View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Illinois Digital Environment for Access to Learning and...

CIR 280
c. 2

STATE OF ILLINOIS
WILLIAM G. STRATTON, *Governor*
DEPARTMENT OF REGISTRATION AND EDUCATION
VERA M. BINKS, *Director*

AUG. 26th 1818

Visual Estimates of Grain Size Distribution In Some Chester Sandstones

David H. Swann
Robert W. Fisher
Mathias J. Walters

DIVISION OF THE
ILLINOIS STATE GEOLOGICAL SURVEY
JOHN C. FRYE, *Chief* URBANA

CIRCULAR 280 1959

ILLINOIS GEOLOGICAL
SURVEY LIBRARY
DEC 18 1959

ILLINOIS STATE GEOLOGICAL SURVEY
3 3051 00003 8210

VISUAL ESTIMATES
OF GRAIN SIZE DISTRIBUTION
IN SOME CHESTER SANDSTONES

Digitized by the Internet Archive
in 2012 with funding from
University of Illinois Urbana-Champaign

http://archive.org/details/visualestimateso280swan

VISUAL ESTIMATES
OF GRAIN SIZE DISTRIBUTION
IN SOME CHESTER SANDSTONES

David H. Swann, Robert W. Fisher, and Mathias J. Walters

ABSTRACT

This investigation was undertaken to develop a rapid semiquantitative method for obtaining large amounts of reasonably accurate data on the distribution and size of grains in consolidated sandy rocks.

Grain size distribution was estimated by measuring, under a stereoscopic microscope fitted with an eyepiece micrometer, grains judged visually to represent the maximum size and the fifth, sixteenth, fiftieth, eighty-fourth, and ninety-fifth percentiles.

Although less accurate than mechanical analyses, the visual method proved as accurate as the sample-to-sample variability of most sediments warrants. For routine examination of well samples, estimation of only the maximum and median grain sizes provided two significant size measures plus a measure of sorting and could be completed in two minutes. The phi scale was used to record the estimates.

Portions of the method were applied in a study of sandstones of Chester (Mississippian) age in southern Illinois. Grain size in the Tar Springs Sandstone correlates with local variation in sand-shale ratio. The relation apparently holds for other sandstones of Chester age. Grain size distribution in the Aux Vases Sandstone could support any of three possible origins of the formation: 1) from the Ozarks to the west; 2) the lower part from the Ozarks and the upper from the Canadian Shield to the north; or 3) all from the Shield. In the last case the lower coarser beds in the west correlate with sandy beds to the east that have been placed in the underlying Ste. Genevieve Limestone. Stratigraphic, mineralogic, and cross-bedding evidence from other studies favor the third interpretation.

INTRODUCTION

Size and sorting of individual grains in a sandstone are important factors in determining the rock's characteristics as an oil or water reservoir, its economic uses, and its geologic history. Many individual size analyses of ancient sandstones are available but there are very few published maps, cross sections, or profiles showing geographic or stratigraphic variations of grain size in consolidated rocks. Neglect of this important factor in facies and stratigraphic studies can be attributed to the wide gap between quantitative analysis of grain size by sieve or settling methods on one hand and qualitative description of grain size on the other.

Quantitative methods demand good samples. Quantitative or semiquantitative analyses of unconsolidated materials are readily made, and size studies based on scores, or even hundreds, of samples of Recent and Pleistocene deposits are reported. However, the difficulties of collecting and disaggregating suitable samples of most consolidated Paleozoic sediments preclude their analytical study on so ambitious a scale. The amount of material needed for standard quantitative techniques restricts their application to outcrop or core samples. Although well logs and cuttings from oil tests furnish voluminous general information on the sedimentary rocks of this country, they have provided comparatively little data on grain size.

Qualitative descriptions of grain size leave much to be desired. Such phrases as "fine-grained silty sandstone," "fine- to coarse-grained sandstone," "medium-grained, poorly sorted sandstone" are helpful but lack precision. Experienced subsurface geologists differentiate certain sandstones in cuttings readily and consistently, yet apply identically worded descriptions to them. A more accurate vocabulary is needed to express the differences the expert discerns but does not adequately describe.

This paper discusses methods of deriving quickly and easily more precise grain size information from well cuttings. The method favored is the visual selection of grains representing the maximum grain size found in a sample plus the fifth, sixteenth, fiftieth (median), eighty-fourth, and ninety-fifth percentiles. Although the grains are picked subjectively, their widths, which correspond in essence to their sieve size, are measured objectively under a stereoscopic microscope fitted with an eyepiece micrometer. For routine well sample examination, picking only the maximum and median sizes gives two significant size measures and a useful measure of sorting in minimum time. The phi scale, the logarithmic scale of grain size that has been used largely for statistical studies of mechanical analyses, is particularly efficient for recording such estimates.

Sands, sandstones, siltstones, and the acid-insoluble residues of carbonate rocks of Illinois and neighboring states, ranging in age from Cambrian to Recent, have been studied. Most of the examples are from sandstones of the Chester (Mississippian) Series. Data on repeatability of maximum size estimates and the relation of sand-shale ratios to grain size in the Tar Springs Sandstone were contributed by Fisher. Walters made the study of geographic variation in the Aux Vases Sandstone.

ACKNOWLEDGMENTS

Part of the material presented has served as the basis for masters' theses submitted to the University of Illinois by Fisher (1956) and Walters (1958). The theses were prepared under the guidance of J. L. Hough, Professor of Geology, and David H. Swann, of the Stratigraphy and Areal Geology Section of the Illinois State Geological Survey. Prof. Robert A. Wijsman, Department of Mathematics, University of Illinois, contributed advice.

A number of staff members of the Illinois State Geological Survey also have assisted. Elwood Atherton, T. C. Buschbach, G. H. Emrich, Howard Schwalb, and J. B. Vercellino participated in the development of the methods reported. Mechanical analyses by D. L. Biggs, M. E. Ostrom, Buschbach, and Swann have been compared with visual estimates.

MEASUREMENT OF SAND GRAINS

Comparison of Methods

Sediment grains are measured in three general ways. They can be sized by dropping them through a graduated series of sieves, their effective sedimentation size can be computed from their hydraulic settling rate, or they can be measured directly by comparing them visually to a scale. Sieving procedures are most commonly used for sands, settling for silts and clays, and direct measurement for gravels, although modifications extend the range of each method. Hydraulic settling methods resemble most closely the actual deposition of sediments, but the reliability and convenience of sieve analysis make it the standard of comparison in the sand size range.

If all sedimentary particles were spheres of the same mineral, different methods of measurement would give comparable results. However, differences in grain shape and density affect the methods unequally. Poole (1957) compared the three methods in an analysis of sand-sized sediments and artificial mixtures that differed markedly in the shape and specific gravity of the constituent grains.

Hydraulic analyses are reported in terms of the effective sedimentation diameter - the diameter of a quartz sphere that would settle at the same rate as the grain. Grain density directly affects the hydraulic method because the heavier minerals settle faster and thus seem larger than quartz grains of the same actual size. To a lesser degree specific gravity affects the results of such methods as the standard sieve technique in which results are reported by weight rather than volume by indicating more sediment in those sizes in which heavy grains are concentrated, typically the finer sand and coarser silt. Large amounts of heavy minerals in some sediments might require adjustment of volume estimates to weight percentages, but the amount in Paleozoic sandstones of the Illinois region is so small that differences in density can be disregarded.

Grain shape is a more critical factor than density in measuring grain size of the rocks under consideration. Rod-shaped grains are found to be smaller by sieve analysis than by hydraulic analysis because shaking and tapping the sieve tilts the grains so that they present their minimum cross section to the screen and slip through regardless of their length. The effective sieve size of a grain is determined solely by this minimum cross section and is equal to or slightly smaller than the intermediate axis. It involves neither specific gravity nor length. Plate-shaped grains seem larger in sieve analyses than in hydraulic analyses, for no matter how thin and light a plate is, it cannot slip through a hole whose diagonal is smaller than the plate's diameter. Blades approaching plate proportions appear larger when measured by sieving; those approaching rod shape appear larger when measured hydraulically.

In any practical visual method applied to loose sand grains, the effects of shape are exaggerated. The loose particle tends to lie on its side with its greatest cross section exposed, its short axis pointing up toward the observer, and only the long and intermediate axes directly visible. The width is the intermediate axis and is more pertinent than the length. It is the available visual element closest to both the effective sedimentation diameter and the effective sieve size.

Throughout this study, size or diameter (unless otherwise specified) refers to the intermediate axis or the width of the loose grain as seen under a microscope. This visual size is equal to or slightly larger than the effective sieve diameter of a grain. The discrepancy is greatest in thin blades or in plates that fall diagonally through square screen holes but are seen in the microscope broad side up. Visual methods intensify the tendency of the sieve method to record thin plates or flakes as larger than sedimentationally equivalent spheres. The mica minerals are the most common flake-like minerals in the sand size range, and micaceous rocks such as the higher Pennsylvanian sandstones are difficult to evaluate by visual estimate.

If the orientation of a grain is not known (for example, a grain on the surface of a sandstone chip), the relation of the measured width to the effective sieve diameter is more complex than it is for isolated grains. The apparent width of the grain may have any value between the true intermediate and short axes, the apparent length any value between the intermediate and long axes.

However, the width tends to lie closer to the intermediate than to the short value, just as in two dimensions the length of the projection of an ellipse at an angle of forty-five degrees to the axes is proportionately closer to the long than to the short axis of the ellipse. In addition, quartz sand grains tend to be roller-shaped rather than blade-shaped, to have subequal short and intermediate axes.

The error involved in using the width seen in a random view as an approximation to the common visual or sieve diameter is not serious. In Paleozoic sandstones of the Illinois region, the typical error does not exceed a fifteenth part of the value measured, a tenth of a grade unit, despite the occurrence of rare plates or blades on edge with sieve diameters much greater than their apparent width.

Grade Scales and Notations

Most properties of sediments that vary with grain size are expressed more simply in terms of proportions or ratios of grain size than in terms of the arithmetic scale. To find a difference in properties comparable to that between grains 0.30 and 0.10 mm in diameter, a grain 1.80 mm across should be compared with another one-third that size, about 0.60 mm across, rather than with a grain 0.20 mm smaller or 1.60 mm in diameter. Thus ratio scales, in geometric rather than arithmetic progression, are most appropriate for describing sedimentary grains.

The limits of the grades commonly used by geologists in this country for description of grain size differ by the constant ratio 1:2. Coarse sand is half the size of very coarse sand; medium sand is half the size of coarse. The National Research Council (Lane et al., 1947), modifying scales proposed by Udden (1898, 1914) and Wentworth (1922), standardized the boundary between gravel and cobbles at 64 mm, between sand and gravel at 2 mm, between silt and sand at 1/16 mm ($62\frac{1}{2}$ microns), and between clay and silt at 1/256 mm (approximately 4 microns). The clay (down to 1/4096 mm or 1/4 micron) and silt ranges are each divided into four grades: very fine, fine, medium, and coarse. Sand and gravel each have an additional fifth grade, very coarse.

This grade scale is now used as a matter of course in describing sand grains or arenaceous rocks. It is applied not only to typical or average grains in a sediment, but also to the largest or smallest grains, or is even used in describing sorting (for example, when half of a sediment falls in a single size grade).

However, the grade scale in its standard form is cumbersome to subdivide or to manipulate statistically. The midpoint of any grade is an irrational number, the lower class limit multiplied by the square root of two. Other equal subdivisions involve higher roots. There are many advantages in assigning to each unit of the grade scale a number than can be divided decimally and that preserves the constant ratio feature of the scale. A logarithmic scale is such a device, because equal increments on the scale correspond to equal ratios between the original numbers. The grade scale can be changed into a logarithmic scale by assigning successive whole numbers to successive grades. Because the grades differ by a ratio of 1:2, the resulting logarithmic scale is to the base 2 rather than to the base 10 of common logarithms.

Krumbein (1934) suggested the application to the size grades of a logarithmic scale which he named the phi (ϕ) scale (pronounced "fee" by most sedimentary petrographers). In one sense this is not a new scale but rather an adaptation of a numerical notation to the named grades of the standard scale. In Krumbein's phi notation, the diameter in millimeters is replaced by its negative logarithm to the base 2. Use of the negative logarithm changes the sign so that all measurements less than a millimeter are positive, those more than a millimeter are negative. This is done so that positive numbers occur in the range commonly used. Phi numbers increase as grains get smaller. A sand grain exactly 1.0 mm in diameter has a value of 0.00 ϕ, a grain one-half mm in diameter has a value of 1.00 ϕ. The relation of phi values, millimeters, and size grades in the range of greatest interest is given in tables 1 through 3 in the appendix and shown graphically in figure 1.

A straight line on semilogarithmic graph paper plotted through any two points taken from table 1 in the appendix provides a simple phi-millimeter conversion chart. Phi values are read on the arithmetic scale and millimeters or microns on the logarithmic scale. Several conversion charts and nomographs have been published (Krumbein, 1934; Krumbein and Pettijohn, 1938; Inman, 1952). The phi-millimeter conversion table by Pàge (1955) is particularly convenient because of its long range and accuracy.

The phi notation has nearly replaced the use of millimeters in statistical studies of grain sizes and in many nonstatistical listings of quantitative size analyses. However, the named grade size is still used almost universally for qualitative field or subsurface descriptions of sandstones. The number of sieve meshes per inch is used in much industrial work in the sand sizes, and sedimentation diameter in microns (thousandths of a millimeter) is used in describing clay fractions. An arithmetic scale is adequate for describing single measurements, although they can be stated equally well in the phi notation. However, because of the tremendous range in the size of sedimentary particles, an arithmetic unit that is reasonable in one part of the range will be either too gross or overly precise in other parts. In the phi notation the size of the unit varies proportionately with the quantity being measured and is suitable throughout the range.

For such a simple mathematical operation as averaging, the phi scale is more appropriate than the measurement in millimeters. The difference between the arithmetic average (mean) of grains 0.41, 0.49, and 0.53 mm in diameter (0.477 mm) and the logarithmic mean (0.474 mm) is so slight that the added complexity of computing seems not worth while when compared with errors in the measurements. However, the arithmetic mean is always larger than the logarithmic mean; thus there is a systematic coarsening of size data averaged as millimeters and converted to grade sizes. An absurd case demonstrates the need for logarithmic treatment. Sand at the coarse end of a grade scale unit, say a coarse sand at 0.999 mm, will control the arithmetic average when combined with a single sample of medium, fine, or very fine sand, silt, or the finest clay, so that the average of the two sediments will remain above 0.500 mm, still in the coarse sand class. If phi values are averaged, the answer is equivalent to the logarithmic mean of the measurements in millimeters; the average of <u>any</u> fine sand (2 to 3 ϕ or 0.25 to 0.125 mm) with <u>any</u> coarse sand (0 to 1 ϕ or 1.0 to 0.5 mm) is in the medium sand range (1 to 2 ϕ or 0.5 to 0.25 mm).

There is no logical or practical reason for not recording size of sedimentary particles directly in phi units. This is no more difficult than recording in millimeters, and in most instances is easier. Subsequent operations are greatly simplified if the phi scale is used from the start. In early stages of this study measurements were recorded in millimeters, but the conversion from millimeter to phi value (or the equivalent operation of plotting measurements in millimeters on a logarithmic scale) took place at progressively earlier stages as the study advanced and finally only phi values were recorded.

The phi scale is particularly well suited for recording grain size as approximated in the field or subsurface laboratory by comparing the sample to a standard. This is because judgments such as "a little coarser" or "much finer" are more closely allied to ratios than to arithmetic differences. Most subsurface laboratories have a comparison slide or card made with sieve fractions corresponding to the units of the standard grade scale. (Many workers use such slides or a micrometer only when in doubt about the assignment of a sample, that is, only when the sample is near a size-grade boundary. This concentrates attention on the grade boundaries rather than on the sediment and carries the implication that accuracy is important near grade boundaries but has less significance in the rest of the size range. Uniform application of numbers instead of the named grade system will promote attention to accuracy throughout the range.)

