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# ANALYSIS OF PARTIALLY ELECTRODED PIEZOELECTRIC ACTUATORS WITH NONUNIFORM THICKNESS FOR THE PURPOSE OF REDUCING ACTUATING SHEAR STRESS CONCENTRATION BY ANSYS

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An elastic plate with thin piezoelectric films bonded on its two major surfaces is a typical smart structure which is used in actuation and sensing. The useful deformation of the smart structure is caused by the shear stress transferring from the actuators to the elastic plate. This shear stress is concentrated at two ends of the interface between the actuator and the plate. This concentration may induce undesirable delamination of the actuator from the plate. It was theoretically proved that actuators with partially covered electrodes have a much less concentrated actuating shear stress than that with fully ones. An actuator with nonuniform thickness was also found that may reduce the concentration of shear stress. The previous theoretical results were based on two simplified models. However, it is very difficult to get an analytical solution when the actuator owns the two characteristics at the same time, i.e. not only with partially covered electrodes, but also with an in-plane varied thickness. In this paper, we turn to use ANSYS to obtain useful numerical results from cases which are nearly impossible to be solved analytically. We study the effect of the electroded area of the actuator on reducing the concentrated shear stress. Moreover, we investigate the shear stress distribution under different variation of the actuator thickness. An optimal thickness profile is obtained. This work is considered as a frontier of smart structure.

**Keywords:** Piezoelectric; Actuator; ANSYS

## 1. INTRODUCTION

The interfacial shear stress between the piezoelectric film and elastic plate is induced by expanding or contraction of the piezoelectric film, also referred to as actuator. It was theoretically proved that this interfacial shear stress concentrates at two ends of actuator [1-4]. In fact, the concentration of shear stress always exists for a film on a substrate when the film expands or contracts due to different effects [5,6]. It was theoretically proved that ending the electrodes a short distance from the edge of actuator reduces the stress concentration [1,2,4]. It was also shown analytically that actuators with nonuniform thickness may also reduce the concentration of shear stress [7]. The above results were obtained based on simple models. It is very difficult to get an analytical solution when a more complicated model is employed. Finite element analyses of piezoelectric actuator on a substrate were reported in [8-11]. However, the effects of actuator thickness and partially covered electrodes on reducing the shear stress concentration have not been studied by finite element method.

It is an important criterion to measure an actuator which can produce a large displacement or deflection, but with a low concentrated stress. In this paper, we study the effect of increment or decrement of the electroded area of the actuator on reducing the concentrated shear stress by using ANSYS. We also investigate the shear stress distribution under different variation of the actuator thickness and an optimal profile of the actuator thickness is obtained.

## 2. STRUCTURE AND MODEL

For the purpose of investigating shear stress concentration, a 2D plane strain model is employed as shown in Fig. 1. A piezoelectric actuator is bonded on the upper surface of an elastic plate. The elastic plate is fixed at the left end. The actuator is far from the two ends of the elastic plate, thus the shear stress distribution under the actuator is not influenced by the boundary conditions at the two ends according to the Saint Venant's principle. The electrodes are deposited on the upper and lower

surfaces of the actuator. The thicknesses of electrodes are much smaller than that of actuator, so the very small effect caused by the thickness and stiffness of electrodes is ignored in this model.

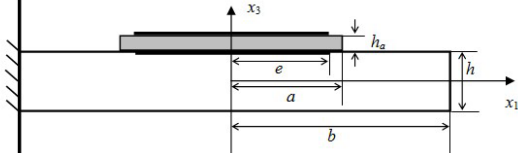


Figure 1. Uniform actuator

### 3. NUMERICAL RESULTS AND DISCUSSION

The actuator is made of piezoelectric ceramic PZT-5H which is poled in the thickness direction. The elastic plate is made of aluminum alloy 6061-T6. 1 V voltage is applied on the upper surface of the electrode, and the lower surface of the electrode is grounded.  $a=5$  cm,  $b=10$  cm,  $h=1$  cm. Quadrilateral element PLANE13 for coupled fields is used to mesh the actuator and quadrilateral PLANE182 is used to mesh the elastic plate. The element size is unified as 0.5 mm for all the following cases.

