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Applications of Cobb-Douglas Production Function in Construction Time-Cost Analysis

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by

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Cobb-Douglas Production Function was widely used in economics and productivity studies across many sectors. The function's quantitative modeling of resource inputs and production outputs is appealing to the research domain of construction management. In this thesis we explored this function's application in construction schedule crashing and project risk analysis related to duration of construction projects. Existing research on construction time-cost tradeoff issues rarely explore the origin of the crashing cost which is defined as the cost needed to shorten the project to the desired duration. In the existing literature crashing cost function was either assumed without much justification, or came from ad-hoc regression analysis of historical data of some actual projects. Cobb-Douglas production function, which defines the portion of labor and equipment needed based on the production rate, provides a much-needed piece to modeling the cost functions in the construction time-cost tradeoff problem during the schedule crashing process. This new perspective fills a gap of existing time-cost tradeoff research by considering project duration, labor and equipment cost as parameters of the Cobb-Douglas production

function. A case study was presented to show how the proposed framework works. Case results are presented based on deterministic values of Cobb-Douglas function's parameters ($\alpha, \beta, c_L,$ and c_K). Presented results show each of labor and equipment's portion in total cost. Here, more than that we had earlier, we can analyze different options based on just one of these factors to find the optimum solution. Not only, the total labor and total equipment cost of a project is available, labor and equipment cost of each activity's option can be calculated and compared with other options. After that, sensitivity analysis on the mentioned parameters was conducted to further explore the model's sensitivities to these parameters. Inflation factors in labor and equipment cost were incorporated in the sensitivity analysis based on assumed costs' fluctuation range. The results show that the total cost highly depends on the summation of α and β. The conclusion of the thesis is that utilizing Cobb-Douglas production function in construction crashing cost analysis expands our understanding of crashing cost sources and the portion of each of elements. Moreover, from sensitivity analysis results, it is concluded that labor and equipment efficiencies have significant effects on total cost of a project. Cost inflation analysis makes managers aware of uncertain market which influences total cost and duration of a project.

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CHAPTER ONE

Introduction

Time-cost tradeoff problem (TCTP) has been extensively studied in construction scheduling research (Feng et al. 1997; Hegazy 1999, Feng et al. 2000; Ng and Zhang 2008) to find out the best solutions in terms of minimizing the cost while shortening the project duration. Many different algorithms and assumptions were used in searching for the best solutions which are explained in detail later.

Despite many existing researches on TCTP, very little study can be found in exploring or explaining the source or the origin of the cost increase during activity crashing. Instead, in many studies, the cost functions associated with crashing were assumed, or based on historical data, or based on simulation results. Being able to explain quantitatively where the increased crashing costs come from is important to better understand the theoretical fundamental of TCTP.

There has been a lack of theoretical base to model the cost functions associated with activity crashing. Evensmo and Karlsen (2008) were among the few researchers tried to explain the origin of cost increase during activity crashing. However, in their study, they only discussed the causes of cost increasing due to labor input changes. A significant limitation in their approach is the lack of consideration of changes of equipment inputs during crashing activities.

Considering many construction crews are composed of both labor and equipment, it is necessary to identify or develop new models to consider both labor and equipment

changes during the activity crashing procedure, so we can more accurately model the crashing cost functions.

Cobb-Douglas production function (CDPF) (Eq. 1) (Cobb and Douglas 1928; Varian 1992) has been widely used in research on economics (Meeusen and van den Broeck 1977, Dennis et al. 2010), technology progress (Sircar and Choi 2009), and productivity (Banker and Natarajan 2008; Pendharkar et al. 2008).

$$Q = A \cdot L^\alpha K^\beta \qquad \text{(Eq. 1)}$$

Where: Q = production rate; L = labor input; K = capital/equipment input; A = technology; α and β are the output elasticity of labor and capital respectively.

In particular, CDPF models production rate, labor inputs, equipment/capital inputs and technology efficiency in a very elegant formation, which can be used to explain many types of production activities. Some important features of CDPF (Fig. 1) are very useful in explaining the origin of the crashing cost under many different situations.
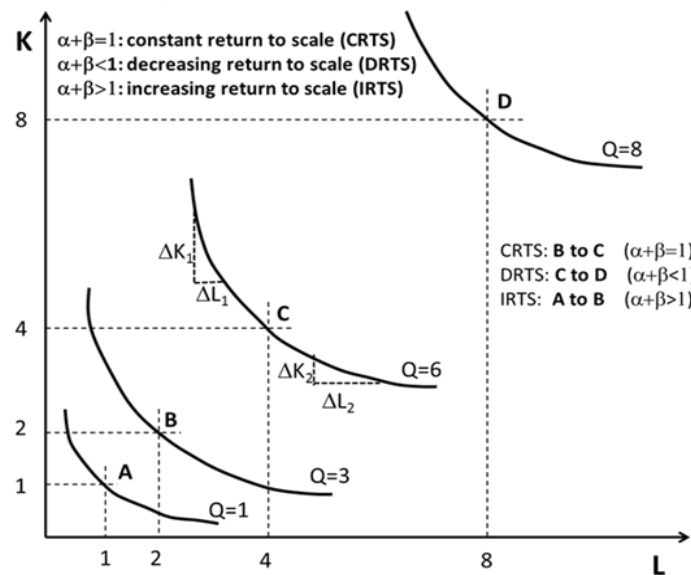

Fig.1. Illustrative isoquant curves from CDPF

An Isoquant (Varian 1992) is a contour line drawn through the set of points at which the same quantity of output is produced while changing the quantities of inputs. Fig. 1 shows a set of isoquants for a production function with two inputs of capital (K) and labor (L). K is equivalent to or interchangeable with equipment in this study. One important feature of CDPF is reflected by the summation value of α and β. When the summation of α and β is less than 1 which is called Decrease Return to Scale (DRTS), the double inputs of L and K will generate less than double output of Q. This is illustrated by case from point C to D in Fig. 1. When the summation of α and β is equal to 1, Constant Return to Scale (CRTS), the double inputs of L and K will generate double output of Q, which is illustrated by case from point B to C. When the summation of α and β is greater than 1, Increase Return to Scale (IRTS), the double inputs of L and K will generate more than double output of Q, which is illustrated by case from point A to B.

Another important feature related to this study is the efficiency of substituting part of labor (ΔL) input for part of equipment input (ΔK). As we can see in the isoquant curve of $Q=6$, the efficiency of substituting K for L is decreasing (from $\Delta K_1 \Delta L_1$ to $\Delta K_2 \Delta L_2$) as more labor replacements are added to generate the same amount of Q, in this case $Q=6$.

Construction activity crashing can be achieved through increasing A, technology, or by increasing inputs of L and/or K in fixed period of time. In both cases the Q will increase in unit time period, which is essentially activity crashing. In this paper, we limit our scope of crashing within the allocation of L and K, while assuming construction technology (A) remains the same during the crashing process. This will allow us to focus our discussions on how to best utilize L and K in crashing.

CDPF provides a feasible tool to model construction schedule crashing activities, especially modeling the time-cost trade-off problem. The origin or source of the cost increase in construction TCTP can be theoretically explained using CDPF. The duration crashing is achieved through increasing either L or K or both, so to increase Q in a given time. Applying CDPF with cost functions of both labor and equipment provide a potential way to incorporate detailed labor and equipment costs and utilization information into the time-cost optimization model in construction TCTP.

To this end, in the paper, a new framework for TCTP in construction using CDPF and GA is proposed. A case study is presented using the proposed framework and the results are discussed. Finally, sensitivity analysis on all production function's factors is done. The proposed CDPF framework for TCTP provides a new perspective for research in construction TCTP by enabling further analysis on optimizing labor and equipment allocations during the activity crashing process. This approach enables the project managers to further understanding his options in allocating appropriate combinations of labor and equipment based on the CDPF. This additional capacity is a major contribution of this study, which has not been reported in existing publications to the best of our knowledge.

CHAPTER TWO

Literature Review

Existing researches on construction TCTP can be classified based on cost functions, solution methods, objective functions, and models.

2.1. Construction Crashing Time-Cost Function

The relation between cost and the time has been well studied in various studies. Cost functions such as linear (Bazaraa and Shetty 1979, Fulkerson 1961), nonlinear (Moussourakis and Haksever 2010), discrete (Kelly, 1961, this research), convex (Lamberson and Hocking 1970, Demeulemeester et al. 1993), and concave (Berman, 1964) have been implemented in the studies on TCTP hitherto.

2.2. Objective Function

Objective function in construction crashing cost analysis may vary significantly. Some researchers consider multi-objective function and assume priority for time and cost, and based on that, they try to find the optimum solution. Some consider minimizing total project cost. Some try to minimize total project cost or duration subject to predefined constraints.

Moussourakis and Haksever (2010) considered three objective functions including minimization of project completion subject to a crash budget constraint, minimization of the total project cost, and minimization of the total cost under late completion penalties. They used nonlinear time-cost functions.

Leu et al. (2001) tried to determine the project completion time regarding project total cost which includes both direct and indirect costs. Some other authors consider a limited budget for the project and try to minimize the project duration regarding the budget constraints (Buddhakulsomsiri and Kim 2006).

Jaskowski and Sobotka (2006) proposed a multi-criteria objective function which considers both time and cost together, and they end up with a Pareto set. They used an evolutionary algorithm to compare different solutions based on fitness values.

One of the main assumptions or constraints, which are commonly considered in the literature, is the limitation of resources. Jaskowski and Sobotka (2004) try to minimize the project completion time regarding the limitation on resources. They also proposed an evolutionary algorithm to assign recourses to activities in a proper time.

Other authors chose a different approach to cope with this problem. To avoid delay in projects, Lin et al. (2011) predict construction project completion time using historical data. They used regression model to forecast the future projects completion time. Chen et al. (2011) developed a cash-payment model for forecasting the cash flow. They evaluated their model by comparing two historical real dataset.

2.3. Models

There are three main categories in existing literature regarding models or assumptions in time-cost trade-off problem. 1) Deterministic relationship between time and cost was assumed in Gerk and Qassim (2008), Moussourakis and Haksever (2010), Pollack-Johnson and Liberatore (2006). 2) Stochastic relation between time and cost was assumed

in Aghaie and Mokhtari (2009), Cohen et al. (2007), Ke et al. (2009). 3) Fuzzy relation between time and cost was assumed by Ghazanfari et al. (2009).

Gerk and Qassim (2008) considered both activity overlapping and substitution in their model. Pollack-Johnson and Liberatore (2006) assumed discrete time-cost trade-off activity like as we do in this study. In both studies, predefined budget were assumed. Moussourakis and Haksever (2010) assumed nonlinear time cost function which is more realistic than linear ones. Some other authors, like Diaby et al. (2011), took similar approach in terms of cost functions. They proposed a geometric programming, and then try to solve it.

Aghaie and Mokhtari (2009) proposed a nonlinear mix integer programming to increase the probability of completion of the project in a given due date. They also assume that each activity duration follow an exponential distribution. Ke et al. (2009) proposed integrating stochastic simulation and genetic algorithm to increase the probability of completion of a project by a specific due date. Cohen et al. (2007) wanted to minimize the expected cost related by the project.

Ghazanfari et al. (2009) assumed fuzzy variables. Via Possibility Goal Programming, the cost was minimized while considering the minimum duration. The main contribution of this fuzzy approach is the use of vagueness in the cost function during the project execution. Zheng and Ng (2005) also presented fuzzy set theory regarding the uncertainty included in TCTP problem. They also used GA as a meta-heuristic algorithm to develop a Pareto set of time and cost.

2.4. Solution Method

Large-scale time-cost trade off problem is often NP-hard. The methods developed to tackle this problem varied from exact approach such as dynamic programming (DP) (Robinson 1965) to heuristic and meta-heuristic algorithms. Yang (2007) proposed a particle swarm algorithm to complete the project for all kind of linear or nonlinear cost function, discrete or continuous, and concave or convex.

Feng et al. (1997) proposed a genetic algorithm to draw a Pareto set for a discrete time-cost trade-off. They consider a multi-objective criteria problem to find the optimum solution, which ended up with a Pareto set. Aghaie and Mokhtari (2009) proposed an ant colony optimization for stochastic crashing problem. They also assume a discrete time cost function problem. As stated earlier, they have assumed that the time-cost relationship is stochastic. They use Monte-Carlo simulation to cope with this problem.

One recent example of exact solution method for large-scale problems using benders decomposition-based exact algorithm is introduced by Hazir et al. (2010). Skutella (2998) proposed an approximation algorithm which is an effective algorithm for large scale problems.

Meta-heuristic algorithms have been introduced in the recent years even in other areas of construction project management. Zhang and Ng (2012), who use this kind of algorithm to develop DSS for TCTP, and Bozejko et al. (2012) are good example for that.

The proposed approach of this study and other recent related researches are summarized in Table 1, to provide a context of our contribution to this research domain.

Table 1. Related works

Authors	Cost Function	Objective Function	Deterministic	Stochastic	Fuzzy	Exact	Heuristic	Meta-Heuristic	Source of Crashing Cost	Approach
Feng et al. (2000)	Discrete	Multi-objective				✓		✓		GA-Simulation
Zheng et al. (2004)	Discrete	Multi-objective	✓					✓		GA
Zheng and Ng (2005)	Discrete	Optimal balance of time and cost			✓			✓		Fuzzy set theory-GA
Pollack-Johnson and Liberatore (2006)	Discrete	Min Project duration considering quality	✓							Quality Management
Cohen et al. (2007)	Linear	Completion by a due date		✓						Robust optimization
Evansmo and Karlsen (2008)	Linear	Determine the impact of labor cost on total project cost	✓						✓	-
Aghaie and Mokhtari (2009)	Non-linear integer math. Programming	Completing the project by a due date		✓				✓		Ant Colony Optimization and MC Simulation
Ke et al. (2009)	Discrete	Min Expected Cost		✓				✓		Chance constraint optimization-GA
Ghazanfari et al. (2009)	Non-linear	Min crashing cost-Min duration			✓					Possibility goal programming
Moussourakis and Haksever (2010)	Non-linear	minimizing project completion- minimizing total project cost- minimizing total cost under late completion penalties	✓			✓				-
Hazir et al. (2010)	Discrete	minimizing project completion considering the budget	✓			✓				Benders decomposition
Diaby et al. (2011)	Non-linear	Project completion	✓			✓				Geometric programming
This research	Discrete	Optimizing construction crashing cost considering labor and equipment cost	✓			✓		✓	✓	Hybrid Genetic Algorithm & Cobb-Douglas Function

2.5. Risk Analysis

Previous research has mainly focused on examining the impacts of risks on one aspect of project strategies with respect to cost (Chen *et al.*, 2000), time (Shen, 1997) and safety (Tam *et al.*, 2004).

Many authors have classified construction risk factors into different groups. Chen et al. (2004) proposed 15 risks concerned with project cost and divided them into three groups: resources factors, management factors and parent factors. Shen (1997) identified eight major risks accounting for project delay and ranked them based on a questionnaire survey with industry practitioners. In this study, we consider two main groups of risk sources which are internal and external ones.

One other aspect which can be considered here is that there are two main approaches in the literature to evaluate risk issues: the first one is using historical data (Uher and Toakley (1999)) or conducting surveys through expert for gathering their idea and data in their businesses (Tam *et al*. (2004)). This approach is called data-driven analysis.

In the second groups of research (like this study), authors use a model to analyze the impact of various sources of risk associated with construction management. This approach is called model-driven analysis.

CHAPTER THREE
Methodology

3.1. Problem Description

In this study, an objective function containing both cost and time (which are normalized to be comparable) used to do time-cost trade off analysis. Cobb-Douglas production function is applied to each activity regarding labor cost and equipment cost. Time and cost weights in the objective function show users' priorities and preference for either duration or cost. The main idea of this study extracted from Shen et al. (2012).

As stated earlier, we assume discrete time-cost relationship; so, for each activity option based on the amount of work needed, we obtained total cost, labor and capital cost.

3.2. Model

In this study, the amount of work needed for each activity is *W*. When an activity is crashed (the time is shortened from normal duration), production output rate Q is increased through the increased inputs of labor (L) and/or equipment (K).

Considering the production function of an activity as $Q = F(L, K)$, and normal time, t_0, then the production output rate to accomplish the activity is:

$$Q_0 = \frac{W}{t_0} \quad \text{(Eq. 2)},$$

where Q_0 is the normal production output rate; W is the total production output during t_0 duration.

During crashing when duration was shortened from t_0 to t_1, the corresponding output becomes: $Q_1 = \dfrac{W}{t_1}$ (Eq. 3),

where Q_1 is the crashed production output rate; t_1 is the crashed duration.

Therefore, to reduce the activity duration by $t_0 - t_1$, output rate is increased by $Q_1 - Q_0$ per time unit, due to the extra inputs of labor and/or equipment.

Also we assume that the technology impact (A) is constant and equal to 1 for simplicity without affecting the results and conclusions, since the thesis is concerned with the allocation of existing labor and equipment resources during crashing, not with introducing new technology into crashing. In most existing literature (Sircar and Choi 2007) technology changes were often measured (by changes of A) over long period of time, normally 5 to 20 years.

