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LEARNING TO TEACH MATHEMATICS WITH
REASONING AND SENSE MAKING

by

Amy L. Nebesniak

A DISSERTATION

Presented to the Faculty of
The Graduate College at the University of Nebraska
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LEARNING TO TEACH MATHEMATICS WITH
REASONING AND SENSE MAKING

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This study uses teacher research to examine teacher learning in the context of instructional coaching. The author, a mathematics instructional coach, engaged in an intense three-week coaching relationship with a high school Algebra teacher. A detailed description of the teaching and learning of quadratics that took place during this research provide information about what and how a teacher learns to teach mathematics with reasoning and sense making. Mapping the terrain of quadratics deepened the teacher's understanding of the mathematical content and encouraged him to adapt his textbook in order to build mathematical reasoning. Through the coaching process, the teacher also enhanced his specialized content knowledge and developed pedagogical reasoning skills when faced with teaching dilemmas. Finally, a discussion about instructional coaching considers an instructional coach's role in regard to teacher learning.

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To Patrick

Your continuous love and encouragement
gave me the confidence to follow this dream.

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INTRODUCTION

American high school students are struggling to learn mathematics. Key indicators include the low mathematical achievements of American high schools, the below-average performance in problem solving compared with that of other industrialized countries, and the practice of repeating basic mathematics year after year by students deemed unable to grasp higher mathematics. (Silva, Moses, Rivers & Johnson, 1990) These indicators are even more apparent in minority students.

What can be done to improve student success? What is the leading factor influencing students' understanding of mathematics? The teacher determines what a student learns in a math class and how well the student understands the mathematics. In other words, what a student learns depends on whom the student has as a teacher (Colvin & Johnson, 2007; Schmoker, 2011). More specifically, a teacher's instructional techniques are recognized as the major contributing factor in a mathematics student's success or failure (National Council of Teachers of Mathematics (NCTM), 2000). Students achieve more when they have access to high-quality teaching (Loucks-Horsley, Stiles, Love, Mundry, & Hewson, 2003; National Mathematics Advisory Panel, 2008; Wright et al., 1997). The teachers who demonstrate high-quality mathematics instruction experience greater success with their students as compared to teachers who do not incorporate effective instructional techniques into their classroom (NCTM, 2000). Since a teacher's instruction is the greatest influence on student achievement, then "improved classroom instruction is the prime factor to improve student achievement gains" (Odden & Wallace, 2003, p. 64).

A teacher's preparation and continued professional development influences what his students learn (Adler, Ball, Krainer, Lin & Novotna, 2005). Professional development is often cited as the path that will bring about change in teachers' instructional practices, attitudes, and beliefs as well as students' learning outcomes (Guskey, 2002). Through a systematic process, teachers deepen their understanding of content and instructional techniques (Desimone, Porter, Garet, Yoon, & Birman, 2002). Professional development not only introduces new ideas or strategies, but also provides teachers with support (i.e., opportunity to problem solve, continued encouragement) as they implement the new strategies (Guskey, 1986). One strategy that features effective professional development, and thus a possible solution to the issue of students struggling, is coaching.

Instructional coaching is a specific professional development strategy directly aimed at “improving instruction to improve learning” (Saphier & West, 2009, p. 47). Coaching incorporates numerous characteristics of effective professional development (National Staff Development Council (NSDC), 2001), including being site-based and on-going (Barr, Simmons, & Zarrow, 2003), focusing on the specific curriculum being implemented (Joyce & Showers, 1996), collaborating amongst peers (Becker, 1996), and being supportive rather than evaluative and judgmental the teacher (Shanklin, 2006). How I define coaching, using both my own practice and the literature on coaching, is described in greater detail in chapter two.

Algebra

Algebra is a core element of mathematics and all students, “regardless of personal characteristics, backgrounds, or physical challenges” (NCTM, 2000, p. 12), should be

given the opportunity to learn algebra. Algebra is becoming a graduation requirement in high schools across the United States (Chazan, 2008) and is deemed by NCTM as one of the content standards to be taught from kindergarten through grade 12. Algebraic concepts are continuously utilized in subsequent mathematics courses, making it a gateway course to all other mathematics. Moses and Cobb's (2001) identify Algebra as the gatekeeper to all other mathematics. Since Algebra is the gateway into abstract thinking (Achieve, 2008), students need to master algebraic concepts in order to become math literate (Moses & Cobb, 2001).

Algebra is a strong predictor of graduation from high school (Quint, 2006), therefore making the course a gatekeeper into life beyond secondary schooling. The completion of advanced mathematics beyond Algebra has a direct impact on future income (Rose & Betts, 2004) and has a greater influence on whether students will graduate from college than any other factor – including family background (Adelman, 2006). Knowledge of algebra concepts is necessary for numerous careers, which now include many traditional “blue collar” jobs (ACT, Inc., 2006). Therefore, the success of a student in Algebra has implications that reach far beyond the high school classroom.

Defining Algebra

A definition of algebra largely depends on the context being discussed. Since the setting of this research is the high school classroom, the term algebra, when used in this study, will refer to those concepts emphasized in a traditional high school Algebra course, or “school algebra” (Katz, 2007). Algebra can be viewed as having two major conceptual goals: generalized arithmetic and functional thinking (Kilpatrick & Izsak, 2008). Algebra is generalized arithmetic involving a variety of mathematical skills and

manipulations with unknown values. For the majority of the nineteenth and twentieth centuries in America, algebra has been an extension of arithmetic (Kilpatrick & Izsak, 2008), yet algebra involves more than symbol manipulation and generalized arithmetic. Learning algebra means understanding the properties of symbol manipulation (NCTM, 2000) and being able to apply those properties to various situations. For example, not only are students expected to know how to factor the polynomial $x^2 - 1$, but they also need to understand how that same algebraic manipulation can be used in the factorization of $4899 = 70^2 - 1$ (Saul, 2008).

Current high school Algebra courses also include a second major emphasis: functional thinking. The study of functions was first incorporated into school algebra in the late 1900s (Kilpatrick & Izsak, 2008). A high school Algebra course allows students to build on earlier experiences with linear functions, introduce other nonlinear relationships (quadratic, exponential, radical functions), and exploring various ways to represent a function (i.e., graphically, algebraically). Students develop conceptual understanding of each class of function and learn to recognize the defining characteristics of various functions (NCTM, 2000). The focus on functions in algebra provides students with the skills to mathematically model various situations as a way to analyze and explain what is happening around them.

Teaching Algebra

If we want Algebra students to understand algebraic concepts, apply their knowledge, have confidence in their mathematical knowledge and be successful in the classroom, then Algebra teachers need to implement high-quality instruction (NCTM, 2000; Wright et al., 1997). My goal here is not to create an exhaustive list of what

constitutes high-quality Algebra instruction. Instead, I have chosen to highlight a few of the elements of instruction that research shows improve student achievement. These key elements include teacher content knowledge, student conceptual understanding, characteristics of reasoning and sense making skills, connections between mathematical concepts, and the planning that takes place prior to instruction.

Deepening content knowledge. A more in-depth understanding of mathematics helps a teacher be more effective (Ball, Hill, & Bass, 2005; Ma, 1999). Algebra teachers should deeply understand the mathematical concepts, procedures, and reasoning skills central to Algebra (Papick, 2011). Therefore, one way to improve mathematics education is to deepen teachers' understanding of mathematical knowledge for teaching (Bruning, 2004). Little research has been conducted on what specific content knowledge is necessary for an Algebra teacher to effectively teach algebra, although the research that is available has focused on knowledge of linear equations, functions, and slope (Fey et al., 2007; Resek et al., 2007). The implication is that by helping teachers develop a greater understanding of the mathematics they teach (i.e., equations, functions, and slope), teachers will more effectively teach the content to students and increase student understanding (Resek et al., 2007). Thus an emphasis has been put on helping teachers know mathematics at a depth and in detail far beyond simply knowing how to perform the basic mathematical procedures (Ball, Hill, & Bass, 2005).

Conceptual understanding. An important reform in mathematics education is the call for a focus on teaching conceptually (Kilpatrick, Swafford, & Findell, 2001). Without conceptual understanding, learning math procedures is meaningless since students are not able to or know when to apply math skills (Seeley, 2009). Instruction

focused on conceptual understanding helps students learn the mathematics at a deeper level (NCTM, 2009; National Mathematics Advisory Panel, 2008), increases students' retention of the information (NCTM, 2009), and increases the likelihood that students can transfer their knowledge to new learning situations (Shepard, Hammerness, Darling-Hammond, Rust, et al., 2005).

Teaching for understanding goes beyond procedures (Kilpatrick & Izsak, 2008), although procedural knowledge and computational fluency are still important aspects of mathematics instruction (Hiebert et al., 1997; National Mathematics Advisory Panel, 2008; Rickard, 1998; Seeley, 2009). Rather than limit mathematics to procedures and computation, teachers who teach conceptually facilitate deeper investigations into a mathematical idea (Kilpatrick et al., 2001). When focusing instruction on the conceptual meaning of mathematics, a teacher is combating a “memorize and repeat” environment while helping students understand and think mathematically. The goal of conceptual instruction is to build understanding of big picture mathematics (NCTM, 2000) rather than teaching disjointed lessons focused on procedural skills.

Reasoning and sense making. Reasoning and sense making are at the heart of learning mathematics conceptually, and have recently received greater attention in the high school setting (Graham et al., 2010; NCTM, 2009). Incorporating thinking skills into mathematics courses is essential to students' understanding and success (Council of Chief State School Officers (CCSS)), 2010). This implies that covering topics is not enough; students need to experience mathematics and create meaning for themselves by reasoning about the math and making sense of what is happening within the mathematics (NCTM, 2009). Instruction focused on reasoning (using evidence to draw conclusions)

and sense making (connecting new knowledge to existing knowledge) increases students' understanding of mathematics (Graham et al., 2010; NCTM, 2009). Teachers infuse reasoning and sense making into the curriculum by choosing specific mathematical tasks, facilitating math discussions (Graham et al., 2010), and posing questions that urge students to think about what is happening mathematically and explain their thinking (NCTM, 2009).

Teaching based on student learning. Students are more likely to understand and retain mathematics when it is taught as an intricate network of concepts and skills that build on one another (NCTM, 2009). Effective mathematics teachers assess students' current knowledge and build new concepts on prior knowledge (Shell et al., 2009). By connecting new learning to prior knowledge, teachers are helping students make connections among concepts and build schema (Bruning, 2004). These connections and schema will allow students to access new knowledge more easily in the future. Effective instruction begins with what the students know and understand and explicitly builds new connections. If math is taught and learned in these ways, new topics would no longer be seen as isolated ideas, but rather a continuation of previous material.

Why should effective instruction emphasize the connections between curricular concepts, as well as the connections between prior knowledge and new learning? Shell et al. (2009) explains:

What is stored together stays together; what is retrieved together stays together. Knowledge is not contained in a single neuron. Knowledge is a connected pattern of neurons. Higher, more complicated forms of learning, like concept formation

or skill development, are essentially about connections. *This means that effective teaching and instruction are about insuring that students are attending to the proper connections.* (Italics in original, p. 15)

The more ways information is connected, the more ways it can be accessed. When a teacher helps students link new concepts to other mathematical understandings, the students are given a higher chance of retrieving and using the new knowledge at a later time due to the vast number of neurological connections. Teachers are essentially helping students retain what they learn by teaching with connections to prior knowledge (NCTM, 2009).

Planning. When planning mathematics instruction, one tends to focus on the actions that take place in the classroom during a lesson. Yet researchers argue that the events that occur outside the classroom such as lesson planning and reflecting on instruction are what matter most (Morris, Hiebert, & Spitzer, 2009; Smith & Stein, 2011). Lesson preparation is an essential factor in growth and improvement in teaching (NCTM, 2007). Planning before a lesson is the time when a teacher thinks about strategies for teaching conceptually, the kinds of connections among mathematics that can be fostered through instruction, and how to promote student reasoning and sense making.

An important aspect of the planning phase is determining high-quality questions that promote student understanding. Questions that are worth asking, the kind that really assess student learning, are not all created on-the-spot. Instead these questions need to be carefully formulated prior to teaching the lesson (William, 2007). Planning instruction involves anticipating students' responses in order to promote productive math discussion within a lesson (Smith & Stein, 2011). This planning includes thinking about the

mathematical representations that are most useful in learning the mathematics, strategies students will employ to solve a problem, and difficulties students might encounter.

My Study

My study is about trying to understand what it takes to teach Algebra with reasoning and sense making. With coaching as the context and me in the role of instructional coach, I attempt to examine the role of reasoning and sense making as it pertains to teaching mathematics. I am not seeking to prove one way of teaching mathematics is better than another, or that instructional coaching should be done in a certain fashion. Instead, the purpose of this study is to provide insight into what a teacher's learning looks like when supported by an instructional coach. My hope is to tell the story of my experience and connect what I learned to mathematics teaching, learning, research in math education, and professional development.

Identifying my Problem of Practice

The research presented here was conducted, analyzed, and written as I simultaneously worked full-time as an instructional coaching. In addition to physically positioning myself as the coach, I also utilized various intellectual perspectives throughout my research. The knowledge I gained during my seven years as a mathematics teacher influenced my work as a coach and, therefore, the research discussed in this paper. Knowledge I gained through my teaching experiences, knowledge such as what teaching Algebra for understanding means and the importance of focusing on students' learning, is part of this story and research. I also embodied a researcher role throughout my inquiry, stepping back to analyze what was happening and how that related to other educational literature.

How I used the various perspectives (teacher, coach, researcher) to address my inquiry can be seen throughout the paper. Each perspective added information and a new spin on the data collected. If I had tried to isolate one of these roles, the research would not be complete. By approaching my inquiry in a variety of ways, I am able to tell a more complete story of what happened when I worked with an Algebra teacher. Each role is focused on learning and teaching, with a specific emphasis on Algebra. The various roles I assumed provide a different viewpoint to the learning and teaching taking place. All of the perspectives have a specific emphasis on Algebra.

Since Algebra is a gateway course to future mathematics and life beyond high school, it is important that students learn algebraic concepts (Graham, Cuoco, & Zimmerman, 2010). As I gained knowledge of just how important it is for students to be successful in Algebra, I formed the first phase of my problem of practice. Not all students were being successful in Algebra. NCTM (2000) clearly states that all students should learn algebra. Yet not all students were achieving this goal. I began to wonder how to help students learn and be successful in Algebra. How do we increase Algebra students' achievement?

If a student's understanding of Algebra is largely dependent upon the teacher, then improving the Algebra teacher's instruction stands to greatly increase student achievement (Collins, 2010; Wright et al., 1997). Although improving teachers' instruction seemed to be the "answer" to my problem of how to increase student achievement in Algebra, this only led me to a second phase of my problem of practice (see Figure 0.1). How do teachers improve their instruction in order to improve student learning? What can be done to support teachers as they work to improve their practice?

I viewed coaching as a way to improve Algebra teachers' instruction, which in turn could increase students' achievement in Algebra. This thinking took me to a third phase in my problem of practice inquiry process (see Figure 0.1). How is coaching done effectively? What does it mean to coach an Algebra teacher to improve instruction? As a secondary mathematics coach, I was particularly interested and motivated to better understand how to coach teachers. Therefore my research aimed to find answers to this problem.

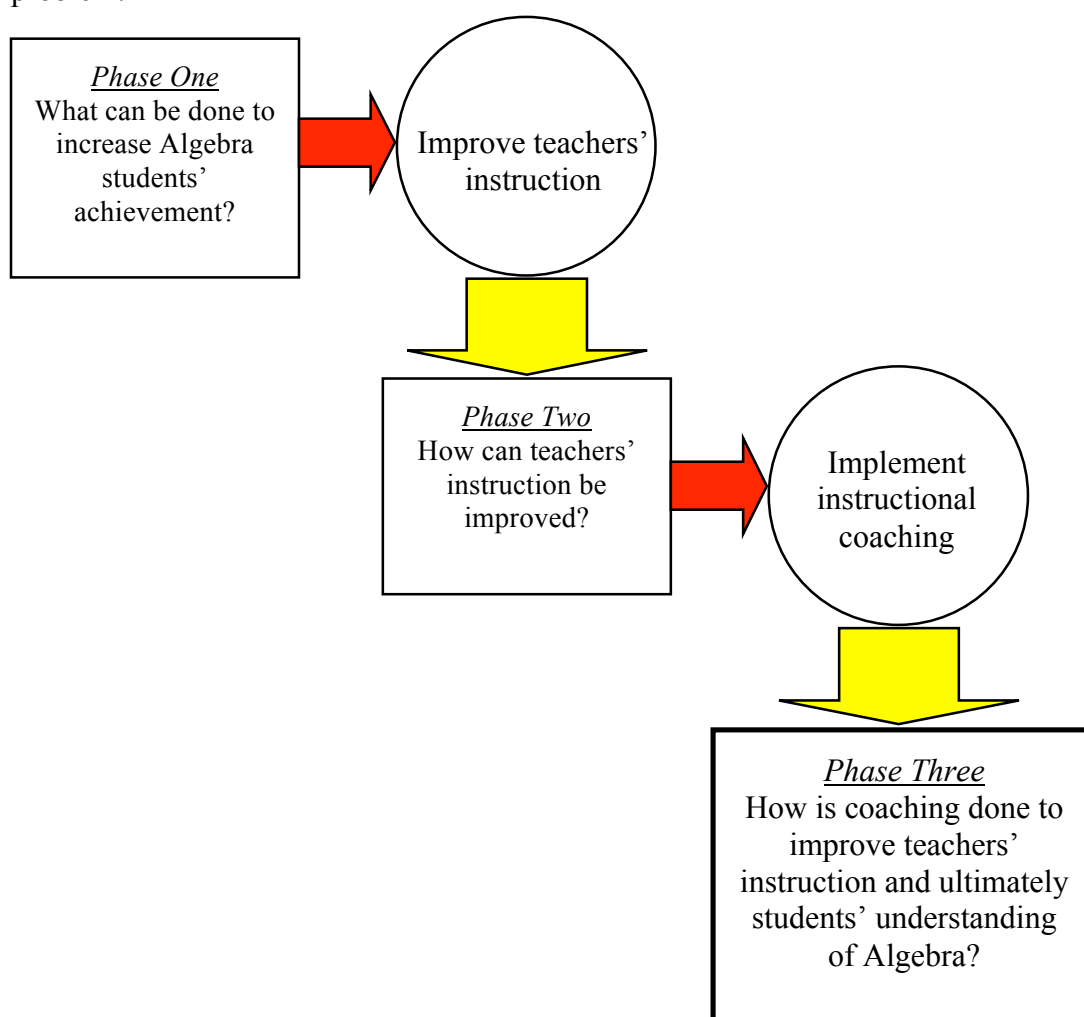


Figure 0.1: The diagram represents the phases that my inquiry underwent as I searched for ways to address my problem of practice.

Responding to this Problem of Practice

My interests in learning about how to help more students be successful in Algebra stemmed from both my previous experiences as an Algebra teacher and my work as a math instructional coach. Studying how teachers change their instruction to increase student achievement would not only help me improve my own coaching practice, but also the instruction of the Algebra teachers I coached. Being a teacher who experienced change in my own instruction gave me a greater understanding of what and how teachers learn to teach Algebra, specifically to struggling students. I had a deep, personal connection to both the algebra content and the instructional strategies used to teach algebra since I spent seven years teaching Algebra.

As an instructional coach, I was in a position to respond to these problems of practice. Since I am a practitioner researcher, I know, see, and feel things related to coaching and teacher change that outside researchers would not experience. I am in a place where rich, detailed data from an insider's perspective can be collected to gain clarity of my problems of practice. Being new to the coaching position put me in an advantageous position, as I was not calloused or accustomed to the small details and multi-faceted nature of the coaching process. I brought a fresh perspective to the coaching role and was, therefore, fully attentive to the experience. Studying my own practice, along with being new to the coaching role, helped me to notice complexities in coaching.

CHAPTER 1: REEXAMINING MY TEACHING PRACTICE

In order to better understand my practice as a secondary mathematics coach helping someone learn to teach algebra, I felt a deeper understanding of my teaching practice as an Algebra teacher was necessary. To help teachers improve their teaching, I first wanted to determine what I believed about teaching and learning, and establish the means by which I gained such knowledge. I needed to ask questions about how my own knowledge and practice were constructed and used (Cochran-Smith & Lytle, 2009). Why did I believe what I did about students? How did I think about and conceptualize Algebra? What was my knowledge of effective instruction and how did I come to gain that knowledge?

