Necessity of Dark Matter in Modified Newtonian Dynamics within Galactic Scales

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To test modified Newtonian dynamics (MOND) on galactic scales, we study six strong gravitational lensing early-type galaxies from the CASTLES sample. Comparing the total mass (from lensing) with the stellar mass content (from a comparison of photometry and stellar population synthesis), we conclude that strong gravitational lensing on galactic scales requires a significant amount of dark matter, even within MOND. On such scales a 2 eV neutrino cannot explain the excess of matter in contrast with recent claims to explain the lensing data of the bullet cluster. The presence of dark matter is detected in regions with a higher acceleration than the characteristic MOND scale of $\sim 10^{-10}$ m/s^2. This is a serious challenge to MOND unless lensing is qualitatively different [possibly to be developed within a covariant, such as Tensor-Vector-Scalar (TeVeS), theory].

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The standard (ΛCDM) cosmological paradigm is based on cold dark matter (CDM), a cosmological constant Λ, and classical general relativity. Despite its enormous success and simplicity, competing models have been proposed, the main reason being the still unknown dark energy component and the undetectability of dark matter. To explain the observed flat rotation curves, Milgrom [1] proposed modified Newtonian dynamics (MOND), based on the relation $f(|\vec{a}|/a_0)\vec{a} = -\vec{\nabla}\Phi_N$, between the acceleration $\vec{a}$ and the Newtonian gravitational field Φ_N. The constant $a_0 \approx 10^{-10}$ m/s^2 is motivated by the acceleration found in the outer regions of the galaxy where the rotation curve is flat. When f, assumed to be a positive smooth monotonic function, equals unity, usual Newtonian dynamics holds, while when it approximately equals its argument, the deep MOND-like regime applies.

MOND has been successful in explaining the dynamics of disk galaxies; it is less successful for clusters of galaxies. It was promoted [2] to a relativistic field theory by introducing a tensor, a vector, and a scalar field (TeVeS). TeVeS has been criticized as lacking a fundamental theoretical motivation. Recently, it has been argued [3] that such a theory can emerge naturally within string models.

Here we calculate within MOND the deflection angles for two generic density profiles and compare them with those predicted in standard lensing. We calculate the mass of the lenses and estimate the amount of dark matter required. We find that despite the alternative gravitational falloff, the masses predicted by MOND are very similar to those predicted within standard gravitational lensing theory. We conclude that MOND within galactic scales needs a considerable amount of dark matter.

We consider a homogeneous and isotropic three metric with the density parameters "tweaked" to the values in a MOND-like cosmology. The outcome of our lensing analysis depends only weakly on the cosmology, for a reasonable range of cosmological parameters. A different background cosmology mainly results in the change of the critical surface mass density [4].

Assuming that the deflection of photons is twice that of nonrelativistic particles and that the photon path is nearly linear, the deflection angle α as a function of the impact parameter b can be written, for a given cumulative mass profile $M(<r)$, as (see, e.g., [5]):

$$\alpha(b) = -\frac{4Gb}{c^2}\int_0^\infty f^{-1/2}\left(\frac{GM(<\sqrt{b^2+z^2})}{[b^2+z^2]a_0}\right) \times \frac{M(<\sqrt{b^2+z^2})}{[b^2+z^2]^{3/2}}\,dz. \tag{1}$$

When the function $f(x)$ in the integrand is removed, we recover the expression of the deflection angle in standard lensing. The function $f(x)$ "modulates" this deflection along the path of the particle depending on the ratio between the local acceleration, $GM(<r)/r^2$, and the MOND-like characteristic acceleration, a_0. We will henceforth use Eq. (1) to calculate the deflection angle. In standard lensing $f(x)$ is set to unity, while in MOND we first adopt [6] $f(x) = x[1+x^2]^{-1/2}$.

