# Turbulent friction in flows over permeable walls

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[1] The experimental results of Nikuradse and the concept of hydraulically smooth, transitional, and rough flow regimes are commonly used as a benchmark for data interpretation and modeling of hydraulic resistance. However, Nikuradse's experiments were carried out in pipes with impermeable rough-walls whereas many geophysical flows occur over permeable walls and thus the permeability effects need to be quantified and accounted for. On the basis of our own experimental results, it is shown that wall permeability influences flow resistance dramatically and that the conventional 'hydraulically-rough regime', for which the friction factor depends only on the ratio of the roughness size to the flow thickness, does not apply to flows over permeable walls. Indeed, even at high Reynolds number (*Re*), the friction factor progressively increases with increasing *Re*. Possible mechanisms that explain this behavior, as well as the implications of these results for modeling of the friction factors and hyporheic exchange in porous-bed rivers are discussed. **Citation:** Manes, C., D. Pokrajac, V. I. Nikora, L. Ridolfi, and D. Poggi (2011), Turbulent friction in flows over permeable walls, *Geophys. Res. Lett.*, *38*, L03402, doi:10.1029/2010GL045695.

## 1. Introduction

[2] The traditional approach to quantify flow resistance in wall bounded flows is to use a friction coefficient or factor that can be defined in a variety of ways. Among the common definitions, the Darcy-Weisbach friction factor that represents the ratio of frictional forces to inertial forces $f = 8\frac{u_*^2}{U_b^2}$, where $u_* = \sqrt{\frac{\tau_0}{\rho}}$ is the friction velocity, $\tau_0$ is the mean shear stress acting at the wall surface, and $U_b$ is a bulk velocity (i.e., averaged over the whole flow thickness). In his pioneering studies on flow resistance in rough pipes, *Nikuradse* [1933] elucidated that $f$ is basically a function of two parameters: (i) the roughness Reynolds number, $Re_* = \frac{u_* d}{\nu}$ (where $\nu$ is the kinematic viscosity and $d$ is the characteristic size of the roughness elements) that quantifies the ratio of the roughness size $d$ to the thickness of the viscous sub-layer, which scales with $\frac{u_*}{\nu}$; and (ii) the relative submergence $\frac{D}{d}$, where $D$ is the pipe diameter. Note that some follow up results, such as the Moody's diagram, are based on a global $Re = UD/\nu$ instead of the roughness $Re_*$. *Nikuradse* [1933] observed that, with increasing $Re_*$ a turbulent flow in a pipe experiences three different flow regimes, namely the *smooth*, *transitional* and *hydraulically-rough* regime. For a given relative submergence, the friction factor decreases with Re in the smooth regime, then increases in the transition regime until it reaches a plateau in the hydraulically rough regime. This behavior is commonly interpreted as a result of the roughness elements being progressively exposed to the turbulent flow, with the viscous sub-layer thickness diminishing with increasing $Re_*$.

[3] The experimental results of *Nikuradse* [1933] and the proposed concepts of hydraulically-smooth, transitional, and rough-wall (bed) flow regimes have been a major breakthrough in fluid mechanics and are now commonly used for data interpretation and prediction of flow resistance in wall bounded flows. However, these results were obtained from experiments involving flows in impermeable-wall pipes, whereas many geophysical flows occur over permeable boundaries.

[4] It is intuitive to expect that, in terms of flow resistance, a permeable wall behaves differently from an impermeable one. Indeed, for the latter case, the effective roughness felt by the flow is limited to the size of the roughness elements characterizing the wall surface. In contrast, for the case of a permeable wall, this limit is set by the thickness of the wall itself. Furthermore, it is plausible that momentum penetration within a permeable wall increases with $Re_*$. As a result of this, the effective roughness and the associated friction factor should increase with $Re_*$ until, in principle, the whole wall thickness is exposed to fluid shear. The described concept is partly confirmed by some studies from the literature [*Ruff and Gelhar*, 1972; *Zagni and Smith*, 1976; *Zippe and Graf*, 1983; *Manes et al.*, 2009] although none of these provides compelling evidence to show that friction factors increase due to momentum penetration.