My knowledge and beliefs about teaching Algebra, leading up to the research I conducted in the Spring of 2011, were rooted in my years of experience as an Algebra teacher and instructional coach. I conducted informal action research in my own classroom over the years to address the problems I encountered in my teaching or in my students' learning. I approached my teaching with inquiry and gathered information to gain insight and ultimately improve my practice (Mills, 2007). When I was experiencing a problem, I would often times change various factors I thought influenced that problem and assess what happened. The continual process of inquiry helped me gain knowledge about teaching and learning as I stepped back and questioned what was going on and how my actions as a teacher influenced what was happening (Cochran-Smith & Lytle, 2009). This process helped me become a reflective teacher and ultimately helped me form the beliefs and understanding I held about teaching and learning (Zeichner & Liston, 1996).

Lessons from My Teaching Experiences

At times I feel that I learned more than I taught during my first nine years as an educator. My experiences as a teacher and instructional coach within six different schools in two districts (see Figure 1.1) helped me become an educator who focused on students' learning and strived to teach mathematics for understanding. I came to recognize that the instruction I planned significantly impacted my students' understanding and that planning instructional tasks could be positively impacted through collaboration with colleagues. Being involved with math education beyond my own classroom, whether it was serving on district committees or formally furthering my education through graduate work, broadened my view of teaching and learning mathematics. These experiences helped me better understand how my role as teacher or coach influenced the teaching and learning of mathematics. I characterize myself as a learner and am continuing to learn as I shift from teacher to coach, once again putting myself in the novice role. Each of these phases of teaching and learning are explained throughout this study in greater detail.

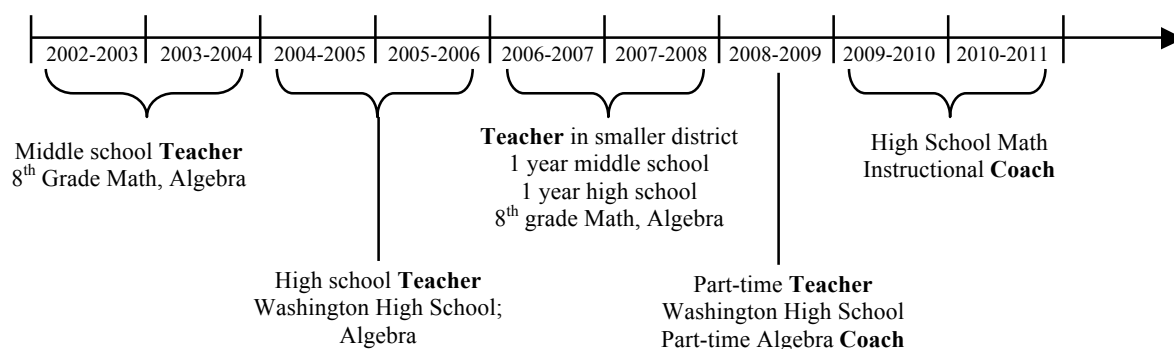


Figure 1.1: This timeline includes my teaching and coaching experiences from 2002 to 2011.

Whose Learning Matters?

My teaching career began in the fall of 2002 at a middle school in a Midwestern school district of approximately 30,000 students where I taught eighth grade math, including Algebra. I had just earned my Bachelors Degree in Middle School Education, specializing in the areas of mathematics, natural science, and social science. All through my K-12 educational career I enjoyed mathematics. I found comfort in the linear thinking found in mathematics and enjoyed knowing that there would always be a correct answer. I received good grades in my math courses all through my secondary education as I took Algebra as an eighth grader and continued to enroll in accelerated math courses.

Entering into my teacher preparation program, my experiences with mathematics shaped the way I thought about doing and teaching mathematics (Ball, 1988). When I began college, I thought I understood math. As I look back, I now realize that I did not understand mathematics. What I viewed as understanding was actually knowledge of how to use a specific skill to complete numerous problems that were identical. For example, I left high school knowing that in order to add or subtract fractions I needed to first obtain common denominators. I could add or subtract fractions quickly and correctly. And I perceived this knowledge of fractions to be “understanding” the mathematics. Yet it was not until my methods courses that I realized that I did not know why common denominators were necessary or conceptually what I was doing when I obtained common denominators.

Due to my enjoyment of the subject matter, my intentions as an education major were always to teach mathematics. My undergraduate college required course work in three subject areas to be endorsed in middle school education. Since science incorporated

the most mathematics, I had chosen natural sciences as my second specialized area and focused the majority of the coursework for this emphasis area on physical science. I chose the third subject area of social science as part of my certification because of what I could learn about teaching and learning. My coursework for this specialization largely consisted of educational psychology and adolescent psychology courses, which I could apply to the profession of teaching. I had completed my student teaching experience in the same school district the previous spring.

As I transitioned from college student to mathematics teacher, I thought I would be making a major shift from being a learner, which I had been for the prior 16+ years of my life, to being a teacher. I was over-confident in my abilities to teach, which is a common perspective among new teachers (Burke, 1987). Yet in my first year of teaching the realities of being a teacher quickly told me I still had a lot to learn about teaching in general and about teaching mathematics specifically. As a new teacher I had to learn the logistical aspects of being a teacher, such as how to run the copy machine and how to refer students to the administration. I was also working to understand the culture of the building and how to juggle the numerous responsibilities of teaching. And most importantly, during the first year I realized I needed to learn how to plan effective lessons that engage students and help them understand mathematics.

After my first year of teaching, I went forth with the title of teacher and with the understanding that I was still a learner. On the one hand, I was able to master some aspects of teaching rather quickly, such as how to make copies and what my role should be in a staff meeting. On the other hand, teaching mathematics was a part of my profession that I began to realize I would forever be learning. For example, even if I

learned how to implement one new strategy to help students learn, I did not think that was sufficient. I was not content. I felt like I needed to learn another teaching strategy. And the same thing happened when teaching various math topics. Even when I had learned a new technique to help students understand a math concept, I recognized that there were several other concepts I needed to be teaching better as well. As I learned more about teaching, specifically teaching mathematics, I quickly realized I would continually be improving as an educator.

Being a teacher and a learner is the frame of mind I have maintained since my first year in education. I continue to view myself as a learner and I believe that I can continually learn more about mathematics and about ways to help students make meaning of the mathematics I teach. And I am sure I will learn things I am not yet even aware of needing to learn. The more I learn, the more I realize there is still more to learn. I have maintained a learner perspective as a way to keep improving myself as an educator, fulfilling my belief that there is always something to learn that will help me be a more effective educator. This inquiry as stance perspective caused me to continually ask myself questions about my practice and look for ways to investigate and learn more about my practice.

Viewing myself as a teacher and a learner may have caused me to largely focus on my own learning rather than my students' learning during my first couple years of teaching. My tunnel vision blinded me from considering the students' learning (Hall & Simeral, 2008) when I thought about teaching. I was aware that there were more effective ways to teach mathematics, yet I did not know what to do to change my practice. I was experiencing a "knowing-doing gap" (Hall & Simeral, 2008, p. 69),

which essentially caused me to largely focus on my learning as a teacher. Daily, I was asking myself questions about my teaching rather than the students' learning. What teaching strategy would help me teach? How could I better organize this idea? What did I need to do to teach that skill better?

Towards the end of my second year of teaching I began to understand that teaching math was about more than what I did. Teaching was about student learning. That change of perspective was largely a result of my participation in a Math Learning Team. In the fall of 2003, my second year of teaching, I was asked to be on a Math Learning Team with six other, more experienced teachers. The six other eighth grade math teachers were outstanding, highly respected educators from across the district, each having between seven and twenty years of experience teaching middle school mathematics. The purpose of the Math Learning Team was to build leadership capacity with the district's middle schools by engaging potential leaders from each school in discussions and literature aimed at creating effective math instruction.

Each month we were provided a substitute for our classroom and were granted professional leave to participate in the Math Learning Team. The curriculum specialist provided us with readings and led discussions centered on how to teach mathematics effectively. The readings were focused on teaching strategies for middle school mathematics and often were specific to Algebra content. As members of the Learning Team, we discussed what we learned from reading the articles and talked about how we could apply the information to our own classrooms. Through the readings and my interactions with these other teachers, I came to realize that I was focusing too much on my own learning. The others in the Math Learning Team would talk about how they

made changes in their instruction based on the students' understanding. The teachers would discuss how their students learned this or did not quite understand that. I began to realize I needed to start paying more attention to my students' learning. The questions I asked myself during my daily teaching started focusing on student learning. What did the students learn? How are they thinking about this idea? What does their understanding of the concept tell me about the instruction?

I began to learn how to implement small changes in my instruction and to ultimately increase student learning. I questioned aspects of my teaching that influenced student learning and searched for ways to address these problems. One instance of my inquiry process aimed at student learning dealt with reteaching students who had not initially learned a math concept. When I began teaching, I was unsure of how to reteach and reassess students who did not master an objective. The district expectation was that teachers identify the students who would benefit from relearning a skill, reteach the students that skill, and then reassess the students' knowledge. I knew from research that student achievement improves when assessment data is used to give students additional instruction and reinforce the skill with which they originally struggled (Baker, Gersten, & Lee, 2002). Yet I did not know how to implement the reteach-relearn-reassess process into my teaching. Thus the knowing-doing gap surfaced in my practice. I knew that helping students relearn the material was important, but I did not know how to do the reteaching and relearning in my classroom. After brainstorming with other math teachers and observing another colleague implement the reteach-relearn-reassess cycle in her classroom, I designed a reteach-relearn-reassess process of my own and began

implementing it with my students. Several students were able to learn objectives not initially mastered and improve their grades.

Although I knew better, I struggled at times to maintain students' learning as the purpose of my teaching. As a new teacher still working to become familiar with the curriculum and the organizational skills required to teach a class, I put a lot of extra time and energy into determining and evaluating my own actions as a teacher (Bullough, 1989). These actions pulled me back into the self-absorbed view of teaching. At times when I became so frustrated with students' lack of progress in class, I could not understand why my teaching was not being successful. It was usually these instances when I was reminded that my teaching was about the students' learning, not my teaching. The focus of my teaching did not change overnight; it has been a growing process for me as I continue to learn what it means to focus teaching on student learning. In fact, I continue to evolve in how I perceive my own learning and teaching.

From Egg Crates to Teacher Planning Centers

As a new teacher, initially I was surprised by how isolating the teaching profession was. I was in my own classroom all day, physically separated from my colleagues similar to how eggs are separated from one another in an egg crate (Lortie, 1975). Besides the occasional adult interaction I received when I ventured out of my room to use the restroom or get my mail in the office, I primarily interacted with 12-13 year olds. I wanted more than sporadic conversations with other educators. I wanted to work with other teachers – be colleagues who helped one another plan lessons and create teaching materials. A month or two into my first year of teaching, the other eighth grade math teacher went on maternity leave (for the remainder of the school year). Like me,

the teacher's long-term substitute was in her first year of teaching. As two new teachers, we began meeting on a daily basis as a means of survival. We were anxious to "acquire the essential practical knowledge needed to function effectively in an unfamiliar environment" (Ewing & Manuel, 2005, p. 8). During our weekly collaboration we talked about how many days to spend on a topic, how we would teach the new material, what homework to assign, problems to put on the quiz, and how to deal with classroom management issues. We relied on one another as we paced our instruction, decided on notes to give students, and wrote assessments. By talking on a daily basis we were able to combat the isolation we felt as teachers (Hargreaves, 1993).

Looking back, the substance of our collaboration during that first year (and my second year with a different teacher in her third year of teaching) did not delve into the deeper aspects of instruction. The other eighth grade math teacher and my conversations focused on the surface aspects of teaching that were easy to discuss and compromise on. Teaching aspects that we could find a direct solution to included how to grade homework and when to give the quiz. The type of collaboration I experienced during my first two years of teaching matched my self-centered frame of mind I had at the time. My colleague and I were concerned about what we were doing as teachers (deciding on problems to assign for homework, determining which day to give the chapter test) rather than what the students were learning (determining instruction based on formative assessments, creating mathematical tasks to encourage student understanding). As I came to understand the importance of focusing on student learning as opposed to my own learning, I began to ask myself questions about the collaboration I was experiencing. Could our collaboration focus on student understanding? What if we used our students'

work to make instructional decisions? How could our collaboration influence student learning? As my focus of teaching shifted to students' learning, I began to realize that there was more to gain from this collaboration.

The third and fourth years of my career were spent teaching in the same district, but at a different building. I taught Algebra at Washington High School¹ for the next two years. (Washington High is also the setting where this study took place.) Since I was endorsed to teach middle school, fourth through ninth grade, the majority of my classes were filled with ninth graders. Therefore, I mainly taught ninth grade Algebra courses in the high school rather than upper level mathematics courses such as Advanced Algebra or Pre-Calculus. I also taught two Algebra Extended courses, which targeted sophomores, juniors, and seniors who had not passed Algebra in previous years. The Algebra Extended course was the algebra curriculum taught over two years, which meant the pacing was half the speed of a typical Algebra course.

The math department at Washington High School had already established a structure to promote collaboration among teachers. The physical arrangement of the building largely influenced the collaboration. All math teachers had a cubical area and desk within a large room called the teacher planning center (TPC). This is where teachers worked when they were not in class with students. The TPC was a familiar space for the math teachers that nurtured teacher collaboration (Wenger, McDermott, & Snyder, 2002) and battled the typical "egg crate" structure of schools (Lortie, 1985, p. 14). Initially a few of the other Algebra teachers and I informally discussed our classes in the TPC. Our informal collaboration consisted of quick, daily conversations about

¹ Pseudonyms are used for all teachers and schools.

how the students did with the lesson that day or small changes we made to help students better understand. Over time, our collaboration became more purposeful and scheduled. Our informal conversations still occurred daily, but we also engaged in more formal collaboration every two or three days. The formal collaboration was a scheduled time when all of the Algebra teachers met before or after school to plan lessons and discuss how we wanted to teach the concepts.

The informal and formal collaboration I experienced with the Algebra teachers at Washington High School was different from what I had experienced my first two years of teaching. We did discuss curriculum pacing, lesson structure and homework assignments, but also went deeper into our planning of how to teach the mathematics. Our collaboration matched our collective focus on student learning and had a greater focus on mathematical understanding (National Council of Supervisors of Mathematics (NCSM), 2008). We talked about the mathematics we wanted students to learn and how we could best facilitate that learning. As suggested by National Council of Teachers of Mathematics (NCTM) (2007), the way to introduce a concept and the problems to use as examples were the center of many of our conversations. In addition to planning for upcoming lessons, we spent time reflecting on lessons we taught. As we shared what we taught the previous days, my colleagues and I worked together to determine what our students understood, how to build on their understanding, and what instructional strategies were effective.

The collaboration I experienced with my colleagues at Washington High School made me a stronger math teacher. I enjoyed discussing instructional techniques with other teachers and reflecting on what we taught. Although I could plan a lesson on my

own, I preferred to brainstorm with someone else to come up with better ideas about how to teach the mathematics. At the end of each day, I relied on the informal discussions with my colleagues that helped me debrief about what my students did and did not learn from our planned lessons. These collaborative interactions helped me learn about myself as a teacher. I had come to view collaboration on how to teach mathematics as an important aspect of teaching, which is a belief I still hold strongly today.

Instruction Makes All the Difference

During my third year of teaching the associate principal asked me for my thought on how to support students who struggle in Algebra. Washington High was not seeing much success with the current Algebra course offered for high school students who struggle (Algebra Extended). Algebra Extended was a two-year course that covered Algebra curriculum at a slower pace. My associate principal knew I primarily taught Algebra so he asked me what I thought we should do. Together the administrator, a few colleagues and I came up with the idea of Algebra Block. Students who struggle would still get an extended amount of time to master the concepts, yet it would take place in one year. The idea was that students who learned the material in one year (as opposed to two years) would be more successful in retaining the math knowledge. The students would have two class periods back to back. We believed the two-hour block would allow students to be introduced to a concept and experience a significant amount of practice all in the same day (Rettig & Canady, 1998). The students would have the opportunity to immediately apply their new knowledge with the extended instructional time. Fitting Algebra Block courses into the master schedule would be the most difficult element of

our plan. Special scheduling would need to be done within the building's master teaching schedule so students stayed with the same teacher two consecutive class periods.

Two teachers and I piloted the Algebra Block course during the 2005-2006 school year. We collaborated daily by discussing the best way to introduce concepts, the most effective sequencing of problems to influence student understanding, and the types of questions we wanted to ask to help build mathematical knowledge. As I began teaching the 100-minute Algebra Block class, I started to realize that my instruction had to change. Simply having students for two consecutive class periods, rather than one, was not going to be enough. The students and I would be completely bored if I tried to teach two periods in a row using a traditional United States math lesson structure, similar to what Stigler and Hiebert (1999) found in their study. I quickly realized that two hours that included the elements of traditional instruction, including completing the warm-up, checking homework, showing students how to solve new problems, having students individually try similar problems, checking seatwork and assigning homework (Stigler & Hiebert, 1999), was not effective. I needed to approach teaching math and practicing the skills differently, in addition to the extra time. Leinwand (2009) wrote:

We've all heard that 'the most important variable in determining the quality of education is the teacher'. Of course that's true. But the next (and far more important) message is that it is instruction – what teachers actually do to present mathematical ideas and to structure learning – that makes all the difference. (p. x)

I was beginning to realize that I would need to significantly change my lessons if I wanted my students to participate in class and ultimately learn Algebra. Although we

continued to talk about how to introduce mathematical ideas and problems to use, my colleagues piloting the Algebra Block course and I began to focus our collaboration on how to incorporate instructional strategies that would engage our students for the full 100 minutes. We worked diligently throughout the year to find new and different instructional techniques such as breaking up direct instruction into smaller chunks, using scavenger hunts and stations to infuse movement, and incorporating “games” like Jeopardy or Math-ketball to engage students.

Although I was excited about my newfound knowledge of the different instructional strategies my colleagues and I created, I still felt as though I was missing something. The instructional techniques we used did help keep students engaged by allowing them to move and by changing the activity often. However, I had a suspicion that the students were engaged in the activity rather than the mathematics. The students were doing the problems, but were they really thinking about the mathematics? I questioned the focus of the students’ engagement. The problems we gave the students and the questions we asked in these engaging activities were largely focused on mathematical procedures. The problems did not ask students to explain their reasoning or apply skills to new situations, which would have required students to engage in the mathematics. Also, the tasks we were having students complete were still largely individualized and did not require students to interact or discuss mathematics with one another.

It was with this nagging feeling of missing a critical instructional piece that I began my fifth year of teaching. The nagging feeling stemmed from a fear that my students were engaged in the activities rather than the mathematics. My husband was

offered a teaching position in another Midwestern city and I accepted a job teaching eighth grade math and Algebra in a smaller school district that served approximately 8,000 students. During new teacher orientation, I was introduced to cooperative learning as an instructional technique. All teachers throughout the district were trained in Kagan Cooperative Learning strategies (Kagan, 1994) and were encouraged to use these strategies in their classrooms. I was excited about cooperative learning and how the teaching strategy engaged students in the mathematics they were learning.

I began using these cooperative learning techniques in my classroom on a weekly basis at first. Once I became more comfortable with cooperative learning, I incorporated the strategy into the formative assessment portion of my lessons daily. The results were powerful. My students' involvement in the learning process increased, which affected their understanding of the mathematics (Nebesniak & Heaton, 2010). As they cooperated with their peers to complete mathematical tasks, their focus was on the mathematics and understanding the content rather than the activity itself. The students could regularly be heard discussing mathematics and explaining the mathematics used to obtain solutions. Students experienced increased confidence in their math abilities (Nebesniak & Heaton, 2010) and a stronger sense of community as they worked together to learn mathematics. For me, cooperative learning reaffirmed how instructional strategies aimed at engaging students in mathematics can influence students' learning of mathematics.

From Fixing Errors to Encouraging Mathematical Thinking

For the majority of my first four years of teaching I saw each section in the book as an individual topic and focused my lessons on procedures. The notes I gave my

students, largely through direct instruction, were riddled with linear steps. Solving a linear system had five steps. Graphing a linear equation was four steps. Each new objective had a different set of steps and the only connections I made were through the procedures (i.e., instances such as solving multistep equations where after completing steps one and two, they continue with the steps used to solve two-step equations). For four years I taught isolated mathematical procedures. And even though students need to master certain math skills, I now know that learning mathematics solely through procedures makes learning new topics harder since the new knowledge is not linked to any previously learned concepts or skills (Kilpatrick et al., 2001). In addition, procedural instruction does not help students build conceptual understanding and they therefore struggle to apply the math skills to other situations (Seely, 2009).

I did not expect my students to think about or reason through the mathematics. They simply needed to follow the steps I gave them. If the students did not understand the math, had a question, or made a mistake, I quickly jumped in to fix their error and refer them back to the procedure that would lead them to the answer. I was a “fixer” of procedures rather than a facilitator of knowledge. I was creating students who mimicked the procedures I modeled for them, rather than mathematical thinkers.

