

STRESS INTENSITY FACTORS OF THREE PARALLEL EDGE CRACKS UNDER BENDING MOMENTS

A.E Ismail

Faculty of Mechanical and Manufacturing Engineering, Universiti Tun Hussein Onn Malaysia (UTHM), Batu Pahat, 86400 Johor, Malaysia

Email: emran@uthm.edu.my

Phone: +607 4533724; Fax: +607 4536080

ABSTRACT

This paper reports the study of stress intensity factors (SIF) of three edge cracks in a finite plate under bending moments. The goal of this paper was to analyze the three edge crack interactions under such loading. According to literature survey, several studies can be found in literature discussing on mode I SIF. However, most of this study obtained the SIFs using tensile force. Lack of SIF reported discussing on the SIFs obtained under bending moments. ANSYS finite element program was used to develop the finite element model where singular elements were used to model the cracks. Different crack geometries and parameters were utilized in order to characterize the SIFs. According to the present results, crack geometries played significant role in determining the SIFs and consequently induced the crack interaction mechanisms.

Keywords: Stress intensity factor (SIF); multiple edge cracks; crack interactions.

INTRODUCTION

Parallel edge cracks are one of the most common flaws especially in industrial failures (Daud et al., 2011). Accurate stress analysis of these components in the presence of cracks and evaluation of fracture design parameters are needed for reliable estimation of the crack growth rates and residual fracture strength (Ismail et al., 2012). This paper discusses three parallel cracks in a sheet with finite width where the sheets are subjected to the bending moment. The crack will be appeared on the surface component which can reduce the component durability (Jiang et al., 1990; Jiang et al., 1991). According to the literature survey (Ismail et al., 2012), most of the work conducted to analyze the crack with the assumption that only a single crack can be used to represent such problems. In many cases the multiple cracks also can be carried out and must be studied to increase the safety factor of the component structure (Lam and Phua, 1991). This paper discusses three parallel cracks in a sheet with finite width. This sheet is subjected to bending moment in the longitudinal direction remote from the cracks. Then, crack interactions are analyzed and discussed.

DISPLACEMENT EXTRAPOLATION METHOD

The finite element method is an appropriate approach to calculate the stress intensity factor (SIF) for linear elastic fracture mechanics problems. In order to determine the SIFs, a displacement extrapolation method (Gustavo et al., 2000) is used in this study. Several other works have implemented a similar method are also available (Aslantas, 2003; Aslantas et al., 2006). After obtaining the elastic finite element solution of the

particular problem, nodal displacements between two crack faces are determined and used to compute the SIFs as follows

$$K_I = \frac{2G\sqrt{2\pi}}{1+\kappa} \frac{|v_b - v_d|}{\sqrt{r}} = \frac{2G\sqrt{2\pi}}{1+\kappa} \frac{|\Delta v|}{\sqrt{r}} \quad (1)$$

$$K_{II} = \frac{2G\sqrt{2\pi}}{1+\kappa} \frac{|u_b - u_d|}{\sqrt{r}} = \frac{2G\sqrt{2\pi}}{1+\kappa} \frac{|\Delta u|}{\sqrt{r}} \quad (2)$$

$$K_{III} = 2G\sqrt{2\pi} \frac{|w_b - w_d|}{\sqrt{r}} = 2G\sqrt{2\pi} \frac{|\Delta w|}{\sqrt{r}} \quad (3)$$

where, K_I , K_{II} and K_{III} are the respective mode I, II and III SIFs, Δv , Δu and Δw are the relative nodal displacements between two crack faces in the direction of y -axis, x -axis and z -axis, respectively, and G is the modulus of rigidity. For plain strain condition, $\kappa = 3-4\nu$, where, ν is the Poisson's ratio. All the SIFs obtained from the analysis are converted into normalized values in order to ensure the generality of the results. A normalized SIF, F , can be defined as follows (Toribio et al., 2009)

$$F_{I,a} = \frac{K_{I,a}}{\sigma_a \sqrt{\pi a}} \quad (4)$$

$$F_{I,b} = \frac{K_{I,b}}{\sigma_b \sqrt{\pi a}} \quad (5)$$

$$F_{II} = \frac{K_{II}}{\tau_{xy} \sqrt{\pi a}} \quad (6)$$

$$F_{III} = \frac{K_{III}}{\tau_{xy} \sqrt{\pi a}} \quad (7)$$

where, σ_a , σ_b and τ_{xy} are the axial, bending and shear stresses, respectively and a is a crack depth.

