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Mixed convection boundary layer flow near stagnation-point on vertical surface with slip*

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Abstract This paper considers the steady mixed convection boundary layer flow of a viscous and incompressible fluid near the stagnation-point on a vertical surface with the slip effect at the boundary. The temperature of the sheet and the velocity of the external flow are assumed to vary linearly with the distance from the stagnation-point. The governing partial differential equations are first transformed into a system of ordinary differential equations, which are then solved numerically by a shooting method. The features of the flow and heat transfer characteristics for different values of the governing parameters are analyzed and discussed. Both assisting and opposing flows are considered. The results indicate that for the opposing flow, the dual solutions exist in a certain range of the buoyancy parameter, while for the assisting flow, the solution is unique. In general, the velocity slip increases the heat transfer rate at the surface, while the thermal slip decreases it.

Key words dual solution, heat transfer, mixed convection, stagnation-point, slip

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1 Introduction

The two-dimensional stagnation-point flow of an incompressible viscous fluid on a vertical sheet has attracted the attention of researchers since the past several decades because of its wide applications in industry and practical applications. Some of the applications are cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown, solar central receivers exposed to wind currents, and many hydrodynamic processes[1].

Significant numbers of investigations have discovered the existence of dual solutions for the problem of the stagnation-point flow toward a vertical plate. Ramachandran et al.[2] studied the steady laminar mixed convection in two-dimensional stagnation-point flows around heated surfaces by taking both cases of an arbitrary wall temperature and arbitrary surface heat flux variations. They found that a reverse flow developed in the buoyancy opposing flow region, and the dual solutions existed for a certain range of the buoyancy parameter. This problem was

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then extended by Devi et al.[3] to the unsteady case, where they obtained the results similar to those reported by Ramachandran et al.[2]. Furthermore, Nazar et al.[4] extended this problem to a micropolar fluid and considered the small and large values of the ratio of the external velocity over the stretching velocity. The steady two-dimensional stagnation-point flow of an incompressible viscous fluid towards a stretching sheet was considered by Zhu et al.[5], where the solutions were obtained by the homotopy analysis method. Hassanien and Gorla[6] studied the stagnation-point flow of micropolar fluids over non-isothermal surfaces. The case of the unsteady mixed convection flow of a micropolar fluid was studied by Lok et al.[7], where they found the smooth transition from the initial unsteady-state flow to the final steady-state flow. Very recently, Ishak et al.[8] reported the existence of dual solutions for both assisting and opposing flows of an electrically conducting fluid past a vertical permeable flat plate. It can be pointed out here that less work has been done on the stagnation-point flow with the slip effect at the boundary. The similarity solutions of the Navier-Stokes equations for the stagnation-point flow towards a flat plate with slip were found by Wang[9], where the solutions are applicable to the slip regime of rarefied gases. Later, Wang[10] extended this problem to the one including the heat transfer aspect, while Ariel[11] studied the stagnation-point flow of a viscoelastic fluid. Labropulu and Li[12] studied the stagnation-point flow of a second-grade fluid with slip. Recently, Zhu et al.[13] considered the steady two-dimensional magnetohydrodynamic stagnation flow towards a nonlinearly stretching surface. The governing partial differential equations were transformed to the ordinary differential equations by the scaling group transformation before being solved analytically by the homotopy analysis method. The slip effects on the mixed convective flow and heat transfer from a vertical plate were considered by Cao and Baker[14], where they reported the local non-similarity solutions. Considering other aspects, Fang et al.[15] solved the problem of the slip flow over a permeable shrinking surface (without the heat transfer aspect) using a second-order slip flow model and presented an exact solution of the governing Navier-Stokes equations. Very recently, Fang and Zhang[16] studied the heat transfer over a shrinking sheet with mass transfer. The flow was induced by a shrinking sheet with a linear velocity distribution from the slot.

