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GEOMETRY, WORKING MEMORY AND INTELLIGENCE

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Contents

1. Thesis structure and goals

Geometry is a fundamental part of mathematical learning. Since ancient time the study of geometry was considered as one of the most important subjects in school. In the arcade of the famous school of Athens, where Plato taught, it was written that entry was not permitted to people who did not know geometry. In the Renaissance period, geometry was part of the '*quadrivium*', which was considered a needed work preparatory for a serious study of philosophy. Nevertheless, despite geometry is one of the main areas of mathematical learning, the cognitive processes underlying geometry-related academic achievement have not been studied in detail.

The present dissertation has three important aims. First, to investigate the relationship between various aspects of geometry and visuospatial working memory (VSWM). Second, to investigate whether the children with nonverbal learning disabilities (NLD) symptoms present difficulties in various aspects of geometry. Third, to investigate the relationship between various aspect of geometry, working memory (WM) and intelligence (*g*).

In the second chapter, a general overview of the relationship between geometry, WM and *g* is provided. Since geometry concerns the study of the space, it requires a particular involvement of spatial abilities. Thus, WM, and in particular VSWM should be crucially involved. In addition, solving geometrical problems requires to reason and to determine a solution among various alternatives. Thus, *g* should be crucially involved in solving geometrical problems.

In the third chapter, the relationship between academic achievement in geometry, intuitive geometry (i.e., a part of geometry, which seems to be independent from the culture), and VSWM will be examined. Two studies will be presented.

In the first study, the involvement of VSWM in intuitive geometry and in school performance in geometry at secondary school was tested. A total of 166 pupils were administered: (1) six VSWM tasks, comprising simple storage and complex span tasks; (2) the intuitive geometry task devised by Dehaene, Izard, Pica, and Spelke (2006), which distinguishes between core, presumably independent form the culture, and culturally-mediated principles of geometry; and (3) a task measuring academic achievement in geometry.

In the second study, VSWM and intuitive geometry were examined in two groups aged 11–13; one with children displaying symptoms of NLD, and the other, a control group without learning disabilities. The two groups were matched for general verbal abilities, age, gender, and socioeconomic level. The children were presented with simple storage and complex-span tasks involving VSWM and with the intuitive geometry task devised by Dehaene and colleagues (Dehaene et al., 2006).

In the fourth chapter, we report a study on the relationship between geometry, WM, and intelligence aimed to determine the model of WM which provided the best fit to the data and to examine the strength of the relations between WM and intelligence (part I) and the relationship between geometry (intuitive geometry and geometrical achievement), WM and *g* testing several models (part II).

In the last chapter a general overview of the important theoretical and applied implications of the three studies will be discussed. The limits of the present dissertation and possible future researches will also be outlined.

2. Cognitive processes involved in geometrical cognition

Geometry, one of the oldest sciences, concerns the study of size, shape, length, relative position of figures and the properties of space. Despite representing one of the major areas of mathematical learning, the cognitive processes that are involved in geometry and those that affect academic achievement have not been studied in detail. In particular, two aspects are crucially involved in geometrical learning: working memory and intelligence.

2.1. Working memory

A first conceptualization of working memory was proposed by Baddeley and Hitch (1974). In the Baddeley and Hitch model, the central executive is the component responsible for controlling resources and monitoring information processing across informational domains. Moreover, storage of information is provided by two domain-specific slave systems: the phonological loop, which provides temporary storage of verbal information, and the visuospatial sketchpad, specialized in the maintenance and manipulation of visual and spatial representations. Further specifications of the model (Baddeley, 2000; Cornoldi & Vecchi, 2003) have maintained the distinction between central modality-independent and specific verbal and visuospatial components. Moreover, Cornoldi and Vecchi (2000, 2003) proposed a distinction between two continua. In the horizontal continuum, tasks are processes distinct for modality. In the vertical continuum, tasks are distinct depending on the involvement of the attentional control. Thus, there are tasks which are mainly passive (in which only recall of information is required) and active (in which processing and manipulation of the stimuli is required).

Other authors have argued that there is no difference between short-term memory (STM) and WM, proposing an unique model of WM. In fact, the possibility that WM and STM can reflect different or unique factors is still debated (e.g., Colom, Rebollo, Abad, & Shih, 2006). Indeed, it has been argued that STM storage and not cognitive control account for the relationship between WM and intelligence (e.g., Colom et al., 2006).

Alternatively, a modality dependent model postulates that WM is supported by two separate pools of domain-specific resources for verbal and visuospatial information (e.g., Shah & Miyake, 1996). In this model, each domain is independently capable of manipulating and keeping information active. Research involving adult participants supports this distinction (Friedman & Miyake, 2000).

On the contrary, a single-resource framework proposed that WM is a unitary system by nature (Cowan, 2001). This model postulates a unitary system involved principally in attentional control. Cowan defines WM as a limited-capacity attentional focus, which operates across areas of activated long-term memory. According to this model, long-term memory can be seen in three components: the larger portion that has relatively low activation at any particular point in time, a subset that is currently activated as a consequence of ongoing cognitive activities and perceptual experience, and a smaller subset of the activated portion that is the focus of attention and conscious awareness.

A similar account, a domain-general model, suggests that working memory capacity is limited by controlled attention. The Engle and collaborators model (e.g., Engle, Tuholski, Laughlin, & Conway, 1999), similarly to Cowan's model, conceive of the contents of stores as temporarily activated representations in long-term memory, as links to existing representations in long-term memory (Engle, 2010). Thus, in this model there is no distinction between modality (e.g., verbal and spatial), but only between STM (independently from the modality), and WM (which is dependent on control-of-attention capacity). This model is consistent to the Cornoldi and Vecchi model (2003). In fact, the distinction between STM and WM is well represented by the distinction of the tasks along the vertical continuum.

2.1.1. Working memory and geometry

Academic achievement in geometry is considered one of the most important areas of mathematical learning, and it is linked to a student's future academic and professional success

(Verstijnen, Van Leeuwen, Goldschmidt, Hamel, & Hennessey, 1998). Pupils attending secondary schools must be able to utilize concepts, definitions, theorems, etc., and have the ability to apply this knowledge when solving problems that are typically presented in language form. It therefore is important to examine whether differences in intuitive geometry and other underlying cognitive mechanisms may have a crucial role in predicting school achievement in geometry.

The WM system, in which specific storage components (i.e., the 'slave' systems) sub-serve a central component responsible for controlling information processing (Baddeley, 1986), could be involved in geometry. A large body of research has shown that WM predicts success in school-related tasks, such as reading comprehension (Daneman & Carpenter, 1980), mathematical achievement (Bull, Espy, & Wiebe, 2008; Fürst & Hitch, 2000; Geary, Klosterman, & Adrales, 1990; Hitch, 1978; Passolunghi, Mammarella, & Altoè, 2008) and arithmetical problem-solving (Passolunghi, Cornoldi, & De Liberto, 1999; Passolunghi & Siegel, 2001, 2004). More specifically, the WM component involved in retaining and processing visuospatial information (VSWM) appears to be involved a children's ability to count (Kyttälä, Aunio, Lehto, Van Luit, & Hautamäki, 2003), performance in multi-digit operations (Heathcote, 1994), nonverbal problem-solving (Rasmussen & Bisanz, 2005) and mathematical achievement (Bull et al., 2008; Jarvis & Gathercole, 2003; Maybery & Do, 2003).

Moreover, VSWM can predicts a person's success in geometry-related activities. To give an example, the capacity to hold and manipulate visuospatial information has been shown to specifically predict success in architecture and engineering (Verstijnen et al., 1998). This makes VSWM the prime candidate for seeking cognitive mechanisms supporting both intuitive geometry, which is supposed to be independent from culture, and school achievement in geometry, though the latter will be associated with many other variables influencing mathematical achievement in school (e.g., language, calculation, problem-solving, motivation, meta-cognition, etc.; Aydın & Ubuz, 2010). In addition, considering the sub-components of VSWM will make it possible to understand which components of VSWM are related to intuitive geometry and achievement in geometry.

2.2. Intelligence

A general conceptualization of intelligence is: a general mental capacity that involve the ability to reason, plan, solve problems, think abstractly, comprehend complex ideas, learn quickly and learn from experience (Gottfredson, 1997, p. 13). The importance of intelligence relies on the fact that it predicts important outcomes like occupational (Schmidt & Hunter, 2004), and academic attainment (Alloway & Alloway, 2010). In addition, the modern workplace runs largely on the cognitive abilities of the workforce (Hunt & Madhyastha, 2012).

Spearman first conceptualized the notion of general intelligence. Spearman observed that different tasks were typically related to each other. Thus, he hypothesized that the common factor in with each task represented a general factor (*g*; Spearman, 1904). On the contrary, the specific variance not common between factors was due to specific factors (*s*), but differently from *g*, they were not common to all measures (Spearman, 1927). This hypothesis was strongly criticized by Thurstone who believed in the existence of primary mental abilities rather than a unique specific factor (Thurstone, 1938). For this reason, the battle between unique and multiple intelligence' supporters began and continues until now.

Similarly to Thurstone, Guiford was convinced of the existence of multiple intelligences. In his previous formulation, the 'structure of intellect' was composed by 120 independent abilities (Guilford, 1967). He later expanded these abilities to 150 (Guilford, 1985). Guilford was sharply criticized by Arthur Jensen who claimed that Guilford's results were weak (Jensen, 1998). Despite whether or not Guilford's results were correct, it should be noted that if one ability was too few, then 120 were too many (Kaufman, 2009).

The theoretical battle between the unique and multiple conceptualization of intelligence continues. Vernon (1950) was the first to conceptualize a hierarchical intelligence model. At the top of this hierarchy Vernon posed the Spearman's general factor; and below *g* were several specific group factors (Vernon, 1950). Because Vernon's theory accounted for a general factor and group factors, it was seen as a reconciliation between Spearman's two factors theory and Thurstone's primary mental abilities theory.

Further progress was represented by the Cattell and Horn's Gf-Gc theory. Cattell and Horn first propose the two types of intelligence: Gf and Gc (fluid and crystallized intelligence respectively; e.g., Horn & Cattell, 1966). Gf refers to solving problems, which are new to the person. On the contrary, Gc refers to solving problem, which are familiar to the person.

John B. Carroll, taking advantage of the Gf-Gc distinction, theorized a three strata theory. Carroll (1993) reached the conclusion that the structure of intelligence consists of three strata: narrow (first stratum), broad (second stratum) and general (third stratum). This theory is hierarchical, like Vernon's model, and includes Gf and Gc like Cattell-Horn model.

An useful synthesis is represented by the CHC model. The CHC model (Cattell-Horn-Carroll) includes both Cattell-Horn and Carroll models. This approach poses in the second stratum fluid intelligence (Gf), crystallized intelligence (Gc) as well as quantitative reasoning (Gq), reading and writing ability (Grw), short-term memory (Gsm), long-term storage and retrieval (Glr), visual processing (Gv), auditory processing (Ga), processing speed (Gs). Moreover, this model has over seventy abilities in the third stratum and just one abilities in the first stratum (*g*). The CHC model is now considered one of the psychometric model of intelligence. However, this model has some limitations. First, the CHC model does not consider working memory. In fact, the WM criterion studies suggest WM may be a significant causal factor working behind the scenes when complex cognitive performance is required (e.g., Gf or g) (McGrew, 2009). Second, the classical CHC model should be compared to other models like the Cattell–Horn extended Gf–Gc theory, or g-VPR model.

Recently the g-VPR model was proposed (Johnson & Bouchard, 2005 In this model, there are four strata: 1) the *g* factor on the top; 2) three factors just below *g* (i.e.; verbal perceptual and rotation); 3) below the second stratum, a stratum with verbal, scholastic, fluency, number, content memory, perceptual speed, spatial, and image rotation; 4) a broad stratum. This model have received a great interest and fits better compare to the classical CHC model and Vernon's hierarchical model (Johnson & Bouchard, 2005; Johnson, Nijenhuis, & Bouchard, 2008).

2.2.1. Intelligence and geometry

Intelligence and academic achievement are distinct constructs and specific cognitive factors are important to explain specific aspects of achievement. The relationship is also supported by empirical evidence: Studies have found a good correlation between achievement tests and the *g*-factor (Frey & Detterman, 2004). In fact, solving mathematical problems, and in particular geometrical problems, requires problem solving and therefore intelligence.

Geometrical problems typically require to determine a solution to a problem and this capacity is related to higher-order control (Clements & Battista, 1992). For this reason, it is very interesting to study the relationship between WM (and the particular visuospatial domain), intelligence, and achievement in geometry. In fact, intelligence, and in particular visuospatial abilities, are very important in geometrical achievement.

2.3. Aims of the present dissertation

The present dissertations has three important aims, i.e. investigate: i) the relationship between various aspects of geometry and VSWM; ii) whether NLD children presents deficits in various aspects of geometry; iii) the relationship between various aspect of geometry WM and intelligence.

3. Geometry and visuospatial working memory

In this chapter, the relationship between geometry and working memory will be outlined. Two studies will be discussed. The first study describes the relationship between visuospatial working memory (VSWM), intuitive geometry and geometrical achievement in secondary school. The second study, describes the relationship between VSWM, intuitive geometry and geometrical achievement in children with nonverbal disabilities children.

3.1. Study 1: Visuospatial Working Memory in Intuitive and in Academic Achievement Geometry

3.1.1. Study 1 Introduction

Although geometry is one of the main areas of mathematical learning, along with calculation and arithmetical problem-solving, the cognitive processes underlying geometry-related academic achievement have not been studied in detail. The psychological aspects of geometry have received attention from both developmental psychologists (e.g., Piaget, 1960; Piaget & Inhelder, 1967) and educational psychologists (Clements & Battista, 1992; Clements, 2003, 2004; Crowley, 1987; Owens & Outhred, 2006; van Hiele, 1986). As regards the underlying cognitive mechanisms, the involvement of spatial abilities and imagery in geometry has also been analyzed (Bishop, 1980; Brown & Presmeg, 1993; Piaget & Inhelder, 1967) but, to the best of our knowledge, no research has attempted to investigate the role of visuospatial working memory (VSWM) in geometry. The present study tried to fill this gap by examining the involvement of different components of VSWM in the learning of various aspects of geometry.

3.1.1.1. Geometry at school and the intuitive (core and culturally-mediated) principles of geometry

The intuitive knowledge of geometry has been examined in a number of studies. For example, Rosch (1975) showed that, when people in a Stone-Age culture with no explicit education in geometry were asked to choose the "best examples" of a set of shapes (i.e., a group of quadrilaterals and near-quadrilaterals), they usually selected a square and a circle, even when the set contained variants closely resembling them (for instance, the set containing squares also included square-like shapes that were open, or had curved sides, or contained non-right angles), suggesting that people have a preference for closed symmetrical shapes (Bornstein, Ferdinandsen, & Gross, 1981).

In the same vein, Dehaene et al. (2006) devised a test to analyze the intuitive comprehension of certain basic concepts of geometry. Their test was based on a series of arrays of six images, each representing an intuitive concept of geometry: five images fitted the target concept (i.e., they were correct), while one contradicted it. Participants included Amazon Indians and North Americans who were asked, each in their own language, to point to the "ugly" image. The results revealed that:

a. core intuitions of geometry can be identified, since the Amazon Indian group succeeded remarkably well with concepts of topology (e.g., connectedness), Euclidean geometry (e.g., lines, points, parallelism, and right angles) and geometrical figures (e.g., squares, triangles, and circles). Dehaene et al. (2006) consequently considered these concepts as the *core principles* (CP) of geometry;

b. adults who had received no schooling in geometry and young children (from both geographical groups) revealed a similar competence in these CP of geometry, i.e. the Amazonian children's performance did not differ from that of the American children. The American adults performed significantly better in all the tests, however, going to show that cultural differences emerge when it comes to non-core principles of geometry. To be more

> precise, the group of Amazon Indian adults performed poorly (on a level comparable with the North American and Amazonian children) in items assessing geometrical transformations, when participants had to use concepts such as translations, symmetries, and rotations. The authors concluded that all of these items entail a mental transformation from one shape into another and might thus require *culturally-mediated principles* (CMP) of geometry.

Spelke, Lee, and Izard recently suggested (2010) that knowledge of geometry is founded on at least two distinct, core cognitive systems; the first is used to represent the shapes of large-scale navigable surface layouts, the second represents small-scale movable forms and objects. Empirical evidence of this latter system emerged from developmental studies showing that infants are sensitive to variations in angle (Schwartz, Day, & Cohen, 1979; Slater, Mattock, Brown, & Bremner, 1991) and length (Newcombe, Huttenlocher, & Learmonth, 1999). The system for representing small-scale movable forms and objects would therefore capture abstract geometrical information representing the shapes of objects that vary in length and angle, but not direction. The system fails to distinguish a form from its mirror image, for instance, and it reveals qualitative continuities during the course of human development (Izard & Spelke, 2009), as well as across cultures (Dehaene et al., 2006).

In sum, these studies have shown that some aspects of geometry are 'intuitive': (1) primitive (Rosch, 1975), (2) very early developed (Spelke et al., 2010), (3) not dependent by culture and formal instruction (Dehaene et al, 2006). Moreover, Dehaene and colleagues (2006) have shown that is possible to assess experimentally intuitive geometry. Although, they did not explore the relationship between intuitive aspects and other aspects, which are independent from culture or schooling (i.e., working memory or intelligence), or aspects dependent on formal instruction (i.e., achievement in geometry).

3.1.1.2. The organization of VSWM

It has been demonstrated that the VSWM system is not unitary. Many studies (see Logie, 1995) have supported a distinction between the visual and spatial subcomponents of VSWM, the former referring to the recall of shapes and/or textures, the latter to the recall of spatial locations and sequences. An alternative approach-that is less widely acknowledged, but has recently received support-(Cornoldi & Vecchi, 2003; Cornoldi, De Beni, & Mammarella, 2008; Mammarella et al., 2006; Mammarella, Borella, Pastore, & Pazzaglia, 2012; Pazzaglia & Cornoldi, 1999)-distinguishes between visual WM tasks that involve memorizing shapes, textures and colors, spatial sequential tasks requiring the recall of a sequence of spatial locations, and spatial simultaneous tasks demanding the recall of an array of simultaneously-presented locations. It has also been suggested that a distinction should be drawn between many different types of WM process based not only on the format/content of the information, but also on the degree of controlled attention involved. This latter distinction has been described in many ways, e.g. by differentiating between simple storage and complex span tasks (Unsworth & Engle, 2005), or between passive processes (as in simple storage tasks) and active processes (as in complex span tasks) (Cornoldi & Vecchi, 2003), where the former involve retaining information that has not been modified after encoding, while the latter require some transformation and manipulation of the information and presumably correlate more closely with an individual's degree of success in geometrical tasks requiring the manipulation of visual information.

3.1.1.3. Study design

The present study was designed primarily to seek any relationships between VSWM, intuitive geometry, and academic achievement in geometry among secondary school students. Second, we aimed to investigate whether different components of VSWM relate differently to CP and CMP of geometry, as defined by Dehaene et al. (2006). To do so, we administered both the intuitive geometry task (Dehaene et al., 2006) and the MT advanced battery, a standardized test assessing achievement in geometry (Cornoldi, Pra Baldi et al., 2010) devised for secondary school students, which includes items of the type contained in the PISA tests (OECD, 2007). We chose to test secondary school students because the PISA tests are only administered to this age group, and because these students will have presumably nearly completed their learning of the cultural and educational aspects of geometry, since any further education may well contain no geometry (in Italy at least, where this study was carried out).

To assess VSWM, we used three simple storage tasks (one visual, one spatial-sequential, and one spatial-simultaneous) and three complex span tasks. The distinction between simple storage and complex VSWM tasks was particularly crucial for the purposes of this study because performance in geometry is related not simply to maintenance, but also to the manipulation of information; complex span tasks could therefore provide important information, while the contribution of simple storage tasks could prove less relevant.

Our study thus examined the involvement of VSWM in intuitive geometry and sought to ascertain whether both VSWM and intuitive geometry affect academic achievement in geometry. Judging from previous evidence, intuitive geometry concepts can be divided into CP and CMP (Dehaene et al., 2006; Spelke, et al., 2010). We examined whether students' achievement in geometry was supported by both CP and CMP of geometry, as well as by VSWM. We also investigated whether the CMP of geometry (the learning of which is mediated by experience) require the support of VSWM.

The pattern of relationships was examined using path analysis models in successive steps to compare the adequacy of different models in describing the relationships between variables.