α and β are also known as the labor and capital's share of output which identify the contribution of labor and capital in the total production. Different combinations of α and β for different activities can be estimated by project managers based on the historical data, for example via regression method (Mateescu 2010). Although both CRTS and DRTS (for example, due to working space constraints) are possible scenarios, to simplify the discussion without compromising the main topic, we limit our discursion in this thesis to a CRTS scenario, in which $\alpha + \beta = 1$ (Fig. 1) in the first section. Then, we do sensitivity analysis and consider DRTS in the last section.

In many existing production research (Felipe and Adams 2005), $\alpha = 0.3$, $\beta = 0.7$ are often reported. Since the specific value of A, α and β are treated as case parameters and will not affect the proposed overall framework, so we assume $A = 1$, $\alpha = 0.3$, $\beta = 0.7$. Then we get:

$Q = L^{0.3}K^{0.7}$ (Eq. 4)

Let assume W = 10000 for this specific activity and normal time is t_0 = 25 days, then we have:

$$Q_0 = \frac{W}{t_0} = \frac{10000}{25} = 400 \quad \text{(Eq. 5)}$$

Accordingly, if the crashed duration t_1 and t_2 equal to 20 day and 16 days respectively, then we have:

$$Q_0 = \frac{10000}{20} = 500 \qquad \text{(Eq. 6)}$$

$$Q_0 = \frac{W}{t_0} = \frac{10000}{25} = 625 \quad \text{(Eq. 7)}$$

The cost function of an activity is defined as:

$$TC = c_1 L + c_2 K \qquad \text{(Eq. 8)}$$

Where *TC:* total cost of the activity; c_1: salary rate; *L* labor input quantity; c_2: equipment rental rate; *K:* equipment input quantity; labor cost *LC= c_1L;* equipment cost *KC= c_2K.*

Eq. 8 can be illustrated by the isocost lines shown in Fig.2, in which lines TC_1, TC_2 and TC_3 are cost lines. The tangent points (such as points A and B) between the isocost lines and the isoquant curves are the minimal cost of producing *Q*=3 (point A) and producing *Q*=6 (point B). The line connecting all the tangent points is referred as production expansion line, which represent the minimal cost solution to expand the production if the cost function (c_1 and c_2) hold the same.
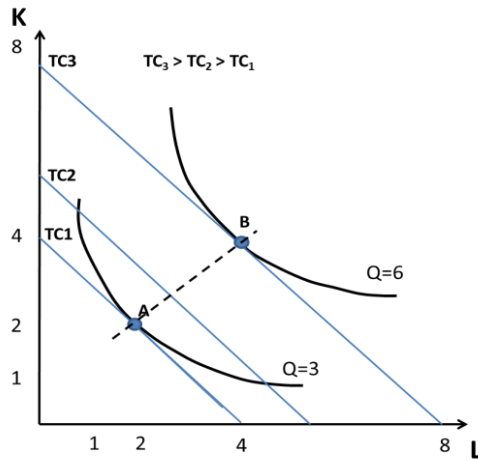
Fig. 2. The concept of isocost lines and the minimal cost of producing Q

If we assume labor cost is 30 dollars per labor unit, capital cost is 90 dollars per equipment unit, Eq. 8 will become:

$$TC = 30x_1 + 90x_2 \qquad \text{(Eq. 9)},$$

where x_1 and x_2 represent L and K respectively.

Using Q_0, Q_1 and Q_2 to derive isoquant curves and Eq. 9 to draw isocost lines, we can find the minimal total cost (TC_{min}) for Q_0, Q_1 and Q_2, in which Q_0 represent production rate in a normal duration, and Q_1 and Q_2 represent production in the crashed durations (Fig. 3).
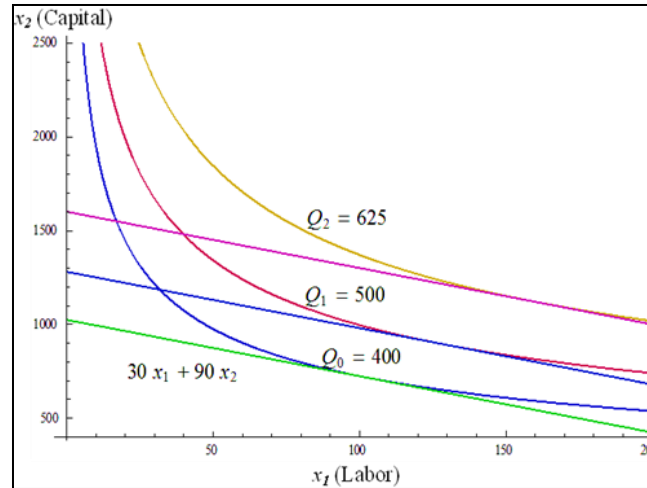
Fig. 3. A simple example of finding minimal activity cost using isocosts and isoquants

The results are: while $t_i = \{25, 20, 16\}$; $TC_{i\ min} = \{848, 1062, 1314\}$ and LC_i, $KC_i = \{(477, 371); (598, 464); (739, 575)\}$. The results are graphically shown in Fig. 3, where straight lines represents isocosts from Eq. 9 and curve lines represent isoquants derived from CDPF with 3 different Q values. The TC_{min} for each duration (or Q) is the tangent point of each isoquant curves.

Considering the total crashing cost for all activities in a network, the mathematical model to find combination of labor and equipment factors minimizing the total cost for all involved activities is represented as:

$Min \quad c_{1i} * L_i + c_{2i} * K_i$

$S.t.$

$\quad Q_i = L_i^{\alpha} * K_i^{\beta} \quad \forall i \in S \qquad$ (Eq. 10)

In order to present a multi-objective function which contains cost and duration we define the objective function as:

$\quad Z_i = w_1 C_i + w_2 T_i \qquad$ (Eq. 11)

Where w_1 and w_2 are defined based on the decision makers' preferences and $w_1 + w_2 = 1$. C_i and T_i are normalized scores of cost and duration respectively, and both have values from 0 to 1. C_i and T_i are defined as:

$$C_i = \frac{\max Cost - C_{Solution}}{\max Cost - \min Cost} \quad \text{(Eq. 12)}$$

$$T_i = \frac{\max Time - T_{Solution}}{\max Time - \min Time} \quad \text{(Eq. 13)}$$

So the best score for either cost or duration is 1, when the solution is the minimal. And the worst score is 0, when the solution is maximal. The larger Z values represent the better overall solutions.

CHAPTER FOUR

Hybrid Genetic Algorithm

Time-cost trade off like lots of other real world problems (travelling salesman, facility location, and etc.) is a NP-hard problem. So a Hybrid Genetic algorithm is applied and explained in this section. It is expected that the evolutionary algorithm will improve the efficiency in searching optimal solutions especially for large-scale time-cost trade off problems.

4.1. GA Background

Genetic algorithm (GA) was developed by John Holland (1975). GA is a population based searching technique. Its main idea came from natural evolution. There are various hard optimization problems such as Travel Salesman Problem (TSP), job shop scheduling, covering, that can be solved by genetic algorithm. GA, like other meta-heuristic algorithms such as Tabu Search (Glover 1989), searches the whole space containing two conflicting operation exploring the whole space while trying to improve the quality of current solution in its neighborhood through finding the local optimum.

Genetic algorithm contains two main operations: crossover and mutation. In the crossover phase, GA produces new offspring from two parents which are chosen from the population. The second operation in GA, which helps GA to search the all search space and not just the local optimums, is mutation. GA like other artificial intelligence

algorithms avoids trapping in local optimum solutions through its operation called mutation which will be explained later. The new generations are compared with the existing solutions; they may be replaced if they have a better fitness value.

4.2. HGA

One of the main problems associated with GA is the initial solution in the population. First is related to the population size which is usually determined by trial and error. According to Golberg (1989), the population size is normally between 30 and 500. The other issue is the quality of chromosomes or initial solutions generated in population (which are usually randomly generated).

Hybrid genetic algorithms (HGA) (El-Mihoub et al. 2006) was proposed to overcome limitations of most meta-heuristic algorithms by adding local searches, adding learning methods, etc., (Revees 1994, Thierens et al. 1998) to make it more efficient. In this study, a well-known local search called 2-opt is used to overcome the mentioned problems. The 2-opt local search is expected to improve the randomly generated population solutions. 2-opt was first introduced in Croes (1958) as a local search for traveling salesman problem. Later, it is modified and used in other operation research applications (McGovern and Gupta 2003, Buffa et al. 1964) as an effective way to address the limitations of pure GA.

The main reason to use 2-opt is that and hybridizing the algorithm is to avoid the generation problems. 2-opt procedure improve the initial solutions.

HGA details are illustrated in Fig. 4. In the initialization phase, algorithm starts with randomly generated chromosomes based on population size (as an input).
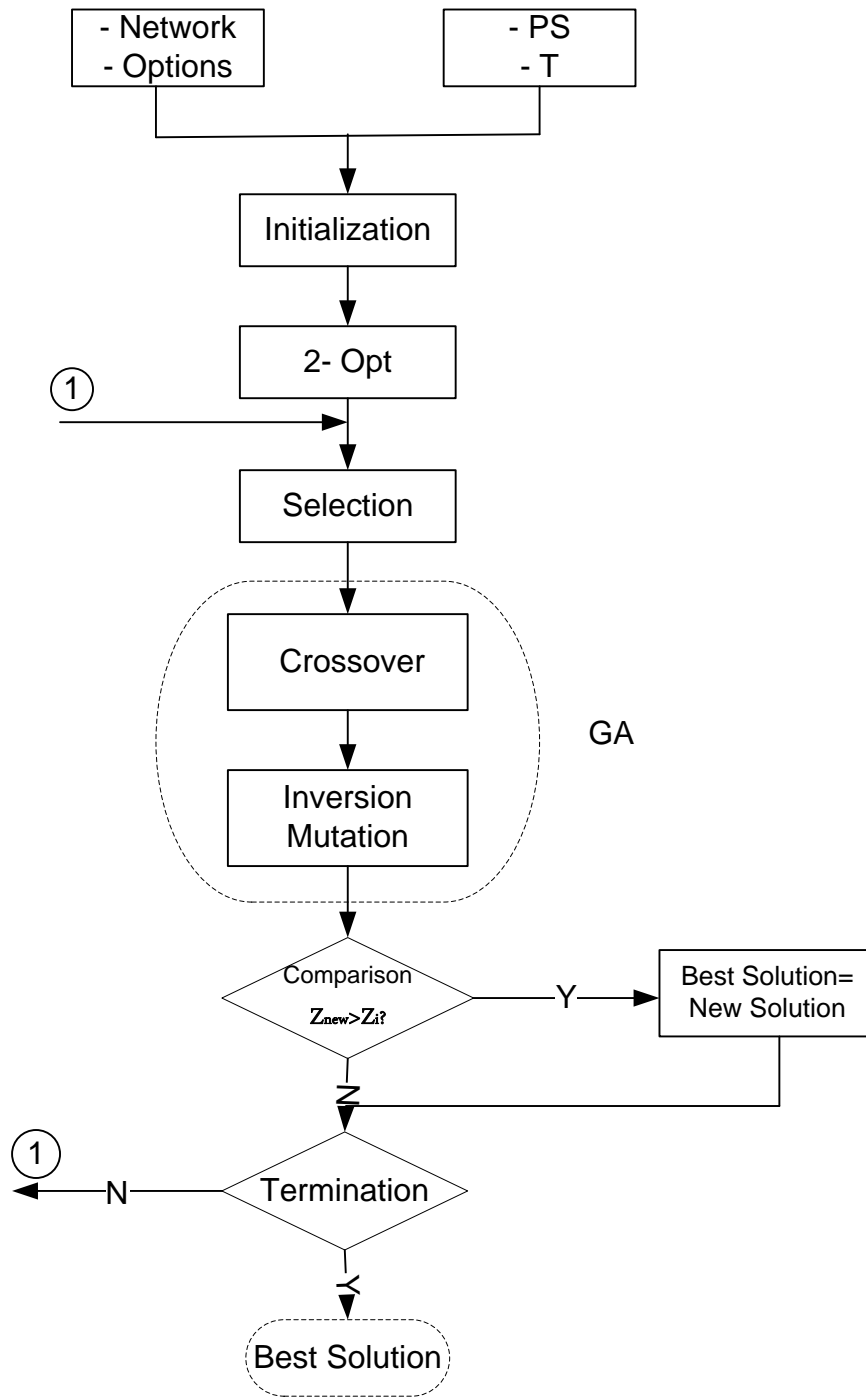
Fig. 4. The flowchart of HGA

4.2.1 Notations

There are some notations that we need to define before developing a genetic algorithm:

- PS: Population size (which is usually defined based on the problem size)

- T: Number of generation before termination

- Z_i: Objective value (fitness value) of the ith solution

PS and T are presented in Figure 4 as inputs of genetic algorithm. They are determined before the algorithm starts. Although, as discussed earlier, PS has an impact on the performance of genetic algorithm and may cause problems in case that it is defined improperly, it is determined mostly by trial and error. It is the same story for defining T as number of iteration must be implemented before the time that the algorithm is terminated. With try and error, it is defined based on trade-off between run time and the efficiency of solutions.

In the presented algorithm, PS is equal to 100, and T is assumed 300. Z_i which will be used in different phases of the algorithm is the objective value or fitness value of the ith solution. Z_i is the basis for comparing the solutions.

4.2.2 Initialization

Regarding the PS, we generate chromosomes. Number of genes in each chromosome is equal to the number of activities of the project. For each activity (gene), a random integer number is generated based on the number of activity's optiosn regarding the fact that we consider discrete form of time-cost function in this study. For example, if each activity has three options, the generated number could be 1, 2, or 3. The same process is

done for all activities in a chromosome. The whole process repeated PS times (number of chromosomes in the initial generation). So now, we have PS (population size) randomly generated chromosomes.

4.2.3 Procedure of 2-opt

Here, for the second phase in the proposed HGA which works as an improvement phase for the initial solutions that are generated in the population, we develop a 2-opt procedure which generates all possible combination of two randomly selected genes and their neighborhoods. That is, for each chromosome (initial solution), it generates 5 offspring. In the first one, it just changes the positions of selected genes. After then, all possible swaps are checked. Although it increases the run time, it avoids the problems which are caused by generating random population. Figure 5 shows the details of how 2-opt works. To better understanding of this figure, it should be noted that each string presents a solution. Each column presents an activity option. For example, in this specific figure, it is assumed that there are 13 activities. So each row (string), shows assigned option to the activities.

Fig. 5. 2-opt algorithm

As illustrated in the above figure, first two activities are selected randomly, and their assigned options are changed. Then for each of those two, the same procedure is done with its neighbors (previous and next activity).

Then, the fitness value of parent and offspring (all together would be at most 6 solutions) are compared and the best one will be replaced by the current solution (parent).

4.2.4 Selection

While in most articles in the literature, authors propose an algorithm which randomly selects the chromosomes from the population to do the GA operations on them (like Zheng et al. 2004), chromosome selection from the population is based on their objective value (fitness value) in this study. Roulette wheel selection (Goldberg, 1989) is applied here to come up with this drawback in the literature. First, it avoids that not only the best chromosome in the population selected. Second, it is not selected randomly. That is, the chromosome (solution) selection is proportional to their objective values. So, the better a chromosome is regarding its objective value, the more probability has to choose. The best two chromosomes which is selected in this phase, are transformed based on GA operators in the next steps to be improved.

The roulette wheel selection method is summarized in the following steps:

Step 1: Calculate the total fitness of all chromosomes in the population:

$$S = \sum_{i=1}^{PS} Z_i \qquad \text{(Eq. 14)}$$

Step 2: Calculate the selection probability for each chromosome, which is proportional to the fitness value of that chromosome to total fitness of all chromosomes in the population:

$$p_i = \frac{z_i}{s} \qquad \text{(Eq. 15)}$$

Step 3: It should be noted that

$$\sum_{i=1}^{PS} p_i = 1 \qquad \text{(Eq. 16)}$$

So, the summation of selection probabilities from the first chromosome to the ith chromosome is called cumulative probability. Calculate the cumulative probability for each chromosome:

$$q_i = \sum_{j=1}^{i} p_j \qquad \text{(Eq. 17)}$$

Step 4: generate $0 < r \leq 1$

Step 5: chromosome i^{th} will be selected if $q_{i-1} < r \leq q_i$. That is the difference between q_{i-1} and q_i is exactly the same as p_i.

$$q_i - q_{i-1} = p_i \qquad \text{(Eq. 18)}$$

4.2.5 The Crossover

The crossover was introduced by Gen and Cheng (1997). In the first step, two parents from the population are randomly chosen, and then the required transformations

are done on them to have new offspring. According to the fact that, each chromosome (solution) contains specific options for each activity, after choosing a substring in order to do crossover, the genes are exchanged to have new offspring. The main purpose of crossover is to do local search to find a better solution in parents' neighborhood. Crossover needs two parents (solutions) to do its procedure.

The procedure to generate offspring from parents is as follows:

Step 1: Select a substring from the first parent randomly.

