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# Linear Modal Analysis of L-shaped Beam Structures - Parametric Studies

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**Abstract.** Linear modal analysis of L-shaped beam structures indicates that there are two independent motions, these are in-plane bending and out of plane motions including bending and torsion. Natural frequencies of the structure can be determined by finding the roots of two transcendental equations which correspond to in-plane and out-of-plane motions. Due to the complexity of the equations of motion the natural frequencies cannot be determined explicitly. In this article we nondimensionalise the equations of motion in the space and time domains, and then we solve the transcendental equations for selected values of the L-shaped beam parameters in order to determine their natural frequencies. We use a numerical continuation scheme to perform the parametric solutions of the considered transcendental equations. Using plots of the solutions we can determine the natural frequencies for a specific L-shape beam configuration.

## 1. Introduction

There has been continuous interest in the dynamics of L-shaped coupled structures since the 1960s. Roberts and Cartmell in [1,2] studied autoparametric resonances within an L-shape beam structure. Balachandran and Nayfeh performed nonlinear dynamic analysis considering only in-plane motions [3]. Warminski *et al* formulated in [4] the third order partial differential nonlinear equations of an L-shaped beam structure with different flexibilities in the two orthogonal directions, without taking into account rotary inertia effects. Ozonato *et al.* studied post-buckled chaotic vibrations of an L-shaped beam structure considering nonlinear motions but only in plane bending [5]. In [7] Georgiades *et al.* extracted the linear equations of motion of an L-shaped beam structure, considering the inextensionality conditions and rotary inertia terms and in [8] they performed a linear modal analysis of an L-shaped beam structure. The natural frequencies of a given L-shaped beam structure can be determined by the roots of two transcendental equations which correspond to in-plane bending and out-of-plane motions.

In this article we start from the equations of motion, which we nondimensionalise in space and time. Then the nondimensional frequencies can be determined by the roots of the two transcendental equations. Using a numerical continuation technique we perform a parametric study to determine the nondimensional natural frequency of the first mode for in-plane and out-of-plane motions for various sets of parameters.

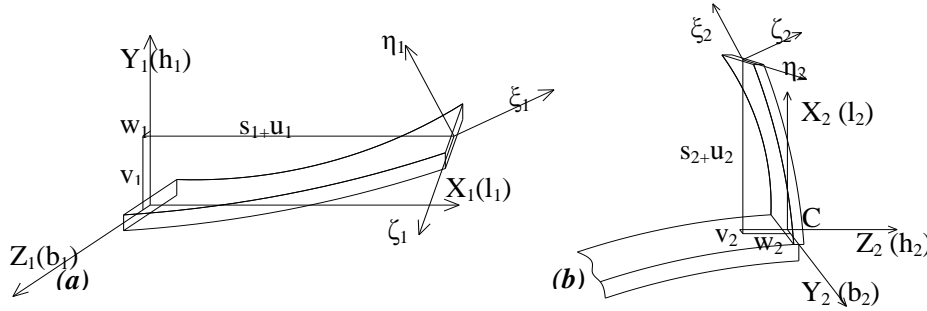
## 2. Theory

### 2.1 Equations of motion

Let us consider beams made of isotropic material and constant cross section with respect to the longitudinal direction. Initially, we nondimensionalise the equations of motion in space and time, using,

$$t = a_i \tau \text{ and } s = l_1 S. \quad (1a,b)$$

noting that the scaling in time is different for the two different motions, with  $i = 1,2$  for in-plane motion and out-of-plane motion respectively. For in-plane bending we use the following nondimensionlised parameters,



**Figure 1.** Indication of axis orientations, displacements and dimensions for (a) the primary beam, and (b) the secondary beam.

$$a_1 = \sqrt{\frac{\bar{D}_{\zeta_1} m_1 l_1^4}{D_{\zeta_1}}}, \bar{m}_2 = \frac{m_2}{m_1}, \bar{l}_2 = \frac{l_2}{l_1}, \bar{D}_{\eta_2} = \frac{D_{\eta_2} \bar{D}_{\zeta_1}}{D_{\zeta_1}}, \quad (2a-d)$$

for which  $\bar{D}_{\zeta_1}$  can have any arbitrary value.

