requires some level of abstraction in order to study pure semantic notions. Abstract argumentation systems [11, 16, 2, 3] are formalisms for argumentation where some components remain unspecified, being the structure of an argument the main abstraction. In this kind of system, the emphasis is put on the semantic notion of finding the set of accepted arguments. Most of these systems are based on the single abstract concept of *attack* represented as an abstract relation, and extensions are defined as sets of possibly accepted arguments. For two arguments  $\mathcal{A}$  and  $\mathcal{B}$ , if  $(\mathcal{A}, \mathcal{B})$  is in the attack relation, then the acceptance of  $\mathcal{B}$  is conditioned by the acceptance of  $\mathcal{A}$ , but not the other way around. It is said that argument  $\mathcal{A}$  *attacks*  $\mathcal{B}$ , and it implies a priority between conflicting arguments.

The simplest abstract framework is defined by Dung in [11]. It only includes a set of abstract arguments and a binary relation of attack between arguments. Several semantics notions are defined and the Dung's argument extensions became the foundation of further research. Other proposals extend Dung's framework by the addition of new elements, such as preferences between arguments [2, 7] or subarguments [14]. Other authors use the original framework to elaborate new extensions [12, 5]. All of these proposals are based on varied abstract formalizations of arguments and attacks.

In this scenario, the combination of time and argumentation is a novel research line. In [13] a calculus for representing temporal knowledge is proposed, and defined in terms of propositional logic. This calculus is then considered with respect to argumentation,

where an argument is defined in the standard way: an argument is a pair constituted by a minimally consistent subset of a database entailing its conclusion. This work is thus related to [4].

In [9, 10] a novel framework is proposed, called *Timed Abstract Framework* (TAF), combining arguments and temporal notions. In this formalism, arguments are relevant only in a period of time, called its *availability interval*. This framework maintains a high abstract level in an effort to capture intuitions related with the dynamic interplay of arguments as they become available and cease to be so. The notion of *availability interval* refers to an interval of time in which the argument can be legally used for the particular purpose of an argumentation process. Thus, this kind of timed-argument has a limited influence in the system, given by the temporal context in which these arguments are taken into account.

Timed abstract frameworks capture the previous argument model by assigning arguments to an availability interval of time. In [10] a skeptical, timed interval-based semantics is proposed, using admissibility notions. As arguments may get attacked during a certain period of time, defense is also time-dependant, requiring a proper adaptation of classical acceptability. In [9], algorithms for the characterization of defenses between timed arguments are presented.

In [9] a natural expansion of timed argumentation frameworks by considering arguments with more than one availability interval is introduced. These arguments are called *intermittent arguments*, available with (possibly) some repeated interruptions in time. In all of these scenarios arguments may become relevant, or cease to be so, depending on time-related factors.

This paper is organized as follows. In the next section we recall time representation notions, where time-intervals are presented. Thereafter, the terminology used in this work are defined, towards the presentation of our Timed Abstract Argumentation Framework with intermittent arguments in Section 3. The notion of stable extension is presented in Section 4. The relation among steadiness and dynamics is analyzed in Section 5. Finally, conclusions and future work are discussed.

## 2 Time representation

In order to capture a time-based model of argumentation, we enrich the classical abstract frameworks with temporal information regarding arguments. The problem of representing temporal knowledge and temporal reasoning arises in a lot of disciplines, including Artificial Intelligence. There are many ways of representing temporal knowledge. A usual way to do this is to determine a *primitive* to represent time, and its corresponding *metric relations* [1, 15]. In this work we will use *temporal intervals of discrete time* as primitives for time representation, and thus only metric relations for intervals are applied.

**Definition 1** [*Temporal Interval*] An interval is a pair build from  $a, b \in \mathbb{Z} \cup \{-\infty, \infty\}$ , in one of the following ways:

- $[a, a]$  denotes a set of time moments formed only by moment  $a$ .
- $[a, \infty)$  denotes a set of moments formed by all the numbers in  $\mathbb{Z}$  since  $a$  (including  $a$ ).

