THE EXPERIENCE OF SECURITY IN MATHEMATICS

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In this paper, we report some findings from an investigation of a topic related to affect and mathematics which is not well-represented in the literature. For some mathematicians, mathematics itself is a source of security in an uncertain world, and we investigated this feeling and experience in the case of 19 adult mathematicians working in universities and schools in Greece. The focus reported here is on ways that a relationship with mathematics offers a sense of permanence and stability on the one hand, and an assurance of novelty and progress on the other.

Keywords: Anxiety; Fear; Mathematicians; Mathematics; Security

La experiencia de la seguridad en matemáticas

En este trabajo presentamos algunos resultados de una investigación sobre un tema relacionado con el afecto y las matemáticas que no está bien representado en la literatura. Para algunos matemáticos, las matemáticas en sí mismas son una fuente de seguridad en un mundo incierto; nosotros investigamos este sentimiento y experiencia en el caso de 19 matemáticos adultos que trabajan en universidades y escuelas de Grecia. El foco de este artículo es en las maneras en que una relación con las matemáticas ofrece un sentido de permanencia y estabilidad por una parte, y garantía de novedad y progreso, por otra.

Términos clave: Ansiedad; Matemáticas; Matemáticos; Miedo; Seguridad

In his introduction to a Research Forum on Affect at the 28th Conference of the International Group for the Psychology of Mathematics Education, Hannula (2004) wrote that emotions "have an important role in human coping and adaptation" (p. 108). The literature concerning emotional responses to mathematics is dominated by investigations into negative responses to instruction and testing, and by constructs such as mathematics anxiety, fear of failure, and mathematics avoidance (Zan, Brown, Evans, & Hannula, 2006). However, there is another side to this coin, with many individuals attesting to a positive response to mathematics, and deriving satisfaction, or pleasure, from it, for various reasons. Bertrand Russell, for example, speaks for those who find a pure, cold beauty in the subject:

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> *Mathematics, rightly viewed, possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show.* (Russell, 1919, p. 60)

The affective issue under investigation in this particular paper is concerned with an important component of "human coping": for some mathematicians, mathematics contributes to their sense of security, and thereby to their well-being. This is an under-researched topic in the domain of positive mathematics-affect. We explored the experience, for some individuals, of mathematics itself as a "safe place". The aim of the research was to explore the concept of security, as it emerges from the relationship of mathematicians to mathematics. To speak of mathematics as offering a haven of some kind may seem strange, if one thinks of mathematics as a body of knowledge. In the next section we shall draw out conceptions of mathematics that could be appealing to certain individuals in terms of security, and describe our conceptualization of security for the purposes of this study. We then proceed with an account of our findings from interviews with a sample of mathematics professionals.

Literature Review and Theoretical Framework

In this section, we frame our investigation in terms of the nature of mathematics itself, and of security as a psychological phenomenon.

The Nature of Mathematics

A wide range of perspectives about the nature of mathematics has evolved since early Greek civilization up to the present time (Davis & Hersh, 1980; Friend, 2007). According to Plato, material objects are mere shadows of an ideal counterpart, or "form". Mathematical objects are paradigmatic exemplars of forms, and they pre-exist, awaiting human discovery. In early modern philosophy, Descartes (1596-1650) continued the Platonic tradition, privileging reason over the sense-experience (Hutchins, 1952, p. 3). The subsequent scientific and industrial revolutions led to the quest for secure foundations for mathematics, and notably to the formalist perspective that mathematics could be reduced to a few axioms and deductive rules as the source of all mathematical knowledge. This vision was undermined by Gödel's proof that any such system complex enough to include arithmetic is necessarily incomplete. A notable response to the collapse of the formalist project is Lakatos' (1976) position, that mathematical knowledge (in keeping with Popper's view of science) is a human and fallible enterprise. This view of mathematics as a human, social construct, negotiable and consensual, is emphasized in social constructivism as a philosophy of mathematics (Ernest, 1998). We pause here to comment that these recent ontologies of mathematics seem to place the mathematician on shifting sand, but nevertheless give them agency in an unfolding mathematical story. On the other hand, Platonism accords well with the mathematician's experience of “discovery” (Huckstep & Rowland, 2001), is consistent with a stable and dependable mathematical universe, and is frequently considered the default metaphysical position regarding mathematics (Friend, 2007).

