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An Efficient Water-Filling Algorithm for Power Allocation in OFDM-Based Cognitive Radio Systems

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Abstract—**In this paper, we present a new water-filling algorithm for power allocation in Orthogonal Frequency Division Multiplexing (OFDM) – based cognitive radio systems. The conventional water-filling algorithm cannot be directly employed for power allocation in a cognitive radio system, because there are more power constraints in the cognitive radio power allocation problem than in the classic OFDM system. In this paper, a novel algorithm based on iterative water-filling is presented to overcome such limitations. However, the computational complexity in iterative water-filling is very high. Thus, we explore features of the water-filling algorithm and propose a low-complexity algorithm using power-increment or power-decrement water-filling processes. Simulation results show that our proposed algorithms can achieve the optimal power allocation performance in less time than the iterative water-filling algorithms.**

Keywords-*cognitive radio, orthogonal frequency division multiplexing, water-filling algorithm, power allocation.*

I. INTRODUCTION

With the rapid development of wireless communications, frequency spectrum is becoming a very precious resource, and scarcity of the spectrum is a serious problem. Traditionally, spectrum allocation is governed by the Federal Communication Commissions (FCC) which regulates the usage of the radio spectrum in the US. In some cases, the spectrum bands are not efficiently utilized because licensed users do not always occupy their spectrum and unlicensed users are not allowed to operate in such spectrum bands. This governance leads to unbalanced spectrum utilization [1].

Joseph Mitola III in [2] proposed Cognitive radio (CR) systems to exploit the unbalanced spectrum utilization and by allowing Secondary Users (SUs) to use the idle spectrum of licensed users or Primary Users (PUs) to gain a higher spectrum utilization. Here, one of the challenges is to detect the available spectrum bands of the PUs [3]. In this approach, the energy detection-based spectrum sensing method is the most common spectrum-sensing technique due to its low computational complexities and easy implementation [4][5]. In the opportunistic spectrum access (OSA) mode, the SUs access the spectrum when the PUs do not use it concurrently [6]. Compared to the spectrum sharing mode, in which the PUs and SUs can share the spectrum channel simultaneously, the OSA mode causes less interference to the PUs [12].

When using Orthogonal Frequency Division Multiplexing in cognitive radio networks, the power allocation schemes for the spectrum resources will be very flexible and convenient [7]. However, it becomes very challenging to allocate power to individual subchannels in the OFDM-based cognitive radio networks. In traditional power allocation problems, water-filling algorithms are prevalent [14] [15]. Because additional interference constraints must be considered in cognitive radio networks, the water-filling algorithms are always performed iteratively to solve the power allocation problem [16] [17]. In this paper, we study the properties of the water-filling algorithm and propose a linear iterative algorithm to reduce computational complexity.

The rest of the paper is organized as follows. The system model is introduced in Section II. Our study of some new properties of the water-filling and the low-complexity power allocation theory are then proposed in Section III. In Section IV the numerical results are given. Finally, our conclusions are presented in Section V.

II. SYSTEM MODLE

A. Traditional Water-Filling Method

Consider an OFDM communication system [13]:

$$y_n[m] = h_n x_n[m] + w[m], \quad n = 0,1, \dots N-1 \quad (1)$$

where $x_n[m]$, $y_n[m]$ and $w_n[m]$ are the input, output and noise signals in each subchannel, respectively. h_n is the channel gain for each subchannel with the total power constraint P_{total}. Assuming the transmit power in each subchannel is P_n, the maximum rate of reliable communication using the OFDM channel is

$$C = \sum_{n=0}^{N-1} \log\left(1 + \frac{P_n |h_n|^2}{N_0}\right) \text{ bit/symbols} \quad (2)$$

where N_0 is the power density of the noise. Therefore the power allocation can be chosen so as to maximize the rate in (2). The power allocation, thus, is the solution to the optimization problem:

$$C_N := \max_{P_0, \dots, P_{N-1}} \sum_{n=0}^{N-1} \log\left(1 + \frac{P_n |h_n|^2}{N_0}\right) \quad (3)$$

Subject to

$$\sum_{k=0}^{N-1} P_n = P_{total} \quad P_n \geq 0, \ n = 0, \dots, N-1 \quad (4)$$