[5] Despite the results reported in the literature, the effects of wall permeability on flow resistance are often neglected. The most striking example comes from studies dedicated to the modeling of friction factors in gravel bed rivers where hydraulically-rough conditions are generally assumed and the dependence of $f$ on $Re_*$ is commonly neglected. One of the aims of this letter is to raise the attention to this issue. Furthermore, this letter contributes to the current knowledge by (i) providing a series of experiments where flow resistance was investigated in turbulent open channel flows over gravel beds to complement the experimental results for artificial walls already available in the literature, (ii) discussing the most appropriate definition of friction factors for flows over permeable walls, and (iii) discussing the physical mechanisms responsible for the increase of friction factors even within a range of *Re*, where one would expect a hydrodynamically rough behavior.

## 2. Description of the Experiments

[6] All the experiments were performed in a water flume at the University of Aberdeen. The flume is 11 m long and

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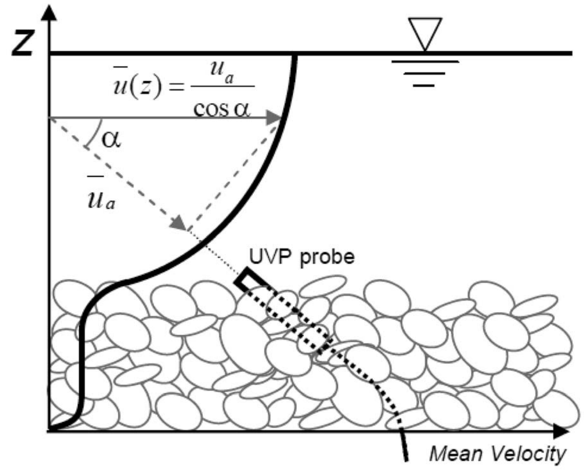


**Figure 1.** Display of the UVP probe within the permeable rough bed.

0.4 m wide. Bed slope, water discharge and flow depth were measured for three bed conditions: (i) a smooth perspex bed; (ii) a single layer of gravel grains with nominal diameter $d$ = 10 mm; (iii) a 6 cm thick layer of gravel grains with the same diameter as in (ii). The permeability of the porous rough bed was $k = 2.4 \times 10^{-8}$ m$^2$. We chose gravel as the permeable material because it is widely encountered in geophysical flows and many of the results presented herein are directly relevant to river hydrodynamics. We will refer to (i) as the smooth bed, (ii) as the rough-impermeable bed, and (iii) as the rough-permeable bed. For bed condition (iii), in addition to the bulk characteristics (i.e., bed slope, flow rate, depth), mean velocity profiles were also measured above the bed by means of a Met-Flow Ultrasonic Velocity Profiler (UVP). A 4MHz UVP probe was inserted within the bed with a known angle and with the emitting head facing the free surface (Figure 1). Once the probe was submerged by the flow, the system could measure velocities along the direction parallel to the physical axis of the probe (i.e., $\overline{u_a}$). Mean streamwise velocities were then computed by assuming that the mean vertical velocity is zero and by using the probe inclination angle, i.e., $\overline{u}(z) = \overline{u_a} \cos \alpha$, (Figure 1). For each experiment, UVP allowed horizontal mean velocity measurements at 128 positions along the whole flow depth with a spatial resolution of roughly half a millimeter. More details on the UVP system and the flume used in these experiments are described elsewhere [*Metflow*, 2000; *Manes et al.*, 2009].

