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Hydromagnetic flow and heat transfer adjacent to a stretching vertical sheet with prescribed surface heat flux

Fazlina Aman · Anuar Ishak

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Abstract The similarity solution for the problem of mixed convection boundary layer flow adjacent to a stretching vertical sheet in an incompressible electrically conducting fluid in the presence of a transverse magnetic field is presented. It is assumed that the sheet is stretched with a power-law velocity and is subjected to a variable surface heat flux. The governing partial differential equations are first transformed into a system of non-linear ordinary differential equations, before being solved numerically by the Keller-box method. The numerical results obtained are then compared with previously reported cases available in the literature as well as the series solution for certain values of parameters, to support their validity. The effects of the governing parameters on the flow field and heat transfer characteristics are obtained and discussed.

List of symbols

a, b	Constants
B	Magnetic field
B_0	Uniform magnetic field
C_f	Skin friction coefficient
f	Dimensionless stream function
g	Acceleration due to gravity
Gr_x	Local Grashof number
k	Thermal conductivity
M	Magnetic parameter
Nu_x	Local Nusselt number
Pr	Prandtl number
q_w	Wall heat flux
Re_x	Local Reynolds number
T	Fluid temperature
T_w	Plate temperature
T_∞	Ambient temperature
u, v	Velocity components along the x and y directions, respectively
U_w	Stretching velocity
x, y	Cartesian coordinates along the surface and normal to it, respectively

Greek symbols

α	Thermal diffusivity
β	Thermal expansion coefficient
η	Similarity variable
θ	Dimensionless temperature
λ	Buoyancy or mixed convection parameter
μ	Dynamic viscosity
ν	Kinematic viscosity
ρ	Fluid density
σ	Electrical conductivity
τ_w	Wall shear stress
ψ	Stream function

Subscripts

w	Condition at the wall
∞	Ambient condition

Superscript

$'$ Differentiation with respect to η

F. Aman
Faculty of Science, Art and Heritage, Universiti Tun Hussein Onn Malaysia, 86400 Parit Raja, Batu Pahat, Johor, Malaysia

A. Ishak (✉)
School of Mathematical Sciences, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia
e-mail: anuarishak@yahoo.com

1 Introduction

Flow and heat transfer of a viscous and incompressible fluid induced by a stretching surface has attracted the interest of many researchers in view of its applications in many industrial manufacturing processes. Some examples are in the extrusion of a polymer in a melt-spinning process, metals and plastics, the boundary layer along material handling conveyers, the cooling and/or drying of papers and textiles, glass blowing, continuous casting and spinning of fibers. The two-dimensional steady flow of an incompressible viscous fluid caused by a linearly stretching plate was first discussed by Crane [1]. Since then, many authors have studied various aspects of this problem. For instance, the effect of suction or injection on the flow field and heat transfer characteristics was studied by Gupta and Gupta [2], Ali [3] and Ali and Al-Yousef [4]. Ali [3] considered the case when the sheet is stretched with a power-law velocity, and then Ali and Al-Yousef [4] extended this problem to a vertically stretching sheet. Magyari et al. [5] studied the heat and mass transfer characteristics of the boundary-layer flows induced by continuous surfaces with rapidly decreasing velocities. This problem was then extended by Ali [6] to a vertical surface, where the effect of buoyancy force was taken into consideration. Quite recently, Partha et al. [7] studied the similar problem, by considering exponentially stretching surface. The temperature field in the flow over a linearly stretching surface subject to a variable surface temperature was studied by Grubka and Bobba [8], while Elbashbeshy [9] considered the heat transfer characteristics on a stretched surface subject to a power-law velocity and variable surface heat flux. Grubka and Bobba [8] showed that Crane's solution to the boundary layer equations also happens to be an exact solution to the Navier–Stokes equations.