Step 2: Produce an offspring by copying the substring into the corresponding positions in that.

Step 3: Place the genes other than the chosen substring from the first parent, into the unfilled positions of the offspring from left to right.

Step 4: Repeat Steps 1–3 to produce another offspring by exchanging the two parents.

Afterward, Z_is are compared to choose the better one between each offspring and the related parent. This process is shown in Fig. 6.

Fig. 6. The Crossover

4.2.6 The Inversion Mutation

There are various procedures are introduced for mutation in different research areas; one of those is inversion mutation. The inversion mutation firstly introduced by Gen and Cheng (1997). In this phase, a substring is randomly selected, and all included genes are flipping. As stated earlier, mutation operation is developed to exploit all over the search space to find the global optimum instead of looking for local optimum in the neighborhood of the current solution. Inversion mutation is used to diversify the solution in the solution space, which should be done basically by a mutation operation. The inversion mutation procedure is shown in figure 7.
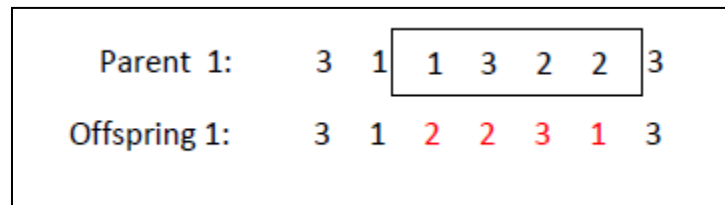

Fig. 7. The Inversion Mutation

This procedure starts with choosing a substring (set of activities' options) from the parent (solution). The selected genes (activity options) are flipping. Regarding this procedure, the solution is diversified. In doing so, we expect the GA searches the whole space. Indeed it avoids GA of trapping in a local optimum.

CHAPTER FIVE

Case Analysis

A case study is presented in this section to demonstrate the application context of the proposed approach. The network has been chosen from (Liu et al. 1995) and is illustrated in Fig. 8.
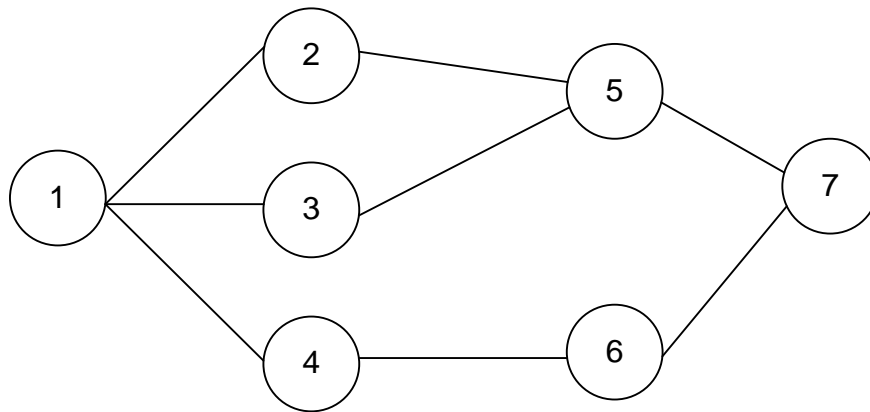

Fig. 8. The project network (Liu et al. 1995)

We adopted the duration options and the activity network from Liu's paper (Liu et al. 1995) for each activity. But we assigned estimated workload (W) to each activity in order to use CDPF to better analyze the best allocations of labor and equipment. We also identified typical unit costs for labor (C_L) and equipment (C_K) for each activity from RS Means Construction Cost Data Book 2009. After knowing the W for each activity, associated Q of each of the three options are obtained using Eq. 2 and Eq. 3 as described in Section 3 of the thesis.

Then, L and K (the amount of labor and equipment inputs respectively) are determined according to the cost minimization function constrained to Cobb-Douglas function, which is presented as Eq. 10 in Section 3. The assumptions are $\alpha = 0.3$, $\beta = 0.7$. The total labor cost for each activity (TLC) is obtained from $C_L \times L$; and the total equipment cost for each activity TKC is obtained from $C_K \times K$. Total cost (TC) is equal to the summation of TLC and TKC. All data used in this case are presented in the Table 2.

Table 2. The case project information

Activity	Option	W	T (day)	Q	CL ($)	CK ($)	L	K	TC ($)	TLC ($)	TKC($)
1	1	5000	14	357.1	40	100	374.8	349.8	49975.3	14992.6	34982.7
1	2	5000	20	250.0	40	100	262.4	244.9	34982.7	10494.8	24487.9
1	3	5000	24	208.3	40	100	218.6	204.1	29152.2	8745.7	20406.6
2	1	500	15	33.3	50	70	23.3	38.9	3885.4	1165.6	2719.8
2	2	500	18	27.8	50	70	19.4	32.4	3237.8	971.4	2266.5
2	3	500	20	25.0	50	70	17.5	29.1	2914.0	874.2	2039.8
3	1	600	15	40.0	45	80	33.1	43.4	4960.0	1488.0	3472.0
3	2	600	22	27.3	45	80	22.5	29.6	3381.8	1014.5	2367.3
3	3	600	33	18.2	45	80	15.0	19.7	2254.5	676.4	1578.2
4	1	6000	12	500.0	75	70	263.3	658.2	65819.1	19745.7	46073.4
4	2	6000	16	375.0	75	70	197.5	493.6	49364.3	14809.3	34555.0
4	3	6000	20	300.0	75	70	158.0	394.9	39491.5	11847.5	27644.1
5	1	4500	22	204.5	60	60	113.0	263.7	22606.6	6782.0	15824.6
5	2	4500	24	187.5	60	60	103.6	241.8	20722.8	6216.8	14505.9
5	3	4500	28	160.7	60	60	88.8	207.2	17762.4	5328.7	12433.7
6	1	5500	14	392.9	55	20	106.9	686.2	19604.7	5881.4	13723.3
6	2	5500	18	305.6	55	20	83.2	533.7	15248.1	4574.4	10673.7
6	3	5500	24	229.2	55	20	62.4	400.3	11436.1	3430.8	8005.2
7	1	4700	9	522.2	65	30	168.0	849.2	36392.3	10917.7	25474.6
7	2	4700	15	313.3	65	30	100.8	509.5	21835.4	6550.6	15284.8
7	3	4700	18	261.1	65	30	84.0	424.6	18196.1	5458.8	12737.3
Optimum Solution			83						128522.9	38556.9	89966.1

TC: Total Cost; TLC: Total Labor Cost; TKC: Total Capital Cost; T: duration options; Q: production quantity

5.1. Computational Results

HGA is applied to data which are presented in Table 2 to find the optimum/near optimum solution reflected by maximal Z value. Then the solutions are compared with the optimum solution which is obtained from checking all possible combinations of activities' options. The HGA has been run for 10 times for each case, and in each case, the result is the same as the optimum solution, which shows that it works properly.

Assuming the decision makers' priority (utility function) for time and cost is: , $w_1 = 0.6$, and $w_2 = 0.4$, then the optimum solution is shown in Table 3 and the selected option of each activity is boxed in Table 2.

Table 3. Optimal Solution Options

Activity	1	2	3	4	5	6	7
Option	3	1	1	3	3	3	2

The 83-day duration of the project is obtained from the critical path (using Fig. 8.) of the optimal solution options. Total cost of the project is obtained from the summation of total cost of the selected option of each activity in the optimal solution, which is equal $128,523 (see Table 2 for details of the selected option in each activity) including both labor ($38,556.9) and equipment ($89966.1) costs.

Objective value of this solution is calculated using Eq. 11, 12, and 13 as follows.

$$C = \frac{\max Cost - Cost\ Solution}{\max Cost - \min Cost} = \frac{203243.32 - 128523.41}{203243.33 - 121207.38} = \frac{74719.92}{82035.95}$$

$$= 0.9108$$

$$T = \frac{\max Time - Time\ Solution}{\max Time - \min Time} = \frac{103 - 83}{103 - 60} = \frac{20}{43} = 0.4651$$

Max and min of cost of the project obtained by assigning the options with max cost and min cost to all activities respectively. The same story is true for max and min time. As stated earlier,

$w_1 = 0.6$, and $w_2 = 0.4$

The objective value is calculated as:

$$Z_i = w_1 C_i + w_2 T_i = 0.6 \times 0.9108 + 0.4 \times 0.4651 = 0.7325$$

Fig.9. illustrates all possible solutions. In case that we do not consider utility function (priority) for time and cost, a Pareto set could be drawn like Fig. 9. For example, in this specific case, if we do not assume any specific preference for w_1 and w_2, then the solution varied on the dash line in Fig. 7. All the possible solutions on this line do not dominate the other ones; but when we assume some specific values for w_1 and w_2, we will have just one answer based on that specific w_1 and w_2 priorities. Pareto set optimality have been used extensively by authors in different aspects of construction project management. Aiyin et al. (2011) have used Pareto set for cash flow planning in construction project management. They develop a multi objective cash flow consists of cash balance and interest paid to have a Pareto set.
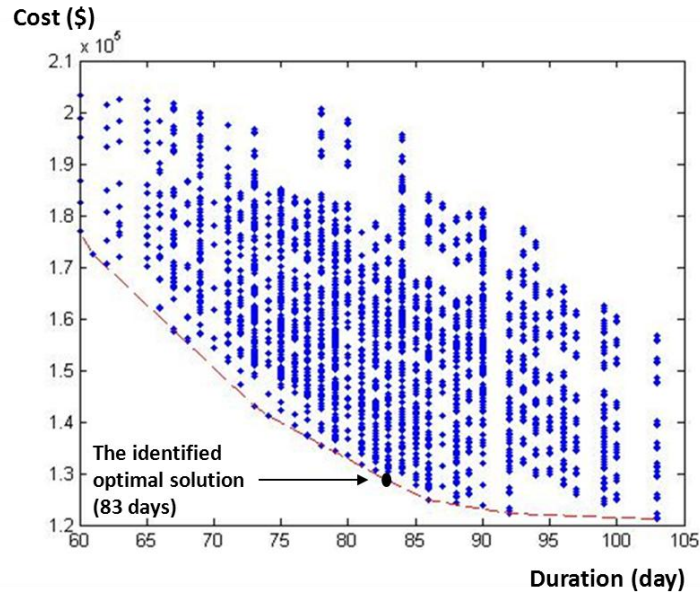
Fig. 9. All possible solutions.

Figure 10 depicts how changes of w_1 (and also w_2 since $w_2 = 1 - w_1$) affect the objective Z values. As expected either w_1 or w_2 becomes 1, Z value would be 1. The reason is that in that case, for example when w_1 is equal to 1, it means that managers just prefer cost rather than time. So, the algorithm chooses the solution with minimum cost and therefore maximum cost. Based on Eq. 12 and Eq. 13, C_i will be 1 and T_i will be 0. When the decision maker weights time and cost equally, Z value will be at the lowest point. So the optimal solution identified using HGA will depends on the decision maker's preference of time or cost.
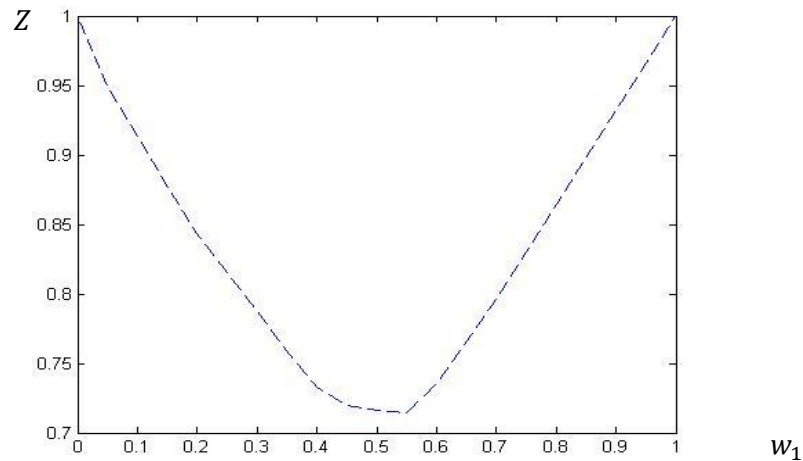
Fig. 10. Trend of Z considering w_1

With using the proposed approach, not only we obtain the optimal objective value, but also we understand what the respective allocations of labor and equipment resources are. This will provide us with much needed capacity in TCTP to better evaluate all the possible resource allocation scenarios.

5.2. Sensitivity Analysis

In this section, we do sensitivity analysis on $\alpha., \beta$, labor cost (c_L), and capital cost (c_K) The main reason is that in the real world, in most cases, we are not sure about the future prices of labor and capital (regarding Cobb-Douglas production function). Moreover, efficiency of the labor and capital is different from a project to the other one.

Sensitivity analysis is applied to be ensuring of risk associated with labor and equipment costs and efficiencies. That is to say, in the real world construction projects, we cannot guarantee a specific value for the summation of labor and equipment

efficiencies ($\alpha + \beta$). The same story is true about the labor and equipment costs (c_L, c_K). So, in the planning phase, managers or decision makers to avoid the associated risks which are arisen from uncertainties in determining labor and equipment cost and efficiencies, it is reasonable to analyze different scenarios for those.

To do so, sensitivity analysis is conducted here, to have some ranges for cost and duration instead of just one value with their respected probability. So, the conditions of the project and its relation to these factors are more clarified for managers or decision makers to make decisions. In doing so, managers or decision makers can avoid associated risks with construction management, which have roots in these parameters uncertainty.

Construction managers cope with different kind of risks associated with all phases of a construction project such as planning, scheduling, design, and etc., which have impact on time, cost, quality, and environment (PMI, 2004). Many authors in the literature have investigated various approaches to analyze risk, identify the possibility of occurrence, impact of its disruption and etc (Uher and Toakley (1999)). Risk management is defined properly by Uher (2003) as: "a systematic way of looking at areas of risk and consciously determining how each should be treated. It is a management tool that aims at identifying sources of risk and uncertainty, determining their impact, and developing appropriate management responses".

Risk analysis has been investigated in many other research areas like supply chain management (Tuncel and Alpan 2010), manufacturing systems (Choi and Chiu 2012), and etc.

A significant portion of the risks is associated with uncertainty of labor and/or equipment productivities and costs. For example fluctuations in both labor and equipment

productivities can be affected by many dynamic factors, such as weather, construction site conditions, the working spaces, crew compositions. In addition, market fluctuations in labor and equipment costs also poses great risks for project cost control. Although construction material cost fluctuation can also be significant sometimes, this paper is focused only on risk analysis associated with labor and equipment productivity and cost factors. Since productivity factors of labor and equipment can be controlled to some extend internally, they are considered internal risk factor in the paper. And by the same token, since the labor and equipment cost rates are determined by the market they are considered as external factors.

For long time there has been a lack of systematic approach to evaluate the construction project risks associated with productivity factors due to the lack of appropriate quantitative models to model the relationships between the production rate and the efficiency of labor and equipment inputs. Since the external risks are also tied to the internal risks there is a need to identify proper modeling tools to be able to evaluate both internal and external risks in related to the risks of project cost control.

The authors believe that Cobb-Douglas production function which has been widely used in economics and manufacturing can be utilized to perform quantitative evaluations of the cost control risks both internally and externally. In this study, we focus on the two main elements of Cobb-Douglas function, labor and equipment, and their relationships with the production rates through two different ways: 1) labor and equipment elasticity, and 2) market cost of labor and equipment. Long term construction projects are more susceptible to external market cost risk sources.

So according to the Cobb-Douglas production, which is the main approach of this study, time and cost of the project can be varied in two ways considering the impact of labor and equipment factors:

1) In the first category, we analyze the impact of the labor and equipment efficiency. In reality, we are not sure about the labor and equipment efficiency which is presented as elasticity in Cobb-Douglas production function. This means various internal sources may affect the workers or equipment efficiency. Hiring unskillful workers or other equipment that are not that much compatible with the specific project can decrease the efficiency. For example, we may use new and sometimes more technical equipment which the labors were not taught to use. In the literature, authors like Mateescu (2010) did regression analysis to have a forecast of what are labor and equipment efficiencies (α, β). So here, we do sensitivity analysis for better understating how they impact the duration and total cost. We consider that the summation of α, β DRTS which is more common in reality. So, we will be aware of how would be the allocation of labor and equipment when they are varied.

2) The other scenario that can be assumed for the sensitivity analysis is the impact of labor and equipment costs. One of the main issues that construction project managers are coped with is cost inflation from the planning date to the date that start constructing. In the literature it is known as *market influence*.

To effectively manage overall project cost, managers or decision makers must continuously evaluate changes in market conditions in relation to cost and time impacts against the project baseline scope, cost, and schedule.

These price variations generally were attributable to global energy prices, primarily petroleum. Rising fuel prices spurred by a strong global economy, particularly in asia, raised petroleum prices. That in turn increased the cost of diesel which is an essential commodity for all construction projects, from the quarrying of aggregates, to excavation, to the delivery of materials to a construction site.

The price sensitivity of the international markets illustrates that global events pose risks to the ability of agencies in achieving their management objectives.