3.1.2. Method

3.1.2.1. Participants

The study involved 166 students (125 boys and 41 girls) in their last two years at secondary school (mean age=17.84; SD=.74) in northern Italy. The mean age of participants in 12th grade was

17.35 (SD=.73) and for those in 13th grade it was 18.03 (SD=.65). Participants were attending schools where geometry had an important role, i.e. secondary schools that focused on science or specialized in land surveying, or technical and industrial colleges.

3.1.2.2. Materials and procedures

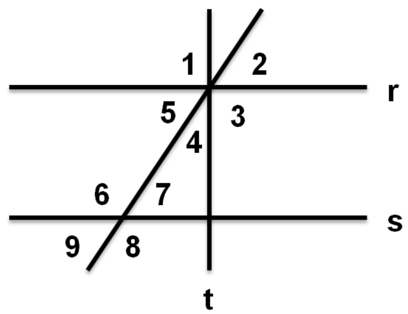
Participants were tested in two phases, i.e. a group session in the classroom lasting approximately 20 minutes, and an individual session approximately one hour long in a quiet room away from the classroom.

During the first phase, we administered a school achievement test (the geometry items in the MT advanced battery) to the whole class (Cornoldi, Pra Baldi et al., 2010). In the second phase, we administered the following tasks on an individual basis: the intuitive geometry task (Dehaene et al., 2006) and six VSWM tasks in the following fixed order: (1) simultaneous dot matrix task; (2) dot matrix task; (3) nonsense shapes task; (4) visual pattern test, active version; (5) sequential dot matrix task; and (6) jigsaw puzzle task.

Measures of geometry

Test on achievement in geometry. The MT advanced geometry task is a paper-and-pencil test that includes the six multiple-choice questions from the MT advanced battery (Cornoldi, Pra Baldi et al., 2010) concerning school-based geometry education. This battery was developed on the basis of the PISA tasks (OECD, 2007) and was designed for use in comparing individual performance with typical school standards in Italy. Participants were asked to solve a series of geometrical problems (see an example in Figure 3.1) and the mean percentage of the correct answers was considered. All the students in the class took about 20 minutes to complete the test.

‘r’ and ‘s’ are parallel lines cut by a perpendicular line ‘t’



Which of the following is false:
(a) The sum of ∠ 3 and ∠ 4 is congruent to ∠ 8;
(b) The sum of ∠ 7 and ∠ 4 is a right angle;
(c) The sum of ∠ 6 and ∠ 5 is a straight angle;
(d) The sum of ∠ 3 and ∠ 4 is congruent to the sum of ∠ 3 and ∠ 5.

Figure 3.1 *Example of the academic achievement task*

Intuitive geometry task. The intuitive geometry task (Dehaene et al., 2006) was programmed using E-Prime 1.1 software, and the items were randomly presented on a computer. Participants were presented with forty-three items split into seven concepts: topology, Euclidean geometry, geometrical figures, symmetrical figures, chiral figures, metric properties, and geometrical transformation. At the beginning of the procedure, a masking screen appeared for 2000 ms before the randomly presented stimuli appeared (see Appendix). Each stimulus remained on the screen until the participant had given a response. The items consisted of an array of six simultaneously-presented images, five of which instantiated a given concept, while one image violated it. For each item, participants were asked to identify the odd one out (which appeared in a random position among the other five images).

Three different scores were calculated: one was the total mean percentage of correct responses; the second (as in Dehaene et al, 2006) was a score representing the CP of geometry (i.e. the mean percentages of correct answers for images relating to topology, Euclidean geometry, and geometrical figures, for a total of 21 items); and the third was a score representing the CMP of geometry (i.e., the mean percentages of correct answers for images relating to symmetrical figures, chiral figures, metric properties, and geometrical transformation, for a total of 22 items). Figure 3.2 shows some examples of the concepts presented.

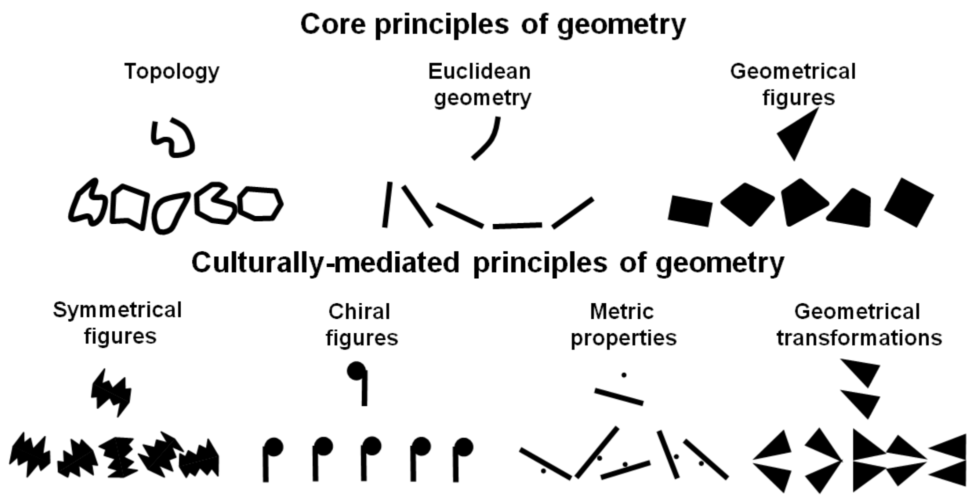


Figure 3.2 *Examples of the intuitive geometry task*

VSWM measures

Participants were presented with six tests (4 computerized, 2 paper-and-pencil); five of them are part of an Italian standardized VSWM test battery (Mammarella, Toso, Pazzaglia, & Cornoldi, 2008), while the dot matrix test was derived from Miyake, Friedman, Rettinger, Shah, and Hegarty (2001). Three tests were passive, simple storage tasks, and three were active, complex span tasks. The simple storage tasks were classifiable as visual (the nonsense shapes task), spatial-sequential (the sequential dot matrix task), or spatial-simultaneous (the simultaneous dot matrix task) (Mammarella, Pazzaglia, & Cornoldi, 2008; Pazzaglia & Cornoldi, 1999). The complex span tasks were the *jigsaw puzzle task (*adapted from Vecchi & Richardson, 2000), the dot matrix task (drawn from Miyake et al., 2001), and active version of the visual pattern test (VPTA, derived from Della Sala, Gray, Baddeley, & Wilson, 1997). Examples of these materials are shown in Figure 3.3.

The six tests were administered adopting a self-terminating procedure (starting with the easiest, the tests became increasingly complex and participants continued as long as they were able to solve at least two of three problems for a given level). For scoring purposes, items on the second level of difficulty scored 2, on the third level they scored 3, and so on. The final scores corresponded to the sum of the last three correct responses. For instance, a participant who solved two problems on the fourth level and one on the fifth scored 4+4+5 = 13 (see Mammarella, Toso et al., 2008; Mammarella, Lucangeli, & Cornoldi, 2010). Before administering each task, participants were given two practice trials with feedback. The tests were administered during a single individual session in a quiet room at the students' school.

For the simple storage tasks, participants had to decide whether a set of figures/locations was the same as, or different from a previously-presented set: after a first stimulus had been shown, either the same stimulus or one in which just one element had changed appeared, followed by a screen containing two letters, U (*uguale*=same) and D (*diverso*=different), and participants responded by pressing one of the two keys on the keyboard. The complex span tasks involved not only recognizing but also processing the information presented.

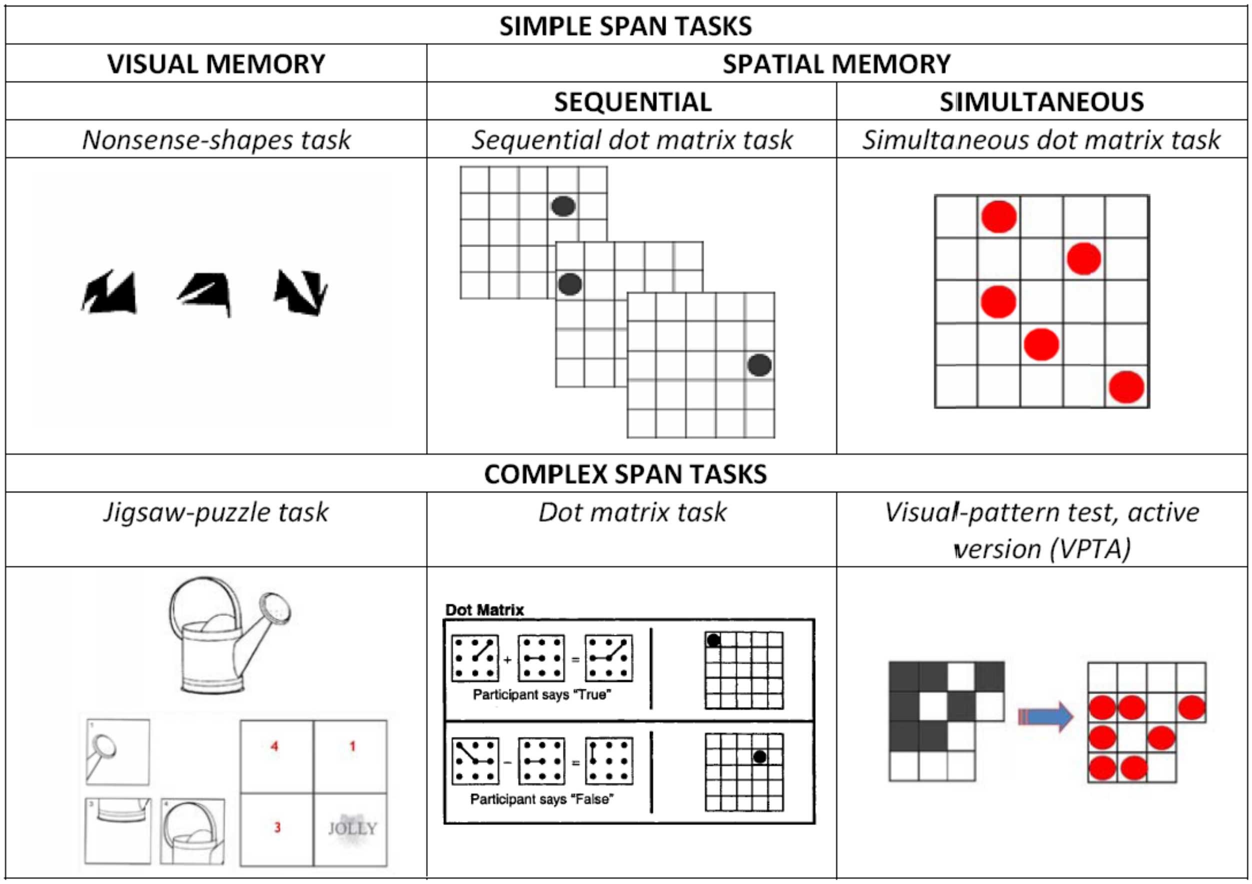


Figure 3.3 *Examples of the materials used to assess visuospatial working memory.*

3.1.3. Results

Descriptive statistics for each test are presented in Table 3.1. The scores are expressed as the percentages of correct responses for geometrical measures, while for VSWM they are given by the sum of the three highest levels of difficulty reached by the subject. Table 3.1 also shows the test reliabilities.

Table 3.1

Descriptive statistics and reliability

	Tasks	Reliability	M	SD	Skewness	Kurtosis
Geometry	MT advanced geometry task*	.66	66.77	21.98	-.41	-.53
	Intuitive geometry task	.65	86.41	7.32	-.71	.46
Simple storage tasks	Nonsense shapes	.89	13.55	6.00	-.08	-.74
	Sequential dot matrix	.91	18.75	4.51	-1.12	1.77
	Simultaneous dot matrix	.90	21.20	4.80	-1,74	2.02
Complex span tasks	Jigsaw puzzle	.84	26.65	4.20	-.90	-.88
	Dot matrix task	.79	10.49	1.79	-2.02	4.49
	VPTA	.89	24.66	4.64	-.68	-.33

Note. * Dependent variable in percentage

3.1.3.1. Model estimation.

Path analysis models were computed with the LISREL 8.8 statistical package (Jöreskog & Sörbom, 1993). We used the fit indices recommended by Jöreskog and Sörbom (1993), such as the root-mean-square error of approximation (*RMSEA*), the non-normed fit index (*NNFI*), and the comparative fit index (*CFI*). Like Schreiber, Stage, King, Nora, and Barlow (2006; see also Schermelleh-Engel, Moosbrugger, & Müller, 2003), we considered substantively interpretive models with a non-significant chi-square, an *RMSEA* below .05, an *NNFI* above .97, and a *CFI* above .97 as a good fit.

3.1.3.2. Preliminary analysis.

Possible differences related to gender and school year were measured: for the former, only the effect of the dot matrix task ($F[1,164]=8.93$, $p=.003$, $\eta_p^2=.05$) was significant (males did better than females); for the latter, only the effects of the MT advanced geometry task ($F[1,164]=11.46$, $p=.001$, $\eta_p^2=.65$), and of the nonsense shapes ($F[1,164]=5.81$, $p=.017$, $\eta_p^2=.03$) were significant (13th graders performed better than 12th graders in both cases).

3.1.3.3. Path analysis

Normality was taken into consideration. Mardia's measure of relative multivariate kurtosis (MK) was obtained using PRELIS (Jöreskog & Sörbom, 1993). The MK was 1.09, which implies a non-significant departure from normality ($-1.96 < z < 1.96$; Mardia, 1970).

For the purposes of our analysis, we considered the VSWM tasks as independent variables and the geometry achievement test (the MT advanced geometry task) as a dependent variable. We sought the best model first (models 1 to 4), considering only the total score for the intuitive geometry task as the mediator variable, then (models 5 and 6) we distinguished between the CP and CMP of geometry (as in Dehaene et al., 2006).

Table3.2

Correlation matrix for all measures

Variables	1	2	3	4	5	6	7	8	9	10
Achievement in geometry										
1. MT advanced geometry task	1									
Intuitive geometry										
2. Intuitive geometry	.35**	1								
3. Core principles of geometry	.24**	.54**	1							
4. Culturally-mediated principles of geometry	.32**	.96**	.30**	1						
Simple storage tasks										
5. Nonsense shapes	.08	.13	.01	.15	1					
6. Sequential dot matrix	.07	.19*	.00	.22**	.06	1				
7. Simultaneous dot matrix	.08	.13	.10	.12	.17*	.11	1			
Complex span tasks										
8. Jigsaw puzzle	.22**	.26**	.10	.27**	.10	.08	.10	1		
9. Dot matrix	.09	.17*	.10	.16*	.12	.17*	.04	.09	1	
10. VPTA	.14	.11	.11	.09	.03	.08	.14	.33**	0.16*	1

Note. * $p < .05$, ** $p < .01$.

We began our analysis by assessing the full model involving all the variables. Then we gradually deleted some of the variables, taking their weight and our hypotheses into account. The initial model thus involved the nonsense shapes, sequential dot matrix, simultaneous dot matrix, jigsaw puzzle and dot matrix tasks, and the VPTA tests as independent variables. The total score for the intuitive geometry task served as the mediator and the MT geometry achievement task as the dependent variable.

Path model 1 was saturated. The fit was completely adequate (Table 3.3, 3.4; Figure 3.4).

In Path model 2, we deleted the direct effects of nonsense shapes, sequential dot matrix, simultaneous dot matrix, dot matrix tasks and VPTA on MT geometry achievement, since the relationships between these variables and MT geometry achievement were not significant. The fit indices of the model were perfect (Table 3.3), but the relationships between the nonsense shapes, simultaneous dot matrix and VPTA variables, and the intuitive geometry task were not significant (Table 3.4).

In Path model 3, the nonsense shapes, simultaneous dot matrix and VPTA were deleted. The fit indices of the model were perfect (Table 3.3).

In Path model 4, the dot matrix task and the non-significant correlation between the sequential dot matrix and jigsaw puzzle were deleted (Figure 3.4, Table 3.4). In this model, the sequential dot matrix and jigsaw puzzle, in conjunction with the mediation of the intuitive geometry task, predicted the MT geometry achievement; the intuitive geometry task and the jigsaw puzzle directly predicted MT geometry achievement. The resulting fit indices were excellent (Table 3.3). This model explained 14% of the MT geometry achievement variance. In Path model 4b, we attempted to delete the direct effect of the jigsaw puzzle on the MT geometry achievement task, but the fit indices became worse (Table 3.3). Since the model 4b was nested in the model 4a, we calculated the chi-square difference between the two models, $\chi^2_D(1)=4.11$, $p=.043$ (right tail), finding the fit of the model 4a statistically better than that the model 4b. We therefore opted for the Path model 4a.

In Path model 5a, CP and CMP of geometry were introduced as separate mediator variables (instead of single mediator variables of intuitive geometry). Based on the fit indices, this model was unacceptable (Table 3.3). In Path model 5b, we introduced a direct path from CP to CMP of geometry and the fit indices improved significantly (Table 3.3), but the path from VSWM to CP, and the direct effect of CP on the MT advanced geometry task were poor.

In Path model 6a, we considered CP as an independent variable, and we also deleted the non-significant correlations between the independent variables (Figure 3.6). In this model, the CP, the sequential dot matrix, and the jigsaw puzzle, with the mediation of CMP of geometry, were able to predict MT geometry achievement; CP and CMP of geometry, and the jigsaw puzzle task also directly predicted MT geometry achievement (Table 3.4). The fit indices were very good (Table 3.3). This model explained 14% of the variance for the MT geometry achievement task. In Path model 6b, we attempted to delete the direct effect of the jigsaw puzzle task on the MT-advanced geometry task, and the fit indices were good. Nevertheless, since model 6b was nested in model 6b, we also calculated the chi-square difference between the two models; the chi-square was significant (χ^2_D[1]=4.05, p=.044 [right tail]), showing that the fit for the model 6a was statistically better than for the model 6b. In Path model 6c, we tested a model including CP with a path on the sequential dot matrix and the jigsaw puzzle task with a path on the CMP of geometry with a path on the MT advanced geometry task, but the model did not converge. We consequently selected the Path model 6a.