All of the above-mentioned works, however, did not consider the situations where hydromagnetic effects emerge. Hydromagnetic flow and heat transfer problems are found to be more important in industrial processes. Many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes stretched [10]. The properties of the final product depend to a great extent on the rate of cooling. The final product of desired characteristics can be achieved as the rate of cooling is being controlled when drawing such sheets in an electrically conducting fluid subject to a magnetic field. The steady flow of an elastic stretching sheet in the presence of an uniform magnetic field of an electrically conducting fluid was investigated by Pavlov [11]. Further study was made by Chiam [12], who investigated the stretching sheet with a power-law velocity distribution and having a variable magnetic field of a special form that results in a similarity solution. Mahapatra and Gupta [13] were then investigated the steady two-dimensional stagnation-point flow of an incompressible viscous electrically conducting fluid over a stretching surface in the presence of an uniform transverse magnetic field. Anjali Devi and Thiyagarajan [14] studied the effect of magnetic field on the flow and heat transfer characteristics of a nonlinearly stretching surface of variable temperature. Ishak et al. [15] extended this problem to a vertical surface in which the effect of buoyancy force could not be neglected. Moreover, Liu [16] investigated the hydromagnetic flow over a linearly stretching sheet by considering both prescribed surface temperature and prescribed surface heat flux conditions, and reported the solutions in terms of Kummer's functions. The steady two-dimensional magnetohydrodynamic stagnation-point flow towards a stretching vertical sheet with variable surface temperature has been studied by Ishak et al. [17].

In view of the above-mentioned investigations, we present in this paper the study of mixed convection hydromagnetic flow and heat transfer over a stretching vertical sheet with prescribed surface heat flux. To the best of our knowledge, this problem has not been studied before.

2 Problem formulation

Consider the steady, two-dimensional mixed convection flow adjacent to a stretching vertical sheet immersed in an incompressible electrically conducting fluid in the presence of a transverse magnetic field $B(x)$. The stretching velocity and the surface heat flux are assumed to be of the forms $U_w(x) = ax^m$ and $q_w(x) = bx^n$, respectively, where a, b, m and n are constants with $a > 0$ and $b, m, n \geq 0$. The magnetic Reynolds number is assumed to be small, and thus the induced magnetic field is negligible. Under the foregoing assumptions and applying the usual boundary layer and Boussinesq approximations, the governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u + g\beta(T - T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are the velocity components along the x and y axes, respectively. Further, ν, ρ, α, β, σ, T, T_∞ and g are the kinematic viscosity, fluid density, thermal diffusivity, thermal expansion coefficient, electrical conductivity, fluid temperature, ambient temperature and acceleration due to gravity, respectively. The boundary conditions are

$$u = U_w, v = 0, \frac{\partial T}{\partial y} = -\frac{q_w}{k} \text{ at } y = 0, \quad u \to 0, T \to T_\infty \text{ as } y \to \infty, \tag{4}$$

where U_w, q_w and k are the velocity of the stretching sheet, the surface heat flux and the thermal conductivity, respectively. To obtain similarity solutions of Eqs. (1)–(4), we assume that the variable magnetic field $B(x)$ is of the form $B(x) = B_0 x^{(m-1)/2}$. This form of $B(x)$ has also been considered by Chiam [12], Anjali Devi and Thiyagarajan [14] and Ishak et al. [15], among others.

The continuity equation (1) is satisfied by introducing a stream function ψ such that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. The momentum and energy equations can be transformed into the corresponding non-linear ordinary differential equations by the following transformation:

$$\eta = \left(\frac{U_w}{\nu x}\right)^{1/2} y, \quad f(\eta) = \frac{\psi}{(\nu x U_w)^{1/2}}, \quad \theta(\eta) = \frac{k(T - T_\infty)}{q_w}\left(\frac{U_w}{\nu x}\right)^{1/2}. \tag{5}$$

The transformed nonlinear ordinary differential equations are:

$$f''' + \frac{m+1}{2} f f'' - m f'^2 - M f' + \lambda\theta = 0, \tag{6}$$

$$\frac{1}{Pr}\theta'' + \frac{m+1}{2} f\theta' - n f'\theta = 0, \tag{7}$$

subject to the boundary conditions (4) which become

$$f(0) = 0, \quad f'(0) = 1, \theta'(0) = -1, \quad f'(\eta) \to 0, \quad \theta(\eta) \to 0 \text{ as } \eta \to \infty, \tag{8}$$

where primes denote differentiation with respect to η, $Pr = \nu/\alpha$ is the Prandtl number and $M = \sigma B_0^2/(\rho a)$ is the magnetic paramater. In Eq. (6), λ is the mixed convection parameter defined as $\lambda = Gr_x/Re_x^{5/2}$, where $Gr_x = g\beta q_w x^4/(k\nu^2)$ is the local Grashof number and $\mathrm{Re}_x = U_w x/\nu$ is the local Reynolds number. It can be shown that λ is independent of x when $n = (5m-3)/2$, and thus similarity is achieved under this restriction.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{2\tau_w}{\rho U_w^2}, \quad Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, \tag{9}$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{10}$$

with μ being the dynamic viscosity. Using the similarity variables (5), we obtain

$$\frac{1}{2} C_f Re_x^{1/2} = f''(0), \quad Nu_x/Re_x^{1/2} = 1/\theta(0). \tag{11}$$