#### 3.1. Uniform thickness actuator

First we investigate uniform thickness actuator with fully covered electrodes. The uniform thickness of actuator  $h_a=2$  mm,  $e=5$  cm. Figure 2 shows the distribution of shear stress  $T_{31}$  along the interface between the actuator and the elastic plate ( $x_3=h/2$ ). The maximum shear stresses (3437 Pa) occur at the two ends of the actuator. The maximum displacement of the plate is at the top right corner (0.108  $\mu\text{m}$ ).

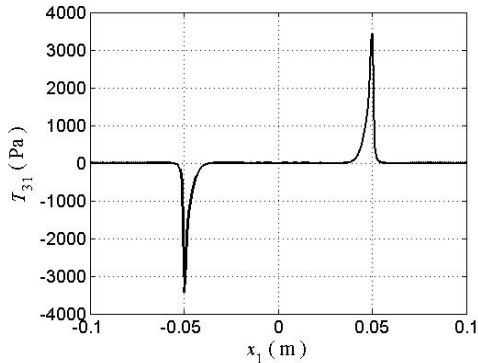


Figure 2. Shear stress distribution of uniform actuator with fully covered electrodes

If the same thickness actuator is partially covered with electrodes ( $e=4$  cm), the distribution of shear stress  $T_{31}$  is shown in Fig. 3. The concentration of stress at two ends is obviously reduced to 2661 Pa under the partially covered electrodes, as predicted in [1,2,4]. The maximum

displacement at the top right corner is reduced to 0.0916  $\mu\text{m}$ . Figure 3 also shows that if the length of electrodes decreases to  $e=3$  cm, the concentration of stress is not further reduced, but the maximum deflection drops to 0.069  $\mu\text{m}$ .

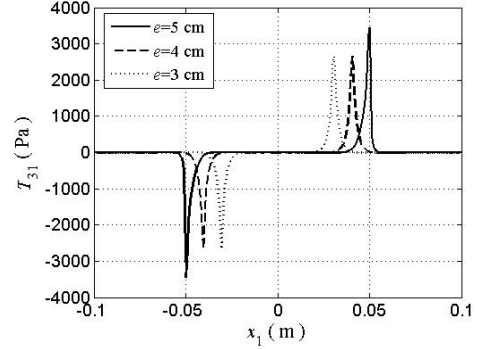


Figure 3. Shear stress distribution of uniform actuator with covered electrodes of different  $e$

If the actuator is fully covered with  $e=5$  cm, but with different  $h_a$ , the shear stress  $T_{31}$  at the interface near the left end of the plate is shown in Fig. 4. The concentration of stress at two ends is reduced to 2489 Pa if  $h_a$  increases from 2 mm to 3 mm, but maximum displacement is also reduced to 0.0856  $\mu\text{m}$ . The stress concentration is 1920 Pa if  $h_a=4$  mm, and the maximum displacement further decreases to 0.069  $\mu\text{m}$ . It is shown from Fig. 4 that shear stress concentration becomes smaller with increasing thickness, however the actuation capability of the actuator also decreases.