For the out-of-plane motion we use the following nondimensionlised parameters,

$$a_2 = \sqrt{\frac{\bar{D}_{\eta_1} m_1 l_1^4}{D_{\eta_1}}}, \bar{m}_2 = \frac{m_2}{m_1}, \bar{l}_2 = \frac{l_2}{l_1}, \bar{D}_{\zeta_2} = \frac{D_{\zeta_2} \bar{D}_{\eta_1}}{D_{\eta_1}}, \bar{I}_{\eta_1} = \frac{I_{\eta_1}}{m_1 l_1^2},$$

$$\bar{I}_{\xi_1} = \frac{I_{\xi_1}}{m_1 l_1^2}, \bar{D}_{\xi_1} = \frac{D_{\xi_1} \bar{D}_{\eta_1}}{D_{\eta_1}}, \bar{I}_{\xi_2} = \frac{I_{\xi_2}}{m_1 l_1^2}, \bar{D}_{\xi_2} = \frac{D_{\xi_2} \bar{D}_{\eta_1}}{D_{\eta_1}}, \quad (3a-i)$$

where  $\bar{D}_{\eta_1}$  can have any arbitrary value. Before proceeding with the parametric study we define the following dimensionless ratios,

$$\beta_1 = \frac{b_2}{b_1}, \beta_2 = \frac{b_2}{h_1}, \beta_3 = \frac{h_1}{l_1}, \beta_4 = \frac{h_2}{l_1}. \quad (4a-d)$$

Therefore using these ratios (eq. 4a-d) and considering that both beams are made of the same material (aluminium with  $\nu = 0.33$ ) and considering the transcendental equations given by [8] we determine the nondimensional natural frequencies.

### 3. Results of Parametric Study

The roots of the transcendental equations have been determined using the optimisation toolbox within Matlab. In both cases, we used a numerical continuation technique. According to this technique the initial input value for the optimisation technique was the one from the already determined nearest parameter value, but using a very fine mesh for the variation of each parameter in a specific range. In this way we tried to avoid jumps from the roots of the transcendental equations which would correspond to the first mode to other roots which correspond to higher modes. In order to scale the system of equations appropriately, and to avoid singular values in parts of the transcendental equations, we used the following nondimensional rescaling value for the stiffnesses:

$$\bar{D}_{\zeta_1} = \bar{D}_{\eta_1} = 1000.$$

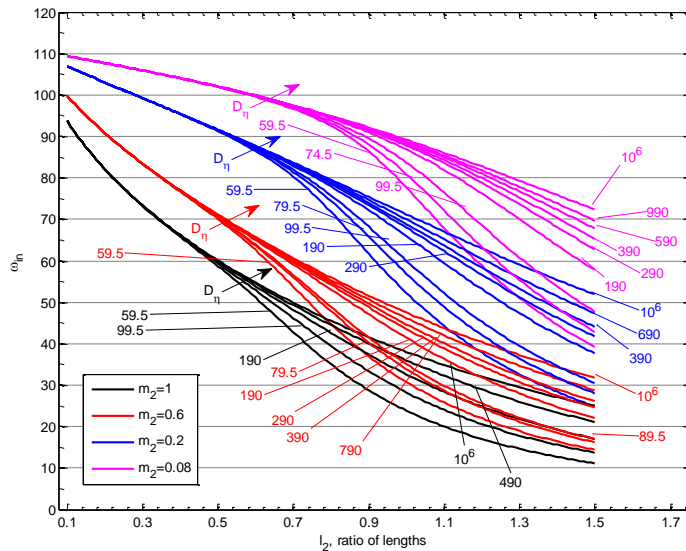
Therefore, for a specific configuration of the L-shaped beam structure, and considering the results from the parametric study, the natural frequencies in rad/sec are given by,

$$\omega_{in,0} = \frac{\omega_{in}}{a_1}, \text{ and } \omega_{out,0} = \frac{\omega_{out}}{a_2}. \quad (26a,b)$$