- $(-\infty, b]$  denotes a set of moments formed by all the numbers in  $\mathbb{Z}$  until moment  $i$  (including  $b$ ).
- $[a, b]$  denotes a set of moments formed by all the numbers in  $\mathbb{Z}$  from moment  $i$  until moment  $j$  (including both  $a$  and  $b$ ).
- $(-\infty, \infty)$  a set of moments formed by all the numbers in  $\mathbb{Z}$ .

The moments  $a, b$  are called endpoints. The set of all the intervals defined over  $\mathbb{Z} \cup \{-\infty, \infty\}$  is denoted  $\mathcal{Y}$ .

For example,  $[5, 12]$  and  $[1, 200]$  are intervals. If  $X$  is an interval then  $X^-, X^+$  are the corresponding endpoints (i.e.,  $X = [X^-, X^+]$ ). An endpoint may be a point of discrete time, identified by an integer number, or infinite.

We will usually work with sets of intervals (as they will be somehow related to arguments). Thus, we introduce several definitions and properties needed for semantic elaborations.

In the following section we present Timed Abstract Argumentation Frameworks with intermittent arguments.

### 3 Timed Argumentation Framework

As remarked before, in Timed Argumentation Frameworks [9] the consideration of time restrictions for arguments is formalized through an *availability function*, which defines a temporal interval for each argument in the framework. This interval states the period of time in which an argument is available for consideration in the argumentation scenario. The formal definition of our timed abstract argumentation framework follows.

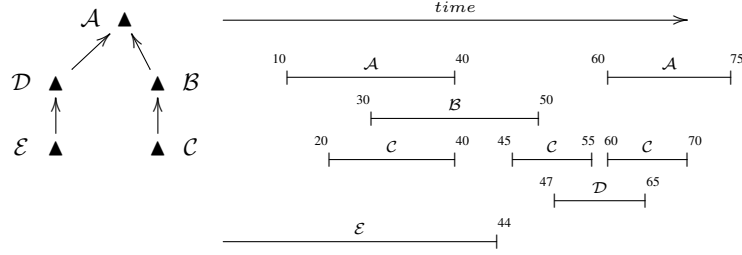
**Definition 2** A *timed abstract argumentation framework (TAF)* is a 3-tuple  $\langle \text{Args}, \text{Atts}, \mathcal{A}v \rangle$  where  $\text{Args}$  is a set of arguments,  $\text{Atts}$  is a binary relation defined over  $\text{Args}$  and  $\mathcal{A}v$  is the availability function for timed arguments, defined as  $\mathcal{A}v : \text{Args} \rightarrow \wp(\mathcal{Y})$ .

**Example 1** The triplet  $\langle \text{Args}, \text{Atts}, \mathcal{A}v \rangle$ , where  $\text{Args} = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\}$ ,  $\text{Atts} = \{(\mathcal{B}, \mathcal{A}), (\mathcal{C}, \mathcal{B}), (\mathcal{D}, \mathcal{A}), (\mathcal{E}, \mathcal{D})\}$  and the availability function is defined as

$\text{Args}$	$\mathcal{A}v$	$\text{Args}$	$\mathcal{A}v$
$\mathcal{A}$	$\{[10, 40], [60, 75]\}$	$\mathcal{B}$	$\{[30, 50]\}$
$\mathcal{C}$	$\{[20, 40], [45, 55], [60, 70]\}$	$\mathcal{D}$	$\{[47, 65]\}$
$\mathcal{E}$	$\{(-\infty, 44]\}$		

is a *timed abstract argumentation framework*.