[7] For bed conditions (ii) and (iii), experiments were conducted by keeping the flow depth constant and varying the bed slope. In analogy with the experiments of *Nikuradse* [1933], this allowed maintaining a constant relative submergence $H/d$ (where $H$ is the flow depth defined as the distance between water surface and the bed-roughness tops, Figure 2), while varying $Re_*$ and therefore highlighting the effects of $Re_*$ on friction factors. This procedure was repeated for two and three values of $H/d$ for the impermeable and permeable bed configurations, respectively. For each submergence, the upper limit of $Re_*$ was bounded by either the onset of sediment motion or the Froude number exceeding 1. Thus, in order to focus specifically on the effects of boundary friction on flow resistance, only subcritical conditions were considered. The range of $Re_*$ investigated in the experiments with beds (ii) and (iii) was from 127 to 597, i.e., it corresponded to the conventional hydraulically-rough regime.

[8] Experiments with the smooth and rough-impermeable beds (i.e., bed conditions (i) and (ii)) were mainly performed to reproduce results already available in the literature and hence verify the reliability of the experimental set up. The main focus of this paper is on the results obtained from the experiments with the rough-permeable bed (bed condition (iii)).

## 3. Definition of Friction Factors in Flows Over Permeable Walls

[9] Let us consider the case of a uniform, steady and turbulent open channel flow over a permeable bed. The flow

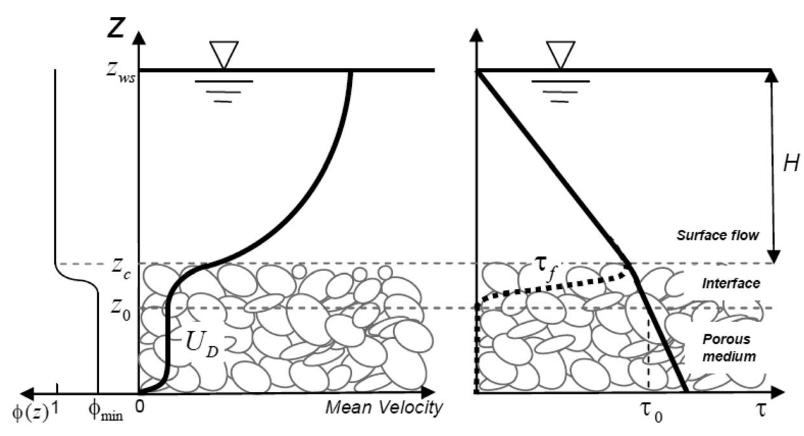


**Figure 2.** Parameters of interest for defining friction factors in flows over permeable beds. (left) A hypothetic profile for the mean velocity (thick solid line) and for the porosity function $\phi(z)$ (thin solid line) over and within a permeable bed. $U_d$ and $\phi_{\min}$ are the undisturbed velocity within the bed and the bulk porosity of the bed respectively. (right) The solid line shows the momentum supply between any level $z$ and the free surface. For $z_0 < z < z_c$ the momentum supply is partially absorbed by the drag and partially transferred by the fluid shear stress (dashed line). The momentum supply at the level $z_0$ is denoted with $\tau_0$.