#### 3.1 In-plane motion

In the case of in-plane motion we performed a parametric study considering variations of  $\bar{m}_2$ ,  $\bar{D}_{\eta_2}$  and  $\bar{l}_2$ . Figures 2a-d depict the nondimensional natural frequencies which correspond to the first natural

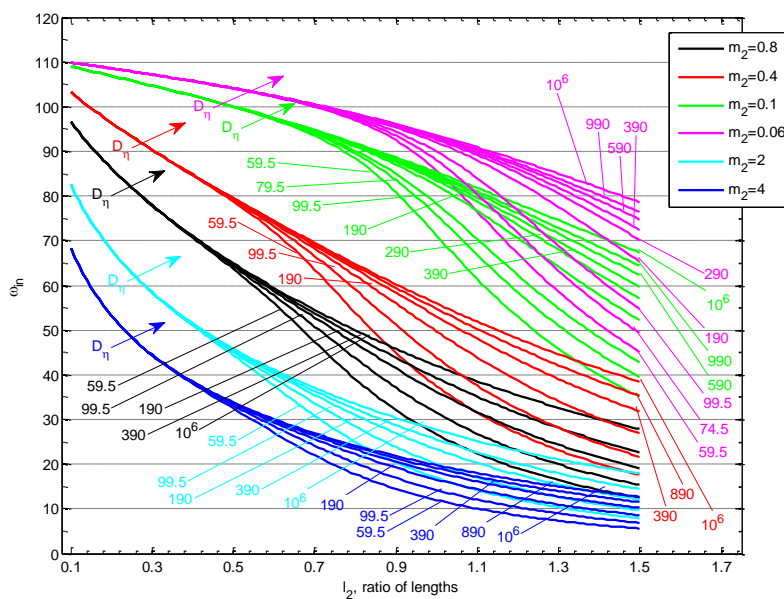
frequency for the in-plane bending. There are two characteristics in Fig. 2a-d which clearly demonstrate: a) the stiffening effect with the increase of bending stiffness, or decrease of mass, and b) that for low length dimensions ( $\bar{l}_2$ ) the secondary beam behaves like a tip-mass, and the bending stiffness is not playing any essential role.



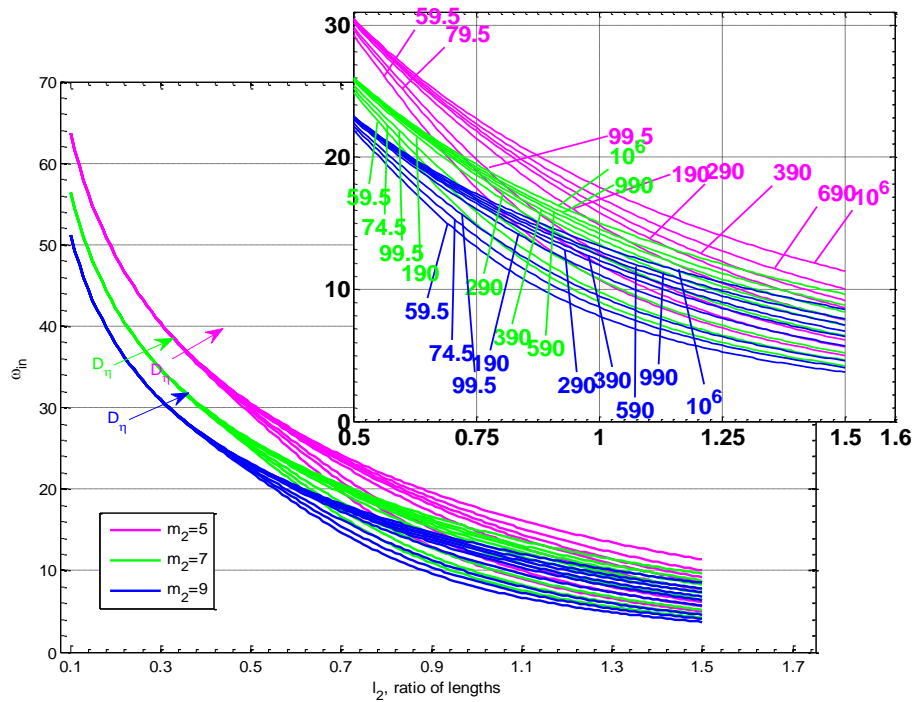
**Figure 2a.** Nondimensional natural frequencies of the first mode for in-plane motion.