METHODOLOGY AND MODEL VALIDATION

ANSYS commercial finite element (FE) analysis software is used to model the parallel edge cracks through the use of ANSYS Parametric Design Language (APDL) as in Figure 1(a). Due to the symmetrical problem, half finite element model is sufficiently used to analyze the problem as in Figure 1(b) where singular finite element is used around the crack tips as revealed in the enlarged area of Figure 1(b). Two important parameters are used in this work such as b/a and a/W . Both parameters are in the range of 0.25 to 2.00 and 0.1 to 0.5, respectively. The calculation of stress intensity factor is based on the theory of displacement extrapolation method (Gustavo *et al.*, 2000). In order to validate the results, the present model must be compared with the previous

results as produced by Jiang et al. (1990). Unfortunately, there are unavailable stress intensity factors under bending moment, especially for multiple parallel edge cracks. Therefore, the SIFs obtained under tensile force are used for validation purposes. Figure 2(a) shows the meshed edge crack model subjected to axial stress and Figure 2(b) represents the stress distribution due to the crack interactions. Figure 5 shows the model validations where it is showed that the present model is well in agreement with the previous model. Therefore, it is assumed that the present model can be validated significantly using the results obtained from Jiang et al. (1990).

For the present analysis, it is assumed that plain strain finite element model is used where PLANE83 element type in ANSYS library is utilized. In order to bend the plate remotely, an independent node is created on the top of the plate. Then, all nodes situated on the top edge of plate are connected to this newly created node. Bending moment is applied to this node and then consequently creating mode I crack mechanism as shown in Figure 2(c). Due to symmetrical problem, stress intensity factor (SIF) for crack 3 is not accounted since it is similar to crack 1 (outer crack). Middle crack in this work is called crack 2. All the SIFs are normalized according to Eqs. (4)-(7). The normalized SIF is also called geometrical correction factor. It is important to normalize the SIFs for the sake of analysis generalization.

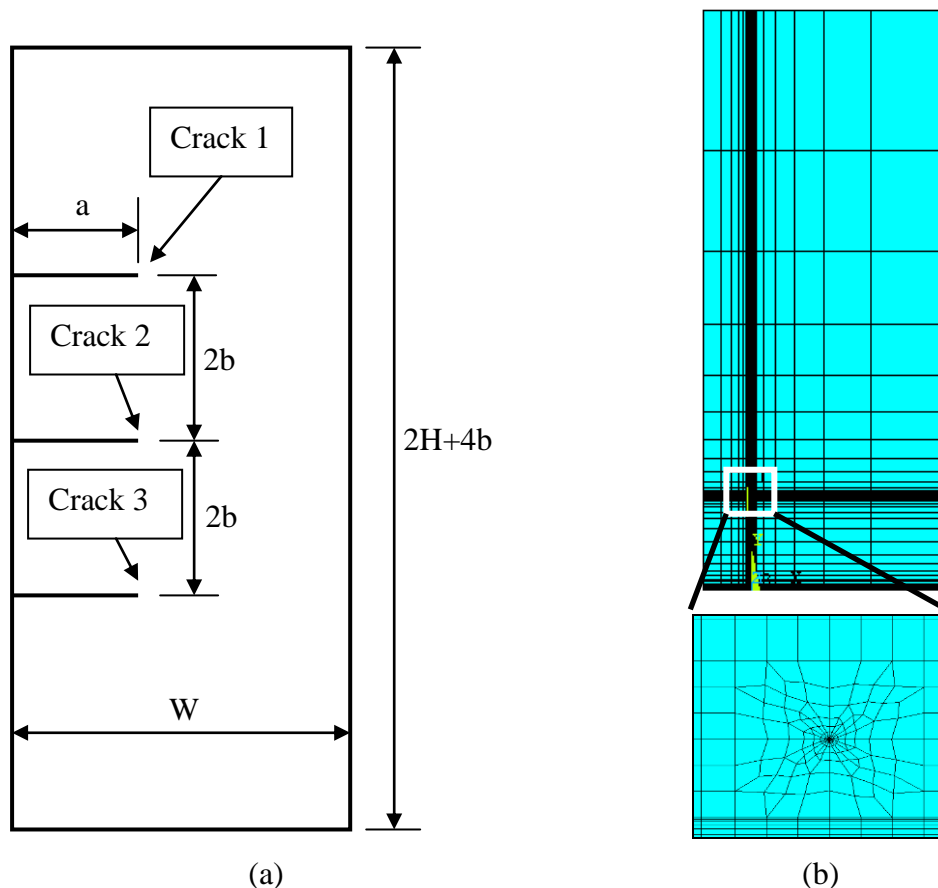


Figure 1. (a) Parallel edge cracks in a finite plate and (b) Half symmetrical finite element model of parallel edge cracks with enlarged crack tip area.