We note that when $m = 1$ (linearly stretching sheet) and $\lambda = 0$ (forced convection flow), Eq. (6) has an exact solution [18]

$$f(\eta) = \frac{1}{\sqrt{1+M}}\left(1 - e^{-\sqrt{1+M}\eta}\right), \tag{12}$$

and the solution for the energy Eq. (7) is given by

$$\theta(\eta) = \frac{1}{\gamma} e^{-\frac{Pr}{\sqrt{1+M}}\eta} \frac{F(\gamma - 1, \gamma + 1, -\gamma e^{-\sqrt{1+M}\eta})}{F(\gamma - 1, \gamma, -\gamma)}, \tag{13}$$

where $\gamma = Pr/(1 + M)$, and $F(a, b, z)$ denotes the confluent hypergeometric function [19] with

$$F(a, b, z) = 1 + \sum_{k=1}^{\infty} \frac{a_k}{b_k} \frac{z^k}{k!},$$

$$a_k = a(a+1)(a+2)\cdots(a+k-1),$$

$$b_k = b(b+1)(b+2)\cdots(b+k-1).$$

Further, from Eqs. (12) and (13), the skin friction coefficient $f''(0)$ and the surface temperature $\theta(0)$ can be shown to be given by

$$f''(0) = -\sqrt{1+M}, \quad \theta(0) = \frac{\sqrt{1+M}}{Pr} \frac{F(\gamma - 1, \gamma + 1, -\gamma)}{F(\gamma - 1, \gamma, -\gamma)}. \tag{14}$$

3 Results and discussion

The transformed Eqs. (6)–(8) have been solved numerically using a finite-difference scheme known as the Keller-box method, described in the book by Cebeci and Bradshaw [20]. This method is unconditionally stable and has a second-order accuracy. In the presence of buoyancy force ($\lambda \neq 0$), similarity solution to the present problem may arise if the temperature exponent parameter n and the velocity exponent parameter m are related by $n = (5m-3)/2$. For simplicity, we consider Prandtl number unity throughout the paper, except for comparisons with previously investigated cases. We expect that the results are qualitatively similar with other values of Pr of O(1). To assess the accuracy of the present method, the values of the surface temperature $\theta(0)$ are compared with those of previous studies as well as the series solution for several values of parameters. The comparison as presented in Table 1 shows a favorable agreement.

Table 1 Comparison with previously published data for the values of surface temperature $\theta(0)$

λ	m	Pr	M	Elbashbeshy [9]	Liu [16]	Present results	Eq. (14)
0	0.6	1	0			1.8894	
			1			2.2780	
	1	0.72	0	1.2253		1.2367	1.236657472
		1	0	1		1	1
		10	0	0.2688		0.2688	0.2687685151
		6.7	0		0.333303	0.3333	0.3333030612
			0.5		0.339715	0.3398	0.3397152199
			1		0.345377	0.3454	0.3453771711
			5		0.380930	0.3809	0.3809302053
1		1	0			0.9240	
			2			1.0665	

Figures 1 and 2, respectively, illustrate the effect of some values of magnetic parameter M on the velocity and temperature distributions when the other parameters are fixed to unity. In Fig. 1, it can be seen that an increase in the transverse magnetic field tends to reduce the boundary-layer thickness, hence causes the fluid motion to slow down, which implies an increase in the velocity gradient at the surface. Thus, the skin friction coefficient $f''(0)$ increases as the magnetic parameter M increases. Conversely, as shown in Fig. 2, the effect of magnetic field is to enhance the fluid temperature in the boundary layer, which in turn increases the surface temperature $\theta(0)$. Thus, the heat transfer rate at the surface $1/\theta(0)$ decreases as M increases.