The objective function (3) is convex in the powers and this optimization problem can be solved by the Lagrangian method. Consider the expression

$$\mathcal{L}(\lambda, P_0, \ldots, P_{N-1}) := \sum_{n=0}^{N-1} \log\left(1 + \frac{P_n |h_n|^2}{N_0}\right) - \lambda \sum_{n=0}^{N-1} P_n \tag{5}$$

where λ is the Lagrange multiplier. The Kuhn-Tucker condition for the optimal solution is

$$\left.\begin{array}{ll} \frac{\partial \mathcal{L}}{\partial P_n} = 0 & \text{if } P_n > 0 \\ \frac{\partial \mathcal{L}}{\partial P_n} \leq 0 & \text{if } P_n = 0 \end{array}\right\} \tag{6}$$

Define $x^+ := \max(x, 0)$. The power allocation can be expressed as

$$P_n^* = \left(\frac{1}{\lambda} - \frac{N_0}{|h_n|^2}\right)^+ \tag{7}$$

which is the optimal solution if the Lagrange multiplier λ satisfies the condition

$$\sum_{n=0}^{N-1} \left(\frac{1}{\lambda} - \frac{N_0}{|h_n|^2}\right)^+ = P_{total} \tag{8}$$

The inverse of the Lagrange multiplier can be regarded as a water level. Generally, the water level can be found by the binary search method [14].

B. OFDM-based cognitve radio system

We consider the scenario in which multiple SUs are allowed to share the spectra designated for a PU network shown in Fig.1. Even though the existence of PU can be correctly detected, some techniques have to be considered to mitigate interference. In [10], the SU may have different interference and detection ranges as shown in Fig. 1, where the SU transmitter (Tx) is placed in the center of the operating circle, R_D is the radius of the detection range, and R_I is the radius of the interference range.

In this case, SUs may not detect the existence of the second PU (PU2) transmitter because of large separation between them, but they can interfere with the PU2 receiver when the SU is transmitting. Meanwhile, it is difficult for the SU to detect the PU receiver. We can convert the problem from detecting the primary receiver to detecting the primary transmitter [11]. Thus, our system model is shown in Fig 1. The PU has a circular protection region, whose radius is represented by r_p, in which the SU's power cannot exceed a certain threshold. Since the PU's transmitting power is also under the same total power constraint, the PU receivers in such an area centered by the PU transmitter can receive the signals transmitted by the PU transmitter. So the requirement for SU is to protect the PU receivers in the same area. When the PU receiver is located in the shaded area illustrated in Fig. 1, the SU will not interfere with the PU receiver even if the threshold is exceeded.
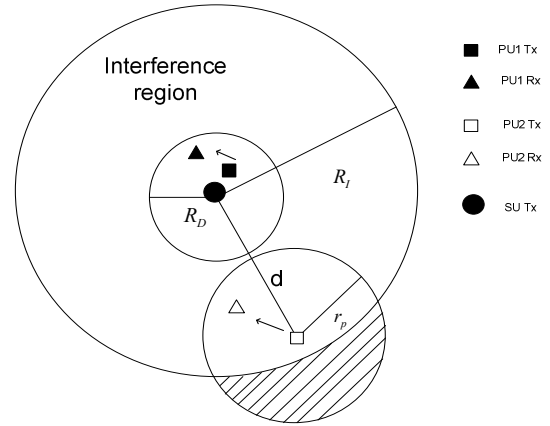

Figure 1. System model of cognitive radio

In this model, we allocate each subchannel to an individual PU. There are N subchannels corresponding to N PUs in the networks. Each subchannel consists of L_j (j=1, 2, …, N) different subcarriers which have different channel gains. So the total number of subcarriers is $M = \sum_{j=1}^{N} L_j$. Therefore, in this OSA cognitive radio model, the SU Tx cannot transmit signal when the PUs are detected. While the PUs are not detected we have to make sure the transmit power is under a certain threshold. This condition can be formulated in [8] as

$$P_j \leq G_j \tag{9}$$

Where

$$G_j \equiv \begin{cases} 0 & \text{PU}_j \text{ is detected} \\ \eta(d_j - r_p^j)^{\beta_j} & \text{PU}_j \text{ is not detected} \end{cases} \tag{10}$$

where P_j is the allocated power for SUs in subchannel j and η is the threshold. For simplicity, the threshold in all sub-channels is assumed to be the same. G_j is the interference constraint for subchannel j, d_j is the distance between the SU transmitter and the PU$_j$ transmitter, and r_p^j is the radius of the protection region in subchannel j. β_j denotes the path attenuation factor. We modify this model to suit an OFDM-based cognitive radio system as follows.