We compare observations of strong lensing systems (which are most often elliptical galaxies) with realistic mass profiles. Spherical symmetry is assumed. In addition to the "no-dark-matter" interpretation of the rotation curves in disk galaxies, we assume that in MOND the stellar mass content represents the full mass budget; the contribution of other baryonic components such as gas or dust is minimal in early-type systems. Their characteristic surface brightness profile can be represented by a Hernquist 3D density profile [7]. The cumulative mass profile is $M(<r) = Mr^2/(r+r_h)^2$, where M is the total mass of the galaxy and r_h the core length scale, related to the projected 2D half-mass radius by $R_e = 1.8153 r_h$. This density model has a logarithmic slope $(d\log\rho)/(d\log r) \propto -1$ towards the center, changing to -4, as $r \longrightarrow \infty$. This is our first model.

The Navarro-Frenk-White (NFW) profile [8] is our second model. The cumulative mass profile diverging logarithmically, we assume a truncation radius r_{virial}. This profile has two free parameters, the core length scale r_s, and the virial radius. Their ratio is the concentration $\mathcal{C}$. Cosmological simulations [9] suggest concentrations on galaxy scales to be $\mathcal{C} \sim 10$. Denoting by x the ratio $x \equiv r/r_{\mathrm{virial}}$, the cumulative mass function of the NFW profile reads

$$M(<r) = M \frac{\ln(1 + \mathcal{C}x) - \frac{\mathcal{C}x}{1+\mathcal{C}x}}{\ln(1 + \mathcal{C}) - \frac{\mathcal{C}}{1+\mathcal{C}}}. \qquad (2)$$

The lens equation $\beta = \theta - \alpha(\theta)D_{\mathrm{LS}}/D_S$ relates the actual position of the background source β, with the position θ of the images. For a given cosmological model, the angular diameter distances from the lens to the source, and from the observer to the source, D_{LS} and D_S, respectively, are obtained from the observed redshifts. The deflection angle α depends on the mass profile of the system and the impact parameter. A characteristic aspect of strong gravitational lensing is that one image appears inside the Einstein radius r_E and the other one outside. The difference between MOND-like and standard lensing lies mostly in the position of the image outside r_E.

Figure 1 illustrates our methodology in estimating the masses of galaxies from lensing data. HE1104-1805 is extracted from the CfA-Arizona Space Telescope Survey (CASTLES [10]) sample. It consists of a galaxy at redshift $z_L = 0.73$ with a background quasistellar object at $z_S = 2.32$. A gray-scale map of the HST-NICMOS F160W image is shown on the right panel, retrieved from the CASTLES web page [11]. This is a double system with the image positions located at 2.09 and 1.10 arcsec on either side of the lensing galaxy. The left panel of Fig. 1 shows the correlation between the actual position β of the quasistellar object and the total mass of the lensing galaxy, assuming a Hernquist profile with the projected 2D half-mass radius being equal to the observed half-light radius of the lensing galaxy. Each set of lines—dashed (MOND) or solid (standard lensing theory)—are the results for each image. The compatible solution corresponds to the crossing of the lines, shown in the figure with a star symbol. This gives the true position of the source and the mass of the galaxy. For comparison, the values from Refs. [12] (for conventional lensing theory) and [13] (for MOND) are given as a shaded region and an arrow, respectively.

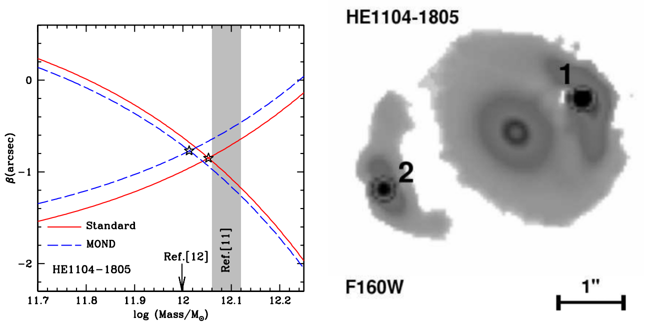

FIG. 1 (color online). *Left*: graphical representation of the lens equation in standard lensing (solid lines) and MOND (dashed lines). Each line corresponds to one of the two images of the background source. The distant one (number 2) corresponds to the lower set of lines (i.e., a more discrepant result between standard and MOND-like lensing). The intersection point of the two lines gives the position of the source and the total mass (Hernquist profile assumed). *Right*: NIR HST-NICMOS gray-scale image of the lensing system (from the CASTLES database).