In this paper, we find the similarity solution for the problem of the mixed convection boundary layer flow near the stagnation-point on a vertical surface with the slip effect at the boundary. The effects of the buoyancy and slip parameters on the skin friction coefficient and the heat transfer rate at the surface are analyzed and discussed. Cao and Baker[14] considered the uniform free stream velocity and the uniform surface temperature that do not admit the self-similar solution.

2 Problem formulation

Consider the steady two-dimensional laminar boundary layer flow of a viscous and incompressible fluid near the stagnation-point on a vertical surface. It is assumed that the velocity of the free stream is $u_e(x)$, the temperature of the plate is $T_w(x)$, and the temperature of the ambient fluid is T_∞. The boundary layer equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\mathrm{d}u_e}{\mathrm{d}x}+\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are the velocities in the x and y directions, respectively, g is the acceleration due to gravity, T is the fluid temperature, β is the thermal expansion coefficient, and α is the thermal diffusivity. The boundary conditions are given by (see Wang[10], Andersson[17], and Mukhopadhyay[18])

$$\begin{cases} u = L\dfrac{\partial u}{\partial y}, \quad v = 0, \quad T = T_{\mathrm{w}} + S\dfrac{\partial T}{\partial y} \quad \text{at} \quad y = 0, \\ u \to u_{\mathrm{e}}, \quad T \to T_{\infty} \quad \text{as} \quad y \to \infty, \end{cases} \tag{4}$$

where L is the slip length, and S is a proportionality constant. Furthermore, we assume

$$u_{\mathrm{e}} = ax, \quad T_{\mathrm{w}}(x) = T_{\infty} + bx, \tag{5}$$

where a and b are constants.

To obtain the similarity solution, we introduce the following similarity transformation (see Wang[10], Andersson[17], and Ishak et al.[19]):

$$\begin{cases} \eta = \left(\dfrac{a}{\nu}\right)^{\frac{1}{2}} y, \\ \psi = (a\nu)^{\frac{1}{2}} x f(\eta), \\ \theta(\eta) = \dfrac{T - T_{\infty}}{T_{\mathrm{w}} - T_{\infty}}, \end{cases} \tag{6}$$

where η is the independent similarity variable, θ is the dimensionless temperature, and ψ is the stream function defined as

$$u = \frac{\partial \psi}{\partial y}, \quad v = \frac{-\partial \psi}{\partial x},$$

which identically satisfy the continuity equation (1). Substituting Eq. (6) into Eqs. (2) and (3), we get the following nonlinear ordinary differential equations:

$$f''' + ff'' - f'^2 + 1 + \lambda\theta = 0, \tag{7}$$

$$\frac{1}{Pr}\theta'' + f\theta' - f'\theta = 0 \tag{8}$$

subject to the boundary conditions

$$\begin{cases} f(0) = 0, \quad f'(0) = \delta f''(0), \quad \theta(0) = 1 + \gamma\theta'(0), \\ f'(\eta) \to 1, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty. \end{cases} \tag{9}$$

Here, the primes denote the differentiation with respect to η, $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $\delta = L(\frac{a}{\nu})^{\frac{1}{2}}$ is the velocity slip parameter, $\gamma = S(\frac{a}{\nu})^{\frac{1}{2}}$ is the thermal slip parameter, and $\lambda\,(= \frac{g\beta b}{a^2})$ is the mixed convection parameter. It is worth mentioning that $\lambda > 0$ corresponds to the assisting flow, $\lambda < 0$ corresponds to the opposing flow, and $\lambda = 0$ corresponds to the forced convection flow.

The physical quantities of interest are the skin friction coefficient C_{f} and the local Nusselt number Nu_x, which are proportional to $f''(0)$ and $-\theta'(0)$, respectively.