This is more plausible in long term projects specifically. There may be some other government rules, environmental issues, economical ones and etc. which cause problems or delay in construction project management.

For example, government and political rules can cause changes in the labor and equipment prices. One of the main issues that affect prices in all over the world is the oil price which highly affects the equipment cost. All together may affect the budget. To be aware of how they affect the total budget sensitivity analysis on their price changes may help us. In this case, we assume an inflation rate for both labor and equipment costs. So, in planning phase, managers can get an insight from this kind of analysis that how their vestment can be varied. And also, it helps them to have a better insight of how much budget needed for that specific project.

Construction projects are mostly long term projects, so there are different sources of uncertainty which affect the decision of managers or decision makers. Indeed, how much

a project takes longer to be done; it is more susceptible to be affected by market sources of uncertainty.

We use Monte-Carlo simulation in order to better understanding the sensitivity of objective value, total budget needed to do the project, and the optimum duration considering stochastic labor and equipment cost and elasticity. Simulation works based on a model which tries to maximize the objective value (Equation 11). Indeed, there are two main approaches in the literature. Most authors analyze the risk associated with construction project management based on the probability of occurrence and the cost of occurrence (Zavadskas et al. 2008). using historical data or in some cases they use expert recognition of a system. So they approach is called data driven analysis. In some other cases, based on a model, authors try to analyze the systems which are called model-driven analysis (this research).

5.2.1. Simulation

One of the main tools to analyze uncertainty is simulation. According to the fact that we assume there is an uncertainty in either elasticity or cost of the labor and equipment, we applied Monte-Carlo Simulation to analyze the uncertainty.

Simulation models, also called Monte Carlo methods, are computerized probabilistic calculations that use random number generators to draw samples from probability distributions. The objective of the simulation is to find the effect of multiple uncertainties on a value quantity of interest (such as the total project cost or project duration). Monte Carlo methods have many advantages.

They can determine risk effects for cost and schedule models that are too complex for common analytical methods.

In each iteration of the simulation, we encounter with a bunch of optimizations calculations. That is, for each activity based on the labor and equipment elasticity and cost, the optimum amount of labor and equipment is derived. Then based on the objective function (equation 11), the optimum combination of activity options is achieved. Then, the procedure starts to find the optimum solution of time and cost associated with the project considering that specific level of uncertainty.

In this stage Cobb-Douglas function will be applied for analyzing the sources of uncertainty in construction crashing cost. According to the equation 10, we encounter with this optimization model in making decision of the best options for activities.

$$Min \quad c_{1i} * L_i + c_{2i} * K_i$$

$$S.t.$$

$$Q_i = L_i{}^\alpha * K_i{}^\beta \quad \forall i \in S$$

As stated earlier, in the real world, there is always a source of uncertainty about the parameters in this model which have impact on the decisions. As it is shown in the above formulas, which is compatible with reality, there is an uncertainty about α, β, c_L, and c_K.

Sensitivity analysis will be conducted through Monte-Carlo Simulation on different scenario for these parameters. Now, it is necessary to define reasonable ranges for each of these to do simulation.

First, to analyze the internal sources of uncertainty which cause labor and equipment become less efficient, stochastic α and β are considered. In most cases in the real world summation of α and β is decrease return to scale (DRTS). That is, the double inputs of L

and K will generate less than double output of Q. Rarely, this happens that this summation is 1, which is assumed in case analysis. In this study, based on experts point of view, this summation is assumed between 0.6 and 1. Moreover, each of these parameters (α and β) are can vary from a project to the other one. So, in this study, it is assumed that α have 20 to 40 percent of total $\alpha + \beta$. So, rest of that is β.

In addition, to analyze the model sensitivity to c_L, and c_K (market influence which impact the labor and capital cost), the model is studied in three different conditions:

- High inflation rate for labor cost (c_L) and low inflation rate for capital cost (c_K)

- Low inflation rate for labor cost (c_L) and high inflation rate for capital cost (c_K)

- High inflation rate for labor cost (c_L) and high inflation rate for capital cost (c_K)

Finally, both internal and external cost are considered together. In doing so, we analyze the three above mentioned conditions regarding DRTS efficiency for labor and capital.

In the following table, different scenarios which are assumed for sensitivity analysis are summarized:

Table 4. different scenarios for sensitivity analysis

	α	β	$\alpha + \beta$	c_L	c_K	internal	external
DRTS	20-40 %	60-80%	0.6-1	-	-	✓	
L: high inflation rate K: low inflation rate	-	-	-	5-15 %	3-5 %		✓
L: low inflation rate K: high inflation rate	-	-	-	3-5 %	5-15 %		✓
L: high inflation rate K: high inflation rate	-	-	-	5-15 %	5-15 %		✓
L: high inflation rate K: low inflation rate & DRTS	20-40 %	60-80%	0.6-1	5-15 %	3-5 %	✓	✓
L: low inflation rate	20-40 %	60-80%	0.6-1	3-5 %	5-15 %	✓	✓

K: high inflation rate & DRTS							
L: high inflation rate							
K: high inflation rate & DRTS	20-40 %	60-80%	0.6-1	5-15 %	5-15 %	✓	✓

5.2.2. Simulation Results

In this section, according to the problem definition, different scenarios assumed for sensitivity analysis. For each scenario, we use Monte Carlo simulation to analyze the budget needed regarding that specific scenario.

5.2.2.1. Decrease Return to Scale (DRTS) Efficiency

In the first case, we just consider internal sources that decrease the efficiency of labor and equipments associated with construction projects. The reason for this assumption is that in the real world mostly adding more labors and equipments do not exactly add that much value to the production rate (Q). So, it is reasonable to assume that $\alpha + \beta$ is DRTS. To do so, we assume uncertainty of $\alpha + \beta$. In this case, we assume that $0.6 < \alpha + \beta < 1$ which is more compatible with the real world. In fact, in most cases in the real world the production function is decreasing return to scale. For example, let assume a case that there is not enough space to do a task. If we add a labor to workers not only he may not add value to the work; but also, he may cause problems, and other feels they cannot work convenient. Moreover we assume that $20\% < \alpha < 40\%$ of the $\alpha + \beta$. The rest of that would be β. Figure 11 through 14 depicts total cost, labor cost, capital cost, and duration

respectively. As expected, labor and capital costs have the same trend with the total cost although they are in different ranges. They construct the total cost together, so the model allocate
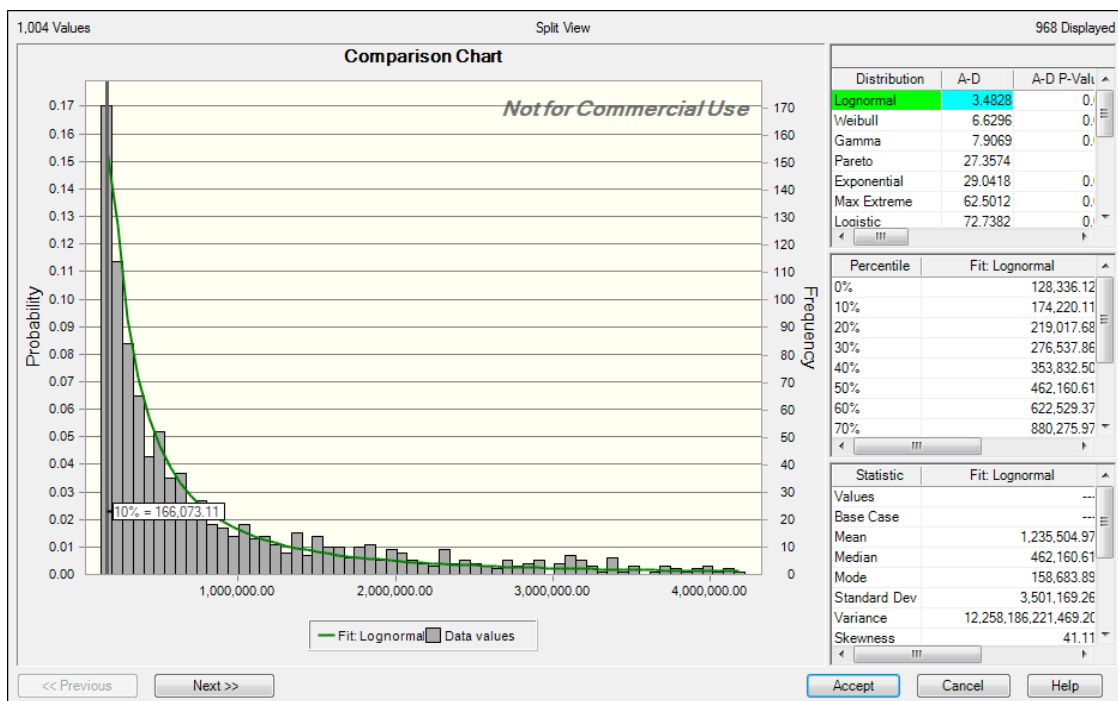
Fig. 11. Total Cost (DRTS Efficiency)

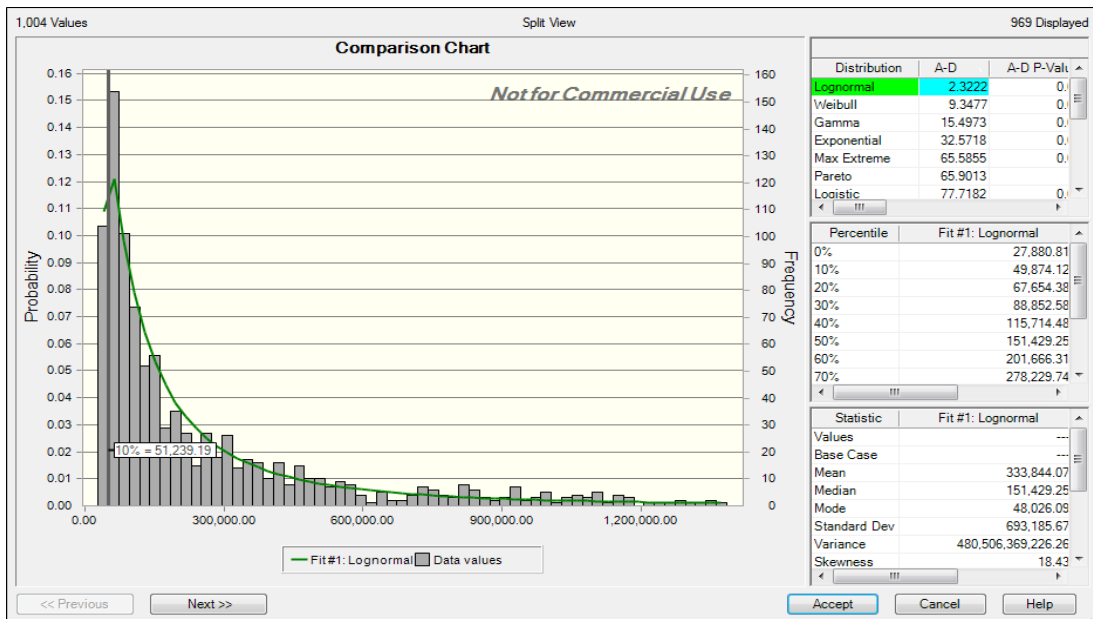**Fig. 12.** Labor Cost (DRTS Efficiency)
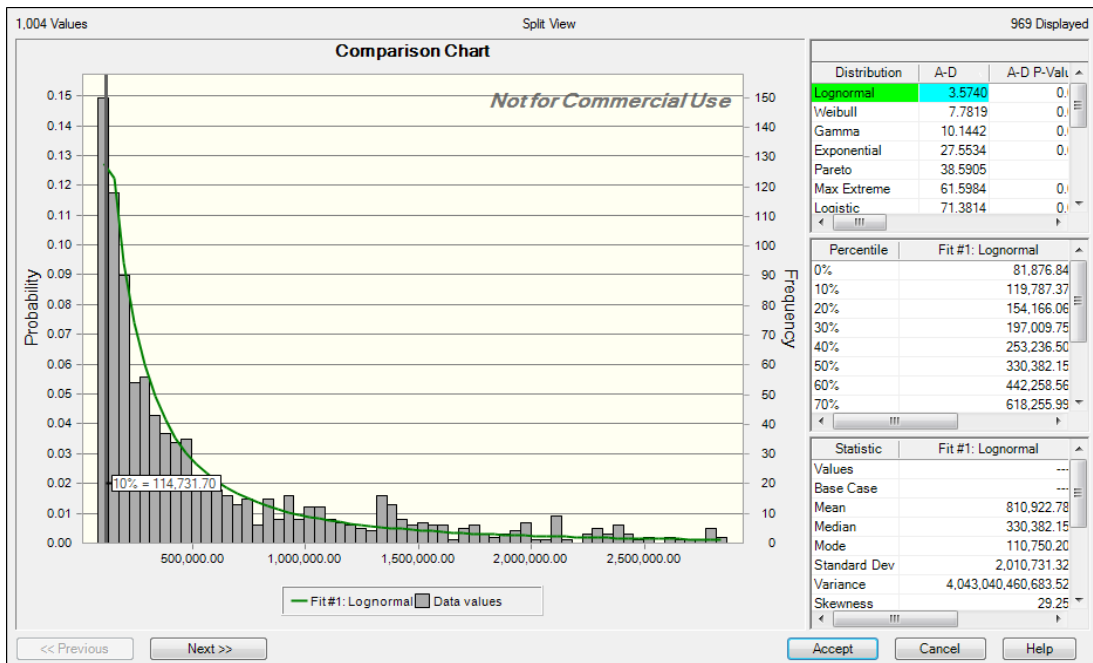

Fig. 13. Capital Cost (DRTS Efficiency)

Fig. 14. Duration (DRTS Efficiency)
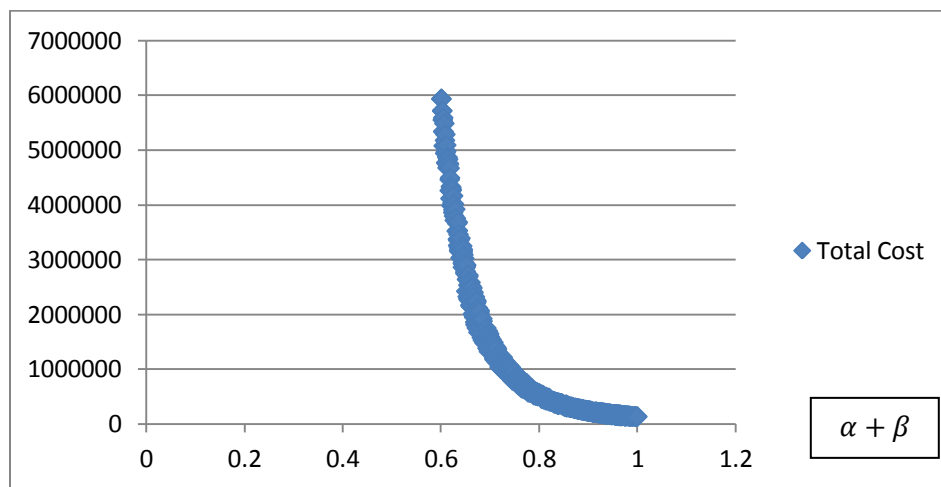
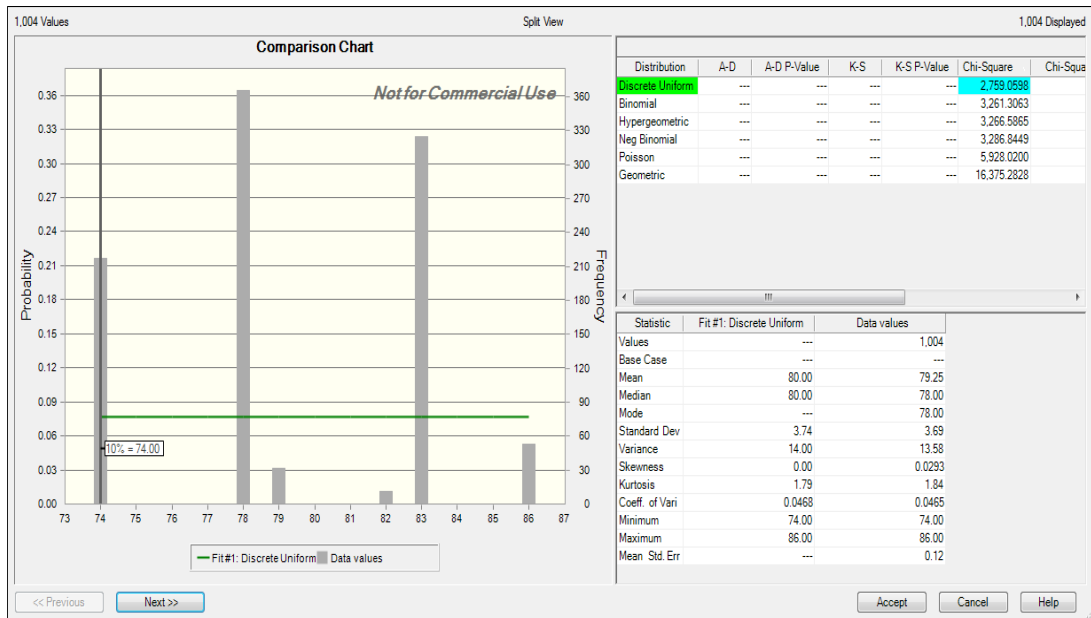**Fig. 15.** Total cost regarding $\alpha + \beta$