Security

Maslow (1970) has proposed that human needs are organised on five priority levels. In this hierarchy, Maslow includes security within a more general “safety” category, along with stability, structure, order, and freedom from fear. This category is located in the second level of Maslow’s hierarchy, preceded only by physiological needs related to survival. In this research, and this paper, we operationalise the concept of security in a two-stage process: (a) by reference to dictionary definitions of security as “freedom from fear or anxiety” (e.g., www.merriam-webster.com); and (b) a typology of fear due to Riemann (1970), who proposed four types of personal need, organised into two opposing pairs. Each type of need brings with it an associated fear. The first pair opposes the need to be an individual against the need to be part of a group: the corresponding fears are fear of assimilation [our translation] and fear of isolation and loneliness. The second pair opposes the need for stability with the need for development: the corresponding fears are fear of change and fear of confinement and stagnation.

As an indication of the relevance and potential application of Riemann’s framework to the topic under investigation here (security in mathematics), consider Mendick’s (2005) account of the “identity work” of two young persons, expressed in terms of their enjoyment of mathematics. Mendick comments that “Phil finds a security in mathematics that enables him to construct himself as intellectually mature and as distant from his working-class, minority-ethnic self” (p. 175). Phil’s security can readily be construed in terms of his response to fear of assimilation—through his engagement and success in mathematics, he positions himself as distinct and distinctive, in terms of his distance from his origins and his intellectual capacity.

Methods

In this section we describe the data collection and analysis used in the study here reported.

Data Collection

We explored the concept of security with adult mathematicians, since they could be expected to have a well-developed relationship with mathematics, and to be able to articulate it. The participants’ professional mathematical roles were in teaching and/or research in Greece. Nine were in university positions: Faidra, Paraskevi, Themis, Vasilis, Sofoklis, Periklis, Alvertos, Dimitris, Kleitos (pseudonyms). Another 10 participants were teaching in secondary schools: Stamatia, Eleftheria, Aris, Sokratis, Avgoustis, Marios, Nestoras, Fanis, Thodoris, Loukas (pseudonyms). This was an opportunity sample, determined by existing connections with one university department and several schools. Only 4 of the 19 participants were female (the first two in each of the lists above), reflecting the population of mathematicians in Greece (Kotarinou, 2004). Most of the participants had substantial professional experience (15 years or more). Themis, Vasilis, Sofoklis, Periklis, Faidra and Eleftheria had been in post between 2 and 8 years.

The first autor conducted a semi-structured interview with each participant, aiming to probe for unconscious feelings which might be difficult to access directly, but could be hinted at during a conversation (Rubin & Rubin, 1995). The interviewer approached the topic indirectly, by discussing with the participants their relationship with mathematics in a general way. This approach minimised the risk of participant discomfort on being asked to disclose personal information (Robson, 2002). The interviews, which mostly lasted up to 30 minutes, were audio-recorded and transcribed in Greek. The semi-structured interviews were organised around the following four themes: (a) the participant's personal history regarding mathematics; (b) their views about mathematics; (c) the relevance of mathematics in everyday life; and (d) their feelings about mathematics. The interviewer had a repertoire of questions from which she drew in a flexible way.

Data Analysis

The scale of the data analysis task was such that it could be handled manually. In a first pass over the interview data, utterances were coded as relevant, or probably relevant, to one of Riemann's four types of fear. Sometimes just one type could be applied to a whole paragraph, at others to only part of a sentence. For example, Stamatia's analysis of mathematical modes of thinking included characteristics referring: (a) to communication, which was connected with fear of isolation; (b) to structured thought, which was connected with fear of change; and (c) to creativity, which was connected with fear of stagnation. In a second pass over the data, the initial fear-type codings were reconsidered, and changed in some cases, and some additional utterances were coded. Several cases of multi-coding arose, even including coding some utterances to opposing fears, as two sides of the same coin. Subsequently, as the data were revisited again and again, a method of constant comparison was applied in a more fine-grained coding, giving rise to broad themes and related sub-themes, associated with each type of fear. The themes and sub-themes related to fear of change are listed in Table 1, by way of illustration. Note that whereas Themes 1-3 emphasize aspects of mathematics and mathematical activity that have the potential to offer protection against change, Themes 4-8 acknowledge interconnections with other fears, and limitations in safeguarding against change.