The framework of Example 1 can be depicted as in Figure 1, using a digraph where nodes are arguments and arcs are attack relations. An arc from argument  $\mathcal{X}$  to argument  $\mathcal{Y}$  exists if  $(\mathcal{X}, \mathcal{Y}) \in \text{Atts}$ . Figure 1 also shows the time availability of every argument, as a graphical reference of the  $\mathcal{A}v$  function. It is basically the framework's evolution in time. Endpoints are marked with a vertical line, except for  $-\infty$  and  $\infty$ . For space reasons, only some relevant time points are numbered in the figure. As stated before, the availability of arguments is tied to a temporal restriction. Thus, an attack to an



**Fig. 1.** Framework of Example 1

argument may actually occur only if both the attacker and the attacked argument are available. In other words, an attack between two arguments may be *attainable*, under certain conditions. *Attainable attacks* are attacks that will eventually occur in some period of time. In order to formalize this, we need to compare time intervals, using the previously defined metric relations.

**Definition 3** Let  $\Phi = \langle \text{Args}, \text{Atts}, \text{Av} \rangle$  be a TAF, and let  $\{\mathcal{A}, \mathcal{B}\} \subseteq \text{Args}$  such that  $(\mathcal{B}, \mathcal{A}) \in \text{Atts}$ . The attack  $(\mathcal{B}, \mathcal{A})$  is said to be *attainable* if  $I_{\mathcal{A}}$  overlaps  $I_{\mathcal{B}}$ , for some  $I_{\mathcal{A}} \in \text{Av}(\mathcal{A})$  and  $I_{\mathcal{B}} \in \text{Av}(\mathcal{B})$ . The attack is said to be *attainable in*  $\text{Av}(\mathcal{A}) \cap \text{Av}(\mathcal{B})$ . The set of intervals where an attack  $(\mathcal{B}, \mathcal{A})$  is attainable will be noted as  $\text{IntSet}((\mathcal{B}, \mathcal{A}))$

Note that an attack is attainable if the availability of both the attacker and the attacked argument overlaps in at least one moment of time.

**Example 2** Consider the timed argumentation framework of Example 1. The attacks  $(\mathcal{D}, \mathcal{A})$  and  $(\mathcal{B}, \mathcal{A})$  are both attainable in the framework. Attack  $(\mathcal{D}, \mathcal{A})$  is attainable since  $[47, 65] \cap [60, 75]$  is non-empty, with  $[47, 65] \in \text{Av}(\mathcal{D})$  and  $[60, 75] \in \text{Av}(\mathcal{A})$ . Attack  $(\mathcal{B}, \mathcal{A})$  is attainable since  $[30, 50] \cap [10, 40]$  is  $[30, 40]$ . Recall that  $[30, 50] \in \text{Av}(\mathcal{B})$ ,  $[10, 40] \in \text{Av}(\mathcal{A})$ . The attack  $(\mathcal{C}, \mathcal{B})$  is also attainable. Since  $\text{Av}(\mathcal{C}) = \{[20, 40], [30, 50]\}$  and  $\text{Av}(\mathcal{B}) = \{[30, 50]\}$  then we can assure the attainability of the attack because  $\text{Av}(\mathcal{C}) \cap \text{Av}(\mathcal{B})$  is non-empty. The attack is then attainable at  $\{[30, 40], [45, 50]\}$ , i.e. in  $\text{Av}(\mathcal{C}) \cap \text{Av}(\mathcal{B})$ . The attack  $(\mathcal{E}, \mathcal{D})$  is not attainable, since the intersection among  $(-\infty, 45]$  and  $[47, 65]$  is empty. The arguments involved in this attack are never available at the same time.

The set of all the attainable attacks in the framework  $\Phi$  is denoted  $\text{AttAtts}_{\Phi}$ . It is also possible to define the attainability of attacks at a particular timed intervals, as shown next.

**Definition 4** Let  $\Phi = \langle \text{Args}, \text{Atts}, \text{Av} \rangle$  be a TAF, and let  $\{\mathcal{A}, \mathcal{B}\} \subseteq \text{Args}$  such that  $(\mathcal{B}, \mathcal{A}) \in \text{Atts}$ . The attack  $(\mathcal{B}, \mathcal{A})$  is said to be *attainable at*  $I$  if:  $I \cap \text{Av}(\mathcal{A}) \neq []$  and the following condition holds:  $I \cap I_{\mathcal{A}}$  overlaps  $I_{\mathcal{B}}$ , for some  $I_{\mathcal{A}} \in \text{Av}(\mathcal{A})$  and  $I_{\mathcal{B}} \in \text{Av}(\mathcal{B})$ .