Table 3.3

Values of selected fit statistics for path models

Model	χ_M^2	df_M	p	RMSEA	LL	UL	NNFI	CFI
1	0	0	1	0	0	0	1	1
2	1.19	5	.95	0	0	.01	1.26	1
3	0.15	2	.93	0	0	.01	1.21	1
4a	0.005	2	1	0	0	0	1	1
4b	4.12	3	.25	.04	0	.14	.95	.97
5a	14.76	3	.002	.15	.08	.23	.37	.81
5b	0.001	2	1	0	0	0	1	1
6a	0.002	4	1	0	0	0	1	1
6b	4.05	5	.54	0	0	.09	1.03	1

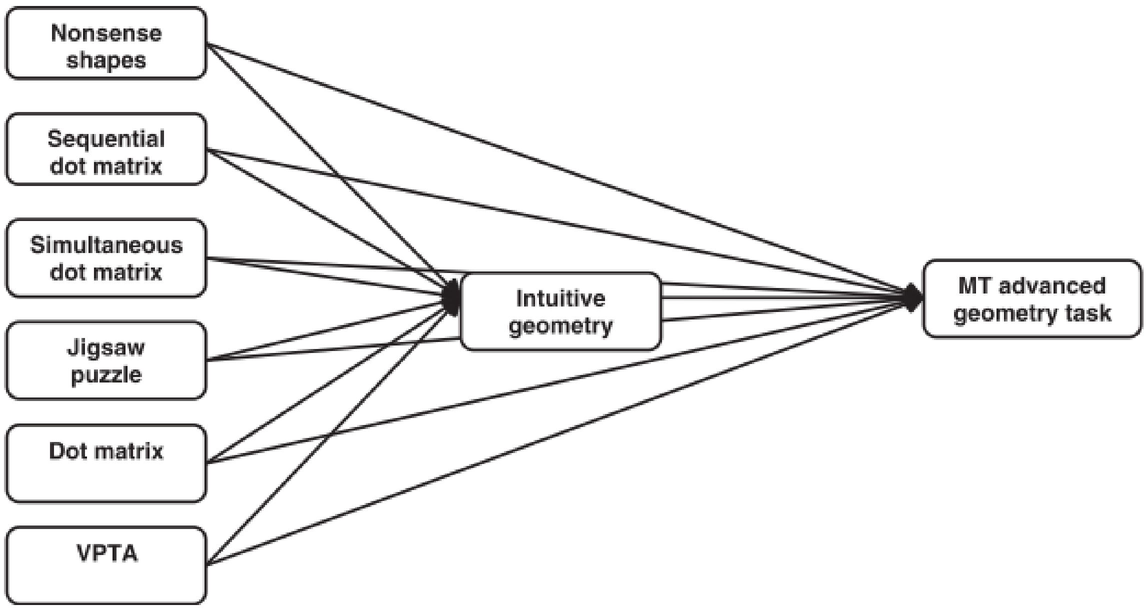


Figure 3.4 *Conceptual diagram of path model 1*

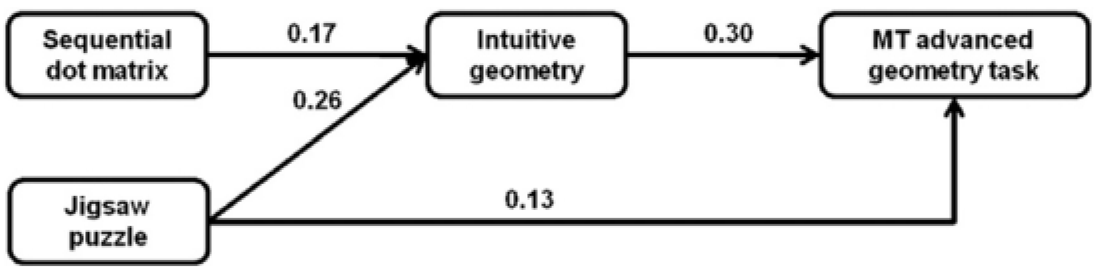


Figure 3.5 *Standardized solution of path model 4a.*

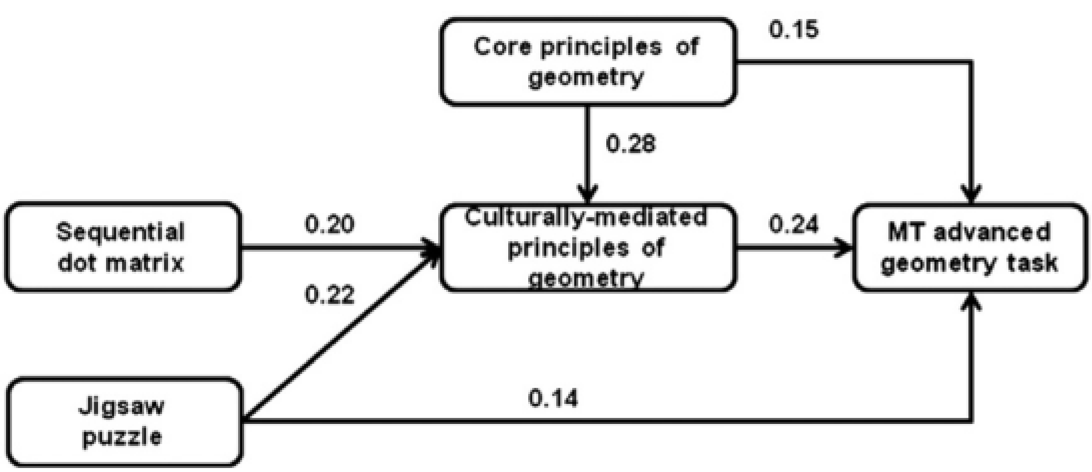


Figure 3.6 *Standardized solution of path model 6a.*

Table 3.4

Direct and indirect effects predicting academic achievement in geometry, and regression weight.

Dependent variable	Independent variable	Direct effect		Indirect effect		Total
	Path model 1	*B*	*Z*	*β*	*Z*	*R²*
MT advanced geometry task	Intuitive geometry	.31	3.93**			.15
	Nonsense shapes	.02	.33	.02	.96	
	Sequential dot matrix	-.01	-.17	.04	1.70*	
	Simultaneous dot matrix	.02	.23	.02	1.02	
	Jigsaw puzzle	.11	1.36	.07	2.33**	
	Dot matrix	.01	.18	.03	1.39	
	VPTA	.07	.89	.00	-0.10	
	Path model 2	*B*	*Z*	*β*	*Z*	*R²*
MT advanced geometry task	Intuitive geometry	.31	4.10**			.14
	Nonsense shapes			.02	.96	
	Sequential dot matrix			.04	1.71*	
	Simultaneous dot matrix			.03	1.03	
	Jigsaw puzzle	.13	1.76*	.07	2.37**	
	Dot matrix			.04	1.39	
	VPTA			.00	-0.10	
	Path model 3	*B*	*Z*	*B*	*Z*	*R²*
MT advanced geometry task	Intuitive geometry	.31	4.16**			.14
	Sequential dot matrix			.05	1.99**	
	Jigsaw puzzle	.13	1.78*	.08	2.54**	
	Dot matrix			.03	1.26	
	Path model 4a	*B*	*Z*	*B*	*Z*	*R²*
MT advanced geometry task	Intuitive geometry	.31	4.17**			.14
	Sequential dot matrix			.05	2.03**	
	Jigsaw puzzle	.13	1.79*	.08	2.61**	
	Path model 5a	*B*	*Z*	*B*	*Z*	*R²*
MT advanced geometry task	Core principles of geometry	.15	2.06**			.13
	Culturally-mediated principles of geometry	.24	3.20**			
	Sequential dot matrix			.05	1.79*	
	Jigsaw puzzle	.14	1.83*	.08	2.57**	
	Path model 6a	*B*	*Z*	*β*	*Z*	*R²*
MT advanced geometry task	Core principles of geometry	.15	1.97**	.07	2.41**	.14
	Culturally-mediated principles of geometry	.24	3.06**			
	Sequential dot matrix			.05	2.09**	
	Jigsaw puzzle	.14	1.83*	.05	2.18**	

Note. * p <.05; ** p<.01 (one tail)

3.1.4. Discussion

In this study, we investigated the relationships between VSWM, intuitive geometry and academic achievement in geometry in secondary school students.

In particular, we expected to find a relationship between VSWM and intuitive geometry, and we hypothesized that both intuitive geometry and VSWM could predict academic achievement in geometry. To investigate these issues, the total score obtained in the intuitive geometry task devised by Dehaene et al. (2006) was used as a mediator variable. The final path model showed that only two of the six VSWM tasks considered were significantly related to the intuitive geometry task, namely a complex span task (jigsaw puzzle) and a simple storage task assessing spatial-sequential memory (sequential dot matrix). Only the jigsaw puzzle task related directly to academic achievement in geometry (i.e., the score in the MT advanced geometry task), whereas the sequential dot matrix task indirectly predicted academic achievement in geometry.

Our second hypothesis, based on the distinction made by Dehaene et al (2006) between the CP and CMP of geometry, was that VSWM could be more implicated in the acquired principles than in the CP of geometry, while both these aspects of intuitive geometry would be related to academic achievement in geometry. In the final path model, only the jigsaw puzzle task directly predicted academic achievement in geometry. More specifically, the VSWM tasks only related to CMP of geometry, while none of them related to the CP of geometry. Both the core principles and the culturally-mediated principles of geometry were related to academic achievement in geometry, but the latter CMP attributes had a stronger (β=.24) relationship with academic achievement than the CP of geometry (β=.15). Although the total variance in academic achievement in geometry explained by the model was not particularly high (producing a result consistent with the observation that many other variables can influence achievement in geometry; Aydin & Ubuz, 2010), the final model showed a very good fit and provided a picture of the relationship between VSWM, intuitive geometrical concepts, and academic achievement in geometry that is plausible and consistent with our predictions. Our results confirm the existence of a relationship between VSWM and geometry, but introduce the novel finding that this relationship is not involved in all the tasks. Some VSWM tests did not correlate significantly with performance in geometry, showing for example that the ability to retain a shape or a pattern of locations is not crucial to success in geometrical tasks. The most powerful VSWM test for predicting performance in intuitive geometry tasks and academic achievement in geometry was the jigsaw puzzle, which requires that participants not only memorize but also manipulate visual information (Cornoldi & Vecchi, 2003). Its relationship with the CMP of geometry can be explained by the finding that the items used in the study by Dehaene et al (2006) in which the Amazon Indian adults failed involved geometrical transformations, with participants having to rotate, translate, or mentally manipulate one shape to convert it into another. In a more recent study comparing adults with children 4-10 years old, Izard and Spelke (2009) demonstrated that it is only after adolescence that young people are able to detect directional relationships, a skill requiring discrimination of mirror and rotated images.

The jigsaw puzzle task not only supported CMP of geometry, but was also directly related to academic achievement in geometry. It is worth noting that the task we used to test academic achievement in geometry included items in which participants had to remember theorems or geometrical rules, as well as visualizing and manipulating visuospatial information to solve the geometrical problems. In contrast with the other two complex span tasks, which involved manipulating spatial locations, the jigsaw puzzle task seems the most suitable for representing operations that are also required in the task for testing academic achievement in geometry.

The second VSWM task entered in the final path model was the sequential dot matrix task, which is believed to assess passive spatial-sequential processes (Cornoldi & Vecchi, 2003). It involves recognizing increasing numbers of locations presented one after the other. This task did not directly predict academic achievement in geometry, but it did appear to be related to the CMP of geometry. The specific contribution of the test to the CMP of geometry could be due to the geometrical requirement involved in memorizing the exact sequence of successive visuospatial operations.

It is worth noting that none of the VSWM tasks was related to the CP of geometry. This may be because the CP of geometry need no support from VSWM. The CP of geometry could develop without any need for either experience or other underlying cognitive structures, as in the case of other aspects of mathematics (Spelke & Kinzler, 2007; Spelke, 2004).

A number of crucial issues would need to be considered in future research. For a start, only VSWM tasks were administered to the participants in our study, based on the assumption that VSWM processes might be stronger predictors of achievement in geometry than verbal WM processes. Further research should consider the role of verbal WM, however, given that formal education in geometry involves using verbal rules, formulas, theorems, and so on), as well as numerous other factors that presumably affect the acquisition of geometrical knowledge (Aydin & Ubuz, 2010), as indirectly demonstrated by the limited percentage of variance explained by our path models. In addition to VSWM, further studies should analyze the role of visuospatial abilities, such as spatial visualization and mental rotation skills in academic achievement in geometry. Finally, reasoning and fluid intelligence may also have a central role in accounting for a part of the variance affecting the acquisition of geometry. Second, our findings might be explained by our sample selection procedures and consequent choice of task for assessing academic achievement in geometry. As previously mentioned, we chose secondary school students because they have received the highest level of compulsory schooling in geometry, and we were thus able to study the role of both CP and CMP of geometry. It would be reasonable to expect different results when testing young children, for instance, when their cultural background and schooling would have a lower weight. Our students were also attending schools where geometry had an important role, so our findings cannot be generally applied to pupils at different types of school. Because the types of secondary school that we considered are attended mainly by boys, our sample also contained more males than females, though the only significant effect of gender was found in the dot matrix task, which was not included in our final path models. This aspect may nonetheless be a limitation of our study.

Finally, our findings also have educational and clinical implications. First of all, they can provide teachers and educators with information on which cognitive processes support students learning geometry. To give an example, knowing that complex VSWM tasks can directly predict academic achievement in geometry could help teachers to suggest activities that do not overshadow their students' VSWM capacity. Secondly, shedding light on the mechanisms influencing academic achievement could help us to understand why some students fail in geometry and how we can help them to cope with their difficulties. Thirdly, assessing visuospatial abilities in general, and VSWM in particular, could make it easier to identify children who might meet with difficulties in learning geometry later on. Consistently with these observations, research is underway to examine the cognitive deficits underlying difficulties in learning geometry. In particular, Mammarella, Giofrè, Ferrara and Cornoldi (2012) found that young children with poor visuospatial skills failed in both intuitive geometry and VSWM tasks; and Hannafin, Truxau, Vermillion and Liu (2008)found that students with weak spatial abilities performed worse than students with strong spatial abilities in terms of their academic achievement in geometry.

In conclusion, our study shows that the academic achievement in geometry of secondary school students can be predicted: (1) indirectly by VSWM tasks, which support CMP of geometry; (2) directly by a complex VSWM task (the jigsaw puzzle task); and (3) by CP and CMP of geometry, the latter showing a stronger relationship with academic achievement than the former.

3.2. Study 2. Intuitive geometry and visuospatial working memory in children showing symptoms of nonverbal learning disabilities.

3.2.1. Introduction

Geometry has long been considered one of the most important forms of mathematical knowledge. Although geometry is included in all the mathematical curricula in the world, in ternational assessments report that many students present difficulties in learning geometry (OECD, 2007). Despite this, a cognitive profile of students with difficulties in learning geometry has never been studied. The present study offers a first contribution in this direction by examining a particular group of learning-disabled children who were hypothesized to present difficulties in geometry.

Learning disabilities occur in approximately 5% of school-age children (Lyon, 1996), and two major subtypes of learning disabilities have been described: children with linguistic disabilities and children with nonverbal learning disabilities (NLD) (Drummond, Ahmad, & Rourke, 2005). A nonverbal learning disability refers to a difficulty in processing visuospatial information or other types of nonverbal information. NLD was also described and labelled either as a dysfunction of the right hemisphere (Gross-Tsur, Shalev, Manor, & Amir, 1995; Nichelli & Venneri, 1995; Weintraub & Mesulam, 1983) or as a visuospatial learning disability (Cornoldi, Venneri, Marconato, Molin, & Montinari, 2003; Mammarella & Cornoldi, 2005a; 2005b). However, the formulation of an inclusive set of characteristics and classifications is still under debate (Roman, 1998; Spreen, 2011). Rourke (1989, 1995) elaborated a model on the NLD syndrome with extensive studies that demonstrated a specific pattern of neuropsychological assets and deficits. Briefly, this pattern includes bilateral tactile-perceptual and coordination deficits, substantially deficient visuospatial abilities, deficits in novel problem solving and concept formation, poor arithmetic skills (Harnadek & Rourke, 1994; Mammarella et al., 2010), and strong word reading with poor reading comprehension of spatial descriptions (Mammarella et al., 2009). To this list, behavioural descriptions frequently added deficient social perception and judgment, interaction verbosity of a repetitive nature, and problems in adapting to novel situations (Rourke & Tsatsanis, 2000).

Although different criteria are used for diagnosing children with NLD, there is a general agreement (Solodow et al., 2006) that the main symptoms of NLD are the presence of a discrepancy between verbal and nonverbal IQ, visuospatial and motor coordination impairments, and school difficulties, in particular in the mathematical area. In a cross-country study comparing the characteristics of British and Italian children who had received a diagnosis of NLD, Cornoldi and collaborators (Cornoldi et al., 2003) validated a rapid screening measure for teacher identification of children with NLD symptoms, focusing on their visuospatial difficulties. Mathematical difficulties were considered as a typically associated symptom, but not as a defining feature of NLD.

Another critical factor underlying the difficulties encountered by children with NLD seems to be related to visuospatial working memory (VSWM) deficits (Cornoldi, Dalla Vecchia, Tressoldi, & Vecchia, 1995; Cornoldi, Rigoni, Tressoldi, & Vio, 1999; Mammarella & Cornoldi, 2005a, 2005b). According to Logie (1995), VSWM is a specific working memory component, responsible for the maintenance and processing of visual (e.g., colour, shape, texture) and spatial (e.g., position of an object in space) information. VSWM has been specifically explored in children with NLD, and evidence showed that they are impaired in both simple storage (i.e., passive) and complex-span (i.e., active) tasks, but that different NLD children may present different specific weaknesses. Simple storage tasks refer to the retention of information that has not been modified after encoding, while complex-span tasks require transformation and manipulation of stored information. Regarding simple storage tasks, for example, Mammarella et al. (2006) observed, in a group of NLD children, a double dissociation between spatial-simultaneous tasks (those requiring them to recall spatial locations presented at the same time) and spatial-sequential tasks (those requiring them to recall spatial locations presented one after the other). Furthermore, a specific analysis of two NLD cases (Cornoldi, Rigoni, Venneri, & Vecchi, 2000) offered evidence in favour of the dissociation between simple storage and complex-span tasks in VSWM. However, a series of studies revealed that children with NLD are usually more impaired on complex-span tasks requiring an active manipulation of stored information than on simple storage tasks (see, for example, Cornoldi et al., 1995, 1999). In sum, these results show that: (a) it is important to consider VSWM, in its different components, for identifying different subtypes of NLD, and (b) VSWM deficits might explain why NLD children fail in a range of activities (e.g., mathematics, drawing, spatial orientation, geometry, etc.) assumed to involve VSWM.

Concerning the failures of NLD children in mathematical tasks, deep attention has been dedicated only to the case of calculation. For example, considering the relationship between VSWM and arithmetic in children with NLD, Venneri, Cornoldi, and Garuti (2003) compared children with NLD to controls in arithmetic calculations. Their results revealed that the group with NLD had more severe difficulties with written calculation, especially when borrowing and/or carrying were involved. The authors hypothesized that NLD children do not have a generalized problem with calculation *per se*; instead, their problems derive from dealing with specific processes, including VSWM, which governs calculation. In a further study, Mammarella et al. (2010) found that children with NLD performed significantly worse than did children with typical development in VSWM tasks and in arithmetic tasks associated with visuospatial processes, as, for example, carrying errors, partial calculation errors, and column confusions. Moreover, their results confirmed that an arithmetic difficulty may be associated with NLD, but also suggested that a VSWM difficulty may be primary in NLD. In fact, using VSWM tasks as covariates, differences in arithmetic skills disappeared, and a discriminant analysis showed that a VSWM task—and not arithmetic performances—was able to contribute to identification of NLD children.

It is worth noting that in the psychology literature, the role of VSWM in arithmetic is still controversial: some studies have failed to find evidence for a role of VSWM components in mental calculation (Logie, Gilhooly, & Wynn, 1994; Noël, Désert, Aubrun, & Seron, 2001), but others have demonstrated the involvement of VSWM in arithmetic (Bull et al., 2008; DeStefano & LeFevre, 2004; Holmes & Adams, 2006; Trbovich & LeFevre, 2003). The involvement of visuospatial abilities and VSWM in mathematics could be even greater in geometry than in calculation, and it could be emphasized in the case of NLD children. In fact, geometry requires by definition the treatment of spatial information of two- and three-dimensional patterns. However, to our knowledge, there are no systematic research studies analysing the relationship between geometry and VSWM in general, and in particular, in children with NLD. Furthermore, evidence is necessary to support the hypothesis that VSWM is critical in learning geometry. In fact, it has been suggested that success in geometrical tasks is not so critically related to spatial abilities as one would intuitively predict, as many other factors may be crucial, including verbalization, abstract reasoning, metacognition, motivation, and others (Aydin & Ubuz, 2010).

An important point to be considered, when examining the relationship between geometry and underlying cognitive processes, is that geometry is a broad area with many facets. In fact, geometrical competence can involve both intuitive concepts, as well as aspects more associated with schooling. The concept of *intuitive geometry* has been recently introduced by Dehaene and collaborators (Dehaene et al., 2006). These authors investigated whether some principles of geometry can be considered as core culture-free concepts (see also Spelke et al., 2010) by examining the spontaneous geometrical knowledge of an Amazonian group that was not exposed to a geometrical instruction. Dehaene and colleagues (2006) hypothesised that people might possess primitive principles of geometry, similar to the case for numerical knowledge. In fact, in the numerical field, a growing number of studies have shown that infants seem to respond to the numerical properties of their visual world without the benefit of language acquisition (Koechlin, Naccache, Block, & Dehaene, 1999; Starkey & Cooper, 1980; Starkey, Spelke, & Gelman, 1990; Xu & Spelke, 2000). To look at the particular case of geometry, Dehaene et al. (2006) compared Amazonian indigenes and American children/young adults in intuitive knowledge of geometry, and their results revealed that the Amazonian group succeeded remarkably well with the intuitive concepts of topology (e.g., connectedness), Euclidean geometry (e.g., line, point, parallelism, and right-angle), and geometrical figures (e.g., square, triangle, and circle). Dehaene and colleagues (2006) consequently considered these concepts as primitive core concepts of geometry. Furthermore, they found that the Amazonian adult group performed poorly in items assessing geometrical transformations, in which subjects have to use concepts like translations, symmetries, rotations. The authors concluded that these items all imply a mental transformation of one shape to another, and they might require culturally mediated, non-innate concepts of geometry. The first study showed that VSWM has a critical role in intuitive geometry and that both VSWM and intuitive geometry contribute to academic achievement in geometry. It is also suggested that VSWM is more critical in supporting the acquisition of culturally mediated concepts of geometry than in the acquisition of the primitive core ones.