The fraction labeled "medium sand" on a comparison slide includes grains whose effective sieve diameter ranges from 1.0 ϕ to 2.0 ϕ (0.50 to 0.25 mm). This represents better sorting than occurs in natural sediments. Although the distribution of sizes within this fraction depends on the character of the original sample, the assumption that 1.5 ϕ (0.35 mm) is the average will not be far amiss. A sample with typical grains about like those of the medium sand on the slide can also be assigned a value of 1.5 ϕ. A sample somewhat finer than the medium but considerably coarser than the fine sand of the comparison slide might receive a value of 1.8 ϕ, whereas a third sample somewhat coarser than the fine sand comparison slide but finer than the medium might be assigned a value of 2.3 or 2.2 ϕ.

In informal use of a comparison slide in this manner, about three subclasses of well sorted sandstone can be recognized within a named grade class. That is, it generally will be possible to rank well sorted sandstones consistently in order of size if their measures of central tendency differ by 0.3 or 0.4 ϕ. In poorly sorted sediments the typical grain diameter is much more difficult to determine, and perhaps even full phi units are too narrow to apply consistently. On the other hand, the diameters of single grains can readily be determined to an accuracy of 0.2 ϕ by using a comparison slide. To attain greater accuracy it is necessary to use a micrometer.

In summary, the logarithmic or phi size scale has many advantages both over the logarithmic but non-numerical descriptive Udden-Wentworth-NRC grade scale it amplifies and over the numerical but non-logarithmic millimeter scale. It shares the equal-ratio feature of the grade scale but permits and promotes greater accuracy and allows mathematical summarization of large masses of size data. Sedimentary processes, relations of one grain to another, and many mass properties of sediments vary as proportional or ratio functions of grain size. The phi unit adjusts automatically to the magnitude measured, whereas several different arithmetic units are necessary to encompass the wide size range shown by sedimentary particles. The logarithmic scale is a more natural choice for rapid estimation of grain sizes by comparison to a standard than is the arithmetic scale. It is as easy to convert micrometer scale readings to phi values as to millimeter equivalents. As is pointed out later, the distribution of grain size in many rocks resembles the normal probability distribution if sizes are expressed in logarithmic or phi terms, but departs widely from normal distribution if arithmetic measurements are used.

Eyepiece Micrometer

An eyepiece micrometer is a glass disc on which has been engraved a scale 5 or 10 millimeters long divided into 50, 100, or 200 parts. The disc is placed in the image plane of the eyepiece of a compound microscope and held by retaining rings or stops. In this study the coarsest, longest scale available for any microscope proved to be the most convenient. Confusion over which line was actually being seen at the edge of a grain when the finer micrometers were used was more troublesome than estimating fractions of the space between the markings of the coarser scales. A scale extending across most of the field of view allows measuring large grains at greater magnifications than does a shorter scale and does not interfere with other operations.

Precise positioning of the disc in the eyepiece is important. In eyepieces with a positive stop this is automatic, but in eyepieces in which the disc support is threaded care must be taken that the support is adjusted to sharp focus at the same point that is the most comfortable focus for the field. Unless the scale is continuously sharp during normal operation of the microscope so that no change in muscular eye tension is needed to bring it into clear relief, the micrometer will seldom be used.

The use of a micrometer in the eyepiece of a stereoscopic microscope is somewhat different from its use in a single tube microscope. The choice of placing the scale in the left or right eyepiece is personal. More complex is the problem presented by variation in scale in various parts of the field.

If the micrometer is in the right eyepiece, grains on the right of the field measure larger than grains on the left. If it is in the left eyepiece the reverse is true. At the lower powers of magnification with large fields of view the linear distortion is about four percent at either right or left edge of the field compared to measurements near the center. The distortion across the entire field is therefore about 0.1 ϕ. There is no appreciable linear distortion from top to bottom of the field. With higher magnifications the field is smaller and the distortion at either edge is only about one percent. However, in any case it is well to make calibrations and measurements near the center of the field or with the scale in a vertical 12 to 6 o'clock position that automatically centers the object.

Possible combinations of microscope bodies, eyepieces, objective lenses, and micrometers are innumerable. In general, the nominal enlargement of component parts cannot be used to find the relation between units on the micrometer scale and actual distances on the stage. Each combination must be calibrated separately. Calibration is discussed in the appendix and a phi-micrometer conversion table for one popular microscope is given as tables 2 and 3. The tables can be used only for the specific microscope, lens, and micrometer combinations given.

Measuring Single Grains

Grains from many types of sediments and rocks were measured under various conditions during this study. Whether the grain is dry, under water, or mounted in balsam, whether it is viewed by transmitted or reflected light, whether it is unwashed, washed, or acidized makes little difference in the value found. However, it is simpler to distinguish single grains from aggregates when they are clean, under a liquid of moderate refractive index, and viewed by transmitted light, than when they are dry, dusty, and seen by reflected light. Grains partially buried beneath other grains are not measurable, but a grain at the surface of a piece of consolidated sandstone will give very nearly the same value it would if it were isolated.

Disaggregation of consolidated sandstones to the point where individual grains can be measured visually is much easier than the complete preparation needed for sieve or hydraulic analysis. A small percentage of sediment left in aggregates can be disregarded in visual measuring, but disaggregation must be essentially complete if the grains are to be sieved.

The grain selected is brought near the middle of the microscope field and its width measured with the micrometer in the 12 o'clock position (or with the middle part of the scale if the micrometer is at some other attitude).

Study of loose grains is simple because an isolated sand grain under water in a watch glass is almost certain to be lying in its most stable position with its greatest cross section exposed. If the grains are dry, gentle tapping of the tray or slide brings most of the grains to their stable positions. The width of the grain is measured, as this is the intermediate axis of the grain and essentially the effective sieve diameter. In a grain with irregular outline the width is not necessarily perpendicular to the long axis but is the minimum distance between parallel lines tangent to the grain outline.

The measurement is made in the units of the micrometer scale. If the scale is rather coarse, estimates to a fifth or tenth of the unit should be made, particularly on smaller grains. As the units on the micrometer scale are arbitrary, all readings should be converted to phi units by a chart such as table 2 or 3 or to other standard measurements before they are recorded.

Measuring a single sand grain is simple, but the decision as to which particular grains in a sandstone should be measured is complex. One answer is obvious - measure them all, or at least enough that a reasonably accurate curve of the distribution of sizes in the sandstone can be drawn. The objection to this answer is also obvious - it takes too long. The number of measurements of loose grains suggested for mechanical analysis under the microscope varies from a few hundred to several thousand (Krumbein and Pettijohn, 1938, p. 296). The necessity of converting the resultant number frequency of different sized grains to the volume or weight frequency used in other procedures is a further drawback. Visual methods of size analysis generally have been considered the most time consuming of all.

By the method proposed here only a few grains are measured in any sample, but those grains are selected to give the most information. The general distribution of grain sizes in sediments determines which grains should be chosen.

GRAIN SIZE DISTRIBUTIONS

The distribution of grain size by weight percentages in a typical Chester sandstone is illustrated in figure 1 by a histogram of a mechanical analysis by Biggs and Lamar (1955, table 4, sample 10B). Superposed on the histogram is the distribution curve of the same data, which can be considered the best fitting or most probable continuous curve that keeps the area in each bar constant but replaces the flat tops of the bars by a single flowing curve. It is the limit that the histogram would approach were the sieve intervals made smaller and smaller, but it is actually constructed by taking values from the cumulative curve plotted on probability paper (Krumbein, 1938; Otto, 1939). The distribution curve constructed from data derived from an analysis of this sample for which sieves of different sizes were used should be very like this one, although the histogram would be different.

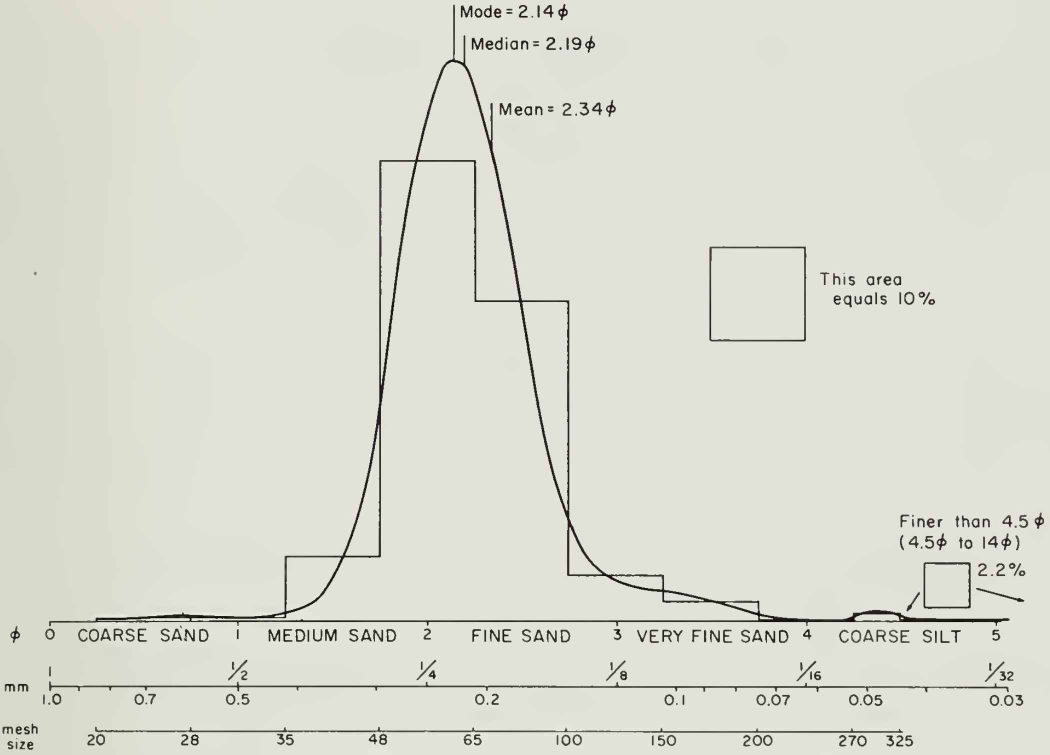

Fig. 1. - Histogram and size-distribution curve showing mechanical analysis of a composite sample of Cypress Sandstone 10 to 22 feet below top of road cut on Illinois Highway 37 at north edge of Cypress, NW¼ SE¼ NW¼ sec. 20, T. 13 S., R. 2 E., Johnson County, Illinois. Sample 10B of Biggs and Lamar (1955).

The phi distribution curve of a mechanical analysis resembles the "bell-shaped" curve of the normal probability function. The statistical measures or parameters that have been devised for similar distributions can be applied to it. In essence these parameters describe actual distributions in terms of their departures from the theoretical "normal" curve of random events. Many different measures of average or central tendency, of sorting or spread, of dissymmetry or skewness, and of peakedness or kurtosis have been devised to accomplish this. Some of them are based on percentiles, some on moments, some on other properties of the distribution.

Measures of central tendency include the mode, median, and mean. These are identical in a perfectly symmetrical distribution but not in most real distribution patterns.

The mode is the grain size at the high point on the distribution curve (fig. 1). It is the parameter used in describing the central tendency that represents the most common grain size by weight or volume in the sediment. If there are two high points separated by a saddle, the distribution is bimodal; if several, it is polymodal.

In the mechanical analysis in figures 1 to 3, the secondary mode at about 4.4 ϕ is artificial. It is caused by the adjustment of data from the three techniques

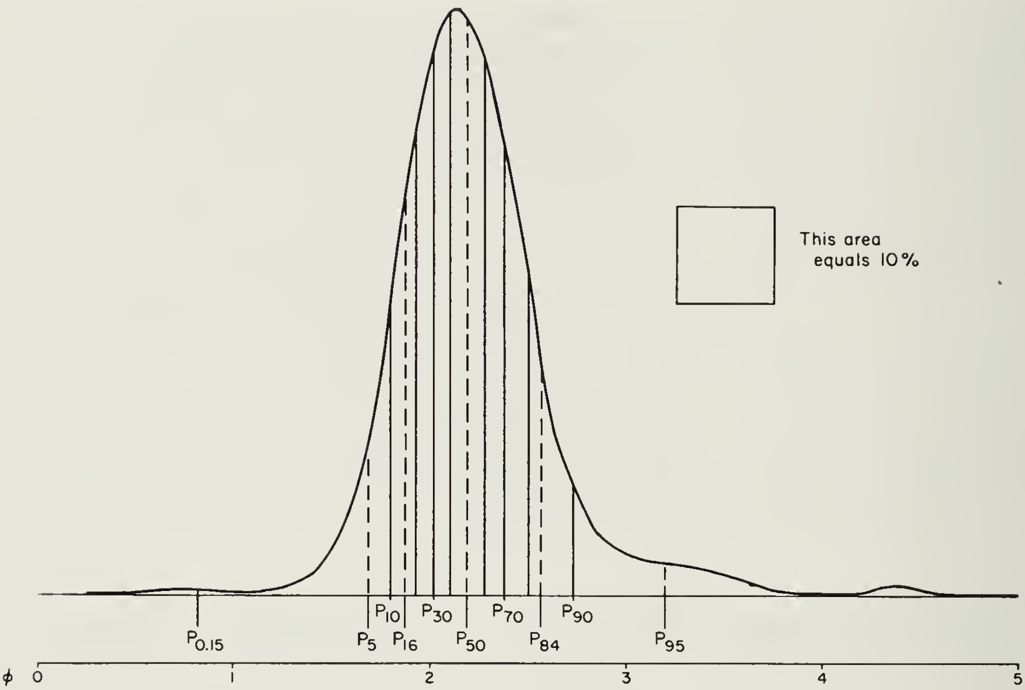

Fig. 2. - Distribution curve from figure 1 showing the ten-percentile divisions by solid lines, and other significant percentiles by dashed lines.

of dry sieving, wet sieving, and settling used in the analysis rather than by an actual concentration of grains of this size. There is a real but slight secondary mode about the thickness of a line above the base at about 10 ϕ, far to the right of the part of the curve reproduced. It results from the clay and limonite that coat the sand grains.

The mode has a definite physical meaning in sediments, but finding its precise value is difficult. Determinations of the mode vary more between duplicate analyses or even repeated plotting of the same data than do the other common parameters expressing central tendency. This is because the apparent position of the mode is extremely sensitive to slight changes in the shape of the line that must be drawn connecting the analytical datum points to form a continuous curve.

The median, which splits the distribution into coarser and finer halves, can be determined much more accurately than the mode by plotting cumulative data on probability paper. It shares with the mode the weakness that both must be determined by inspection rather than by purely mathematical treatment of the original analytical data, and therefore both are difficult to study theoretically. For instance neither the mode nor the median of a mixture can be foretold from the values for the components.

The median is a member of a family of phi values, including quartiles, deciles, and percentiles, that divide the distribution into fourths, tenths, hundredths, or other equal parts. For uniformity, measurements of this type are converted to percentiles. The median is the fiftieth percentile, P_{50}. Some sedimentary petrographers have used ϕ_{50} to distinguish the phi value of the median and reserved P_{50} for the millimeter value, but the distinction no longer seems necessary and the standard symbol for percentile is used in this paper to represent the phi value.

Percentiles are named from the coarse end; the fifth percentile, P_5, is the phi value finer than 5 percent and coarser than 95 percent of the sediment.

Figure 2 shows the distribution curve of figure 1 divided into tenths or deciles, that is, into units of ten percentiles each. The original histogram of figure 1 shows the distribution divided into units of equal width (0.5 ϕ) but of varying height and, therefore, area. In figure 2 the distribution is divided into units of equal area, each representing 10 percent of the sample, whose width is varied to compensate for the difference in height. The percentiles are crowded together in the central hump of the curve and spread out in the tails.

Several specific percentiles, $P_{0.15}$, P_5, P_{16}, P_{50}, P_{84}, and P_{95}, are indicated on figure 2. Though the curve of figures 1, 2, and 3 could be described by giving the phi values of many combinations of percentiles, this particular group is perhaps the best combination for describing simply but accurately most grain size distributions. Their advantage is that from them can be constructed graphic parameters similar to another entirely different family of descriptive measure, one based upon the moments of the distribution, the mathematical analogs to the mechanical moments of vector properties.

