

Probabilistic Analysis of Surface Crack in Round Bars under Tension Loading

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Abstract. This paper presents the probabilistic analysis of surface crack in round bars subjected to tension stress using Monte Carlo Simulation (MCS). A probabilistic model based on an elastic finite element analysis (FEA) was developed to evaluate the failure probability obtained using a *K*-estimation method. This method is based on the stress intensity factor (SIF) where the failure occurred when the SIF was assumed to exceed the critical SIF. It was found that, the *K*-estimation method was adequate to evaluate the failure probability of the bars when compared with the results obtained using the probabilistic FEA approach. The advantage of *K*-estimation methods was the reduction of the computational time and cost significantly.

Introduction

Probabilistic fracture mechanics (PFM) is used to evaluate the fracture response and reliability of cracked structures. The PFM has incorporated the statistical uncertainties of mechanical, geometrical and loading properties to determine the probability of failure of the components. Currently, there are many methods and applications for probabilistic fracture mechanics in oil and gas, nuclear, automotive, naval, aerospace, and other industries. Nearly all of which have been developed based on linear elastic fracture mechanics models [1-3]. Probabilistic fracture mechanics also provides a more rational means to describe the actual behavior and reliability of structures than traditional deterministic methods [4]. Monte Carlo simulation (MCS) is normally used to estimate the probability of failure. However, the use of MCS requires longer computational time when incorporated with finite element analysis (FEA). Therefore, an alternative method such as *K*-estimation is proposed to reduce the computational time and then, they are compared with the results obtained from FEA. In this work, *K*-estimation method is based on the linear elastic fracture mechanics. Then, the *K*-estimation method is coded into ANSYS finite element analysis by incorporating with the statistical characteristics of mechanical, geometrical and loading properties. Then, the failure probability of the considered crack geometries are presented and discussed.

K-estimation Method

The SIF under bending moment, $K_{I,a}$ is estimated using Eq. 1 and the bending stress, σ_b can be represented as Eq. 2.

$$K_{I,a} = F_{I,a} \sigma_a \sqrt{\pi a} \quad (1)$$

$$\sigma_a = \frac{4P}{\pi D^2} \quad (2)$$

where, $F_{I,a}$ is the geometrical correction factor are obtained from [5-7], P is a tension force, D is the diameter of bar and a is the crack length or crack depth. Due to the uncertainties of mechanical, geometrical and loading properties, a Monte Carlo Simulation (MCS) is used in the probabilistic

analysis to determine the probability of failure. Table 1 lists all the random variables in the probabilistic analysis. Consider a cracked structure with uncertain mechanical and geometric characteristics that is subjected to random loads. Denote by X an N -dimensional random vector with components X_1, X_2, \dots, X_N characterizing all uncertainty in the system and load parameters. Let K be a relevant fracture parameter that can be calculated from elastic finite element analysis. If K is a valid fracture parameter, then it can be applied to determine the failure probability of the cracked structure.

Table 1: Random variables used in probabilistic analysis [3]

Random variables	Mean, μ	COV*	Distributions
Elastic modulus, E	222 MPa	0.08	Normal
Crack depth, a	Arbitrary	0.10	Normal
Diameter, D	50 mm	0.01	Normal
Toughness, K_{IC}	24.8 MPa \sqrt{m}	0.05	Lognormal
Poisson ratio, ν	0.3	0.03	Normal
Tension stress, P	Arbitrary	0.05	Normal

*COV = Coefficient of variation

Suppose that the structure fails when $K > K_{IC}$. This requirement cannot be satisfied with certainty, because K depends on input vector X which is random and K_{IC} itself is a random variable. Hence, the performance of the cracked structure should be evaluated by the reliability, P_s or its complement, the probability of failure, P_f ($P_s = 1 - P_f$) defined as [8] in Eq. 3

$$P_f = \Pr[g(X) < 0] = \int_{g(X) < 0} f_x(X) dX \quad (3)$$

where $f_x(X)$ is the joint probability density function of X and $g(X)$ is the performance function given by Eq. (4)

$$g(X) = K_{IC} - K_{I,a} \quad (4)$$

Substitute Eq. 1 and Eq. 2 into Eq. 4 then produce

$$g(X) = K_{IC} - F_{I,a} \left(\frac{4P}{\pi D^2} \right) \sqrt{\pi a} \quad (5)$$

Codes of APDL are developed to incorporate the performance function as in Eq. 5 with the all random variables listed in Table 1. Then, MCS with Latin Hypercube Sampling is used to generate the random numbers and therefore calculating the probability of failure. The results of probability of failure from ANSYS are given in terms of cumulative density function (CDF).