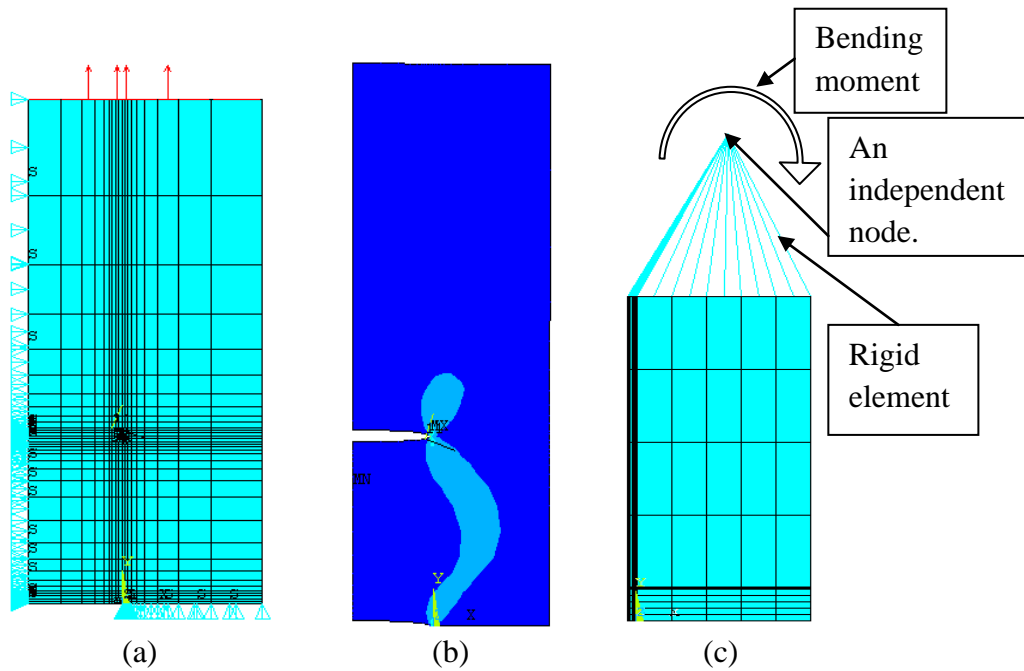


Figure 2. (a) Finite element model under axial stress, (b) The stress distribution and the crack interaction of parallel edge cracks and (c) remotely applied bending moment.

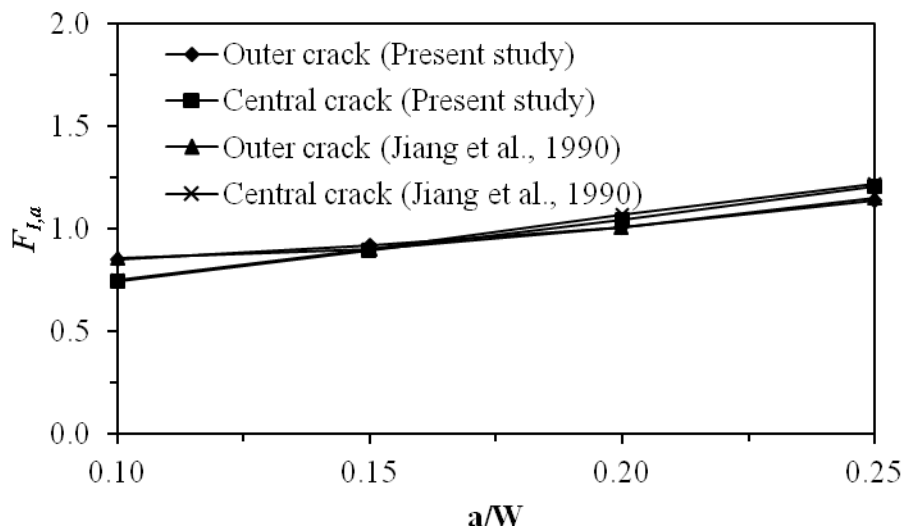


Figure 3. Validations of the present finite element models with the previous model developed by Jiang et al. (1990).