The first moment, the center of gravity of the distribution, is the mean, the familiar average obtained by summation and division. All of the moment measures can be calculated mathematically from the original data in a manner somewhat analogous to the first moment although the calculation is tedious (Inman, 1952; Krumbein and Pettijohn, 1938; any statistics text). Moment measures have been very thoroughly explored in statistical disciplines. They take the entire distribution into account, whereas most other measures reflect differences in some parts of the distribution but not in others.

In the more common types of distribution the mean is the most stable central statistic, varying less in duplicate analyses than any other. Its weakness in the study of some sediments, aside from the fact that its calculation is more laborious than the graphic solution for the median, lies in the leverage that the extremes of the distribution exert upon it and upon the other moment measures. Figure 1 illustrates this effect, showing how a small amount of clay pulled the mean well to the right. The usefulness of the mean for bimodal distributions and for open-ended analyses with an appreciable amount of sediment beyond the finest point determined is thus lessened.

The measure associated with the second moment is the standard deviation, which describes the sorting or spread of the data. It is the square root of the average of the squares of the distances of all points under the distribution curve from the mean, and is analogous to mechanical torque. It, like the first moment or mean, is a linear measure and is reported in phi units. About two-thirds of the sample in the theoretical normal distribution is within one standard deviation on either side of the mean.

The higher moments are less stable than the mean and the standard deviation. That is, they vary much more between duplicate analyses than does the mean. Also they are ratios, pure dimensionless numbers, whereas the mean and median are linear, measured in phi units.

The measure associated with the third moment, the skewness, is a cubic function of the distribution. It describes departure from symmetry and is positive for a distribution like that of figure 1 where values are closely bunched left of the central hump but spread out in a long tail to the right. A phi distribution with a sharp cut-off on the fine side but a long tail to the coarse left side has negative skewness.

The measure associated with the fourth moment, the kurtosis, is a fourth-power function of the distribution. Its value is 3.0 for the normal distribution. A curve with a large, broad, square-shouldered central hump and small tails is platykurtic, with a value less than 3.0. A curve with a high, relatively thin central spike and extended tails is leptokurtic, with a kurtosis measure greater than 3.0.

The problem in visual size analysis is to reproduce the essentials of the distribution curve. The reverse problem of choosing the most significant measures readily determined from the curve of a mechanical analysis is helpfully discussed by Inman (1952). He discards statistics expressing arithmetic relations as less significant than the logarithmic measures implicit in the phi scale. The parameters based on the mathematical moments are very significant but they are discarded as too laborious to compute and too strongly influenced by the extreme portions of the distribution that are most susceptible to error. Measures based on quartiles, easy to compute and widely used since their adaptation to sediments by Trask (1932), are limited in that they describe only the central half of the distribution and disregard both tails entirely. On the other hand, measures dependent upon the extremes of the distribution are handicapped by the difficulty of obtaining reliable data on the parts of a sediment beyond the fifth and ninety-fifth percentiles.

Inman concludes that parameters resembling the standard mathematical moment measures, but computed from a handful of significant points rather than from the entire distribution, offer the best compromise between ease of compilation and significance. Folk and Ward (1957) accept Inman's conclusions but modify his specific parameters in the light of their experience with strongly bimodal sediments that are pronouncedly non-normal with curves very unlike the bell-shaped curve of the probability function. All six of Inman's and three of Folk and Ward's four parameters (table 4 in the appendix) are calculated from the phi values of only five points on the cumulative frequency curves - P_5 (the fifth percentile), P_{16}, P_{50}, P_{84}, and P_{95}.

These five points are chosen for visual estimates of size distributions. An additional sixth point, the maximum grain size, is particularly well suited to visual estimates. It is standardized at about $P_{0.15}$ under optimum conditions with good samples.

These specific percentiles were chosen partly on theoretical and partly on practical grounds. In a normal distribution (but only exceptionally in actual distributions) 68.3 percent of the entire sample occurs within one standard deviation on either side of the mean, that is, between about P_{16} and P_{84}. A reasonable approximation to the standard deviation can therefore be made by taking half the total spread between the two points. In the theoretical normal curve, although not in other distributions, the inflection points between the central convex portion of the curve and the concave tails are at one standard deviation on either side of the mean. Thus there is a natural division between the central bulk and the extended tails of a normal distribution at about P_{16} and P_{84}. As P_{16}, P_{50}, and P_{84} are near the midpoints of the coarsest, middle, and finest thirds of the distribution, respectively, their average provides an approach to the mean.

In a normal distribution, 95.4 percent of the sample lies within two standard deviations of the mean, and thus P_2 and P_{98} in theory should well represent the extremes of the distribution (Cadigan, 1954; Tanner, 1958). But these points, particularly P_{98}, have the practical disadvantage of lying beyond the range of many analyses. In analyses that do extend this far, the variability of duplicates is much greater toward the extremes than between P_5 and P_{95}. Data on the fine end of a complete mechanical analysis are profoundly affected by analytical procedure.

Many sedimentary petrology laboratories use dry sieving followed by hydraulic analysis of the material passing through the finest sieve to the pan. Other laboratories follow American Foundrymen's Association or other commercial or engineering sand analysis procedures in which a wet sieve separation of fines from sand sizes is followed by drying and sieving of the sand and by hydraulic analysis of the still wet fines fraction. The initial wet sieving yields one to four or more percent fines from "clean" sandstones in which only a small fraction of a percent passes a dry 4.0 ϕ or 4.25 ϕ sieve. This material is not coarse or medium silt, but very fine silt and clay that clings to the dry grains. Sandstones that seem to be unimodal with reasonably normal distribution if analyzed dry first, are bimodal with a secondary mode in the 8-12 ϕ range, positively skewed and leptokurtic if wet-sieved first. This laboratory effect is dominant in the mechanical analysis beyond P_{95} for most sandstones and beyond P_{90} for a few.

The choice of P_5 and P_{95} is dictated by the practical need for reliability in measurements. These points lie 1.65 standard deviations respectively coarser and finer than the mean in the theoretical normal distribution and thus encompass 3.3 standard deviations.

Although points finer than P_{95} cannot be used profitably, the maximum grain size is peculiarly well suited to visual estimation. To obtain a true absolute maximum it would be necessary to search an entire sand body for the largest grain. In a normal distribution, 99.7 percent of the sample lies within three standard deviations on either side of the mean. A practical maximum can be established at $P_{0.15}$. An occasional exceptionally coarse grain representing about one part in seven hundred of the sediment can then be disregarded. The maximum so defined would be three deviations from the mean in the theoretical normal distribution. Most sandstones in the Illinois region are positively skewed, with less spread on the coarse than the fine end, so the difference between maximum ($P_{0.15}$) and median (P_{50}) is appreciably less than three standard deviations.

Conditions may be such that a point other than $P_{0.15}$ is more suitable for use as maximum size. In conglomeratic sandstones, or in samples diluted with much caved material, a single large grain may represent much more than one part in seven hundred of the available material. The maximum grain seen probably will then be more than fifteen hundredths of a percentile. Differences in sample size are not particularly important in the nonconglomeratic sandstones of this region that have a quite rapid, coarse cut-off, so the difference between, for example, $P_{0.1}$ and P_1 is much less than in a normal distribution.

VISUAL ESTIMATES OF THE DISTRIBUTION OF GRAIN SIZES

Complete Estimates

A visual estimate of the distribution of grain sizes in a sandstone is made by measuring individual grains estimated to represent the maximum size (about $P_{0.15}$), the size limiting the coarsest twentieth (P_5), the limit of the coarsest sixth (P_{16}), the median (P_{50}), the limit of the finest sixth (P_{84}), and the limit of the finest twentieth (P_{95}). Figure 3 repeats, as a solid line, the phi distribution curve of figures 1 and 2 and shows the position of these significant percentiles computed from the analysis. The position of these same percentiles, estimated visually for a split of the same sample, is indicated on the dashed distribution curve derived from the estimates.

Preparation of sandstone samples for visual analysis is simple compared to the complete disintegration essential for mechanical analysis. A small percentage of sandstone grains left in aggregates can be overlooked in visual examination. Some samples used in this study had been prepared and analyzed mechanically by standard methods (Krumbein and Pettijohn, 1938; Milner, 1940; Biggs and Lamar, 1955). Many well samples, particularly of the more friable sandstones, are so disaggregated in drilling that no further preparation is necessary before visual determination. Some calcareous sandstones disintegrate sufficiently if a few drops of acid are put on them. More commonly a few representative chips are crushed gently under water in a watch glass with a pencil eraser or the wooden handle of a teasing needle. Enough sand grains can be rubbed from many hand specimens with the fingers, a needle, or forceps, though a properly prepared and split crushed sample will be more representative than a few hundred grains rubbed off a surface.

Involuntary sorting in picking the pinch of material to go under the microscope is a big factor in the experimental error of this method. Use of a microsplit is justified if estimates for good unconsolidated or disintegrated samples are to be compared with mechanical analyses but not justified for most well samples from which a few chips are used and the entire crushed yield is examined. In well samples of unconsolidated or friable sands, the standard practice of using an extra tray and examining the middle cut of the sample as it is poured from container to tray or from tray to watch glass seems satisfactory.

The choice of individual grains to measure is admittedly subjective. The maximum grain size is the most nearly objective, and is normally the first measurement made on a sample. It is the only point in the distribution that can be estimated visually as accurately as it can be determined from a mechanical analysis. Estimates of maximum grain size made in this study have varied from some that were deliberately a full percentile off the true maximum to others in which large samples were diligently searched and the maximum grain found was within the coarsest tenth, or even hundredth, of a percentile. Theoretical considerations make the value $P_{0.15}$, which is coarser than all but one part in seven hundred of sediment, the standard to be used unless very small samples or coarse grains make such an extreme value impractical.

Estimates of maximum grain size in sandstones are most valid if they are confined to quartz and similar detrital minerals. Fossils, mica, and authigenic grains of glauconite, pyrite, siderite, or calcite are disregarded.

The median grain size, P_{50}, is in general the most reliable of the percentile estimates other than the maximum and is normally picked second. A median grain is picked subjectively by imagining the sample divided into two piles, one consisting of grains larger than a certain size (or even than a certain specific grain) and the other of grains smaller. The mental image is adjusted until the size of the two imaginary piles is equal, and the width of the grain serving to discriminate the two is measured. The judgment is easiest if the sandstone is well sorted, with a single mode near the median. As the percentiles are crowded near the mode (fig. 2), a relatively large error in picking the percentile will result in only a small error on the phi scale. The median estimate is difficult if the sand is bimodal, particularly when the median is situated in a deep saddle between the modes. In such cases the median itself is of little significance, for a minor change in the proportions of the two components responsible for the modes would cause a big change in the position of the median.

Other percentile points are picked in an analogous manner. The fifth percentile, P_5, is commonly the third point estimated, chosen by picking the size

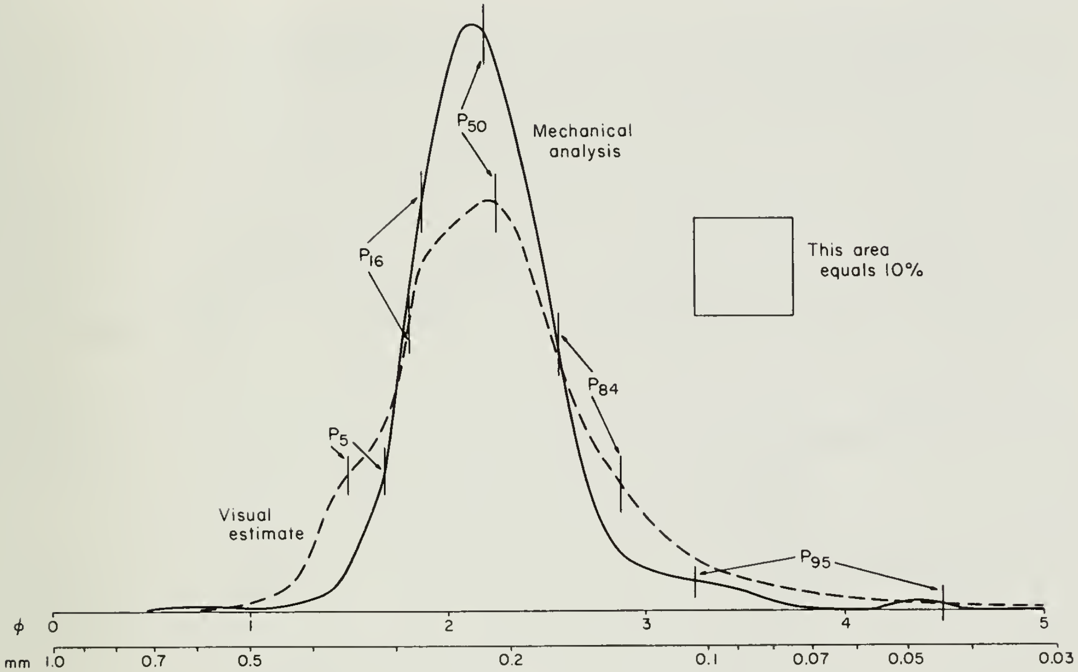

Fig. 3. - Distribution curves and percentiles of mechanical analysis (solid line) and visual estimate (dashed line) of the sample used for figure 1.

exceeded by a twentieth of the sediment. In most sediments it is more nearly repeatable and nearer the mechanical analysis than are the measures at P_{16}, P_{84}, and particularly at P_{95}. The measures at P_{16} and P_{84} are made by picking grains delimiting the extreme sixths of the sediment. The P_{95} estimate is the most doubtful of the group. If the visual estimate is to be compared to a mechanical analysis, the P_{95} estimate may be deliberately modified in an effort to compensate for the characteristics of the procedure used in the mechanical analysis. To match an analysis in which wet separation was used as a first step, 2 to 4 percent is credited to dust on the grains, and the P_{95} value is picked at a point apparently coarser than only 1 to 3 percent of the sediment. To match dry sieve analyses the intentional bias is reversed, because very fine grains cover a much greater area on the watch glass than do the same bulk or weight of larger grains and, unless conscious care is taken, there is a tendency to overestimate their amount.

The width of the selected grain is measured by the eyepiece micrometer with the grain near the center of the microscope field. Micrometer readings are converted to the phi scale by use of a conversion chart similar to those in tables 2 and 3. Conversion to the closest tenth of a phi unit is all that is generally warranted, although maximum grain size in many sandstones, and perhaps the median in a very few extremely well sorted ones, can be determined with sufficient accuracy to justify greater detail.