Figures 3 and 4, respectively, show the samples of velocity and temperature profiles for some values of velocity exponent parameter m (and temperature exponent parameter n) with fixed values of Pr, λ and M. It is clear from these figures that the parameter m results in decreasing manner of both the velocity and temperature distributions inside the boundary layers. Thus, both the skin friction coefficient $f''(0)$ and the local Nusselt number $1/\theta(0)$ increase as m increases. It is evident from Figs. 1–4 that the far fields boundary conditions (8) are satisfied asymptotically, which support the validity of the numerical results obtained.

Figures 5 and 6 elucidate the variations with λ of the skin friction coefficient $f''(0)$ and the local Nusselt number $1/\theta(0)$ for some values of m (and n) when $Pr = 1$ and $M = 1$. As shown in these figures, the solution does not exist for the opposing flow ($\lambda < 0$). For $\lambda = 0$ (buoyancy force is absent), the value of $f''(0)$ is negative for all values of Pr and M considered. Physically, negative values of $f''(0)$ mean the sheet exerts a drag force on the fluid, and positive values of $f''(0)$ mean the opposite. The negative values of $f''(0)$ when $\lambda = 0$ is not surprising since for this case, the formation of the boundary layer is solely caused

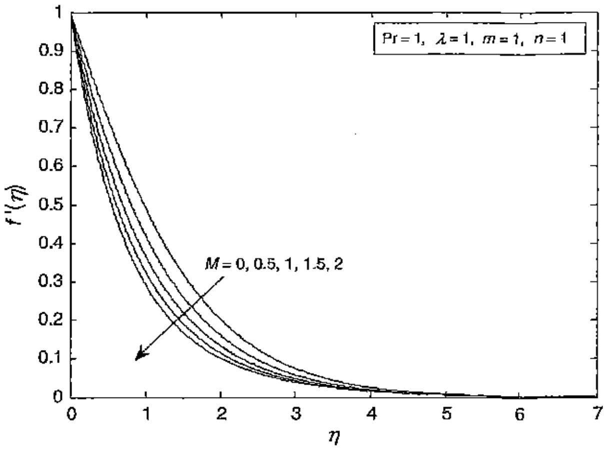

Fig. 1 Velocity profiles $f'(\eta)$ for some values of magnetic parameter M

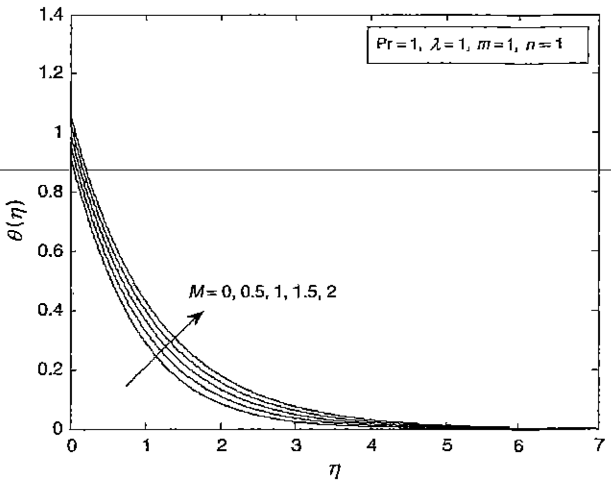

Fig. 2 Temperature profiles $\theta(\eta)$ for some values of magnetic parameter M

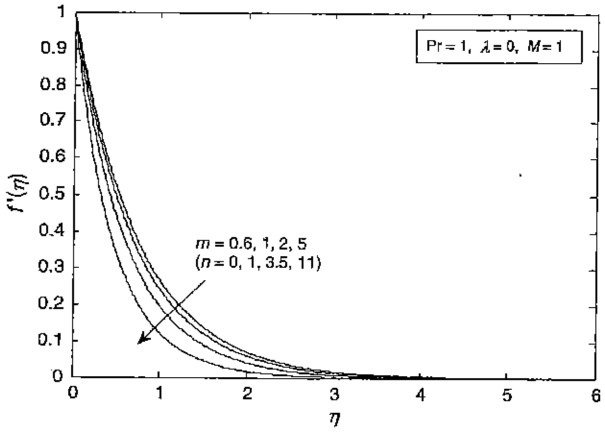

Fig. 3 Velocity profiles $f'(\eta)$ for some values of m (and n)