Finite Element Modeling

Finite element model (FEM) is constructed using ANSYS through the use of Ansys Parametric Design language (APDL). Due to the symmetrical analysis, quarter FEM is used in order to reduce the computational efforts as shown in Fig. 1(a) and it is also used in the probabilistic analysis. MPC184 elements or rigid element beams are used. These elements are connected from an independent node to the bar. Then, tension forces are applied directly on the cross-sectional area of the bar. Singular elements are used around the crack tip to ensure stress/strain singularities. Mid-side nodes of the singular elements are shifted to quarter position from the crack tip and stress intensity factors (SIF) are determined along the crack front. The results of SIFs can be obtained elsewhere [9-11]. Different crack aspect ratio, a/b range between 0.2 to 1.2 and relative crack depth,

a/D between 0.1 and 0.3 are considered. The crack geometries considered in this work are based on the experimental observations [11]. The present model is then compared with the previous model from the literature for validation purposes. It is found that the present model is well in agreements with others as shown in Fig. 1(b).

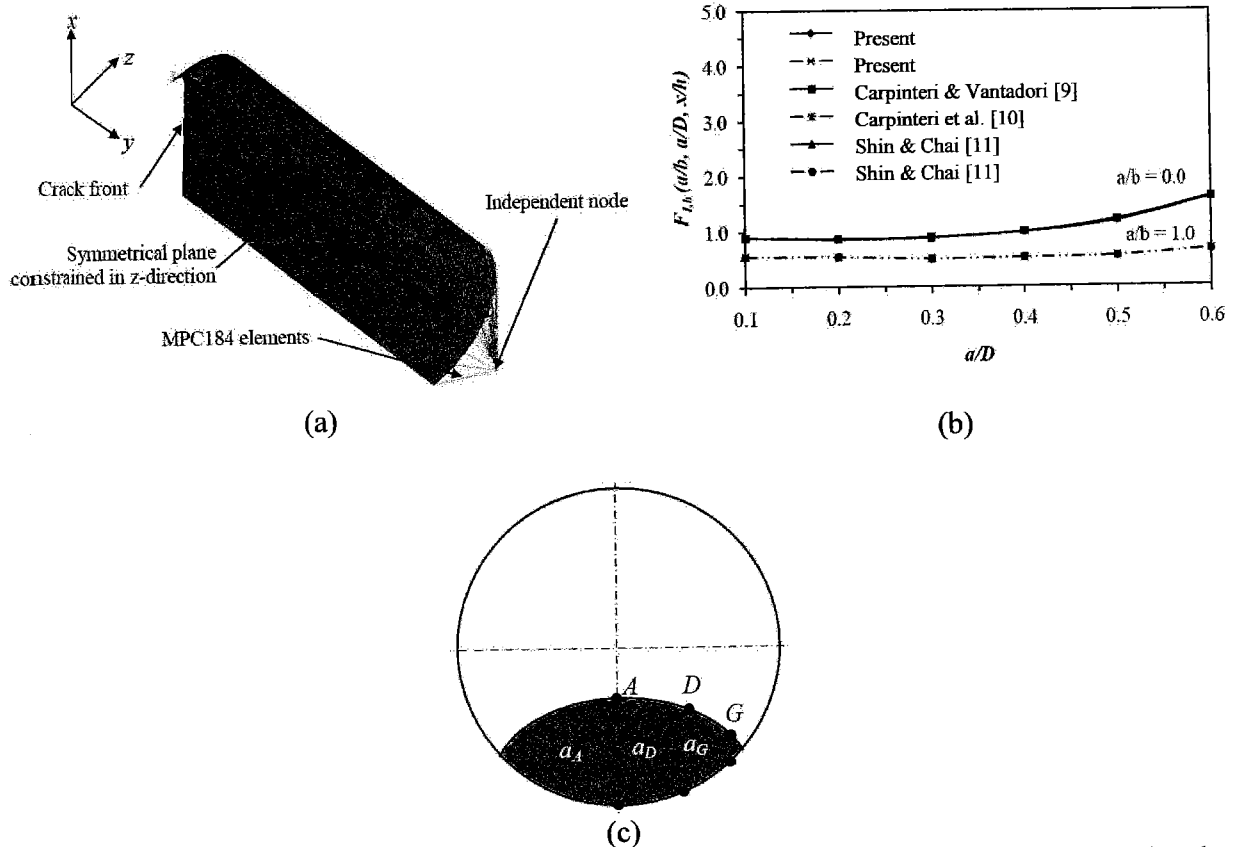


Figure 1: (a) Quarter finite element model, (b) Result validations of finite element model and (c) The failure probability point calculated along the crack front

Results and Discussion

Fig. 2 shows the probability of failure subjected to tension stress, $P_{f,a}$ plotted against tension force, P . Two methods are used to estimate the $P_{f,a}$ such as FEA and K -estimation. The results reveal that the $P_{f,a}$ are in agreement to each others. The $P_{f,a}$ are estimated for three points situated along the crack front which are A, D and G as depicted in Fig. 1(c). According to the Fig. 2, higher $P_{f,a}$ is obtained at point A compared with other positions. However, when a/D is increased the curves of $P_{f,a}$ become closer to each other. The band of $P_{f,a}$ for $a/D < 0.2$ is larger than the $P_{f,a}$ for $a/D > 0.2$. This is indicated that for deeper cracks result higher $P_{f,a}$ and the different positions along the crack front are not affected the $P_{f,a}$. Increasing the a/D also influenced the $P_{f,a}$, for $a/D = 0.1$, the curves of $P_{f,a}$ are almost vertical compared with the curves obtained using $a/D > 0.1$. The reliability of the cracked bars is affected even lower applied bending moment is used. Fig. 3(a) shows the comparison in CPU time between FEA and K -estimation. It is revealed that the analysis using K -estimation method solves the problem faster than FEA. Therefore, K -estimation is used for further probabilistic analysis. In the practical point of view, K -estimation method provided an excellent analysis tool to estimate the failure probability of the components. This is due to the fact that the K -estimation method capable to predict the probability of failure similar to FEA results. Crack aspect ratio, a/b played an important role in determining the $P_{f,a}$. It can be observed that, there is not significant different between $P_{f,a}$ for $a/b = 0.2$ and 0.4 . However, the for $a/b \leq 0.4$ produced higher $P_{f,a}$ relative to $a/b = 0.6$. It is shown that at point G, lower SIFs are obtained for $a/b = 0.6$ compared to $a/b < 0.6$.