3 Results and discussion

The nonlinear ordinary differential equations (7) and (8) subject to the boundary conditions (9) were numerically solved using the shooting method described by Zheng et al.[20] for some values of the velocity slip parameter δ, the thermal slip parameter γ, and the buoyancy or mixed convection parameter λ, while the Prandtl number Pr is fixed at 0.7 (such as air) except for the comparisons with the previously reported cases. The comparisons of the values of the reduced skin friction coefficient $f''(0)$ and the reduced local Nusselt number $-\theta'(0)$ with those obtained by Ramachandran et al.[2], Devi et al.[3], Hassanien and Gorla[6], and Lok et al.[7] for several values of Pr when the slip effect is absent ($\delta = 0$ and $\gamma = 0$) are listed in Tables 1 and 2, respectively. It is observed that the results show the very good agreement.

Table 1 Values of $f''(0)$ for different values of Pr when $\lambda = 1$, $\delta = 0$, and $\gamma = 0$

Pr	Ramachandran et al.[2]	Devi et al.[3]	Lok et al.[7]	Hassanien and Gorla[6]	Present results
0.7	1.7063	1.7064	1.7064	1.70632	1.7063
1	–	–	–	–	1.6754
7	1.5179	1.5180	1.5180	–	1.5179
10	–	–	–	1.49284	1.4928
20	1.4485	1.4485	1.4486	–	1.4485
40	1.4101	–	1.4102	–	1.4101
50	–	–	–	1.40686	1.3989
60	1.3903	1.3903	1.3903	–	1.3903
80	1.3774	–	1.3773	–	1.3774
100	1.3680	1.3680	1.3677	1.38471	1.3680

Table 2 Values of $-\theta'(0)$ for different values of Pr when $\lambda = 1$, $\delta = 0$, and $\gamma = 0$

Pr	Ramachandran et al.[2]	Devi et al.[3]	Lok et al.[7]	Hassanien and Gorla[6]	Present results
0.7	0.7641	0.7641	0.7641	0.76406	0.7641
1	–	–	–	–	0.8708
7	1.7224	1.7223	1.7226	–	1.7224
10	–	–	–	1.94461	1.9446
20	2.4576	2.4574	2.4577	–	2.4576
40	3.1011	–	3.1023	–	3.1011
50	–	–	–	3.34882	3.3415
60	3.5514	3.5517	3.5560	–	3.5514
80	3.9095	–	3.9195	–	3.9095
100	4.2116	4.2113	4.2289	4.23372	4.2116

The effects of the velocity slip parameter δ on the reduced skin friction coefficient (the wall shear stress) $f''(0)$ and the reduced local Nusselt number (the heat transfer rate at the surface)

$-\theta'(0)$ are shown in Figs. 1 and 2, respectively. Moreover, the effects of the thermal slip parameter γ on $f''(0)$ and $-\theta'(0)$ are, respectively, shown in Figs. 3 and 4. It is evident from these four figures that the dual solutions exist for the buoyancy opposing flow ($\lambda < 0$). For the assisting flow ($\lambda > 0$), the solution is unique. We identify the upper and lower branch solutions in the following discussion by the way that they appear in Figs. 1–4, i.e., the upper branch solution has the higher values of $f''(0)$ and $-\theta'(0)$ for a given λ than the lower branch solution. It can be seen that for the upper branch solution, as the buoyancy parameter λ increases, both $f''(0)$ and $-\theta'(0)$ increase due to the increase of the velocity caused by the buoyancy forces. The opposite trend can be observed for the lower branch solution. In general, for the upper branch solution, an increase in the velocity slip parameter δ yields the decrease of the wall shear stress and the increase of the heat transfer rate at the surface. In contrast, Fig. 4 shows that the heat transfer rate at the surface decreases with the increasing values of the thermal slip parameter. It is seen from Figs. 2 and 4 that $-\theta'(0) > 0$ for all values of the parameters considered in this study, which means that the heat is transferred from the hot surface to the cool fluid.