RESULTS AND DISCUSSION

Tables 1 and 2 list the geometrical correction factors for outer and central multiple edge cracks under bending moments, respectively. Figures 4 and 5 show the plot of SIF against a/W when b/a is varied for cracks 1 and 2, respectively when compared with non-interacting cracks. It is showed that the SIF increased when the cracks become deeper. These characteristics are widely reported in literature (Jiang et al., 1990 and 1991).

Table 1. Geometrical correction factor, $F_{I,b,o}$ for outer edge multiple cracks.

b/a	a/W								
	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.25	0.7980	0.8110	0.8360	0.8770	0.9400	1.0130	1.1130	1.2410	1.4060
0.50	0.8400	0.8760	0.9150	0.9690	1.0410	1.1300	1.2420	1.3810	1.5560
1.00	0.9440	0.9780	1.0260	1.0880	1.1630	1.2530	1.3580	1.4940	1.6470
2.00	1.0750	1.1050	1.1430	1.1860	1.2390	1.3040	1.3910	1.4980	1.6520

Table 2. Geometrical correction factor, $F_{I,b,c}$ for central edge multiple cracks.

b/a	a/W								
	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.25	0.5290	0.5290	0.5410	0.5680	0.6000	0.6780	0.7710	0.8990	1.0750
0.50	0.6000	0.6280	0.6620	0.7180	0.7990	0.9080	1.0490	1.2310	1.4420
1.00	0.7600	0.8010	0.8640	0.9620	1.0720	1.1930	1.3260	1.4780	1.6440
2.00	0.9950	1.0540	1.1180	1.1790	1.2390	1.3060	1.3930	1.4980	1.6540

Figure 4 shows the stress intensity factors (SIFs) of single edge crack under bending moment. It is found that the effects of crack eccentricities are played an important role in increasing the SIFs. However, the increment of SIFs occurred for the cracks situated within $b/a \leq 0.50$. For the cracks with $b/a \geq 2.0$, it is seemed that the SIFs decreased as a function of a/W compared with central crack ($b/a = 0.0$). The decrement of SIFs is due to the fact when the location of crack is away from the central line of finite plate, bending moment or stress decreased.

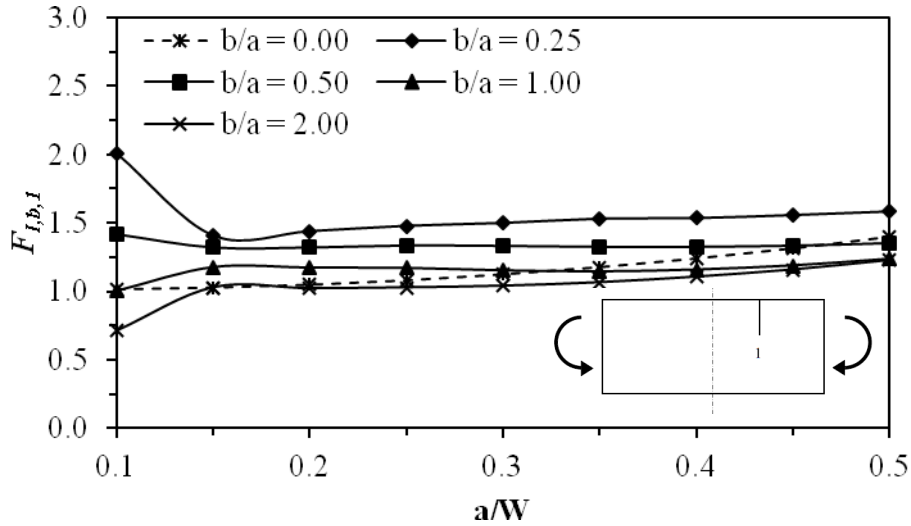
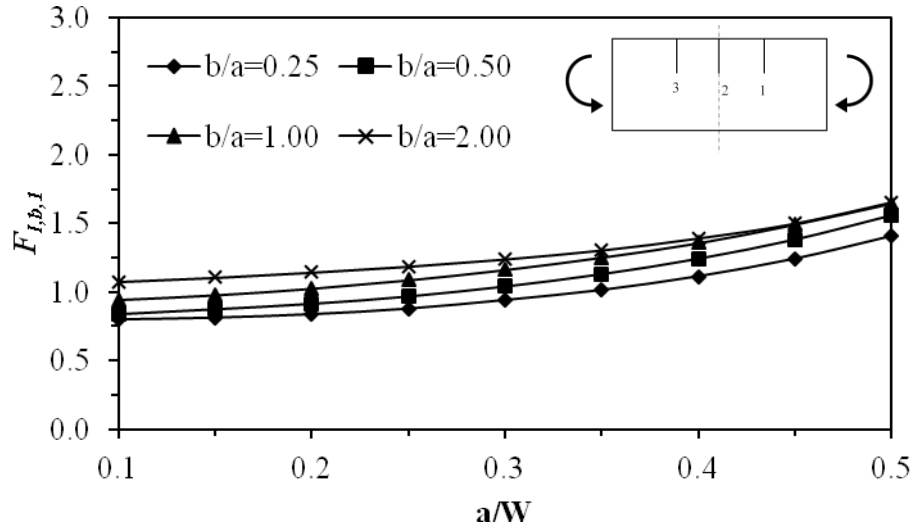