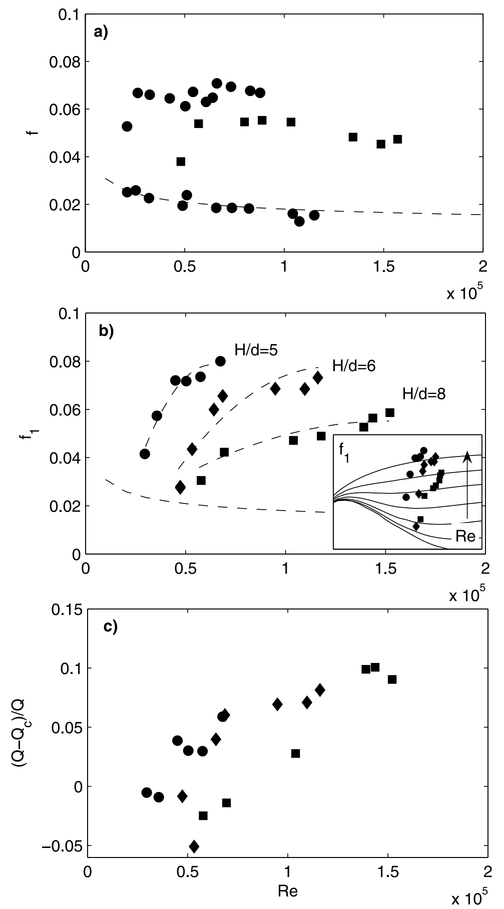


**Figure 3.** (a) Friction factors for the impermeable-rough bed; (b) friction factors for the permeable-rough bed; (c) residual discharge for the permeable-rough bed experiments. The dashed lines in Figures 3a and 3b correspond to the Blasius-Karman relationship for smooth walls. The inset in Figure 3b shows the friction factors for the permeable-rough bed (symbols) overlapped with the Moody diagram (solid lines). Relative relative roughness (submergence) increases (decreases) in the direction of the arrow.

domain can be split in three distinct regions, here called (1) the *porous medium* region where the flow can be studied by means of porous media hydrodynamics; (2) the *surface* flow region that often exhibits boundary layer type behavior; and (3) an *interface* region where the surface flow and the porous medium region interact (Figure 2). According to this flow partition, we define $z_c$ as the height of the roughness tops and $z_0$ as the level at which the total fluid stress $\tau_f$, imposed by the surface flow, vanishes within the porous bed. It is reasonable to assume that this level coincides with the elevation where the mean velocity profile within the porous bed becomes constant (Figure 2) (although in granular beds of particular regular packing this level may also correspond to a local minimum [*Pokrajac et al.*, 2007]).

[10] The other parameters for our considerations include: $h = \int_{z_0}^{z_c}\phi(z)\,dz$ (where $\phi(z)$ is the wall porosity function, Figure 2), i.e., the effective height of fluid volume contained within the interface region; $\tau_0 = \rho g S(H + h)$, which is the total weight per unit surface of the fluid contained between $z_0$ and $z_{ws}$, projected along the mean flow direction, where $\rho$ is the fluid density, $g$ is the gravity acceleration, $S$ is the bed slope, and $H$ is the flow depth; the Double Averaged mean velocity profile $\langle\overline{u}\rangle(z)$, i.e., averaged first in time and then in space within thin boxes parallel to the bed (overbar and angular brackets stand for time and spatial averaging respectively; for more details on the Double Averaging Methodology (DAM) see *Nikora et al.* [2007]); the bulk mean velocity $U_c = \frac{1}{H}\int_{z_c}^{z_{ws}}\overline{u}(z)dz$ within the surface flow region; the bulk mean velocity $U_L = \frac{1}{h}\int_{z_0}^{z_c}\langle\overline{u}\rangle(z)\phi(z)dz$ within the interface region; and the undisturbed velocity $U_D$ within the porous media region, which comes from the balance between gravity and drag forces. The above velocities relate to the specific bulk discharge as $q = Q/B = U_cH + U_Lh + U_DD$, where $B$ is the width of the flume and $D = \int_0^{z_0}\phi(z)\,dz$ is the effective thickness of the porous bed below the interface region (Figure 2).

[11] In open-channel flow hydrodynamics, friction factors have been introduced primarily for engineering applications and thus their definition follows the need of quantifying the amount of water passing through a stream cross-section per unit time at given flow depth and bed slope. In the case of flows over permeable beds, a friction factor can be defined in several ways depending on the particular needs. If we are interested in a combined flow rate through the *surface* and the *interface* flow regions, then a friction factor may be defined as $f = 8\frac{u_*^2}{U_b^2}$, where $U_b = \frac{U_cH+U_Lh}{H+h}$ [*Poggi et al.*, 2009] and, assuming 2-D flow conditions, the friction velocity can be estimated as $u_* = \sqrt{gS(H+h)}$.