The present study was devoted to exploring the geometrical competencies and the role of VSWM in children displaying some symptom of NLD who were hypothesized to encounter difficulties in geometry. It should be noted that our NLD group had not received a clinical diagnosis, but was identified through a school screening. In particular, our NLD group displayed the most typical symptoms of NLD, both reported by their teachers through the Short Visuospatial (SVS) Questionnaire (Cornoldi et al., 2003), and recognised through two subtests (i.e., one spatial and one verbal) of the Primary Mental Ability (PMA) battery (Thurstone & Thurstone, 1963). Our screening did not use mathematical difficulties as a criterion for identifying children with NLD, thus avoiding the risk of a circularity of looking for mathematical difficulties in children identified on the basis of a mathematical difficulty.

The main aims of the current study were as follows: first, we wanted to examine whether the mathematical difficulties of NLD children could be extended to the case of intuitive geometry; second, we looked for further support for the presence of VSWM deficits of NLD children, mainly in complex-span tasks; and third, we examined whether the hypothesized VSWM deficits are critical in explaining group differences in intuitive geometry, thus offering indirect evidence for the assumption that skill in geometry is supported by VSWM.

To reach these aims, in the present study, different measures of VSWM and intuitive geometry were administered to the group of NLD children and to a control group matched for verbal general abilities, age, gender, and socioeconomic level. For VSWM, three simple storage tasks (visual, spatial-sequential, spatial-simultaneous) and three complex-span tasks were used. The simple storage tasks were selected on the basis of the Cornoldi and Vecchi model (2003), distinguishing between visual, spatial-sequential, and spatial-simultaneous. The complex-span tasks were selected from the literature and chosen to ensure a variety of task types, mapping different processes. To examine geometry, we administered the intuitive geometry task (Dehaene et al., 2006). As already mentioned, this test involves trials assessing different concepts of geometry. This articulation offered the possibility of individuating the aspects where the specific visuospatial deficit of NLD could have a greater impact and, in contrast, the aspects where abstract reasoning, supported by language, could compensate for the visuospatial deficit.

3.2.2. Method

3.2.2.1. Participants

The initial screening involved a sample of 278 children (143 M, 135 F) aged 11 to 13 years (mean=149.69 months; SD=10.61), with 99 children from sixth grade, 100 from seventh grade, and 79 from eighth grade.

The identification of NLD children and the control group (CG) was carried out on the basis of difficulties detected by their teachers through the SVS Questionnaire (Cornoldi et al., 2003). General verbal and visuospatial abilities were evaluated using the Verbal Meaning and Spatial Relations subtests of the Primary Mental Ability Test (PMA, Thurstone & Thurstone, 1963), respectively. For all children, parental consent was obtained prior to testing. Children referred to as having a very low socioeconomic level were not included in the groups.

The inclusion criteria of NLD children were the following: (a) visuospatial scores on the SVS Questionnaire lower than 20th percentile; (b) scores lower than 2 SD in the Spatial Relations subtest of the PMA; and (c) average scores in the Verbal Meaning subtest of the PMA. In contrast, the inclusion criteria of the CG group were the following: (a) scores equal to or higher than 50th percentile in the visuospatial score of the SVS Questionnaire; and (b) average performance in both PMA subtests (Spatial Relations and Verbal Meaning).

Our sample was composed of 16 NLD children (9 M and 7 F), sixth-, seventh-, and eighth-graders, aged between 11 and 13, and 16 control group (CG) children (9 M and 7 F) matched for age, schooling, gender, PMA Verbal Meaning subtest scores, and socioeconomic level as assessed by their teachers (Table 3.5).

Table3.5

Characteristics of children showing symptoms of NLD and controls (CG): mean, standard deviations (SD), confidence intervals (CI 95%), and statistical analyses.

	NLD (N=16)			**CG (N=16)**			**Statistical Analyses**		
	Mean (SD)	CI 95%		Mean (SD)	CI 95%		*F(1,30)*	*p*	η^2
		I.L.	S.L.		I.L.	S.L.			
Age (months**)**	146.25 (9.52)	141.18	152.32	146.06 (9.21)	141.15	150.97	.003	.955	.0001
Svs Questionnaire									
Visuospatial score	22.81 (2.88)	21.28	24.35	33.38 (3.34)	31.59	35.16	91.64	.001	.75
PMA									
Verbal meaning	13.81 (4.92)	11.19	16.43	13.88 (2.78)	12.39	15.36	.002	.965	.0001
Spatial relations	2.75 (2.79)	1.26	4.24	15.06 (3.99)	12.94	17.19	102.22	.001	.77

As shown in Table 3.5, the two groups did not differ significantly in terms of age or scores on the PMA Verbal Meaning subtest. As expected, the differences between groups were significant on the visuospatial scores of the SVS Questionnaire and on the PMA Spatial Relations subtest. Each child's socioeconomic level was estimated by teachers using a four-point scale (1=high socioeconomic level; 2=medium-high; 3=medium-low; 4=very low), and the two groups did not differ in this estimation (*U* Mann-Whitney p=.37).

3.2.2.2. Materials and Procedure

Participants were tested in an individual session lasting approximately one hour in a quiet room outside the classroom. In order to avoid biasing of performance in any test through effects of practice or fatigue, test presentation order was counterbalanced according to a randomized Latin square. Children were presented with six VSWM tasks and the intuitive geometry task (Dehaene et al., 2006).

Visuospatial working memory tasks. Participants were presented with six tasks (four computerized, two paper-and-pencil). Five of them were part of an Italian standardized VSWM test battery (Mammarella, Toso, et al., 2008), while the dot matrix test was derived from Miyake, Friedman, Rettinger, Shah, and Hegarty (2001). Three tests were simple storage tasks (i.e., passive), while three were complex-span tasks (i.e., active). Moreover, the simple storage tasks were distinguished as visual, spatial-sequential, and spatial-simultaneous (Cornoldi & Vecchi, 2003; Mammarella, Pazzaglia, et al., 2008; Pazzaglia & Cornoldi, 1999).

The six tests used a self-terminating procedure: they were administered starting with the simplest series and rose in complexity, and participants continued as long as they were able to solve at least two items out of three at a given level. For scoring, we used the absolute scoring method, because is the method predominantly used in child WM research (Hornung, Brunner, Reuter, & Martin, 2011). Moreover, items at the second level had a value of 2, at the third level a value of 3, and so on; final scores were the sum of the values for the three final correct responses (for example, if a participant successfully solved two items at the fourth level and one at the fifth, then the score was 4+4+5=13). Before administering each task, participants were given two practice trials with feedback.

Simple storage tasks. In simple storage tasks, participants had to decide whether a series of figures/locations were the same as or different from the one previously presented: following a first stimulus presentation, either the same stimulus or one with a change of just one element was presented. This was followed by a response screen containing two letters, U (*uguale*=same) and D (*diverso*=different): participants had to respond by pressing one of two keys on the keyboard.

The *nonsense shapes task*, involving passive visual working memory, was based on the presentation of nonsense figures varying from two to eight, according to the complexity level. At the beginning of each trial, a blank screen appeared for 1000 ms, followed by another blank screen for 500 ms, and then the nonsense figures (3000 ms), followed by another blank screen for 500 ms. After presentation of a fixation point for 1500 ms, either the same series of figures or a series differing in one figure was presented for the recognition task.

The *sequential dot matrix task* involved passive spatial-sequential working memory. In this task, a gray screen was presented for 1000 ms followed by a 5x5 matrix shown to participants for 250 ms. Immediately afterward, red dots appeared in various cells of the matrix one at a time for 1000 ms, followed by a 250-ms interval. The number of red dots varied from two to eight, according to the complexity level. After a delay of 500 ms after the last red dot appeared, a fixation point of 1000 ms, and another delay of 500 ms, the same sequence or one with one red dot in a different order was presented at the same rate. Participants had to decide whether the sequence of dots was the same as that just presented, or if there were a change in order.

The *simultaneous dot matrix task* involved passive spatial-simultaneous working memory. The same display as that used in the sequential dot matrix task was used (5x5 matrices), but this time the red dots appeared simultaneously. In the test, participants had to decide if the new pattern of red dots was the same as that just presented, or whether one red dot appeared in a different location. After a blank screen of 1000 ms, a 5x5 matrix appeared on the screen for 500 ms, and then a variable number (two to eight, depending on the complexity level) of red dots appeared for 2500 ms, followed by another delay of 500 ms. After a fixation point of 1000 ms, the same arrangement of dots, or one with a red dot in a different location, was presented.

Complex-span tasks.

The *jigsaw puzzle task* (adapted from Vecchi & Richardson, 2000) tests the ability to manipulate a visual shape. It consists of a series of drawings derived by Snodgrass and Vanderwart (1980). Each drawing is fragmented into two to ten numbered pieces forming a puzzle. Drawings represent common, inanimate objects with a high value of familiarity and of image agreement. Each whole drawing is presented for 2000 ms, together with its verbal label, and is then removed. The material of each puzzle and the response sheet (a blank matrix with a number of cells corresponding to the number of pieces) are then displayed in front of the participant with the pieces set out in a nonordered way. Puzzles have to be solved not by moving the pieces but by writing down or pointing to the corresponding number of each piece on a response sheet. The level of complexity is given by the number of pieces composing each puzzle (e.g., from 2 to 10).

The *dot matrix task* (derived from Miyake et al., 2001) tests the ability to simultaneously process visuospatial information and maintain information in the visuospatial store. The test required participants to verify a matrix equation while simultaneously remembering a dot location in a 5x5 matrix. Each trial contained a set of matrix equations to be verified, each followed by a 5x5 matrix containing one dot. In the matrix equation display, a simple addition or subtraction equation was presented. Participants were given 4500 ms to verify whether the sum (or subtraction) of two successively presented segments was correctly described by a third presented pattern. Immediately afterward, a 5x5 matrix containing a dot in one cell was automatically displayed on the screen for 1500 ms. After a sequence of two to five equations and matrices, participants had to recall (in any order) which cells of the 5x5 matrix had contained dots (by clicking in the empty cells with the mouse).

The *visual pattern test, active version* (VPTA) (derived from Della Sala et al., 1997) tests the ability to maintain simultaneously presented spatial information and make a simple transformation of it. Participants were presented with matrices created by filling half cells of a matrix for 3000 ms. The matrices increased in size from smallest (4 squares at first level, with 2 filled cells) to largest (20 squares at final level with 10 filled cells). After the presentation phase, when participants memorized the filled squares, the initial stimulus was removed, and participants were presented with a blank test matrix on which they had to reproduce the pattern, filling in the cells corresponding to the positions in a row *below* the row filled in the presentation matrix (whose bottom row was always empty). For example, if in the presentation matrix the second square in the first row was filled, the participant had to fill in the second square in the second row. The level of complexity was defined as the number of filled cells in the matrix (2 to 10).

Intuitive geometry task.

In the *intuitive geometry task* (Dehaene et al., 2006), items were randomly presented by a computer. At the beginning of the procedure, a masking screen appeared for 2000 ms, followed by the stimuli (randomly presented). Each stimulus remained on the screen until subjects gave a response. Items consisted of an array of six images, five of which instantiated the desired concept, while the remaining one violated it. For each stimulus, participants were asked to point to the odd-one-out see Appendix).

Participants were presented with 43 items split into seven concepts: topology (for example, closed vs. open figures), Euclidean geometry (e.g., concepts of straight lines, parallel lines, etc.), geometrical figures (e.g., squares, triangles, and so on), symmetrical figures (for example, figures showing horizontal vs. vertical symmetrical axes), chiral figures (in which the odd-one-out was represented by a mirrored figure), metric properties (for example, the concept of equidistance), and geometrical transformation (e.g., translations and rotations of figures).

Different scores were computed: first, the total mean percentage of correct responses, and second, the mean percentages of correct responses derived from the seven concepts of geometry.

3.2.3. Results

3.2.3.1. Differences in Intuitive Geometry and VSWM

The mean scores obtained by the NLD and control group are presented in Table 3.6. Moreover, the mean percentages of correct responses of the two groups in the seven concepts of geometry are reported in Figure 3.7.

A one-way ANOVAs comparing the two groups showed significant differences on the total score of geometry ($F(1,30)=10.08$, $p=.003$, *Cohen*'s $d=1.12$). As the ranges of scores and of deviations were different for the different geometry subtests, we computed separate one-way ANOVAs. NLD children performed significantly more poorly than did the CG in Euclidean geometry ($F(1,30)=6.93$, $p=.013$, *Cohen's d*=.93) and in geometrical transformation ($F(1,30)=7.77$, $p=.009$, *Cohen's d*=.99). No differences between groups were found in topology ($F(1,30)=3.08$, $p=.09$, *Cohen's d*=.62); geometrical figures ($F(1,30)<1$, *Cohen's d*=.26); symmetrical figures ($F(1,30)<1$, *Cohen's d*=.21); chiral figures ($F(1,30)= 2.83$, $p=.10$, *Cohen's d*=.59); and metric properties ($F(1,30)=2.26$, $p=.14$, *Cohen's d*=.53).

For VSWM tasks, two multivariate analyses of variance (MANOVAs) were performed: the first, compared simple storage tasks (nonsense shapes, sequential dot matrix, and simultaneous dot matrix tasks) by group and the second complex-span tasks (jigsaw puzzle, dot matrix, and VPTA) by group. We chose to calculate two separate multivariate analyses of variance, since the two categories of tasks are clearly distinguishable (see for example, Cornoldi & Vecchi, 2003; Cowan, 2005; Miyake & Shah, 1999). We did not find a significant difference between groups in simple storage tasks ($F(3,28)=.69$, $p=.41$, $\eta^2=.02$), while the comparison on complex-span tasks revealed a main effect of group ($F(3,28)=5.70$, $p=.004$, $\eta^2=.38$). Univariate tests of significance showed that NLD children and CG scored significantly differently in all three of the tests: the jigsaw puzzle task ($F(1,30)=6.11$, $p=.019$, *Cohen's d*=.87), the dot matrix task ($F(1,30)=10.09$, $p=.003$, *Cohen's d*=1.12), and the VPTA ($F(1,30)=11.14$, $p=.002$, *Cohen's d*=1.18).

Table 3.6

Reliability, maximum possible score, mean scores at the VSWM tasks and, mean total score for the intuitive geometry task (standard deviations in brackets)

			NLD	**CG**	
VSWM tasks	Reliability	Max Possible	*M (SD)*	*M (SD)*	*Cohen's d*
Simple storage tasks					
Nonsense shapes	.83	24	11.5 (6.67)	13.31 (5.61)	.29
Sequential dot matrix	.91	24	14.43 (6.58)	18.63 (3.09)	.82
Simultaneous dot matrix	.90	24	17.19 (4.78)	19.06 (5.09)	.38
Complex span tasks					
Jigsaw puzzle	.84	30	11.25 (6.02)	16.25 (5.41)	.87
Dot matrix	.79	12	5.44 (3.61)	8.63 (1.75)	1.12,
VPTA	.89	30	8.06 (7.07)	15.13 (4.65)	1.18
Intuitive geometry task					
Total score	.65	100%	61.63 (12.07)	72.67 (6.92)	1.12

Note. children showing symptoms of NLD; controls (CG).

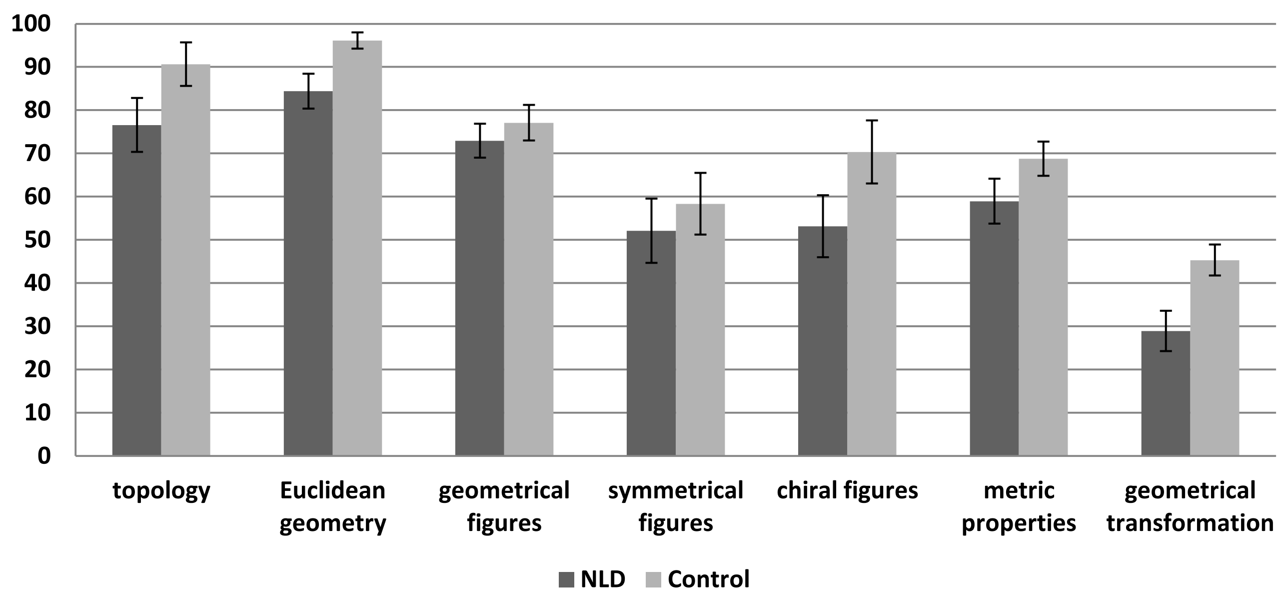


Figure 3.7 *Mean percentages of correct responses on the seven concept of geometry in children showing symptoms of NLD and CG.*

Note. Error bars represent standard errors.

In order to analyse the relationship between VSWM and intuitive geometry, we decided to examine the effect of VSWM on the total score of intuitive geometry using ANCOVAs. Specifically, we compared the total score of intuitive geometry of the two groups with ANCOVAs, considering the complex-span tasks as covariates. Using both the dot matrix task and the VPTA as covariates, the difference between groups was no longer significant, respectively: $F(1,28)=2.99$, $p=.09$, ($R^2=.40$) and $F(1,28)=1.92$, $p=.18$, (R =.33); whereas the difference was still significant using the jigsaw puzzle as covariate: $F(1,28)=4.75$, $p=.038$.

3.2.3.2. Discriminant Function Analysis

In order to find tasks with highest discriminative power in distinguishing between NLD children and CG, a discriminant analysis was performed to identify the variables most capable of making this distinction and to predict the probability of different participants belonging to each group. Before conducting the discriminant function analysis, issues related to sample size and multivariate normality were addressed (Tabachnick & Fidell, 2007). The criterion that the sample size of the smallest group should exceed the number of predictors was met. Group size was equal, ensuring multivariate normality. The discriminant function analysis was carried out with the stepwise method, using the six VSWM tasks and the total score of intuitive geometry. The tasks included in the analysis were the VPTA and the dot matrix task, for which Wilks' λ (Lambda) = .63, indicating that these were the variables best separating the two groups. The discriminant function analysis had a reliable association with children with NLD and CG: $\chi^2(2)=13.32$, $p<.001$. The VPTA and the dot matrix task were able to correctly classify into groups 68.8% of NLD children (i.e., 11/16) and 87.5% of CG children (i.e., 14/16).

3.2.4. Discussion

The relationship between VSWM and intuitive geometry was analysed as a contribution to the study's main goal of examining difficulties in developing geometrical competence. This was approached by administering VSWM and intuitive geometry tasks to NLD children matched with controls for age, gender, verbal abilities, and socioeconomic level. As noted above, our NLD children showed most of the typical symptoms, but they had not been diagnosed as having nonverbal learning disability; thus, our results cannot be directly generalised to children with a clinical diagnosis of NLD.

The first main result of the study was that the NLD children's difficulties in mathematics are extended also to the case of intuitive geometry. The result may seem rather obvious, as geometry has an evident visuospatial component and NLD are characterized by a visuospatial weakness, but previously this fact had never been documented. Furthermore, the geometrical failure of NLD children was differentiated; in fact, they performed significantly more poorly than did controls specifically in two subtests: Euclidean geometry and geometrical transformation.