In N different subchannels, the interference constraints are based on the different PUs. Then, we have N different interference constraints. The power allocation problem can be formulated as follows.

$$P^* = \arg \max \sum_{i=1}^{M} \frac{B}{N} \log_2(1 + P_i H_i) \tag{11}$$

$$\text{s.t.} \quad P_i \geq 0 \tag{12}$$

$$\sum_{i=1}^{M} P_i \leq P_{total} \tag{13}$$

$$\sum_{i \in I_i} P_i = F_j \leq G_j \tag{14}$$

where P_i is the allocated power for each subcarrier, B is the bandwidth of the channel, $H_i = |h_i|^2 / (N_0 B N^{-1})$ is the channel-to-noise ratio (CNR) for subcarrier i, I_j is a set that consists of all of the subcarriers belonging to the same sub-

channel j, and F_j is the total allocated power to subchannel j. We find that without equation (14) the remaining equations form the traditional water-filling problem we discussed in section II.A. Equation (14) adds an interference constraint to each subchannel. The power allocation for this problem is subject to the total power constraints and individual subchannels' power constraints. We define this problem as a *dual-layer constraint power allocation problem*. To make both constraints meaningful $P_{total} < \sum_{j=1}^{N} G_j$ must be satisfied [9].

III. PROPOSED LOW COMPUTATIONAL COMPLEXITY POWER ALLOCATION ALGORITHM

As mentioned in the last section, the power allocation problem in cognitive radio systems cannot be solved simply by applying the traditional water-filling method. The algorithm proposed in [8] adopts the iterative water-filling to solve the problem. The algorithm, which is called the iteratively partitioned water-filling (IPWF) algorithm, initially allocated all of the subchannels in a set A. The algorithm begins by running the water-filling in set A and generating a water level and power allocation vector. It then extracts the subchannels under the condition $F_j \geq G_j$ to set B, and repeats this process in the new set A until no subchannels are needed to extract to set B. By this iterative process, the water level for subchannels in set A will converge to a common water level $\hat{w}$. Additional water-filling operations are required to calculate the unique water levels for all of the extracted subchannels in set B. The IPWF must be performed many times, which is not a linear operation. Therefore, high computational complexity will be incurred. In the worst case, only one subchannel can be extracted in an iteration and there is only one subchannel in set A. In this case, to classify the set A and set B, $(N-1)$ runs of water-filling algorithm have to be performed. To determine the unique water levels of the subchannels in set B, an additional $(N-1)$ runs of the water-filling algorithm must be performed. Thus, the water-filling algorithm must be run $2(N-1)$ times.

By exploring the properties of the water-filling, we propose a low computational complexity power allocation algorithm which requires performing only a single water-filling calculation. This algorithm not only greatly reduces the computational

A. Properties of the water-filling algorithm

We consider a water-filling problem with N channels and a total power constraint P_{total}. The water level is w. The allocated power vector is $P = \{P_i, \ i = 1,2, \ldots, N\}$. The CNR of each channel is H_i $(i = 1,2, \ldots, N)$. When some power is added or subtracted from the total power P_{total}, how can the power allocation problem be solved with power constraints with the total power constraints $P_{total} + \Delta$ or $P_{total} - \Delta$? Should we have to perform another water-filling calculation? Our answer is no. To solve this power allocation problem we treat it as *power increment water-filling* or *power decrement water-filling*, respectively. First we consider power increment water-filling. The channels are divided into three sets shown in Fig. 2.

$$Q = \{P_i | H_i^{-1} \leq w\},$$
$$R = \left\{P_i | w < H_i^{-1} < w + \frac{\Delta}{N}\right\},$$
$$S = \left\{P_i | w + \frac{\Delta}{N} < H_i^{-1}\right\}.$$

The following lemma 1 can solve the power increment water-filling without performing water-filling.