Table I compares our mass estimates with the MOND-like analysis of Ref. [13] and with the standard nonparametric approach of Ref. [12] (where spherical symmetry is not assumed). The masses are quoted in units of $10^{10} M_\odot$ for a ΛCDM cosmology and, in brackets, for the open cosmological model of Ref. [13]. A Chabrier [14] initial mass function is considered for the stellar masses quoted from Ref. [12]. The resulting synthetic population, constrained by the photometry of the lensing galaxy in the optical (F814W) and NIR (F160W) passbands, is used to determine the stellar mass content. The sample studied here comprises only double systems to be suitable for a 1D approximation of the lens and serves to show the differences between MOND and standard lensing.

Table I shows a small difference in the mass estimates between the two different cosmologies considered here, despite their density parameters being quite different. This is because the angular distance is mostly unaffected by the change in the parameters. There are also some noticeable differences between a Hernquist and a NFW ($\mathcal{C} = 10$) model for the distribution of mass in the lensing galaxy. Nevertheless, the differences found are not large enough to affect our conclusions. One could always argue for a Hernquist profile as this is the model that a baryon-only MOND-like cosmology would favor, given that the projected mass distribution resembles the typical de Vaucouleur profile of early-type galaxies. However, recent lensing work on clusters, most noticeably the bullet cluster [15], has been used to postulate a 2 eV neutrino which would be important on scales of galaxy clusters, not on galactic scales [16]. We present the NFW profile to illustrate the robustness of our claims in rejecting the hypothesis of a 2 eV neutrino.

The top panels of Fig. 2 compare the mass estimates between standard theory and MOND for both density profiles: Hernquist (hollow dots) and NFW with $\mathcal{C} = 10$ (filled dots). The mass differences are shown as a function of conventionally calculated mass (left panel) and R_{LENS}/R_e (right panel). The difference between the conventional theory and MOND-like predictions stays mostly within 10%. This is especially noteworthy in systems with $R_{\mathrm{LENS}}/R_e \gtrsim 2$. Notice that the lensed images probe accelerations slightly above the MOND-like threshold. For instance, in lens HE1104-1805 (figure 1), image 2 (right

TABLE I. Mass estimates (in $10^{10}M_\odot$ units) for ΛCDM cosmology: $(\Omega_m, \Omega_\Lambda, \Omega_k) = (0.3, 0.7, 0)$. The masses in brackets correspond to the open cosmology of Ref. [13]: $(\Omega_m, \Omega_\Lambda, \Omega_k) = (0.03, 0.36, 0.51)$.

	Hernquist		NFW ($\mathcal{C} = 10$)		Ref. [12] (standard)		Ref. [13]
Lens	Standard	MOND	Standard	MOND	Standard	$M_{\rm STAR}$	MOND
Q0142 − 100	32.37(34.58)	29.28(31.56)	29.67(31.70)	26.63(28.74)	$24.9^{31.7}_{20.2}$	$20.9^{30.8}_{13.0}$	29.9
HS0818 + 1227	50.99(54.03)	46.31(49.50)	48.14(51.01)	43.38(46.42)	$67.4^{73.6}_{60.7}$	$16.2^{21.2}_{12.6}$	—
FBQ0951 + 2635	4.07(4.16)	3.82(3.91)	3.28(3.35)	3.07(3.14)	$4.7^{5.7}_{3.6}$	$1.1^{2.1}_{0.5}$	3.6
BRI0952 − 0115	7.33(5.25)	6.62(4.80)	8.37(3.42)	7.48(3.10)	$4.5^{4.9}_{4.2}$	$3.5^{4.0}_{2.7}$	4.3
Q1017 − 207	9.93(10.95)	9.04(10.04)	9.57(10.55)	8.64(9.61)	$4.8^{6.2}_{4.5}$	$4.3^{13.0}_{1.4}$	14.7
HE1104 − 1805	112.93(123.11)	103.17(113.25)	89.63(97.71)	81.28(89.29)	$122.0^{130.0}_{115.0}$	$22.8^{51.2}_{12.7}$	99.6

panel) is located on the lens plane at a point with a local acceleration of 4.5×10^{-10} m/s^2 (using the MOND mass estimate in Table I for a Hernquist profile), which explains why the difference between the solid lines (standard lensing) and the dashed lines (MOND) in the leftmost panel is so small.