[12] As far as our experiments are concerned, $f$ could be easily estimated for the impermeable-rough bed case, because $U_b$ and $u_*$ are known. Indeed, $Q$, $H$, $B$, $S$ were measured and $h$ is constant since $z_0$ coincides with the flume bottom. In contrast, for the permeable-rough bed case, $f$ is difficult to estimate because the position of $z_0$ is likely to be flow-dependent and is not known a priori. As an alternative, a definition can be implemented, focusing on the assessment of the *surface* flow rate defined by $U_c$, i.e., $f_1 = 8\frac{gSH}{U_c^2}$, where $U_c$ in our experiments was computed from the integration of the mean velocity profile measured by UVP. The interrelation between $f$ and $f_1$ can be presented as:

$$f = 8\frac{u_*^2}{U_b^2} = 8\frac{gS(H+h)^3}{(U_cH+U_Lh)^2} = f_1\frac{\left(1+\frac{h}{H}\right)^3}{\left(1+\frac{U_L}{U_c}\frac{h}{H}\right)^2} = f_1\frac{\epsilon_1^3}{\epsilon_2^2}. \quad (1)$$

Since the ratio between interface and surface mean velocity is less than unity, i.e., $\frac{U_L}{U_c} < 1$, then $\frac{\epsilon_1^3}{\epsilon_2^2}$ is greater than one. This means that $f_1$ always underestimate $f$ and the difference increases with increasing $\frac{h}{H}$ and hence with increasing momentum penetration within the bed.

## 4. Results and Discussion

[13] Figures 3a and 3b report plots of the friction factor *vs* bulk Reynolds number $Re = U_bH/\nu$. $Re$ is used in place of $Re_*$ because it allows inclusion of smooth-bed data, and also for consistency with already published data. It is encouraging to see that for the impermeable bed configurations, friction factors behave as reported in the literature. The smooth bed data follow the well known Blasius-Karman relationship. The rough-impermeable bed data at different submergences are constant within experimental uncertainty, except for the point at the lowest Reynolds numbers that

shows some transitional behavior. This indicates that the flow was mostly at the hydraulically-rough regime. Friction factor curves for the permeable rough bed configuration are very similar to those reported by *Zagni and Smith* [1976] and *Ruff and Gelhar* [1972]. In particular, friction factors display increasing trends at any Reynolds number without reaching a hydrodynamically rough plateau (this is particularly evident for the curve associated with the highest relative submergence, which contains the widest range of *Re*). At low *Re* such increasing trend may still be justified by standard transitional behavior, but not at higher *Re* where $Re_*$ approaches values close to 400–500 and therefore viscous drag acting on the grains located at the bed surface is expected to be negligible compared to pressure drag. It is worth noting that the increasing trend of $f_1$ reported in Figure 3b would be even steeper for $f$, because, as discussed in section 3, $f_1$ underestimates $f$ as *Re*, and hence momentum penetration, increases. Therefore, the increasing trend in $f_1$ is a robust result, i.e., it does not depend on how exactly friction factors are defined.

[14] We now discuss a range of possible physical mechanisms that may explain the *Re* dependence of the friction factors observed in Figure 3b. As introduced in section 1, the increasing trend of friction factors can be explained by a progressively deeper momentum penetration within the porous bed as *Re* increases. In turbulent flows over porous media, the effective hydrodynamic roughness is not related to the size of the grains composing the bed, but rather to the thickness of the interface region, i.e., $h$. Therefore, it is intuitive to state that an increase in $h$ (i.e., an increase in momentum penetration) corresponds to an increase in the effective roughness height and, in the case of flows with constant depth, to a decrease in relative submergence $H/h$. In a standard Moody diagram, the decrease in relative submergence implies a jump from a lower to a higher line of relative roughness and therefore to an increase in the friction factor (inset of Figure 3b).