The set of attainable attacks of  $\Phi$  at interval  $I$  is denoted  $\text{AttAtts}_{\Phi}^I$ .

**Example 3** Consider the timed argumentation framework of Example 1. The set  $AttAtts_{\Phi}$  is:  $\{(\mathcal{D}, \mathcal{A}), (\mathcal{B}, \mathcal{A}), (\mathcal{C}, \mathcal{B})\}$ . The set  $AttAtts_{\Phi}^{[35,40]}$  is  $\{(\mathcal{B}, \mathcal{A}), (\mathcal{C}, \mathcal{B})\}$ . The attack  $(\mathcal{D}, \mathcal{A})$  is in  $AttAtts_{\Phi}$  but it is not in  $AttAtts_{\Phi}^{[35,40]}$ , since  $[35, 40] \cap [47, 65]$  is the emptyset. The attack  $(\mathcal{B}, \mathcal{A})$  is in  $AttAtts_{\Phi}$  and is also in  $AttAtts_{\Phi}^{[35,40]}$ , since  $[35, 40] \cap [10, 40] = [35, 40]$  and  $[35, 40] \cap [30, 50] = [35, 40]$ . Note that  $[10, 40] \in Av(\mathcal{A})$  and  $[30, 50] \in Av(\mathcal{B})$ .

The definition of attainability of attacks can be attached to particular time points too. The set of attainable attacks of  $\Phi$  at moment  $i$  is denoted  $AttAtts_{\Phi}(i)$  and is defined as  $AttAtts_{\Phi}(i) = AttAtts_{\Phi}^{[i,i]}$ .

## 4 Semantics for Timed Argumentation

In [9, 10, 8] several semantic notions for timed frameworks are introduced. Admissibility semantics are captured by considering temporal defense. As attacks may occur only on a period of time (that in which the participants are available), argument defense is also occasional. In [10] a skeptical, timed interval-based semantics is proposed, using admissibility notions. The classical definition of acceptability is adapted to a timed context. The complexity of this adaptation lies on the fact that defenses may occur sporadically and hence the focus is put on finding *when* the defense takes place. For example, an argument  $\mathcal{A}$  may be defended by  $\mathcal{X}$  in the first half of an availability interval, and later by an argument  $\mathcal{Y}$  in the second half. Although  $\mathcal{X}$  is not capable of providing a full defense, argument  $\mathcal{A}$  is defended while  $\mathcal{A}$  is available. In other words, defenders *take turns* to provide a defense.

In [8] a notion of stable extension is introduced which considers the global evolution of a timed framework. This requires the definition of the notion of t-profile: a pair formed by an argument and a set of intervals in which this argument is considered. Since arguments are related to time, a t-profile of an argument  $\mathcal{X}$  is the formal reference of  $\mathcal{X}$  within several frames of time, which are subintervals of the original availability intervals of  $\mathcal{X}$ . Hence, an argument is not considered stand-alone in a specific moment of time, but associated with a set of intervals. A t-profile attacks another t-profile if an attack is formally defined between its arguments and at least one interval of time of each profile is overlapping. A set of t-profiles is a collection of arguments which are considered within different intervals of time, not necessarily overlapping. The timed notion of *stable extension* is later defined, not as a set of arguments but as a set  $S$  of t-profiles such that every t-profile denotes intervals of time in which a given argument attacks other available arguments. Hence, the arguments in these t-profiles may collectively form a stable set. Notoriously, an argument  $\mathcal{X}$  may appear in t-profiles *inside* and *outside* this timed stable set simultaneously, but with different intervals of time since an argument may become attacked or not as time evolves. Thus, an argument may get in and out a stable set depending on time, but in any point of time a set of arguments is characterized which attacks any other argument not included in that set.

