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Norman R. Simon
University of Nebraska-Lincoln, nsimon@unl.edu

Terry J. Teays
University of Nebraska-Lincoln, tteays1@jhu.edu

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THE STRUCTURAL PROPERTIES OF CEPHEID VELOCITY CURVES

NORMAN R. SIMON AND TERRY J. TEAYS
Behlen Laboratory of Physics, University of Nebraska-Lincoln
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ABSTRACT

Fourier decompositions are performed on the velocity curves of 11 classical Cepheids. The progression of curve shape with period is described in terms of combinations of the lower order Fourier coefficients. These quantities are shown to change with pulsation period in a manner similar to that already demonstrated for Cepheid light variations (Simon and Lee). We recommend further velocity observations, particularly in the period range $10 \leqslant P \leqslant 16$ days.

Subject headings: stars: Cepheids — stars: pulsation

I. INTRODUCTION

Recently, Simon and Lee (1981) reported on the Fourier decomposition of the light curves of a large sample of classical Cepheids. It was demonstrated that combinations of the low-order Fourier coefficients could be used to describe quantitatively the progression of curve shape with period. In the present investigation we use the same technique to treat Cepheid velocity curves. Because of the relative paucity of velocity data, our sample in this case consists of only 11 stars whose velocity curves, and thus Fourier decompositions, are generally less accurate than were the light variations. Nonetheless, we shall be able to see in the velocities a progression very similar to that found for the light, including the sharp break at ~ 10 days characteristic of the Cepheid resonance (Simon and Schmidt 1976).

II. THE FOURIER DECOMPOSITIONS

Our fitting scheme is described by Simon and Lee (1981). For the velocities, we fit to a different form of Fourier series, viz.,

$$v = A_0 - A_i \sin\left[i\omega(t - t_0) + \phi_i\right], \qquad (1)$$

where, for a given fit, the index i runs from 1 to $i_{\max}$. In the present investigation, $i_{\max} = 4$ or 8. Criteria for the sufficiency and appropriateness of the fits are as given by Simon and Lee (1981). However, because of the small sample of velocity observations, a number of fits that would have been considered marginal according to Simon and Lee (1981) have been accepted here.

Following Simon and Lee (1981), the time t in equation (1) is used in the form

$$t = \mathrm{JD} - \tau,$$

where JD is the time of the observation in Julian days. The quantities τ and t_0 (eq. [1]) are constants in the fit.

Table 1 lists the stars whose velocity curves we have studied. References to the observations and to the values of τ are given in the "Source" column. Other columns give the period, observed amplitude, the number of observed points presented to the fitting routine, the order of the fit (i.e., $i_{\max}$), and the standard deviation. As explained by Simon and Lee (1981), the periods listed in Table 1 should not be considered definitive.

The Fourier coefficients (A_i, ϕ_i) up to fourth order, as well as the quantity t_0, are displayed in Table 2. For those stars with observations from more than one source, the data were combined as follows. First, each set was presented separately to the fitting routine and its zeroth order (i.e., unperturbed) velocity determined. This quantity was then subtracted from each observation before the different sets were put together for analysis.

The footnote "b" in the first column of Table 2 indicates that an eighth-order fit was constructed for the corresponding stars. The additional coefficients for these cases appear in Table 3. The footnote "a" in the first column of Table 2 indicates a marginal fit in the sense that coefficients of order higher than two should not be considered fully reliable. These coefficients have been included merely for completeness. The coefficients of first and second order, on the other hand, are acceptable. It is these coefficients that shall interest us in what follows.

For the light curves in Simon and Lee (1981), the Hertzsprung progression was quantified in terms of combinations of the low-order Fourier coefficients, viz.,

$$R_{21} = A_2/A_1, \quad \phi_{21} = \phi_2 - 2\phi_1, \quad \phi_{31} = \phi_3 - 3\phi_1.$$

In the present case, we lack sufficient data to employ the last of these.