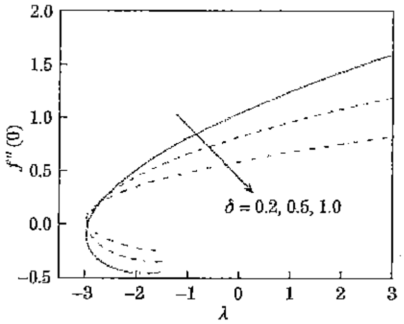

Fig. 1 Variation of skin friction coefficient $f''(0)$ with buoyancy parameter λ for different values of velocity slip parameter δ when $Pr = 0.7$ and $\gamma = 1$

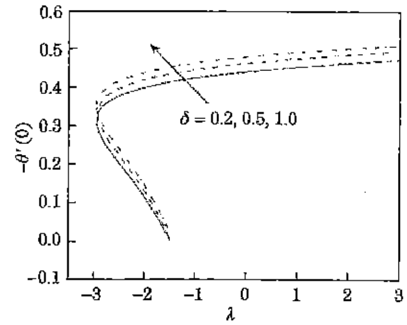

Fig. 2 Variation of local Nusselt number $-\theta'(0)$ with buoyancy parameter λ for different values of velocity slip parameter δ when $Pr = 0.7$ and $\gamma = 1$

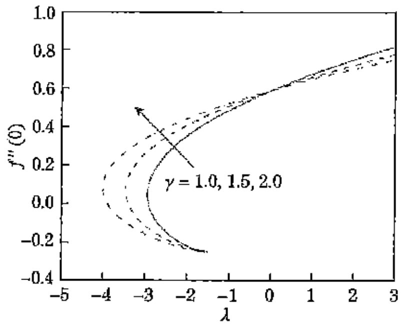

Fig. 3 Variation of skin friction coefficient $f''(0)$ with buoyancy parameter λ for different values of thermal slip parameter γ when $Pr = 0.7$ and $\delta = 1$

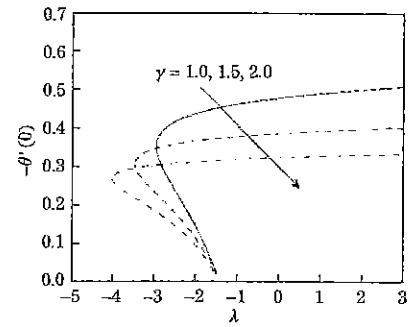

Fig. 4 Variation of local Nusselt number $-\theta'(0)$ with buoyancy parameter λ for different values of thermal slip parameter γ when $Pr = 0.7$ and $\delta = 1$

For each selected values of δ and γ (see Figs. 1–4), there is indeed a critical value λ_c of λ, for which the solution exists. Based on our computations, we find that $\lambda_c = -2.9600$, -2.9361, and -2.9613, respectively, for $\delta = 0.2$, 0.5, and 1.0 (see Figs. 1 and 2), and $\lambda_c = -2.9613$, -3.4890, and -4.0266, respectively, for $\gamma = 1.0$, 1.5, and 2.0 (see Figs. 3 and 4). It is worth mentioning that the computations have been performed until the point where the solution does not converge, i.e., the calculations have been terminated at that point. It can also be pointed out here that the effect of the slip is to reduce the range of λ, for which the solution exists.

Figures 5 and 6, respectively, present the samples of the velocity $f'(\eta)$ and the temperature $\theta(\eta)$ profiles for the selected values of the velocity slip parameter δ when $\lambda = -2.0$ for both the upper and the lower branch solutions, while the velocity and temperature profiles for different values of the thermal slip parameter γ are depicted in Figs. 7 and 8, respectively. It is obvious that the upper branch solution displays the thinner boundary layer thickness compared with the lower branch solution. It can be seen that the velocity gradient at the surface decreases as δ increases for the upper branch solution and as a result decreases the reduced skin friction coefficient $f''(0)$. Figures 5–8 show that the boundary conditions (9) are asymptotically satisfied, which supports the validity of the obtained numerical results besides supporting the existence of the dual solutions presented in Figs. 1–4.