The graphs of figures 3 and 4 allow comparison of a number of visual estimates to mechanical analyses of the same samples. The correspondence between the curves is typical of the work of a geologist familiar with subsurface laboratory techniques but using the microscope only sporadically. Better correlation results if a geologist works regularly with the stereoscopic microscope to estimate dis-

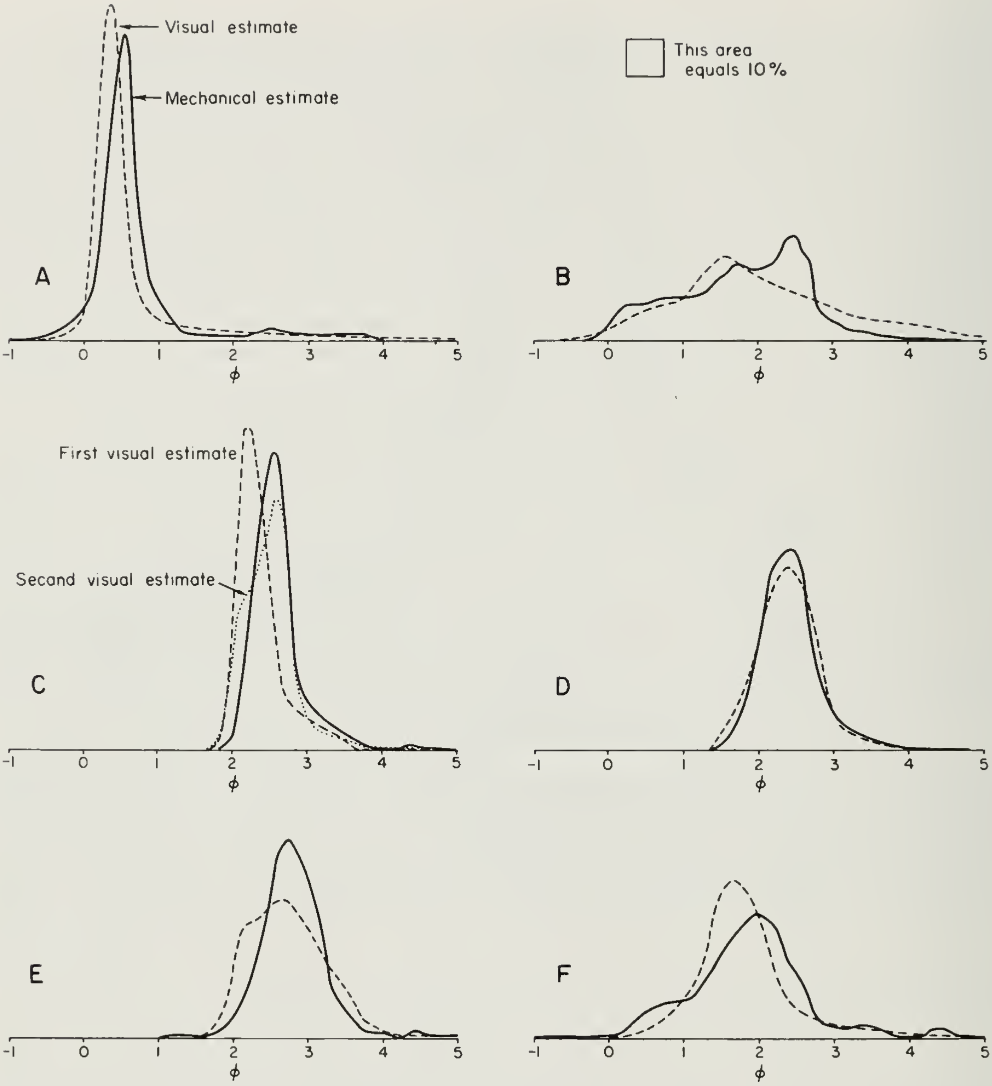

Fig. 4. - Size-distribution curves of mechanical analyses (Biggs and Lamar, 1955) (solid lines) and of visual estimates (dashed lines). Stratigraphic units represented and sample numbers used by Biggs and Lamar are A, Degonia (Chester), sample 3A; B, Caseyville (Pennsylvanian), sample 20A; C, Caseyville, sample 6C; D, Tar Springs (Chester), sample 4A; E, Hardinsburg (Chester), sample 24A; F, Caseyville, sample 9A.

tributions and checks his determinations occasionally with mechanical analyses. The mechanical analyses are much more exact. No comprehensive study has been made of the errors in estimating entire distribution but two geologists active in developing the method commonly repeat estimates, check each other's estimates, and compare estimates to sieve analyses, all within 0.1 ϕ for maximum size and within 0.2 ϕ for median (P_{50}) and fifth percentile (P_5) on reasonably well sorted sandstones. A study of duplicated estimates of maximum grain size is reported in the following section on partial estimates. The P_{16}, P_{84}, and P_{95} estimates are progressively less accurate, at least compared with complete mechanical analyses that use a preliminary sieving. Picks of P_{95} as far as 1.0 or even 2.0 ϕ from the mechanical analysis occur (fig. 3). Beginners' estimates of P_{16}, P_{50}, or P_{84} run consistently coarser than do the mechanical analyses.

Results of duplicated mechanical analyses reported in the literature are quite varied. Some record operator variability in mechanical analyses as great as our errors of estimation; others show median or mean checking within 0.02 ϕ. The total range of results on repeat analyses by an experienced analyst who has used carefully regulated procedure and technique rarely exceeds 0.05 ϕ, at least between P_5 and P_{95}. In general the estimating error appears to be 5 to 10 times the usual analytical error.

The range of the median or the mean in mechanical analyses of different parts of various single sand or sandstone bodies that have been sampled in detail varies from 0.3 ϕ in uniform sand bodies to 2.0 ϕ or more in the most heterogeneous ones. The typical error of the estimation is thus generally much less than the variation from sample to sample within the single sand body.

The shape of an analytical curve, which is summarized in terms of deviation, skewness, and kurtosis, is considered more useful in diagnosing the origin of a deposit than is the absolute size, the position of the curve with respect to the phi scale which is summarized by mode, median, or mean. Comparison of the solid and dashed curves in figures 3 and 4 shows that the shape of a mechanical analysis curve is not as adequately reproduced by the visual estimates as is its position.

Measures of sorting or deviation calculated from the visual estimates generally have values that range from about 75 percent to about 135 percent of the deviations calculated from the mechanical analysis. The heights of the curves are rough reciprocal functions of the deviations, so the height is in itself a visual expression of the deviation. Cursory examination of the literature indicates that standard deviations of various samples within single sand bodies vary from about half of to more than double this range, although the situation is complicated because deviation is much greater in composite samples than in spot samples of the same deposit. Visual estimates obviously can separate the classic textbook sorting examples of beach sand and till on the basis of graphic standard deviation. They will distinguish sandstones that differ as widely in sorting as do typical samples of St. Peter Sandstone and Glenwood Sandstone (both Ordovician). However, it seems doubtful that visual sorting measures are precise enough to be of help in distinguishing many sandstones.

The graphic analogs of the higher moment measures, skewness and kurtosis, are very imprecisely determined by the visual estimates. Again, the textbook examples of radically skewed distributions can be recognized, or a sharply peaked unimodal sand with high kurtosis can be distinguished from a bimodal sand with very low kurtosis. However, the range of these parameters calculated from duplicate estimates of the same sample (see fig. 4C) is much grosser than the values used by Mason and Folk (1958) to discriminate Recent sands deposited in beach, sea dune, and aeolian flat environments.

Estimation of the phi values of these six percentiles generally takes less than five minutes per sample. Computing parameters based on them by use of the formulae in table 4 takes a little longer, and preparation of curves such as those of figures 3 and 4 to facilitate comparison to mechanical analyses much longer.

Well Samples

The material available in well sample sets is too small in amount and too contaminated to justify mechanical analysis or even complete visual estimation. Cuttings vary from single sand grains to rock fragments half an inch long. Caving, a limiting factor in some uses of rotary samples, is of minor significance in this study because drilling mud filters into and leaves a cake protecting permeable sandstone in normal rotary operation. Individual sand grains or sand blebs in shale may cave, and glacial drift is not entirely cased out of occasional Illinois wells. Individual grains from friable sandstone that fail to drop out in the mud-settling pits and are recirculated with the mud are a more likely source of contamination, as are the few grains or chips always delayed in their trip up the annulus by dropping out on ledges or being caught in eddies.

Accidental sorting before, during, and after sampling is a serious factor in the study of incoherent or friable sandstones, but not of indurated formations. Small isolated sand grains may be carried on in the mud stream through many of the sampling devices where larger grains tend to drop out. Washing to remove drilling mud also removes silt and fine sand and concentrates the larger grains.

The time it takes for a chip to come to the surface in the mud stream varies with such factors as depth, pump capacity and pressure, and hole diameter, but in typical Illinois situations is about a minute for each hundred feet of depth. As Illinois rocks drill at from one minute or less to thirty minutes per foot, a sample collected at the surface when the bit was at 3000 feet and labeled 2295 to 3000 may contain material from as deep as 2999 feet or may contain nothing below 2970 feet.

The maximum and median sizes are the most readily and accurately estimated of the six elements used in approximating the complete distribution. The amount of finer material is more apt to be distorted either by contamination or selective removal during the complicated course from subsurface formation to microscope field than is the size of the largest grains. The poorer the sample, the greater the advantage of the maximum size over the other measures.

The mode is perhaps less distorted by minor additions or subtractions of material than the median, but the two are generally closer together than the typical error in visually estimating either. Each has been used to some extent in this study. The advantages of consistently using the percentile measure, the median, appear to outweigh the advantages of the greater resistance of the mode to distortion, at least in ordinary unimodal sandstone.

Partial Estimates

For routine well sample descriptions the visual estimation of two points on the distribution curve, the maximum ($P_{0.15}$) and the median (P_{50}), offers the best combination of speed, accuracy, and ease of recording. Estimation of these two values takes only two minutes per sample, which is about all the time that can be allowed if 1000 to 2000 feet of samples are described daily. Hesitation in picking either value suggests that more than one component is present, either a single bimodal or polymodal sandstone, or a mixture of different beds or sandstone bodies in the sample.

The maximum and median are useful values in interpreting the environment of deposition. The maximum is an index of the energy that was available to move sand, and the median is an index of the normal effective energy level. Their difference is a measure of sorting. It is three times the standard deviation in the

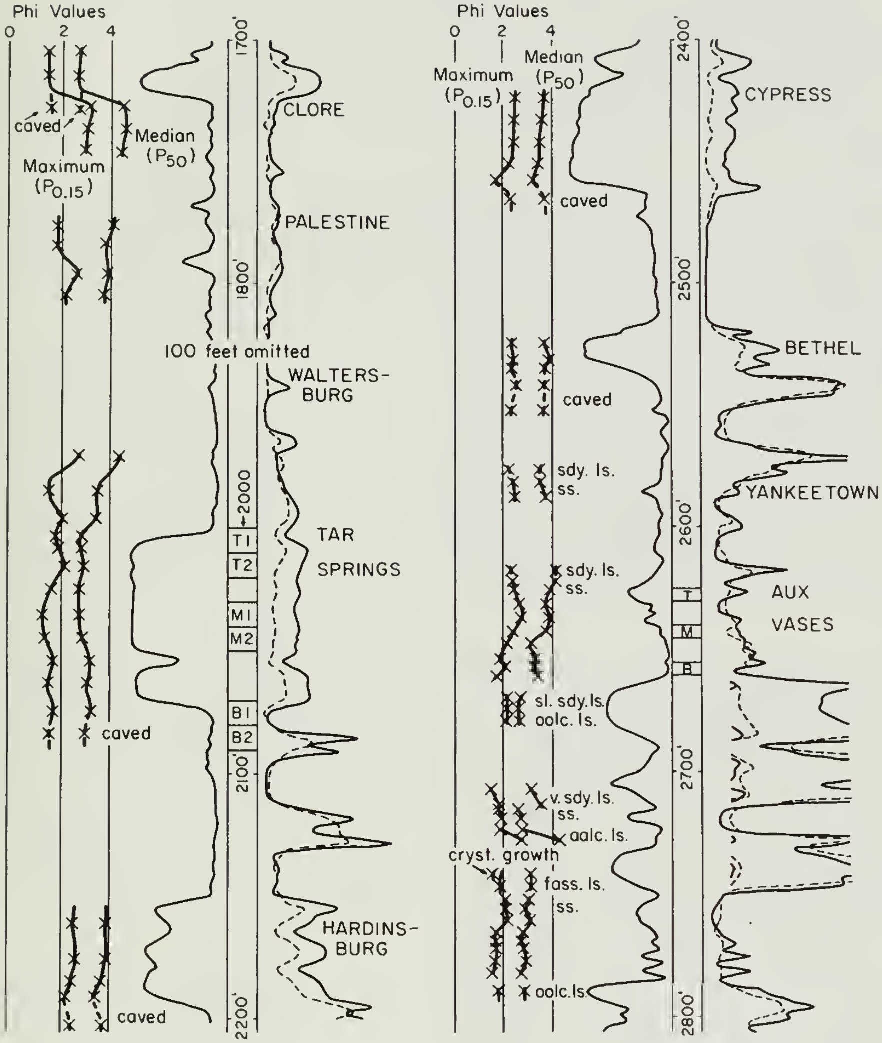

Fig. 5. - Log of Bell and Zollier's well 4, Zeigler Coal and Coke Co., NW$\frac{1}{4}$ SW$\frac{1}{4}$ sec. 12, T. 7 S., R. 1 E., Franklin County, Illinois, showing maximum and median grain size estimates of quartz grains in sandstones and sandy limestones. The relative position of samples used in the Tar Springs and Aux Vases studies is marked T, M, and B for top, middle, and bottom, and 1 and 2 for first and second series.

normal distribution, but less than that in most unimodal Illinois sandstones. Figure 5 is an example of one method of recording graphically the estimates of maximum and median grain size in a well. Lag corrections used in figure 5 are drawn from both the drilling time and the electric log.

Repeatability of Size Estimates in Well Samples

Repeatability of estimates of maximum grain size was evaluated in a study of the Tar Springs Sandstone, a typical oil-producing sandstone in the Chester (upper Mississippian) Series.

Thirty-two wells, distributed in six groups representing separate areas within the Illinois Basin, were selected for study (fig. 6). The wells are listed by Fisher (1956).

To get a wide range of maximum grain sizes the wells within each group were selected to show great variation in sand-shale ratio, as determined from electric logs, in the belief, confirmed by the study, that the two factors are somewhat related. A series of three samples, from depths near the top, middle, and bottom of the formation, was taken from each well and was studied first because it had been determined that the average of the maximum diameters of sand grains from all depth intervals in a formation was quite similar to that of the sand grains from the top, middle, and bottom samples of the formation. This first series is indicated on figure 5 by the symbols T1, M1, and B1, although the well illustrated was not one of the 32 used in this particular study.

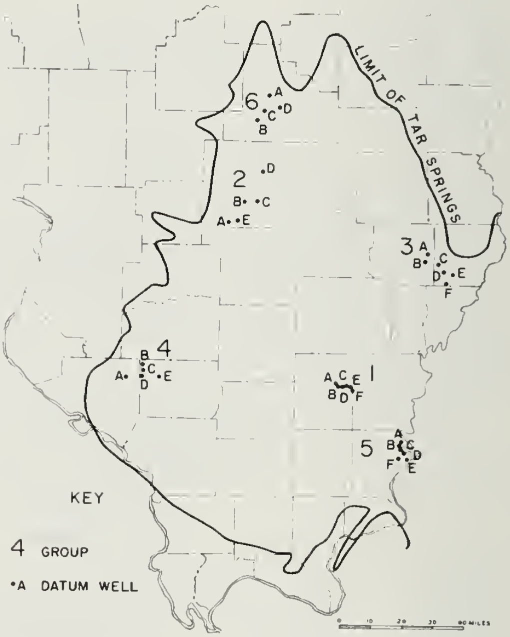

Fig. 6. - Wells used in study of maximum grain size in the Tar Springs Sandstone.

The split of the sample to be measured was obtained from the envelope of cuttings by pouring about half of the cuttings out and catching the next gram or so on a watch glass, washing, and pouring off excess water. The typical sample included several times as much cavings as actual sand grains. The entire sample on the watch glass was searched for sand grains. The Tar Springs is poorly cemented and most samples included many loose grains freed during drilling. The width of the largest grain was recorded as the maximum diameter for that depth. An exception occurred when one or two grains were abnormally large. That is, if one or two grains were about .50 mm whereas the rest of the large grains were in the vicinity of .30 mm, the coarsest grains were disregarded and the next largest diameters were recorded. This kept the results of minor contamination from affecting the value recorded. Measurements were recorded in hundredths of a millimeter, although phi units would have been used had this particular study been made later.

After all three samples from a single well were measured, the measurements were averaged arithmetically and the average maximum diameter recorded for the well. Again, the logarithmic mean obtained by average phi values would have been preferable, though the difference is too small to affect the conclusions.