The subtest on Euclidean geometry seems to represent a core concept of geometry. According to Spelke et al. (2010), natural geometry is founded on at least two evolutionarily ancient and cross-culturally universal cognitive systems that capture abstract information about the shape of the surrounding world: two *core systems* of geometry. The first represents the shapes of large-scale, navigable surface layouts, while the second represents small-scale, movable forms and objects. Empirical evidence regarding the origins of this latter system—which also involves concepts of Euclidean geometry (e.g., line, point, parallelism, and right-angle)—comes from developmental studies demonstrating that infants are sensitive to variations of angle (Schwartz et al., 1979; Slater et al., 1991) and length (Newcombe et al., 1999). This system therefore shows qualitative continuity not only over human development (Izard & Spelke, 2009) but also across cultures (Dehaene et al., 2006). It has to be noted that the Euclidean geometry subtest was relatively easy for all children, and that also NLD children, despite performing significantly poorer than controls, obtained a high percentage of correct responses. For this reason, some caution is needed in interpreting this finding, and further research should examine whether this problem is more evident with younger NLD children.

Geometrical transformation is the second subtest in which we observed significant differences between NLD children and controls. Different from the previous subtest, the performances of both groups were not particularly high, although they were both above the chance level (which is represented by the value of 16.6% as reported by Dehaene et al., 2006). The geometrical transformation subtest is considered by Dehaene and colleagues (2006) as involving a culturally mediated concept of geometry, since children have to individuate the odd-one-out among six images representing modifications based on translations, symmetries, rotations, and so on. In a recent study comparing adults and 4- to 10-year-olds, Izard and Spelke (2009) showed that, at all ages, children were able to detect angle and length relationships, but failed to detect directional relationships (i.e., requiring discrimination of rotated images) before adolescence. According to Spelke and colleagues (2010), the core systems of geometry are able to capture Euclidean distance and angle, but not geometrical transformation, and therefore fail to distinguish, for example, a shape from its mirror image. In our study the group differences were particularly small in the subtests involving geometrical figures, symmetrical figures, and metric properties, probably because NLD children could rely on their preserved linguistic skills. In particular, geometrical figures could be named and the odd-one-out could be rejected on the basis of the different verbal label. Similarly, the metric properties could also be found through a verbalisation process.

In sum, our results suggest that NLD children experience particular difficulty with two specific aspects of intuitive geometry: the first represents a core principle of geometry, while the second is a culturally mediated concept necessary in order to perform geometrical transformations such as mental rotations.

Regarding the second main aim of our research (i.e., finding further support for a VSWM impairment in children with NLD), results revealed that NLD children performed significantly more poorly than did CG on complex-span tasks, but not on simple storage tasks. This result is in agreement with previous research (see for example, Cornoldi et al., 1995; Cornoldi et al., 1999; Mammarella & Cornoldi, 2005a). The crucial role of VSWM in children with NLD has been extensively demonstrated in the last thirty years, but the present results support the hypothesis that NLD children encounter difficulties in VSWM tasks, especially when information must not only be maintained, but also actively processed.

Finally, our third main aim was to analyse whether, to some extent, a VSWM deficit might explain the failure of NLD children in intuitive geometry. This relationship was supported by the observation that NLD children actually presented both types of problems. Moreover, our covariance analyses showed that, when the contribution of two complex-span tasks was eliminated, the difference in the intuitive geometry score between groups also disappeared. Specifically, both the dot matrix task and the VPTA as covariates removed the differences between NLD children and controls in the intuitive geometry task. Mammarella and coworkers (Mammarella, Pazzaglia et al., 2008), in an attempt to classify visuospatial complex-span tasks, hypothesised that both the dot matrix task and the VPTA involve spatial working memory processes that are sequentially and simultaneously presented, respectively. Differently, the jigsaw puzzle task seems to involve, to a greater extent, visual working memory processing; in fact, participants have to make complex transformations of visual information. Our results thus confirmed that a spatial working memory difficulty may be primary in children with NLD, offering further support for the assumption that VSWM is critical for some aspects of mathematical cognition, such as intuitive geometry. Both Dehaene et al. (2006) and Giofrè, Mammarella, Ronconi, and Cornoldi (2013) suggested that intuitive geometry can be distinguished by core and culturally mediated concepts. Furthermore, Giofrè and coworkers (2013) showed that the role of VSWM is critical in supporting culturally mediated concepts of geometry. However, in the current study, a differential pattern of difficulties between these two types of concepts of geometry did not emerge. The novel finding of the present study is then represented by the crucial role of spatial complex-span tasks in intuitive geometry. In fact, as already mentioned, using both the dot matrix task and the VPTA as covariates, the differences between NLD children and the controls in the intuitive geometry task disappeared.

Taking up this point, our study concluded by exploring whether specific tasks contributed to the identification of NLD children. The discriminant function analysis demonstrated that the dot matrix task and the VPTA were the instruments most useful in this sense: using these VSWM tasks, 68.8% of the NLD children were correctly classified, confirming that, in general, assessment of VSWM is important for analysis of these children. Thus, results from the discriminant function analysis strengthened the hypothesis that VSWM difficulty is primary in explaining the performances of children at risk of NLD, and that failures in intuitive geometry are mediated by their impairment in spatial working memory tasks. However, it is worth noting that 31.2% of children with NLD were incorrectly classified, thus suggesting that some other variables could be crucial in the identification of children with NLD, such as, motor coordination, visuo-constructive and visuospatial abilities (Gross-Tsur, et al., 1995; Rourke, 1995; Roman, 1998; Drummond, et al., 2005). Hence, further research is needed to confirm and extend the present results, to study many other factors, such as motivation and metacognition, that presumably affect the acquisition of geometrical knowledge (see, for example, Aydin & Ubuz, 2010) and to overcome the limitations of the present research.

For example, a limitation of our study concerns the specific range of VSWM and geometry tasks that were assessed. In particular, the area of geometry is very large, and different aspects could be examined, also with reference to different ages and instructional requirements. Further research should also consider the role of other working memory components and spatial ability tasks. Moreover, the small sample size tested in the current study suggest some caution in interpreting our findings. Despite these limitations, we think that the present study has the merit of opening a window on issues that were until now neglected—specifically, the issue of children's having a learning difficulty related to the area of geometry and on their underlying cognitive mechanisms.

In conclusion, our research demonstrates that, in general, children showing symptoms of NLD performed more poorly than did the CG on intuitive geometry and complex-span tasks involving VSWM and that two of the three complex-span tasks used in the current research are appropriate for identifying children with NLD. Furthermore, the fact that these two VSWM tasks accounted for group differences in the intuitive geometry task lends support to the hypothesis that VSWM is involved in geometry and that VSWM deficits in NLD children mediate their difficulties in intuitive geometry.

4. Geometry, working memory and intelligence

In this chapter, we will extend expand the findings of our previous studies. In particular, we did not consider the important role of verbal working memory and intelligence (g). In fact, WM and g are closely related, but separable constructs. Thus, it is possible that including g in the analysis will results in an increment of the proportion of explained variance in geometrical achievement. Indeed, in the first study, the portion of the explained variance indicates that other factors may have a crucial role.

For these reasons, we performed a third study in which we included geometry g and WM. In the first paragraph, we will focus on the relationship between g and WM. In the second paragraph, we will focus on the relationship between geometry, working memory and g.

4.1. Study 3 part I. The Structure of Working Memory and Its Relation to Intelligence in children

4.1.1. Study 3 part I introduction

Working memory (WM) is a limited-capacity system that enables information to be stored temporarily and manipulated (Baddeley, 2000); it is involved in complex cognitive tasks such as reading comprehension (Borella, Carretti, & Pelegrina, 2010; Carretti, Borella, Cornoldi, & De Beni, 2009; Daneman & Merikle, 1996) and arithmetical problem solving (Passolunghi & Mammarella, 2010, 2011; Passolunghi & Pazzaglia, 2004). Intelligence is the ability to reason, plan, solve problems, think abstractly, understand complex ideas, learn quickly, and learn from experience (Gottfredson, 1997, p. 13).

Various models of WM have been suggested. In a tripartite model originally proposed by Baddeley and Hitch (1974), there is a central executive responsible for controlling the resources and monitoring information-processing across informational domains. The storage of information is mediated by two domain-specific slave systems, i.e. the phonological loop (used for the temporary storage of verbal information), and the visuospatial sketchpad (specialized in retaining and manipulating visual and spatial representations). This model has met with a broad consensus (Baddeley, 2012), and further developments of the model (Baddeley, 2000; Cornoldi & Vecchi, 2003) have maintained the distinction between a central, modality-independent component and separate verbal and visuospatial components.

Other authors have argued, however, that there is no difference between short-term memory (STM) and WM, and suggested a single model of WM. Whether WM and STM reflect the same or different factors is still debated (e.g., Colom et al., 2006), and some researchers claim that it is storage capacity (i.e., STM), not cognitive control, that accounts for the relationship between WM and intelligence (e.g., Colom et al., 2006).

An alternative, modality-dependent model is based on the assumption that WM is supported by two separate sets of domain-specific resources for handling verbal and visuospatial information (e.g., Shah & Miyake, 1996), each of which would be independently capable of manipulating the information and keeping it active (i.e., readily accessible). Research on adults supports this distinction (Friedman & Miyake, 2000).

A different approach, described using a domain-general model, suggests that WM capacity is limited by controlled attention. It has been argued that the residual variance in verbal WM reflects controlled processing, which is uniquely linked to general fluid intelligence (Engle et al., 1999). This model and the Baddeley and Hitch (1974) model share the same central component for coordinating ongoing information processing (called controlled attention and the central executive, respectively) and the storage of information in subsystems. Both models also envisage domain-specific storage components. The model developed by Engle et al. is also consistent with the concept postulated by

Cornoldi & Vecchi (2003) of a vertical continuum reflecting different degrees of attentional control (from a passive storage of information to its active processing and manipulation).

Regarding the structure of WM in children, some authors (Engel De Abreu, Conway, & Gathercole, 2010) have supported a distinction between STM and WM (i.e., a modality-independent model), while Alloway and colleagues (Alloway, Gathercole, & Pickering, 2006) claimed that the tripartite model is the most appropriate across various age ranges.

The relationship between WM and intelligence, in both adults and children, is still debated. Some studies indicate that WM and intelligence are closely-related but separable constructs (Engle et al., 1999). One meta-analysis showed a correlation of r=.48 between WM and intelligence (Ackerman, Beier, & Boyle, 2005), though the correlation between latent variables is typically higher, r=.72 (Kane, Hambrick, & Conway, 2005); this incomplete overlap suggests that these two constructs are not isomorphic (Conway, Kane, & Engle, 2003).

Research on children has produced less robust evidence on the relationship between intelligence and WM. It has been argued that WM, not intelligence, is the best predictor of literacy and numeracy (Alloway & Alloway, 2010), and that child prodigies may have only a moderately high level of intelligence, but perform extremely well in WM (Ruthsatz & Urbach, 2012). These findings would indicate that WM and intelligence are dissociable, casting doubts on the relationship between intelligence and WM in children.

Many studies on the relationship between STM, WM and intelligence in children have their limitations, however. For a start, they use only a single task (e.g., Raven) to assess intelligence, whereas performance in different measures (preferably having different formats) should be considered to reduce the specific effects of a given test and treat intelligence as a latent construct (Süß & Beauducel, 2005). Second, not all studies have distinguished between (verbal and spatial) STM and WM, making it impossible to compare the different models. Third, the absolute credit score has been applied to WM tasks, which is fine in clinical settings, while the partial credit score is more reliable and appropriate in correlational studies (Conway et al., 2005).

In the present study, we explored the nature of the relationship among STM, WM and intelligence in 4th- and 5th-graders, examining: i) different models of WM in children, using confirmatory factor analyses (CFA); ii) the relationship between WM and intelligence, and the strength of their association, using structural equation modeling (SEM).

Our first aim was to use CFA to elucidate the structure of WM in children by comparing the following models: (1) a one-factor model that sees WM as a single construct; (2) a model distinguishing between a visuospatial and a verbal component, without distinguishing between STM and WM (the modality-dependent model; Shah & Miyake, 1996); (3) a two-factor model distinguishing between WM and STM, without distinguishing between content domains (the modality-independent model; Engle, et al., 1999); (4) a three-factor model (Baddeley's model; 1986), expanded to include a distinction between two STM components (verbal *vs.* spatial), and including a WM component too (i.e., a tripartite model).

The second aim of our study was to test the relationship between WM and intelligence using a SEM approach. We examined whether STM and WM carry a similar weight, irrespective of the demands of the memory tasks administered, in terms of the processes and presentation format involved in predicting intelligence. Previous findings on this issue are unclear: some studies suggest that, when the variance that WM and STM have in common is controlled, only the residual WM factor reveals a significant link with intelligence (Engel De Abreu et al., 2010); other research indicates that the relationship between WM and intelligence is explained primarily by STM (Hornung et al., 2011). In addition, some authors have argued that both storage and executive processes in the WM system can independently predict intelligence (Tillman, Nyberg, & Bohlin, 2008), but well-controlled WM processes have a higher predictive power in typically-developing children than poorly-controlled STM processes (Cornoldi, Giofrè, Calgaro, & Stupiggia, 2012). The inclusion of both verbal and visuospatial STM tasks in our study also gave us an opportunity to see whether the latter are more closely related to intelligence than the former, as Kane and coworkers suggested (Kane et al., 2004).

4.1.2. Method

4.1.2.1. Participants

We collected data for 183 children, but 7 of them had extremely low scores on the Raven's Colored progressive matrices (below the 5th percentile of the Italian norms, Belacchi, Scalisi, Cannoni, & Cornoldi, 2008) and were excluded from further analyses. A total of 176 typically-developing children (96 males, 80 females; M_{age}=9.27, SD=.719) in 4th and 5th grade at school were thus included in the final sample.

4.1.2.2. Materials

Intelligence tasks

Colored progressive matrices (CPM*;* Raven, Raven, & Court, 1998). The children were asked to complete a geometrical figure by choosing the missing piece from among 6 possible solutions. The patterns gradually became more difficult. The test consisted of 36 items divided into three sets of 12 (A, AB, and B). The score corresponded to the sum of correct answers.

Primary mental ability, reasoning (PMA-R; Thurstone & Thurstone, 1965). This task assesses the ability to discover rules and apply them to verbal reasoning. It is a written test in which the children had to choose which word in a set of four was the odd one out, e.g. *cow, dog, cat, cap* (the answer is *cap*). There was only one correct answer. The test included 25 sets of words and children were allowed 11 minutes to complete it. The score was the sum of the correct answers.

Primary mental ability, verbal meaning (PMA-V; Thurstone & Thurstone, 1965). In this written test, the children had to choose a synonym for a given word from among four options, e.g.

small: (a) *slow*, (b) *cold*; (c) *simple*; (d) *tiny* (the answer is *tiny*). There was only one correct answer. The test included 30 items and had to be completed within 12 minutes. The score was the sum of the correct answers.

Working memory tasks

Simple span tasks (syllable span task, SSPAN; and digit span task, DSPAN). These tasks examined short-term memory abilities involving the passive storage and recall of information (Cornoldi & Vecchi, 2000; Swanson, 1993). Syllables/digits were presented verbally at a rate of 1 second per item, proceeding from the shortest series to the longest (from 2 to 6 items). There was no time limit for recalling the syllables or digits in the same, forward order. The score was the number of digits/syllables accurately recalled in the right order.

Matrices span tasks (derived from Hornung et al., 2011). Short-term visuospatial storage capacity was assessed by means of two location span tasks. The children had to memorize and recall the positions of a blue cell that appeared briefly (for 1 second) in different positions on the screen. After a series of blue cells had been presented, the children used the mouse to click on the locations where they had seen a blue cell appear. The number of blue cells presented in each series ranged from 2 to 6. There were two different conditions: in the first, the targets appeared and disappeared in a visible (4x4) grid in the center of the screen (*matrices span task, grid [MSTG])*; in the second, the targets appeared and disappeared on a plain black screen with no grid (*matrices span task, no grid [MSTNG]*). The score was the number of cells accurately reproduced in the right order.

Categorization working memory span (CWMS; Borella, Carretti, & De Beni, 2008; De Beni, Palladino, Pazzaglia, & Cornoldi, 1998). The material consisted of a number of word lists containing four words of high-medium frequency. The word lists were organized into sets of word lists of different length (i.e., from two to five words to recall). The children were asked to read each word aloud and to press the space bar when there was an animal noun. After completing each set, they had to recall the last word in each list, in the right order of presentation. The score was the number of accurately recalled words.

Listening span test (LST; Daneman & Carpenter, 1980; Palladino, 2005). The children listened to sets of sentences arranged into sets of different length (containing from 2 to 5 sentences) and they had to say whether each sentence was true or false. After each set, the children had to recall the last word in each sentence, in the order in which they were presented. The score was the number of accurately recalled words.

Visual pattern test, active (VPTA; Mammarella et al., 2006; Mammarella, Giofrè et al., 2012). This tests the ability to maintain and process spatial information simultaneously presented on a computer screen. Eighteen matrices, adapted from the Visual Pattern test (Della Sala et al., 1997), of increasing size (the smallest with 4 squares and 2 cells filled, the largest with 14 squares and 7 cells filled) contained a different number of cells to remember (from 2 to 7). After the matrices had been shown for 3 seconds, they disappeared and the children were presented with a blank test matrix on which they were asked to reproduce the pattern of the previously-seen cells by clicking in the cells corresponding to the same positions but one row lower down (the bottom row in the presentation matrix was always empty). The score was the number of accurately placed cells.

4.1.2.3. Procedure

The tasks were administered as part of a broad study on the relationship between cognitive structures and academic achievement.

The children were tested in two phases, one involving a group session in their classroom that lasted approximately 1 hour, the other in individual sessions lasting approximately 90 minutes, in a quiet room away from the classroom.

During the group session, the intelligence tests were administered in a fixed order (CPM, PMA-V, PMA-R). At the individual sessions, the WM tasks were administered in the following fixed order:

(1) syllable span task; (2) matrix span task, grid; (3) visual pattern test, active; (4) categorization working memory span; (5) digit span task; (6) matrix span task, no-grid; (7) listening span task. At the individual sessions, all tasks were presented on a 15-inch laptop and were programmed using E-prime 2 software. Each task began with two training trials, then the simplest level of the task, and their complexity gradually increased thereafter, using three trials for each level of complexity. The partial credit score was used for scoring purposes (see Conway et al., 2005).

4.1.3. Results

4.1.3.1. Statistical Analysis

The assumption of multivariate normality and linearity was tested using the PRELIS package and all the CFA and SEM analyses were estimated with the LISREL 8.80 software (Jöreskog & Sörbom, 2002, 2006).

No multivariate outliers were found (Mahalanobis distance; $p < .001$). The measure of relative multivariate kurtosis was 1.066. This value is considered relatively small, so the estimation method that we chose to use (maximum likelihood) is robust against several types of violation of the multivariate normality assumption (Bollen, 1989).

Model fit was evaluated using various indices following the criteria suggested by Hu and Bentler (1999). In particular, a model was judged to have a good fit if it had: an insignificant χ^2_M goodness-of-fit statistic; a root mean square error of approximation (*RMSEA*) nearing .06; a standardized root mean square residual (*SRMR*) $\leq$ .08; a non-normed fit index (*NNFI*) and a comparative fit index (*CFI*) $\geq$ .96. The Akaike information criterion (*AIC*) was used to compare the fit of non-nested models. To take the children's different ages into account, a partial correlation analysis was conducted with age in months partialed out (Alloway et al., 2006). Partial correlations were used in all the analyses. Descriptive statistics, correlations, and partial correlations with Cronbach's alphas are presented in Table 4.1.

Table 4.1

Correlations, means (M), standard deviations (SD), and reliabilities for the measures of g and WM.