Lemma 1

Assume that all of the channels satisfy $H_i^{-1} < w + \frac{\Delta}{N}$ or $H_i^{-1} > w + \frac{\Delta}{N} + \frac{\sum_{P_i \in R}(H_i^{-1}-w)+\sum_{P_i \in S}\frac{\Delta}{N}}{|Q|+|R|}$, then the new water level becomes $w' = w + \frac{\Delta}{N} + \frac{\sum_{P_i \in R}(H_i^{-1}-w)+\sum_{P_i \in S}\frac{\Delta}{N}}{|Q|+|R|}$. And the powers in each channels are $P_i' = P_i + w' - H_i^{-1}$ for $P_i \in Q \cup R$ and $P_i' = 0$ for $P_i \in S$.

Proof:

If $w \geq H_i^{-1}, \forall i \in N$, then

$$P_i = w - H_i^{-1} \quad \forall i \in N \qquad (15)$$

and

$$w = \frac{1}{N}(P_{total} - \sum_{i \in N} H_i^{-1}) \qquad (16)$$

combine (15) and (16)

$$P_i = \frac{1}{N}(P_{total} + \sum_{i \in N} H_i^{-1}) - H_i^{-1} \qquad (17)$$

Equation (17) is a linear equation between P_i and P_{total}. When the Δ power is added, the updated power allocation vector becomes $P_i' = P_i + \frac{\Delta}{N}$.
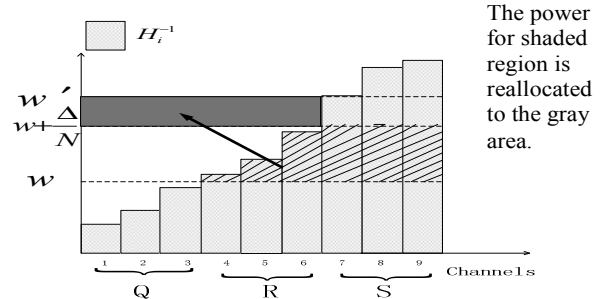

Figure 2. Illustration of power increment water-filling

Now consider the situation with $< H_i^{-1} \ \exists \ i \in N$. For example, in Fig. 2 the inverse of CNRs from channel 4 to 9 are greater than the water level. Without loss of generality, we sort the inversion of CNRs in ascending order. Channels of 1, 2 and 3 belong to set Q, channel 4, 5 and 6 belong to set R, and channel 7, 8 and 9 belong to set S. When the increment power Δ is added, each channel obtains additional power $\frac{\Delta}{N}$. However, when some inversions of the CNRs are higher than the water level w, the increment power cannot be allocated in those channels. The shadowed part denotes the power which cannot be added into the corresponding channels. To ensure the entire increment power equal to Δ , power in the shadowed part must be re-allocated into the better channels 1,2,3,4,5 and 6. As arrow-head pointed in Fig. 2, the power of shaded area in set S is reallocated to the gray area in new channels belonging to set Q and R.. The power of shaded area P_s can be denoted by

$$P_s = \left(\sum_{P_i \in R} H_i^{-1} - w\right) + \sum_{P_i \in S} \frac{\Delta}{N} \qquad (18)$$

As assumed

$H_i^{-1} > w + \frac{\Delta}{N} + \frac{\sum_{P_i \in R}(H_i^{-1}-w)+\sum_{P_i \in S}\frac{\Delta}{N}}{|Q|+|R|}$, then we have

$H_i^{-1} > w + \frac{\Delta}{N} + \frac{P_s}{|Q|+|R|}$ or $H_i^{-1} < w + \frac{\Delta}{N}$, which means the inversion of the CNRs of each channel in set S will still be higher than the new water level w' after P_s is reallocated to the pointed gray area in Fig 2. So the new water level becomes

$$w' = w + \frac{\Delta}{N} + \frac{\sum_{P_i \in R}(H_i^{-1}-w)+\sum_{P_i \in S}\frac{\Delta}{N}}{|Q|+|R|} \qquad (19)$$

And $P_i' = P_i + w' - H_i^{-1}$ for $P_i \in Q \cup R$ or $P_i' = 0$ for $P_i \in S$.