A major shift occurred in how I thought about and implemented math instruction due to my involvement in a two and a half year University of Nebraska-Lincoln graduate program. I began the Math in the Middle program² in the summer between my third and fourth year of teaching (June, 2005) and earned my Masters degree in Teaching, Learning, and Teacher Education prior to my sixth year of teaching (August, 2007).

² <http://scimath.unl.edu/MIM/>

During the program, I learned a lot about mathematics, including how concepts are connected, how middle school math knowledge extends into future mathematical topics, and how mathematics can be applied to real life. I also constructed a large amount of knowledge about the teaching of mathematics. I came to understand how mathematics itself can be engaging, how learning could be fostered through deep thinking about a few carefully chosen, rich math problems, and how powerful it can be to have students share various approaches to solving a problem. These were features of mathematics and mathematics teaching that I needed to learn about the most since they were the issues I had been wrestling with in my own teaching. Through these graduate courses, I began realizing that I conceptually knew very little about mathematics, especially algebra. And I knew even less about how to teach mathematics conceptually. As I struggled with various problems, I began to understand that open-ended mathematical tasks created an environment where I could make sense of mathematics. Several of the mathematical problems I tried to solve through Math in the Middle helped me recognize just how flawed my own teaching and the mathematical tasks I incorporated into my instruction were. I needed to be engaging my students in more mathematical thinking.

As part of my Masters program coursework, I began to reflect upon my own understanding of mathematics and teaching mathematics, specifically algebra. In my fifth year of teaching Algebra I began to see the content in richer ways. Each year I learned a little more about the big mathematical ideas embedded in algebra and how algebra concepts fit together. I started to see algebra not as a collection of procedures, but instead being about generalized arithmetic and functional thinking (Kilpatrick & Izsak, 2008). Solving equations (linear, quadratic, linear systems) was no longer a “study

of the 24th letter of the alphabet” (Katz, 2007, p. 41), but instead was an investigation into what variables represented (Saul, 2008). And functions in algebra became a way to represent and understand the world.

As I thought about algebra more conceptually, I also worked to renovate my teaching. My instruction became more focused on helping my students conceptually understand mathematics. I took what I had learned during my first four years of teaching algebra about procedures, skills, and critical processes and used the new conceptual learning I gained to start redefining what teaching mathematics looked like in my classroom. For the first time in my teaching career I was attempting to teach the overarching goals of an algebra concept before teaching students “how to do” the algebraic procedures.

An example of how I worked to transition from teaching lessons to teaching mathematics is best shown through an example. A lesson on graphing linear functions in slope-intercept form during my first four years of teaching consisted of me giving the students the formula ($y = mx + b$) and explaining that m is the slope (which the students were taught a few days earlier) and b is the y-intercept. The students then practiced identifying the slope and y-intercept and graphed several functions in that form. Compare that lesson to the way I taught the mathematics my fifth year of teaching. During that year I used a real-life story of several children running a race. Some of the children in the race got to start a certain distance in front of the starting line, some started right on the starting line, and others began behind the line. (The starting point indicated the y-intercept.) Almost all of the children ran at a different pace. (The children’s pace was the rate of change or slope.) My students were asked to work in their teams to graph

each of the children on the same graph and to make observations about the graph. I encouraged students to make connections to what they knew about graphs and tables of values. My goal was to set the stage for why slope-intercept form was an important idea in mathematics and to help students recognize the big idea of graphing linear functions.

Some Algebra concepts have been easier for me to learn how to teach for conceptual meaning than others. I believe a variety of factors play into this, which include my personal knowledge and comfort level with the concept, the amount and quality of resource materials available to me, and the connections I see a concept has with other Algebra skills. In the example above, the concept of linear functions in slope-intercept form was easier for me to incorporate into my instruction since I had numerous resources available to me and I understood several direct applications of the concept students could relate to, such as running a race. An example of a concept I struggled to teach for understanding rather than procedures was the concept of quadratics. My own lack of confidence with quadratics, limited access to resources for conceptual teaching tasks, and less obvious ways to connect quadratics with real-life examples being less obvious to me added to my struggle. In the five subsequent years I worked to change my instruction, I had done little to better my teaching of quadratics.

By my fifth year of teaching I had a vision for how I wanted to teach math. I envisioned myself using high-quality mathematical tasks to encourage students to create their own meaning (Henningesen & Stein, 1997). I wanted students to use prior knowledge of mathematics to create new understanding and use their math reasoning skills to tackle unfamiliar problems. My job would no longer be to fix the students' mistakes, provide steps for procedures, or fill the students' math knowledge voids.

Instead, I would ask students questions to get them to think about the mathematics (Fairbairn, 1987) in order to address their own mistakes and build their own knowledge. I would ask questions such as what do you think, why do you think that and what is going on here? I wanted to model mathematical thinking and emphasize reasoning behind various mathematical procedures. I imagined my students thinking about and making sense of the mathematics for themselves rather than mimicking the steps of the procedures I gave them.

Since 2007, the year I came to recognize that I wanted to teach mathematics for understanding, I have been working to move my instruction in this direction. The learning process has been continuous. The more I work to incorporate conceptual understanding into my teaching, the more I learn about how various concepts can be connected and how students learn. The transition of my teaching has been difficult some days and relatively seamless others. I continue to learn what it means to teach mathematics for meaning and I continue to collaborate with my colleagues to teach mathematics this way.

Continuing as a Learner

My growth as a teacher did not occur in isolated episodes as it may appear. In actuality, the changes in my understanding of teaching and learning occurred concurrently (see Figure 1.2). For example, around 2004 when I began to view teaching as focusing on my students' learning I was also starting to collaborate with my peers around ways to engage students. My instruction became more directed on students involving in their learning and my teaching became more centered on students as engaging learners. In 2006-2007, I began thinking of mathematics as connected concepts

that I wanted to teach through big ideas. This thinking led me to start viewing my students as learners of mathematical concepts. My collaboration soon after shifted to a greater mathematical focus, with my students' conceptual understanding the large motivator. The changes I experienced were not as linear as represented in Figure 1.2 and are approximate dates. In fact, a spiral or interwoven figure for each timeline of understanding would better describe my learning process. I would move forward in my thinking in some ways, but remain behind in others. The changes I experienced were not immediate or complete. I did not simply decide one day to teach mathematical big ideas rather and automatically transform my classroom. The change process has been slow and winding, which is not evident in Figure 1.2.

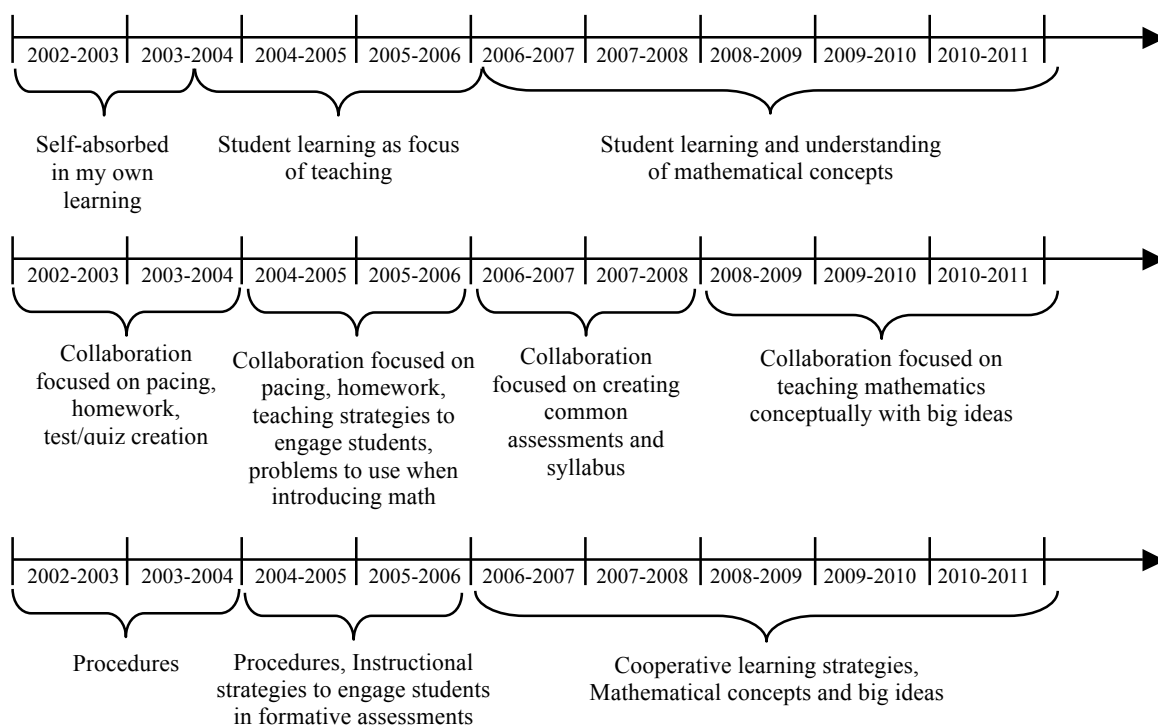


Figure 1.2: The three timelines include my concurrent learning from 2002 to 2011.

I completed the Math in the Middle program in August 2007 as I began my sixth year of teaching. I was proud of all I had accomplished and learned throughout my graduate program, yet I felt incredibly disappointed to think I was finished learning. Teaching is complex and I did not feel as though I had yet figured out teaching mathematics. Some teachers earn their Masters degree to move over on the pay scale and never take any more classes. That was not me. I was not satisfied with being done with classes. I felt that there was more for me to learn about mathematic and teaching mathematics. As a way to continue exploring and learning, I applied for a doctoral program.

I was accepted into the Ed.D. Cohort: Scholars of Educational Practice³ at the University of Nebraska-Lincoln in January, 2009. Through this doctoral program, I continued to learn and grow as an educator. The literature my cohort members, instructors, and I read and the discussions we had helped me to better understand the complexities in teaching and learning. I began to realize that there is not one right way when it comes to teaching. The journey taught me a great deal about myself as a scholarly practitioner as I worked to use literature to inform my practice.

The Ed.D. program has also pushed me to view inquiry as stance. The idea of inquiry as stance, or making my own practice problematic and then investigating the problems to inform my practice (Cochran-Smith & Lytle, 2009), was a way for me to embrace educational research. I started looking at my own practice not only as the practitioner, but also as a researcher. The Ed.D. program continued to feed my desire to learn, helped me improve as an educator, and pushed me to study my own practice.

³ <http://cehs.unl.edu/tlte/graduate/eddc cohort.shtml>

My Role as a Teacher

My learning has not ceased since I began my teaching career (see Figure 1.3). My continual involvement with colleagues, committees, and college courses has helped me become the educator I am today. I have discovered that to be an effective teacher and improve my practice, I need to be a continuous learner (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010). I have learned that focusing on student understanding will help create more effective instruction and collaboration centered on student learning will result in deep, meaningful discussion about how to teach mathematics. I have made radical improvements in my own teaching over the years, moving from procedural lessons that aimed to fix students' lack of understanding to conceptual instruction meant to help students construct their own mathematical understanding. College courses, along with my personal experiences, have facilitated much of my learning over the years.

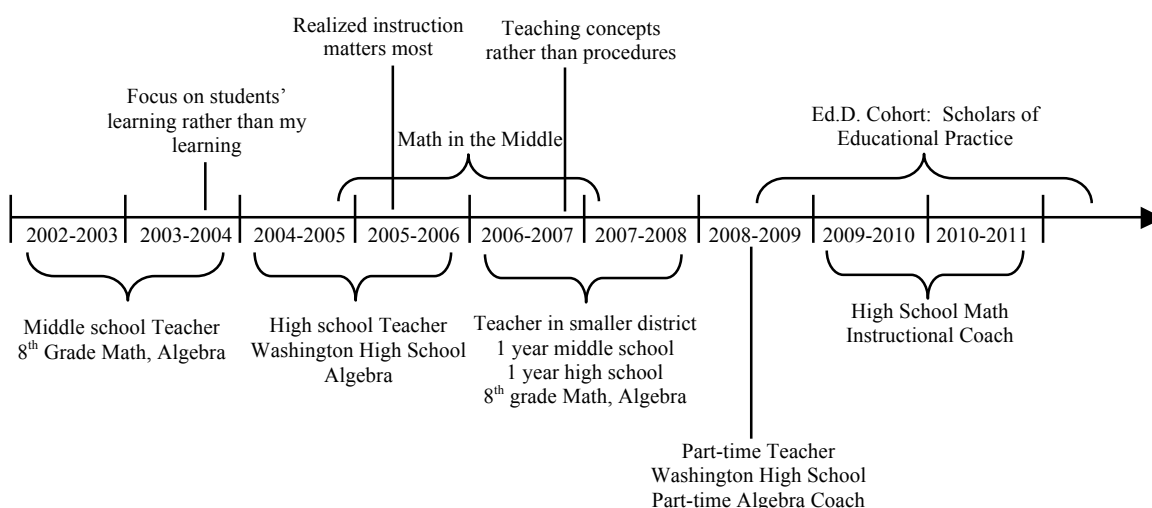


Figure 1.3: The timeline includes key changes in my thinking from 2002 to 2011.

The learning process has not been linear or smooth, as it may appear in Figure 1.3. For example, I did not one day switch my practice from focusing on my own learning to focusing on students' learning. Instead, I gradually began to think about my students' learning more and more as I planned lessons and went about my daily teaching. Even though there were significant moments when I realized what changes needed to be made in my teaching, the changes in my practice took time.

What I learned as a mathematics teacher is important because it speaks to who I was as a classroom teacher. The story of my math teaching helps tell the story of how my understanding of teaching and learning evolved. Looking back and reexamining my teaching practice better equipped me to investigate my questions about my coaching practice. I now have a better understanding about my own view of teaching Algebra, which I am able to use when I investigate other teachers' views of teaching Algebra.

Examining my own learning as a mathematics teacher has also prepared me to examine my learning as an instructional coach. What am I learning as I coach other teachers? How do I learn? What problems am I experiencing in my practice as a coach? And how can I change my own practice? How is coaching similar to teaching mathematics? I am better prepared to investigate questions about my coaching practice after examining my own teaching experiences.

CHAPTER 2: COACHING AS I UNDERSTOOD IT

After teaching mathematics full-time in middle and high schools for six years, I became a secondary math coach. For the first school year as a coach, 2008-2009, I coached part-time and still taught two Algebra classes (see Figure 2.1). In the fall of 2009 I became a full-time instructional coach working in three high schools. During this time, through December of 2010, I was specifically hired to work with Algebra teachers. There were approximately 25 teachers in the three buildings that I coached. Beginning in January of 2011 I was assigned to Washington High School full-time to work with Algebra and Geometry teachers in an effort to increase the school's graduation rate. This was during Spring 2011, the semester when my research was conducted.

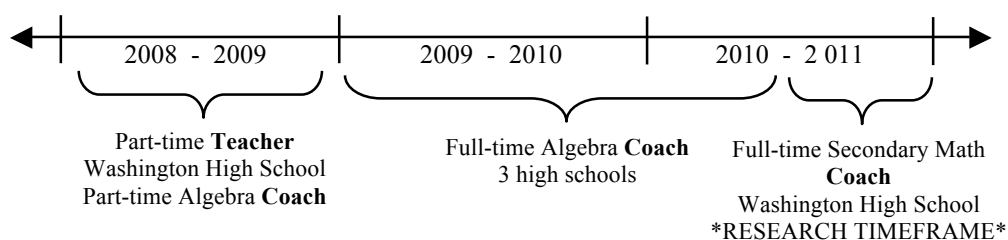


Figure 2.1: The timeline includes my coaching experiences from 2008 to 2011.

When I transitioned from my own classroom to the coaching role, I was removed from an environment where I felt confident and put into an unfamiliar setting. I was once again a novice, which I found to be confusing and uncomfortable (Chval et al., 2010). As I became more familiar with my new role as an instructional coach I started to develop a definition of coaching, which is still evolving. Chapter Two explores how my definition of coaching developed, starting from my first exposure to the coaching role, and how I

applied that definition to my coaching practice the first year and a half as a full-time Algebra coach.

Initial View of Coaching

The word coach comes from the Hungarian word *kocsi szeker*, meaning “large kind of carriage”⁴. In the English language today the word coach is still used to describe vehicles used for transportation, such as stagecoach, coach bus, and railroad coach. Similar to how a carriage carries a person from one destination to another, a coach is also a person who carries others from one destination to another. The basic work of an athletic coach is to help athletes improve their fundamental skills in a sport (Osborne, 1999; Wooden, 2004). In the business arena, executive coaching is an intervention aimed to change managers’ behavior by helping executives develop leadership skills and become more effective in their position (Feldman & Lankau, 2005). Life coaches are extensions from the fields of counseling and psychology. A life coach provides an on-going partnership with people to help them enhance the way they perform in various aspects of life and overall improve the quality of their lives (Williams & Menendez, 2007). Coaching in each of these areas has the fundamental purpose of moving people. A person being coached starts at a certain skill level and is moved to a higher or improved level.

As a way to better understand and define coaching, I interviewed Dr. Tom Osborne on August 26, 2010. At the time of the interview, Dr. Osborne was serving as the athletic director at the University of Nebraska-Lincoln. Coach Osborne was the head football coach at the University of Nebraska-Lincoln from 1973 to 1997 during which he

⁴ www.etymonline.com

experienced numerous successful seasons. My reasons for interviewing Coach Osborne prior to collecting data on my own practice was to gain insight into a successful coach's thoughts on effective coaching.

Through my experiences with coaching over the past four years, my review of coaching literature, and my interview with Coach Osborne, my personal definition of coaching had been constructed and revised numerous times. How I perceived coaching changed from when I first experienced coaching as a teacher in a smaller district to the time of this study when I worked as a secondary math instructional coach in a large, Midwestern school district. In addition, my understanding of coaching grew as I read more research. The following is my attempt to construct a clearer definition of coaching based on my experiences and the literature.

From Appraisal to Improving Instruction: Definition of Coaching From Experience

My definitions of coaching were initially formed through my various experiences with coaching both as a teacher and a coach. Each experience provided me with a different perspective on coaching and caused me to describe the role of a coach a little differently for each coaching situation. My definition of coaching went from coaching as an appraisal process to coaching as a support for implementation of curriculum to coaching as a method for improving instruction.

Coaching as appraisal. My first experience with an instructional coach occurred in a previous district where I was a high school Algebra teacher. The fourth through eighth grade math coach visited my classroom once each semester. She emailed me approximately one week ahead of time, asking if she could observe me teach on a specific day during a specific class. The actual reason for the coach coming to my

classroom was never clear to me. I was told the observation of my teaching and my class was so the coach could just see how things were going in my class. I assumed she was coming to see if I was implementing the curriculum appropriately. My impression was also that the coach wanted to see that I incorporated a good lesson that contained the three parts of a lesson the district deemed appropriate: a warm-up, direct instruction, and closure. Yet what, specifically, she came to observe was not explained to me. Even today I cannot say what she was observing exactly.

On the day of the lesson, the coach would come to my room a couple minutes before the start of class. I would quickly inform her of my objective for the day and verbally give her an outline of the lesson. We would not plan or discuss the lesson together before class. While I taught, the coach sat in the back of the room taking notes on what was happening. She wrote down what students did, what I did, anything she observed and felt necessary to share with me. Sometimes these observations were focused on what the students did or did not understand. Other times the observation she shared with me was specific to my instruction. As I taught, I was unsure of what the coach was looking for or how she decided what information to write down. Occasionally, she would join a group of students during instruction or guided practice and help them learn the material. After the lesson she shared her notes with me, explaining what she thought I did well and ideas to make the lesson better. I would listen to her suggestions and feedback. She would then thank me for allowing her to come to my class. This process occurred once a semester.

The teachers in the high school did not respect the coach. The overall sense in the math department was that the coaching process was a waste of time and a random event

that teachers were required to participate in to appease administrators and district leaders. Many of these feelings were compounded when, on several occasions, the coach went to the administration with concerns (i.e., the teacher was giving too much homework, the students did not have their books in class, a teacher was behind in the pacing of the curriculum) after being in a teacher's classroom. The coach's strong link to the administration was a concern for teachers, leading to minimal trust between the teachers and the coach (Mraz, Algozzine, & Watson, 2008).

Since the coach and I had not established a strong, trusting relationship, I was not willing to open myself or my practice up to her (McLymont & da Costa, 1998; Noyce Foundation, 2007). I made sure to "put on a show" when she came to my classroom. I worked hard to teach a fun, creative lesson that I thought would make an impression. My belief was that I needed to impress the coach so she thought I was a good teacher. I thought I needed to deliver the perfect lesson and all my students needed to leave the lesson with mastery. I did not want her to go to an administrator or district leader with any negative observations about my instruction. For me, the coaching process was about showing how good of a teacher I was. I did not feel like coaching helped me learn or grow as a teacher.