Table 1
Fear of Change Themes and Sub-Themes

Themes	Sub-themes
1. Mode of thought	Precision, connectedness, systematization, orderliness, verifiability, consistency, sense-making, realism, real life
2. Inferences	Certainty, reliability, one reality, real life
3. Art	Harmony, beauty, balance
4. Assimilation	Self-awareness and mode of thought, self-fulfilment and mode of thought, self-confidence and mode of thought
5. Isolation	Historical continuity and reliability, precision and one reality, omnipresence and connectedness
6. Stagnation	Change and creativity, change and mental activation; change and diversity
7. Limitations to assimilation	Realism
8. Limitations to change	Mode of thought

FINDINGS

In this paper, we restrict our report to those findings from the analysis of the interview data that shed light on the participants' views with regard to the second of Riemann's opposing pairs: fear of change and fear of stagnation. The analysis is restricted here to those themes (like mode of thought, inferences, and art in Table 1) that relate specifically to the fear-type under examination, rather than those that indicate interconnections and limitations. In the case of fear of stagnation, these were creativity, problem solving, and diversity.

Fear of Change

First, we will report the participants' views which we associated with fear of change. These views explain how mathematics could make the participants feel that they were protected against, or ready to confront, the unexpected changes of life.

Mode of Thought

The participants perceived the mathematical mode of thought as precise, interconnected, systematic, rule-governed, verifiable, non-random, absolute, and sense-making. We present some examples in the following lines.

Fanis: Mathematics makes you feel secure because it reveals harmony and orderliness. Every system functions with certain rules; if you violate them, then the system collapses… It makes sense how one [statement] is linked to another… What seems complicated and difficult can be broken down into the links that produce it… it is not randomly produced.

Aris: In order to prove… you need absolutely rational thought, absolute logic.

Dimistris: You start from a point arbitrarily, but everything you say afterwards is established.

Avgoustis: Mathematics is precise; its results are verifiable… you know if you were right or wrong, you have no doubt.

The interviewees also believed that these attributes of mathematical thought could be transferred to everyday life, and improve it.

Thodoris: I say to students: maybe you won't use mathematics after [school], but from your mathematical experience you may acquire a mode of thought.

Alvertos: Mathematics reformulates the problem you want to solve, until it becomes comprehensible. The same you do with a real problem. You distinguish and organise [the data] in hierarchies depending on the values you have in your head.

Sokratis: If you've been taught by mathematics and if you've conquered your passions… you can see more clearly, and consequently you are better equipped to confront [a problem] successfully.

Thus mathematics was seen as being ordered itself, and inculcating orderly behaviour.

Inferences

The interviewees asserted that the mathematical mode of thought starts from sound foundations and leads to certain, reliable and permanent conclusions. We present some examples in the following lines.

Vasilis: Mathematics is logic; there are axioms and a stable basis […] mathematics doesn't change; what has been found remains as it is.

Alvertos: You may say that proving makes the knowledge secure.

Stamatia: Mathematical thought engenders and answers "whys"… through indubitable arguments.

Loukas: [In mathematics] for every problem we can obtain one unique correct solution.

The participants transferred this certainty to real life, where mathematical "sound foundations" was translated into pragmatic "realistic assumptions".

Aris: Mathematics influences our decisions…; [it allows us to judge] what our abilities are, so that we make correct choices.

Fanis: Mathematics helps you… to put the assumptions in order and to reach the best possible solutions.

Sofoklis: You can distinguish between right and wrong… in life, contradiction is allowed to some extent; but even though you may not be able to prove something, you'll be able to exclude something [else].

The logic inherent in mathematics was especially prized for the "certainty" guaranteed, from both Platonist or formalist points of view, and the same modes of inference were seen as valuable in everyday affairs.

Art

The orderliness of mathematics suggests harmony and balance, and these in their turn imply beauty. The interviewees judged mathematics as beautiful. We present some examples in the following lines.

Sofoklis: There are proofs which display harmony… I love the logic hidden in mathematics and its beauty… There are questions which simply emerge and they are beautiful.

Dimitris: After solving a problem, I imagine the solution as a work of art.