Beyond the previous time-based semantics, since at any moment in the evolution of the timed framework there may be active arguments with available attacks, it is possible

to apply Dung’s classical semantics at any timepoint. When arguments become available or cease to be so, these semantic consequences may change. This is addressed in the following section.

## 5 Steadiness in dynamic argumentation

A timed argumentation framework is a natural model for argumentation dynamics, where the set of arguments is not fixed and evolves through time, *i.e.* arguments may appear or disappear from the framework. These evolution may cause changes in the semantic consequences of the overall set of arguments and how an argument impacts on the outcome of the argumentation framework depends naturally on the particular semantics. We are mainly interested in the study of periods of time in which some semantic properties are unaffected by this argument dynamics. We will refer to these as *steady intervals* of a timed framework. For instance, the introduction of a new argument may not change any argument extension of a given semantics, as shown in the following example.

**Example 4** Consider the TAF of Figure 2. Argument  $A$  is attacked by arguments  $B$  and  $C$ , although in different moments in time. However, argument  $D$  provides a defense for  $A$  whenever it is attacked. Hence, although an argument cease to exist and another one begins,  $\{A, D\}$  is an admissible set in  $[10, 30]$ .

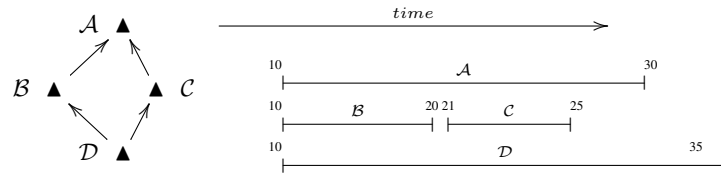


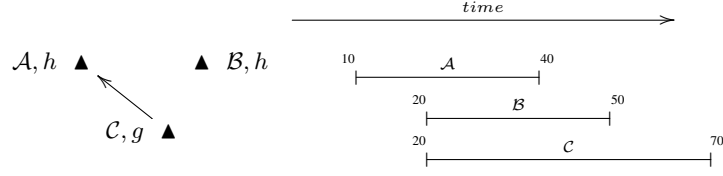
Fig. 2. Steady notions

There is also another view of steady intervals when conclusions of arguments are taken into account. Since arguments support conclusions, then this conclusions are kept in time. However, a new argument may change argument extensions while preserving the set of conclusions supported by those arguments.

**Example 5** Consider the TAF of Figure 3. Argument  $A$  supporting conclusion  $h$  is free of attackers in  $[10, 20]$ . Later on, it is attacked by argument  $C$  supporting conclusion  $g$  in  $[20, 40]$ . However, conclusion  $h$  is also supported by argument  $B$  which is not attacked by  $C$  and although  $A$  lacks of defenders, conclusion  $h$  is sustained in  $[10, 50]$ .

The following definitions provide basic notions for the study of the evolution of a timed framework.

**Definition 5** Let  $A$  be an argument and let  $I$  be an interval.



**Fig. 3.** Steady notions

- $Args(i)$  is the set of arguments available at a given timepoint  $i$ .
- An argument  $\mathcal{A}$  is said to occur in  $I$  if  $\mathcal{A} \in Args(k)$  for some  $k \in I$ .
- An argument  $\mathcal{A}$  is said to fully occur in  $I$  if  $\mathcal{A} \in Args(k)$  for all  $k \in I$ .
- An argument  $\mathcal{A}$  is said to partially occur in  $I$  if  $\exists k \in I$  such that  $\mathcal{A} \notin Args(k)$ .

**Definition 6** Two intervals  $I_1$  and  $I_2$  are consecutive if  $I_1^+ = I_2^- - 1$ .

In a timed argumentation framework, arguments may be available or cease to be so as times goes by. An interval in which no changes occur in the framework is said to be *static*, as defined next.

**Definition 7** A static interval for a TAF is a period of time  $I = [i, j]$  such that  $\forall k, m \in I, Args(k) = Args(m)$ . A maximal static interval is a static interval not included in another static interval.