My definition of coaching, based on my experience with a coach as a teacher, was similar to an observation conducted by an appraising administrator. The coaching process included the same three components as an administrator observation: Pre-conference (I give a quick description of what I will be teaching), Lesson Observation (coach/principal sitting in the back of the room taking notes), and Post-conference (coach/principal reading the notes explaining what I did well and what I could do better)

(Garmston, 1993). The only difference between coaching and an appraisal was that after the observation the administrator formally evaluated my teaching and put her notes into my personnel file. Everything else was very similar. The entire coaching process was centered on the teacher working hard to deliver a stellar lesson in order to receive positive comments from the coach. As a teacher in this particular situation, I viewed coaching as an informal observation of my teaching rather than a way to improve my practice.

Coaching as curriculum support. In 2008 when I was hired as a part-time instructional coach in my current district I was impacted by my prior experience with coaching. I was hired as a part-time Algebra instructional coach for a curriculum (Transition to Advanced Mathematics, Johns Hopkins University) being implemented in select classrooms. The curriculum was part of a study being conducted by Johns Hopkins University to research student achievement in high school Algebra⁵ and included a focus on the impact of various course structures and teacher supports on student learning. As part of the study, another Algebra coach and I devoted half of our school day to supporting seven Algebra teachers implementing the curriculum. The arrangement of my coaching starkly differed from the math coach I had previously worked with. First of all, I worked with a small group of teachers on a weekly basis rather than the once-a-semester arrangement I had experienced as a teacher working with a coach. The weekly coaching fostered a more trusting relationship between the teachers and me and helped me better understand an individual teacher's needs. Also, my previous coach focused on observing my lessons. My focus as a coach was to support teachers as they implemented

⁵ More details about the Johns Hopkins study can be found at <http://ies.ed.gov/funding/grantsearch/details.asp?ID=495>.

a new curriculum. Another key difference was that I taught Algebra part-time in the same building that I was a part-time coach. The direct connection with the school and the teachers allowed me the benefits of greater knowledge of the building and larger availability for teachers (Barr, Simmons, & Zarrow, 2003).

As a way to gain the necessary knowledge needed to survive in this unfamiliar environment (i.e., teaching new curriculum), the focus of our coaching interactions was on lesson planning. Four teachers at Washington High School⁶ and I did general long-term planning together. We worked together to determine the sequencing, timing, and objectives of lessons one week at a time. I usually planned alongside the teachers and created templates that helped organize our ideas. Then, a teacher and I would plan specific lessons together for the day I would be in her class. I would make instructional suggestions, discuss the mathematics with her, and help create materials for class. During class I provided support in the form of co-teaching, modeling instructional strategies or lessons, collecting data on instruction the teacher desired more information about, or simply helping students who had questions or struggled with the mathematics. The last action, helping students, was similar to acting as a teacher's aide or paraeducator and is a role I now understand should be avoided as a coach (Bean & Eisenberg, 2009). I worked very closely with these teachers planning lessons and reflecting on student learning on a weekly basis. Since I also taught part-time at Washington High, I was visible and available to the teachers on a daily basis, building a strong relationship with each teacher. My relationship with the teachers grew because we engaged in a professional learning

⁶ I taught at Washington High School from 2004-2006, and again part-time during the 2008-2009 school year. Washington is also the setting of my research study.

community together, I ate lunch with them every day, and I worked alongside them as a teacher part-time.

My definition of coaching during this year was to work with a small group of teachers for a significant period of time (Shanklin, 2006) as they implemented new curriculum (Toll, 2006). Coaching consisted of planning lessons with teachers, creating materials for the teachers' classes, increasing communication between teachers (Hansen, 2009), and providing resources. I thought a coach largely served as the facilitator and organizer of a small, collaborative community of Algebra teachers (Rectanus, 2006).

Coaching as improving instruction. After completing my first year as a part-time instructional coach for a small group of teachers, I was hired as a full-time coach for 25 Algebra teachers. The following is the job description that was posted in the spring of 2009 for my current position as a full-time secondary math coach.

Secondary Math Coach 1.0 FTE

This position will work as part of a team with two other secondary math coaches and the curriculum specialist for mathematics to promote instructional growth among teachers and increase student learning in mathematics. Responsibilities include collaboratively planning lessons with teachers; observing teachers' lessons and providing detailed formative feedback and structured reflection; and designing and delivering necessary professional development on research-based instructional practices. The position seeks a teacher with outstanding instructional skills, deep conceptual understanding of mathematics, and content-specific pedagogical knowledge.

Starting in the fall of 2009, I became a full-time instructional coach for the district. Not only did my position and the number of teachers I worked with grow, but my view of coaching expanded as well.

Since instructional coaching was now being implemented district-wide, the math curriculum specialist, four math coaches, and I felt it was important to more clearly outline how coaches could support teachers (see Figure 2.2). To increase understanding

of coaching across the district we (coaches and the curriculum specialist) created and distributed the graphic to teachers and administrators to show how a coach could work with them (Bean & DeFord, n.d.). The figure was purposely non-linear, emphasizing the point that individual teachers could enter into the coaching process where they felt the greatest personal need (Bean & Eisenberg, 2009; Shanklin, 2006). Each of the four quadrants of the district coaching graphic are explained in greater detail. Examples of how the items were incorporated into coaching are also included.

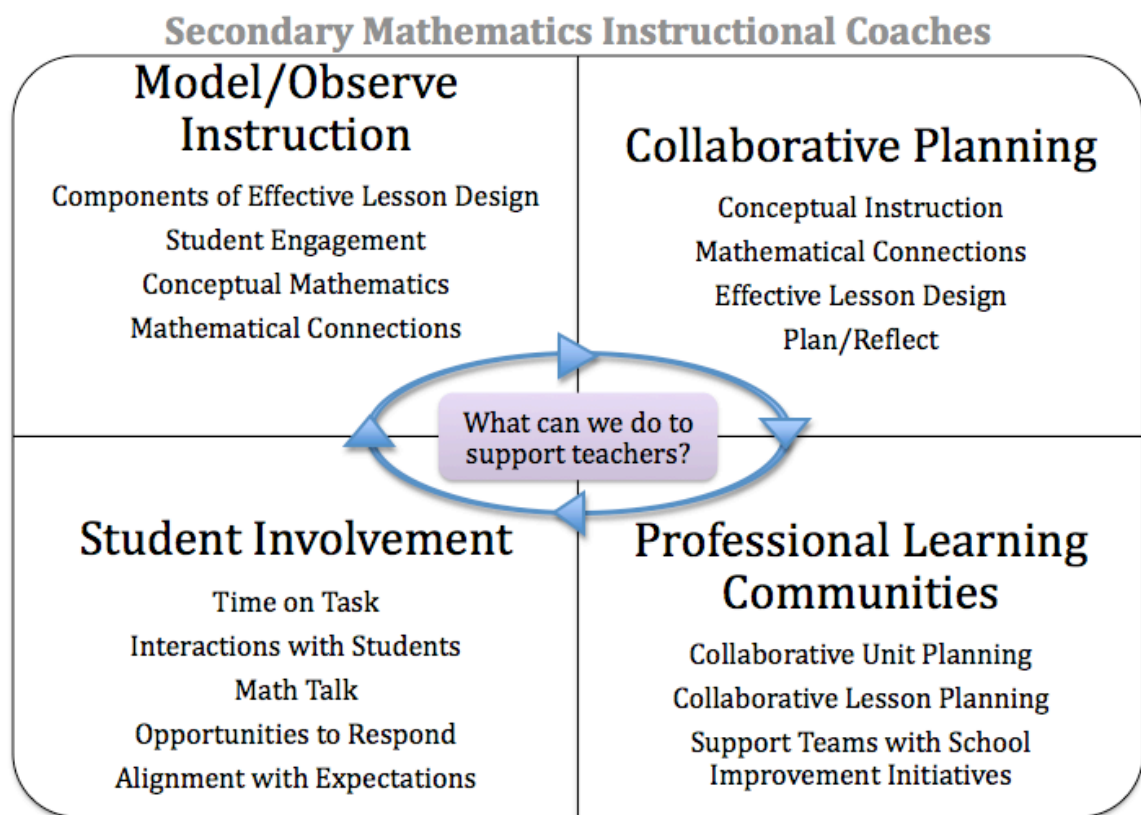


Figure 2.2: The graphic was used by my school district to communicate the role of instructional coaches with teachers and administrators.

Collaborative planning. The first three components of collaborative planning listed on the graphic were conceptual instruction, mathematical connections, and effective lesson design. These elements were also the focus of effective mathematics lessons (Nebesniak, in press). When having purposeful conversations centered on teaching conceptual mathematics with connections and research-based lesson design, I assisted teachers in building a collection of effective instructional strategies (West, 2009). The plan/reflect component referred to the planning discussions that occurred prior to a lesson and the reflective conversations that took place following the teaching of the planned lesson. The plan/reflect element of coaching was non-existent in the coaching I experienced as a teacher two years prior to becoming a full-time coach.

Professional learning communities (PLCs). High school teachers in my district were given one hour each week to meet in professional learning communities (PLCs). PLCs are widely seen as a way to improve teachers' instruction (Schmoker, 2006). PLCs were a place where I had access to a group of Algebra teachers at one time. Planning for math teaching in this environment tended to be long-term and looked at key features of big mathematical ideas within a chapter. Although some PLCs chose to collaboratively plan specific math lessons, much of the PLC time was devoted to deciding the pacing of the unit.

Student involvement. Student involvement, although not worded as an action on the coaching graphic (see Figure 2.2) was included when coaches observed a need for greater student engagement in the secondary math classrooms. The action that could accompany the title would be "Increase Student Involvement." The five subtopics are fundamental tools used in achieving effective classroom management (Sprick, Knight,

Reinke, & McKale, 2006). As a coach, I collected data using these tools during a teacher's class to help the teacher and me identify what we could work on in order to increase student involvement.

Model/observe instruction. Modeling occurred when a teacher asked to watch me teach a lesson or when a teacher seemed apprehensive about teaching a topic and I offered to teach. I used the technique of modeling to demonstrate to teachers how to teach in a particular manner (Poglinco et al., 2003). I often modeled for teachers to demonstrate how to present an effective lesson, ways to teach mathematics conceptually, and how to connect the mathematics being taught to prior or future knowledge. Even though I demonstrated these aspects of teaching from time to time, the majority of my modeling as a coach focused on using cooperative learning instructional strategies to engage students.

Initially, I thought having a formal outline of my coaching responsibilities in the form of the graphic was comforting. I finally had some language to use when I explained to teachers and administrators what coaching could offer. Yet soon after creating the graphic, I began to think the graphic did not completely capture the role of coaching. The components listed on the coaching menu were elements of my work as a coach, but I was finding that these elements were interconnected rather than separated into domains. For example, a coach and teacher may focus on getting students more involved in the mathematics. Together, we would collaboratively plan a lesson to increase involvement, I would model or observe the lesson, and then we would reflect on the lesson together.

Another limitation of the coaching graphic was that it did not include all of my responsibilities as a coach. An example of a key component of my coaching role that

was not specifically included in the coaching menu was the pilot and implementation of a new Algebra curriculum. My role in the implementation was to collaborate with the other secondary coaches (one Algebra coach and one middle school math coach) to create a syllabus, assessments, teaching tips and grading procedures to guide the Algebra teachers as they implemented the new curriculum. I was involved in getting feedback from the teachers about assessments, pacing, and the textbook. With the teachers' feedback, the coaches and I modified the assessments, syllabus, and teaching tips we had created. As part of our coaching duties, my colleagues and I also aligned the curriculum with other math courses and the state standards (Hull, 2009). During the summer of 2010 a significant portion of our coaching time was spent training every Algebra and Advanced Algebra teacher throughout the district. When training the teachers, we as coaches demonstrated the components of the new curriculum, explained how to use the assessments and grading guidelines, showed how the syllabus was developed, and helped get teachers familiar with the on-line curriculum resources (Walpole & Blamey, 2008).

In addition, my coaching role included "learning facilitator" when I began offering professional development workshops for middle school and high school teachers (Killion, 2009). The coaches worked with the curriculum specialist to determine what professional development sessions on research-based instructional practices to offer teachers. The coach offered suggestions for professional development sessions and the curriculum specialist decided if the suggested topics would be relevant for the teachers. The majority of the staff development I offered centered on cooperative learning as an instructional strategy, since I had attended conferences for cooperative learning (Kagan, 1994), worked for three years to implement cooperative learning into my own classroom,

and had conducted action research to learn more about the influence of cooperative learning on students' involvement in the mathematics classroom (Nebesniak & Heaton, 2010). My cooperative learning workshops lasted half of a day and were designed to engage teachers through cooperative learning activities, teach new instructional strategies, and briefly touched on relevant research about the benefits of cooperative learning (Vogt & Shearer, 2007). I viewed the staff development sessions as lessons on pedagogy. The teachers were the students and I was the teacher. One reason I viewed staff development in this way was that the physical set-up of these workshops resembled a classroom. The students were at tables and I was up in front of the class. I also viewed these sessions as lessons in pedagogy because the teachers and I all came to the session with the belief that I had some knowledge that I was going to teach them and they were going to learn. For my staff development sessions, I created an (extended) "lesson" for two purposes. First, I wanted to teach teachers about cooperative learning. Secondly, I wanted to demonstrate how to implement an effective lesson and instructional practices.

My definition of coaching when I was a full-time coach for the one and a half years leading up to my research centered on the idea of a coach helping a teacher improve his or her instruction. I believed that instruction improved when a coach did the following: planned with a teacher, modeled lessons, led staff development sessions, and organized and led the implementation of new curriculum. I saw coaching as a way to help teachers professionally develop and improve their instruction.

Coaching to Improve Instruction: Definition of Coaching From Literature

My experiences as a teacher and as a coach helped me to construct my own definition of coaching. I began to more clearly articulate the role of a coach. Literature on coaching also provided me with insights as I continued to create a definition.

In many school districts today, coaches can be heard introducing themselves as a Literacy Coaches, Technology Coaches, Math Coaches, or more specifically Algebra Coaches. And often all of these coaches refer to themselves under the generic title of Instructional Coach. The definition of a Math Coach or Instructional Coach depends on the initiative of each individual school or district. The coaching role can be a hybrid of peer coaching (Joyce & Showers, 1996), cognitive coaching (Garmston, 1993), and content coaching (West, 2009) and often incorporates aspects of all three coaching models.

In their search for a clear definition of coaching, Vogt & Shearer (2007) concluded that a singular definition of coaching might be naïve and undesirable. They argued that a coach is meant to fulfill a supporting position, which is dependent upon the needs of the ones being supported. Therefore, a single, all-encompassing definition of a coach would not allow educators (or business people) the opportunity to modify the role to fit their specific needs (Vogt & Shearer, 2007). Although a specific definition is not recommended, too broad of a definition puts a coach in danger of not succeeding due to extremely high expectations (Shanklin, 2006). Below is an explanation of how I viewed instructional coaching prior to the Spring of 2011. My definition of coaching, based on the coaching literature available, included a glance at the problem I thought coaching was trying to combat, why coaching was suggested as a possible solution (situating coaching

in the context of professional development), and the coaching process used to address the problem.

What is the problem? As mathematics educators, we want our students to know, like, and be able to apply the mathematics being taught to them. Yet, we are falling short of this goal (Stigler & Hiebert, 1999). The reality is that not all students are achieving success in mathematics. Teachers, administrators, parents, and researchers generally agree that when it comes to student achievement, teachers matter most. Students achieve more when they have access to high-quality teaching (National Council of Teachers of Mathematics (NCTM), 2000; National Mathematics Advisory Panel, 2008). The teacher and the quality of teaching have a larger effect on student performance than curriculum, technology, assessments, class size, student placement, or class offerings (National Commission on Teaching and America's Future, 1996; Wright, Horn, & Sanders, 1997). This means that the single best way to improve student achievement is by improving teacher effectiveness (Hall & Simeral, 2008; Wright et al., 1997). Wei et al. (2009) concluded, "Efforts to improve student achievement can succeed only by building capacity of teachers to improve their instructional practice and the capacity of school systems to advance student learning" (p. 1).

Since teachers are the most important factor in improving student achievement, focusing attention on improving teachers' instruction should lead to increased student learning (Leinwand, 2009; National Foundation for the Improvement of Education, 1996). How do we improve teachers' instruction? The opportunity for in-service teachers to improve instruction is largely provided by district leaders and in the form of professional development. Professional development is seen as a major contributor to the

improvement of teacher instruction and professional beliefs (Guskey, 2002). Professional development traditionally comes in the form of a full day or half-day workshops in which a large group of teachers listen as a speaker or trainer instructs them on new ideas, methods, or materials (Vogt & Shearer, 2007). Having an expert do one-shot training is not effective at changing teachers' instruction (Knight, 2007;) and only about 10% of teachers transfer strategies presented in a workshop to their classroom (Joyce & Showers, 2005; Bush, 1984).

The other 90% of teachers who struggle to change, modify, or implement new strategies provided through professional development (Costa & Garmston, 1991; Hull, Balka, & Miles, 2009) may struggle due to the fact that the process of change in teaching is complex, multifaceted, and involves a combination of factors. In addition, individual teachers possess different sets of prior knowledge, vary in their experiences, and react in their own ways to change. For those reasons, professional development needs to be created on an individualized basis and address a specific teacher's needs (Kise, 2006; Kise 2009; Shanklin, 2006).

The first part of my definition of coaching directly related to my problem of practice. In my mind, coaching was born out of a desire to help improve teachers' instruction. This goal to improve teachers in order to improve students' achievement in Algebra, or any other content, was the drive behind professional development. Unfortunately, the traditional implementation of professional development (one-time workshops with little connection to teachers' classrooms and no differentiation for teachers' varying needs) was leading to very little change in teachers' instruction. Therefore, the need for coaching was created. So for me, coaching was a professional

development strategy formed to improve students' understanding by improving teachers' instruction through individualized work between a coach and a teacher.

Situating coaching in the context of professional development. There are some fundamental characteristics of professional development that foster teacher change successfully (National Staff Development Council (NSDC), 2001; Wei, Darling-Hammond, Andree, Richardson, & Orphanos, 2009). New professional development models, such as instructional coaching, incorporate components of effective professional development to improve teacher effectiveness and ultimately increase student achievement (Olson & Barrett, 2004). Effective professional development is school-based (National Foundation for the Improvement of Education, 1996), related to classroom instruction and student learning (American Educational Research Association, 2005; Loucks-Horsley, Stiles, Love, Mundry, & Hewson, 2003), and actively involves teachers in the professional development activity (Desimone, Porter, Garet, Yoon, & Birman, 2002). Approximately 90% of teachers will be able to transfer a new skill into their classroom if professional development includes theory, demonstrates an instructional strategy, allows teachers to practice within the training, provides the teacher with feedback, and supports the implementation of a new skill with coaching (Joyce, 1987).

Coaching can be referred to by a variety of different names including content coaching, differentiated coaching, cognitive coaching, and math coaching to name a few. Putting all of the types of coaching aside, Killion (2009) provided a general definition of coaching.

Called by many different titles, teacher leaders in this role are primarily school-based professional development specialists who work with individuals and teams to design and facilitate appropriate learning experiences, provide feedback and support, and assist with implementation challenges. Their work centers on refining and honing teachers, and their indicator of success is student academic success. (p. 9)

Tying back to the origin of the word coach and the definitions provided in the literature, I understood coaching as the process of moving mathematics teachers' thinking about teaching and their abilities to implement effective mathematical instruction. Coaching was about transporting teachers from where they were performing to a higher level of effective performance.

By incorporating elements of effective professional development, coaching addresses the concerns of traditional, one-time professional development workshops. Putnam and Borko (2000) recommend professional development be situated within teachers' practice. One of the most important components of coaching is the coherence that is fostered when a coach is site-based, working with teachers using their actual curriculum, assessments, and student data (American Education Research Association (AERA), 2005; Guiney, 2001). When coaches are site-based and working with school leaders to construct a definition of coaching and goals that address issues specific to the teachers' instruction in that school, there is an improvement in teachers' practices (Guiney, 2001; Shanklin, 2007). Coaching focused on teachers' current curriculum increases the likelihood that teachers will adopt the instructional strategies discussed during professional development (Cohen & Hill, 2001).