### 3.2 Out-of-plane motion

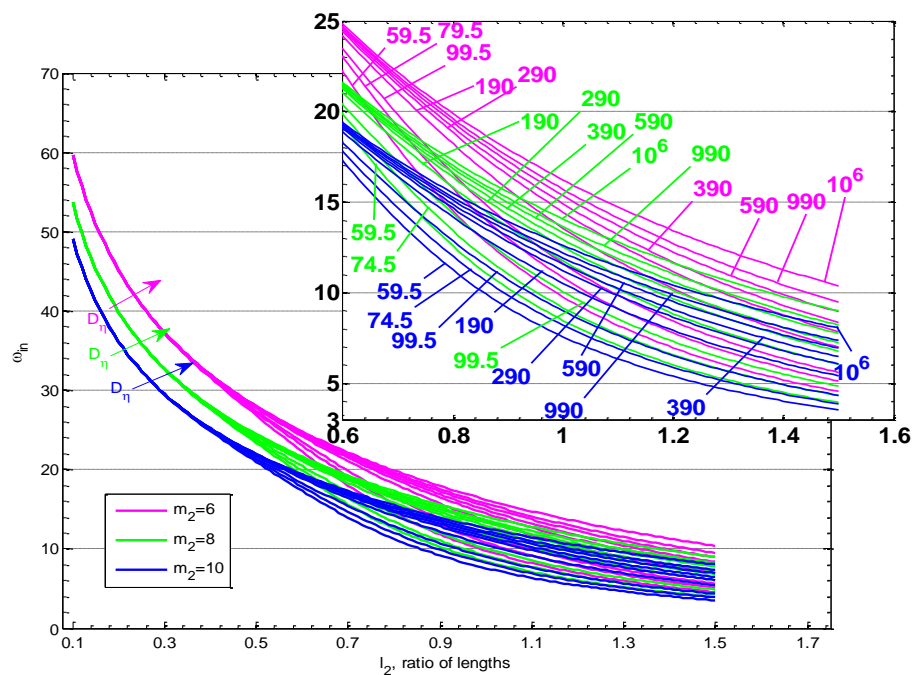
In Figure 3-5 the nondimensional natural frequencies of the first mode are depicted for out of plane motion for  $\beta_1=9,1,0.1$  respectively. In these Figures the left side corresponds to  $\bar{l}_2 \in [0.1,0.8]$  and the right side Figures correspond to  $\bar{l}_2 \in [0.8,1.5]$ , noting that the parametric values of the pair  $(\beta_2, \beta_4)$  are also indicated. The convergence of many curves to one with a decrease in  $\bar{l}_2$ , for which the secondary beam behaves like a tip mass, is evident only in Figs. 3-5 and in those cases of very small dimensions for the secondary beam in comparison to the primary beam dimensions.



**Figure 2b.** Nondimensional natural frequencies of the first mode for in-plane motion.



**Figure 2c.** Nondimensional natural frequencies of the first mode for in-plane motion.



**Figure 2d.** Nondimensional natural frequencies of the first mode for in-plane motion.

**4. Conclusions**

In this article a parametric study was performed for the determination of the nondimensional natural frequency of the first system mode for in-plane and out-of-plane bending, for various sets of parameters. For the case of in-plane bending the effects related to an increase in stiffness or a decrease in mass were demonstrated. When the dimensions of the secondary beam are much smaller than those of the primary beam then the system behaves like a cantilever beam with a tip mass.

