Nanomagnetic skyrmions

Ralph Skomski
University of Nebraska-Lincoln, rskomski2@unl.edu

Zhen Li
University of Nebraska-Lincoln

Rui Zhang
University of Nebraska-Lincoln

Roger D. Kirby
University of Nebraska-Lincoln, rkirby1@unl.edu

Axel Enders
University of Nebraska–Lincoln, a.enders@me.com

See next page for additional authors

Authors
Ralph Skomski, Zhen Li, Rui Zhang, Roger D. Kirby, Axel Enders, Daniel Schmidt, Tino Hofmann, Eva Schubert, and David J. Sellmyer

Nanomagnetic skyrmions

Ralph Skomski,[1,a)] Zhen Li,[1] Rui Zhang,[1] Roger D. Kirby,[1] A. Enders,[1] D. Schmidt,[2] T. Hofmann,[2] E. Schubert,[2] and D. J. Sellmyer[1]
[1]*Department of Physics and Astronomy and Nebraska Center for Materials and Nanoscience, University of Nebraska, Lincoln, Nebraska 68588, USA*
[2]*Department of Electrical Engineering and Nebraska Center for Materials and Nanoscience, University of Nebraska, Lincoln, Nebraska 68588, USA*

(Presented 31 October 2011; received 24 September 2011; accepted 24 October 2011; published online 22 February 2012)

Magnetic skyrmions and other topologically protected nanostructures are investigated. Since skyrmions are mathematical rather than physical objects, they describe a wide variety of physical systems, from simple magnetic domain walls to complicated quantum phases with long-range many-body entanglement. Important distinctions concern the skyrmions' relativistic character, their quantum-mechanical or classical nature, and the one- or many-body character of the wave functions. As specific examples we consider magnetic nanospirals, where the topology of a vortex-like spin state is protected by magnetostatic interactions, and edge currents in dilute magnetic semiconductors and metallic nanodots. Our analysis militates against giant orbital moments created by a mesocopically enhanced spin-orbit coupling. © *2012 American Institute of Physics*. [doi:10.1063/1.3672079]

I. INTRODUCTION

Skyrmions are mathematical objects that have fascinated the magnetism community for many years, covering a number of physically very different phenomena. Skyrme's original idea[1] emerged in nuclear physics, where skyrmions are specific solutions of a nonlinear sigma model, used to describe baryons. Today, the term is also used for a wide variety of nonlinear phenomena, such as dislocations in liquid crystals, vortices in superconductors, magnetic domain walls,[2] and quantum states in topological insulators.[3]

By definition, skyrmions are a homotopically non-trivial soliton-like solutions of nonlinear field equations, typically characterized by topological invariants such as winding numbers. Figure 1 compares a trivial spin structure (a) with two soliton-like skyrmionic spin configurations (b),(c). The structures (b) and (c) are homotopically equivalent, that is, they can be continuously deformed into each other. Homotopic nonequivalence is often imposed by boundary conditions, as in Fig. 1.

Some skyrmions are relatively simple classical objects, but many skyrmion-like phenomena are based on highly nontrivial quantum physics. These include anyons,[4] fractional quantum Hall effect,[4,5] quantum phases,[6] quasi-particles with fractional statistics,[5,7] and topological insulators.[3] These systems involve features such as a long-range entanglement of many-body wave functions[6] and Berry phases.[8]

Topological solitons are very stable entities, because their topological nature protects them against perturbations. The robust character of skyrmions is potential useful in technology. For example, anyons are considered as potentially useful for fault-tolerant quantum computers.[9]

The aim of this paper is to show that nanoscale magnetic skyrmions form a multifaceted world with a rich and diverse physics. Key distinctions are classical physics versus quantum mechanics, relativistic versus nonrelativistic interactions, and single-particle versus many-particle effects. As explicit examples, we discuss magnetic nanospirals and possible giant orbital moment in magnetic nanoparticles.