Figure 4. Stress intensity factors (SIFs) of single edge cracks under bending moment for crack 1.

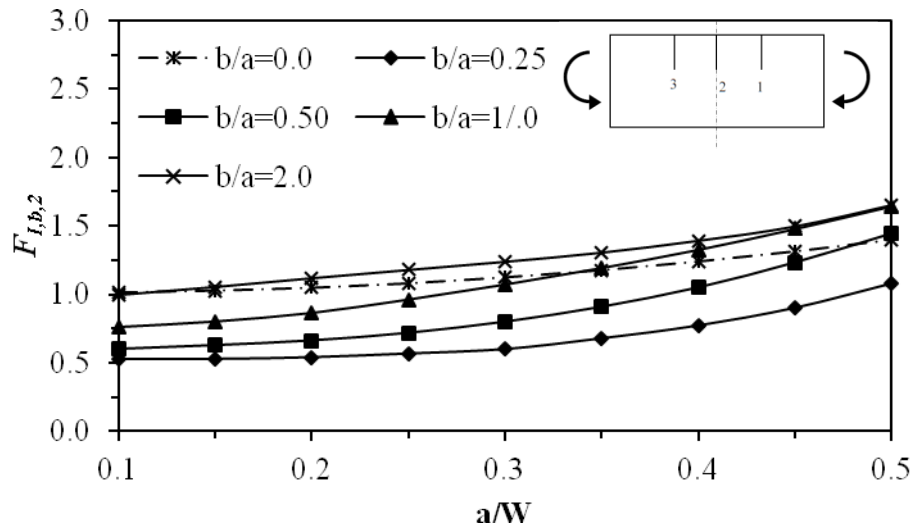
Figure 5 reveals the SIFs distribution for multiple edge cracks subjected to bending moment for cracks 1 and 2, respectively. Figure 5(a) shows the SIFs of multiple edge cracks for crack 1. It is found that the SIF increased in the similar pattern due to the

interactions and stress relaxations among the cracks. As the results, the SIFs for multiple cracks are lower than the offset single cracks. This is due to the stress field redistribution when the present of cracks.

As expected, the SIFs of multiple cracks reduced when the cracks interacted to each others as in Figure 5(b). It is also found that greater amount of interactions occurred when closer crack intervals are used. However, such interactions are diminished when the neighbouring crack is situated greater than $b/a \geq 2.0$.



(a)



(b)

Figure 5. Stress intensity factors (SIFs) of multiple edge cracks under bending moment for (a) crack 1 and (b) crack 2.

CONCLUSION

This paper present the calculation of stress intensity factors of three parallel edge cracks in a finite plate subjected to remote bending moment. Several main conclusions can be drawn from this work are as follows:

- (a) The results show that due to the presence of multiple crack edge cracks, the stress distribution is relaxed and therefore, the stress intensity factors for all cracks decreased.
- (b) When the distance between the cracks is increased, interaction is seemed to be diminished and it is can be neglected especially for swallow crack.

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