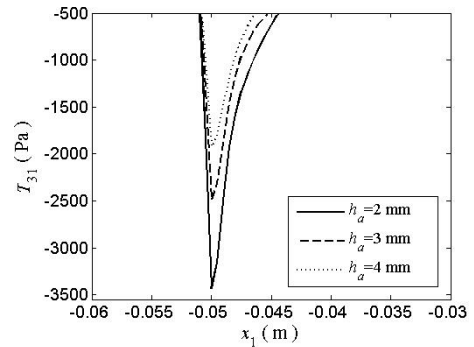


Figure 4. Shear stress distribution of uniform actuator with different thickness

#### 3.2. Nonuniform thickness actuator

We next investigate the effect of nonuniform-thickness actuator on the stress concentration at two ends. Based on the theoretical conclusion that concentrated shear stress is reduced when the actuator is thicker near the edges [7], we build a model with step thickness as shown in Fig. 5. The edges of actuator are raised to two steps where  $h_a=4$  mm. The middle thickness  $h_m=2$  mm. It is

observed from Fig. 3 that the width of shear stress concentration is about 2 cm, so the stress concentration would be trapped in the two steps if  $a_m=2$  cm, the rest dimensions remain unchanged. The distribution of shear stress  $T_{31}$  at  $x_3=h/2$  is shown in Fig. 6.

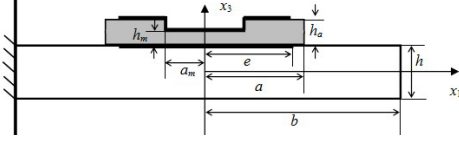


Figure 5. Actuator with step thickness

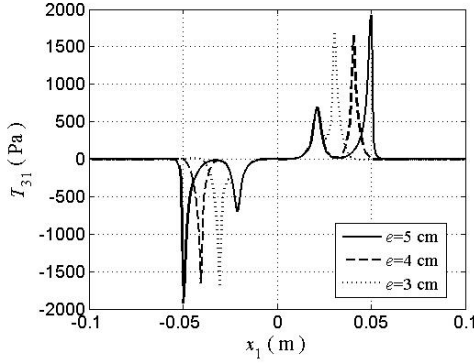


Figure 6. Shear stress distribution of step-thickness actuator with different  $e$

If the actuator is fully covered with electrodes  $e=5$  cm, the concentration of stress at two ends is 1920 Pa (44% of concentrated stress is reduced compared with uniform actuator with fully covered electrodes). The maximum displacement at the plate tip is  $0.0856 \mu\text{m}$  (79% of that for fully covered uniform actuator). If the electrode is partially covered with  $e=4$  cm, the maximum shear stress is 1645 Pa (38% of concentrated stress is reduced compared with uniform actuator with  $e=4$  mm), the maximum displacement is  $0.0769 \mu\text{m}$  (84% of that for uniform actuator with  $e=4$  mm). If  $e=3$  cm, the maximum shear stress is increased a little to 1671 Pa, however, the maximum displacement is further reduced to  $0.0620 \mu\text{m}$ .

We conclude that step-thickness actuator has smaller concentration at the two ends and at the same time it maintains the actuation capability at a certain level. However, further decreasing  $e$  will not further reduce the stress concentration. The effect of partially covered electrodes on reducing the stress concentration has its limit. The above results are illustrated in Table 1. We also observe from Fig. 6 that there are small stress concentrations at  $x_1=\pm a_m$  due to the sudden jump of actuator thickness.

Table 1. Concentrated stress (Pa) and maximum displacement ( $\mu\text{m}$ )

Actuator	$e=5$ cm	$e=4$ cm	$e=3$ cm
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Uniform	3437	2661	2661
( $h_a=2$ mm)	0.108	0.0916	0.069
Step-thickness	1920	1645	1671
	0.0856	0.0769	0.0620
Smooth concave	1918	1645	1658
	0.0822	0.0734	0.0586

We also investigate the effect of the width of the step. A step-thickness actuator with fully covered actuators but with different  $a_m$  is shown in Fig. 7, if  $a_m=3$  cm the maximum shear stress is not reduced (1921 Pa), but the maximum displacement increases from  $0.0856$  to  $0.0934 \mu\text{m}$ . If we further narrow the step width to  $a_m=4$  cm, the maximum shear stress increases to 1973 Pa, the maximum displacement increases to  $0.101 \mu\text{m}$ . It is also shown that the stress concentration at  $x_1=\pm a_m$  is 559 Pa when  $a_m=2$  or 3 cm, but raised to 661 Pa when  $a_m=4$  cm. It can be concluded from Fig.7 that smaller step width has little effect on reducing the stress concentration, but obviously generates larger actuation capability.