II. RELATIVISTIC AND COULOMB EFFECTS

A major distinction is between nonrelativistic (or electrostatic) interactions, such as exchange, and relativistic interactions, such as spin-orbit coupling (SOC). The latter are weaker by a factor of order α^2 than the former, where $\alpha = 1/137$ is Sommerfeld's fine-structure constant, but they play an important role in many skyrmionic effects. Magnetic domain walls, as in Fig. 1(b) involve magnetic anisotropy, which originates from spin-orbit coupling and is therefore a relativistic effect. Noncollinearities due to relativistic Dzyaloshinski–Moriya (DM) interactions occur in ordered[10,11] and disordered[12,13] structures with broken inversion symmetry. Figure 2 illustrates how DM interactions yield noncollinear spin structures, including some types of skyrmions. Two ways of writing the DM interactions are $\mathscr{H} = \gamma\, \mathbf{M} \times \nabla \mathbf{M}$ (Ref. 11) and $\mathscr{H} = D_{ij} \cdot \mathbf{S}_i \times \mathbf{S}_j$,[12] where the DM vector $\mathbf{D}_{ij}$ depends on the atomic positions $\mathbf{R}_i$ and $\mathbf{R}_j$ of the involved atoms. For long-wavelength spin spirals of the type $M_x = {}_{Mo}\cos(kz)$, $M_y = M_o \sin(kz)$, and $M_z = 0$, the two definitions are equivalent with $\mathbf{D} = \mathbf{k}\, \gamma$.

However, many skyrmions are protected by interatomic exchange in combination with geometrical boundary conditions. Examples are Fig. 1(b) and skyrmions formed by adding a ↓ electron to completely filled two-dimensional ↑ subband.[14] The latter is a correlation effect and means that rapid magnetization changes are opposed by an exchange-type energy, similar to Fig. 1(c). In fact, most noncollinear

a)Author to whom correspondence should be addressed. Electronic mail: rskomski@neb.rr.com.

0021-8979/2012/111(7)/07E116/3/$30.00

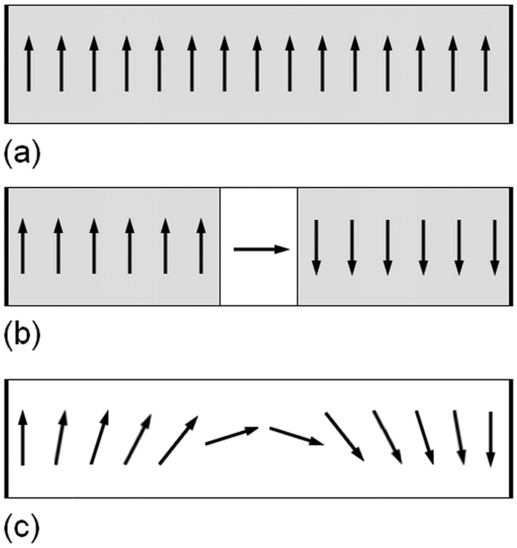

FIG. 1. Simple topological excitations in a nanomagnet: (a) ground state of a ferromagnetic chain, (b) domain wall, and (c) delocalized spin change. The configurations (b) and (c) are protected, that is, (a) cannot be reached by continuous deformation.

spin structures encountered in practice are caused by competing exchange and may mask weaker relativistic effects.[13] Examples are the helimagnetism of the heavy rare-earth elements[10] and the noncollinearities in many Cr and Mn alloys. Interestingly, Figs. 1(b) and 1(c) are homotopically equivalent, that is, there is a continuous deformation between structures without (c) and with (b) relativistic interactions.

Other examples of geometrically protected spin structures are magnetic nanotubes[15] and *nanospirals*, Fig. 3, which can be produced by glancing-angle deposition (GLAD) onto a rotating substrate.[16] The materials can be nonmagnetic (e.g., Si) or magnetic (e.g., Co), and the spirals (helices) have nanoscale feature sizes with wire diameters below 50 nm. The exchange energy E_{ex} in long and thin spirals is obtained by integration over the exchange-energy density

$$E_{ex} = A \int \left(\frac{\nabla \mathbf{M}}{M_s} \right)^2 dV, \qquad (1)$$

where A is the exchange stiffness of the magnetic material. This energy depends on the geometry of the spirals. Assuming that $\mathbf{M}$ is parallel to the wire axis (that is, protected by

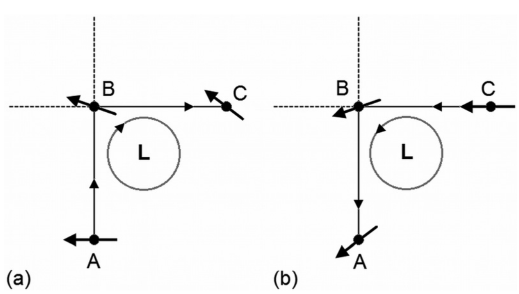

FIG. 2. DM interactions, spin-orbit coupling, and broken inversion symmetry. The electrons of (a) and (b) move in opposite directions but exhibit the same clockwise spin precession along the path ABC.