A static interval is the first notion of steadiness in a timed framework. In the framework of Figure 2 the intervals  $[10, 15]$ ,  $[10, 20]$ ,  $[21, 25]$  and  $[26, 35]$  are all static intervals.

**Definition 8** Let  $I_1, I_2$  be two consecutive maximal static intervals. The pair  $(I_1^+, I_2^-)$  is said to be the changing leap of  $I_1$  to  $I_2$

Note that the changing leap denotes a transition, since something has occurred that breaks static periods of time. This is stated in the following proposition.

**Proposition 1** Let  $I_1$  and  $I_2$  be two consecutive maximal static intervals. Then  $Args(I_1^+) \neq Args(I_2^-)$ .

What is really interesting about changing leaps is the ability to affect semantic consequences. For instance, a single new argument may cause several arguments to be dropped out of argument extensions. The following definition, inspired from [6], characterizes the set of all the argument extensions induced by a given semantic.

**Definition 9** Let  $\mathcal{S}$  be an argumentation semantics. The set  $\mathcal{E}_{\mathcal{S}}(i)$  is the set of all the extensions under semantic  $\mathcal{S}$  at timepoint  $i$ .

In the timed framework of Figure 2, given  $\mathcal{S} = \text{admissibility}$ , then  $\mathcal{E}_{\mathcal{S}}(15) = \{\{\mathcal{A}, \mathcal{D}\}, \{\mathcal{D}\}\}$  and  $\mathcal{E}_{\mathcal{S}}(32) = \{\{\mathcal{D}\}\}$ .

Definition 9 leads to a semantic notion of steady intervals: those in which the set of extensions induced by a given semantics does not change over time. Thus, in every timepoint of the interval the set of extensions is the same. This is formalized in the following definition.

**Definition 10** Let  $\mathcal{S}$  be an argumentation semantic for TAF. A steady interval for  $\mathcal{S}$  is a period of time  $I = [i, j]$  such that  $\forall k, m \in I, \mathcal{E}_{\mathcal{S}}(k) = \mathcal{E}_{\mathcal{S}}(m)$ . A maximal steady interval for  $\mathcal{S}$  is a steady interval not included in another steady interval. The set of all the extensions during a steady interval  $I$  is denoted  $\mathcal{E}_{\mathcal{S}}^I$ .

In this kind of intervals the framework is *steady* since it is not semantically disturbed by changes in the set of arguments, if any. In the timed framework of Figure 2, the intervals  $I_1 = [10, 30]$  and  $I_2 = [31, 35]$  are steady intervals for admissibility.

**Definition 11** Let  $I_1$  and  $I_2$  be two maximal steady intervals for semantics  $\mathcal{S}$ . The changing leap of  $I_1$  and  $I_2$  is said to be a semantic leap of  $\mathcal{S}$ . A changing leap that is not a semantic leap is said to be irrelevant to  $\mathcal{S}$ .

Not every change in the set of arguments is a semantic leap. In the timed framework of Example 2 argument  $\mathcal{B}$  is *replaced* by argument  $\mathcal{C}$ , but this situation does not affect the admissible set  $\{\mathcal{A}, \mathcal{D}\}$ . However, if no change occurs, then naturally the semantic consequences remain unchanged, as stated in the following proposition.

**Proposition 2** Every static interval is a steady interval.

A semantic leap is interesting since it denotes a changing leap with an impact in the outcome of the framework. It is interesting to identify arguments introduced and discarded by the semantic leap.

**Definition 12** Let  $I_1, I_2$  be two consecutive steady intervals. The sets  $AV_{in}(I_1, I_2)$  and  $AV_{out}(I_1, I_2)$  are defined as

- $AV_{in}(I_1, I_2) = \text{Args}(I_2^-) \setminus \text{Args}(I_1^+)$
- $AV_{out}(I_1, I_2) = \text{Args}(I_2^+) \setminus \text{Args}(I_1^-)$

Although semantic extensions may change between two consecutive steady intervals, an argument may still be included in some extension. Since it spans between two interval, this argument is not added nor deleted in the semantic leap.