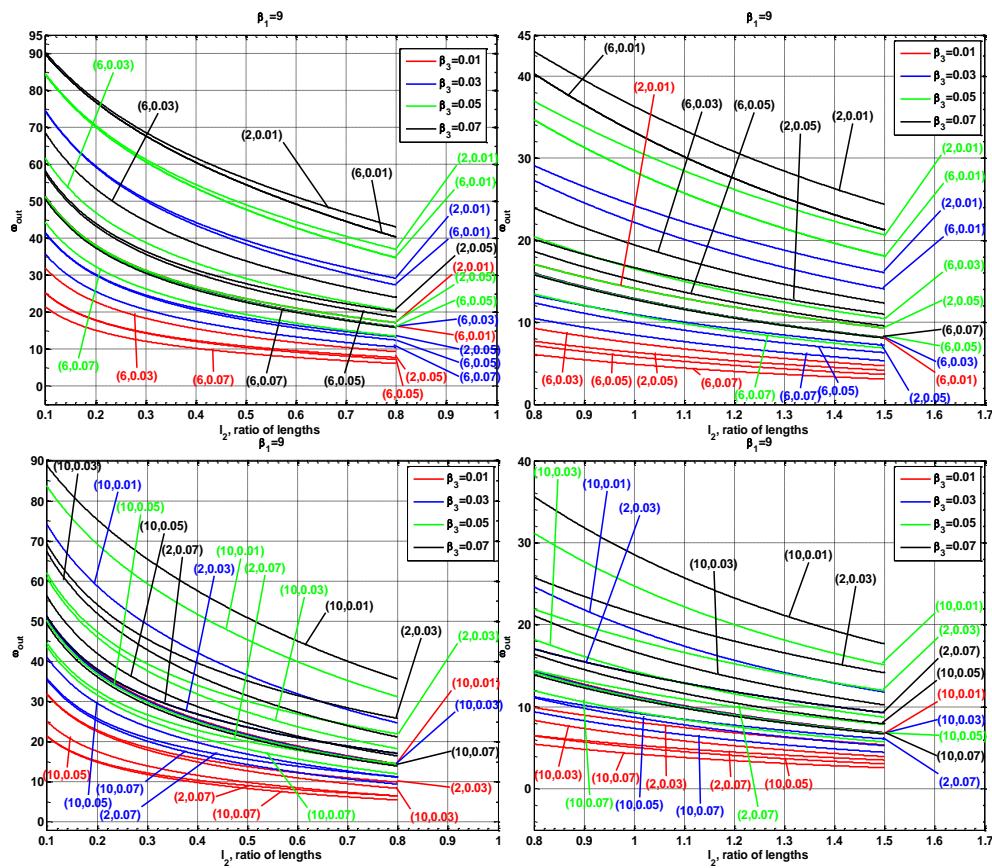


Figure 3. Nondimensional frequencies for the first mode of out-of-plane motions with  $\beta_1=9$ .