The amount of error introduced in splitting and measuring was tested by measuring new splits from the same envelopes several weeks later. A scatter plot

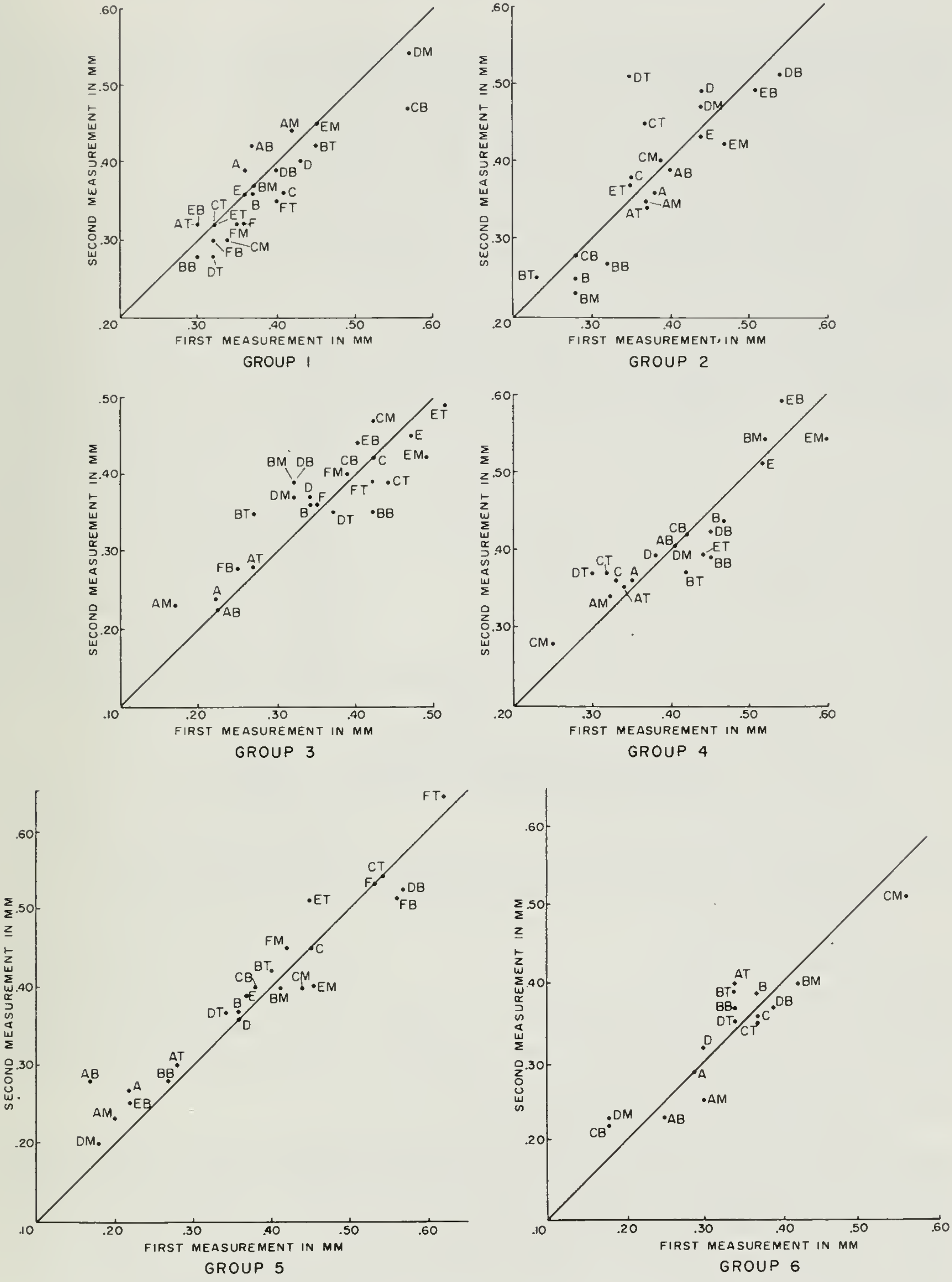

Fig. 7. - Control check by a single operator of measurements of maximum grain size on two splits of the same well samples. Where two letters are used, the first identifies data from the wells labeled in the groups on figure 6 and the second indicates an individual sample from near the top (T), middle (M), or base (B) of the Tar Springs Sandstone in the well. The single letter indicates the average of all three samples. Samples yielding identical results in both measurements are indicated on the diagonal lines.

of the first measurement against the second measurement on an arithmetic scale resulted roughly in a straight line, the ideal result of a graph of this type (fig. 7). The results indicate that the error introduced by splitting effect is relatively unimportant. The second measurement was within 0.2 ϕ of the first in 83 percent of the single samples and 97 percent of the well averages.

Variation between operators also was tested by having a second operator measure yet another split from the same samples. The second operator was completely unfamiliar with the technique used and worked from a written set of instructions. The results obtained (fig. 8) indicated that the technique demands little training and emphasized again that the operator effect and splitting effects are unimportant.

The effect of choosing samples from various depths to represent a well also was studied. A second series of three samples was selected, the location of each sample being 5 to 10 feet deeper than the corresponding sample of the first series (fig. 5, T2, M2, and B2). The time between the two series of measurements was great so that there was little chance for bias due to memory. The results of this test are shown by plotting the two averages of the first series and the averages of the deeper series for each well (fig. 9). As can be seen on the graphs, the error introduced by sampling is small relative to the difference between wells.

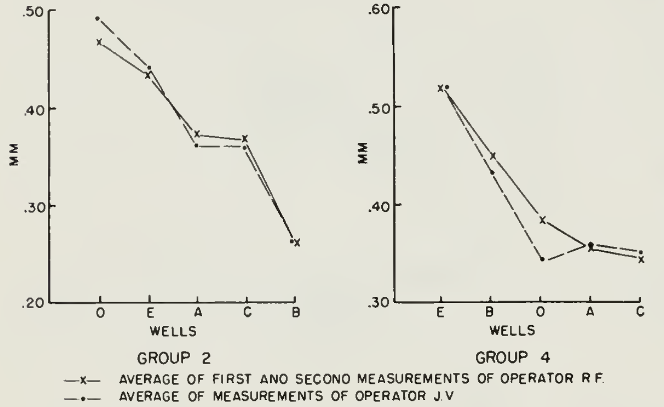

Fig. 8. - Comparison of maximum grain size in the Tar Springs Sandstone as measured by two operators on different splits of the same samples. The well averages are plotted.

Bimodal Distribution

Visual estimates of size percentiles and the parameters derived from them have not proved very successful in describing bimodal sands. Estimates of the median have departed from the mechanical analysis by nearly a full phi unit on certain samples, and either P_{16} or P_{84} have varied that much on other bimodal samples. The problem needs more study, but it seems better to discard the percentile approach for bimodal distributions and substitute an approach on the basis of components, giving approximate amounts or percentages (not percentiles) of each component and the mode characterizing each.

For example, a bimodal sample of Glenwood (Ordovician) Sandstone with maximum grain size ($P_{0.15}$) at -0.3 ϕ is reasonably well described as being composed of two components, one with a mode at 1.2 ϕ making up about 25 percent of the sample, the other with a mode at 3.4 ϕ composing about 75 percent of the sample. In this case, several estimates of the phi value of the coarser mode varied little, though they were coarser than the value of 1.4 ϕ suggested by the mechanical analysis. Estimates of the finer mode varied much more, from 2.7 to 3.6 ϕ, which might be expected because the sieve analysis showed an almost rectangular pattern with nearly equal amounts on the 3.0, 3.25, 3.5, and 3.75 ϕ sieves and a very sharp cut-off beyond 3.75. Estimates of the amount of the coarse component

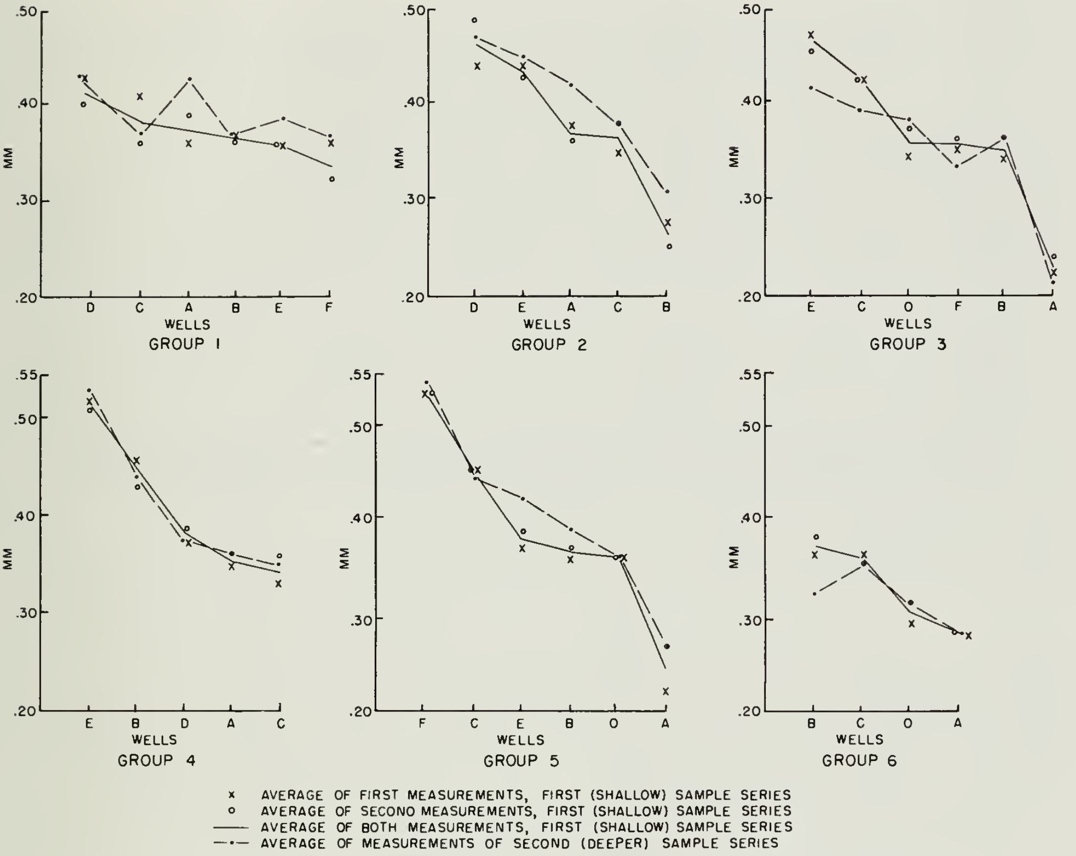

Fig. 9. - Averages of grain size from three levels in the Tar Springs Sandstone compared to averages of a second series made from three levels, each five or ten feet beneath the corresponding level of the first set.

ranged from 10 percent to 40 percent. Despite such variability, distribution curves drawn from component estimates suggest the analytical curve better than curves drawn from percentile estimates of this or other bimodal sands.

GRAIN SIZE AND SAND-SHALE RATIOS IN THE TAR SPRINGS SANDSTONE

In addition to yielding data on the repeatability of the size data from well samples, the study of the Tar Springs Sandstone was designed to investigate the relation of grain size to relative sandiness or shaliness as indicated by sand-shale ratios, and, if possible, to show broad regional patterns of size distribution (fig. 10). The Tar Springs Sandstone was selected because it has some sections that are largely sandstone and others that are largely shale. The formation includes varying proportions of dark to medium gray shale and fine-grained, rather friable sandstone that is yellowish brown in outcrop but light gray to white in subsurface.

No regional pattern of variation in grain size in the Tar Springs Sandstone can be deduced from this study, perhaps because the selection of wells shows a

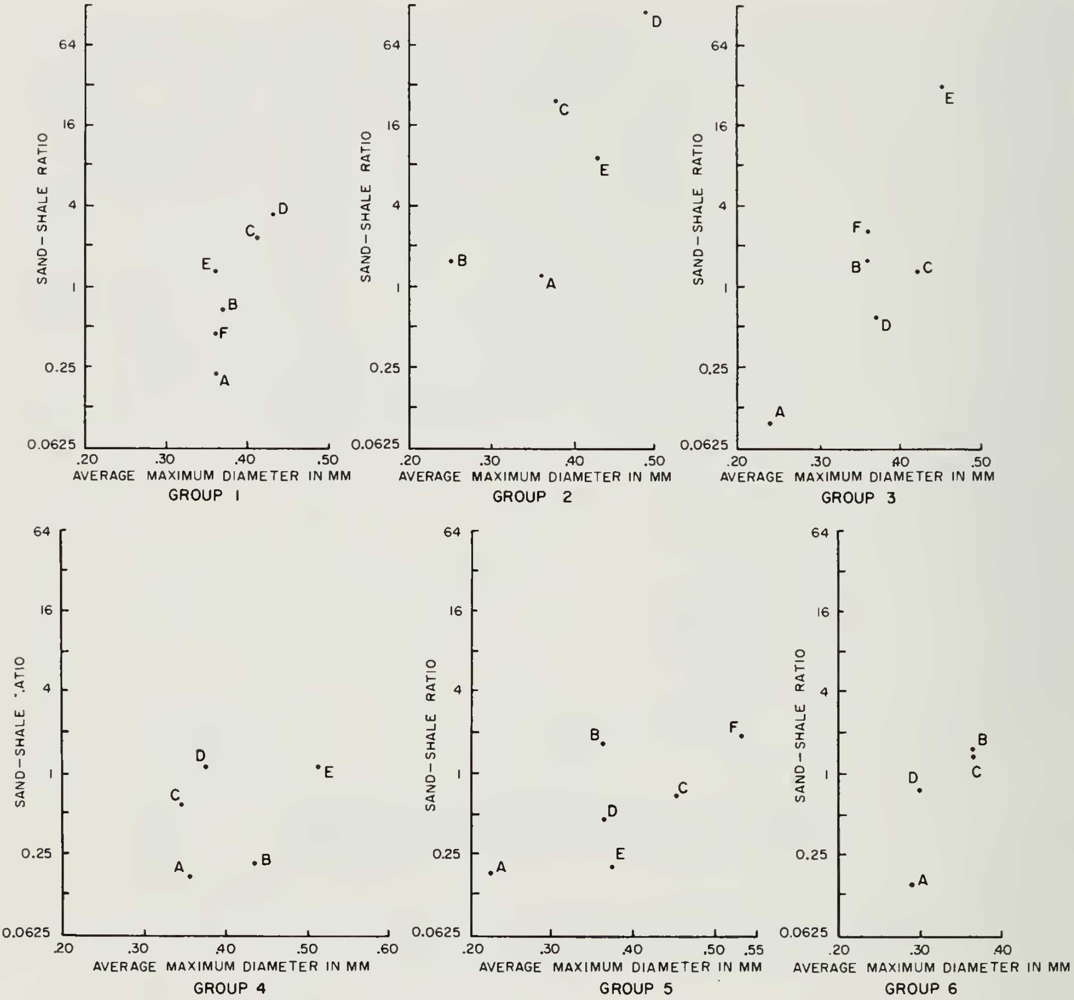

Fig. 10. - Maximum grain size averages and sand-shale ratios of the Tar Springs Sandstone.

wide range of sand-shale ratios within each geographic group. A regional pattern might emerge were the wells restricted to a narrower range of sand-shale ratios, but, on the other hand, environment may be so similar throughout the region that any gradient in size would be too slight to be detected.

There is no consistent pattern of vertical differences in maximum grain size, although some pattern might emerge were other variables held more nearly constant.

The only readily recognized correlation of the grain size data is to relative sandiness and shaliness, as documented by sand-shale ratios. In any one group the maximum grain size tends to be greater in the wells having the most sand, although the correlation is not perfect. The relation suggests that studies of small-scale grain size variation within a pool or township may be useful in helping delimit shale-outs by indicating approach to the sand-shale boundary before the change in grain size has affected the permeability or before there is enough clay mineral present to reduce the electric self-potential. In regional studies, wells with uniform sand-shale ratios should be chosen to reduce the variability caused by relative sandiness and thus emphasize any regional gradients that might exist.

REGIONAL VARIATION IN THE AUX VASES SANDSTONE

Regional changes in grain size in the Aux Vases Sandstone, the lowest of the Chester formations, were studied by visual estimation (Walters, 1958). The lateral equivalence of the very fine-grained sandstone called Rosiclare in the outcrops and "basin Aux Vases" in the oil fields of southeastern Illinois to a coarser phase of the Aux Vases in western Illinois has been generally recognized (Swann and Atherton, 1948), but there has been no quantitative study of the relationship.