According to Figure 15, total cost is presented based on the summation of α and β. The result of the simulation confirms our expectation. As expected, as the summation decreased, the total cost increased; but the interesting point is that the total cost can be increased to 50 times more than the case that the summation is equal to 1. The reason is the specific conditions of Cobb-Douglas production function. That is, since elasticity factors are the power of labor and equipment, even a little change in them, can cause a huge difference to the total production value and so the cost.
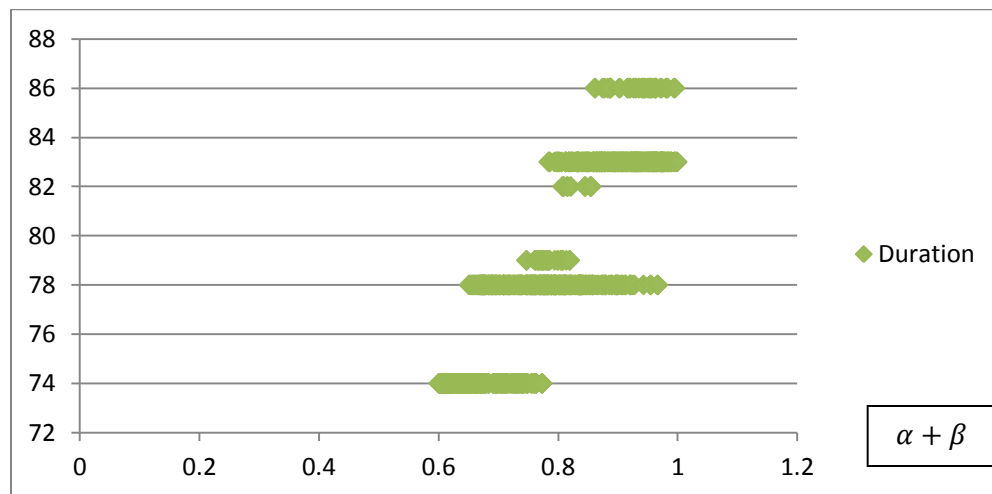

Fig. 16. Duration regarding $\alpha + \beta$

As figure 16 shows, as the summation increased, it tends to increase the duration. It contains two main reasons, first, the specific priorities in this example. But the second and more important one is that increasing the summation causes the conditions be the same as without uncertainty, and it goes to 83 days.

It should be noted that the reason for overlapping in some cases is that other than $\alpha + \beta$, we assume that both α and β are varied in a range.

It may seem irrational in the first observation. Since according to the figure when $\alpha + \beta$ decreased, the duration also decreased. The hidden reason, which should not be ignored, is that we fixed w_1 and w_2 for this case, so in that range it prefers to do the project in shorter time according to our priorities. So here, we present 2 new figures which are depicted based on different w_1 and w_2. In this example, we assume

Table 5. Priorities based on $\alpha + \beta$

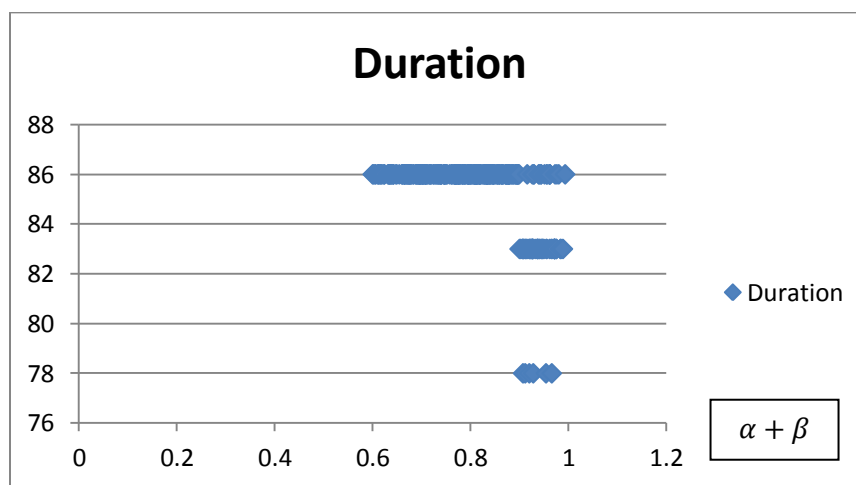| $\alpha + \beta$	w_1
0.6-0.7	0.9
0.7-0.8	0.8
0.8-0.9	0.7
0.9-1	0.6

So, we end up with Figure 17 and 18.

Fig. 17. Duration regarding $\alpha + \beta$ (with various priorities)
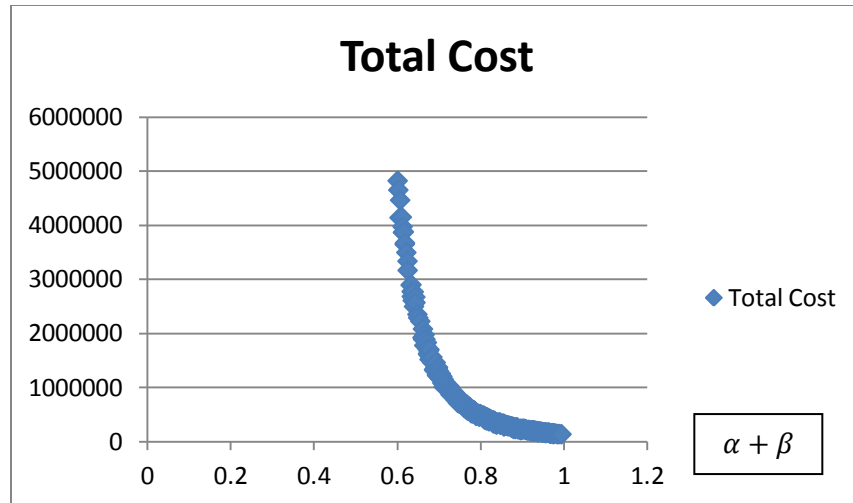
Fig. 18. Total Cost regarding $\alpha + \beta$ (with various priorities)

In this case, it is assumed that we have different priorities for time and cost (w$_1$ and w$_2$). So, as the results show, when $\alpha + \beta$ increases, duration decreases. The reason is that, now we prefer more on cost than time, so it increases the duration of the project to pay less money.

Moreover, Total Cost is also different from the previous results. In this case it ends up with $ 5M, while previously, it ends up with $ 6M.

5.2.2.2. Market Influence (Cost Inflation)

As stated earlier, market can influence the labor and equipment cost. We consider this uncertainty as an external source of uncertainty to do analysis. So, according to equation 10, C_L and C_K are considered as stochastic ones. We assume three different scenarios for this condition.

In the first case, based on expert point of view, we assume that there would be an inflation rate of 5 to 15% for labor cost and 3 to 5% for capital cost. For the second case,

it is vice-versa. And in the last scenario, we assume higher inflation rate for both of labor and capital, 5 to 15%. All of these scenarios are realistic, and in some economic conditions they may be applied.

One of the most interesting points that can be achieved from this section is that, regardless what the labor and capital cost is, the optimum duration based on the objective function (equation 11) is 83 days. Now, it may not be that much surprising according to previous section analysis. As stated, when the summation of α, β tends to be 1, the optimum duration becomes 83 days. So, in these cases, since this summation is equal to 1, the optimum duration would not be changed.

In the following table, results of different scenarios including total cost, labor and equipment cost are summarized.

Table 6. Cost categories based on different scenarios (External Risk)

Cost Scenario	Total Cost	Labor Cost	Capital Cost
L: high inflation rate K: low inflation rate	135866.7	40760.04	95106.66
L: low inflation rate K: high inflation rate	141197.97	42359.45	98838.53
L: high inflation rate K: high inflation rate	144591.8	43377.55	101214.02

As expected, total cost increased from scenario to scenario. The reason is that we assume more inflation from one to one.

It should be noted that from scenario1 to 2, total labor cost increased, while the inflation rate of labor cost in the second scenario is less than that of the first one. The reason is that

because of its higher price, it assigns less labor and more equipment, so the total labor cost is less than the other one.

It should be noted that although these analysis are not specific for the case, but the conditions affect the results. For example, if w_1 and w_2 are changed, duration would be a new one.

5.2.2.3. Considering Both Inflation rate and DRTS Efficiency

In this section, we again assume the same previous three scenarios; but this time $\alpha + \beta$ is again randomly distributed between 0.6 and 1.

In these cases, we consider both internal and external sources of uncertainties. That is, efficiency $(\alpha + \beta)$ is assumed DRTS which is more compatible with the real world, and also we consider inflation for both labor and equipment costs.

As illustrated in Figures 19 through 24, like the case that we just assume uncertainty in efficiency of labor and equipment (internal sources of uncertainties), duration varied. So it can be understood from these figures that when there is an internal source of uncertainty which causes DRTS efficiency, model chooses different duration based on the summation of efficiencies.
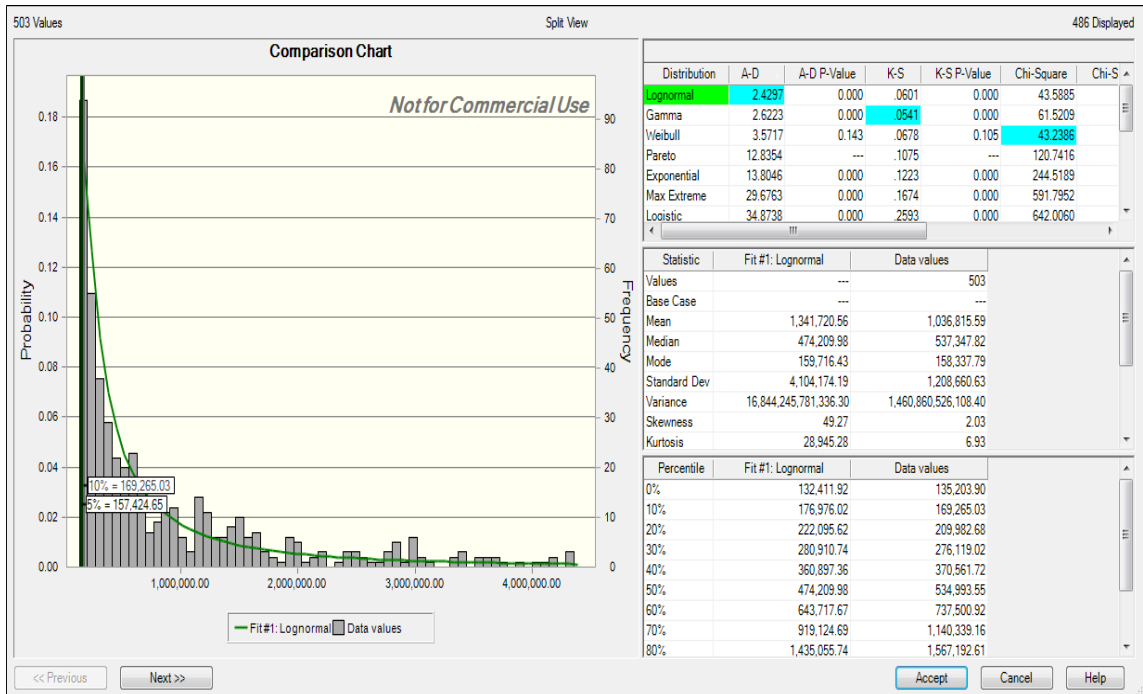
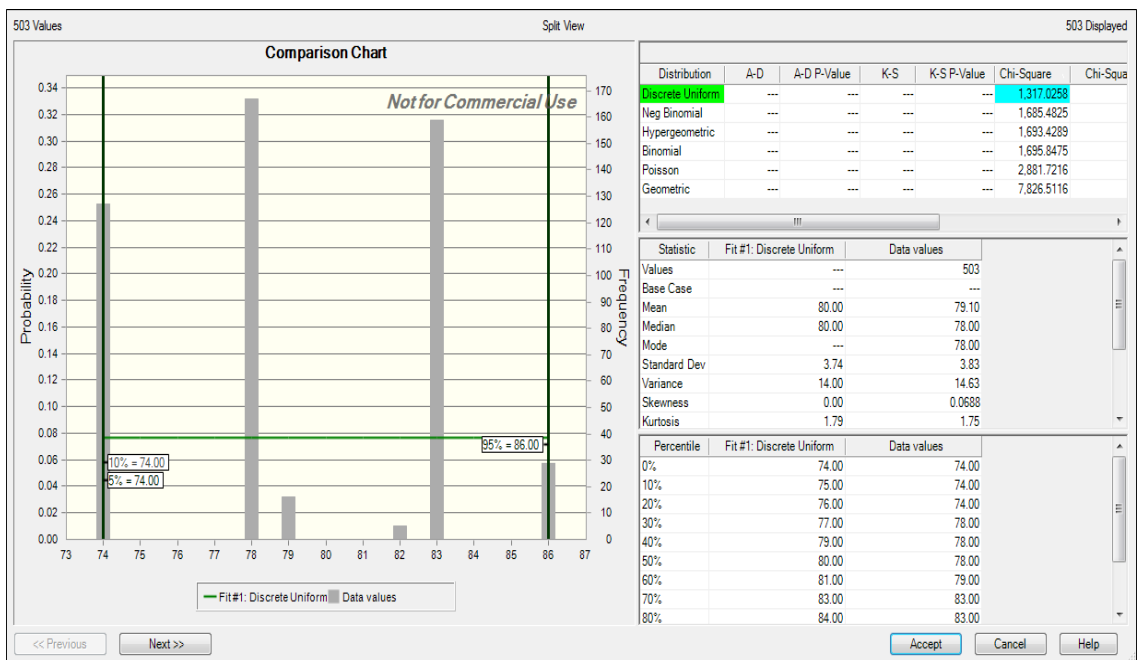**Fig. 19.** Total Cost (L: high inflation rate) (K: low inflation rate)

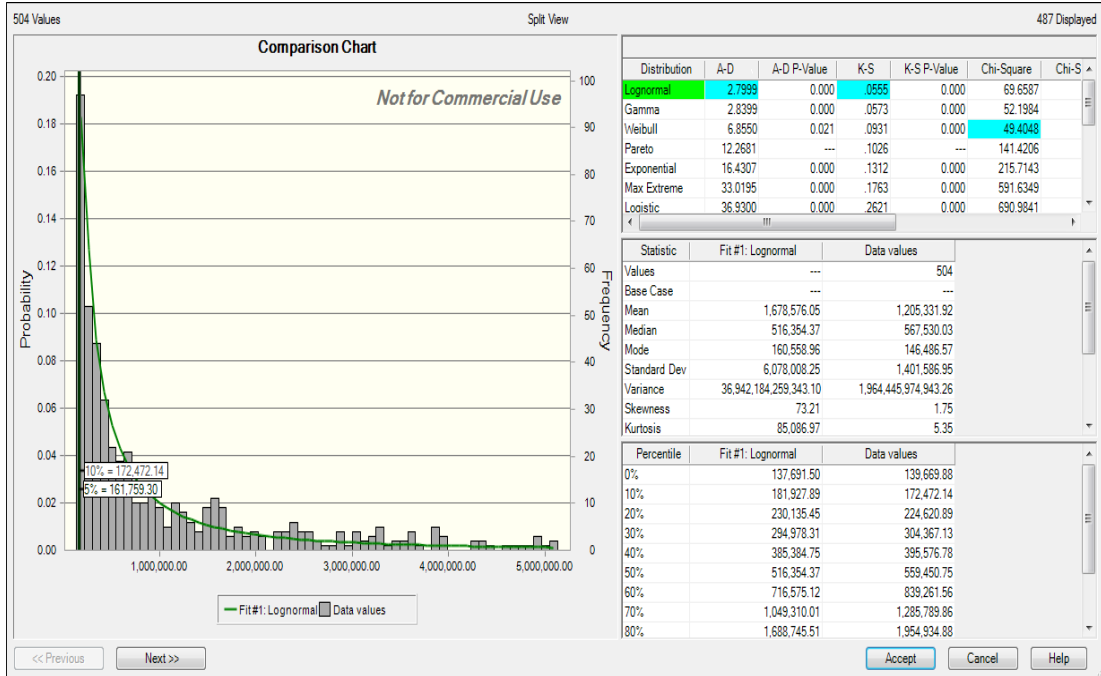**Fig. 20.** Duration (L: high inflation rate) (K: low inflation rate)