[15] Some support to the hypothesis of momentum penetration increasing with *Re* can be found in Figure 3c: since the flow rate within the porous media region is negligible, the ratio $(Q - Q_c)/Q$ (where $Q_c = U_c\ (HB)$) represents an estimate of the amount of water flowing within the interface region normalized with the total discharge $Q$. Figure 3c clearly shows that $(Q - Q_c)/Q$ increases with increasing *Re*, which means that the amount of water flowing within the interfacial region increases as *Re* increases. This behavior is consistent with our hypothesis, although it does not prove it because the increase of $(Q - Q_c)/Q$ could also be explained by an increase of mean velocities within a fixed depth of momentum penetration. It is worth noting that at low *Re*, $(Q - Q_c)/Q$ assumes negative values in Figure 3c because $Q_c$ is computed as if the mean velocity profile measured in the central cross section of the flume extends across the whole width, i.e., side-wall effects are neglected. A correction of $Q_c$ that includes side wall effects was not attempted because we are not interested in the absolute values of $(Q - Q_c)/Q$ but rather in its dependency on *Re*.

[16] Another possible mechanism that may contribute to the *Re* dependence of the friction factors is associated with the nature of drag around sheltered and buried grains. It is plausible that, due to the sheltering effects of the overlying grains, some buried grains may not generate pressure drag even at high *Re*. With increasing *Re*, however, more and more sheltered particles may experience flow separations leading to the growing total drag. This can produce a rise of the friction factors with increasing *Re* even if the depth of momentum penetration (i.e., $h$) remains constant.

[17] At this stage we cannot assess the contributions of the described mechanisms to the observed effect of growing resistance factors with *Re*. However, it is likely that a combination of mechanisms acts to produce the observed effect rather than a single dominant mechanism.

[18] It is important to note that, at sufficiently large *Re*, the conventional picture of a constant friction factor maybe restored because the depth of momentum penetration is limited by the depth of the porous bed. However, the critical *Re* restoring the conventional picture may be sufficiently high that it may not be attained in many practical applications.

## 5. Implications

[19] The common assumption that at $Re_* \gg 70$, river flows are in the hydraulically-rough regime and friction factors are dependent only on relative submergence [see, e.g., *McGahey et al.*, 2009], has to be refined. River beds are often porous and therefore a Reynolds number dependence of friction factors due to permeability must be expected. This result has direct implications for river engineering and earth sciences. For example, it is common practice to estimate resistance coefficients for river flows at periods of low flow rates when it is easy to measure. By assuming hydraulically-rough conditions, the same coefficients are then used to estimate water levels during floods, when the flow *Re* is much higher. Such a procedure may lead to a significant underestimation of water levels because, according to our results, with increasing *Re* the flow resistance exerted by porous beds is significantly enhanced.

[20] Our results also provide explanation for an unresolved issue concerning hyporheic exchange in open channel flows over porous beds. *Packman et al.* [2004] observed that the diffusion coefficients for solute exchange between open channel flows and (flat) gravel or sand beds were proportional to the square of the Reynolds number. The authors could not provide a definitive explanation for this behavior. However, they argued that, with increasing *Re*, momentum transfer between stream and pore-water flows may also increase and in turn enhance scalar diffusion. Our results provide support to this hypothesis. In fact, the increase of friction factors with *Re* observed in our experiments is consistent with the idea of enhanced momentum transfer at the sediment water interface.

## 6. Conclusions

[21] Flow resistance in turbulent boundary layers over porous walls has a peculiar behavior. By means of the experimental results presented herein, we have shown that for a given relative submergence, friction factors increase as *Re* increases without displaying a classical hydraulically rough plateau even when $Re_* \gg 70$. We have demonstrated that such a rise in friction factors is likely to be associated with a progressive increase of momentum penetration into the porous bed and/or with gradual involvement of sheltered bed particles in the generation of pressure drag. We have then discussed the implications of our results in the context

of modeling friction factors and hyporheic exchange in gravel bed rivers.

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