Stigler & Hiebert (1999) argue that in order to improve teachers' instruction, professional development must be built into their daily and weekly routines. Coaching is a way to incorporate professional development into the daily lives of teachers (Corcoran, McVay, & Riordan, 2003; Ross, 1992; Shidler, 2009). And the teachers who spent more time in the coaching process found greater gains in their students' achievement (Viadero, 2010). In addition, the more individualized approach of coaching replaces the "one-size fits all" professional development approach that does not take into consideration individual teachers' distinctive needs (Elish-Piper, L'Allier, & Zwart, 2008; Shanklin, 2006). Coaching tailors professional development to a teacher's specific needs, interests, goals and prior experiences (Elish-Piper, et al., 2008; Kise, 2006).

Another important element of effective professional development is a focus on content knowledge (American Educational Research Association (AERA), 2005; NSDC, 2001). Saxe, Gearhart, & Nasir (2001) found that teachers' instruction improved most when professional development engaged teachers in prolonged collaboration in content and instructional practices surrounding that content. In mathematics, teachers can and should learn more about mathematics in order to improve their mathematics teaching (Hill & Ball, 2004). Coaching is meant to increase teachers' content knowledge and develop a deeper understanding of the big ideas involved in specific content (Driscoll, 2007; West, 2009). The "appreciation of mathematical reasoning, understanding the meaning of mathematical ideas and procedures, and knowing how ideas and procedures connect" (Hill & Ball, 2004, p. 331) are all aspects of teachers' content knowledge. Mathematics coaching can help improve teachers' content knowledge by helping teachers develop a greater understanding of the current content being taught, as well as

mathematics beyond the level being taught and not to use that knowledge in the tasks of teaching. This increase in content knowledge will help teachers acquire a stronger understanding of various branches of math, problem solving techniques, mathematical habits of mind, mathematical thinking, and the means for communicating mathematics (NCTM, 2000).

Building on content knowledge, effective professional development also includes discussing and learning effective instructional practices specific to the content being taught (AERA, 2005; NSDC, 2001). Teachers' knowledge of students' mathematical thinking and learning can be referred to as pedagogical content knowledge (Hill, Ball, & Schilling, 2008). Increasing teachers' pedagogical content knowledge helps teachers more effectively demonstrate, model, and represent mathematics to students (NCTM, 2000). In mathematics education, "teachers need to know mathematics in ways useful for, among other things, making mathematical sense of student work and choosing powerful ways of representing the subject so that it is understandable to students" (Ball, Thames, & Phelps, 2008, p. 404). To address the need for greater emphasis on pedagogical content knowledge, coaches work with teachers to help them develop effective instructional strategies specific to the content they teach using students' thinking, understanding, and written work (Foster, 2007; West, 2009).

Based on my understanding of the professional development and coaching literature, prior to Spring 2011, I viewed coaching as a professional development strategy that encompassed the characteristics of effective professional development. I believed that by working with a coach, teachers would gain knowledge about their content and begin to use more effective instructional strategies as they taught mathematics. The one-

on-one aspect of coaching, along with being site-based, allowed me to work with teachers on a weekly basis in their own classrooms. By combining all of these effective elements of professional development, I believed coaching could help teachers change their instruction.

How coaching is used. The first step in implementing mathematics coaching to improve student achievement is the selection of a math coach. An instructional coaching position is often filled with an accomplished teacher in that content area (Arbaugh, Chval, Lanin, VanGarderen & Cummings, 2010). For example, a successful math teacher moves out of her own classroom and into the math coaching role. The underlying belief is that if a person is successful in a classroom, then as a coach she will be able to share knowledge of teaching with other classroom teachers and their instruction will improve. The assumption is that the experience and understanding an accomplished teacher that has accumulated can be used to help other teachers improve when she is put in the math coaching position (Driscoll, 2007).

To begin, coaches get teachers involved by engaging them in coaching. The enrollment procedure, whether it is a formal interview, informal interview, or administrator referral, is a way to gather information about the individual, explain the coaching process, and begin to develop a relationship between the teacher and coach (Knight 2007). The teachers who enroll in coaching are not all doing so by choice. Some teachers are encouraged or recommended by their administrators or curriculum specialist to work with a coach. Other teachers seek out the coach and are choosing to engage in coaching. After the initial enrollment, the coaching process begins. The coaching process, or coaching cycle, includes a pre-conference planning session,

classroom lesson, and post conference debriefing meeting (Costa & Garmston, 1991).

These three components are described in greater detail in the following section.

Planning Session. During pre-conference planning, a teacher and coach frame the lesson, identifying the goals and key mathematics of the lesson (Foster, 2007). This is the time when a coach becomes familiar with the teacher's understanding of the mathematics, thoughts about effective strategies, and beliefs about students (Staub, West, & Bickel, 2003). The pre-conference planning session is a good time for the coach and teacher to discuss effective instructional strategies and brainstorm potential student challenges and misconceptions that might occur during the lesson (Hansen, 2009). An agreement is also made during this time about the specific roles of the teacher and coach during the lesson (Foster, 2007). Saphier and West (2009) argue that planning conferences with teachers should be a priority over observing classroom lessons or debriefing since they believe the time spent planning has greater potential to improve instruction.

Lesson. The role of a coach during a classroom lesson varies, ranging from modeling instruction, to coteaching with the teacher, to observing the teacher (Staub et al, 2003). These roles of the teacher and coach are decided upon when a lesson is first discussed. Since both the teacher and coach jointly create the lesson during the pre-conference planning, both people accept responsibility for the lesson being taught (Staub et al., 2003). This means no matter what roles the coach and teacher assume during the lesson, both people are held accountable for the outcomes of the lesson.

Modeling a lesson. Modeling instruction is one role the coach can employ when in a classroom. The benefit of modeling is that the teacher can see exactly how a

teaching practice works with his students and is a way for the teacher to observe aspects of teaching that are difficult to describe (Knight, 2007). During a model lesson, the coach leads the class and the lesson while the teacher observes. This strategy is often utilized when a coach begins to introduce a new instructional technique to a teacher (Staub et al., 2003).

A coach should model lessons after a strong rapport has been built with a teacher (Hull et al., 2009). The reason modeling should not be incorporated into the first few coaching cycles is because these demonstration lessons run the risk of sending the message that the coach is the “expert” and the teacher is doing something wrong (Hull et al., 2009). To help avoid this misconception, it is important that prior to the lesson the teacher and coach determine what the teacher should observe. The elements of the lesson that will be observed could include students’ responses, a teacher’s questioning techniques, or the success of a specific instructional strategy (Hansen, 2009). The teacher then looks for those elements during the lesson and possibly records what is taking place. As the coach and teacher engage in the coaching cycle more over time, the coach will begin to phase out modeling and start utilizing a different strategy such as coteaching or observing (Staub et al., 2003).

Coteaching. Coteaching is done when the teacher and coach either teach as a team or in tandem (West, 2009). This means that the teacher and coach could decide to both teach the mathematics, playing off of one another’s instruction and questions. Coteaching may also look more like a hand-off where the teacher leads one component of the lesson and the coach leads another. This second type of coteaching is often used, and even recommended, when the instructional coach wants to model a specific teaching

strategy within the lesson rather than the entire lesson (Knight, 2009). In that case, the teacher will lead the majority of the lesson and the coach steps up when the coach determines it is a time to implement the desired strategy.

Observing the teacher. A lesson observation occurs when the teacher leads the lesson and the coach assumes an observation role. Often times the coach avoids using the term “observe” due to the strong connection to administrators observing during the appraisal process (Knight, 2009). Since the coaching role is intended to be purely non-evaluative (Shanklin, 2006), it is important the teacher does not misinterpret the coach’s role during the coaching observation as being evaluative. The coach is simply a second set of eyes in the room to record information on the lesson. During the pre-conference planning, the coach and teacher decide upon the information that should be collected during an observation (Knight, 2007). The data collection aspect of the observation should focus on student learning. This may include gathering student work, taking notes on students’ dialogue, or recording how students do or do not demonstrate their understanding of the mathematics being taught (Shanklin, 2006).

Debrief Meeting. Hansen (2009) and Foster (2007) believe the post-conference debrief is the most important component of the coaching cycle since that is the time when both the students’ and the teacher’s learning is assessed. During the debriefing session, a coach and teacher use student data collected during the lesson to facilitate teacher self-reflection and change (Peterson, Taylor, Burnham, & Schock, 2009). The teacher and coach approach the debrief as a partnership (Knight, 2009). The debriefing discussion includes the coach focusing questions on student understanding and the teacher and coach discussing how the instruction and student learning were linked (Foster, 2007). When a

coach and teacher engage in a post conference debrief, it is important for the coach to remember that teachers can feel very vulnerable during this phase of the coaching cycle (Hull et al., 2009).

Professional learning communities (PLCs) are an additional support for teachers beyond the coaching cycle. Hall and Simeral (2008) define a PLC as “a collective of educators who always strive to perform at their ultimate potential, working together to learn, grow, and improve the professional practice of teaching in order to maximize student learning” (p. 17). An instructional coach could have a significant role within the PLC. The coach could act as a facilitator among the group of teachers, encouraging discussion about effective lesson design and instruction, and weighing in when differences about instructional choices arise among the teachers (Saphier & West, 2009). During a PLC, a coach should strive to do a lot of listening without forcing her own opinions about instructional strategies or how the mathematics being discussed should be taught (Hansen, 2009).

In addition to being involved in the coaching cycle and PLC work, the coaching role also includes the completion of larger scale tasks aimed at a large group of teachers. For example, a coach may need to address all math teachers within a district or a school’s whole staff (Hansen, 2009; Saphier & West, 2009). Coaches manage and organize curriculum and instructional materials to provide continuity and consistency among teachers (Hull, Balka, & Miles, 2009; Killion, 2009). Often coaches lead the curriculum implementation of new programs or curriculum, (Brigham & Berthao, 2006; Toll, 2006; Vogt & Shearer, 2007) and gather, analyze, and interpret data to inform teachers and district leaders of student performance (Knight, 2007; McKenna & Walpole, 2008).

Instructional coaches are also responsible for organizing and providing more traditional professional development through one-time workshops or trainings in which the coach is viewed as the expert (Hull et al., 2009; Walpole & Blamey, 2008).

Prior to Spring 2011, implementing coaching as a professional development strategy to me meant engaging teachers in the coaching cycle. Prior to engaging in this research study, I believed that the pre-conference planning, lesson modeling/coteaching/observing, and post conference debriefing were the key aspects of coaching that would ultimately improve teachers' instruction. In my mind, the "other" parts of my role as a coach (facilitating PLCs, organizing curriculum, writing assessments) were not as important.

Initial Interpretation of Coaching

Developing a clear understanding of coaching was important for me as a coach because I felt the better I understood my role, the better I would become. The definitions provided the backdrop or context of my work as a practitioner and as a researcher. I was unsure how to apply these definitions to my own practice. What should I do, or not do, during the pre-conference planning session if I want the teacher to learn a new strategy? How do I effectively conduct a post conference debrief? What was I doing as a practitioner that was helping teachers improve instruction? Is there anything I could be doing better as a coach?

From August 2009 through December 2010, I decided to take a closer look at my practice as an instructional coach. I was in the midst of taking courses in research methodology and decided to utilize data collection techniques such as field notes, a

personal journal, and artifacts to examine my practice. This inquiry came prior to my research in the Spring of 2011 and served as a precursor to my formal research study.

The field notes served as my primary technique for data collection and were used to capture what I saw and heard related to my coaching interactions with teachers. I took detailed field notes immediately following meetings and coaching cycles with teachers. The personal journal entries included any interactions with participating teachers, emphasizing our weekly coaching contact. I wrote about my perceptions and feelings surround interactions with a teacher and her school, as well as any topic, issue, or feeling I had concerning my coaching position. Artifacts included, but were not limited to, classroom worksheets and notes, written communication, school or department wide documents, or any other artifacts that put with other data will result in a story of the teacher's learning. These tools helped me construct and analyze my coaching reality, although I understood that what I deemed as the truth may conflict with the version the teachers involved in the coaching tell (Angrosino, 2005).

In order to fully conceptualize how my definition of coaching played out in my practice, I offer the following vignette from my inquiry. My goal is that this vignette will serve as an example of my coaching interactions during my first one and a half years as a full-time instructional coach and will provide insight into what my initial coaching looked like, and to what I did and did not attend. The coaching cycle with Sarah serves as a way to highlight my coaching and my role as a coach prior to my research in Spring 2011.

Vignette: Coaching Sarah

The vignette below details a coaching cycle that occurred on March 30, 2010. Sarah and I had been working together on a fairly regular basis since the end of October. Prior to the lesson from which the vignette below was written, we had been through the coaching cycle 13 times. We had already experienced a variety of coaching interactions including observation with data gathering, modeling of a full lesson, and co-teaching a concept.

Planning. I emailed Sarah to see if I could come to her class Tuesday of the next week. It was typical that I would ask if she would like to participate in a coaching cycle. If I did not initiate the coaching through email or by asking her in person, Sarah would not have contacted me to arrange a coaching cycle. Since I was scheduled to coach in a different building on Monday, Sarah's and my planning occurred via email. Planning over email was rather common for us since I was rarely at her school two consecutive days. On Monday Sarah emailed me an outline of her lesson:

I was wondering if you wanted to lead the fan and pick classifying polynomial activity tomorrow? I have never really done a fan and pick so I didn't know if it worked best in groups of 2,3, or 4? I have a warm up ready that reviews classifying so the students should be ready to go for the fan and pick. I thought we could do that first period. Second period I am going to introduce adding and subtracting polynomials with direct instruction. I was thinking about a simon says for practice followed by their homework assignment. Let me know if that's ok with you? (Personal Communication, March 29, 2010).

I agreed to model the Fan and Pick (Kagan & Kagan, 2009) cooperative learning activity. I also used my experiences with the activity to give her a little background on how different group sizes could potentially influence the outcomes of the activity. I also suggested another formative assessment that I had used in another teacher's class that students really enjoyed. I thought that including the second assessment called Take Off Touch Down (Kagan & Kagan, 2009) would allow us to help students practice the classification terminology and would help us see which students were still struggling. I did not explain to Sarah why we should use Take Off Touch Down. I simply told her it "may be kind of fun" (Personal Communication, March 29, 2010).

Since Sarah did not say anything about how she was planning to teach adding and subtracting polynomials besides using direct instruction, I wanted to delicately prompt a discussion about the mathematics. I wrote in my email, "Simon Says should work great for adding and subtracting polynomials. Hopefully they will just make the connection to it (adding and subtracting polynomials) being combining like terms, which they already know how to do" (Personal Communication, March 29, 2010). Most of the mathematical topics in Sarah's class were taught in isolation, so I wanted to suggest a connection to students' prior knowledge. Finally, I included in my email the idea of showing students polynomials represented both horizontally and vertically as a way to help some students with their computation (see Figure 2.3).

In an email, Sarah confirmed the progression of the day and that I would be leading the two formative activities (Fan and Pick, Take Off Touch Down). She also commented that she likes to teach adding and subtracting polynomials by lining them up horizontally and if its subtraction she always has them "add the opposite and change the

<p><i>Horizontal</i></p> $(12x^3 + x^2) - (18x^3 - 3x^2 + 6)$	<p><i>Vertical</i></p> $\begin{array}{r} 12x^3 + x^2 \\ - 18x^3 - 3x^2 + 6 \end{array}$
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Figure 2.3: I suggested to Sarah that we represent polynomial expressions both horizontally and vertically.

second polynomial's signs" (Personal Communication, March 29, 2010) (see Figure 2.4).

The three emails were the extent of our planning until I got to her school on Tuesday.

We took a couple of minutes to touch base and make sure we agreed on with the plan for the day before the first bell rang signaling the start of her class.

<p><i>Original Expression</i></p> $(12x^3 + x^2) - (18x^3 - 3x^2 + 6)$	<p><i>Equivalent Expression</i></p> $(12x^3 + x^2) + (-18x^3 + 3x^2 - 6)$
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Figure 2.4: Sarah explained that she has students subtract polynomial expressions by adding the opposite.

The Lesson. Immediately after the bell rang, Sarah greeted the students and handed out a worksheet that asked students to rewrite four different polynomials in standard form and then classify based on degree and number of terms. While students worked on the warm-up, Sarah took attendance and completed other "housekeeping" tasks. During this time I walked about the class getting students started on the warm-up and answering students' questions. Sarah brought the students together as a class and they completed the warm-up together as a way of checking their work.

After the warm-up, I led the two formative assessments that were meant to help students practice the terminology. Prior to starting the activities, I had students list the various vocabulary terms by degree classification and by term classification (see Figure 2.5). I had them do this because I noticed several students did not have the correct words with the appropriate classification on their warm-up. I did not address this issue with Sarah or explain to her why I decided to put the terms on the board prior to starting Fan and Pick.

<u>Degree</u>	<u>Number of Terms</u>
<i>constant</i>	<i>monomial</i>
<i>linear</i>	<i>binomial</i>
<i>quadratic</i>	<i>trinomial</i>
<i>cubic</i>	<i>polynomial</i>

Figure 2.5: Students listed the various vocabulary terms by degree and number of terms.

I introduced the Fan and Pick cooperative learning activity to the students, explaining that they would be working in groups of three to verbally classify several polynomials. Using a group of students as an example, I described that one student would fan out the cards that had various expressions. The next student would pick a card and categorize the expression by degree. The third person in the group would then classify the expression by the number of terms. We talked about what it meant for students to coach or help their peers if they were stuck on a problem. During this activity Sarah, Jane (the special education co-teacher) and I walked around and checked for understanding.

Next, I led the Take Off Touch Down activity to continue practicing classification in a more individualized way. I handed each student a polynomial and announced a statement. “Take off if your polynomial is linear. Take off if your polynomial is a trinomial.” Students stood up (take off) if the statement applied to their polynomial. If it was not true of their polynomial, they remained seated (touch down). Sarah and I determine individual students’ understanding by checking their polynomial and if they had stood up or remained seated.

With a few minutes left in the first period of the block class, Sarah began direct instruction on adding and subtracting polynomials. She handed out a piece of paper that had been cut in half vertically and left only the odd numbered problems down the side (see Appendix A). Sarah led students through the first three problems (#1, 3, and 5) and emphasized that they must add the opposite or change the sign of everything in the second set of parentheses. These three problems were all subtraction. She asked students questions as they did the problems together as a class. Her questions were aimed at the procedures. She asked questions such as “What does 12 plus -18 equal?” or “What do I do next?” Overall, students participated in the notes and were engaged in the problems. During this time, I walked around helping kids by asking them questions similar to the questions Sarah asked during her direct instruction.

About halfway through the direct instruction, the ringing bell signaled that students could have a five-minute break in the hall. When students returned from break, Sarah finished leading them through the three problems. She then gave the students a couple of minutes to work on the last two problems on the paper individually. One problem (#7) was addition and the other (#9) was subtraction again. While students

worked individually, Sarah sat up front by her overhead and I walked around checking students' work, praising students for correct simplification, and addressing issues with students who were not correct. After a couple minutes, the timer went off and Sarah did the two problems with the class.

To practice adding and subtracting polynomials, Sarah put students with a partner and they did Simon Says. One student told the partner what to do and how to simplify the expressions. The people writing only wrote what their partner told them to write and only wrote something if it was correct. (I had introduced Sarah to this cooperative learning strategy and we had used it multiple times since the beginning of the year with her students.) After explaining the directions, Sarah handed out another half sheet. This half sheet included the even problems from the worksheet she had cut in half (see Appendix B). During this time, I partnered with a student who had difficulty working with others in the past. Sarah and Jane walked around as students worked on the problems. When a pair was finished, Sarah checked it and then gave them a worksheet with ten problems to complete on their own for homework (see Appendix C).

Debrief. Although Sarah had a plan period immediately following this class, finding time to debrief with her proved to be difficult. She would often tell me in her body language that she had other things she wanted to do. For example, she often sat behind her desk and worked on her computer as I tried to ask her about the lesson (Field Notes, March 12, 2010). Previous experience told me that she would be interested in doing other work after class during her plan period on this particular day, as opposed to reflecting with me. On this day, I decided to try asking her some debriefing questions during the break between the first and second half of class.

I tried to start a conversation about what had happened during the first period.

Amy: How do you think the kids are doing with classifying polynomials?

Sarah: Good. I think they've got it. (Post Lesson Debrief, March 30, 2010)

Since that question did not launch us into a conversation, I tried a few more questions focused on the students and the activities. Each time I asked a question, Sarah responded in a short, quick manner.

Amy: What do you think about the two classifying activities (Fan N' Pick, Take Off Touch Down)?

Sarah: I think they went well.

Amy: Would you do anything different with the activities to make them better?

Sarah: I don't know. I think they were fine.

Amy: Do you think the activities helped with student understanding?

Sarah: Yea, they were good. (Post Lesson Debrief, March 30, 2010)

After more prompting, pushing, and giving her some of my ideas for making the formative assessments better, Sarah stated a writing component to the activities would be beneficial so they would have the practice of writing down the new terms.

