Nonlinear Modal Analysis of an L-Shape Beam Structure

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Since the 1960s there has been continuous interest in the dynamics of L-shaped coupled structures exhibiting certain nonlinearities. Balachandran and Nayfeh performed nonlinear modal analysis considering only the in-plane motions [1]. Roberts and Cartmell studied autoparametric resonances within an L-shaped beam structure [2], and Warminski *et al*. formulated the 3rd order partial differential nonlinear equations of an L-shape beam structure with different flexibilities in the two orthogonal directions [3]. Linear modal analysis of an L-shape beam is discussed in Georgiades *et al*. [4] where it has been shown that the system of linear PDEs can be divided into sets of coupled equations. One set includes two equations which describe the in plane bending motions (v_1,w_2) and another set contains four coordinates including out of plane bending with torsion (w_1,v_2,φ_1,φ_2). In this article we discretise the 2nd order equations of motion in space, using a projection of the total displacements onto the infinite space of the linear mode shapes, with the final aim of determining the NNMs of the nonlinear ODE system [4]. We followed the formulation described in ([3]) for the definition of curvatures, angular velocities and linear velocities, and also considered the angular velocities of the beams for the out-of-plane motions on the basis that they may play significant role, as is shown in linear modal analysis [4]. We express the local displacements in the following series,

$$v_1 = \sum_{i=1}^{\infty} Y_{v_{1,i}}(s_1)\, q_{v_{1,i}}(t), w_1 = \sum_{i=1}^{\infty} Y_{w_{1,i}}(s_1)\, q_{w_{1,i}}(t), \varphi_1 = \sum_{i=1}^{\infty} Y_{\varphi_{1,i}}(s_1)\, q_{\varphi_{1,i}}(t),$$

$$v_2 = \sum_{i=1}^{\infty} Y_{v_{2,i}}(s_2)\, p_{v_{2,i}}(t) - s_2 \sum_{i=1}^{\infty} Y_{\varphi_{1,i}}(l_1)\, q_{\varphi_{1,i}}(t) - \sum_{i=1}^{\infty} Y_{w_{1,i}}(l_1)\, q_{w_{1,i}}(t),$$

$$w_2 = \sum_{i=1}^{\infty} Y_{w_{2,i}}(s_2)\, p_{w_{2,i}}(t) + s_2 \sum_{i=1}^{\infty} Y'_{v_{1,i}}(l_1)\, q_{v_{1,i}}(t),$$

$$\varphi_2 = \sum_{i=1}^{\infty} Y_{\varphi_{2,i}}(s_2)\, p_{\varphi_{2,i}}(t) + \sum_{i=1}^{\infty} Y'_{w_{1,i}}(l_1)\, q_{w_{1,i}}(t) \qquad \text{(1a-f).}$$

Considering the Lagrange formalism and the equations for axial displacement from the inextensionality conditions, then by taking the variation of the modal displacements and velocities, with a truncation of the series to just 1 mode (eq.1 with i=1), the following general form of the equations is obtained (presented in compact form here due to limited space):

$$\ddot{q}_{v_1} = F_1(q_{v_1}, p_{v_2}p_{\varphi_2}, q_{\varphi_1}q_{w_1}, \dot{p}_{v_2}\dot{p}_{\varphi_2}, \dot{q}_{\varphi_1}\dot{p}_{\varphi_2}, \dot{p}_{w_2}\dot{q}_{v_1}, \dot{p}_{w_2}\dot{p}_{w_2}, \dot{q}_{v_1}\dot{q}_{v_1}, \dot{q}_{\varphi_1}\dot{q}_{\varphi_1}, \dot{q}_{w_1}\dot{p}_{v_2}, \dot{p}_{v_2}\dot{p}_{v_2}, \dot{q}_{w_1}\dot{q}_{\varphi_1}, \dot{p}_{v_2}\dot{q}_{\varphi_1}, \dot{q}_{\varphi_1}\dot{p}_{v_2}, p_{v_2}q_{\varphi_1}, q_{\varphi_1}q_{\varphi_1}, p_{w_2}p_{w_2}, q_{v_1}p_{w_2}, p_{v_2}p_{v_2}, q_{v_1}q_{v_1}, p_{\varphi_2}, q_{w_1}p_{v_2}),$$

$$\ddot{q}_{w_1} = F_2(q_{w_1}, q_{\varphi_1}q_{v_1}, p_{v_2}p_{w_2}, \dot{p}_{w_2}\dot{p}_{v_2}, \dot{q}_{v_1}\dot{q}_{\varphi_1}, \dot{q}_{v_1}\dot{p}_{v_2}, q_{v_1}p_{v_2}, q_{w_1}p_{w_2}),$$

$$\ddot{q}_{\varphi_1} = F_3(q_{\varphi_1}, q_{v_1}q_{w_1}, \dot{q}_{v_1}\dot{q}_{w_1}, \dot{q}_{v_1}\dot{p}_{\varphi_2}, q_{\varphi_1}q_{v_1}, \dot{p}_{v_2}\dot{q}_{v_1}, \dot{p}_{w_2}\dot{p}_{\varphi_2}, p_{w_2}p_{\varphi_2}, q_{v_1}p_{\varphi_2}, p_{v_2}q_{v_1}),$$

$$\ddot{p}_{v_2} = F_4(p_{v_2}, p_{\varphi_2}p_{w_2}, q_{w_1}p_{w_2}, q_{v_1}p_{\varphi_2}, \dot{q}_{v_1}\dot{q}_{w_1}, \dot{q}_{\varphi_1}\dot{q}_{v_1}, \dot{p}_{\varphi_2}\dot{p}_{w_2}, \dot{p}_{w_2}\dot{p}_{\varphi_2}, \dot{q}_{w_1}\dot{p}_{w_2}, \dot{p}_{\varphi_2}\dot{q}_{v_1}q_{v_1}q_{w_1}, p_{v_2}q_{v_1}, q_{\varphi_1}q_{v_1}),$$

$$\ddot{p}_{w_2} = F_5(p_{w_2}, p_{v_2}p_{\varphi_2}, q_{w_1}p_{v_2}, \dot{p}_{v_2}\dot{p}_{\varphi_2}, \dot{q}_{\varphi_1}\dot{p}_{\varphi_2}, \dot{q}_{v_1}\dot{q}_{v_1}, \dot{q}_{w_1}\dot{p}_{v_2}, \dot{q}_{w_1}\dot{q}_{w_1}, q_{v_1}q_{v_1}, q_{w_1}q_{w_1}, p_{w_2}q_{v_1}),$$

$$\ddot{p}_{\varphi_2} = F_6(p_{\varphi_2}, p_{v_2}p_{w_2}, p_{v_2}q_{v_1}, q_{\varphi_1}p_{w_2}, \dot{p}_{w_2}\dot{p}_{v_2}, \dot{q}_{\varphi_1}\dot{p}_{w_2}, \dot{q}_{v_1}\dot{p}_{v_2}, q_{\varphi_1}q_{v_1}) \qquad \text{(2a-f).}$$

The expression of the equations in terms of modal displacements, and including the second order nonlinearities, indicates that all 6 considered motions are coupled together. The next step is to perform a numerical nonlinear modal analysis, initially with truncation of the series to just one mode, and then to increase progressively the number of modes in the series of Eq.(1).

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