**Definition 13** Let  $\Phi$  be a TAF. Let  $\mathcal{S}_1$  and  $\mathcal{S}_2$  be two argumentation semantics. Let  $sl_{\mathcal{S}_1}^{i,j}$  and  $sl_{\mathcal{S}_2}^{i,j}$  be the set of all the semantic leaps of  $\Phi$  in  $[i, j]$  for semantics  $\mathcal{S}_1$  and  $\mathcal{S}_2$  respectively. Two argumentation semantics  $\mathcal{S}_1$  and  $\mathcal{S}_2$  are said to be chained in  $[i, j]$  if  $sl_{\mathcal{S}_1}^{i,j} = sl_{\mathcal{S}_2}^{i,j}$

Chained semantics share semantic leaps in a given interval, although the outcome of these semantics may differ. Semantic leaps keep track of what information does not change with the evolution of the framework.

**Definition 14** An argumentation framework is said to be well-formed in interval  $I$  if it is well-formed in any timepoint of  $I$ , i.e. it is cycle-free in that timepoint.

**Proposition 3** If an argumentation framework is well-formed at interval  $I$ , then the grounded and stable semantics are chained in  $I$ .



Proof: As stated in [11], for well-formed argumentation frameworks the grounded and stable semantics coincide. If an argumentation framework is well-formed in interval  $I$ , then any changing leap does not introduce cycles. Hence, any changing leap causing a semantic change in the grounded extension will cause a change in the stable extension and viceversa.  $\square$

**Proposition 4** *Let  $\Phi$  be a TAF. Let  $\alpha$  be a changing leap. If  $\alpha = (i, i + 1)$  is an irrelevant leap for admissibility, but a semantic leap of stable, then  $\Phi$  is not well-formed at timepoint  $i$ .*

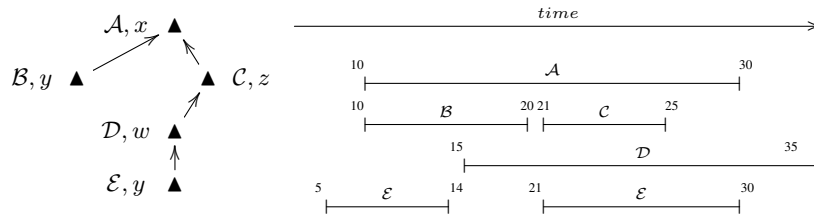
Proof: If  $\alpha$  is irrelevant for admissibility, then no admissible extension is changed after  $\alpha$ . It means that all the arguments in an admissible extension before  $\alpha$  keep their defenders after  $\alpha$ . Thus  $\alpha$  (a) removes an attacked attacker or (b) introduces new attacking arguments which are attacked in turn by arguments in a previous admissible set. Suppose  $\Phi$  is well-formed after  $\alpha$ . Then case (b) does not introduces new argument cycles (the same is trivially true for case (a)). Since a new argument  $\mathcal{A}$  is not introducing cycles, then Since  $\alpha$  is a semantic leap for stable semantics, then  $\mathcal{E}_{stable}(i) \neq \mathcal{E}_{stable}(i + 1)$ .

**Definition 15** *Let  $\mathcal{A}$  be an argument. The supportive interval of argument  $\mathcal{A}$  is a maximal interval  $I = [i, j]$  such that for any timepoint  $m$  in  $I$ , it holds that  $\mathcal{A} \in E$  for some  $E \in \mathcal{E}_S(m)$ .*

A supportive interval for an argument can span several steady intervals and they share endpoints with steady intervals.