After the second part of class and during her plan period, I again attempted to ask Sarah questions. This time I focused on the second half of the lesson. I asked her how she thought the lesson went, if she thought the students understood how to add and subtract polynomials, and why she chose to cut the worksheet in half for notes. She again quickly responded that she thought things went fine. She did not seem interested in talking about the lesson or brainstorming ways to improve the instruction, so I left the room.

As I personally reflected on the lesson and coaching cycle, I felt that Sarah was making some positive changes in her direct instruction. One of my goals in working with Sarah was to help her break free from the traditional direct instruction where she does several problems for the students and they mindlessly write down what she does (Stigler & Hiebert, 1999). Today she tried something different by cutting the worksheet in half and giving the students some problems to try on their own for guided practice. As she led students through the first three problems, she asked them questions and involved them in the problem. Although I noted these positive changes, the majority of my reflective thoughts were fixated on her lack of interactions with students. During her instruction she did not gather feedback about where students were at in their understanding of the concept. For my own coaching purposes, I pinpointed this as a teaching move that I wanted to work on with Sarah.

Initial Coaching

Since I had such a large number of teachers to support during my first year and a half of full-time coaching, I tried to work with as many teachers as possible. After a while, I realized that not all teachers were interested in partnering with a coach. By trying to work with all 25 teachers simultaneously, I was not using my coaching time efficiently. In order to focus my attention, I decided to work closely with three to five teachers at each building. These teachers were chosen based on a variety of factors including the teacher's willingness to work with a coach, the number of Algebra courses the teacher taught, and the teacher's schedule. I continued to maintain a connection to all Algebra teachers by engaging them in a coaching cycle periodically and attending the Algebra PLCs at each building.

Fixing. Since I was not in one school for consecutive days, planning largely took place via email or during a plan period earlier in the same day that the lesson was being taught. Teachers would send me an email about their thoughts on a lesson and I would reply with questions, suggestions, or ideas from other Algebra classes and teachers I had worked with. If I met with a teacher the same day as the lesson, the teacher usually had a lesson already planned and we would talk about ways to “tweak” it or how to help students be most successful. Our attention was on that specific lesson. I would often ask what the students had learned previously to get a better idea of their prior knowledge. Yet I did not know first-hand what they did instructionally leading up to the lesson being planned.

Planning with teachers during the first year and a half as a full-time coach could be described as fixing. I thought my primary role as a coach was to identify the weaknesses in teachers’ lessons and improve the instruction. A teacher shared her lesson with me that she created, and I worked to improve it prior to class. Like with Sarah, when she shared her lesson ideas via email I first tried to make the lesson better by suggesting that she include another formative assessment. Then I tried to change her direct instruction of adding and subtracting polynomials by asking if she was planning to show both horizontal and vertical representations. I believe Sarah also saw me as someone who fixed her lessons, which was evident in her comment, “Let me know if that’s ok with you” (Personal Communication, March 29, 2010).

When I think about my view of planning as a way to fix teachers’ lessons, I now realize the errors in my thinking. Coaching is meant to improve instruction long-term, not fix a lesson immediately (Costa & Garmston, 1991). I now see that the way I was

approaching the planning session contradicted what I had learned as a teacher. If students had incorrect solutions in my Algebra class, I had learned not to jump in and fix their mistakes, but instead question in ways such that they would identify their own mistakes (Lobato, Clarke, & Ellis, 2005). Through this process, my Algebra students began to recognize their own mistakes, determine why their solutions were incorrect, figure out how to improve their work, and ultimately think like mathematicians. As an Algebra teacher, I learned that I should not immediately point out my students' flawed thinking and tell them how to do the mathematics correctly, because I wanted them to problem solve and learn what to do when they get stuck. That was the essence of the contradiction I was experiencing. With the teachers I was coaching, I was pointing out flaws in their lessons and telling them how to correctly plan a lesson. At the time, this approach made sense to me, because it helped the teacher and I see immediate results. We felt like we were changing our instruction. But in reality, in hindsight, I recognize we were only changing our instruction for that particular lesson. I was not seeing a long-term change in instruction. Why would I as a teacher expect my students to reason through their mathematical misunderstanding but as a coach not expect teachers to reason through their pedagogical misunderstandings? I began to see why I never felt like I was making progress with the teachers with whom I worked.

The planning phase of the coaching cycle, as I was experiencing it from August 2009 through December 2010, created another contradiction. From my own teaching experiences I had come to realize that collaboration was an essential part of teaching. My own teaching practices improved when I collaborated daily with my colleagues on ways to introduce and teach mathematical concepts (Darling-Hammond & Richardson, 2009).

The dissonance in my coaching was the fact that my collaboration with teachers was neither intense nor content-focused. Yet I knew intense, content-focused collaboration is what pushed me as a teacher to improve my instruction. How did I expect teachers to improve their instruction if I was not engaging in that very kind of collaboration with them?

Giving answers. My contribution to a teacher's lesson largely revolved around formative assessments and cooperative learning strategies. Math teachers in the district referred to me as the "cooperative learning expert." This view was due to the numerous cooperative learning professional development training sessions I facilitated (Walpole & Blamey, 2008). The teachers I coached tended to view me as the person who could provide a variety of engaging formative assessments or suggestions about how to successfully implement a cooperative learning activity. I answered their requests by showing teachers specific cooperative learning activities to use with their students. Each activity was presented to the teachers as a list of steps or procedures, which they could take and use in their own classrooms.

Initially, I was flattered that teachers believed I had instructional strategies that they wanted to have as well. Then when people started calling me the expert, I became concerned. As a teacher, cooperative learning was one instructional strategy that worked for my students and my teaching. I was not perfect at implementing cooperative learning and I did not have all the answers. In fact, I often told teachers I was able to model cooperative learning only because I had been using it in my own classroom for several years and I had failed so many times at incorporating it that I was finally beginning to

figure out how to use cooperative learning effectively. I did not view myself as an expert.

Teachers I coached on a weekly basis were not hesitant to ask me to model a cooperative learning structure in their class, as seen in the vignette. In her email, Sarah immediately asked me to lead the cooperative learning activity Fan and Pick. Then as we continued in our correspondence, she asked me to model a second formative assessment (Take Off Touch Down). In theory, I would show Sarah how to implement these specific cooperative learning activities in her classroom with her students (Knight, 2007), and then she could mimic the process in future lessons. I maintained this mode of operation with all of the teachers I coached. I modeled the instruction (usually a cooperative learning strategy) the first time and they imitated what I did the next time. Many teachers did mimic me as they implemented the same cooperative learning strategy. Yet teachers were not as quick to imitate other instructional techniques such as questioning students, interactive direct instruction, and classroom management.

As I reflected on my typical role during the lesson component of the coaching cycle, I was once again hit with the realization that I was living a large contradiction. When I was a new teacher, I approached teaching mathematics in a step-by-step manner and focused on the right answer. As I grew in my understanding of mathematical concepts and how to effectively teach mathematics, I came to recognize that only teaching my students procedures was not helping them learn the big ideas of mathematics (NCTM, 2007). I was not helping them learn how to think, reason, and make sense of mathematics (NCTM, 2009). During my first year and a half as a full-time instructional coach, I now see myself approaching coaching as giving answers to teachers by telling

them how to teach lessons. I was giving teachers cooperative learning activities and expecting them to simply repeat the activity. My coaching was rarely focused on the teacher understanding why the activity worked. I began to wonder if my approach to coaching was the best way to help teachers improve their lessons. Was it important for teachers to think about their instruction? Or was it enough for them to observe my teaching and imitate my actions?

Limited discussion of observations. After lessons, I tried to debrief with the teacher during her plan period, after school, or via email. The debriefing of the lesson was often rushed due to a variety of reasons such as teachers attending meetings or coaching athletics. Others had already filled their plan period with other tasks and did not feel like they had time to talk. This was obvious in the example of Sarah. The combination of her short, quick answers to my questions and her body language caused our debriefing session to be limited. We did not engage in a deep, meaningful conversation about student learning or how her instruction did or did not affect student learning (Peterson et al., 2009).

As a coach in my first year and a half, I worked to keep the focus of the post conference debrief on student learning (Staub et al., 2003). However, classroom management and formative assessments often became the center of my debrief discussions with teachers. These were the topics that were the easiest and least controversial to talk about. The sense among the teachers was that it was okay to admit weaknesses in classroom management and formative assessments since all teachers experience frustration in these areas. The teachers and I often talked about topics such as how to get more students engaged (i.e., how to stop Jeremy from talking), what went well

in the activity, and what we would do differently if we could teach the lesson again. If I did have discussions about mathematics with a teacher, those discussions were largely centered on student misconceptions or the current level of student understanding.

During the post conference debrief I tended to concentrate on the teacher's implementation of an activity or classroom management strategies. I was not very concerned with the teacher's understanding of the teaching moves that were made. I did not consider whether the teacher understood how her choices influenced student learning. Yet from my teaching experiences I knew that by focusing on students' understanding of the entire concept rather than right answers, I helped students increase their mathematical knowledge. How was I going to get teachers to increase their math knowledge? What would it look like if I focused on the concept of teaching mathematics?

Facing the Contradictions

Inquiry into my practice of coaching pointed out some startling contradictions within my practice. Contradictions I was not aware of until I began systematically looking at my practice and reflecting on my coaching interactions. I began noticing a large disconnect between what I had come to believe and understand about helping students learn Algebra and what I was attending to as an instructional coach helping teachers learn to teach Algebra in better ways. As an Algebra teacher I had come to realize that my job was to help students learn and understand mathematical concepts. And in order to teach for understanding, I needed to help students attend to the conceptual aspects of mathematics as well as the procedures (NCTM, 2009). I could not jump in and "fix" my students' mistakes, but instead I needed to guide them towards finding and addressing their own misunderstandings. I knew from my teaching

experience and the literature that collaboration among teachers focused on content increased students' understanding (Shuhua, 2004) and such collaboration also increased my own understanding of mathematical concepts.

I knew all of those things as an Algebra teacher, yet I was not seeing how that teaching knowledge applied to my coaching role. There was a definite mismatch between what I believed and understood as an Algebra teacher and my practice as an instructional coach. I initially saw my role as a teacher to be largely distinctive from my role as a coach. Yet was teaching Algebra to students that different from helping teachers implement more effective mathematics instruction? As I reflected on my first year and a half as a full-time coach, my definition of coaching and what I understood as a practicing coach was called into question. Further research would help me gain a deeper understanding of this disconnect. If coaching was more closely related to teaching, I would need to enter into my inquiry of coaching as both a coach and teacher.

CHAPTER 3: UNDERSTANDING THE NATURE OF MY INQUIRY

When I began inquiring about my own practice as a secondary mathematics coach, the unveiling of numerous contradictions in my practice surprised me. As an Algebra teacher I wanted my students to learn mathematics by having me question them and push them to make their own inferences about the math concepts. Yet in my early work as a coach, I found myself telling teachers what they could do better and providing more effective instructional strategies for the teachers to try. As an Algebra teacher, I came to realize the importance of helping students understand mathematical concepts. Yet as a coach, I was not helping teachers understand concepts of teaching.

In my practice as a coach, I was beginning to feel like my practice was in conflict with what I knew about teaching and learning. I felt a significant disconnect between what I believed about education and how I was interacting with teachers. Literature on coaching cautions coaches that appearing to a teacher as the expert could damage the coach-teacher relationship (Hull, Balka, & Miles, 2009). Yet as I coached teachers, I was signaling to them that I was the expert by changing their lesson plans and suggesting instructional techniques to improve their teaching. I realized that how I was interpreting my definition of the coaching role was in conflict with my beliefs about how learning occurs, as well as some of the coaching literature.

Inquiry into Coaching

Developing inquiry in relation to my coaching practice was a process influenced by both theory and practice. I tried a variety of research methods until I found the approach that best fit my practice and the goal of my inquiry. As I worked with the various methodologies and partnered the research ideas with my practice, I came to better

understand the root of my inquiry. I came to understand the actual problem within my practice. The numerous contradictions I recognized are what urged me to look more closely at my practice. Yet it was not until I began to develop a more formal inquiry process that I came to understand the nature of my inquiry. In Chapter Three, I describe my inquiry process as I encountered and contemplated various ways to research my coaching practice.

Too Complex for Numbers

My initial research consideration in the Spring of 2009 was to conduct an AB single-subject design (Richards, Taylor, Ramasamy, & Richards, 1998) in which I would select a handful of teachers and graph their growth over time based on my coaching in three areas: content knowledge, pedagogical content knowledge, and teacher efficacy. I envisioned using a sophisticated rubric (see Appendix D) to evaluate teachers' instruction in the three domains throughout the school year. Every two or three weeks I would evaluate a teacher's instruction using the rubric and plot rubric scores over time on a line graph. By tracking the teachers' progress (or lack of progress) in key instructional domains, while simultaneously coaching them in effective instructional practices, I was hoping to explain how teachers change their instruction. My underlying assumption was that coaching does change instruction, which was reflected in my research questions.

- How does instructional coaching promote teacher change?
- How does instructional coaching influence mathematics teaching?
- How do teachers' mathematical knowledge, pedagogical content knowledge, and beliefs about students change when they work with an instructional coach?

Since quantitative research is highly respected by government agencies and committees (Schneider et al., 2008), I wanted to incorporate more quantitative measures into my study. I thought that my study would be more widely accepted if I incorporated quantitative measures, such as a survey that used a Likert scale to assess teachers' efficacy and the Knowledge of Algebra for Teaching assessment to assess teachers' knowledge⁷. I also found it appealing that quantitative research would allow me to draw conclusions and predictions about the effects of a particular "treatment" (i.e., instructional coaching) on others (i.e., teachers) (Yin, 2009). This type of research design, being predetermined and structured, would remove extraneous influences and distractions to allow my research to focus solely on the data (Merriam, 2009).

Yet I quickly realized that to make statistical inferences, I would need a decent sample size (Urdan, 2005) of more than 30 to make claims of causation. As an instructional coach, I worked with far fewer than 30 teachers. This fact made sampling size a limitation of quantitative research for my problem of practice. Quantitative research methodologies would also limit my findings to numbers (Merriam, 2009), and would therefore lose some of the richness and description of what was taking place. Since my sample size was too small, and because I wanted to provide a richer description of what was happening in the study, I decided quantitative methodology was not a sufficient or appropriate way to investigate my problem of practice.

⁷ The Knowledge of Algebra Teaching (KAT) assessment was developed with support from a grant from the National Science Foundation (NSF REC No. 0337595). Co-PIs for the grant were R.E. Floden, J. Ferrini-Mundy, R. McCrory, M. Reckase, and S. Senk. Further information about the KAT assessment is available at www.msu.edu/~kat.

Another issue arose as I began to construct the rubric to evaluate teachers' instruction. I came to realize that instruction is influenced by a number of different factors and dependent upon numerous things. Thus, it would be very difficult to determine effect of any single factor (Bean & Isler, 2008). Although I already knew this as a practitioner, I expected the literature to provide me with some clear guidelines for effective instruction, which would in turn allow me to use a rubric to evaluate instruction. I never found a clear-cut definition of effective instruction. Since effective instruction is dependent upon a variety of factors (Loucks-Horsley, Stiles, Mundry, Love & Hewson, 2003), identifying the specific characteristics of effective instruction and creating a rubric to quantify instruction would be impossible. The richness and description of instruction would have been lost if the teachers' instruction had been evaluated and a number reported. If I had approached my research in this fashion, I would have been lacking data on an equally important question about what the process of "improving instruction" looked like?

My practice as a coach also informed my decision to abandon the rubric. As a new instructional coach, I was finding that the numerous roles I played occasionally caused confusion and anxiety among teachers. At times I resembled a teacher as I planned lessons with my colleagues and attended the department and staff meetings. In other instances, I was perceived as the expert Algebra teacher as I modeled lessons and facilitated professional development classes. And to complicate my role further, in the eyes of some teachers I became "one of them" as opposed to "one of us" when I worked with administrators and curriculum specialists or facilitated professional development.

The continuous role changes further complicated the coaching process. For example, one day I modeled a lesson for a teacher, which placed me in the expert category. Then when the teacher and I met to plan another lesson, the teacher still viewed me as the expert and was therefore apprehensive to plan a lesson in fear of being "wrong." Or as another example, I was in a teacher's classroom collecting data one period as part of the coaching process. I then met with that teacher's administrator about staffing ideas during the next period. Since collecting data on instruction is largely associated with the appraisal process done by an administrator, the teacher became worried that I was meeting with the administrator to share her classroom data and consequently began to distance herself from me. These examples of apparently clashing roles caused the coaching process to slow down and, in some cases, it was brought to a complete stand still (Knight, 2009).

By using a rubric to evaluate teachers' instruction in my research, I felt like I would be continuing to blur the role of coaching with evaluating. I did not want to risk having the teachers distance themselves from me or the coaching process. I remember being fearful as a teacher that my coach would go to an administrator if I did something "wrong." For these reasons, I became uneasy about using a rubric to score teachers' instruction. Since my role as an instructional coach was purely non-evaluative (Costa & Garmston, 1991; Knight, 2007), I would be complicating and confusing teachers if I began using a rubric to evaluate their instruction. I am the teachers' peer, their colleague, and using a rubric to evaluate teachers' instruction would infringe on the collegial coach-teacher relationship by putting me in an appraisal situation. Since using a rubric to evaluate instruction was not advantageous to my research or my role as a coach, I

abandoned the observation tool and sought a research method that might be beneficial to my practice.

Rich, Colorful Descriptions

I had a methodological break-through as I was searching for a way to use research to gain understanding of my practice while continuing to coach. During an ethnographic methods course I came to understand that research does not need to prove something. It does not need to make a point of how effective or ineffective this intervention or that method was. Some forms of research may simply help us better understand something. And that is exactly what qualitative research does. As Merriam (2009) writes, "Qualitative researchers are interested in understanding the meaning people have constructed, that is, how people make sense of their world and the experiences they have in the world" (p. 13). I honestly felt liberated as a researcher. I no longer felt like quantitative research was my only option.

At this point in my inquiry, I decided to approach my research as a case study of one Algebra teacher working with a coach. By choosing to focus on one teacher in a case study research design, I was better equipped to describe instruction and instructional coaching and answer the question of how coaches help teachers improve their instruction. Case studies provide information on complex situations consisting of numerous factors (Merriam 2009; Stake, 2005), which made it the research method most compatible to my problem of practice. Case study research would allow me to further investigate the many variables involved in instruction and coaching.

Since case study research allowed me to look deeper into the complex happenings of the coaching process by working with and researching the experience of one Algebra

teacher, my research questions naturally changed. My questions became focused on gaining a deeper understanding of the complexities of coaching that occurred between a teacher and coach rather than proving the effects of instructional coaching.

- What happens when an Algebra teacher works with an instructional coach?
- What is involved in the instructional coaching process?
- What are the roles of the teacher, the Algebra instructional coach, and others involved in coaching?
- How is the professional development strategy of instructional coaching and teacher change related?

Case study research would also allow me to provide colorful descriptions (Merriam, 2009) about the complexities of coaching and inform others of the numerous factors that I was encountering as a practitioner. My findings and conclusions could be presented through intricate descriptions of what happened when I coached an Algebra teacher. These descriptions put readers in the context being described so they may learn vicariously about the phenomenon of coaching through my detailed narrative (Stake, 2005). Most important of all, as a case study my research would "not attempt to simplify what cannot be simplified" or "eliminate what cannot be discounted" (Shields, 2007, p. 13), but instead provide a rich description of the complex process of coaching as a means of improving Algebra instruction.

Since qualitative case studies are focused on deep understanding of a complex social situation and incorporate rich descriptions to convey findings (Merriam, 2009), having a researcher who is active in the environment being studied is another strength of case study research (Stake, 1995). I would be active in the sense that I was the

instructional coach working directly with the teacher being studied. My position would be beneficial since case study research requires continuous attention and an insider's viewpoint to fully capture descriptive data. I was a “local” in the coaching context and in a position to tell the story of my daily life as a coach (Fetterman, 2010). Since I would be the primary data collection instrument, I would be able to offer specific details of the data and offer knowledge embedded in context (Merriam, 2009).

A shift in my methodology also allowed me to rethink the questions I wanted to research (see Figure 3.1). Listing my research questions for my initial single-subject research design and my case study research design shows how my questions changed. My original questions assumed that coaching promotes change and influences the teaching of mathematics, when in fact there is currently very little research on the effects of coaching. After changing to a case study framework, I reworded my questions to focus on the story of coaching rather than trying to prove the effectiveness of coaching. Since my research questions evolved to focus more on gaining greater understanding of the complex, chaotic coaching process, my methodology naturally transitioned to qualitative work (Merriam, 2009; Stake, 1995). The focus of my research was now directed towards the act of coaching and the relationship between a coach and teacher.