The type region for the Aux Vases Sandstone, the underlying Ste. Genevieve Limestone, and the overlying Renault Formation is in southeastern Missouri and southwestern Illinois on the western edge of the Illinois Basin. The name Aux Vases is used for the sandstone consistently throughout the oil field area of the Illinois Basin and normal oil-field usage places it at the base of the Chester. However, in outcrops in southeastern Illinois and western Kentucky, the Levias Limestone, which carries a fossil assemblage typical of the Ste. Genevieve, lies above the eastern continuation of the Aux Vases Sandstone. This portion of the Aux Vases is the Rosiclare Sandstone of the outcrop but not the subsurface nomenclature. In the southeastern Illinois outcrop area, both this sandstone and the Levias are considered members of the Ste. Genevieve.

Grains of sand from the Aux Vases Sandstone from 66 oil tests, all drilled with rotary tools, and from 12 outcrop localities were measured. The wells were not picked at random, but were chosen because electric logs or prior sample studies showed a good sandstone phase. Some wells were eliminated even after this preliminary selection, either because they had inadequate sand sections or because of poor samples. Wells with low sand-shale ratios comparable to some used in the Tar Springs study were excluded, thus diminishing one source of variability. Effects of local variability also were reduced by picking three or four wells within a few miles of each other to form a datum group. The average value obtained for the entire group was used in preparing the figures. The specific wells and outcrops used and the measurements obtained for the individual samples are recorded by Walters (1958).

The Aux Vases is normally more coherent than the Tar Springs, and well-cutting chips aggregating several hundred or several thousand grains generally were used for estimating the size parameters. Several representative chips were picked under low power magnification, transferred to a watch glass, acidized with a few drops of dilute hydrochloric acid, and crushed with the handle of a teasing needle. The width of selected grains was then measured at a higher magnification. As the entire yield from the chips was examined, a measure of central tendency as well as the maximum size could be obtained without distortion by accidental sorting during preparation. The measure used was the mode, but there would have been no significant difference in results had the median been used. Later work monitored by occasional mechanical analyses suggests that the visual values for either mode or median obtained without checking probably are coarser in the order of 0.1 to 0.2 ϕ, or half a contour interval of the maps of figure 12, than the median determined by mechanical analysis.

In most cases the samples were unimodal and well sorted so that the mode, the commonest grain size in the sample, was readily estimated. The widths of several "typical" grains were measured in each of several traverses of the watch glass and the average reading was recorded in hundredths of a millimeter, though phi values would have been more appropriate. Because the few chips used may have represented beds with different grain sizes, some samples were definitely bimodal and the two modes were then averaged to obtain the sample value. In other samples, primarily from the southern and western parts of the area, sizes are appreciably larger and the sorting is poorer. In these cases the "mode" was rather broad and the arithmetic midpoint of the modal range was recorded. The maximum grain size recorded for a single sample is the average of the values for the largest grain in each of several fields of view, not the largest grain that might be found by searching the entire sample. It thus corresponds to a percentile probably in the range $P_{0.2}$ to $P_{0.5}$.

Geographic Distribution of Sandstone and Grain Size Variations

Figure 11 shows the thickness and percentage of sandstone in the interval from the base of the Aux Vases Sandstone to the top of the Downeys Bluff Limestone (variously called "upper Renault" and "lower Paint Creek" in different parts of the Illinois Basin). The interval includes not only the Aux Vases Sandstone, but sandstones of Renault and Yankeetown (Benoist) age lying above the Aux Vases, and some shales and thin limestones. The map is redrawn from the basic data used in figures 9 and 10 of Swann and Bell (1958). It is used here instead of one of the Aux Vases alone because the top of the Downeys Bluff can be picked with fair accuracy throughout the Illinois Basin, whereas the Aux Vases-Renault contact can not be precisely determined in some records.

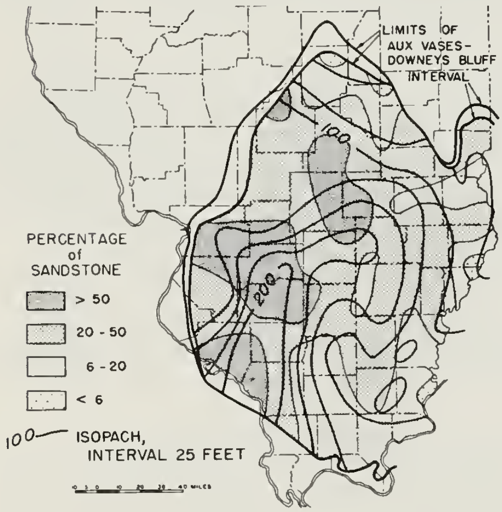

Fig. 11. - Thickness and percentage of sandstone in the interval from the base of the Aux Vases Sandstone to the top of the Downeys Bluff Limestone. Modified from Swann and Bell (1958).

Although the Aux Vases includes only about half the sandstone shown on figure 11, the distribution of the Aux Vases is similar to that of the higher sandstones in the interval, and the general pattern of the Aux Vases is essentially that shown. The section is thin at the north and thickens to the southwest. More than half the section is sandstone in areas in the west and southwest, but less than 6 percent is sandstone at the eastern edge of the state. The amount of sandstone in the interval continues to decrease eastward so there is less than a foot on most of the outcrop belt at the eastern edge of the Illinois Basin in Indiana and Kentucky.

Figures 12A and 12B show in microns the geographic distribution of the average mode and average maximum grain sizes of samples of the Aux Vases Sandstone from wells or outcrops in predominantly sandstone phases in southern Illinois. The contour interval used is the ratio $1/\sqrt[4]{2}$, that is, 0.25 ϕ. The approximate locations of the groups of wells and outcrops also are shown, together with the number of individual samples averaged to produce a smoothed figure for contouring. The average used is the arithmetic mean

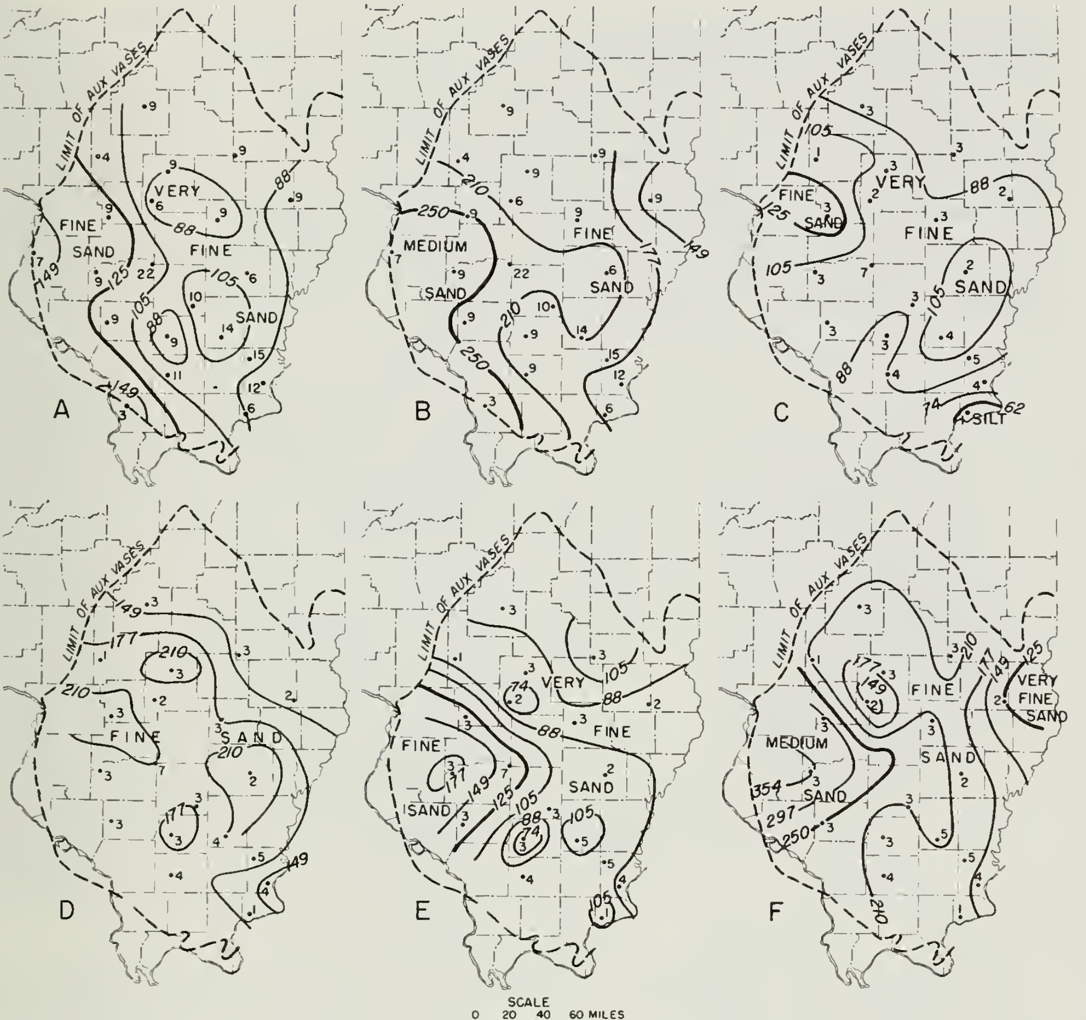

Fig. 12. - Variation in grain size of Aux Vases Sandstone, in microns. The number of individual samples measured and averaged for the datum group is indicated at each point. The contour interval is 0.25 ϕ. A, average mode, entire formation; B, average maximum, entire formation; C, average mode, upper part of formation only; D, average maximum, upper part only; E, average mode, lower part of formation only; F, average maximum, lower part only.

and the values are thus a little coarser than they would have been had phi values been averaged. In general, three samples were taken from each well, one near the top, one near the middle, and one near the bottom of the Aux Vases Sandstone (fig. 5, T, M, and B). In a few instances only one or two samples could be used from a well. The outcrop samples were taken from individual hand samples that had been collected from the outcrop for other purposes. They tend to be coarser than samples from nearby wells. It seems likely that in the selection of a "good" outcrop sample the purer, less shaly, and coarser parts of the section were picked rather than the statistically more representative parts.

A general correspondence of the maps shown in figures 11 and 12 is apparent. Coarser grain sizes in both mode and maximum occur where the Aux Vases - Downeys Bluff interval is sandiest, and finer grain sizes where there is relatively little sandstone in the interval. This extends even to some of the finer details such as the shaly or finer re-entrants in the south-central and north-central parts of the map, despite the use of different datum wells in the two studies.

Figures 12C through 12F segregate data from the top and bottom parts of the formation. The upper part of the formation, shown in figures 12C and 12D, has surprisingly little variation across the entire area. Most of the variation is local rather than regional.

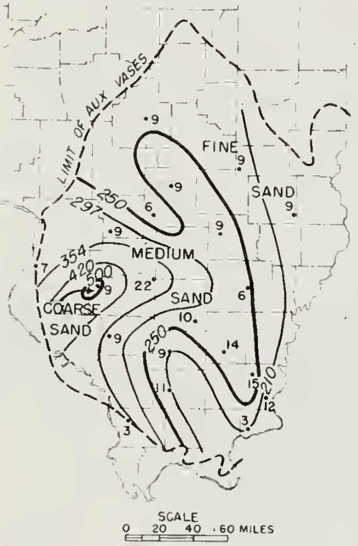

Fig. 13. - Variation of the largest value, in microns, for maximum size of Aux Vases Sandstone grains in a datum group. The number of individual samples from which the largest value was selected is shown beside each datum point. The contour interval is 0.25ϕ.

In contrast, figures 12E and 12F of the bottom of the formation show a well marked gradient. In the eastern part, and to some extent in the northwestern or northwest-central parts, of the area mapped there is little size differentiation between the upper and lower parts of the Aux Vases. It is in the western and southwestern, and to some extent in the northeastern, parts that the lower Aux Vases is much coarser than the upper. Figure 14 depicts the marked difference in the grain size patterns of the upper and lower part of the formation.

Most of the pattern of size distribution for the entire formation is contributed by the presence of the lower, coarser sandstones. It is not clear from the maps, nor from the original data, whether the lower, coarser sandstone of the southwest area is correlative with the lower part of the finer grained sandstone of the east, or whether the fine-grained "upper" of the southwest is equivalent to the entire eastern section.

Figure 13 shows the greatest maximum grain size found in any sample of the datum groups. In only one sample of one well studied were any grains in the coarse sand range. In this sample several grains were almost exactly on the coarse-medium boundary, 1.05 to 0.95 ϕ.

In summary, the Aux Vases is a very fine-grained sandstone except for the lower portion in the western part of the basin that is fine grained. The largest grains are very fine in a few wells in eastern Illinois, fine in most of southeastern Illinois, medium in southwestern Illinois, and fall at the medium-coarse boundary in only one sample from southwestern Illinois.

Interpretation of Aux Vases Grain Size Distribution

For more than 40 years the Aux Vases Sandstone commonly has been thought to be derived from the Ozark Dome (Weller, 1920). Analysis of the problem of the origin of Chester sands in recent years has pointed to a northern source (Siever, 1954; Potter et al., 1958). This source is now generally accepted for the sandstones of the middle and upper Chester, but many geologists working with the oil-bearing sandstones in the lower part of the Chester still postulate that the Aux Vases and the overlying Yankeetown (Benoist) entered the basin from the west and are Ozark-derived.

The grain size distribution maps of the Aux Vases support an Ozark source as well or better than they do a northern source. However, data on cross-bedding of the Aux Vases and other Chester sandstones do not harmonize with an Ozark source (Potter et al., 1958) nor do petrographic studies. The Aux Vases contains more mica and feldspar (H. B. Willman, personal communication) and angular tourmaline (P. E. Potter, personal communication) than could come from any source in the Ozarks.

One interpretation in harmony with the grain size data is that which holds the lower part of the Aux Vases was deposited as a sandstone in the western part of the state while Ste. Genevieve Limestone (Fredonia Member by outcrop terminology, entire Ste. Genevieve in oil-field terminology) was being deposited in the east. The fact that the pre-Aux Vases lenses of sandstone in the Ste. Genevieve ("Rosiclare" of the oil fields and of the Indiana outcrops, or Spar Mountain and other "sub-Rosiclare" sandstones of the southeastern Illinois and Kentucky outcrops) are much coarser than the eastern Aux Vases, and as coarse or coarser than the lower western Aux Vases, (figs. 5, 14) corroborates the correlation of the lower part of the Aux Vases Sandstone in the west to part of the Fredonia Limestone in the east.

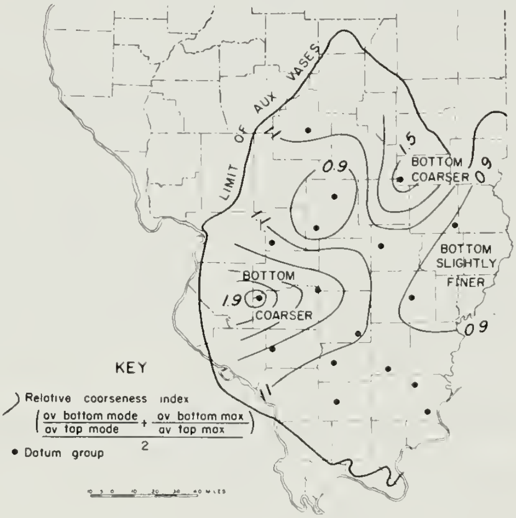

Fig. 14. - Relative coarseness of samples from the lower part of the Aux Vases Sandstone compared to samples from the upper part of the formation. Values larger than 1.0 indicate the lower part is coarser than the upper.

Comparison of figures 12E, 12F, and 13 of this paper with figures 7 and 8 of Swann and Bell (1958), which show lithologic facies in the pre-Aux Vases Ste. Genevieve, also suggests this correlation. Areas with relatively coarse and thick lower Aux Vases are areas where little or no sandstone is recognized in the Ste. Genevieve. However, such areas are immediately surrounded by areas in which the Ste. Genevieve has the most sandstone. The relatively coarse pre-Aux Vases sandstones of the Ste. Genevieve are clearly northern derived (Swann and Bell, 1958; Malott, 1952, for the Indiana "Rosiclare"). The grain size data indicate the possibility that the coarser lower part of the Aux Vases of the west is part of the northern-source "pre-Aux Vases" Ste. Genevieve sandstones of the northern, central, and south-central parts of the basin.