Fig. 21. Total Cost (L: low inflation rate) (K: high inflation rate)
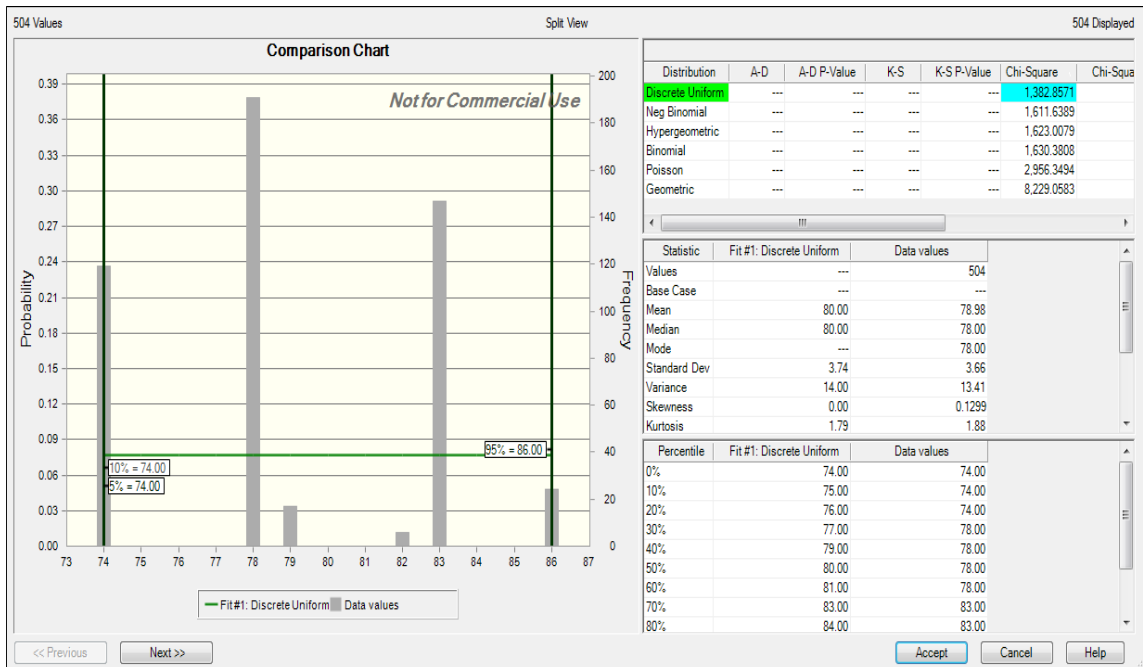

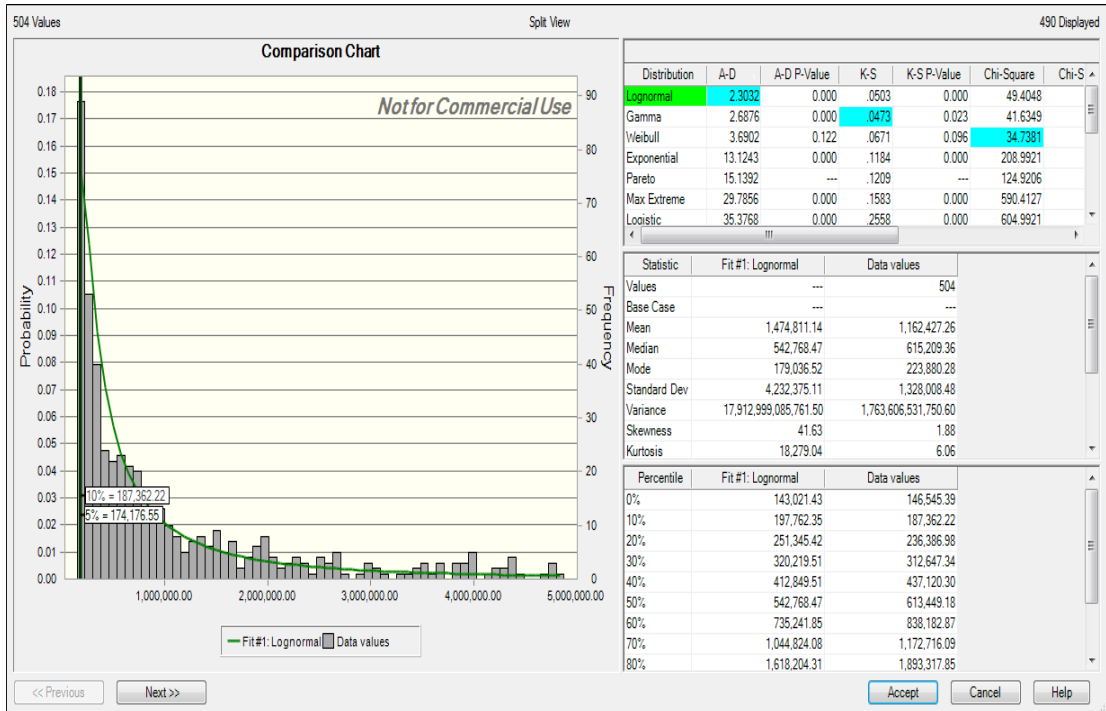**Fig. 22** Duration (L: low inflation rate) (K: high inflation rate)

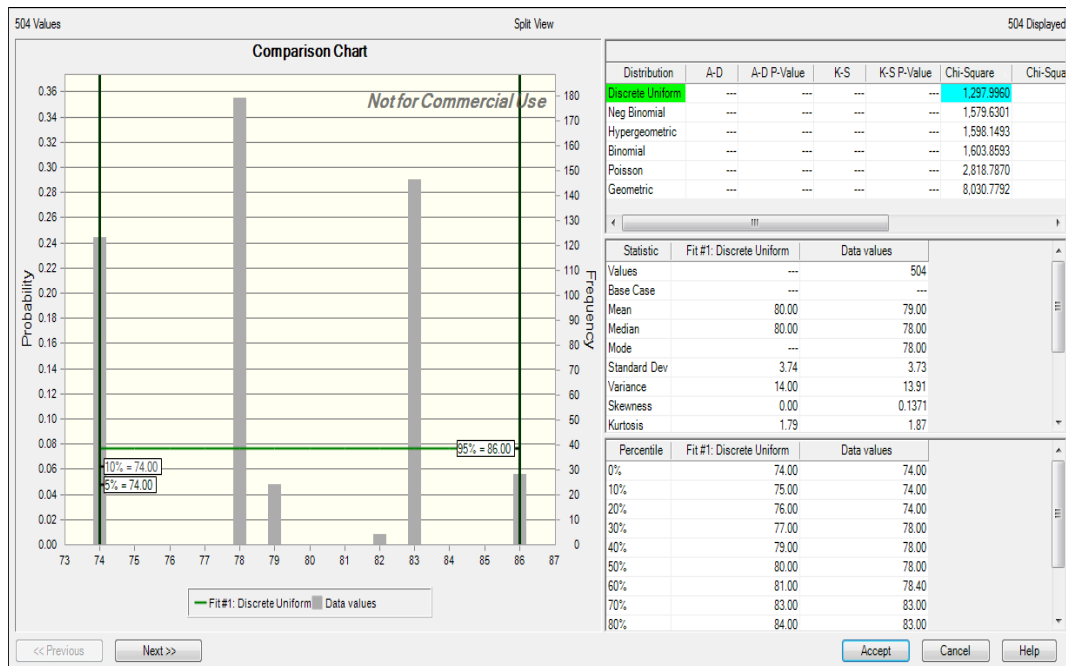**Fig. 23.** Total Cost (L: high inflation rate) (K: high inflation rate)

Fig. 24. Duration (L: high inflation rate) (K: high inflation rate)

According to Figures 25 through 30, as expected trend of the total cost and duration in all cases are the same as the cases when considering just one of internal or external risks. That is, duration tends to be 83 days when $\alpha + \beta$ goes to 1. Moreover, if the elasticity of labor and capital decreases significantly, the total cost arisen irrationally.

According to the fact there are thousands of points in each figure and it seems that the minimum of the total cost figures are 0, in order to avoid confusing of the figures of total cost in this section, table 7 summarizes the minimum and maximum costs of each scenario:

Table 7. Min & Max Cost considering both DRTS efficiency & inflation

Cost Scenario	Min Cost	Max Cost
L: high inflation rate K: low inflation rate	135203.9	6235722.3
L: low inflation rate K: high inflation rate	139669.879	6291923.88
L: high inflation rate K: high inflation rate	146545.386	6625296.95

According to table 6, both the min and max of total cost increase from scenario to scenario. It is clear that since there is more inflation in the third scenario (high inflation rate for both labor and capital cost), it has more cost than the others. Moreover, range of the second scenario is higher than the first one, since equipment usually and also in this case, cost more. So, inflation in its cost has more impact on the total cost.
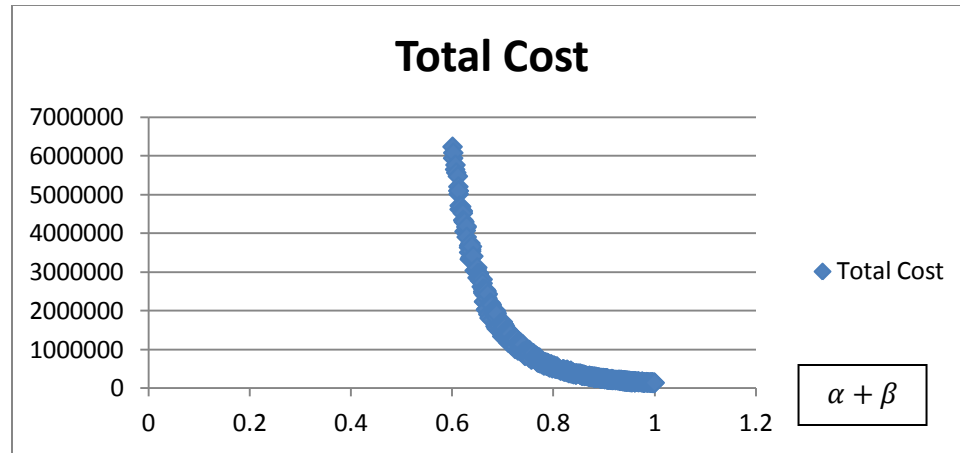
Fig. 25. Total Cost (L: high inflation rate) (K: low inflation rate)
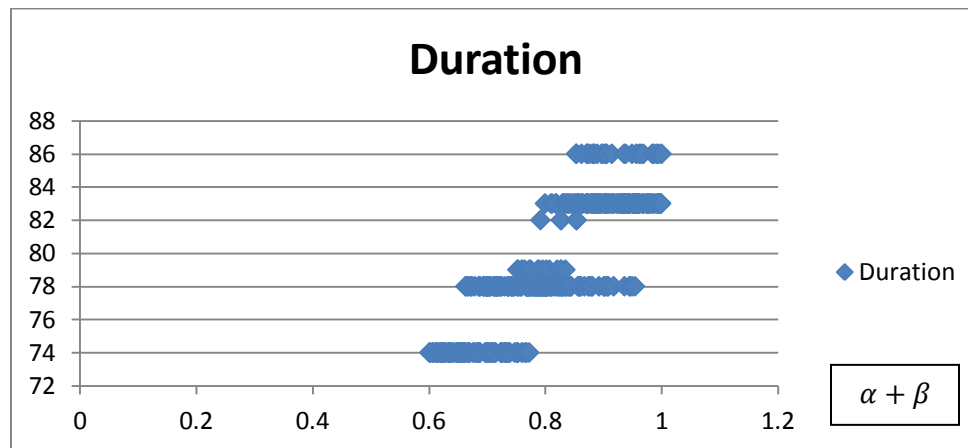

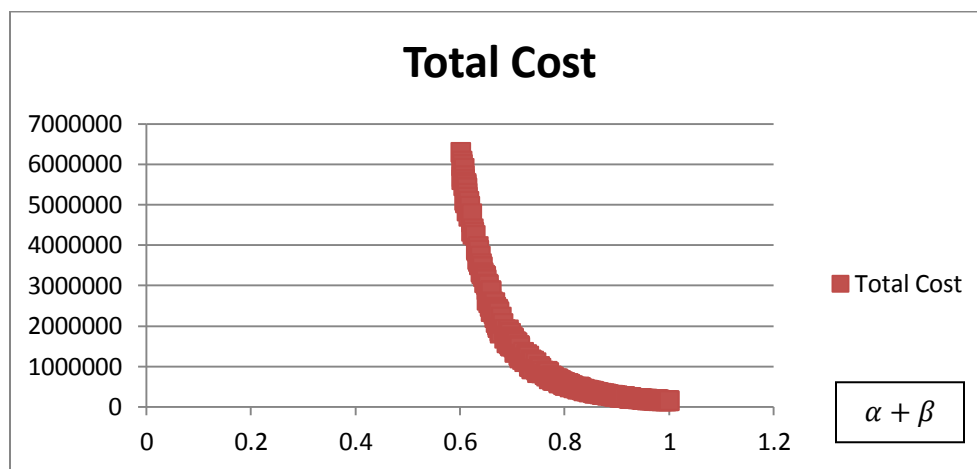**Fig. 26.** Duration (L: high inflation rate) (K: low inflation rate)

Fig. 27. Total Cost (L: low inflation rate) (K: high inflation rate)
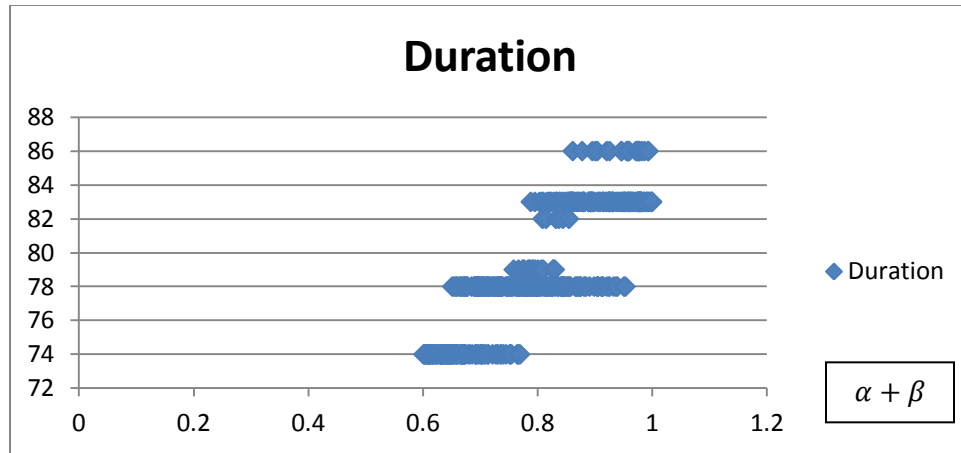
Fig. 28. Duration (L: low inflation rate) (K: high inflation rate)
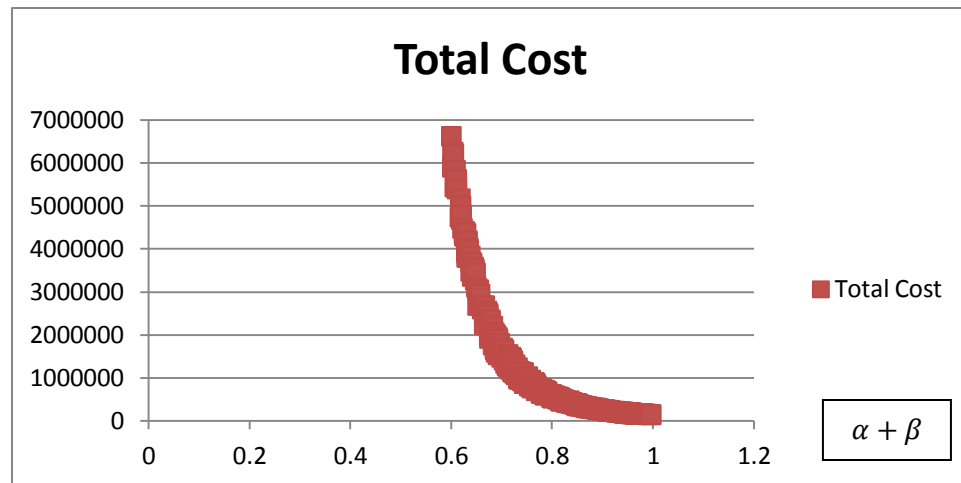

Fig. 29. Total Cost (L: high inflation rate) (K: high inflation rate)
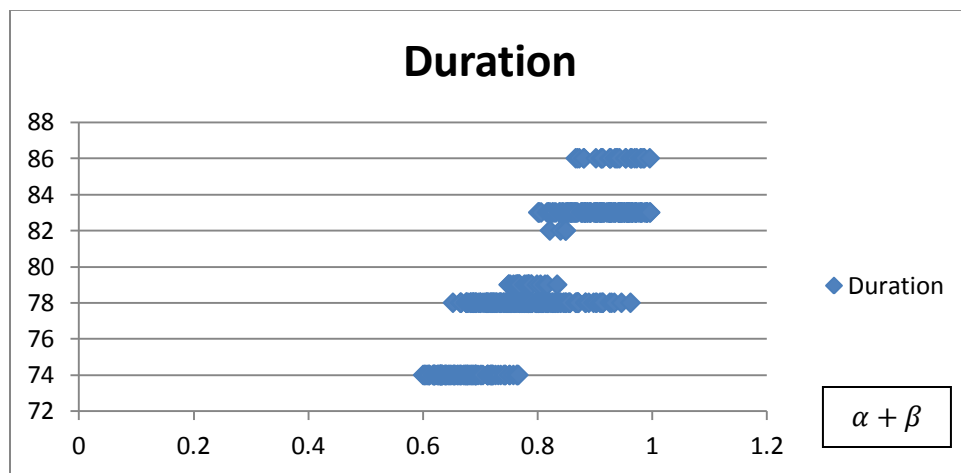
Fig. 30. Total Cost (L: high inflation rate) (K: high inflation rate)

According to Table 8, it should be noted that the average of the total cost is different from cases that consider just one of internal or external sources of uncertainty. That is, in these cases, because of considering DRTS efficiencies, the average cost jumps significantly and they are in $1M range. These data are summarized in the following table:

Table 8. Cost categories considering both DRTS efficiency & inflation

Cost / Scenario	Total Cost	Labor Cost	Capital Cost
L: high inflation rate K: low inflation rate	1036815.59	330478.6	706336.99
L: low inflation rate K: high inflation rate	1162427.26	367175.51	795251.74
L: high inflation rate K: high inflation rate	1205331.92	369867.98	835463. 93

Like the case that we just consider market cost inflation, from scenario 1 to 2, although the inflation rate of labor cost has been decreased, total labor cost increased. The reason is that because of higher cost of capital in this problem, model prefers to utilize more labor because in total, the total cost is less than that if utilize more equipment.