The bottom panels of Fig. 2 put this result in context with the need for dark matter. The figure compares MOND-like lensing mass with stellar mass (solid dots). Our 1D estimates are compared with more detailed nonparametric models from Refs. [12,17]. A typical error bar from these estimates is also shown. Even though some of the systems can be compatible with no dark matter, the MOND-like analysis presented here finds in most cases the need for dark matter at a level around $M_{\rm DM}/M_{\rm STAR} \sim$ 0.5–2. Given that the dust and gas content in early-type galaxies corresponds to a fraction of the stellar mass, we infer the need for dark matter even within MOND. Our analysis shows that dark matter in early-type systems appears in regions with different absolute accelerations compared to disk galaxies. Hence, a theory with a fixed absolute acceleration (such as MOND) cannot explain both early- and late-type systems.

The form of the function $f(x)$, which varies smoothly from the deep MOND-like to the standard regime, is an extra source of uncertainty in the MOND-like mass estimates. If $f(x)$ varies too slowly, lingering close to the conventional regime for too long, MOND-like mass predictions are too high, while if $f(x)$ falls quicker to the MOND-like limit, the need for dark matter would diminish. There is no precise way to determine the exact form of this function. From galactic rotation curves some restrictions can be placed on its form, but there still exists a degree of freedom. Varying the form of $f(x)$, it was found [13] that the predicted masses are not affected considerably and that many of the lenses still give a high dark matter content. Here, we considered two alternatives for the acceleration function, namely $f(x) = x/(1 + x)$ and $f(x) = 1 - e^{-x}$. The MOND mass estimates are lowered by less than 10%. Note that one could manufacture a function $f(x)$ such that MOND can be successful without dark matter; however, such artificially made functions would disregard the data from rotation curves.

Another possible source of uncertainty lies in the absolute value of the acceleration scale a_0. One can increase a_0 by a factor 2 and still be compatible with the rotation curve data [18]. In our case, the mass estimates are lowered by about 10%. A combination of a higher a_0 and a shallower function $f(x)$ can result in mass estimates lower than our

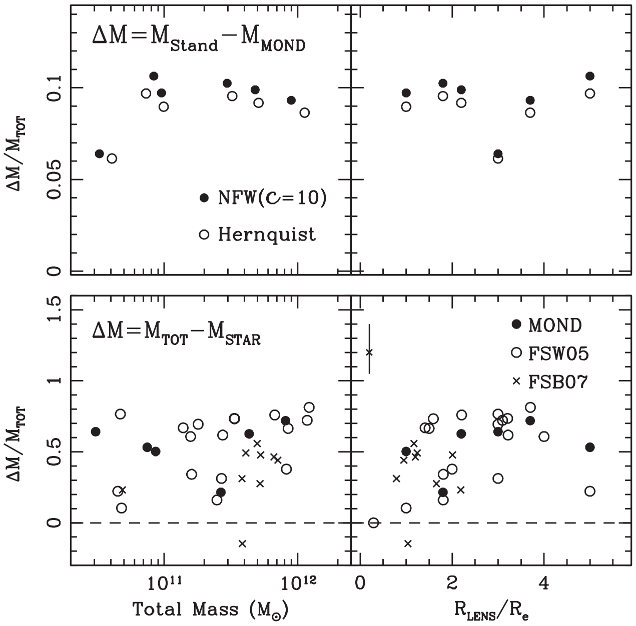

FIG. 2. *Top*: difference between conventional and MOND masses for a NFW model with $\mathcal{C} = 10$ (●) and a Hernquist profile (○). The ratio $\Delta M \equiv M_{\rm std} - M_{\rm MOND}$ is shown as a function of total (standard) mass (left panel) and the ratio between the average lens separation over which lensing masses can be reliably measured, and the observed half-light radius (right panel). *Bottom*: contribution of dark matter to the total mass budget from a comparison between MOND-like lensing and stellar mass for a NFW model with $\mathcal{C} = 10$ (●). We also include more detailed nonparametric conventional mass estimates of strong lenses from Refs. [12,17].

fiducial MOND estimates by about 25% which would still not be large enough to make dark matter unnecessary.