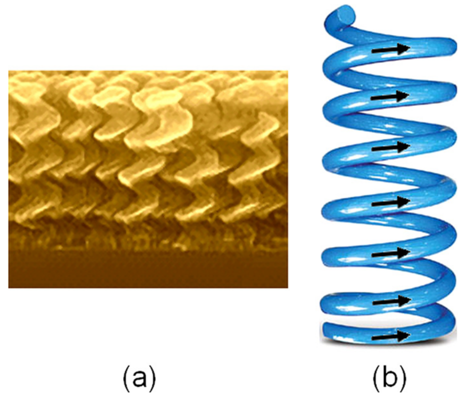

FIG. 3. (Color online) Nanospirals: (a) Si nanospirals produced by GLAD and (b) chirally spin-polarized magnetic nanospiral.

magnetostatic self-interactions in combination with coil geometry), Eq. (1) yields

$$E_{ex} = \pi A \left(\frac{a}{R} \right)^2 L \left(1 - h^2/L^2 \right). \qquad (2)$$

Here a is the wire radius, R is the radius of the coil, L denotes the contour length of the wire, and h is the total height h of the coil. For example, the fictitious extension of a spiral to full length, $h = L$, leads to $E_{ex} = 0$, because the corresponding magnetization $\mathbf{M} = M_z\, \mathbf{e}_z$ is homogeneous.

III. QUANTUM EFFECTS

Quantum-mechanical skyrmions form part of the "new quantum mechanics" based on concepts such as Berry phases[8] and quantum orders.[6] The Berry phase is a geometrical phase factor in the wave function that arises from adiabatically slow variations of a parameter, unrelated to the familiar dynamical phase factor from the time-dependent Schrödinger equation. The phase is essential for the understanding of solid-state phenomena such as orbital magnetism and the anomalous Hall effect,[17] but even more important for the understanding of topological or quantum orders.[6] These orders are caused by nonlocal many-electron quantum entanglement.

Quantum-mechanical skyrmions often involve non-simply connected or "doughnut-like" spaces. A simple example is the Aharonov–Bohm effect, where the Berry phase corresponds to the vector potential and the magnetic flux line forms the "hole" of the doughnut. Other examples are the fractional quantum Hall effect and the related concept of anyons,[4] where the electrons move around each other on

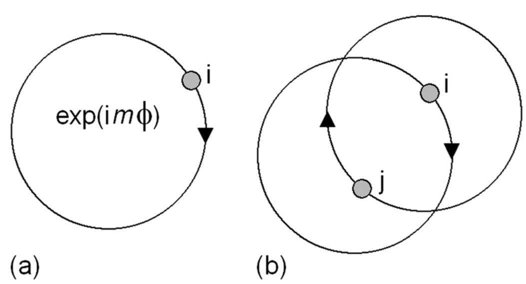

FIG. 4. Nanoscale meaning of the fractional Hall effect: (a) one-electron orbit and (b) many-electron orbits.

doughnut-like mesoscopic trajectories,[6] as illustrated in Fig. 4(b).

Topological insulators, such as $Bi_{1-x}Sb_x$, are electronic materials that have bulk band gaps like ordinary insulators but exhibit protected conducting states on their surface.[3,18] These states involve spin-orbit interaction, which couples the electron's wave vector to the spin and means that electrons on the surface of a topological insulator are not allowed to make U-turns.[19]

IV. ORBITAL MOMENT

An interesting question is whether the unusual and poorly understood orbital-moment behavior of dilute magnetic semiconductors[20] and of nanostructures such as AuFe nanoparticles[21] is a Berry-phase effect. It is well-known that the Berry phase and the orbital moment are closely related[17,22] and that edge states are important in some quantum systems.[6] One possible mechanism is that the spin-orbit coupling at the edges of the sample creates macroscopic currents that yield a big magnetic moment.[23] The corresponding phenomenon is basically a one-electron process, governed by the spin-orbit coupling

$$\mathcal{H} = \frac{\hbar^2}{4m^2c^4}\boldsymbol{\sigma}\cdot(\nabla V \times \mathbf{k}), \tag{3}$$

where $\mathbf{k}$ is the wave vector of the moving electron and V is the electrostatic potential.[24] Note that the usual form of the SOC, in terms of L, is obtained from this equation by introducing spherical potentials $V(\mathbf{r}) = V(r)$. Adding the kinetic energy, which is quadratic in k, to the SOC term of Eq. (3) yields an energy scaling as

$$E = E_o(a_o k)^2 - \alpha^2 E_1 a_o k, \tag{4}$$

where both E_o and E_1 are of the order of a few eV, and $a_o = 0.529$ Å is the Bohr hydrogen radius. The same procedure applied to the energy expression Eq. (2) in Ref. 23 yields

$$E = E_o(a_o k)^2 - \alpha^2 E_1 R k, \tag{5}$$

where R is the lateral feature size, for example, the radius of a nanodot. The only difference between the two equations is the replacement of a_o in Eq. (4) by R in Eq. (5). In other words, Eq. (5) greatly overestimates the effect of the spin-orbit coupling. Note that the spin-orbit interaction of Eq. (3) is a local quantity and depends on spin, potential gradient, and electron velocity, but not on the feature size R.