Telling the Story of Coaching

As a practitioner, I was beginning to realize the complexity of coaching. The numerous coaching roles and responsibilities I had to assume was just one example of how I was coming to understand the complicated coaching process. As I began to recognize more and more of these complexities as a practitioner, I was increasingly

Single-Subject Design Research Questions	Case Study Design Research Questions
<ul style="list-style-type: none"> • How does instructional coaching promote teacher change? • How does instructional coaching influence mathematics teaching? 	<ul style="list-style-type: none"> • What happens when an Algebra teacher works with an instructional coach? • What is involved in the instructional coaching process? • What are the roles of the teacher, the Algebra instructional coach, and others involved in coaching? • How is the professional development strategy of instructional coaching and teacher change related?

Figure 3.1: My research questions changed when I went from a single-subject design to case study methodology.

compelled as a researcher to gather and analyze data, and then tell the story of coaching with description and detail.

As if the numerous roles of a coach did not complicate the coaching process enough, there were several other issues that added to the complexity. One of those issues was the process of building relationships with teachers, department chairs, administrators, and curriculum specialists. Forming relationships within coaching seemed to continuously complicate my coaching experiences. I was spending a significant amount of coaching time trying to navigate individual teachers' and administrators' personalities, while simultaneously trying to engage them in the coaching process. Creating and maintaining coaching relationships was an aspect of the coaching story.

Learning, understanding, and working within the culture of various schools and math departments were another complicated aspect of instructional coaching. Every school and department has a set of unwritten (and written) rules that must be learned and

followed (Stoll, 1998), causing the coaching process to be even messier. Interestingly, the way teachers perceived coaching and interacted with the coaching process seemed to be influenced by the school in which the teachers taught. For example, I engaged in in-depth conversations about mathematics and teaching with the math teachers in the school where collaboration was the norm. When I worked in a different building, where teachers largely isolated themselves to their individual classrooms, I found it much more difficult to even engage teachers in the coaching process. The influence of a school culture intrigued me as a coach.

Finally, the personal feelings I was dealing with as I learned to be an effective coach surprised me. These feelings seemed to depend on the people I was interacting with and the aspects of coaching we were involved in. On numerous occasions I questioned my decision to leave my own classroom and move into the coaching role where I possessed lesser confidence in my professional abilities. Feelings of inadequacy and frustration, stemming from the lack of immediate progress or the absence of measureable teacher learning, at times made me question my coaching role. For example, a few teachers commented that I was not old enough and did not have adequate experience teaching all high school courses to be an instructional coach. Although my curriculum specialist replied by saying that an effective coach is determined by her ability to teach and communicate effective instruction as opposed to her age or course experience, this incident only added to my growing mix of emotions towards being a coach. In another instance I remember feeling like an outsider when the lunchroom conversation was about a person who left the district curriculum department to “be a real

teacher again.” The eyes that darted towards me and the sympathetic apology that followed only magnified how I was seen as different from the others at the table.

The complexities within my coaching practice seemed to become more and more apparent the more coaching experience I obtained. All of these complexities within my practice intrigued me. Since I had decided to conduct a case study for my research, I began to think I could use my research to further investigate these coaching complexities. I began to ask questions that pointed towards my role in coaching. How does a coach build a relationship with a teacher? What does a coach encounter as she transitions into the coaching role? How does the coaching process interact with the cultural norms of teaching?

Finding a Focus: Who is the Learner?

Conducting an in-depth study of one Algebra teacher while coaching him brought focus to my research, yet at the same time added ambiguity. When I first transitioned to case study methodology I was focused on using my research to tell the complex story of coaching as it pertained to this one Algebra teacher. I thought a case study directed towards the coaching relationship between a teacher and a coach would be enlightening, especially since the research field lacked literature specific to coaching research. I thought my research could offer an inside perspective of coaching in practice. I was motivated to learn more about this new professional development strategy and the complexities involved in coaching.

Yet researching the process of coaching, or even the relationship between a coach and a teacher, would not help me gain understanding about how coaching helps teachers improve instruction. I needed a different focus if I wanted to learn how coaching

ultimately improves students' understanding of Algebra. At this point in my inquiry the professors on my doctoral committee were noticing the similar conflicts in my research plans. They encouraged me to change my view of the teacher participant and to think of him as a learner. Initially, I became defensive. Of course I thought of the teacher as a learner. Or at least I thought I did.

When I went back and reexamined my problem of practice, research plan, and research questions, reality hit me. I was being self-centered, similar to how my career started as a teacher. My focus was on what I would learn and how my learning would help me as an instructional coach. I was not concerned about what the teacher learned. This was just like my first two years of teaching when I was so wrapped up in my own teaching that I focused all of my attention on what I was teaching rather than what my students were learning. The same thing happened to me again. This time it took my doctoral committee to help me realize that as a coach I needed to stop concentrating on what I was doing as a coach and begin focus on the teacher's learning.

As a way to help me think differently about my teacher participant, I stepped back from my current coaching routine and observed the teacher as a learner. What were the patterns in his learning and teaching? What does he learn? How does he learn? What kind of learner was he? I hoped that observing the teacher as a learner would help me think differently about the teacher and about my role as an instructional coach.

How Learning Pertains to Teachers as Learners

With this new perspective on teacher learning, I prepared myself to revisit my inquiry. I refocused my research (and practice) on the teacher's learning as a way of improving instruction and students' achievement in Algebra. I began thinking and

reading about what a teacher needs to learn and how he learns. I quickly began to realize that the learning process of a teacher (or adult) parallels that of children. Learning, whether it is an adult or child, includes elements such as conceptual understanding, connections, active involvement, and repetition. Research in adult learning states that, “The art and science of teaching adults – andragogy – may not be significantly different and/or exclusive to adult learning” (Cyr, 1999, p. 8). Yet these learning components do not always look the same for a child and an adult. And even though the learning process is fundamentally constant, I came to realize that teachers should be approached in a different manner from students.

Conceptual understanding. Meaningful work focused on conceptual knowledge benefits the learner both cognitively and motivationally (Shepard, Hammerness, Darling-Hammond, & Rust et al., 2005). Learners are able to understand the content more thoroughly and retain knowledge when they learn concepts at a deeper level. For teachers to commence their learning, an understanding of the underlying reasons *why* they need to learn the skill or concept is important (McKenna & Walpole, 2008). By learning the theory and research (the *why*) behind a new skill or instructional strategy, teachers are more likely to transfer the learning into their classrooms (Shepard et al., 2005).

Connections to prior knowledge and experience. Effective teaching identifies a learner’s current level of prior knowledge and uses that knowledge to build new learning (Shell et al., 2009). Therefore the new material being taught is no longer seen as an isolated, original topic, but rather a continuation of previous material. Teachers are especially interested in making connections between the new knowledge and their prior

knowledge, since adult learners come with a greater amount of experience (Lawler, 2003).

Engagement. Teaching and instruction is largely centered on learners attending to ideas (Shell et al., 2009). Knowledge is created when learners are engaged in a concept and are allowed to be actively involved in the learning process. As opposed to passive learning techniques, the implementation of activity in learning promotes understanding and retention (Zemke, 1995). Since the teaching profession specifically requires teachers to be active, professional development experiences that invite teachers to participate and collaborate encourage learning (Lawler, 2003).

Practice and repetition. Learning takes place when practice and repetition are used to construct new knowledge (Shell et al., 2009). As a learner, simply being exposed to an idea is not enough to constitute learning. The opportunity to practice new skills in a safe environment is a critical component of the learning process (Zemke, 1995). Learners need to be given the opportunity to practice what has been learned if the new knowledge is to remain. This element of the learning process will almost certainly look different depending on the age of the learner. Adults should be given the opportunity to exercise self-direction and choose how they practice new skills and the frequency of the repetition (Ross-Gordon, 2003; Knowles, 1990). For example, teachers may choose to practice a new instructional strategy on their own, with a trusted group of colleagues, or in their classroom with students. The more a learner chooses to practice, the deeper his learning and mastery of the skill (Shell et al., 2009).

Purpose of the Study

Through a deeper look at teacher learning in the Fall of 2010, I came to understand the focus of my case study. If teachers are expected to increase student achievement in Algebra by continually improving instruction, then I needed to take a deeper look at how a teacher learns to be a better teacher and how a coach supports this learning. I would need to study both the product and the process (Alexander, Schallert & Reynolds, 2009) of a teacher's learning (as well as my own learning) and be concerned with both *what* the teacher was learning and *how* he was learning. The process of coaching would serve as the context in which the teacher's learning would be studied.

Since my goal was to deeply understand an Algebra teacher's learning through instructional coaching, a case study approach was still the best methodological choice since the research approach allowed me to use a single case in order to more deeply understand teacher learning (Stake, 1995). As both the instructional coach and the researcher, I was in a beneficial position to closely and intimately study the case of one teacher's learning. I was not an outsider looking in on the teacher's learning, but instead a partner directly involved with the learning process. By writing about the teacher's learning process I hope to provide readers with a deeper understanding of teacher learning as it happens through the process of coaching.

From quantitative methodology to case study, evaluative rubrics to rich descriptions, and understanding coaching relationships to a teacher's learning to teach specific mathematical content, my inquiry finally brought together a research plan to fit my problem of practice. The purpose of this study was to gain understanding of an Algebra teacher's learning, specifically his learning of how to teach Algebra to students

who struggle in mathematics. The context of instructional coaching was used in this study as both a resource for the teacher's learning and as a lens through which to examine teacher learning. As a resource or support, I worked with a teacher as he planned, executed, and reflected on instruction, providing the teacher with opportunities to learn and grow as an educator (West, 2009). As a context, instructional coaching provided me with a close, intimate view of a teacher's learning. Since instructional coaching is a highly individualized professional development opportunity for teachers, I created and modified the coaching to fit the learning needs of the individual teacher (Bean & Eisenberg, 2009) and therefore had a close, strong understanding of the teacher's knowledge and learning. This study will serve the purpose of examining what and how an Algebra teacher learns while receiving my support as an instructional coaching.

CHAPTER 4: METHODOLOGY

Since my research and practice occurred simultaneously, my inquiry can be categorized as teacher research or practitioner inquiry. Teacher research is largely defined by the insider perspective and is based upon a teacher's praxis. It is a systematic study focused on problems from the teacher's classroom (Baumann & Duffy, 2001). My inquiry, or search for how to coach teachers to improve instruction and ultimately increase Algebra students' success, evolved. The various experiences I encountered as a coach changed the way I thought about my inquiry and resulted in research modifications. And as I learned about various research methodologies, I became more knowledgeable about the research method that would be most appropriate for my practice and inquiry.

Since the research I would be conducting was specific to my own practice, I began making research decisions based on my practitioner role. For example, I avoided data collection techniques that placed me in an evaluative role because, as a coach, I am not an administrator and should avoid being portrayed as such (Shanklin, 2006). My practice and my research would be conducted simultaneously. I would be the "insider" collecting data from a coach's perspective and a researcher stepping back to make sense of all the data (Fetterman, 2010).

Practitioner research grows from the desire to close the gap between research and practice in the field of education (Mills, 2007). Kennedy (1997) found that the apparent failure of research to affect teaching practice is due to teachers not finding the research persuasive or relevant to their practice. Since teacher research is situated in the teachers' experiences, the findings are a knowledge that is useful in education (Lytle & Cochran-

Smith, 1992). The teacher researcher brings rich data to the research that outside researchers do not have available to them (Hubbard & Power, 2003). A teacher has knowledge about her environment and practice, which is important elements of the educational process outsiders may not fully understand.

Teacher research, which draws on traditional qualitative research, is gaining respect as a powerful form of inquiry among teachers, researchers and education policy makers (Rust, 2009) and more teachers and practitioners are being encouraged to become teacher researchers (e.g. Cochran-Smith & Lytle, 2009). Several teacher researchers in mathematics education have simultaneously assumed the role of both practitioner and researcher in their pursuit of greater knowledge and understanding of the practice (Ball, 1993; Chazan, 2000; Heaton, 2000; Lampert, 1990, 2001). I am trying to continue in the tradition of educational teacher research by studying my own practice of coaching.

Conducting teacher research does come with limitations. One critique of teacher research is that practitioner inquiry is too specific to one context and therefore does not allow larger generalizations (Wilson, Floden & Ferrini-Mundy, 2001). My intent has never been to generalize my findings. Rather, I am conducting this research in order to gain personal knowledge about instructional coaching and teacher learning.

Also, some scholars believe the dual roles of a teacher researcher cause the research to be egocentric or self-absorbed (Bullough & Pinnegar, 2001) in which the teacher researcher is preoccupied with her own practice. A similar argument is that practitioners will inevitably face conflicts of interest while conducting research and will be compelled to do what is best for the students, thus sacrifice the quest for valid knowledge (Cochran-Smith & Lytle, 2009). The benefits of being an insider outweigh

these concerns. I was able to see what others on the outside cannot see. The data gathered while in the teacher researcher position offered a close, intimate look at the practices being studied.

Details of the Study

The details of my research interest and design initially stem from my practice as a coach. I worked to construct possible solutions to my question about how coaching could be done to improve teachers' Algebra instruction, with the ultimate goal of increasing student achievement in Algebra. I decided to make some changes to my coaching.

Why Change Coaching Approach?

In January 2011, my coaching assignment changed and I began serving the math teachers at Washington High School. This was my chance to experiment with other coaching techniques and approaches in order to address what I found unsatisfactory about my current coaching. This change just happened to coincide with my research timeframe and I decided this would be the best time to try a different coaching approach.

My previous approach to coaching and the coaching practices I had established with teachers was not being effective. Our isolated, once-a-week coaching cycles dictated the focus of the teacher and my conversations and hindered our planning. Since there were several days between when I met with a teacher, I would not know what the teacher had taught the previous five or six days or how the mathematical concepts had been introduced to the student. This made it very difficult for the teacher and me to discuss the actual mathematics and effective ways to teach concepts. Our conversations prior to class were once again stifled since the teacher had already determined what would be taught and how it would be taught. The teacher and I were not able to

collaborate from scratch and I felt restricted in what was appropriate for me to suggest. I was putting myself in a situation where I was unintentionally critiquing the teacher's lesson plan.

The lack of efficient planning time created by this coaching structure was frustrating and I was beginning to feel like I was spinning my wheels. I found myself thinking, what is the point? My personal journal is full of comments like, "I feel like we are taking one step forward and two steps back" (Personal Journal, March 12, 2010) and "Is she changing anything or am I just another person in the room helping?" (Personal Journal, September 5, 2010). I was becoming consumed with what the teacher did or did not do in terms of her instruction, as that was all I could really do in the current coaching structure. As a coach, I was not satisfied with the fact that our conversations largely focused on formative assessments and cooperative learning strategies – the more procedural aspects of teaching. I was putting all of my energy into helping teachers tweak single, isolated lessons.

The major structural change in my schedule now allowed meeting with teachers on consecutive days rather than isolated once-a-week intervals. Since I was assigned to one high school, as opposed to three high schools, I reported to Washington High School daily, giving me the opportunity to interact with teachers on a regular basis. (I was occasionally out of the building fulfilling other coaching duties such as assessment writing, curriculum organization, or planning of professional development classes.) Coaching teachers consecutive days could help me address many of the frustrations I was experiencing with the once-a-week coaching. For instance, meeting with a teacher on a regular basis would allow us to be more focused on truly building students'

understanding of the mathematics since both of us would be experiencing students' learning together. We could build each new lesson upon what occurred the previous day. Our debriefing conversations after a lesson would not only allow us to reflect, but will also lead us into planning for the next lesson. We could build each new lesson upon what occurred the previous day, rather than spend time reviewing and modifying a lesson the teacher already prepared. I also believed that coaching consecutive days would more closely parallel the actual planning, teaching, reflecting cycle that teachers work through on a daily basis, giving the teacher and me room to collaborate on these many aspects of teaching.

Yet the switch came with risks. This coaching approach would be intense and focused, requiring significant time and effort from both the teacher and me. Would a teacher want to put forth that much energy? Would I be able to keep coaching others? I did not know if the consecutive coaching structure would improve instruction, but I felt like modifying my coaching technique was a natural risk to take.

Why Coach Reasoning and Sense Making?

The modification in the coaching structure would allow the teacher and me to be more purposeful in our discussions of teaching and learning mathematics. From my own experiences as a math teacher and research on high-quality math instruction, I knew that specifically focusing my coaching on understanding math at a deeper level (Papick, 2011), teaching topics conceptually (National Mathematics Advisory Panel, 2008), and building upon students' knowledge (National Council of Teachers of Mathematics (NCTM), 2009) were essential aspects of effective mathematics instruction. I thought

one way to address all of these components of instruction was by making reasoning and sense making an important part of the mathematics.

The “cornerstones of mathematic(s)” are reasoning and sense making and should occur in every mathematics classroom every day (NCTM, 2009, p. 7). Reasoning is the process of using evidence to determine conclusions, while sense making uses prior knowledge to develop understanding of a new mathematical concept (NCTM, 2009). Regularly engaging in reasoning helps students build a productive disposition that helps math make more sense to them (Kilpatrick et al., 2001). Although students often understand mathematics before they are able to verbally articulate their understanding (Kilpatrick, et al.), asking students questions such as “What is going on here?” and “Why do you think that?” (NCTM, 2000; NCTM, 2009) encourages deeper thinking about the concept. Incorporating these higher-level thinking questions into the daily mathematical instruction elevates the learning from procedural knowledge to conceptual understanding.

By emphasizing reasoning and sense making in the high school classroom, teachers can “help students organize their knowledge in ways that enhance the development of number sense, algebraic fluency, functional relationships” (NCTM, 2009, p. 5), as well as other mathematical areas. When connecting knowledge, a student is more likely to be able to retrieve the new learning (Bruning, Schraw & Norby, 2011; Shell et al., 2010). If one does not create these connections through logical thinking, the isolated procedures or topics being taught are more difficult to learn (Kilpatrick et al., 2001).

Why Focus Coaching on Quadratics?

The mathematical concept of quadratics presented itself as a mathematical concept on which I could concentrate this research. The new coaching structure (consecutive coaching cycles with a focus on teaching mathematics for reasoning and sense making) had the potential to be cumbersome in terms of the teacher's and my time and energy. I chose to initially experiment with a limited timeframe centered on one math concept or chapter. I wanted the participating teacher to determine the specific algebraic concept to focus our consecutive coaching upon, since it needed to be a topic where he was interested in improving his instruction. And although I would ultimately have the teacher determine the mathematical concept we would be focusing on, I wanted to be sure a concept was chosen with reasoning and sense making in mind.

During the research timeframe, there was one concept being taught in Algebra that included both parts of algebraic reasoning and sense making. Quadratic equations and functions incorporated symbolic manipulation (solving quadratic equations) and functions (graphing quadratic functions). My first thought was to use the chapter on quadratics to coach a teacher for consecutive days, with a focus on teaching with reasoning and sense making. Algebra has two major conceptual goals: generalized arithmetic and functional thinking (Kilpatrick & Izsak, 2008). Students who achieve those two goals in Algebra are better prepared for future mathematics courses (NCTM, 2000). Being able to recognize and manipulate symbolic equations (including quadratic equations), as well as developing a strong understanding of all functions (including quadratic functions) are central aspects of algebra instruction (NCTM, 2000). Therefore

focusing this research on reasoning and sense making in quadratics addressed the goal within my practice and larger, mathematical goals.

Research Questions

The remainder of this chapter is a way to pause my inquiry and focus on my research from a researcher perspective. I explain the data collection instruments I chose to employ, how I used the instruments, and other decisions made during the research timeframe. The reasons previously described, about why specific research decisions were made based on my practice, can be seen interwoven in my discussion of research methodology.

The data collection techniques in my study were chosen based on the data I was interested in collecting. My primary goal was to obtain data on what and how an Algebra teacher learns to teach quadratics with reasoning and sense making while working with an instructional coach. I was also interested in gathering data on how an intense focus on a single algebraic concept (quadratic equations and functions) influences a teacher's and a coach's learning. The following main research questions guided my study.

- What does an Algebra teacher learn about teaching reasoning and sense making of quadratics while working with an Algebra coach?
- How does an Algebra teacher learn about teaching reasoning and sense making of quadratics while working with an Algebra coach?

I used sub questions to clarify and further focus my two main questions.

- What does an Algebra teacher need to learn about reasoning and sense making in quadratics?

- How does an Algebra teacher learn to teach reasoning and sense making to struggling students?
- What is involved in the learning process for an Algebra teacher working with an instructional coach?
- How does an intense focus on a single algebraic concept influence a teacher's learning? A coach's learning?
- What does a teacher find problematic in teaching reasoning and sense making in Algebra?
- How does the context of instructional coaching support an Algebra teacher's learning of reasoning and sense making?
- How does the relationship between a teacher and instructional coach influence a teacher's learning?