Figure 1 displays a plot of ϕ_{21} versus period for the 11 stars in our sample. The resemblance of this plot to the

TABLE 1

LIST OF STARS IN THE SURVEY

Star Number	Star Name	Source	Period	Amplitude (km s^{-1})	Number of Observations	Order of Fit	Standard Deviation (km s^{-1})
1...........	SU Cas	1	1.94933	24.7	135	4	1.88
2...........	R TrA	2	3.389287	31.6	27	4	1.68
3...........	XY Cas	3	4.501697	34.2	47	4	0.956
4...........	V482 Sco	2	4.52786	35.3	24	4	1.53
5...........	δ Cep	4	5.36633	39.7	91	8	0.762
6...........	S TrA	2	6.32344	35.0	23	4	1.82
7...........	η Aql	5	7.18372	42.0	31	4	1.83
8...........	S Sge	6	8.38217	35.9	45	8	0.202
9...........	β Dor	7	9.84238	39.5	156	4	1.96
10...........	ζ Gem	8	10.1535	30.9	46	4	0.794
11...........	X Cyg	9	16.3800	63.4	23	4	2.71

SOURCE.—(1) Abt 1959; Gieren 1976; Beavers 1979; $\tau = 2{,}400{,}000$. (2) Gieren 1981; $\tau = 2{,}444{,}000$. (3) Imbert 1981; $\tau = 2{,}440{,}000$. (4) Shane 1958; $\tau = 2{,}400{,}000$. (5) Wright 1899; $\tau = 2{,}400{,}000$. (6) Herbig and Moore 1952 (Table 5); $\tau = 0$, times as given in Table 5. (7) Applegate 1927; Stibbs 1955; $\tau = 2{,}400{,}000$. (8) Campbell 1901; $\tau = 2{,}400{,}000$. (9) Duncan 1921; $\tau = 2{,}420{,}000$.

TABLE 2

FOURIER COEFFICIENTS (A_i, ϕ_i) (where $i = 1$–4)

Star Number	Star Name	t_0	A_0	A_1	ϕ_1	A_2	ϕ_2	A_3	ϕ_3	A_4	ϕ_4
1[a]	SU Cas	40,000	+1.34(−2)	9.26	4.95	1.65	3.30	9.85(−2)	3.06	2.60(−1)	5.00
2[a]	R TrA	421	−13.33	10.32	1.36	4.44	2.54	1.86	4.37	5.80(−1)	2.74
3	XY Cas	4010	−42.07	14.98	6.68(−1)	4.64	1.17	2.33	1.94	1.36	2.63
4[a]	V482 Sco	421	+7.54	15.55	5.61	4.58	4.73	2.01	3.91	1.25	2.81
5[b]	δ Cep	6000	−16.16	15.69	3.86	6.73	4.42	3.87	5.36	1.86	3.07
6[a]	S TrA	421	+3.99	13.20	6.17(−1)	5.79	1.39	1.93	2.61	1.14	2.81
7	η Aql	5500	−14.46	16.16	2.17	8.14	4.72	3.31	1.13	1.58	3.11
8[b]	S Sge	0	+3.85(−2)	14.31	5.65	7.59	5.80	1.86	5.42	1.49	5.40
9	β Dor	5000	+1.44(−2)	14.09	9.92(−1)	5.00	3.84	2.38	4.71	3.71(−1)	2.80(−1)
10	ζ Gem	30,000	+6.86	12.35	6.09	3.00	1.70	1.98	1.47	1.93(−1)	3.03
11[a]	X Cyg	2000	+9.43	27.00	5.32	5.39	4.58	3.35	2.33	2.56	8.96(−1)

[a] Higher order coefficients ($i > 2$) may not be fully reliable.
[b] Eight-order fit; see Table 3.

TABLE 3

FOURIER COEFFICIENTS (A_i, ϕ_i) (where $i = 5$–8)

Star Number	Star Name	A_5	ϕ_5	A_6	ϕ_6	A_7	ϕ_7	A_8	ϕ_8
5...........	δ Cep	1.13	7.21(−1)	6.17(−1)	4.54	3.99(−1)	2.40	3.11(−1)	6.13(−1)
8...........	S Sge	2.08(−1)	5.28	4.39(−1)	4.94	1.49(−1)	4.80	2.21(−1)	4.71

corresponding one for the light curves in Simon and Lee (1981) is extremely strong. One sees the same moderate rise in ϕ_{21} between 4 and 8 days, followed by a rapid jump near 10 days and subsequent decline at longer periods. (Unfortunately, our present sample contains only one long-period star, X Cyg.)