**Remark 1** *It is possible for two consecutive steady intervals have the same set of warranted arguments, although in different extensions.*

**Proposition 5** *Let  $I = [i, j]$  be a steady interval and let  $\mathcal{A}$  be an argument in some extension  $E$  from  $\mathcal{E}_S^I$ . Then  $I$  is a subinterval of a supportive interval of  $\mathcal{A}$ .*



**Fig. 4.** Steady notions

As shown in Example 5 and in framework depicted on Figure 4 there is another notion of steadiness in timed argumentation frameworks, and it is related to the fact that several arguments may support the same conclusion. In the framework depicted on Figure 4  $y$  is supported on  $[5, 30]$  although in each particular moment of the interval  $y$  is granted through argument  $\mathcal{B}$ ,  $\mathcal{D}$  or both. The formalization of this ideas is currently being explored.

## 6 Conclusions and future work

In this work we presented an analysis over timed argumentation frameworks. The main idea was to find stable time periods, periods of time where extensions do not change. These periods are interesting in frameworks dynamics, since future changes only affects some of these periods instead of affecting the whole framework. Concepts and relations related with this notion were presented. Future work has several directions. Some of them are a deeper analysis of steadiness concept properties; other forms of steadiness are of particular interest. Steadiness of argument's conclusions is currently being analyzed.

## References

1. Allen, J.: Maintaining knowledge about temporal intervals. *Communications of the ACM* (26), 832–843 (1983)
2. Amgoud, L., Cayrol, C.: On the acceptability of arguments in preference-based argumentation. In: 14th Conference on Uncertainty in Artificial Intelligence (UAI'98). pp. 1–7. Morgan Kaufmann (1998)
3. Amgoud, L., Cayrol, C.: A reasoning model based on the production of acceptable arguments. In: *Annals of Mathematics and Artificial Intelligence*, vol. 34, 1-3, pp. 197–215 (2002)
4. Augusto, J.C., Simari, G.R.: Temporal defeasible reasoning. *Knowl. Inf. Syst.* 3(3), 287–318 (2001)
5. Baroni, P., Giacomin, M.: Resolution-based argumentation semantics. In: *Proc. of 2nd International Conf. on Computational Models of Argument (COMMA 2008)*. pp. 25–36 (2008)
6. Baroni, P., Giacomin, M.: On principle-based evaluation of extension-based argumentation semantics. *Artif. Intell.* 171(10-15), 675–700 (2007)
7. Bench-Capon, T.: Value-based argumentation frameworks. In: *Proc. of Nonmonotonic Reasoning*. pp. 444–453 (2002)
8. Cobo, M.L., Martínez, D.C., Simari, G.R.: Stable extensions in timed argumentation frameworks. In: Modgil, S., Oren, N., Toni, F. (eds.) *TAFa. Lecture Notes in Computer Science*, vol. 7132, pp. 181–196. Springer (2011)
9. Cobo, M., Martínez, D., Simari, G.: An approach to timed abstract argumentation. In: *Proc. of Int. Workshop of Non-monotonic Reasoning 2010* (2010)
10. Cobo, M., Martínez, D., Simari, G.: On admissibility in timed abstract argumentation frameworks. In: Coelho, H., Studer, R., Wooldridge, M. (eds.) *ECAI. Frontiers in Artificial Intelligence and Applications*, vol. 215, pp. 1007–1008. IOS Press (2010)
11. Dung, P.M.: On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence* 77(2), 321–358
12. Jakobovits, H.: Robust semantics for argumentation frameworks. *Journal of Logic and Computation* 9(2), 215–261 (1999)
13. Mann, N., Hunter, A.: Argumentation using temporal knowledge. In: *Proc. of 2nd International Conf. on Computational Models of Argument (COMMA 2008)*. pp. 204–215 (2008)
14. Martínez, D.C., García, A.J., Simari, G.R.: Modelling well-structured argumentation lines. In: *Proc. of XX IJCAI-2007*. pp. 465–470 (2007)
15. Meiri, I.: Combining qualitative and quantitative constraints in temporal reasoning. In: *Proceedings of AAAI '92*. pp. 260–267 (1992)
16. Vreeswijk, G.A.W.: Abstract argumentation systems. *Artificial Intelligence* 90(1–2), 225–279 (1997)