The assumption in lemma 1 may not always be guaranteed. So the iterative operation has to be performed. The *power increment water-filling* can be solved by iteratively updating the water level and generating new *power increment water-filling* until the assumption in lemma 1 is satisfied.

Now we consider the *power decrement water-filling*. The amount of power Δ will be subtracted from the total power. Determining the new water level does not require iterations of the water-filling algorithm. It is evident that the computation of power decrement water-filling is simpler than that of power increment water-filling. Here, only two sets are needed to determine, set $C = \left\{P_i | H_i^{-1} \le w - \frac{\Delta}{N}\right\}$, $D = \left\{P_i | H_i^{-1} > w - \frac{\Delta}{N}\right\}$, shown in Fig. 3.

Lemma 2

Assume that $P_i \in C$ satisfies following condition $w - \frac{\Delta}{N} - \sum_{i \in D} \min\left(\left(H_i^{-1}-w+\frac{\Delta}{N}\right),\frac{\Delta}{N}\right) \le H_i^{-1}$, then the

new water level is $w' = w - \frac{\Delta}{N} - \frac{\sum_{i \in D}\min\left(\left(H_i^{-1}-w+\frac{\Delta}{N}\right),\frac{\Delta}{N}\right)}{|C|}$ and $P_i' = w' - H_i^{-1}$, when $P_i \in C$; $P_i' = 0$ when $P_i \in D$.

Proof:

From the proof in Lemma 1, we show that if $w - \frac{\Delta}{N} \ge H_i^{-1} \ \forall \ i \in N$, namely set $D = \emptyset$, then $w' = w - \frac{\Delta}{N}$. When D $\ne \emptyset$, we have to further subtract the shaded part shown in Fig. 3 because this part was not subtracted in the corresponding channels. The shaded part can be denoted as

$$P_s = \sum_{i \in D} \min\left(\left(H_i^{-1}-w+\frac{\Delta}{N}\right),\frac{\Delta}{N}\right) \qquad (20)$$

As assumed with $w - \frac{\Delta}{N} - \sum_{i \in D} \min\left(\left(H_i^{-1}-w+\frac{\Delta}{N}\right),\frac{\Delta}{N}\right) \le H_i^{-1}$ or $w - \frac{\Delta}{N} - P_s \le H_i^{-1}$, the inversions of the CNRs below $w - \frac{\Delta}{N}$ will still be below the new water level after the shadowed area power is subtracted from the area where the arrow head points in the Fig. 3. The new water level is $w' = w - \frac{\Delta}{N} - \frac{\sum_{i \in D}\min\left(\left(H_i^{-1}-w+\frac{\Delta}{N}\right),\frac{\Delta}{N}\right)}{|C|}$ after the further

power re-allocation. Based on the new water level, we have $P_i' = w' - H_i^{-1}$ for $P_i \in C$ or $P_i' = 0$ for $P_i \in D$.

The assumption in lemma 2 may not always be guaranteed. The iteration operation is still needed. The power decrement water-filling can be solved by iteratively updating the water level and generating new power decrement water-filling until the assumption in lemma 2 is satisfied.
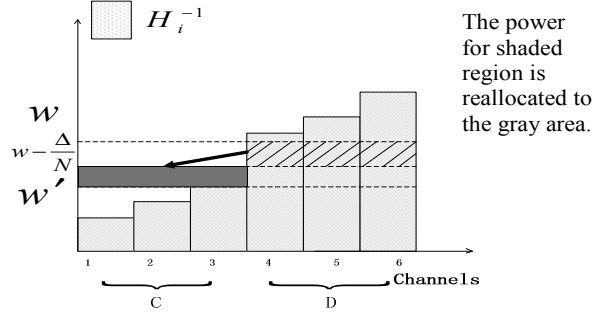