In Figure 2 we plot the amplitude ratio R_{21} versus period. Here, as in Simon and Lee, the scatter is considerably greater than that for ϕ_{21}. Nonetheless, we may note a crude resemblance between the light and velocity data in the relative maximum which occurs before 10 days and the subsequent dropoff in the 10 day vicinity.

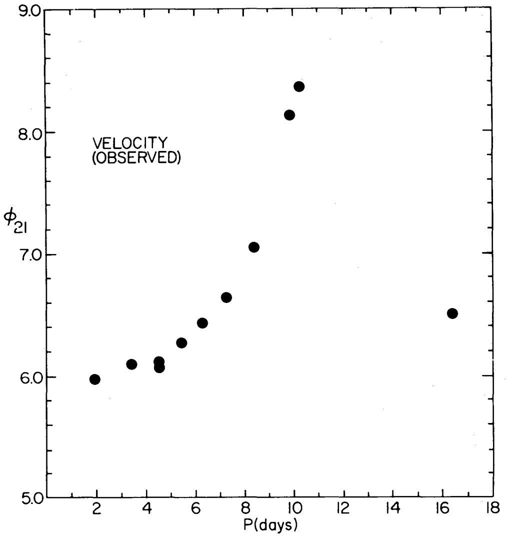

FIG. 1.—Phase difference $\phi_{21} = \phi_2 - 2\phi_1$ vs. period

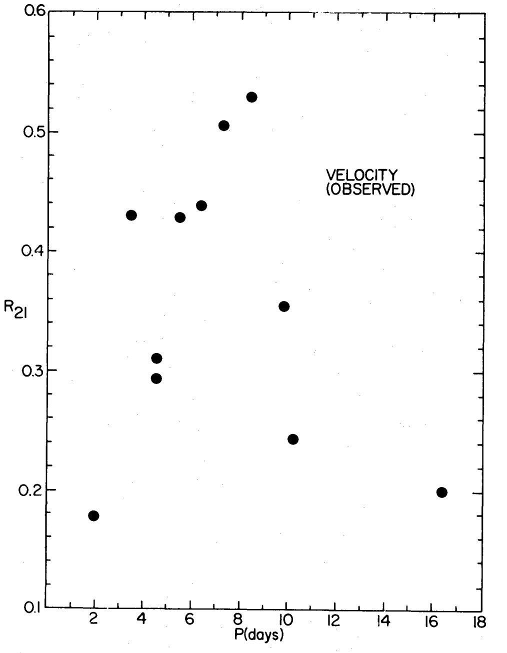

FIG. 2.—Amplitude ratio $R_{21} = A_2/A_1$ vs. period

For the two stars with observations from more than one source, we might try to compare the values of ϕ_{21} and R_{21} obtained by analyzing each source separately with the values coming from the combined data. In the case of SU Cas, unfortunately, the Fourier decompositions from individual data sets did not meet minimum criteria for reliability, as a result of inadequate phase coverage. On the other hand, for β Dor, the larger of the two data sets (133 points) yielded values of ϕ_{21} and R_{21} which differed by $\lesssim 3\%$ from those of the combined set.

III. DISCUSSION

The qualitative correspondence between Cepheid light and velocity curves along the Hertzsprung sequence has long been known. In the present work this correspondence manifests itself in the quantities ϕ_{21} and R_{21} determined by Fourier decomposition. In particular, one notices the sharp break at 10 days, characteristic of the period resonance $P_2/P_0 = 0.5$ (Simon and Schmidt 1976).

It was suggested by Simon and Lee (1981) that Fourier decomposition ought to provide a useful medium for comparing the observed variations of Cepheids with those generated from hydrodynamic models. This comparison may be performed in either light or velocity. However, it is well known that the theoretical velocity curves are generally smoother than the light curves and display fewer numerical artifacts. Thus, from the theoretical side the match is better made in velocity. While the present investigation provides a preliminary basis for such a comparison, the situation would clearly be improved if accurate velocity curves could be determined for a larger sample of stars. A velocity study of Cepheids with periods between 10 and 16 days would be particularly useful to fill a serious gap in the present data. We recommend that such a study be undertaken.

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NORMAN R. SIMON and TERRY J. TEAYS: Behlen Laboratory of Physics, University of Nebraska, Lincoln, NE 68588