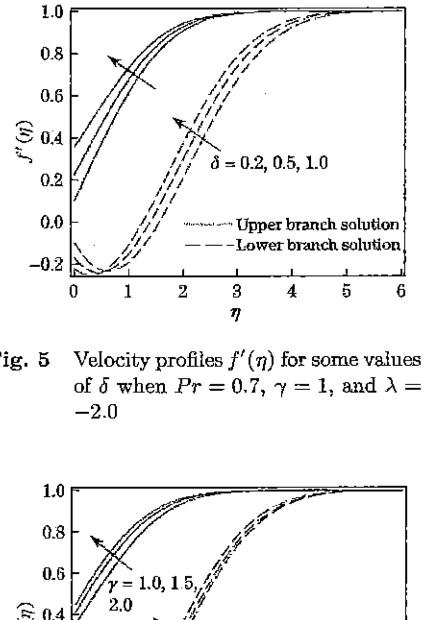

Fig. 5 Velocity profiles $f'(\eta)$ for some values of δ when $Pr = 0.7$, $\gamma = 1$, and $\lambda = -2.0$

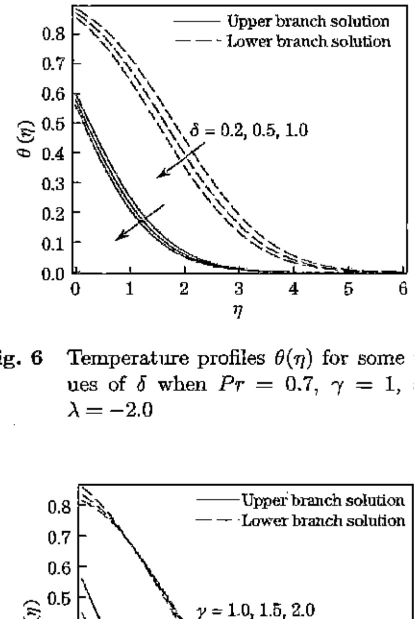

Fig. 6 Temperature profiles $\theta(\eta)$ for some values of δ when $Pr = 0.7$, $\gamma = 1$, and $\lambda = -2.0$

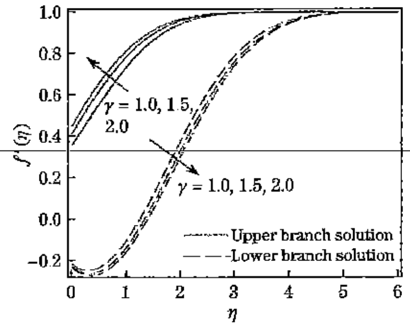

Fig. 7 Velocity profiles $f'(\eta)$ for some values of γ when $Pr = 0.7$, $\delta = 1$, and $\lambda = -2.0$

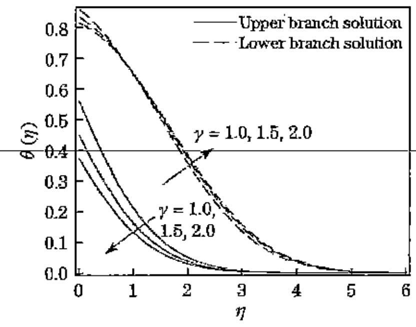

Fig. 8 Temperature profiles $\theta(\eta)$ for some values of γ when $Pr = 0.7$, $\delta = 1$, and $\lambda = -2.0$

4 Conclusions

In this paper, the flow and heat transfer characteristics near the stagnation-point on a vertical surface with the slip effect on the boundary are numerically studied. The boundary layer equations governing the flow are reduced to the ordinary differential equations by the similarity transformation. These equations are then numerically solved to obtain the skin friction coefficient and the local Nusselt number as well as the velocity and the temperature profiles for various values of the velocity slip parameter δ, the thermal slip parameter γ, and the buoyancy parameter λ with a fixed value of the Prandtl number Pr. It can be found that for the opposing flow ($\lambda < 0$), the dual solutions exist for a certain range of the buoyancy parameter, whereas for the assisting flow ($\lambda > 0$), the solution is unique. Moreover, the effect of the slip is to reduce the range of λ, for which the solution exists. In general, the velocity slip increases the heat transfer rate at the surface, while the thermal slip decreases it.

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