Interestingly, the orbital moment follows from the Berry phase,[17] but it can also be obtained directly from the local currents $\mathbf{j} \sim \nabla\psi$,[10] similar to the scenario outlined in Ref. 25 and equivalent to Eq. (3). This avoids the use of ill-defined[17] angular-momentum vectors $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, which is one of the reasons for considering the Berry phase in the context of the orbital moment.

V. CONCLUSIONS

In summary, magnetic skyrmions are nanoscale objects that involve a wide variety system-dependence mechanisms. One specific example is the possibility of giant orbital currents in magnetic nanodots and nanoparticles. In spite of the close relationship between Berry phase and orbital moment—and in spite of the long-range entanglement effects in typical quantum phases, the orbital moment is created locally, involving local currents and local spin-orbit coupling.

ACKNOWLEDGMENTS

The authors are grateful to M. Stocks and J. M. D. Coey for discussing Berry-phase effects in magnetic solids and orbital moments in dilute magnetic semiconductors, respectively. This research is supported by NSF-MRSEC (Grant No. NSF-DMR-0820521) and NCMN.

[1] T. H. R. Skyrme, Nucl. Phys. **31**, 556 (1962).
[2] E. M. Chudnovsky and J. Tejada, *Lectures on Magnetism* (Rinton Press, Princeton, 2006).
[3] P. Roushan, J. Seo, C. V. Parker, Y. S. Hor, D. Hsieh, D. Qian, A. Richardella, M. Z. Hasan, R. J. Cava, and A. Yazdani, Nature **460**, 1106 (2009).
[4] I. J. R. Aitchison and N. E. Mavromatos, Contemp. Phys. **32**, 219 (1991).
[5] F. D. M. Haldane, Phys. Rev. Lett. **67**, 937 (1991).
[6] X.-G. Wen, *Quantum-Field Theory of Many-Body Systems* (University Press, Oxford, 2004).
[7] X.-L. Qi, R.-D.Li1, J.-D. Zang, and Sh.-Ch. Zhang, Science **323**, 1184 (2009).
[8] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984).
[9] Ch. Nayak, Nature **464**, 693 (2010).
[10] R. Skomski, *Simple Models of Magnetism* (University Press, Oxford, 2008).
[11] P. Bak and H. H. Jensen, J. Phys. C **13**, L881 (1980).
[12] K.-H. Fischer and A. J. Hertz, *Spin Glasses* (University Press, Cambridge, 1991).
[13] R. Skomski, J. Phys.: Condens. Matter **15**, R841 (2003).
[14] S. L. Sondhi, A. Karlhede, S. A. Kivelson, and E. H. Rezayi, Phys. Rev. B **47**, 16419 (1993).
[15] Y. C. Sui, R. Skomski, K. D. Sorge, and D. J. Sellmyer, Appl. Phys. Lett. **84**, 1525 (2004).
[16] E. Schubert, Th. Höche, F. Frost, and B. Rauschenbach, Appl. Phys. A **81**, 481 (2005).
[17] D. Xiao, M.-Ch. Chang, and Q. Niu, Rev. Mod. Phys. **82**, 1959 (2010).
[18] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
[19] M. Franz, Nature **466**, 323 (2010).
[20] J. M. D. Coey, Current Opin. Solid State Mater. Sci. **10**, 83 (2006).
[21] J.-H. Wu, J. H. Min, H.-L. Liu, J. U. Cho, and Y. K. Kim, IEEE Trans. Magn. **45**(6), 2442 (2009).
[22] A. G. Aronov and Y. B. Lyanda-Geller, Phys. Rev. Lett. **70**, 343 (1993).
[23] A. Hernando, P. Crespo, and M. A. García, Phys. Rev. Lett. **96**, 057206 (2006).
[24] W. Jones and N. H. March, *Theoretical Solid State Physics I* (Wiley, London, 1973).
[25] J. Zhang, R. Skomski, Y. F. Lu, and D. J. Sellmyer, Phys. Rev. B **75**, 214417 (2007).