The possibility of transgressive overlap of western coarse-grained Aux Vases Sandstone on eastern fine-grained "basin Aux Vases," an idea current among area geologists a decade ago as a compromise between oil-field and conventional outcrop stratigraphy (Swann and Atherton, 1948, p. 11), is clearly controverted by the grain size data presented here, which shows that the finer phase overlaps the coarser.

The grain size data harmonize with any of three theories of Aux Vases source: 1) simple derivation from the Ozarks to the west, 2) derivation of the lower part of the Aux Vases of western Illinois from the west, and the upper part of the Aux Vases of western Illinois and the entire formation in eastern Illinois from the north, and 3) derivation of the entire formation from the north, in which case the coarser lower sandstones of the west must be correlated with part of the Ste. Genevieve Limestone of eastern Illinois, whose sandstone lentils are clearly from a northerly source. The choice of one of these alternatives must be based on other types of data. Interpretations of both cross-bedding and petrographic information favor the third alternative.

REFERENCES

Biggs, D. L., and Lamar, J. E., 1955, Sandstone resources of extreme southern Illinois. A preliminary report: Illinois Geol. Survey Rept. Inv. 188.

Cadigan, R. A., 1954, Testing graphical methods of grain-size analysis of sandstones and siltstones: Jour. Sedimentary Petrology, v. 24, p. 123-127.

Fisher, R. W., 1956, Geographical variation of maximum grain size in the Tar Springs Sandstone: Unpublished master of science thesis, Univ. of Illinois, Urbana, Ill.

Folk, R. L., and Ward, W. C., 1957, Brazos River bar: A study in the significance of grain size parameters: Jour. Sedimentary Petrology, v. 27, p. 3-26.

Inman, D. L., 1952, Measures for describing the size distribution of sediments: Jour. Sedimentary Petrology, v. 22, p. 125-145.

Krumbein, W. C., 1934, Size frequency distributions of sediments: Jour. Sedimentary Petrology, v. 4, p. 65-77.

Krumbein, W. C., 1938, Size frequency distributions of sediments and the normal phi curve: Jour. Sedimentary Petrology, v. 8, p. 84-90.

Krumbein, W. C., and Pettijohn, F. J., 1938, Manual of sedimentary petrology: D. Appleton-Century Co., Inc., New York.

Lane, E. W., et al., 1947, Report of the sub-committee on sediment terminology: Amer. Geophysical Union Trans., v. 28, p. 936-938.

Malott, C. A., 1952, Stratigraphy of the Ste. Genevieve and Chester formations of southern Indiana: Edwards Letter Shop, Ann Arbor, Mich.

Mason, C. C., and Folk, R. L., 1958, Differentiation of beach, dune, and aeolian flat environments by size analysis, Mustang Island, Texas: Jour. Sedimentary Petrology, v. 28, p. 211-226.

Milner, H. B., 1940, Sedimentary Petrology (third edition): Thomas Murby and Co., London, and Nordeman Publishing Co., New York.

Otto, G. H., 1939, A modified logarithmic probability graph for the interpretation of mechanical analyses of sediments: Jour. Sedimentary Petrology, v. 9, p. 62-76.

Page, H. G., 1955, Phi-millimeter conversion table: Jour. Sedimentary Petrology, v. 25, p. 285-292.

Poole, D. M., 1957, Size analysis of sand by a sedimentation technique: Jour. Sedimentary Petrology, v. 27, p. 460-468.

Potter, P. E., Nosow, Edmund, Smith, N. M., Swann, D. H., and Walker, F. H., 1958, Chester cross-bedding and sandstone trends in the Illinois Basin: Amer. Assoc. Petroleum Geologists Bull., v. 42, p. 1013-1046.

Siever, Raymond, 1954, Petrology and sedimentation of upper Chester sandstones: Jour. Sedimentary Petrology, v. 23, no. 4, p. 207-219. Reprinted as Illinois Geol. Survey Rept. Inv. 170.

Swann, D. H., and Atherton, Elwood, 1948, Subsurface correlations of lower Chester strata of the Eastern Interior Basin: Jour. Geology, v. 56, p. 269-287. Reprinted as Illinois Geol. Survey Rept. Inv. 135.

Swann, D. H., and Bell, A. H., 1958, The habitat of oil in the Illinois Basin (p. 447-472) in The habitat of oil: Amer. Assoc. Petroleum Geologists, Tulsa.

Tanner, W. F., 1958, Comparison of phi percentile deviations: Jour. Sedimentary Petrology, v. 28, p. 203-204.

Trask, P. D., 1932, Origin and environment of source sediments of petroleum: Gulf Publishing Co., Houston.

Udden, J. A., 1898, Mechanical composition of wind deposits: Augustana Library Publications No. 1.

Udden, J. A., 1914, Mechanical composition of clastic sediments: Geol. Soc. America Bull., v. 25, p. 655-744.

Walters, M. J., 1958, Regional variation in grain size in the Aux Vases (Mississippian) Sandstone: Unpublished master of science thesis, Univ. of Illinois, Urbana, Ill.

Weller, Stuart, 1914, Geology of parts of St. Clair, Monroe, and Randolph Counties, Illinois: Illinois Geol. Survey unpublished manuscript SW-2.

Weller, Stuart, 1920, The Chester Series in Illinois: Jour. Geology, v. 28, p. 281-303, 395-416.

Weller, Stuart, and St. Clair, Stuart, 1928, The geology of Ste. Genevieve County, Missouri: Missouri Bur. Geol. and Mines, ser. 2, v. 22.

Wentworth, C. K., 1922, A scale of grade and class terms for clastic sediments: Jour. Geology, v. 30, p. 377-392.

APPENDIX

Calibration and Conversion Procedures

In calibrating an eyepiece micrometer in millimeters or microns a reasonably accurate scale is placed on the microscope stage. A millimeter rule is sufficient at low magnifications, but a stage micrometer slide is preferable for higher magnifications. Because of the variation with position in the field, all measurements in calibration should be made near the center of the field or with the micrometer vertical. When a millimeter rule is used it generally will be necessary to measure the length of a millimeter in terms of eyepiece micrometer divisions and take the reciprocal for the length of a single division.

Unless the conversion factor is a very simple one, a conversion chart is more convenient than multiplying individual readings by the factor to obtain actual distances. The most easily used chart for converting to the closest hundredth of a millimeter is one based on the micrometer lengths equivalent to the midpoints between the hundredths of a millimeter.

Though a phi-micrometer table can be constructed by converting the phi numbers to millimeters and millimeters to micrometer divisions, it is simpler to work directly from phi to micrometer units. This can be done by changing the millimeter-micrometer conversion factor to its phi value.

For example, if one micrometer division is 0.0383 mm, it is also 4.707 ϕ. To find the number of scale divisions equal to 1.0 ϕ, find in a table or conversion chart the millimeter value that corresponds to 1.0 ϕ less 4.707 ϕ, that is, to -3.707 ϕ. The number of millimeters found here is 13.056, rounded to 13.1, which in turn corresponds to the number of scale divisions in 1.0 ϕ. In practice it is necessary to compute accurately only the conversions for the divisions of a single grade, a single cycle of phi numbers. These should be in the coarsest range needed. The remainder of the table is prepared by halving the value for each full integer added.

The phi-micrometer table simplest to use is one based on the range of micrometer divisions falling between the midpoints of the phi interval. If the table uses 0.1 ϕ as its interval, the final table would indicate for 1.1 ϕ the micrometer divisions and tenths of a division corresponding to the range from 1.05 to 1.15 ϕ and would take the form 13.5 to 12.7 divisions = 1.0 ϕ, 12.6 to 11.8 divisions = 1.1 ϕ, and so on.

Stereoscopic microscopes with drum-mounted objectives appear to be replacing other types as oil-field equipment, particularly where high magnifications are desired. Tables 2 and 3 give phi-micrometer conversion values for several magnifications of the American Optical Company's Cycloptic drum-type stereoscopic microscope in combination with micrometer 1406A, which is the longest and coarsest (10 mm in 100 parts) micrometer available.

Each table must be used only with the eyepieces shown. Although a nominal magnification of 40 can be obtained on this microscope with 10X eyepieces, the 40X column (table 2) has been calculated for use with the 15X eyepiece and therefore cannot be used for the combination with the 10X eyepiece. Certain magnifications that can be obtained with the same eyepiece but with or without use of an auxiliary field lens (e.g., 20X and 40X with the 15X eyepiece), although not precise equivalents, are so nearly identical that the tables, figured for an intermediate value, may be used either way.

Phi-Percentile Parameters of Grain Size Distributions

Since the introduction of the phi scale (Krumbein, 1934), it has been used in many graphic or algebraic short-cut methods for describing grain size distributions. Equations for some of these are listed in table 4. The studies of Inman (1952), Folk and Ward (1957), and Tanner (1958) are particularly pertinent to the choice of such formulae.

The average of any pair of symmetrically placed percentiles is a measure of central tendency, and many such pairs have been used, but the average of P_{16} and P_{84} (equation 2 of table 4) offers more advantages and is closer to the mean in more types of sediments than any other average. Averaging the median with these two (equation 3) gives a closer approximation to the true mean in extremely non-normal sediments, but the exaggeration that equation 2 gives to the values on the limbs is justified in most Illinois sandstones. Equation 4 is a closer approach to the definition of mean, and the constants used are so simple that its greater accuracy may well offset its complexity. The mode approximation (equation 5) expresses the tendency of the mean to be about three times as far from the mode as from the median in most unimodal distributions, and also is probably useful in defining the primary mode if the secondary mode of a bimodal sandstone is beyond P_{10} or P_{90}.

All the sorting parameters listed (equations 6 to 13) are approximations of the moment standard deviation and would equal it for the theoretical normal curve. The denominator in each is the number of standard deviations separating the percentiles in the normal distribution. A number of proposed non-standard indices of sorting in which the difference between two percentiles is divided by some other factor offer no advantages. Equation 7 is probably the most useful of the simple deviations, but the inclusive deviation (equation 12) is a closer approach to the moment measure in most distributions. It is the average of equations 7 and 9 and thus describes more of the distribution. Equation 13 covers the coarse half of equation 11 and is the form that must be used in the maximum-median partial visual estimate recommended for well samples.

The skewness measures listed (equations 14 to 17) are again all of the same general form, although several non-standard forms have been suggested. The measures are the ratio of two distances - the distance from median to midpoint of two symmetrical percentiles and from midpoint to either percentile. The P_{16} to P_{84} (equation 15) is probably the most logical simple measure, though the inclusive skewness (equation 17) covers more of the distribution. Skewness shown by these measures lies between -1 and +1, and is positive if the median is coarser than the mean.

Kurtosis measures are not as standardized as the others, for no simple method yet devised approaches the fourth moment measure very closely. All the graphic measures proposed compare the spread of two pairs of symmetrical percentiles, but the comparison has taken several different forms, three of which are illustrated here by equations. The ratio of the deviations calculated from two spreads is suggested by Folk and Ward (1957), whose original parameter is given as equation 18. It uses two percentiles, P_{25} and P_{75}, which are not picked in the visual method outlined here, so the analogous equation 19 must be used for the visual method. Despite the similarity in form of equations 18, 19, and 20, the values given by them are not generally comparable. Kurtosis from these equations is 1.0 for the normal curve, less than 1.0 for platykurtic (big square central hump) distributions and more than 1.0 for leptokurtic (emaciated central spike) distributions. The platykurtic values are closely bunched and the leptokurtic ones are spread out widely. A transformation such as that of equations 18a, 19a, and 20a leaves the value for the normal distribution at 1.0 but reduces the relative tendency to bunch. Inman's parameter (equation 21) gives values below 0.65 for platykurtic distribution and above 0.65 for leptokurtic ones.

For the partial visual estimate using the maximum and median, only equations 1 and 13 can be used.

If three percentiles are used in characterizing distributions, they should be P_{16}, P_{50}, and P_{84}, and equations 2 (or 3 if extreme bimodal distributions are expected), 7, and 15 should be used. The most suitable equation if five percentiles, P_5, P_{16}, P_{50}, P_{84}, and P_{95}, are known are 4, 12, 17, and 19. By adding P_{25} and P_{75} to the five, equation 18 can be substituted for 19. Equation 5 is most useful in drawing distribution curves and for correcting the distortions forced on skewed distributions by cumulative curves on probability paper. Equations 18a or 19a are used where kurtosis values for several sediments are plotted against other properties.

TABLES

Table 1. - Phi-millimeter Conversion Table

Phi	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
-0.	1.000	1.072	1.149	1.231	1.320	1.414	1.516	1.624	1.741	1.866
-1.	2.000	2.144	2.297	2.462	2.639	2.828	3.031	3.249	3.482	3.732
-2.	4.000	4.287	4.595	4.925	5.278	5.657	6.063	6.498	6.964	7.464
-3.	8.000	8.574	9.190	9.849	10.556	11.314	12.126	12.996	13.929	14.929
-4.	16.000	17.148	18.379	19.698	21.112	22.627	24.251	25.992	27.858	29.857

Phi	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
+0.	1.000	0.933	0.871	0.812	0.758	0.707	0.660	0.616	0.574	0.536
+1.	0.5000	0.4665	0.4353	0.4061	0.3789	0.3536	0.3299	0.3078	0.2872	0.2679
+2.	0.2500	0.2333	0.2176	0.2031	0.1895	0.1768	0.1649	0.1539	0.1436	0.1340
+3.	0.1250	0.1166	0.1088	0.1015	0.0947	0.0884	0.0825	0.0769	0.0718	0.0670
+4.	0.0625	0.0583	0.0544	0.0508	0.0474	0.0442	0.0412	0.0385	0.0359	0.0335
+5.	0.0313	0.0292	0.0272	0.0254	0.0237	0.0221	0.0206	0.0192	0.0179	0.0167
+6.	0.0156	0.0146	0.0136	0.0127	0.0118	0.0110	0.0103	0.0096	0.0090	0.0084
+7.	0.0078	0.0073	0.0068	0.0063	0.0059	0.0055	0.0052	0.0048	0.0045	0.0042
+8.	0.0039	0.0036	0.0034	0.0032	0.0030	0.0028	0.0026	0.0024	0.0022	0.0021

Table 2. - Phi-Micrometer Conversion Table for American Optical Company's Cycloptic Microscope with 15X Eyepiece and No. 1406A Micrometer Disc (10 mm in 100 divisions)

Conversion factors: 1 micrometer scale division = 0.01907 mm at 80X
0.0383 mm at 40X
0.0769 mm at 20X
0.1570 mm at 10X

	Micrometer Scale Divisions			
Phi	80X	40X	20X	10X
Fine Pebbles				
-3.0				49.3-52.7
2.9				46.0-49.2
2.8			87.5-93.7	42.9-45.9
2.7			81.7-87.4	40.0-42.8
2.6			76.2-81.6	37.4-39.9
-2.5			71.1-76.1	34.9-37.3
2.4			66.3-71.0	32.5-34.8
2.3			61.9-66.2	30.3-32.4
2.2			57.8-61.8	28.3-30.2
2.1			53.9-57.7	26.4-28.2
Very Fine Pebbles (Granules)				
-2.0			50.3-53.8	24.7-26.3
1.9			46.9-50.2	23.0-24.6
1.8		87.9-94.1	43.8-46.8	21.5-22.9
1.7		82.0-87.8	40.9-43.7	20.0-21.4
1.6		76.5-81.9	38.1-40.8	18.7-19.9
-1.5		71.4-76.4	35.6-38.0	17.5-18.6
1.4		66.6-71.3	33.2-35.5	16.3-17.4
1.3		62.1-66.5	31.0-33.1	15.2-16.2
1.2		58.0-62.0	28.9-30.9	14.2-15.1
1.1		54.1-57.9	27.0-28.8	13.2-14.1