CHAPTER SIX
Conclusion & Future Direction

A framework using Cobb-Douglas production function (CDPF) to solve construction time-cost trade-off problem (TCTP) is proposed in this paper. Within the framework, a multi-objective optimization method utilizing Hybrid Genetic Algorithm is presented with a case application. The proposed algorithm tries to improve some deficiencies which were available in the previous studies such as improving the randomly generation of the initial solutions via 2-opt procedure, and using roulette wheel selection method.

Then, sensitivity analysis on budget and duration needed for completion of the project was conducted. A significant advantage of introducing CDPF into TCTP is that CDPF can be used to quantitatively explain the origin of the crashing costs from both labor and equipment perspective, which was a fundamental gap in previous research on TCTP.

The results suggested that, by tying CDPF to TCTP, the proposed approach is capable to identify optimal labor and equipment allocation solution effectively to satisfy the need for duration reduction. Indeed, more than the combination of total cost and duration of a project, now we can analyze what combination of labor and equipment could minimize the total cost.

Furthermore, sensitivity analysis on CDPF parameters demonstrates their impact on total cost and duration of a project. This will provide managers with an insight for budget planning considering different sources of uncertainty.

In this study, in three different categories, sensitivity of the model to the parameters are analyzed. First, only the internal sources of uncertainty are assumed. In doing so, it is assumed that the summation of α and β is DRTS (it is varied between 0.6 to 1). According to the fact that these parameters are have impact as power of labor and equipment regarding equation 1, they highly affect the total cost even to 50 times more than the case the summation of α and β is equal to 1.

In order to analyze the market influence (fluctuation in labor and equipment costs), three different scenarios, high inflation rate for labor cost and low inflation rate for capital cost, the vice versa condition, and high inflation rate for both of them are considered. In doing so, the total cost is influenced with this inflations, but duration stay remains.

Finally, with combining both internal and external sources of uncertainty, the case is analyzed. In this case, the total cost is much more than the previous case even the first case since more than DRTS efficiency, we consider an inflation rate for each of labor and equipment costs.

Although the presented case application is simple, the proposed approach is expected to work efficiently in larger and more complex applications.

In future study, more complex cases can be investigated. Moreover, capacity constraint on labor and equipment can be considered in the future. In this study, it is assumed that we can hire as many labors as we need. The same story is true for equipment, while, in reality, there is a constraint on number of labors or equipments. So, it is more realistic to assume capacity constraint on labor and equipment.

For future study, different elasticity for each activity can be assumed. That is, in this study, it is assumed that α, β are the same for all activities. While, it is more realistic to assume from activity to activity they can be varied. Experts can help decision makers for arranging specific values for α and β. Moreover, as stated earlier, using regression analysis, a range for each of those can be determined. In addition, the summation of α and β can be different from an activity to an activity.

In this study, like most studies in the literature, *A,* the impact of technology, is presumed as 1. In some projects there is a long period between planning and the time to start the project. So, it may be reasonable to assume different *A*. That is, there is high chance that a significant improvement is happened in the technology. For example, in that time, many new types of equipment are launched to do a task in a shorter time.

Furthermore, in this study there is not constraint on time or cost of the project. That is, in this study, a multi-objective function is introduced. But, based on managers' point of view, and specific conditions of a project it can be changed to a problem with a due date, then the objective is to minimize the total cost considering the due date, or the objective could be the minimum time to do a project considering a limited budget.

In addition to the mentioned objective function, according to this study, we are now aware of the portion of labor and equipment cost. In a future, study, the objective function can be assumed as one of these costs (labor or equipment cost) considering a due date.

The conclusion is that introducing Cobb-Douglas function into time-cost tradeoff problem provides us extra capacity to further identify the optimal allocations of labor and equipment resources during crashing. Until now, no one in the literature consider the

impact of these factors in construction crashing cost. Considering these factors, managers can determine their priority to invest on labor and equipment in different projects for crashing. It means that, some managers may prefer to invest more in equipment which can be used in future projects, while sometimes, they just want to take care of the current projects. In doing so, they may prefer to add more labor to do their work in a shorter time.

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APPENDIX A

HGA Code

```
clear
clc

%%%%% Parameters  %%%%%

*** In this section parameters are defined and


*** n: number of activities
*** option: number of activities' options
*** a: alpha
*** b: beta
*** PS: population size

n=7;
option=3;

a=0.7;
b=0.3;

PS=100;

*** W: workload

W=[5000 500 600 6000 4500 5500 4700];



*** Cost: in each row, it defines the labor and equipment costs
respectively for that activity


Cost=[40 100
    50 70
    45 80
    75 70
    60 60
    55 20
    65 30];


% Table=[1  14  48523.399
% 1 20  33966.38
% 1 24  28305.316
% 2 15  3997.706
% 2 18  3331.421
% 2 20  2998.279
```

```
% 3 15   4982.924
% 3 22   3397.448
% 3 33   2264.965
% 4 12   70524.268
% 4 16   52893.201
% 4 20   40596.496
% 5 22   24056.162
% 5 24   22051.482
% 5 28   18901.27
% 6 14   23082.507
% 6 18   17953.061
% 6 24   13464.796
% 7 9    41838.75
% 7 15   25103.25
% 7 18   20919.375];
```

```
*** Table: it determined different options for each activity. Fits
colums shows the activity number, second column it shows duration, and
the last one determined the total cost
```

```
Table=[1    14   357.1428571
1    20   250
1    24   208.3333333
2    15   33.33333333
2    18   27.77777778
2    20   25
3    15   40
3    22   27.27273
3    33   18.18182
4    12   500
4    16   375
4    20   300
5    22   204.5455
5    24   187.5
5    28   160.7143
6    14   392.8571
6    18   305.5556
6    24   229.1667
7    9    522.2222
7    15   313.3333
7    18   261.1111];
```

```
% a=0.7
% b=0.3
% maxt=103;
% mint=60;
% maxc=203243.322;
% minc=121206.829;

% a=0.6
% b=0.4
 maxt=103;
 mint=60;
 maxc=205010.099;
```

```matlab
 minc=120250.725;

 *** w1 and w2 determined the priorities of cost and time respectively.

routs=3;

w1=0.4;
w2=0.6;


%%%% Initialization %%%%%%

*** in this phase, PS chromosomes are randomly generated

for i=1:PS
    for j=1:n
        X(i,j)=floor(option*rand)+1;
    end
end


%%%%%%% Improvement %%%%%%%%%

*** in this stage, as stated in the cntext, 2opt procedure is applied
to improve the randomly generated population

for k=1:PS

    Temp(1,:)=X(k,:);
    opt1=floor(n*rand)+1;
    opt2=floor(n*rand)+1;
    Temp(2,:)=X(k,:);
    temp=Temp(2,opt1);
    Temp(2,opt1)=Temp(2,opt2);
    Temp(2,opt2)=temp;
    Temp(3,:)=Temp(2,:);
    if opt1==1
        Temp(4,:)=Temp(2,:);
        TTemp=Temp(4,1);
        Temp(4,1)=Temp(4,2);
        Temp(4,2)=TTemp;
    else
        if opt1==n
            Temp(4,:)=Temp(2,:);
            TTemp=Temp(4,n);
            Temp(4,n)=Temp(4,n-1);
            Temp(4,n-1)=TTemp;
        else
            TTemp=Temp(3,opt1);
            gh=opt1+1;
            Temp(3,opt1)=Temp(3,gh);
            Temp(3,gh)=TTemp;
            Temp(4,:)=Temp(2,:);
            TTemp=Temp(4,opt1);
            jh=opt1-1;
```

```
            Temp(4,opt1)=Temp(4,jh);
            Temp(4,jh)=TTemp;
        end
    end
    Temp(5,:)=Temp(2,:);
    if opt2==1
        Temp(6,:)=Temp(2,:);
        TTemp=Temp(6,1);
        Temp(6,1)=Temp(6,2);
        Temp(6,2)=TTemp;
    else
        if opt2==n
            Temp(6,:)=Temp(2,:);
            TTemp=Temp(6,n);
            Temp(6,n)=Temp(6,n-1);
            Temp(6,n-1)=TTemp;
        else
            TTemp=Temp(5,opt2);
            ser=opt2+1;
            Temp(5,opt2)=Temp(5,ser);
            Temp(5,ser)=TTemp;
            Temp(6,:)=Temp(2,:);
            TTemp=Temp(6,opt2);
            sre=opt2-1;
            Temp(6,opt2)=Temp(6,sre);
            Temp(6,sre)=TTemp;
        end
    end
```

 *** after that the 2-opt procedure is done, the new solutions (offspring) are compared with the first solution, the best one is chosen.

```
for tt=1:6

    dd1=Temp(tt,1);
    dd3=Temp(tt,2)+3;
    dd6=Temp(tt,3)+6;
    dd9=Temp(tt,4)+9;
    dd12=Temp(tt,5)+12;
    dd15=Temp(tt,6)+15;
    dd18=Temp(tt,7)+18;


    Eeval(tt,1)=Table(dd1,3)+Table(dd3,3)+Table(dd12,3)+Table(dd18,3);
    Eeval(tt,2)=Table(dd1,3)+Table(dd6,3)+Table(dd12,3)+Table(dd18,3);
    Eeval(tt,3)=Table(dd1,3)+Table(dd9,3)+Table(dd15,3)+Table(dd18,3);
    Eeval(tt,4)=Table(dd1,2)+Table(dd3,2)+Table(dd12,2)+Table(dd18,2);
    Eeval(tt,5)=Table(dd1,2)+Table(dd6,2)+Table(dd12,2)+Table(dd18,2);
    Eeval(tt,6)=Table(dd1,2)+Table(dd9,2)+Table(dd15,2)+Table(dd18,2);
```

```matlab
Eeval(tt,7)=Table(dd1,3)+Table(dd3,3)+Table(dd12,3)+Table(dd18,3)+Table
(dd6,3)+Table(dd9,3)+Table(dd15,3);
    Eeval(tt,8)=max(Eeval(tt,4:6));
end
% maxt=max(Eval(:,7));
% mint=min(Eval(:,7));
% maxc=max(Eval(:,8));
% minc=min(Eval(:,8));
%
for ttt=1:6
    Eeval(ttt,9)=(maxt-Eeval(ttt,8))/(maxt-mint);
    Eeval(ttt,10)=(maxc-Eeval(ttt,7))/(maxc-minc);
    Eeval(ttt,11)=w1*Eeval(ttt,9)+w2*Eeval(ttt,10);
end

X(k,:)=Temp(1,:);

    Beste=max(Eeval(:,11));
    [sv,sd]=find(Eeval(:,11)==Beste);
    sv=sv(1);
    sd;
    X(k,:)=Temp(sv,:);



end


%%%%%%% Evaluation %%%%%%%%%%%%%%%

*** in this step, the objective values of all chromosomes in population
are calculated. (Zi)



roond=0;
while roond~=100

Eval=zeros(Genes,Genes);
for kk=1:Genes

Eval(kk,1)=(Table(X(kk,1),3)+Table(X(kk,2)+3,3)+Table(X(kk,5)+12,3)+Tab
le(X(kk,7)+18,3));

Eval(kk,2)=(Table(X(kk,1),3)+Table(X(kk,3)+6,3)+Table(X(kk,5)+12,3)+Tab
le(X(kk,7)+18,3));

Eval(kk,3)=(Table(X(kk,1),3)+Table(X(kk,4)+9,3)+Table(X(kk,6)+15,3)+Tab
le(X(kk,7)+18,3));

Eval(kk,4)=(Table(X(kk,1),2)+Table(X(kk,2)+3,2)+Table(X(kk,5)+12,2)+Tab
le(X(kk,7)+18,2));
```

```matlab
Eval(kk,5)=(Table(X(kk,1),2)+Table(X(kk,3)+6,2)+Table(X(kk,5)+12,2)+Tab
le(X(kk,7)+18,2));

Eval(kk,6)=(Table(X(kk,1),2)+Table(X(kk,4)+9,2)+Table(X(kk,6)+15,2)+Tab
le(X(kk,7)+18,2));

Eval(kk,7)=Table(X(kk,1),3)+Table(X(kk,2)+3,3)+Table(X(kk,3)+6,3)+Table
(X(kk,4)+9,3)+Table(X(kk,5)+12,3)+Table(X(kk,6)+15,3)+Table(X(kk,7)+18,
3);
    Eval(kk,8)=max(Eval(kk,4:6));
end
% maxt=max(Eval(:,7));
% mint=min(Eval(:,7));
% maxc=max(Eval(:,8));
% minc=min(Eval(:,8));

for jj=1:Genes
    Eval(jj,9)=(maxt-Eval(jj,8))/(maxt-mint);
    Eval(jj,10)=(maxc-Eval(jj,7))/(maxc-minc);
    Eval(jj,11)=w1*Eval(jj,9)+w2*Eval(jj,10);
end




%%%%%%% Selection    %%%%%%%%

*** Selection procedure is done on chromosomes in the population. The
better a chromosome is the more chance it has to be selected. Selected
chromosome are used in next steps for GA procedure.


Total=sum(Eval(:,11));

for ii=1:Genes
    p(ii)=(Total-Eval(i,11))/(Total*(100-1));
    q(ii)=sum(p(1:ii));
end

r=rand();
kul=0;
for rr=2:Genes
    if q(rr-1)<r && r<=q(rr)
        chromosome=X(rr,:);
        ss1=rr;
        kul=1;

    end
end

if kul~=1
    chromosome=X(1,:);
        ss1=1;
end
```

```matlab
%hh=X(1,:)
r2=rand();
kel=0;
for pp=2:Genes
    if p(pp-1)<r2 && r2<=q(pp)
        chromosome(2,:)=X(pp,:);
        ss2=pp;
        kel=1;

    end
end

if kel~=1
    chromosome(2,:)=X(1,:);
        ss2=1;
end

if chromosome(2,:)==chromosome(1,:)
    if ss2~=Genes
        ss2=ss2+1;
        chromosome(2,:)=X(ss2,:);
    else
        ss2=ss2-1;
        chromosome(2,:)=X(ss2,:);
    end

end




%      chromosome(1,:);
%      chromosome(2,:);

%%%%%%%      Algorithm     %%%%%%%%%%%

*** from here to the end, GA procedure works containing crossover, and
mutation procedures.

%%%%%%% Cross-over %%%%%%%%%%%%%%%

 *** Order cross-over procedure is done on selected chromosomes. A
random substring from the first parent is selected, and it is
substituted with that substring from the second chromosome (parent).

crossrate=rand();
cross=floor(n*crossrate)+1;

%  if cross==4
%      cross=3;
%  end
% cross
% chromosome(3,:)=chromosome(1,:);
% chromosome(4,:)=chromosome(2,:);
```

```matlab
% mashi=1;
% for ashi=cross:n
%     chromtemp(1,mashi)=chromosome(1,ashi);
% end

chromosome(3,:)=chromosome(1,:);
chromosome(4,:)=chromosome(2,:);

chromtemp=chromosome(1,cross:n);
chromosome(3,cross:n)=chromosome(4,cross:n);
chromosome(4,cross:n)=chromtemp;
```