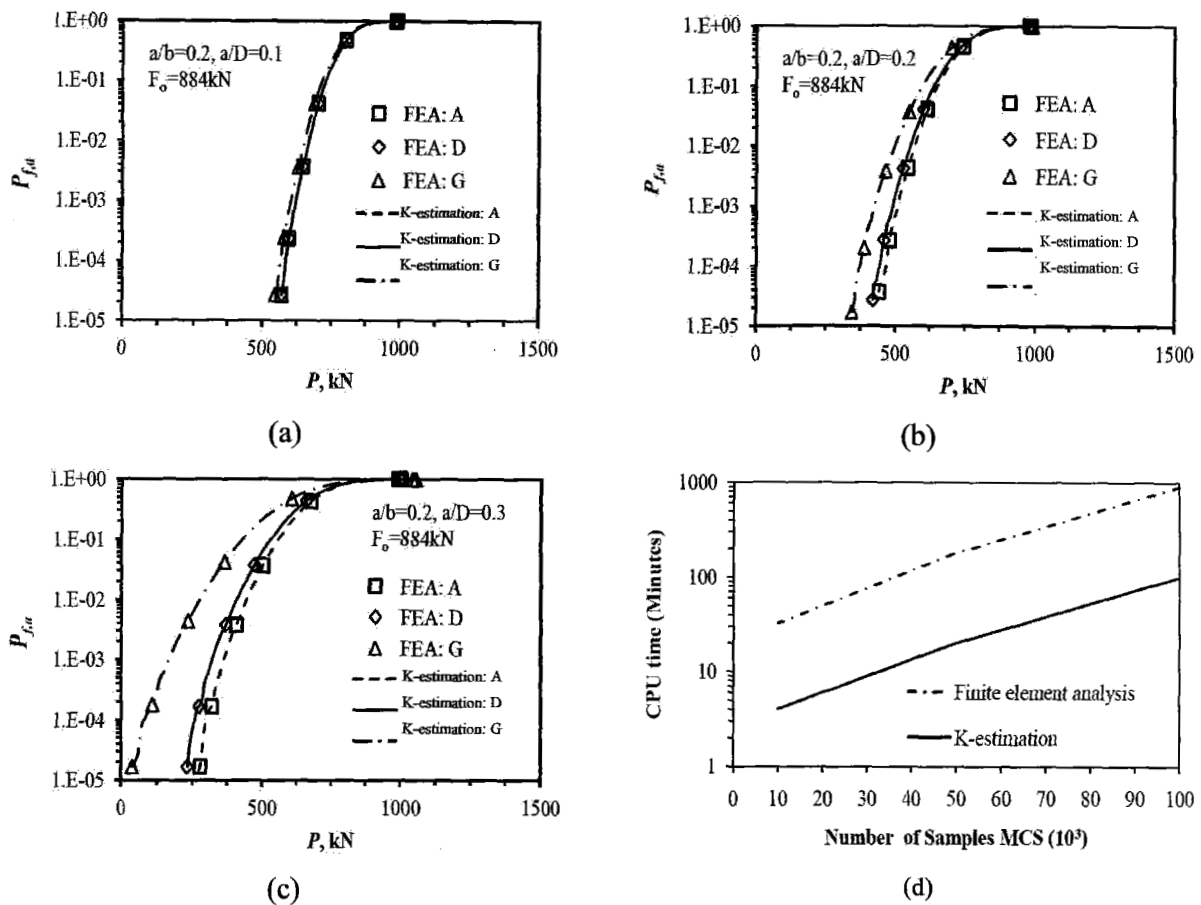


Figure 2: Comparisons of failure probability of crack aspect ratio, $a/b = 0.2$ for different relative crack depth, a/D (a) 0.1, (b) 0.2 and (c) 0.3 and (d) Comparison of CPU time between two methods

Conclusion

Three-dimensional finite element model (FEM) was developed using ANSYS and used to determine the failure probability. Stress intensity factor (SIF) based on the estimation method called K -estimation method was developed to estimate the failure probability. Monte Carlo Simulation (MCS) was used to assist the FEM and K -estimation method to estimate the failure probability. Comparisons were conducted and found that the results were agreed to each others. K -estimation method capable to reduce the computational time compared to FEA results.

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