	Micrometer Scale Divisions			
Phi	80X	40X	20X	10X
Fine Sand				
+2.0	12.7-13.5	6.4- 6.7	3.2- 3.3	1.6
2.1	11.9-12.6	5.9- 6.3	3.0- 3.1	1.5
2.2	11.1-11.8	5.5- 5.8	2.8- 2.9	1.4
2.3	10.3-11.0	5.2- 5.4	2.6- 2.7	1.3
2.4	9.6-10.2	4.8- 5.1	2.4- 2.5	1.2
+2.5	9.0- 9.5	4.5- 4.7	2.3	1.1
2.6	8.4- 8.9	4.2- 4.4	2.1- 2.2	1.1
2.7	7.8- 8.3	3.9- 4.1	2.0	1.0
2.8	7.3- 7.7	3.7- 3.8	1.9	.9
2.9	6.8- 7.2	3.4- 3.6	1.7- 1.8	.9
Very Fine Sand				
+3.0	6.4- 6.7	3.2- 3.3	1.6	.8
3.1	6.0- 6.3	3.0- 3.1	1.5	.7
3.2	5.6- 5.9	2.8- 2.9	1.4	.7
3.3	5.2- 5.5	2.6- 2.7	1.3	.6
3.4	4.8- 5.1	2.4- 2.5	1.2	.6
+3.5	4.5- 4.7	2.3	1.1	.6
3.6	4.2- 4.4	2.1- 2.2	1.1	.5
3.7	3.9- 4.1	2.0	1.0	.5
3.8	3.7- 3.8	1.9	.9	.5
3.9	3.4- 3.6	1.7- 1.8	.9	.4

Very Coarse Sand				
-1.0		50.5-54.0	25.2-26.9	12.4-13.1
.9		47.1-50.4	23.5-25.1	11.5-12.3
.8	88.2-94.5	44.0-47.0	21.9-23.4	10.8-11.4
.7	82.3-88.1	41.0-43.9	20.5-21.8	10.0-10.7
.6	76.8-82.2	38.3-40.9	19.1-20.4	9.4- 9.9
- .5	71.7-76.7	35.7-38.2	17.8-19.0	8.8- 9.3
.4	66.9-71.6	33.3-35.6	16.6-17.7	8.2- 8.7
.3	62.4-66.8	31.1-33.2	15.5-16.5	7.6- 8.1
.2	58.2-62.3	29.0-31.0	14.5-15.4	7.1- 7.5
.1	54.3-58.1	27.1-28.9	13.5-14.4	6.6- 7.0
Coarse Sand				
0	50.7-54.2	25.3-27.0	12.6-13.4	6.2- 6.5
+ .1	47.3-50.6	23.6-25.2	11.8-12.5	5.8- 6.1
.2	44.1-47.2	22.0-23.5	11.0-11.7	5.4- 5.7
.3	41.2-44.0	20.5-21.9	10.3-10.9	5.0- 5.3
.4	38.4-41.1	19.2-20.4	9.6-10.2	4.7- 4.9
+ .5	35.9-38.3	17.9-19.1	8.9- 9.5	4.4- 4.6
.6	33.5-35.8	16.7-17.8	8.3- 8.8	4.1- 4.3
.7	31.2-33.4	15.6-16.6	7.8- 8.2	3.8- 4.0
.8	29.1-31.1	14.5-15.5	7.3- 7.7	3.6- 3.7
.9	27.2-29.0	13.6-14.4	6.8- 7.2	3.3- 3.5
Medium Sand				
+1.0	25.4-27.1	12.7-13.5	6.3- 6.7	3.1- 3.2
1.1	23.7-25.3	11.8-12.6	5.9- 6.2	2.9- 3.0
1.2	22.1-23.6	11.0-11.7	5.5- 5.8	2.7- 2.8
1.3	20.6-22.0	10.3-10.9	5.1- 5.4	2.5- 2.6
1.4	19.2-20.5	9.6-10.2	4.8- 5.1	2.4
+1.5	18.0-19.1	9.0- 9.5	4.5- 4.7	2.2- 2.3
1.6	16.8-17.9	8.4- 8.9	4.2- 4.4	2.1
1.7	15.6-16.7	7.8- 8.3	3.9- 4.1	1.9- 2.0
1.8	14.6-15.5	7.3- 7.7	3.7- 3.8	1.8
1.9	13.6-14.5	6.8- 7.2	3.4- 3.6	1.7

Coarse Silt				
+4.0	3.2- 3.3	1.6	.8	.4
4.1	3.0- 3.1	1.5	.8	.4
4.2	2.8- 2.9	1.4	.7	.3
4.3	2.6- 2.7	1.3	.7	.3
4.4	2.4- 2.5	1.2	.6	.3
+4.5	2.3	1.2	.6	.3
4.6	2.1- 2.2	1.1	.5	.3
4.7	2.0	1.0	.5	.2
4.8	1.9	.9	.5	.2
4.9	1.7- 1.8	.9	.4	.2
Medium Silt				
+5.0	1.6	.8	.4	.2
5.1	1.5	.8	.4	.2
5.2	1.4	.7	.4	.2
5.3	1.3	.7	.3	.2
5.4	1.2	.6	.3	.2
+5.5	1.2	.6	.3	.1
5.6	1.1	.5	.3	.1
5.7	1.0	.5	.3	.1
5.8	.9	.5	.2	.1
5.9	.9	.4	.2	.1
Fine Silt				
+6.0	.8	.4	.2	.1
6.2	.7	.4	.2	.1
6.4	.6	.3	.2	.1
6.6	.5	.3	.1	
6.8	.5	.2	.1	
Very Fine Silt and Clay				
+7.0	.4	.2	.1	
7.5	.3	.1		
8.0	.2	.1		
9.0	.1			

Table 3. - Phi-Micrometer Conversion Table for American Optical Company's Cycloptic Microscope with 10X Eyepiece and No. 1406A Micrometer Disc (10 mm in 100 divisions)

Conversion factors: 1 micrometer scale division = 0.02065 mm at 50X
0.0414 mm at 25X
0.08333 mm at 15X
0.170 mm at 7X

	Micrometer Scale Divisions			
Phi	50X	25X	15X	7X
Fine Pebbles				
-3.0			92.8-99.3	45.5-48.7
2.9			86.6-92.7	42.5-45.4
2.8			80.8-86.5	39.6-42.4
2.7			75.4-80.7	37.0-39.5
2.6			70.3-75.3	34.5-36.9
-2.5			65.6-70.2	32.2-34.4
2.4			61.2-65.5	30.0-32.1
2.3			57.1-61.1	28.0-29.9
2.2			53.3-57.0	26.2-27.9
2.1			49.7-53.2	24.4-26.1
Very Fine Pebbles (Granules)				
-2.0			46.4-49.6	22.8-24.3
1.9		87.1-93.3	43.3-46.3	21.3-22.7
1.8		81.3-87.0	40.4-43.2	19.8-21.2
1.7		75.9-81.2	37.7-40.3	18.5-19.7
1.6		70.8-75.8	35.2-37.6	17.3-18.4
-1.5		66.0-70.7	32.8-35.1	16.1-17.2
1.4		61.6-65.9	30.6-32.7	15.0-16.0
1.3		57.5-61.5	28.6-30.5	14.0-14.9
1.2		53.7-57.4	26.7-28.5	13.1-13.9
1.1		50.1-53.6	24.9-26.6	12.2-13.0

	Micrometer Scale Divisions			
Phi	50X	25X	15X	7X
Fine Sand				
+2.0	11.7-12.5	5.9- 6.2	2.9- 3.1	1.5
2.1	11.0-11.6	5.5- 5.8	2.8	1.4
2.2	10.2-10.9	5.1- 5.4	2.6- 2.7	1.3
2.3	9.5-10.1	4.8- 5.0	2.4- 2.5	1.2
2.4	8.9- 9.4	4.5- 4.7	2.2- 2.3	1.1
+2.5	7.3- 8.8	4.2- 4.4	2.1	1.0
2.6	7.8- 8.2	3.9- 4.1	2.0	1.0
2.7	7.2- 7.7	3.6- 3.8	1.8- 1.9	.9
2.8	6.8- 7.1	3.4- 3.5	1.7	.8
2.9	6.3- 6.7	3.2- 3.3	1.6	.8
Very Fine Sand				
+3.0	5.9- 6.2	3.0- 3.1	1.5	.7
3.1	5.5- 5.8	2.8- 2.9	1.4	.7
3.2	5.1- 5.4	2.6- 2.7	1.3	.6
3.3	4.8- 5.0	2.4- 2.5	1.2	.6
3.4	4.5- 4.7	2.3	1.1	.6
+3.5	4.2- 4.4	2.1- 2.2	1.0	.5
3.6	3.9- 4.1	2.0	1.0	.5
3.7	3.6- 3.8	1.8- 1.9	.9	.5
3.8	3.4- 3.5	1.7	.9	.4
3.9	3.2- 3.3	1.6	.8	.4

Very Coarse Sand				
-1.0		46.7-50.0	23.2-24.8	11.4-12.1
.9	87.3-93.5	43.6-46.6	21.7-23.1	10.7-11.3
.8	81.5-87.2	40.7-43.5	20.2-21.6	9.9-10.6
.7	76.0-81.4	38.0-40.6	18.9-20.1	9.3- 9.8
.6	71.0-75.9	35.4-37.9	17.6-18.8	8.7- 9.2
- .5	66.2-70.9	33.0-35.3	16.4-17.5	8.1- 8.6
.4	61.8-66.1	30.8-32.9	15.3-16.3	7.5- 8.0
.3	57.6-61.7	28.8-30.7	14.3-15.2	7.0- 7.4
.2	53.8-57.5	26.9-28.7	13.4-14.2	6.6- 6.9
.1	50.2-53.7	25.1-26.8	12.5-13.3	6.1- 6.5
Coarse Sand				
0	46.8-50.1	23.4-25.0	11.6-12.4	5.7- 6.0
+ .1	43.7-46.7	21.8-23.3	10.9-11.5	5.4- 5.6
.2	40.8-43.6	20.4-21.7	10.1-10.8	5.0- 5.3
.3	38.0-40.7	19.0-20.3	9.5-10.0	4.7- 4.9
.4	35.5-37.9	17.7-18.9	8.8- 9.4	4.4- 4.6
+ .5	33.1-35.4	16.5-17.6	8.2- 8.7	4.1- 4.3
.6	30.9-33.0	15.4-16.4	7.7- 8.1	3.8- 4.0
.7	28.8-30.8	14.4-15.3	7.2- 7.6	3.5- 3.7
.8	26.9-28.7	13.5-14.3	6.7- 7.1	3.3- 3.4
.9	25.1-26.8	12.6-13.4	6.3- 6.6	3.1- 3.2
Medium Sand				
+1.0	23.4.25.0	11.7-12.5	5.8- 6.2	2.9- 3.0
1.1	21.9-23.3	10.9-11.6	5.5- 5.7	2.7- 2.8
1.2	20.4-21.8	10.2-10.8	5.1- 5.4	2.5- 2.6
1.3	19.0-20.3	9.5-10.1	4.8- 5.0	2.4
1.4	17.8-18.9	8.9- 9.4	4.4- 4.7	2.2- 2.3
+1.5	16.6-17.7	8.3- 8.8	4.1- 4.3	2.1
1.6	15.5-16.5	7.7- 8.2	3.9- 4.0	1.9- 2.0
1.7	14.4-15.4	7.2- 7.6	3.6- 3.8	1.8
1.8	13.5-14.3	6.8- 7.1	3.4- 3.5	1.7
1.9	12.6-13.4	6.3- 6.7	3.2- 3.3	1.6

Coarse Silt				
+4.0	3.0- 3.1	1.5	.8	.4
4.1	2.8- 2.9	1.4	.7	.3
4.2	2.6- 2.7	1.3	.7	.3
4.3	2.4- 2.5	1.2	.6	.3
4.4	2.3	1.1	.6	.3
+4.5	2.1- 2.2	1.1	.5	.3
4.6	2.0	1.0	.5	.2
4.7	1.8- 1.9	.9	.5	.2
4.8	1.7	.9	.4	.2
4.9	1.6	.8	.4	.2
Medium Silt				
+5.0	1.5	.8	.4	.2
5.1	1.4	.7	.3	.2
5.2	1.3	.7	.3	.2
5.3	1.2	.6	.3	.1
5.4	1.1	.6	.3	.1
+5.5	1.1	.5	.3	.1
5.6	1.0	.5	.2	.1
5.7	.9	.5	.2	.1
5.8	.9	.4	.2	.1
5.9	.8	.4	.2	.1
Fine Silt				
+6.0	.8	.4	.2	.1
6.2	.7	.3	.2	.1
6.4	.6	.3	.1	
6.6	.5	.2	.1	
6.8	.4	.2	.1	
Very Fine Silt and Clay				
+7.0	.4	.2	.1	
7.5	.3	.1		
8.0	.2	.1		
9.0	.1			

Table 4. - Selected Formulae for Phi-Percentile Parameters

Property	Nomenclature	Formula
Central Tendency	1. Median	$\mathrm{Mdn} = P_{50}$
	2. Mean (Inman)	$M_2 = \frac{P_{16} + P_{84}}{2}$
	3. Mean (Folk and Ward)	$M_3 = \frac{P_{16} + P_{50} + P_{84}}{3}$
	4. Mean (Swann et al.)	$M_{10} = \frac{P_5 + 2P_{16} + 4P_{50} + 2P_{84} + P_{95}}{10}$
	5. Mode	$\mathrm{Mo} = 3P_{50} - (P_{16} + P_{84})$
Dispersion (Sorting)	6. Quartile standard deviation	$s_{1.35} = \frac{P_{75} - P_{25}}{1.35}$
	7. Inman standard deviation	$s_2 = \frac{P_{84} - P_{16}}{2}$
	8.	$s_{2.58} = \frac{P_{90} - P_{10}}{2.58}$
	9.	$s_{3.3} = \frac{P_{95} - P_5}{3.3}$
	10.	$s_4 = \frac{P_{98} - P_2}{4}$
	11.	$s_6 = \frac{P_{99.85} - P_{0.15}}{6}$
	12. Inclusive standard deviation (Folk and Ward)	$s_I = \frac{P_{84} - P_{16}}{4} + \frac{P_{95} - P_5}{6.6}$
	13. Coarse partial deviation (Swann et al.)	$s_C = \frac{P_{50} - P_{0.15}}{3}$

Table 4. - Continued

Property	Nomenclature	Formula
Dissymetry (Skewness)	14. Modified quartile skewness	$Sk_{50} = \frac{P_{25} + P_{75} - 2P_{50}}{P_{75} - P_{25}}$
	15. First Inman skewness	$Sk_{68} = \frac{P_{16} + P_{84} - 2P_{50}}{P_{84} - P_{16}}$
	16. Modified second Inman skewness	$Sk_{90} = \frac{P_5 + P_{95} - 2P_{50}}{P_{95} - P_5}$
	17. Inclusive skewness (Folk and Ward)	$Sk_I = \frac{P_{16} + P_{84} - 2P_{50}}{2(P_{84} - P_{16})} + \frac{P_5 + P_{95} - 2P_{50}}{2(P_{95} - P_5)}$
Peakedness (Kurtosis)	18. 90/50 kurtosis (Folk and Ward)	$K_{90/50} = \frac{s_{3.3}}{s_{1.35}} = \frac{P_{95} - P_5}{2.44(P_{75} - P_{25})}$
	19. 90/68 kurtosis	$K_{90/68} = \frac{s_{3.3}}{s_2} = \frac{P_{95} - P_5}{1.65(P_{84} - P_{16})}$
	20. 80/50 kurtosis	$K_{80/50} = \frac{s_{2.58}}{s_{1.35}} = \frac{P_{90} - P_{10}}{1.91(P_{75} - P_{25})}$
	18a, 19a, 20a. Transformed kurtosis	$K' = \frac{2K}{K + 1}$
	21. Inman kurtosis measure	$K_{Inman} = \frac{(P_{95} - P_5) - (P_{84} - P_{16})}{P_{84} - P_{16}}$

Illinois State Geological Survey Circular 280
43 p., 14 figs., 4 tables, app., 1959

ILLINOIS
Land of Lincoln

CIRCULAR 280

ILLINOIS STATE GEOLOGICAL SURVEY

URBANA

114