	1	2	3	4	5	6	7	8	9	10
G										
1 CPM	1	.433	.432	.251	.127	.226	.309	.290	.313	.330
2 PMA-R	.449	1	.568	.258	.187	.359	.340	.381	.363	.178
3 PMA-V	.446	.615	1	.272	.240	.402	.485	.393	.387	.322
WM										
4 SSPAN	.261	.277	.291	1	.541	.390	.397	.249	.349	.241
5 DSPAN	.134	.197	.245	.543	1	.462	.386	.261	.315	.256
6 MSTG	.300	.395	.404	.258	.266	1	.544	.299	.266	.378
7 MSTNG	.332	.405	.435	.363	.321	.722	1	.292	.297	.291
8 CWMS	.324	.370	.505	.407	.391	.305	.323	1	.720	.372
9 LST	.247	.399	.446	.401	.464	.315	.302	.560	1	.288
10 VPTA	.335	.189	.322	.246	.259	.376	.295	.297	.382	1
M	28.26	16.30	20.73	41.80	46.28	39.50	29.53	26.57	27.43	59.81
SD	4.93	4.08	7.34	8.58	8.15	10.14	10.03	6.65	6.78	11.75
Reliability	.82	.78	.93	.69	.70	.83	.83	.77	.83	.91

Note. Zero order correlation below and partial correlation (controlling for age) above the diagonal; all coefficients ≥.148 are significant at .05 level; CPM=colored progressive matrices; PMA-R=primary verbal abilities reasoning; PMA-V=primary mental abilities verbal; SSPAN=syllable span; DSPAN=number span; MSTG=matrix span task grid; MSTNG=matrix span task no-grid; CWMS=categorization working memory span; LST=listening span task; VPTA=visual pattern test active; Reliability=Cronbach's alpha

4.1.3.2. CFA models for WM

Model 1 investigated a single WM factor in children (one-factor model; Figure 4.1); it provided a poor fit with the data (Table 4.2).

Model 2 investigated two distinct verbal and spatial factors (modality-dependent model; Figure 4.1), and provided a poor fit with the data (Table 4.2).

Model 3 investigated two distinct STM and WM factors (modality-independent model; Figure 4.1); and provided a poor fit with the data (Table 4.2).

Model 4 tested two STM factors (verbal [-V], and visuospatial [-VS]) and one WM factor (tripartite model; Figure 4.1). It fitted well with the data and proved better than the other models, i.e. it had the best fit and a lower AIC (Table 4.2), suggesting that the structure of children's WM may be well represented by combining two STM components (one verbal and one spatial) with a WM component.

Table 4.2

Fit indices for different confirmatory factor analyses (WM) and SEM analysis (WM and g)

Model	χ^2_M (*df*)	*p*	*RMSEA*	*SRMR*	*CFI*	*NNFI*	*AIC*
WM							
(1)	117.00 (14)	<.001	.20	.10	.80	.70	141.33
(2)	37.45(13)	<.001	.11	.07	.95	.92	68.86
(3)	92.88(13)	<.001	.20	.12	.84	.74	136.95
(4)	17.74(11)	=.088	.06	.04	.98	.97	51.62
WM and g							
(1)	40.88(29)	=.070	.04	.04	.99	.98	89.90

Note. χ^2_M=Model chi-square, RMSEA=Root mean square error of approximation, SMSR=standardized root mean square residuals, CFI=comparative fit index, NNFI=non-normed fit index, AIC=Akaike Information Criterion.

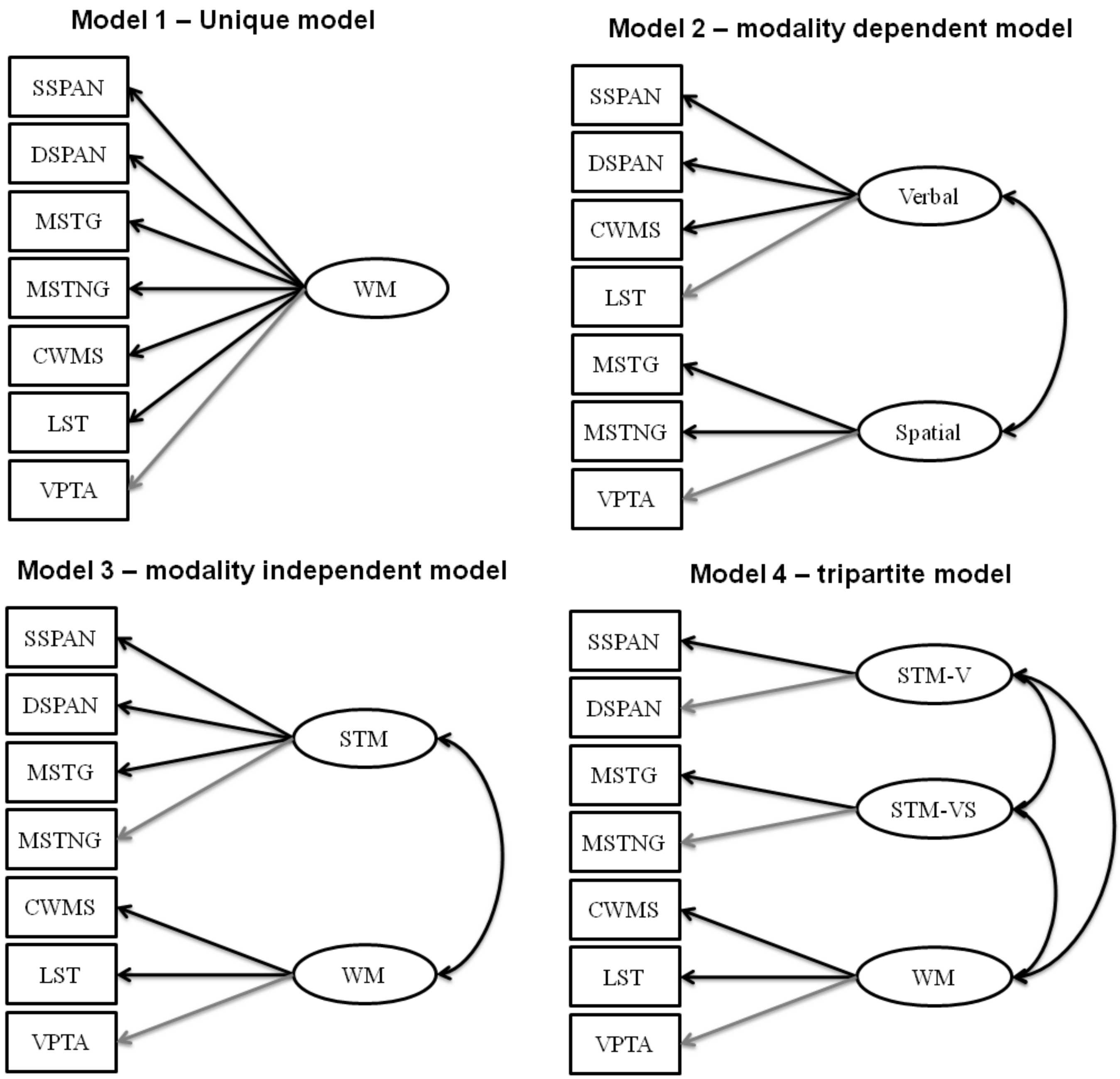


Figure 4.1. *Conceptual diagrams for different WM models.*

Note. Fixed parameters are in gray. *S*SPAN=syllable span; DSPAN=number span; MSTG=matrix span task grid; MSTNG=matrix span task no-grid; CWMS=categorization working memory span; LST=listening span task; VPTA=visual pattern test active. STM-V=short term memory-verbal; STM-VS= short term memory-visuospatial; WM=working memory.

4.1.3.3. SEM model for WM and intelligence.

The next step was to use SEM to test the relationship between WM (our Model 4) and intelligence. We used STM-V, STM-VS, WM (correlated with one another) as exogenous, or independent factors, and *g* as the endogenous, or dependent factor (Figure 4.2). The overall fit of the model was very good, and 65% of the variance in *g* was predicted (Table 4.2). Notably, only STM-VS and WM were significantly related to *g*, while STM-V was not.

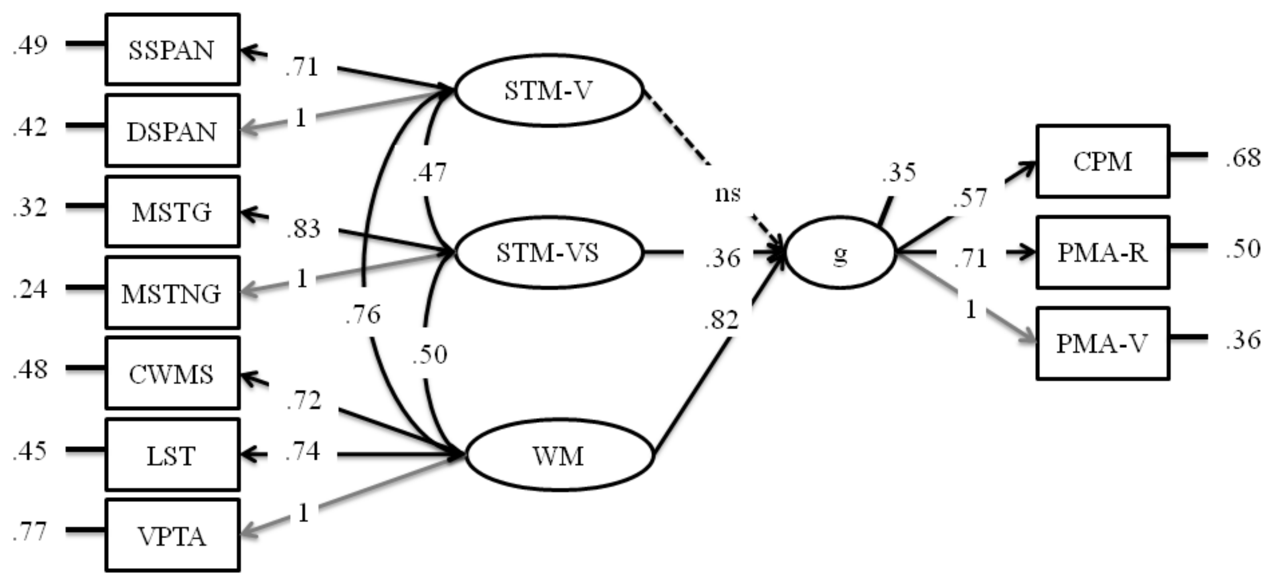


Figure 4.2 *SEM model for WM and g factor.*

Note. Path significant at .05 level are indicated by solid lines. Fixed parameters are in gray. The residual variance components (error variances) indicate the amount of unexplained variance (R^2= 1 - error variance). CPM=colored progressive matrices; PMA-R=primary verbal abilities reasoning; PMA-V=primary mental abilities verbal; *S*SPAN=syllable span task; DSPAN=number span task; MSTG=matrix span task grid; MSTNG=matrix span task no-grid; CWMS=categorization working memory span; LST=listening span task; VPTA=visual pattern test active; STM-V=short term memory-verbal; STM-VS= short term memory-visuospatial; WM=working memory; g=intelligence

4.1.4. Discussion

The purpose of the present study was to investigate the relationship between WM and intelligence in children in 4th and 5th grade. In particular, we examined: *i)* whether children's WM could be seen as a single factor, or separated into different components; and *ii)* which WM component is more closely related to intelligence. Concerning the first issue, much of the research examining the structure of WM has been done within the framework of the Baddeley and Hitch model (1974; Baddeley, 1986). The main results of the present study indicate that our data fitted poorly with the one-factor WM model, and with two-factor models distinguishing either between visuospatial and verbal components (see Shah & Miyake, 1996), or between STM and WM (e.g., Engle et al., 1999). Our findings revealed instead that children's WM can be separated into a WM component and two storage components relying on domain-specific, verbal and visuospatial resources.

As for the second issue, we investigated whether different WM components predict intelligence equally well. We found STM-VS and WM significantly related to intelligence. This result is consistent with findings in adults and confirms that STM-VS (typically involving unfamiliar situations) predicts a unique portion of the variance not explained by active WM (Kane et al., 2004), whereas the verbal component of WM (typically involving more familiar material) is irrelevant. Our results also confirm that WM has a strong positive effect on the *g* factor (.82), while STM only accounts for a small part of *g* (Conway, Getz, & Engel De Abreu, 2011). Unlike previous research (e.g., Kane et al., 2004), our finding cannot be attributable to our *g* factor being biased towards spatial abilities because one of the three measures of intelligence used in our study was verbal (vocabulary) and another (reasoning) both involved language. The lack of a significant relationship between intelligence and verbal STM tasks seems to have two reasons: (i) the simple rote repetition of familiar material is scarcely related to fluid intelligence (Engle et al., 1999); and (ii) not only the Listening span test, but also one of the other two active tasks (i.e., the Categorization working memory span) relied on language. This meant that the contribution of the verbal STM components on *g* disappeared when the influence of these two tasks was partialed out.

The present study showed a strong relationship between WM and *g*. Unlike the picture seen in adults of a close relationship between WM and intelligence (e.g., Engle et al., 1999; Kane et al., 2004), only a moderate relationship between intelligence and WM had emerged in previous research on children (Engel De Abreu et al., 2010; Hornung et al., 2011), whereas WM and *g* shared a substantial portion of the variance (about 65%) in our sample. Our finding is similar to the situation observed in adults and is attributable to two features of our study: first, the partial credit score method maximizes the correlation among factors (Conway et al., 2005); and second, we used several tests (not just one) to measure each factor.

Although it contains some insightful findings, the present study has some limitations. First, we considered only 4^{th} and 5^{th} graders and, although previous studies have suggested that the structure of WM remains much the same, whatever the age group considered (Alloway et al., 2006), our results cannot be generalized to samples of older or younger children. Future studies will be needed to address this issue in other age groups. Second, we did not consider developmental changes, although previous research has shown that verbal and visuospatial WM follow a different developmental trajectory (Gathercole, 1998), so the relationship between WM and *g* may change as a child develops.

To sum up, the present study demonstrates that, in 4^{th}- and 5^{th}-grade children, the WM structure best fitting our data is characterized by a central component and two STM domain-specific storage components, devoted to retaining verbal information in one and spatial information in the other. As for the relationship between WM and intelligence, the WM component and STM-VS do significantly predict intelligence, while STM-V does not. Finally, our findings also suggest that WM is a fundamental part of intelligence.

4.2. Study 3 part II. Relationship between geometry, working memory and intelligence in children.

4.2.1. Study 3 part II introduction

Despite the fact that geometry is one of the main primary areas of mathematical learning, the cognitive processes underlying geometry-related, academic achievement have not been studied in depth. For example, the study of geometry requires the ability to manipulate and retain information in mind. For this reason, working memory (WM), and in particular, visuospatial working memory (VSWM), is crucially involved in the acquisition of geometrical skills (Giofrè et al., 2013; Mammarella, Giofrè, et al., 2012). Moreover, the study of geometry heavily relies on spatial abilities (Piaget, Inhelder, & Szeminska, 1960; Piaget & Inhelder, 1967). In addition, geometrical academic problems typically require one to discover a solution to a problem; this capacity seems to be related to reasoning skills (Clements & Battista, 1992). The present study tried to explore the involvement of WM and intelligence (*g*) in two different aspects of geometry, i.e., intuitive geometry and academic achievement in geometry.

4.2.1.1. Intuitive Geometry and Academic Achievement Geometry

Geometrical may be differentiated in two aspects: the first aspect depends on learning and education; the second aspect, includes abilities, which are suppose to be independent from the culture. In fact, geometry might represent a core knowledge, as suggested by Spelke, Lee and Izard, (2010). It has been argued that some principles of geometry are independent upon the culture (i.e., intuitive) (Dehaene et al., 2006). With 'intuitive' we mean aspects which are: (1) primitive (Rosch, 1975), (2) very early developed (Spelke et al., 2010), and (3) not dependent upon culture and formal instruction (Dehaene et al., 2006). Moreover, Dehaene et al. (2006) have shown that it is possible to assess experimentally intuitive geometry. They tested, Amazonian Indians, without any formal instruction in geometry, and they discovered that: 1) Amazonian Indians succeeded in distinguishing some important geometrical aspect (e.g., right angles, parallelism); 2) performances in the intuitive geometry task of Amazonian Indians children and adults do not differ. The intuitive geometry task was also used to predict a significant portion of the variance in an academic achievement task (Giofrè et al., 2013), and to discriminate between children with nonverbal learning disabilities (who failed in spatial but not in verbal tasks) and with typical development (Mammarella, Giofrè et al., 2012).

This idea of geometry as 'core concept', independent from culture, is not new. For example, Socrates demonstrates that a house slave (Meno), ignorant in geometry, was able to learn a complicated geometrical problem (Plato, 1977). In fact, Socrates claims that Meno 'spontaneously recovered' knowledge he knew from a past life without having been taught. However, was Meno able to solve the geometrical problem because geometry is innate or because he reasoned on the problem? In fact, it is possible that Amazonian Indians were capable to solving simple, geometrical problems because they somehow knew the solution, but also because they were reasoning on the alternatives. Both the hypotheses are, in fact, plausible.

Conversely, academic achievement in geometry represents the student's ability to respond to the typical, geometry questions on the mathematical curriculum (Giofrè et al., 2013). It is considered one of the most important areas of mathematical learning, and it is linked to a student's future academic and professional success (Verstijnen et al., 1998). Academic achievement in geometry is related to a wide range of skills and it is related to high order cognitions (e.g., WM and g). Moreover, academic achievement in geometry is very important in STEM (science, technology, engineering, and mathematics) fields such as Engineering and in Mathematics.

4.2.1.2. Cognitive Processes Involved in Geometry: WM, and *g*

Working Memory

The WM system, in which specific storage components (i.e., the 'slave' systems) sub-serve a central component responsible for controlling information processing (Baddeley, 1986), could be involved in geometry. A tripartite model was initially proposed by Baddeley and Hitch (1974). In this model, the central executive is the component responsible for controlling resources and monitoring information processing across informational domains. Moreover, storage of information is mediated by two domain-specific slave systems: the phonological loop, which provides temporary storage of verbal information, and the visuospatial sketchpad, specializing in the maintenance and manipulation of visual and spatial representations. This model has received a large consensus (Baddeley, 2012) and further specifications of the model (Baddeley, 2000; Cornoldi & Vecchi, 2003) have maintained the distinction between central modality-independent and specific verbal and visuospatial components. WM has been described in many ways, differentiating for instance, between simple storage and complex span tasks (e.g., Unsworth & Engle, 2007), or between passive and active processes (involving simple storage and complex span tasks, respectively) (Cornoldi & Vecchi, 2003).

A large body of research has shown that WM predicts success in school-related tasks, such as reading comprehension (Carretti et al., 2009), mathematical achievement (e.g., Geary et al., 1990; Passolunghi et al., 2008), approximate mental addition (Caviola, Mammarella, Cornoldi, & Lucangeli, 2012), mathematical skills (Alloway & Passolunghi, 2011), and geometrical achievement (Giofrè et al., 2013).

More specifically, the WM component involved in retaining and processing visuospatial information appears to be involved in a child's ability to count (Kyttälä et al., 2003), to perform multi-digit operations (Heathcote, 1994), nonverbal problem-solving (Rasmussen & Bisanz, 2005), in mathematical achievement (Bull et al., 2008; Jarvis & Gathercole, 2003; Maybery & Do, 2003) and academic achievement in geometry (Giofrè et al., 2013).

Intelligence

Intelligence (*g*) involves the ability to reason, plan, solve problems, think abstractly, comprehend complex ideas, learn quickly, and learn from experience (Gottfredson, 1997 p. 13). Intelligence and academic achievement are considered highly related but separable (Kaufman, Reynolds, Liu, Kaufman, & McGrew, 2012) constructs. In fact, language, reasoning, working memory and attentional processes that underlie reading and mathematical operations also underlie intellectual functioning (Deary, Strand, Smith, & Fernandes, 2007; Hunt, 2011). The relationship is also supported by empirical evidence: studies have found a good correlation between achievement tests (such as SAT and ACT) and *g*-factor measures (Frey & Detterman, 2004; Koenig, Frey, & Detterman, 2008), and these results are consistent (typically correlations ranged between .6 and .7; Coyle & Pillow, 2008).

Intriguingly, academic achievement in geometry represents a special case. In fact, academic achievement in geometry requires the ability to solve problems. Solving mathematical problems, and in particular, geometrical problems, require problem solving and therefore, intelligence (Cornoldi, Giofrè, & Martini, 2013). However, not only reasoning, but also spatial abilities should be involved; in many cases, children use spatial skills to solve the problem and to reach the solution (Piaget & Inhelder, 1967).

Spatial ability may be defined as the ability to generate, retain, retrieve, and transform well-structured, visual images. Spatial abilities were crucial in Thurston's conceptualization of primary mental abilities (Thurstone & Thurstone, 1965). Moreover, Guilford (1967) included spatial abilities in his model of intelligence. In addition, Vernon's hierarchical model place spatial-visualization factors immediately below general ability (Vernon, 1950), and Carroll's three stratum theory place Gv (visuo-spatial thinking) in the second stratum (Carroll, 1993). More recently, the g-VPR model postulates that intelligence is highly related to the capacity of rotate stimuli (Johnson & Bouchard, 2005). In addition, spatial ability contributes to learning, the development of expertise, and securing advanced educational and occupational credentials in STEM (Lubinski, 2010). Finally, spatial abilities are closely related to visuospatial working memory (Miyake et al., 2001). For these reasons, we believe that solving geometrical problems require the contribution of g, and WM, in particular complex span tasks involving the visuospatial domain.