Avgoustis: Mathematics is something like music: once you hear it, it sticks in your mind.

Eleftheria: This beauty provides equilibrium in the chaos of an uncertain world. The world is chaotic; through the symmetry of mathematics I find balance.

Fear of Stagnation

Here we report the participants' views which we interpreted in relation to the opposing fear, of stagnation. These views explain how mathematics could make the participants feel that they have powers of self-determination, and ability to change the status quo.

Creativity

Some interviewees affirmed that mathematics gives rise to original creations which shape the present and will influence the future. We present some examples in the following lines.

Alvertos: In mathematics you are expected… to explore existing paths, and potentially to create [new ones].

Eleftheria: Mathematics contributes to contemporary development, it influences the present.

Fanis: Differential geometry and vector spaces are a glance into the future.

Stamatia: Science fiction is born of the womb of the science of mathematics.

The interviewees also believed that mathematical creations adhere to logic but are not restricted by any physical laws or limitations. Kleitos dissociates himself from a view of a pre-determined mathematical universe.

Kleitos: Other sciences discover, while mathematics creates; there isn't something specific you're looking for… mathematics uses the least possible rules.

Stamatia commented on the transfer of mathematical creativity in real life, in a bold assertion of self-determination.

Stamatia: Mathematics is the science that cultivates independence, boldness, and the love to explore the unknown. You dream an imaginary world, and mathematics allows you to make it real.

Problem Solving

The participants observed that mathematical problems can be tackled using various approaches. As examples see the following lines.

Alvertos: Everyone approaches mathematics differently.

Nestoras: I like to read about [mathematical issues] which are examined from different perspectives.

Furthermore, the participants observed that problem-solving offers great intellectual independence and stimulation.

Stamatia: I'm pleased when I watch my students reaching a solution using their brain instead of parroting others' opinions.

Eleftheria: Mathematics keeps you vigilant; your mind doesn't get the chance to be idle.

The participants also commented on problem-solving being an unexpected experience.

Themis: I like it when students see things that I haven't.

Periklis: I see mathematics as an ongoing route and not as something which I've learned and I can rest upon.

These contributions present mathematics as offering scope for novelty and originality, taking pleasure in diversity and in the unexpected.

Diversity

As an occupation, mathematics can give rise to a range of emotions.

Alvertos: Mathematics engenders thousands of feelings; from vanity for one's efforts to surprise, hedonism, fury, and stubbornness.

Mathematics was considered to be a tool, both with respect to other sciences and for organising one's thought, and this tool can be used in many different ways.

Sokratis: It can be a hobby, a profession, a means to get rich, a means of deceit, a means of exploration, and an object of research… [however] applying mathematics is not bloodless; the missiles have been made by mathematicians.

Here, Sokratis disputes Hardy's (1940) claim that mathematics is benign, harmless and practically useless, and mathematicians detached from practical affairs. Like the other informants here, he attests to the endless diversity and variety of experience and emotion derived from mathematics, which we interpret as another safeguard against stagnation.

CONCLUSION

This investigation into feelings of security in mathematics was underpinned by a conception of security as relief from fear, and by Riemann's (1970) focus on four types of fear, in two opposing pairs. In this paper we reported findings relating to fear of change, and fear of stagnation. Mathematics was perceived to offer a balance between these opposing anxieties. As many philosophers have suggested in the past (e.g., Hutchins, 1952), the interviewees believed that what distinguished mathematics from the other sciences, natural or human, was its mode of thought. This mode was considered to lead to an exceptional kind of knowledge, indubitable and unchanging, whether discovered or invented. This infallible knowledge was believed to be continually increasing, to the benefit of both mathematics and other disciplines. The former enjoys results unbound by any physical law, the latter findings which could be used to change the world (Guillen, 1995). Mathematics was perceived as a realm of creation, of beauty and balance, in which everything makes sense. Furthermore, insofar as the mathematical mode of thought can be transferred from mathematical to real-life problems, it was valued as a tool of unique precision, both in handling the unavoidable changes of life (fear of change) and in escaping from undesirable situations (fear of stagnation).

Several lines of further research are suggested by these findings. Perhaps the first fruitful avenue would be to investigate the extent to which these findings might be replicated in other cultures, within and beyond Europe.

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