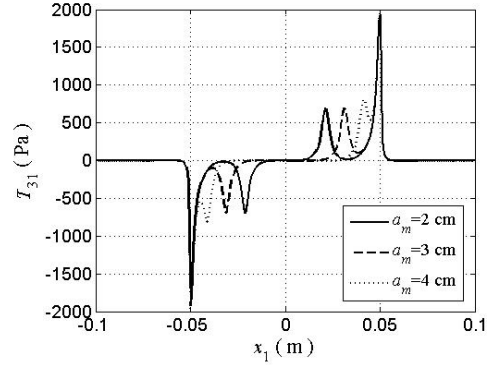


Figure 7. Shear stress distribution of step-thickness actuator with different  $a_m$

### 3.2.2 Smooth concave actuator

In order to reduce the stress concentration due to the sudden jump of actuator thickness, our next model includes a smooth concave actuator in Fig. 8 where  $a_r=2$  cm,  $h_m=0.02$  mm, all the other dimensions remain unchanged. Result is shown in Fig. 9.

For the effect of electroded area on reducing shear stress concentration, the current model has very similar result as shown by the step-thickness actuator. If the electrode is fully covered  $e=5$  cm, the concentration of stress at two ends is 1918 Pa (44% of concentrated stress is reduced compared with uniform actuator with fully covered electrodes), maximum displacement at the plate tip is  $0.0822 \mu\text{m}$  (76% is maintained). If the electrode is partially covered with  $e=4$  cm, the maximum shear stress is 1645 Pa (38% of concentrated stress is reduced compared with uniform actuator with  $e=4$  mm), the

maximum displacement also reduces a little to 0.0734  $\mu\text{m}$  (80% is maintained). If  $e=3$  cm, the maximum shear stress is no more reduced, but increased a little to 1658 Pa, the maximum displacement becomes 0.0586  $\mu\text{m}$ . The above results are also illustrated in Table 1. However, the advantage of current model over the step-thickness actuator is that the small stress concentrations at  $x_1=\pm a_r$  are reduced significantly from 556 Pa in Fig. 6 of previous case to 279 Pa in current smooth-thickness case, nearly 50% of stress is reduced.

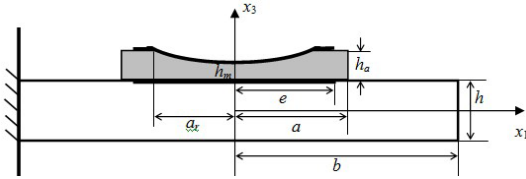


Figure 8. Smooth concave actuator

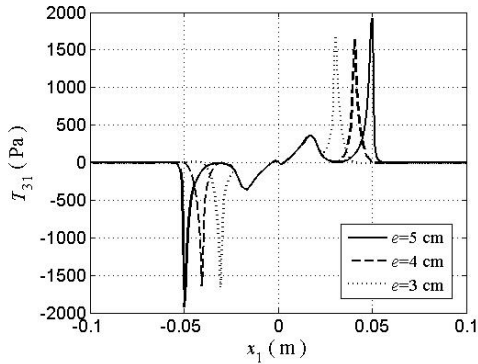


Figure 9. Shear stress distribution of smooth concave actuator with different  $e$

#### 4. CONCLUSION

By performing numerical experiments using ANSYS, we confirm that partially covered electrodes reduces the concentrated shear stress, but further decrement of electrode area would not reduce the concentration any more. The effect of partially covered electrodes on reducing the stress concentration has its limit.

We also investigate the shear stress distribution under step-thickness and smooth concave actuators. Smooth concave actuators generate less stress concentrations and meanwhile still maintain certain level of actuation capacity.

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