4.2.1.3. Goals of the Present Paper

In the present study, we explore the nature of the relationship among WM, g and geometry (intuitive and academic achievement in geometry) in children attending 4^{th} and 5^{th} grades. The main goals were: i) examine whether WM, g, and intuitive geometry predict a significant portion of academic achievement in geometry variance and to determine the model which best suits the data; ii) examine the contribution of g, WM, and intuitive geometry in explaining academic achievement in geometry variance and the strength of this association.

Our primary goal was to determine the model with the best fit using SEM. Specifically, the following models were compared: (1) a model in which WM, intuitive geometry and intelligence are correlated exogenous variables (independent) and they are linked to academic achievement in geometry, which is the only endogenous variable (i.e., dependent variable); (2) a model considering intelligence as mediating the relationship between all the other factors and geometrical achievement in geometry; (3) a model close to the previous model but, in which intuitive geometry, as well as academic achievement in geometry, is considered as an endogenous variable.

The hypothesis underling the first model is that WM, intelligence, and intuitive geometry might be independent factors correlated with each other and explaining a unique portion of the academic achievement in geometry variance. Another possibility is that WM does predict academic achievement only with the mediation of intelligence (tested in Model 2). In previous research, we found that an active, visuospatial working memory task indirectly predicted academic achievement in geometry (Cornoldi & Vecchi, 2003). Thus, it is possible that active working memory and visuospatial short term memory tasks (STM-VS) are significantly related to *g*, which is, in turn related to academic achievement in geometry. Thus, g should mediate the relationship between WM and academic achievement in geometry. This is consistent with a large body of research indicating that only active working memory is related to high order cognition (Engle et al., 1999). In addition, recent results have shown that STM-VS predicts a unique portion of g variance (Kane et al., 2004). Finally, in Model 3 we tested if the intuitive geometry task is predicted by the *g* factor. In fact, in the intuitive geometry task developed by Dehaene (Dehaene et al., 2006) Amazonian Indians could have been able to determine the solution to geometrical concepts for several reasons, and one of these reason may be that they were reasoning on the tasks (more than knowing the right alternative). Thus, a part of the intuitive geometry variance may be related to *g*.

4.2.2. Method

4.2.2.1. Participants

We collected data for 183 participants; seven children had extremely low score on colored progressive matrices (under the 5^{th} percentile of the Italian norms, Belacchi, Scalisi, Cannoni, & Cornoldi, 2008) and we decided to exclude these subjects from further analysis. A total of 176 typically developing children (96 male, M_{age}=9.27, SD=.719), attending the 4^{th} and the 5^{th} grades, were included in the final sample.

4.2.2.2. Materials and procedure

Participants were tested in two group sessions in the classroom that lasted approximately 1 hour, and an individual, approximately 1.5 hours-long session in a quiet room, away from the classroom.

In the first group session, general cognitive ability tests were administered in the following fixed order (CPM, PMA-V, PMA-R). In the second group session, geometrical tasks were administered in the following fixed order (geometrical problems and geometrical questions). In the individual session, we administered first the intuitive geometry task and then working memory tasks (see Study 1 for further details).

Geometry

Intuitive geometry task (I-GEO; Dehaene et al., 2006). Items consisting of an array of six images, five of which were instantiated the desired concept, while the remaining one violated it. For each stimulus, participants were asked to click on the odd-one-out (see Appendix). Items were randomly presented and remained on the screen until subjects gave a response. Participants were presented with 43 items, split into seven concepts: topology (e.g., closed vs. open figures), Euclidean geometry (concepts of straight lines, parallel lines etc.), geometrical figures (e.g., squares, triangles, and so on), symmetrical figures (e.g., figures showing horizontal vs. vertical symmetrical axes), chiral figures (in which the odd-one-out was represented by a mirrored figure), metric properties (e.g., the concept of equidistance), and geometrical transformation (e.g., translations and rotations of figures).

Academic achievement tasks.

Geometrical problems (GEO-P; Mammarella, Todeschini, & Englaro, 2012). In this test, children are required to solve nine geometrical problems. In particular, children had to calculate the area of complex figures, draw lines which are not perpendicular or parallel or solve complex geometrical problems. The test lasts approximately 40-45 minutes.

Geometrical questions (GEO-Q; Mammarella, Todeschini et al., 2012). In this test, children are required to ask to eight geometrical questions. The questions were focused on important geometrical concepts or definitions (e.g., concave, segment, goniometer, and parallelogram). The test lasts approximately 10-15 minutes.

4.2.3. Results

4.2.3.1. Statistical analysis

The assumption of multivariate normality and linearity was evaluated with the PRELIS package and the models were estimated with the software LISREL 8.80 (Jöreskog & Sörbom, 2002; 2006).

The measure of relative multivariate kurtosis was 1.04. This value is considered relatively small. Therefore, and the estimation method that we decided to use (Maximum Likelihood) is robust against several types of the violation of the multivariate normality assumption (Bollen, 1989). Covariance matrix was used (Kline, 2011).

Model fit was evaluated by various indices following the criteria suggested by Hu and Bentler (1999). In particular, a model with: a non significant model chi-square goodness-of-fit statistic (χ^2_M); a root-mean-square error of approximation (RMSEA) close to .06; a standardized root-mean-square residual (SRMR) ≤ .08; a non-normed fit index (NNFI) and a comparative fit index (CFI) ≥ .96 was considered to a have a good fit. In addition, the χ^2_M may not discriminate between good fitting models and poor fitting models (Kenny & McCoach, 2003). Thus, in case of a significant χ^2_M, we considered acceptable a model with relative chi-square/df ratio (χ^2/df) < 2 (Tabachnick & Fidell, 2007). Moreover, the chi-square difference (χ^2_D) and the Akaike Information Criterion (AIC) were used to compare the fit of respectively nested and non-nested models (Kline, 2011). Descriptive statistics, correlations and reliabilities are presented in Table 4.3.

Table 4.3

Correlations, means (M), standard deviations (SD), and reliabilities for measures of g, and WM.

Tasks	1	2	3	4	5	6	7	8	9	10	11	12	13
g													
1 CPM	1												
2 PMA-R	.45	1											
3 PMA-V	.45	.61	1										
WM													
4 SSPAN	.26	.28	.29	1									
5 DSPAN	.13	.20	.25	.54	1								
6 MSTG	.30	.39	.40	.26	.27	1							
7 MSTNG	.33	.41	.43	.36	.32	.72	1						
8 CWMS	.32	.37	.51	.41	.39	.31	.32	1					
9 LST	.25	.40	.45	.40	.46	.31	.30	.56	1				
10 VPTA	.33	.19	.32	.25	.26	.38	.30	.30	.38	1			
Geometry													
11 GEO-P	.30	.28	.15	.26	.22	.21	.17	.27	.29	.21	1		
12 GEO-Q	.32	.25	.33	.26	.18	.10	.16	.25	.28	.26	.33	1	
13 I-GEO	.43	.36	.42	.25	.21	.32	.38	.32	.21	.37	.24	.20	1
M	28.26	16.30	20.73	41.80	46.28	39.50	29.53	26.57	27.43	59.81	5.82	4.09	25.87
SD	4.93	4.08	7.34	8.58	8.15	10.14	10.03	6.65	6.78	11.75	3.47	1.82	6.13
Reliability	.82	.78	.93	.69	.70	.83	.83	.77	.83	.91	.59	.50	.81

Note. Zero order correlation; all coefficients ≥.148 are significant at .05 level; CPM=colored progressive matrices; PMA-R=primary verbal abilities reasoning; PMA-V=primary mental abilities verbal; *S*SPAN=syllable span; DSPAN=number span; MSTG=matrix span task grid; MSTNG=matrix span task no-grid; CWMS=categorization working memory span; LST=listening span task; VPTA=visual pattern test active; GEO-P=geometrical problems; GEO-Q=geometrical questions; I-GEO=intuitive geometry; Reliability=Cronbach's alpha.

4.2.3.2. SEM models

Model 1. In this model we tested geometrical achievement (measured by geometrical problems and questions) as an endogenous variable (dependent) and all the other constructs as exogenous variables (i.e., STM-V, STM-VS, WM and I-GEO). Since, I-GEO has only one indicator, we fixed the error variance of the intuitive geometry task (IGT) to .19 (1-Reliabilty) (see Kline, 2011). The fit of the model was good (Table 4.4; Figure 4.3). Probably due to the high correlation between latent factors, all the paths from exogenous to the endogenous variables were not significant. Thus, we decided to consider other models.

Model 2. Differently to Model 1 we considered *g* as mediating the relationship between working memory (i.e., STM-V, STM-VS, WM), intuitive geometry (I-GEO) and geometrical achievement (GEO-ach) (Figure 4.3). The fit of the model was good and the AIC was lower than model 1 (Table 4.4). Thus this model was preferable compare to Model 1.

Model 3a. In this model we considered intuitive geometry as an endogenous variable (i.e., dependent). The fit of the model was good and the AIC was lower than models 1 and 2. Thus, this model was preferred because more parsimonious compare to Model 1 and 2 (Figure 4.3; Table 4.4). Nevertheless, the path from intuitive geometry (I-GEO) to geometrical achievement (GEO-ach) was not-significant. For this reason, we attempted to eliminate the path.

Model 3b. In this model we eliminated the path from intuitive geometry (I-GEO) to geometrical achievement (GEO-ach) (Figure 4.4). This model revealed a better fit compared to the others (i.e., lower AIC) (Table 4.4). In addition, since model 3b is nested to model 3a the chi-square difference between the two models was calculated (Table 2). We found a non-significant chi-square difference between the two models ($\chi^2_D(1)=0.01$, $p=.92$), which indicates that model 3b is preferable, because more parsimonious. Thus, we retained model 3b (Figure 4.4). In this model the 42% of geometrical achievement variance and 39% of the intuitive geometry variance was predicted by intelligence and WM (with the intelligence's mediation).

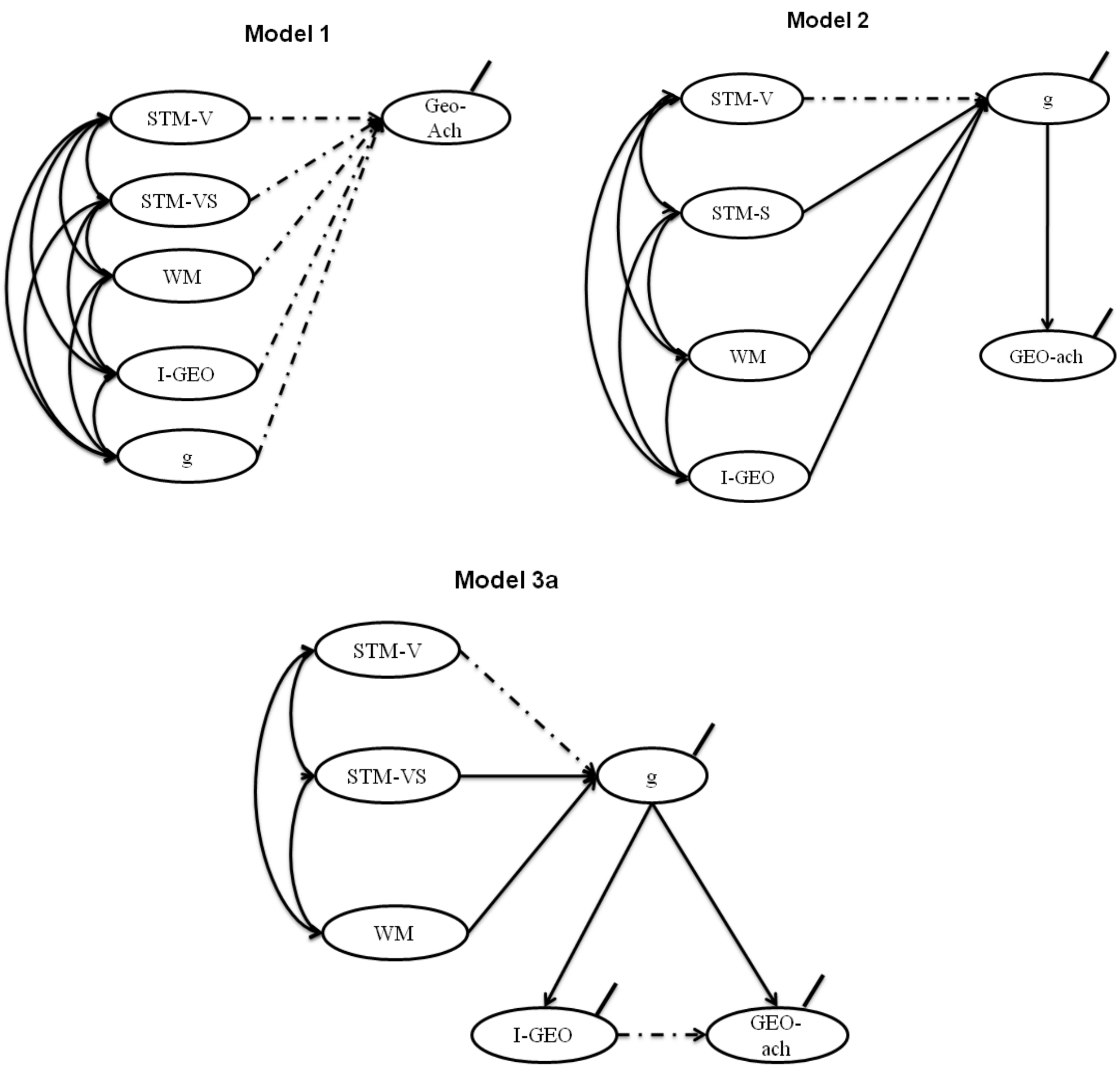


Figure 4.3 Structural models of the relationship between WM, GCA and geometry.

Note. Path significant at .05 level are indicated by solid lines. STM-V=verbal short term memory; STM-S=spatial short term memory; WM=working memory; I-GEO=intuitive geometry; g=general cognitive ability; GEO-ach=geometrical achievement.

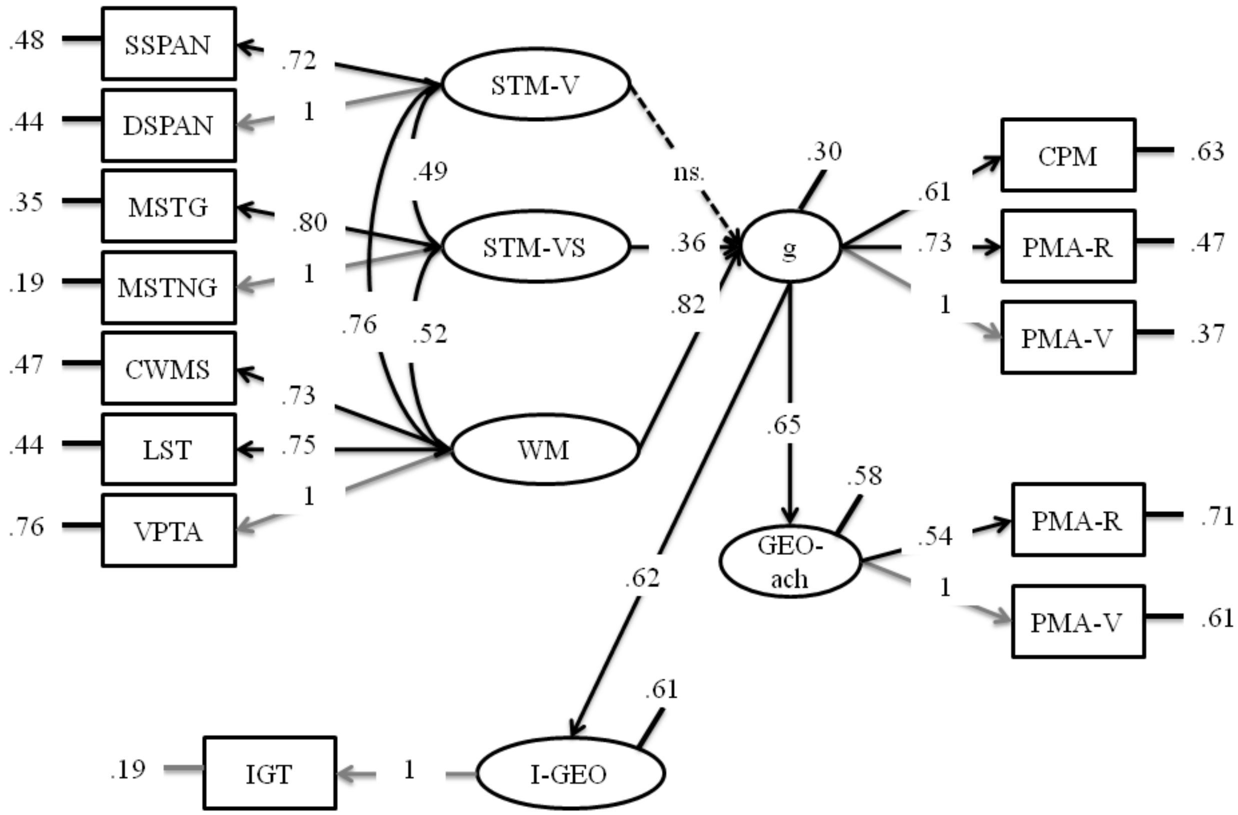


Figure 4.4. *Measurement model of the relationship between WM, GCA and geometry.*

Note. Path significant at .05 level are indicated by solid lines and path in gray are fixed. Ns = not significant. STM-V=verbal short term memory; STM-S=spatial short term memory; WM=working memory; g=general cognitive ability; GEO-ach=geometrical achievement; I-GEO=intuitive geometry; IGT=intuitive geometry task.

Table 4.4

Fit indices

Model	χ^2_M (*df*)	P	χ^2/df	RMSEA	SRMR	CFI	NNFI	AIC
(1)	80.74 (51)	.005	1.58	.06	.05	.98	.97	158.09
(2)	89.90(55)	.002	1.63	.06	.06	.98	.97	157.08
(3a)	92.24(57)	.002	1.61	.06	.06	.98	.97	156.31
(3b)	92.25(58)	.003	1.59	.05	.06	.98	.97	154.41

Note. χ^2_M=model chi-square, RMSEA=Root mean square error of approximation; GFI=goodness of fit index, CFI=comparative fit index, SMSR=standardized root mean square residuals, AIC=Akaike Information Criterion.

4.2.4. Discussion

The main aims of the present study were, i) to examine the best model able to predict the relationship among WM, *g*, and intuitive geometry ; ii) to study the specific contribution of *g*, WM, and intuitive geometry in explaining academic achievement in geometry variance and the strength of this association.

The distinction between intuitive geometry and academic achievement in geometry is supported by previous research findings (Clements, 2003, 2004; Dehaene et al. 2006; Izard & Spelke, 2009; Spelke, et al., 2010). The relationship between intuitive geometry, academic achievement in geometry and visuospatial working memory has been previously tested by Giofrè, et al. (2013) revealing that different VSWM were significantly related to intuitive geometry and academic achievement in geometry. Specifically, an active visuospatial task and STM-VS task predicted intuitive geometry, whereas only a STM-VS with the mediation of intuitive geometry predicted academic achievement.

However, differently from Giofrè, et al. (2013), in the current research not only visuospatial but also verbal STM and WM tasks were presented. Moreover, given that formal education in geometry involves verbal rules, such as formulas and theorems, different measures of intelligence were collected in order to understand the role of other factors that presumably could affect the acquisition of geometrical knowledge (Aydin & Ubuz, 2010).

Different models were carried out in order to study the relationship between WM, *g*, and geometry. In our final model we concluded that intuitive geometry did not mediate the relationship between WM and academic achievement in geometry, as previously observed in Giofrè et al. (2013). Instead, intuitive geometry test was highly related to intelligence and in particular to the Raven's Colored progressive matrices (which are arguably a good measure of fluid intelligence; e.g., Jensen, 1998). On the basis of this result, we should hypothesize that some aspect of intuitive geometry are related to reasoning abilities. Indeed, the intuitive geometry test requires one to make use of basic geometric concepts such as points, lines, parallelism, or right angles to detect the odd-one out in simple pictures (Dehaene et al., 2006) but solving these tasks also requires mentally manipulating the stimuli and, as well as reasoning on the alternatives. Thus, intuitive geometry, which is supposed to be independent from culture as suggested by Dehaene et al. (2006; see also Spelke, et al. 2010), share a large portion of the variance with *g* (which is also been found to largely independent from learning and education; e.g., Bouchard, 2004).