In this Letter we have compared mass estimates for a set of early-type lensing galaxies using both standard lensing analysis and MOND. We used two density profiles, the NFW profile and the Hernquist profile. We used the lensing equations to predict the mass of a system from the image positions for a 1D model (spherical symmetry). Besides the standard paradigm ΛCDM cosmology, other recent alternatives from the literature were considered, including the possible solution presented in Ref. [19] where the addition of massive neutrinos allows a cosmology of $(\Omega_m, \Omega_\Lambda, \Omega_k) = (0.22, 0.78, 0)$ to give an acceptable fit to both the CMB angular power spectrum as well as the high-redshift supernova data. For our purposes, any of the cosmologies discussed give very similar mass estimates, a result which should not come as a surprise since the observational constraints mostly impose limits on the luminosity and angular diameter scales.

We tested MOND by looking at a set of strong gravitational lensing early-type galaxies from the CASTLES survey. The masses predicted in the framework of conventional theory are very close to those from MOND-like lensing, even for galaxies observed out to a few effective radii. Comparing the stellar mass content from a comparison of the observed optical and NIR photometry with stellar population synthesis models we found that a very similar amount of dark matter is needed in both conventional and MOND analysis. This result is in remarkable contrast with the recent attempts to explain the lensing data on cluster scales by introducing a 2 eV neutrino [16]. This component can cluster on Mpc scales but should not cluster on galactic scales to keep the analysis of the rotation curves of disk galaxies unchanged. However, our lenses, which do require dark matter, are studied over length scales comparable to those of the rotation curve analysis. We therefore conclude that either lensing must work in a qualitatively different way within MOND (or more correctly the covariant "parent" theory, such as TeVeS) or dark matter should be considered within MOND even on galactic scales.

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[1] M. Milgrom, Astrophys. J. **270**, 365 (1983).
[2] J. D. Bekenstein, Phys. Rev. D **70**, 083509 (2004); R. H. Sanders, Astrophys. J. **473**, 117 (1996).
[3] N. E. Mavromatos and M. Sakellariadou, Phys. Lett. B **652**, 97 (2007).
[4] P. Schneider, arXiv:astro-ph/0509252; R. Narayan, arXiv:astro-ph/9606001.
[5] D. J. Mortlock and E. L. Turner, Mon. Not. R. Astron. Soc. **327**, 557 (2001).
[6] R. H. Sanders, arXiv:astro-ph/9606089.
[7] L. Hernquist, Astrophys. J. **356**, 359 (1990).
[8] J. F. Navarro, C. S. Frenk, and S. D. M. White, Astrophys. J. **462**, 563 (1996).
[9] R. H. Wechsler *et al.*, Astrophys. J. **652**, 71 (2006).
[10] D. Rusin *et al.*, Astrophys. J. **587**, 143 (2003).
[11] http://cfa-www.harvard.edu/castles/.
[12] I. Ferreras, P. Saha, and L. L. R. Williams, Astrophys. J. **623**, L5 (2005).
[13] H. Zhao, D. J. Bacon, A. N. Taylor, and K. Horne, Mon. Not. R. Astron. Soc. **368**, 171 (2006).
[14] G. Chabrier, Publ. Astron. Soc. Pac. **115**, 763 (2003).
[15] C. W. Angus, H. Y. Shan, H. S. Zhao, and B. Famaey, Astrophys. J. **654**, L13 (2007).
[16] R. H. Sanders, Mon. Not. R. Astron. Soc. **380**, 331 (2007).
[17] I. Ferreras, P. Saha, L. L. R. Williams, and S. Burles, arXiv:0708.2151; I. Ferreras, P. Saha, and S. Burles, arXiv:0710.3159 [Mon. Not. R. Astron. Soc. (to be published)].
[18] R. H. Sanders and S. S. McGaugh, Astron. Astrophys. **40**, 263 (2002).
[19] C. Skordis, D. F. Mota, P. G. Ferreira, and C. Boehm, Phys. Rev. Lett. **96**, 011301 (2006).