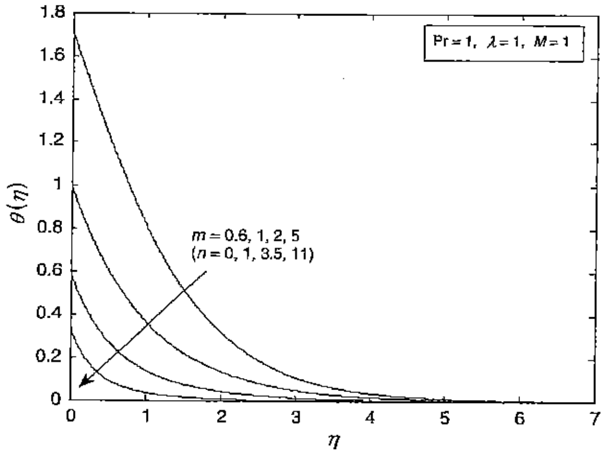

Fig. 4 Temperature profiles $\theta(\eta)$ for some values of m (and n)

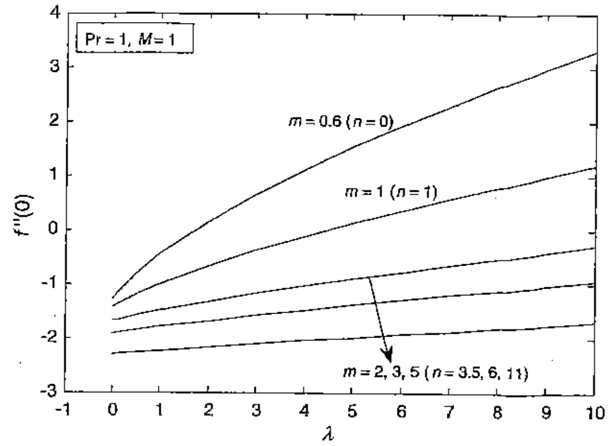

Fig. 5 Variation of the skin friction coefficient $f''(0)$ with λ for some values of m (and n)

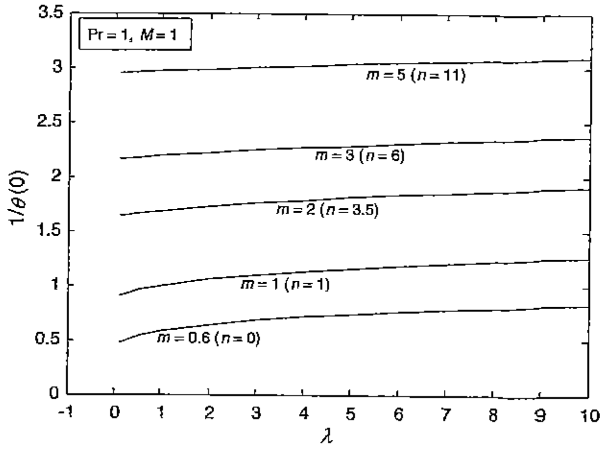

Fig. 6 Variation of the local Nusselt number $1/\theta(0)$ with λ for some values of m (and n)

by the stretching sheet. Different from $f''(0)$ which becomes positive in the present of buoyancy force, the local Nusselt number $1/\theta(0)$ is always positive, which means the heat is transferred from the hot sheet to the cool fluid, regardless of the existence of the buoyancy force. Figure 6 also shows that the heat transfer rate at the surface increases with increasing values of the velocity exponent parameter m as well as the buoyancy parameter λ.

4 Conclusions

In this study, analysis of the hydromagnetic flow adjacent to a vertical sheet with prescribed surface heat flux has been carried out. The governing boundary layer equations are first transformed into a system of ordinary differential equations, before being solved numerically by the Keller-box method. The comparison of the numerical results obtained for certain values of parameters showed a favorable agreement with the existing results from open literature as well as the series solutions obtained in terms of Kummer's functions for forced convection flow over a linearly stretching sheet. The effects of the governing parameters on the flow field and heat transfer characteristics for Prandtl number unity have been analyzed and discussed. It is found that both the skin friction coefficient and the surface temperature increase in the presence of magnetic field. Moreover, the effect of buoyancy force is to enhance the fluid motion, and in consequence increases the skin friction coefficient and the heat transfer rate at the surface.

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