In the present work, we found that WM and *g* explain a large portion of academic achievement in geometry variance. Nevertheless, the opposite can be argued, i.e. studying geometry may increase the ability to solve problems, which are needed in a complex society in which we live. In fact, national intelligence depends on cultural values, social structures, and economic resources (Hunt, 2012). In addition, national intelligence is highly dependent on environmental variables, such as education (Rindermann & Ceci, 2009). Thus, the study of geometry (e.g., trigonometry) is relevant because cognitive abilities are highly dependent on society's mental cognitive artifacts (which are ways of thinking that are used to reason about the phenomena; see Hunt, 2012).

Our final model revealed that geometrical achievement in geometry is predicted by WM and *g*. A similar relationship has been observed between math achievement and WM (e.g., De Smedt et al., 2009). In addition, the relationship between geometrical achievement in geometry, WM and *g*, extend previous results in which WM explained a small (14%) but significant portion of academic achievement in geometry variance (Giofrè et al., 2013). In fact, when the general factor is included in the model the explained variance increased up to the 42%. Furthermore, we found that only *g* has a direct effect on academic achievement variance. Moreover, WM and STM-VS (visuospatial short term memory) indirectly predicts a significant portion of academic achievement variance.

Contrary to previous results, we did not find a direct path form a WM to academic achievement in geometry (Giofrè et al., 2013). It is worth noting that in Giofrè et al. (2013) students attending secondary schools were tested, while in the current research children of the last years of primary school were examined. This finding can be attributable to the fact that our WM and *g* factor were highly related (they share the 65% of the variance). Thus, academic achievement variance may be explained from the shared *g*, WM variance. In fact, in our findings STM-VS and WM were significantly related to intelligence (see Study 3 part I). This result is coherent with findings obtained with adults participants and it confirms that STM-VS, typically involving unfamiliar situations, predicts a unique portion of variance not explained by active WM (Kane et al., 2004), whereas the verbal component of WM, typically involving less unusual material, is not relevant. Furthermore, our results confirm that WM has a strong positive effect on the *g* factor (.82) and STM only accounts for a small part (Conway et al., 2011).

Geometry is included in all the mathematical curricula in the world, and in international assessments like PISA (OECD, 2010). It has been argued that PISA proficiency scores predict educational outcomes (Fischbach, Keller, Preckel, & Brunner, 2012) and that there is a positive effect of geometry education on the improvement of spatial intelligence (Gittler & Glück, 1998). For these reasons, academic achievement in geometry play a crucial role in the development of complex math skills needed in STEM disciplines.

The present study, although it offers some insightful findings, has some limitations. First, we only tested children attending the final two years of primary school. Further studies are warranted to address this issue using other age groups. Second, our WM factor involved two verbal and only one visuospatial working memory tasks. Thus, it is possible that the presence of more than one VSWM task should either increase the portion of variance explained, or determine a direct link to academic achievement in geometry, without the mediation of *g*. Third, other spatial abilities may be related to geometry. In addition to verbal and visuospatial STM and WM tasks, further studies should analyze the role of visuospatial abilities, such as spatial visualization and mental rotation skills in academic achievement in geometry. Finally, as suggested by Aydin and Ubuz, (2010) motivational and metacognitive factors could be associated with both intuitive geometry and school achievement in geometry and should be tested in further studies.

In summary, this study showed how WM, *g*, and academic achievement in geometry are closely related and share a large portion of the variance. In other words, the success in geometry achievement is crucially related to other higher cognitive function, such as WM and *g*. Thus, it can be argued that improving academic achievement in geometry may result in an improvement in individual cognitive artifacts, which in turn may produce and increment in the society's mental cognitive artifacts which are needed in our complex society.

5. General Discussion

5.1. Research overview

Geometry is a fundamental part of the mathematical learning. The principal aim of the present dissertation was to investigate the relationship between geometry, WM, and intelligence in typical and atypical development. In fact, geometry, and in particular geometrical achievement, is highly related to higher order cognition, but this relationship was not investigated in detail. In the present dissertation, we found that a great portion of variance is shared between academic achievement in geometry, WM and intelligence. Thus, we provide an important insight on the processes involved in geometrical achievement.

In the first study, we found a significant relationship between VSWM and academic achievement in geometry. This relationship was mediated by the intuitive geometry task (which measures core and culturally mediated aspects of geometry). We extended these results in the third study.

In the third study, we considered geometry (intuitive and academic achievement in geometry), WM and the *g* factor. We found that a large portion of academic achievement in geometry variance is explained by WM with the mediation of the *g* factor; and that a large portion of intuitive geometry variance was explained by the *g* factor.

In the second study, we focused on a sample of NLD children. Importantly we found that NLD children fail in geometrical task and that NLD children have difficulties on core and culturally mediated concepts of geometry.

5.2. Aims of the present dissertation

5.2.1. Relationship between VSWM and geometry (Study 1)

The first aim of the present dissertation was to investigate, in a secondary school sample, the relationship between various aspects of geometry and VSWM. In the first study, we found that VSWM, with the mediation of the intuitive geometry task, were significantly related to academic achievement in geometry. Both, core and culturally mediated principles of geometry were significantly related to academic achievement. Importantly, only the culturally mediated aspects of geometry mediated the relationship between VSWM tasks and academic achievement in geometry. In addition, one of the VSWM tasks was directly related to the academic achievement in geometry. Moreover, the model accounted for the 14% of the variance. For these reasons, we confirmed our hypothesis that VSWM is related to intuitive and academic achievement geometry. Nevertheless, due to the relatively small amounts of variance explained, we argued that other factors may be involved (e.g., intelligence).

5.2.2. Nonverbal learning disabilities and geometry (Study 2)

The second aim of the present dissertation, was to understand if children with NLD symptoms showed difficulties in intuitive and academic achievement geometry. In fact, we know that NLD children have difficulties in VSWM (Mammarella & Cornoldi, 2005a; 2005b). Thus, we hypothesized that they could also have deficits in geometry, which requires VSWM. In the second study, we administered the intuitive geometry task, various VSWM tasks and a task measuring academic achievement in geometry to children with NLD symptoms as well as a control group. We confirmed our hypotheses; in particular, we found that children with NLD symptoms fail in intuitive geometry task (in both core and culturally mediated concepts). Further, children with NLD symptoms have also difficulties in the academic achievement in geometry. To sum up, we confirmed that NLD children fail in geometry and that geometry is an important predictor of NLD symptoms.

5.2.3. Geometry, intelligence and working memory (Study 3)

The third aim of the present dissertation was to investigate the relationship between geometry, WM and intelligence. In a first part, we considered several WM models and we found that the Baddeley and Hitch (1974) tripartite model provided the best fit to the data. In addition, we found that the relationship between WM and intelligence was extremely high (R^2=.65). In a second part, we included geometry in the analysis. We found that geometrical achievement was indirectly predicted by WM (in particular WM and STM-VS) with the mediation of the *g* factor. Further, including the *g* factor in the analysis the relationship between intuitive and achievement in geometry was no longer significant. These findings, which extend the results obtained in the first study, show that intelligence explains a relevant portion of the academic achievement variance. Notably, the intuitive geometry task share a large portion of the variance with the *g* factor. These two aspects may be somehow related to each other. In fact, these two aspects are not considered to be entirely dependent from culture.

5.3. Theoretical and applied implications

Our findings also have theoretical and applied implications. First of all, they can provide teachers and educators with information on which cognitive processes support students learning geometry. Secondly, shedding light on the mechanisms influencing academic achievement could help us to understand why students sometimes fail in geometry and how we can help them to cope with these difficulties. Geometry has been central to the historical development of mathematics, and concepts such as abstraction generalization, deduction and proof are an important part of our scientific reasoning.

In the first study, we found a relationship between VSWM and geometrical achievement. This finding has important theoretical implications. In fact, spatial abilities and geometry were previously studied (Bishop, 1980; Brown, & Presmeg, 1993; Piaget & Inhelder, 1967). But, the study was the first to consider the important role of VSWM. We found that the 14% of the academic achievement variance was explained by VWSM with the mediation of intuitive geometry. The study also suggested that other factors may be important: one of these factors is intelligence.

In fact, in the third study we found that geometry is highly related to intelligence, extending the findings from the first study. In particular, we explained a relevant portion of the academic achievement in geometry variance (R^2=.42). This result shed light on the mechanisms influencing academic achievement helps us to understand why some students fail in geometry. In fact, geometry is highly related to reasoning and WM. This finding could help us in developing interventions aimed to help children cope with their difficulties in geometry.

In the second study, we considered a sample with atypical development. Previous findings indicated that children with NLD performed significantly worse than did children with typical development in VSWM tasks and in arithmetic tasks associated with visuospatial processes (Mammarella et al., 2010). Since geometry is related to VSWM, we hypothesized that this group may also have difficulties in geometry. Hence, we found that children with NLD symptoms have specifics difficulties in geometry. This study has important clinical implication; in fact, geometrical tasks can be now used as predictors of NLD symptoms .

5.4. Avenues for further studies and limitations

Although the investigation of geometrical abilities is noteworthy as well as highly motivating for the several implications in practical field, the issue leaves open several other aspects that may be addressed in further research.

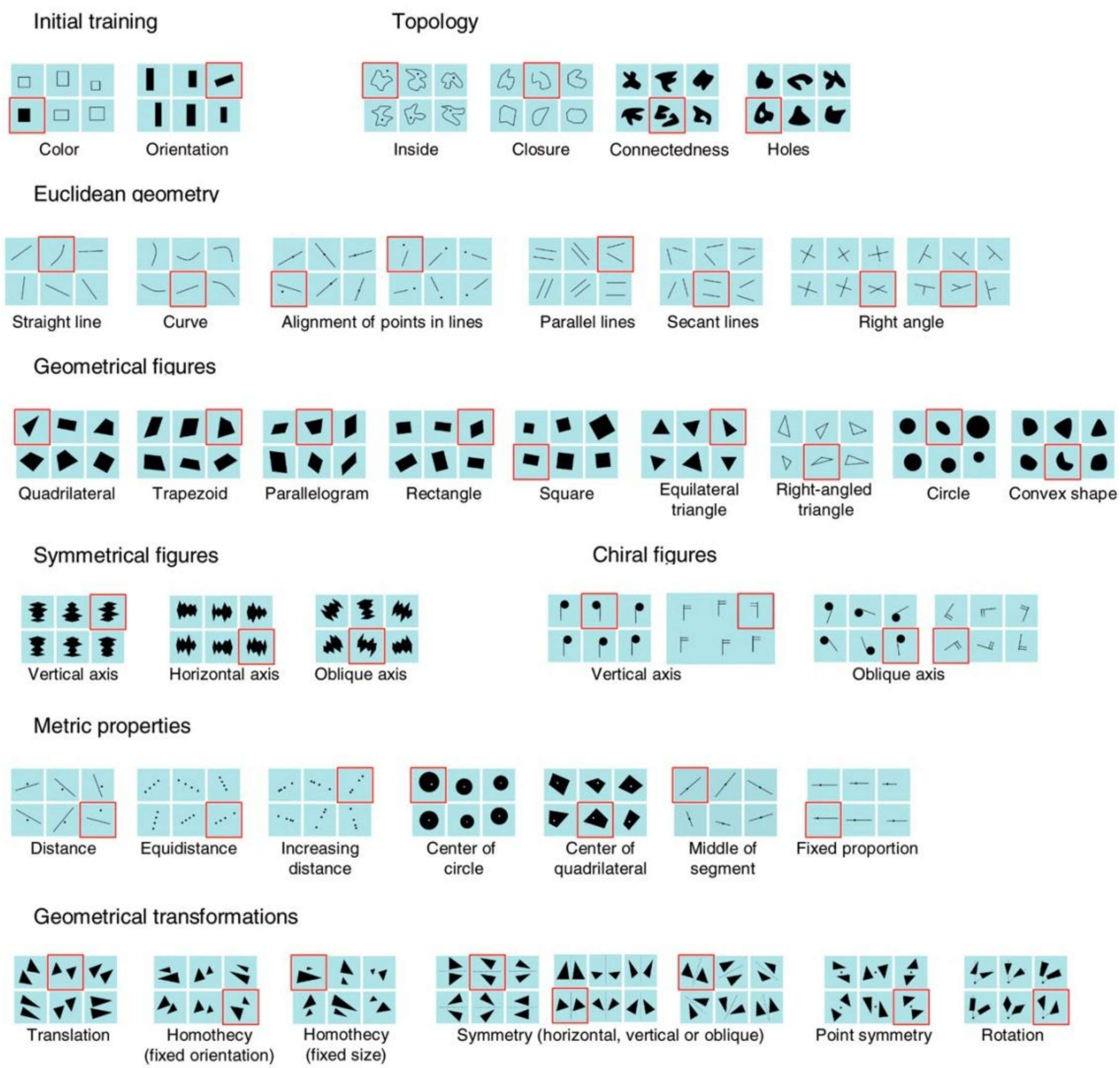
First, we only considered VSWM as a measure of spatial abilities. For example, future research may focus on the relationship between large scale systems measures (such as spatial navigation tasks), and achievement in geometry. In fact, we found that the 42% of the variance in geometrical achievement was explained by WM and *g*. Thus, a large portion of the variance remains unexplained. In fact, it has been argued that meta-cognitive factors are important in geometry (Aydın & Ubuz, 2010). Further studies are needed to clarify the impact of metacognitive abilities in geometrical achievement.

Second, the geometrical curriculum in many countries includes geometry but, unfortunately, often with insufficient results. In Italy, for example, there is a large difference in geometrical achievement between northern and southern Italian regions. In fact, northern regions performed well above international norms in geometry; conversely, southern regions performed poorly (see TIMSS 2011 results; INVALSI, 2012). It has been argued, that differences in academic achievement tasks in Italy can be mainly attributable to genetic factors (Lynn, 2010, 2012). On the contrary, we believe that these differences are largely dependent on environmental factors (Cornoldi, Belacchi, Giofrè, Martini, & Tressoldi, 2010; Cornoldi et al., 2013). For this reason, we believe that compulsory education in geometry should be improved. Thus, it will be interesting to study whether academic achievement in geometry can be improved in poorer regions (such as, for example, southern Italy).

Third, academic achievement in geometry might be improved with specific trainings. In fact, there is considerable empirical evidence that WM training can improve performance not only in WM-related tests (e.g., Beck, Hanson, Puffenberger, Benninger, & Benniger, 2010; Mezzacappa & Buckner, 2010) but also generalized to high order cognition such as reading comprehension or reasoning (e.g., Borella, Carretti, Riboldi, & De Beni, 2010; Carretti, Borella, Zavagnin, & De Beni, 2012). Hence, it will be interesting to understand whether a WM training, and in particular, a VSWM training, may be effective in improving the performance in geometrical achievement. For example, in the case of children with NLD, previous findings on single cases revealed that a VSWM training may also have an impact on their mathematical and geometrical difficulties (Mammarella, Coltri, Lucangeli, & Cornoldi, 2009); however, these effects have never been studied with groups of children with this kind of developmental disability. There is no evidence that trainings in geometry are effective in improving VSWM. In fact, only a positive effect of geometry education on the improvement of spatial abilities has been found (Gittler & Glück, 1998), but there is no evidence on improvement in VSWM.

To conclude, since ancient times, geometry was considered an important part of mathematical learning. In fact, geometry is highly related to spatial abilities (Clements, 2003, 2004), working memory (Giofrè et al., 2013) and reasoning. We believe that geometrical achievement in geometry is related to higher order cognition and to the capacity to reason and solve problems. This ability, is crucial in our complex society and in STEM related fields (Lubinski, 2010). Thus, higher order cognition (Hunt, 2012) and consequently geometrical achievement, should have important socio-economical implications in our post industrial complex society.

6. Appendix: The intuitive geometry task



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Summary in Italian/Sommario

Lo studio della geometria è una parte fondamentale dell'apprendimento matematico ed ha una storia antica. Basti pensare come, ai tempi i cui Platone insegnava, l'ingresso nella scuola di Atene era proibito a coloro che non conoscevano la geometria. La geometria, inoltre, in epoca rinascimentale, faceva parte del 'quadrivium', ed era considerata uno studio necessario per intraprendere gli studi di filosofia. A dispetto dell'importanza che la geometria ha avuto nel passato, i processi cognitivi che sono alla base della geometria non sono ancora stati studiati in maniera dettagliata.

Il presente lavoro di tesi si propone tre obiettivi. Primo, indagare la relazione tra vari aspetti della geometria e la memoria di lavoro visuospaziale (VSWM). Secondo, verificare se ragazzi con sindrome non verbale (NLD) presentino difficoltà in vari aspetti della geometria. Terzo, investigare la relazione tra vari aspetti della geometria, la memoria di lavoro (WM) e l'intelligenza (*g*).

Nel secondo capitolo, viene fornita una panoramica sulla relazione tra geometria, WM e *g*. Dato che la geometria riguarda lo studio dello spazio, essa richiede un coinvolgimento attivo delle abilità spaziali. La WM, ed in particolare la VSWM, inoltre, sono coinvolte in maniera attiva in compiti geometrici. Risolvere problemi geometrici, in aggiunta, richiede di ragionare sul problema e trovare una soluzione tra le tante alternative possibili. Per questa ragione, l'intelligenza (*g*), è coinvolta in maniera attiva nella soluzione di problemi geometrici.

Nel terzo capitolo, viene discussa la relazione tra geometria intuitiva (quella parte della geometria che sembra essere indipendente dalla cultura), geometria scolastica (la geometria che viene insegnata a scuola) e la VSWM. Vengono presentati due studi. Nel primo studio, è stata svolta una ricerca su 166 ragazzi frequentanti gli ultimi due anni della scuola secondaria di secondo grado. Lo studio prevedeva la presentazione di: 1) sei prove di VSWM, .2) una prova di geometria intuitiva (suddivisa in due parti: riguardanti principi core e mediati dalla cultura) 3) una prova di geometria scolastica. Dai risultati emerge come due prove di VSWM sono relate ad aspetti geometrici mediati dalla cultura i quali, insieme con principi ‘core’, che si pensa siano indipendenti dalla cultura, spiegano una porzione significativa di varianza delle prove di scolastica (14%). Nel secondo studio, la relazione tra VSWM e geometria (intuitiva e scolastica) è stata studiata considerando partecipanti con sintomi non verbali (NLD; i quali hanno problemi con prove spaziali, ma non con prove verbali). Lo studio ha preso in considerazione 16 partecipanti con NLD e 16 partecipanti appartenenti al gruppo di controllo. Dai risultati emerge come partecipanti con NLD cadano: i) in prove di geometria intuitiva (in aspetti ‘core’ e mediati dalla cultura), ii) in prove di geometria scolastica. partecipanti con NLD, inoltre, cadono anche in prove di VSWM. I risultati della ‘discriminant function analysis’, infine, confermano come prove di VSWM e geometriche siano importanti nel discriminare sintomi di NLD.

Nel quarto capitolo, viene discussa la relazione tra geometria, memoria di lavoro e intelligenza . Nella prima parte dello studio viene analizzata la relazione tra WM e il fattore *g*. In un primo momento sono stati valutati diversi modelli di WM e il modello tripartito di Baddeley e Hitch (1974) è risultato essere quello che meglio si approssima ai dati (miglior fit). In un secondo momento, abbiamo analizzato la relazione tra il modello tripartito e il fattore *g*. L’analisi dimostra come due componenti della memoria di lavoro (memoria a breve termine verbale e memoria di lavoro) spighino una porzione consistente della varianza di *g* (65%). Nella seconda parte dello studio, vengono confrontati vari modelli concorrenti sulla relazione tra vari aspetti della geometria (intuitiva e scolastica), WM e g. Il modello con il migliore adattamento ai dati mostra come WM, con la mediazione del fattore *g*, spieghi una quota significativa di varianza della geometria scolastica e della geometria intuitiva. In aggiunta, i risultati dimostrano come una quota significativa di varianza sia condivisa tra il fattore generale e la geometria intuitiva.

Nel quinto capitolo, viene presentata una panoramica generale degli studi presentati. Vengono, inoltre, evidenziati i limiti degli studi e i possibili sviluppi per studi futuri.