Figure 3. Illustration of Power decrement water-filling

B. Low computational complexity power allocation algorithm

After the first time water-filling operation, the sub-channels with $F_j \ge G_j$ will be moved from set A to set B [8]. Using the lemma in [8], the calculated water level is also the optimal power allocation solution for the rest of the sub-channels in set A and the sub-channels moved out to set B with respect to the total power constraints $P_{total} - \sum_{j \in B} F_j$ and F_j . The theorem in [8] requires recalculation of the unique water levels in set B with the power constraint G_j. The common water level will be determined by the total power constraint $P_{total} - \sum_{j \in B} G_j$. Therefore, the power allocation problems in each sub-channel in set B can be considered as a *power decrement water-filling* with $\Delta = F_j - G_j$. On the contrary, the set A is a *power increment water-filling* with $\Delta = F_j - G_j$. We defined this entire process as the *power reflowing process*. By iteratively running the power reflowing process, the common water level will be gradually increased, and all of the subchannels in set B will be moved out. In the entire process, the water-filling operation is performed only once.

The details of the algorithm can be summarized in Table 1. By using this algorithm, we can obtain the optimal power allocation solution without performing water-filling calculations multiple times. The *power increment water-filling process* and *power decrement water-filling processes* performed in the algorithm are both linear operations and the complexity will be reduced. Furthermore, we observed that the *power reflowing process* is a process of power reflowing from set B's subchannels to A's subchannels. This is why the common water level will be gradually increased and more subchannels have to be moved to set B. The process is analogous to an unbalanced barometric pressure model. The barometric pressure on the unique water levels is greater than the barometric pressure on the common water level. Some water in sub-channels belonging to set B will be pressed to A's subchannels. When this flowing process is stopped, the steady status is established. Due to the different barometric pressures on the subchannels, the water will not flow from

one subchannel to another even though the water levels of all of the subchannels are not even. So the steady but uneven water levels can still achieve the maximum capacity.

Table 1

Initialization

1. $A = \{ j \mid j = 1, 2, ..., N \}$ $B = \emptyset$ $P = P_{total}$

Running the conventional Water-Filling for set A with P, get the water level w

2. Move the subchannels to set B if $F_j > G_j$, update set A.

   Perform Power decrement on set B

3. Perform Power increment on set A with $\Delta = F_j - G_j$, update the common water level w ,F_j

4. if $\exists F_j > G_j$, goto 2

IV. NUMERICAL RESULTS

In this section, we present the Matlab simulation results for our proposed algorithm. We assume the Added White Gaussian Noise (AWGN) noise density N_0 and the number of subcarriers, *M,* which satisfies the normalization $N_0 B \times M = 1$. For all subcarriers, gain h_i is assumed independent and identically Chi-square distribution. In our simulations, we have different number of PUs ranging from 1 to 40. We fix every subchannel with 32 subcarriers, so that the total subcarriers range from 32 to 1280.

The performance of the proposed algorithm is evaluated in Fig.4. In Fig.4, the computation time for the two kinds of algorithms is plotted. To show the specific increment of the computation time, we use the logarithmic scale for the Y axis. It is shown that the computation time of proposed algorithm is nearly 100 times faster than that of the IPWF algorithm [8].
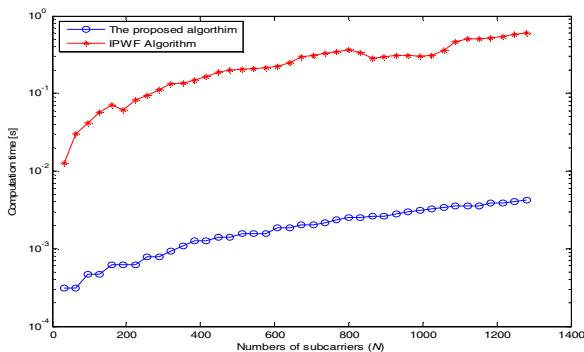

Figure 4. Computation time versus numbers of subcarriers

V. CONCLUSION

In this paper, we proposed an optimal power allocation algorithm for OFDM-based cognitive radio networks with low computational complexity. By exploiting some properties of the water-filling method, we proposed power increment water-filling and power decrement water-filling algorithms with much lower computational complexity than

traditional water-filling. Thus, the algorithm can be used to solve the power allocation problem with high efficiency. In the future, we will study how to apply the proposed algorithms to realistic cognitive radio networks.

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