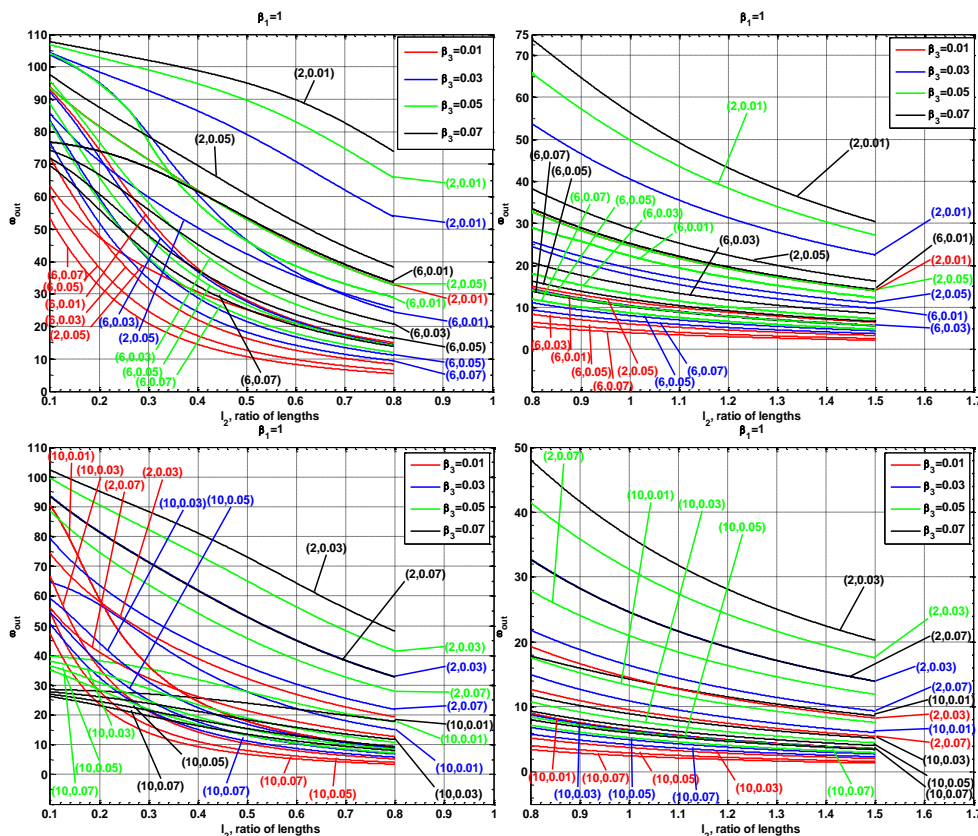
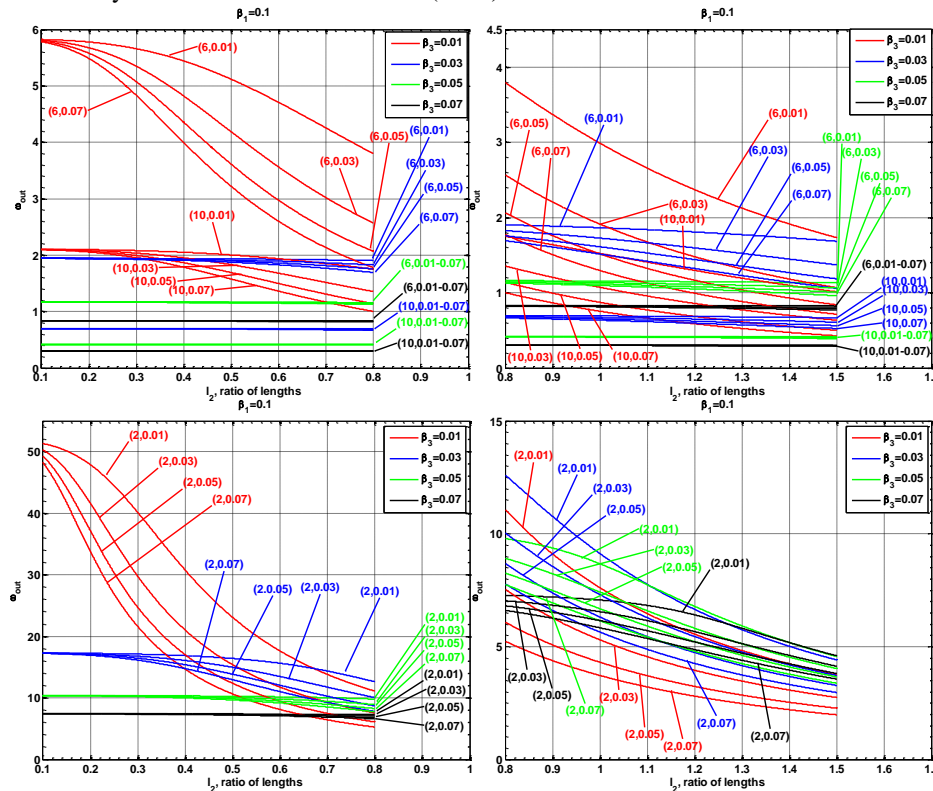


Figure 4. Nondimensional frequencies for the first mode of out-of-plane motions with  $\beta_1=1$ .



**Figure 5.** Nondimensional frequencies for the first mode of out-of-plane motions with  $\beta_1=0.1$ .

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