*** New generated chromosomes are compared with their parents (chromosomes from the population). If the new ones are better, they will be substituted with current chromosomes in the population.

```matlab
for tt=1:4

Eevall(tt,1)=(Table(chromosome(tt,1),3)+Table(chromosome(tt,2)+3,3)+Tab
le(chromosome(tt,5)+12,3)+Table(chromosome(tt,7)+18,3));

Eevall(tt,2)=(Table(chromosome(tt,1),3)+Table(chromosome(tt,3)+6,3)+Tab
le(chromosome(tt,5)+12,3)+Table(chromosome(tt,7)+18,3));

Eevall(tt,3)=(Table(chromosome(tt,1),3)+Table(chromosome(tt,4)+9,3)+Tab
le(chromosome(tt,6)+15,3)+Table(chromosome(tt,7)+18,3));

Eevall(tt,4)=(Table(chromosome(tt,1),2)+Table(chromosome(tt,2)+3,2)+Tab
le(chromosome(tt,5)+12,2)+Table(chromosome(tt,7)+18,2));

Eevall(tt,5)=(Table(chromosome(tt,1),2)+Table(chromosome(tt,3)+6,2)+Tab
le(chromosome(tt,5)+12,2)+Table(chromosome(tt,7)+18,2));

Eevall(tt,6)=(Table(chromosome(tt,1),2)+Table(chromosome(tt,4)+9,2)+Tab
le(chromosome(tt,6)+15,2)+Table(chromosome(tt,7)+18,2));

Eevall(tt,7)=Table(chromosome(tt,1),3)+Table(chromosome(tt,2)+3,3)+Tabl
e(chromosome(tt,3)+6,3)+Table(chromosome(tt,4)+9,3)+Table(chromosome(tt
,5)+12,3)+Table(chromosome(tt,6)+15,3)+Table(chromosome(tt,7)+18,3);
    Eevall(tt,8)=max(Eevall(tt,4:6));
end
% maxt=max(Eevall(:,7));
% mint=min(Eevall(:,7));
% maxc=max(Eevall(:,8));
% minc=min(Eevall(:,8));
%
for ttt=1:4
    Eevall(ttt,9)=(maxt-Eevall(ttt,8))/(maxt-mint);
    Eevall(ttt,10)=(maxc-Eevall(ttt,7))/(maxc-minc);
    Eevall(ttt,11)=w1*Eevall(ttt,9)+w2*Eevall(ttt,10);
```

```matlab
    end

Eevall;

if Eevall(3,11)>Eevall(1,11)
    chromosome(1,:)=chromosome(3,:);
end


if Eevall(4,11)>Eevall(2,11)
    chromosome(2,:)=chromosome(4,:);
end



% offspring4(heur3)=offspring4(heur1);
% offspring4(heur1)=offtemp;
%
% offspring5=offspring4;
% offtemp=offspring5(heur2);
% offspring5(heur2)=offspring5(heur3);
% offspring5(heur3)=offtemp;
%
%  offspring6=offspring2;
%  offtemp=offspring6(heur3);
%  offspring6(heur3)=offspring6(heur2);
%  offspring6(heur2)=offtemp;



%%%%%%        Inversion Mutation   %%%%%%%%%%%%


*** In this procedure, a substring is randomly selected from a parent.
And including substrings are flipping.



chromosome(3,:)=chromosome(1,:);
chromosome(4,:)=chromosome(2,:);

% invmutation=chromosome(1,:);
% invmutation2=chromosome(2,:);
inv1=floor(n*rand)+1;
inv2=floor(n*rand)+1;
if inv1>inv2
    matinv=chromosome(1,inv2:inv1);
    matinv2=chromosome(2,inv2:inv1);
    k=1;
    for inv=inv2:inv1
        chromosome(3,inv)=matinv(inv1-inv2+k);
        chromosome(4,inv)=matinv2(inv1-inv2+k);
        k=k-1;
    end
end
if inv2>inv1
    matinv=chromosome(1,inv1:inv2);
```

```
    matinv2=chromosome(2,inv1:inv2);
    k=1;
    for inv=inv1:inv2
        chromosome(3,inv)=matinv(inv2-inv1+k);
        chromosome(4,inv)=matinv2(inv2-inv1+k);
        k=k-1;
    end
end
```

 *** Generated chromosomes are compared with those in population (parents). If the generated ones are better, they will be substituted with the current ones.

```
for isi=1:4

Evall(isi,1)=(Table(chromosome(isi,1),3)+Table(chromosome(isi,2)+3,3)+T
able(chromosome(isi,5)+12,3)+Table(chromosome(isi,7)+18,3));

Evall(isi,2)=(Table(chromosome(isi,1),3)+Table(chromosome(isi,3)+6,3)+T
able(chromosome(isi,5)+12,3)+Table(chromosome(isi,7)+18,3));

Evall(isi,3)=(Table(chromosome(isi,1),3)+Table(chromosome(isi,4)+9,3)+T
able(chromosome(isi,6)+15,3)+Table(chromosome(isi,7)+18,3));

Evall(isi,4)=(Table(chromosome(isi,1),2)+Table(chromosome(isi,2)+3,2)+T
able(chromosome(isi,5)+12,2)+Table(chromosome(isi,7)+18,2));

Evall(isi,5)=(Table(chromosome(isi,1),2)+Table(chromosome(isi,3)+6,2)+T
able(chromosome(isi,5)+12,2)+Table(chromosome(isi,7)+18,2));

Evall(isi,6)=(Table(chromosome(isi,1),2)+Table(chromosome(isi,4)+9,2)+T
able(chromosome(isi,6)+15,2)+Table(chromosome(isi,7)+18,2));

Evall(isi,7)=Table(chromosome(isi,1),3)+Table(chromosome(isi,2)+3,3)+Ta
ble(chromosome(isi,3)+6,3)+Table(chromosome(isi,4)+9,3)+Table(chromosom
e(isi,5)+12,3)+Table(chromosome(isi,6)+15,3)+Table(chromosome(isi,7)+18
,3);
    Evall(isi,8)=max(Evall(isi,4:6));
end
% maxt=max(Evall(:,7));
% mint=min(Evall(:,7));
% maxc=max(Evall(:,8));
% minc=min(Evall(:,8));

for isit=1:4
    Evall(isit,9)=(maxt-Evall(isit,8))/(maxt-mint);
    Evall(isit,10)=(maxc-Evall(isit,7))/(maxc-minc);
    Evall(isit,11)=w1*Evall(isit,9)+w2*Evall(isit,10);
end




if Evall(3,11)>Evall(1,11)
    chromosome(1,:)=chromosome(3,:);
end
```

```
if Evall(4,11)>Evall(2,11)
    chromosome(2,:)=chromosome(4,:);
end

X(ss1,:)=chromosome(1,:);
X(ss2,:)=chromosome(2,:);


roond=roond+1;
end


*** to choose the best one, all available solutions in the population
are check based on their objective values regarding the predefined
priorities.


Best=max(Eval(:,11));
[cv,cd]=find(Eval(:,11)==Best);
BestSolution=X(cv,:);


for che=1:4
    if Evall(che,11)>Best
        BestSolution=chromosome(che,:);
        Best=Evall(che,11);
    end
end

Best
BestSolution(1,:)
size(cv);
```

APPENDIX B

Simulation Code

```
clear
clc


*** According to the fact that it needs to do some optimization stuff
in this problem in every iteration of the simulation, it needs to set
optimization assumptions correctly regardint the problem.

opts = optimset('fmincon');
opts.LargeScale = 'off';
% opts.MediumScale = 'on';
opts.TolFun = 1.e-6;


format long

global Q
global alpha
global beta
global act
global cost
global rep

*** N: number of iterations

N=1000;

*** cost and time priorities are defined.

w1=0.4;
w2=1-w1;

SERI=zeros(N,14);
% alpha=0.3
% beta=0.7



*** In the new problem, according to the fact that alpha, beta, labor
and equipment costs are stochastic, and they are generated randomly in
their predefined ranges in each iteration, total cost is achivied after
optimization calculations. So table here is different from the previous
problem and the third row is empty which is set after optimization
calculations.

Table=[1    14
1   20
1   24
```

```
2   15
2   18
2   20
3   15
3   22
3   33
4   12
4   16
4   20
5   22
5   24
5   28
6   14
6   18
6   24
7   9
7   15
7   18];
```

*** For the optimization purpose, regarding the total number of
activities, initial points are defined.

```
X00=[100 400
    150 350
    200 300
    50 60
    30 40
    45 55
    65 70
    75 40
    20 30
    200 400
    300 350
    290 310
    440 520
    250 350
    270 330
    230 300
    280 180
    170 70
    125 145
    175 335
    180 200];
```

*** For each activity option in case that labor and equipment costs are
determined, FACTORCOST shows these costs.

```
FACTORCOST=[40   100
40   100
```

```
40  100
50  70
50  70
50  70
45  80
45  80
45  80
75  70
75  70
75  70
60  60
60  60
60  60
55  20
55  20
55  20
65  30
65  30
65  30];
```

*** TotalQ shows the Q rate of each activity's option which is achieved from w/t.

```
TotalQ=[357.1
250.0
208.3
33.3
27.8
25.0
40.0
27.3
18.2
500.0
375.0
300.0
204.5
187.5
160.7
392.9
305.6
229.2
522.2
313.3
261.1];

%   albeta(rep)=1;
%     alpha(rep)=0.7;
%     beta(rep)=albeta(rep)-alpha(rep);
%
%
% for act=1:21
%         act;
%         x0=X00(act,:);
```

```
%            A=[];
%            b=[];
%            Aeq=[];
%            beq=[];
%             lb=[];
%             ub=[];
%          cost(act,:)=FACTORCOST(act,:);
%          Q(act)=TotalQ(act);
%          x(act,:)= fmincon(@Cobb,x0,A,b,Aeq,beq,lb,ub,@mycons);
%      %   x(act,:)= fmincon(@Cobb,x0,A,b,Aeq,beq,@mycons)
%          %x =
fmincon(@(x)cost(1)*x(1)+cost(2)*x(2),x0,A,b,Aeq,beq,lb,ub,@mycons);
%          LaborCost(act)=x(act,1)*cost(act,1);
%          CapitalCost(act)=x(act,2)*cost(act,2);
%          Table(act,3)=LaborCost(act)+CapitalCost(act);
%
% end


 *** the simulation process starts from this point.


for rep=1:N
rep;


% INFL(rep,1)=rand()*0.05+1.10;
% INFL(rep,2)=rand()*0.05+1.10;

% INFL(rep,1)=rand()*0.10+1.05;
% INFL(rep,2)=1;



INFL(rep,1)=1;
INFL(rep,2)=1;



%INFL(3)=rand()*0.03+1.03;
%INFL(4)=rand()*0.03+1.04;



SERI(rep,15)=INFL(rep,1);
SERI(rep,16)=INFL(rep,2);
%A(rep)=2;
%while A(rep)>1


*** One of the main steps in the simulation process is to define alpha,
beta and summation of those. Based on the predefined ranges for each of
those, random numbers are generated here.


     albeta(rep)=0.4*rand()+0.6;
     alpha(rep)=(0.2*rand()+0.2)*albeta(rep);
     beta(rep)=albeta(rep)-alpha(rep);
```

```matlab
if albeta(rep)>=0.6 && albeta(rep)<0.7
     w1=0.1;
else
    if albeta(rep)>=0.7 && albeta(rep)<0.8
        w1=0.2;
    else
        if albeta(rep)>=0.8 && albeta(rep)<0.9
            w1=0.3;
        else
            w1=0.4;
        end
    end
end

    w2=1-w1;



%     albeta(rep)=1;
%     alpha(rep)=0.3;
%     beta(rep)=albeta(rep)-alpha(rep);



    SERI(rep,5)=alpha(rep);
SERI(rep,6)=beta(rep);
    SERI(rep,14)=albeta(rep);

% alpha(rep)=0.3;
% beta(rep)=0.7;
% albeta(rep)=1;



%end


*** In each iteration, based on the determined values for alpha, beta,
labor and equipment costs, the optimum values of labor cost, equipment
cost, and total cost are calculated.


for act=1:21
        act;
        x0=X00(act,:);
         A=[];
         b=[];
         Aeq=[];
         beq=[];
          lb=[];
          ub=[];
        cost(act,1)=FACTORCOST(act,1)*INFL(rep,1);
        cost(act,2)=FACTORCOST(act,2)*INFL(rep,2);
        Q(act)=TotalQ(act);
        x(act,:)= fmincon(@Cobb,x0,A,b,Aeq,beq,lb,ub,@mycons);
    %   x(act,:)= fmincon(@Cobb,x0,A,b,Aeq,beq,@mycons)
```

```
        %x =
fmincon(@(x)cost(1)*x(1)+cost(2)*x(2),x0,A,b,Aeq,beq,lb,ub,@mycons);
        LaborCost(act)=x(act,1)*cost(act,1);
        CapitalCost(act)=x(act,2)*cost(act,2);
        Table(act,3)=LaborCost(act)+CapitalCost(act);

end
Table;
kkk=1;
```

*** Based on the determined values for parameters, optimum objective value is achives via checking all possible combinations of activities' options.

```
for i=1:3
    for ii=1:3
        for iii=1:3
            for j=1:3
                for jj=1:3
                    for k=1:3
                        for kk=1:3


Eval(kkk,1)=(Table(i,3)+Table(ii+3,3)+Table(jj+12,3)+Table(kk+18,3));

Eval(kkk,2)=(Table(i,3)+Table(iii+6,3)+Table(jj+12,3)+Table(kk+18,3));
    Eval(kkk,3)=(Table(i,3)+Table(j+9,3)+Table(k+15,3)+Table(kk+18,3));

Eval(kkk,4)=(Table(i,2)+Table(ii+3,2)+Table(jj+12,2)+Table(kk+18,2));

Eval(kkk,5)=(Table(i,2)+Table(iii+6,2)+Table(jj+12,2)+Table(kk+18,2));
    Eval(kkk,6)=(Table(i,2)+Table(j+9,2)+Table(k+15,2)+Table(kk+18,2));

Eval(kkk,7)=Table(i,3)+Table(ii+3,3)+Table(jj+12,3)+Table(kk+18,3)+Tabl
e(iii+6,3)+Table(j+9,3)+Table(k+15,3);
    Eval(kkk,8)=max(Eval(kkk,4:6));
    X(kkk,1)=i;
    X(kkk,2)=ii;
    X(kkk,3)=iii;
    X(kkk,4)=j;
    X(kkk,5)=jj;
    X(kkk,6)=k;
    X(kkk,7)=kk;




    kkk=kkk+1;
```

```
                              end
                          end
                      end
                  end
              end
          end
end
```

*** In order to calculate the objective value, according to equation 11 and 12, max and min of time and cost are needed. They are calculated in this step.

```
 maxt=max(Eval(:,8));
 mint=min(Eval(:,8));
 maxc=max(Eval(:,7));
 minc=min(Eval(:,7));

 SERI(rep,12)=maxt;
  SERI(rep,13)=mint;
   SERI(rep,11)=minc;
    SERI(rep,10)=maxc;



for ss=1:3^7

%     if 59<Eval(ss,8) && Eval(ss,8)<70
%         Eval(ss,7);
%       Eval(ss,7)=Eval(ss,7)*INFL(1);
%  else
%      if 69<Eval(ss,8)&& Eval(ss,8)<80
%                 Eval(ss,7);
%          Eval(ss,7)=Eval(ss,7)*INFL(2);
%      else
%          if 79<Eval(ss,8)&& Eval(ss,8)<90
%                    Eval(ss,7);
%              Eval(ss,7)=Eval(ss,7)*INFL(3);
%          else
%                      Eval(ss,7);
%              Eval(ss,7)=Eval(ss,7)*INFL(4);
%          end
%      end
%    end
%
%

    Eval(ss,9)=(maxt-Eval(ss,8))/(maxt-mint);

    Eval(ss,10)=(maxc-Eval(ss,7))/(maxc-minc);

    Eval(ss,11)=w1*Eval(ss,9)+w2*Eval(ss,10);
```

```
end




*** Comparing all possible combination of activities' options, the
optimum objective value and corresponding activities' options are
chosen.




Best(1)=max(Eval(:,11));

[cv,cd]=find(Eval(:,11)==Best(1));
CV=cv(1);
cd;
Eval(CV,9);
Eval(CV,7);
Eval(CV,8);
Eval(CV,3);
Eval(CV,10);
BestSolution=X(CV,:);

%%% SERI 1:time  2:cost   3:labor cost   4:capital cost
Eval(CV,8);

SERI(rep,1)=Eval(CV,8);
SERI(rep,2)=Eval(CV,7);
LCost=0;
pop=0;
 for asas=1:7

    LCost=LaborCost(3*pop+BestSolution(1,asas))+LCost;
    pop=pop+1;
 end
 SERI(rep,3)=LCost;
 SERI(rep,4)=SERI(rep,2)-SERI(rep,3);

SERI(rep,7)=Best;
SERI(rep,8)=Eval(CV,9);
SERI(rep,9)=Eval(CV,10);

%SERI(rep,10)=Eval(CV,11);

end
100

INFL;
Table;
```

*** this function is one of the functions needed by MATLAB software to calculate the optimum value of total cost.

In this function constrints of the model which is the CD function are presented.

```matlab
function [c ceq] = mycons(x)

global Q
global alpha
global beta
global act
global rep

c=[];
ceq=(x(1)^alpha(rep))*(x(2)^beta(rep))-Q(act);
```

*** This function defined the objective function of the model. This function is called in the optimization process to calculate the optimum value of total cost.

```matlab
function f = Cobb(x)

global cost
global act


f=cost(act,1)*x(1)+cost(act,2)*x(2);
```