My main and sub research questions not only focused my research, but helped me construct data collection instruments.

Data Collection Instruments

As a teacher researcher, the data collection techniques I used needed to fit into my practice since I would be collecting data while simultaneously coaching. With regards to the data collection tools used in teacher research, Rust (2009) writes:

Because it is intimately embedded in practice and in the time frames of teachers' lives in classrooms, teacher research describes a form of qualitative inquiry that draws on techniques that are generally already part of the instructional tool kit of most practitioners. These include classroom maps, anecdotal records, time-sampled observations, samples of student work, drawings and photographs, audio

and video recordings, interviews, conversations, surveys, and teachers' journals. Generally, these are used by teachers over time to answer questions about practice. (p. 1883)

As Rust notes, the data collection techniques I chose to incorporate into my research were largely tools that I used informally in my day-to-day coaching. The data I collected was initially gathered from my coaching perspective and then I tried to make sense of the same data using an outsider or researcher perspective (Fetterman, 2010). The tools I used to gather data and how they pertained to my research specifically are described below.

Field notes. During my work with one Algebra teacher, I took extensive field notes in order to capture the teacher's learning in a variety of situations. Since my field notes were long-term, embedded in the everyday happenings of the context, and based upon the relationship between the teacher and me, I was specifically engaged in participant observation (Angrosino, 2005). These field notes served as my primary technique for data collection.

I kept extensive field notes for each of the three components of the coaching cycle: preconference planning, classroom lesson, and post conference debrief. Following each section of the coaching cycle, I immediately sat down and wrote about what I observed and experienced. I made it a point to look for evidence of learning, although it did not always appear on the surface. For example, I paid close attention to the teacher's expression of mathematical knowledge necessary to teach Algebra with reasoning and sense making, such as presenting an algebraic idea, encouraging and responding to students' "why" questions, connecting algebraic concepts to previous (middle school) or future (Geometry, Advanced Algebra, etc.) topics, and choosing

various representations (algebraic, graphic, etc.) for a specific purpose (Ball, Thames & Phelps, 2008). Field notes were recorded for each coaching cycle the teacher and I experienced during Spring 2011.

In addition to taking field notes on the coaching cycle, I also took field notes on other teacher or school interactions that provide information about the participating teacher's learning. For example, each week the teacher and I participated in a Professional Learning Community (PLC) meeting (Eaker, DuFour, & DuFour, 2002) with other Algebra teachers. During these meetings I closely observed the teacher, his actions, and what he chose to contribute to the conversations. Immediately following the PLC meeting I wrote notes.

Videotaping lessons. Video served as a record of what took place instructionally. During the actual class I was engaged as a practitioner, modeling instruction or co-teaching as the coach. Since I did not observe or take field notes during class, the videotaped lessons allowed me to observe the lesson at a later time. The videos of each lesson (16 lessons specific to quadratics and five lessons scattered throughout the rest of the semester) offered a record of the class. While watching the videotapes, I completed notes based on my observations to supplement the field notes I had written the day of the individual lessons. These notes detailed what happened during class and specifically explained what and how the students and teacher were learning.

I arranged to have every lesson taught during the intense data collection focused on quadratics videotaped. The videos largely focused on the teacher and followed him around the classroom to capture what he did and said. At times the video did focus on student work or a group of students problem solving. I also videotaped two lessons prior

to quadratics and three lessons after. The lessons prior to quadratics were part of our weekly coaching cycle and were used to help me better understand the participant as a teacher prior to the intense coaching. The videotaped lessons after quadratics served as a way to understand his teaching after the intense coaching since I was no longer coaching the teacher.

Personal journal. Personal journal entries specifically focused on the participating teacher and school, as well as my own learning as a coach. An entry was included at the conclusion of a coaching cycle, and also followed any field notes written about an interaction with the participating teacher or school. My written reflections continued to focus largely on the teacher's learning. For example, after taking extensive field notes on a weekly coaching cycle, I used three prompts for my personal reflections. I did not limit my thoughts to these questions if I felt more needed to be written regarding the teacher's learning. The writing prompts were created based on my research questions.

What do I think the teacher learned this week?

How do I know that is what he learned?

How did he learn it?

The personal journal provided me the space to reflect on what I thought the teacher was learning and how I thought he was learning.

Artifacts. Artifacts, or other written documentation, are sources of data that may or may not be influenced by the researcher (Merriam, 1998). In my study, artifacts included but were not limited to classroom materials (worksheets and notes), student work, lesson plans, written communication with the teacher or others in association with

the teacher, school or department wide documents, or any other documents that I determined related to the participating teacher's learning.

Post lesson debrief interviews. A post lesson debriefing session was conducted during the post-observation component of the coaching cycle. As part of my coaching practice, I used the debrief to encourage the teacher to reflect on the lesson taught. The post lesson debrief could be considered an unstructured interview, a process for understanding complex behavior without a prearranged list of questions (Fontana & Frey, 2005). Each of these reflective conversations was audio taped and transcribed.

I used more open-ended questions to begin our discussion, with the remainder of the debriefing sessions being mostly conversational or natural dialogue (Fetterman, 2010). I asked open-ended questions that focused on student work or understanding, using specific examples of what was observed in the lesson. These types of questions are suggested in the coaching literature as ways to increase teacher reflectivity (Peterson, Taylor, Burnham, & Schock, 2009). I also gave the teacher generic prompts such as, "What did you feel about the lesson?" and "What are you thinking?" At times I did ask questions specific to the research, such as, "How did today's lesson compare to how you taught graphing quadratics last year?" and "What have you taken from our work together over the last three weeks?" I allowed the discussion to develop and flow according to the direction the teacher took us. Since the debrief was an important component of the coaching process, using the debriefing discussion for my practice was a priority.

Formal interviews. Formal, semi-structured interviews took place at the beginning and end of the research timeframe. I entered into these interviews with a set of questions I wanted to be sure to ask (see Appendix E), but was open to our discussion

moving in whichever direction the teacher took the conversation. I prepared interview questions such as:

- In one or two sentences, how would you summarize our work together the last 1.5 years?
- What do you feel are your strengths as a teacher?
- In one or two sentences, how would you summarize our work together this semester?
- Can you think of any moments throughout the semester that relate to what you highlighted as things you want to work on (in your initial interview)?

These interviews were used as a way to gain a greater understanding of the teacher's perspective on teaching, learning, and the coaching process (Fontana & Frey, 2005). The formal interview in January, 2011 was used to gain greater insight into the teacher's history with Algebra and teaching, previous learning experiences in terms of teaching Algebra, and his view on our previous coaching interactions. At the end of May, 2011, I conducted another formal interview to gain a greater understanding of his overall perception of our work over the semester, beliefs about his learning, his perspective on what he learned, and what specifically he thought helped him learn.

Data Collection Throughout Spring 2011

Using the previously described data collection techniques, I conducted research from January through May, 2011 (see Figure 4.1). The activities that occurred, the frequency of the various data collected, and my role as a teacher and researcher varied throughout the five months. These variations can generally be separated into three

categories based on their relation to the intense, consecutive coaching I did focused on the concept of quadratics: Prior to Quadratics, Quadratics, and Post Quadratics.

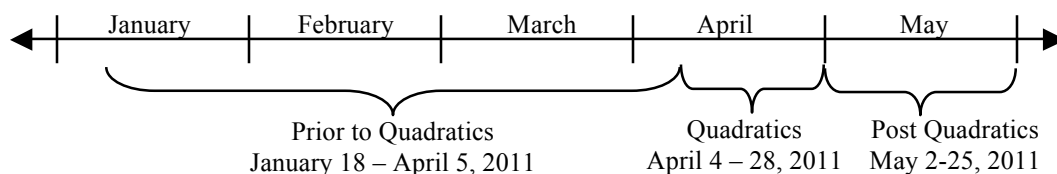


Figure 4.1: This Spring 2011 timeline displays the three stages of data collection.

One event that remained consistent throughout the research timeframe was my involvement in the weekly professional learning community (PLC) meeting that took place on Tuesday afternoons from 2:15-3:15 pm. I joined the participant teacher, along with five other Algebra teachers, as a collaborative participant during PLC time as we discussed issues related to teaching Algebra. Some of these issues included pacing, summative assessment rubrics, formative assessment activities, and instructional techniques. (Since teachers met in PLC, students were released from school one hour early every Tuesday. Consequently, classes on Tuesdays were each approximately 10 minutes shorter.) The PLC meetings are included in the tables below when the meeting provided data on the teacher's learning and the field notes and personal journal for each weekly coaching cycle included data from the PLC.

Prior to quadratics. From January through the first week of April, I engaged in a weekly coaching cycle with the teacher participant. Since I was now in the same building for consecutive days, I was able to meet with the teacher a day or two before the day I was scheduled to be in his classroom to discuss the mathematics being taught and

plan the instruction that would take place. As I began my research, I started with a get-acquainted period in which I stepped back and looked at my coaching and my participating teacher to better understand who and what I was studying (Fetterman, 2010). I needed to make the familiar strange in order to move forward with a clear view of teacher learning in the context of instructional coaching (Luker, 2008). My goal during these first four weekly coaching cycles was to better understand who the teacher was as a teacher and as a learner by not influencing the participant or activity (Angrosino, 2005). My goal was also to gain insight into the participant's teaching, including his current teaching routines and his learning. The teacher and I did very little planning together since I wanted to gather information about him without influencing the data. For the first four weeks of my research I gathered field notes, a personal journal, audiotaped interviews of the post-conference debriefing session, and artifacts to better understand the teacher's teaching and learning.

The teacher and I participated in the coaching cycle nine times during this phase (see Figure 4.2). Each week I took field notes, wrote personal journal entries, collected artifacts, and conducted post lesson debriefing sessions. The grayed dates in the table signal the get-acquainted period, where I attempted to act solely as a research observing the teacher. The initial, formal interview was conducted on January 28, 2011.

Quadratics. The consecutive days of coaching, which focused on reasoning and sense making in quadratics, took place from April 4 through April 28, 2011. During this time, the teacher and I engaged in the coaching cycle on a daily basis, resulting in 14 planning-lesson-debrief cycles (see Figure 4.3). Each lesson included field notes, personal journal entries, artifacts, and videotaped lessons. Eleven of the 14 coaching

Prior to Quadratics	Date	Event	Data Collection
January	21	Coaching Cycle	Field Notes, Personal Journal, Artifacts, Post Lesson Debrief
	25	Coaching Cycle	Field Notes, Personal Journal, Artifacts, Post Lesson Debrief
	28	Initial Interview	Formal Interview
	31	Coaching Cycle	Field Notes, Personal Journal, Artifacts, Post Lesson Debrief
February	10	Coaching Cycle	Field Notes, Personal Journal, Artifacts, Post Lesson Debrief
	16	Coaching Cycle	Field Notes, Personal Journal, Artifacts, Post Lesson Debrief
	22	Coaching Cycle	Field Notes, Personal Journal, Artifacts, Post Lesson Debrief
March	3	Coaching Cycle	Field Notes, Personal Journal, Artifacts, Post Lesson Debrief
	7	Coaching Cycle	Field Notes, Personal Journal, Artifacts, Post Lesson Debrief
April	5	Coaching Cycle	Field Notes, Personal Journal, Artifacts, Post Lesson Debrief

Figure 4.2: Prior to quadratics (January 21 through April 5), I gathered data during our weekly coaching cycles.

cycles also included post lesson debriefing sessions, which were audio taped and transcribed. Also included in the quadratics phase were four days devoted to planning and preparing for the quadratics lessons. The field notes for these four days include descriptions of the discussions the teacher and I had about teaching quadratics with reasoning and sense making. I also included notes on the discussions I had with the Algebra PLC, other math colleagues, and a mathematics professor as we worked together to better understand quadratics. The data collected during the quadratics phase is examined and analyzed in detail in chapter six.

Quadratics	Date(s)	Event	Data Collection
April	4	Quadratics Planning	Field Notes, Personal Journal, Artifacts
	5	Quadratics Planning (PLC)	Field Notes, Personal Journal
	6	Quadratics Planning	Field Notes, Personal Journal, Artifacts
	7	Quadratics Planning	Field Notes, Personal Journal, Artifacts
	8	Quadratics Coaching Cycle	Field Notes, Personal Journal, Artifacts, Videotaped Lesson, Post Lesson Debrief
	11-13	Quadratics Coaching Cycle	Daily Field Notes, Personal Journal, Artifacts, Videotaped Lesson
	14-15	Quadratics Coaching Cycle	Daily Field Notes, Personal Journal, Artifacts, Videotaped Lesson, Post Lesson Debrief
	18-21	Quadratics Coaching Cycle	Daily Field Notes, Personal Journal, Artifacts, Videotaped Lesson, Post Lesson Debrief
	25-28	Quadratics Coaching Cycle	Daily Field Notes, Personal Journal, Artifacts, Videotaped Lesson, Post Lesson Debrief

Figure 4.3: During quadratics (April 4 through April 28), I gathered data during our daily coaching cycles.

Post quadratics. In the weeks after the intense, consecutive coaching cycles experienced during the quadratics phase, I interacted very little with the teacher participant. I did not meet with the teacher prior to the videotaped lessons and was not present in the classroom, but instead watched these videos later and took field notes on what took place in these lessons (see Figure 4.4). The participating teacher gathered artifacts, specifically student work, from his classroom for approximately two weeks in May. The final structured interview was conducted on May 25 to conclude the data collection.

Post Quadratics	Date	Event	Data Collection
May	10	Lesson	Videotaped Lesson, Field Notes
	13	Lesson	Videotaped Lesson, Field Notes
	17	Lesson	Videotaped Lesson, Field Notes
	2-17	Lessons	Artifacts
	19	Conversation with Nathaniel	Field Notes, Personal Journal
	25	Final Interview	Formal Interview

Figure 4.4: After quadratics (May 10 through May 25), I gathered data through videotaped lessons and a formal interview.

Use and organization of data collection tools. Each data collection tool (field notes, personal journal, videotaped lessons, artifacts, post lesson debriefing session, and formal interviews) was employed during each of the three phases of my research time frame, with the exception of the formal interview, which was not used during the quadratics period. The specific use and frequency of each data collection tool was dependent upon the phase in relation to the teaching of quadratics (see Figure 4.5).

As a researcher, I used a data collection template as an organized way to bring several of these data collection instruments together (see Appendix F). Prior to the intense data collection during quadratics, the data collection template was used on a weekly basis. During the teaching of quadratics, I used the template daily to record field notes, personal journal entries, post lesson debrief transcriptions, and artifacts. Post quadratics I rarely saw a need for the template since I did not interact with the teacher except for watching his videotaped lessons and the final formal interview.

Data Collection Tool	Use of Each Data Collection Tool		
	Prior to Quadratics January 18, 2011 – April 5, 2011	Quadratics April 4, 2011 – April 28, 2011	Post-Quadratics May 2, 2011 – May 27, 2011
Field Notes	-Weekly coaching cycle. -Weekly PLC meetings. -Periodically for interaction with Nathaniel or colleagues related to his learning.	-Daily coaching cycle. -Weekly PLC meetings. -Periodically for interaction with Nathaniel or colleagues related to his learning.	-Periodically for interaction with - Nathaniel or colleagues related to his learning. -Three times based on videotaped lessons.
Personal Journal	-Weekly coaching cycle. -Weekly PLC meetings. -Periodically for interaction with Nathaniel or colleagues related to his learning.	-Daily coaching cycle. -Weekly PLC meetings. -Periodically for interaction with Nathaniel or colleagues related to his learning.	-Periodically for interaction with Nathaniel or colleagues related to his learning.
Videotaped Lessons	-Twice	-Daily	-Three Times
Artifacts	-Weekly coaching cycle. -Periodically for interaction with Nathaniel or colleagues related to his learning.	-Daily coaching cycle. -Periodically for interaction with Nathaniel or colleagues related to his learning.	-Periodically for interaction with Nathaniel or colleagues related to his learning.
Post Lesson Debrief	-Weekly	-Daily	N/A
Formal Interviews	-Once	N/A	-Once

Figure 4.5: The table describes the use of each data collection tool during the three phases of data collection.

Data Analysis

During the weeks prior to quadratics, I simultaneously collected and analyzed the data. I was able to refine and focus what data was collected while in the field by incorporating an ongoing analysis process (Merriam, 2009). The use of periodic analytic

memos during this time also gave me the space to reflect on issues within the data and help me to think about larger themes or patterns. Due to the intensity of data collection during the teaching of quadratics, simultaneous analysis and analytic memos were not feasible. Therefore analysis of the quadratics data, and any data collected post-quadratics, was done at the conclusion of data collection.

Coding was used to further analyze my field notes, interview transcripts, and post lesson debriefs. I used verbatim words in the raw data to eventually identify emerging categories or themes. After reading through the raw data several times, I began to chunk sentences or paragraphs together in order to identify patterns in the data. These chunks were then assigned a two to three word phrase taken directly from the data. Some of the common phrases taken from the data were: connection between concepts, reasoning and sense making, through collaborative discussions, and pushing instruction. Once all of the information was initially coded by verbatim phrasing, I went back through and compiled the codes into categories based on themes that emerged from the data (Merriam, 2009). These categories, along with categories from coding on other pieces of data, were then grouped together to create themes or sub-themes. These themes led to key findings as I worked to answer my research questions. The major themes related to teacher learning that emerged in the data were curriculum, mathematics, and teaching.

The personal journal and artifacts were analyzed through less structured inductive methods. I started by looking for trends or repeated words in these data collections related to teacher learning. The emerging categories and themes were then merged with the themes from the coding process previously described.

Since evidence of teacher learning is inferred, I was sure to notice in the data when the teacher did something instructionally different from previous teaching or when the teacher openly displayed an “ah-ha” moment. The categories and themes I looked for as I coded the data were centered on my two main research questions. First, I looked for *what* the teacher learned and triangulated that data with various data sources. Did the teacher approach the concept differently? How did he engage a student in the understanding? For example, if the teacher discussed something he learned in his interview, I triangulated that with my field notes of the classroom lesson, the videotaped lesson, or other artifacts looking for evidence of what he said he learned.

Secondly, I looked for data focused on *how* the Algebra teacher learned about reasoning and sense making when working with an Algebra coach. Once the learning had been identified (either through an interview or field notes), I used interview questions, artifacts, and my personal journal to try to determine how the learning occurred. What may have occurred during the coaching cycle to prompt the learning? What did the teacher feel helped him learn?

Context of the Study

Once again, the various roles I assumed in my inquiry, my perspectives as a teacher, instructional coach, and researcher, influenced my research. This study’s context can be represented as a funnel, starting with the broad context of reasoning and sense making and narrowing down to one teacher participant (see Figure 4.6). Each aspect of the context is introduced and a discussion is offered for why the decision was made to include that context in my study.

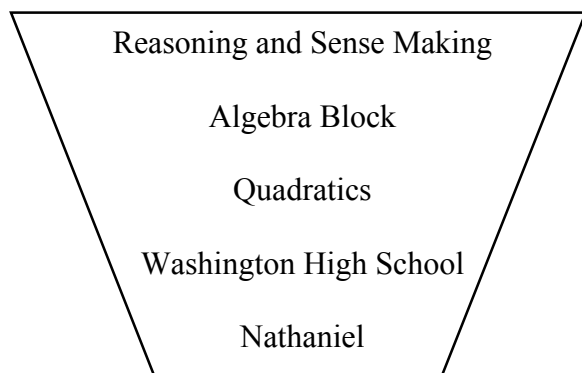


Figure 4.6: The context of my study can be represented as a funnel.

Reasoning and Sense Making

Reasoning and sense making were chosen as a focus of the coaching interactions due to the over-emphasis on procedural skills in the United States' math classrooms (Stigler & Hiebert, 1999). Reasoning and sense making can address several of the reforms in mathematics instruction shown to help students learn and understand mathematics, including teaching conceptually (NCTM, 2000) and making connections between math concepts (NCTM, 2009). An “emphasis on reasoning and sense making will help students appreciate (Algebra)” (Graham et al., 2010, p. 2) and facilitate a greater conceptual understanding of mathematical concepts (NCTM, 2000).

Algebra Block

The teacher participant in my study taught Algebra Block, Geometry, and Accelerated Mathematics.⁸ In the curriculum guide adopted by the school district (2010), the course description for Algebra and Algebra Block was as follows:

⁸ Accelerated Mathematics is a course designed for students who have not had the opportunity to learn prerequisite skills for Algebra. All of the students in the class were

Algebra is the first course in the traditional college preparatory sequence. Course topics include equation solving, linear sentences, linear inequalities, lines, slope, graphing, exponents and powers, polynomials, systems of equations, quadratic equations, functions, and statistics. Algebra Block is a double period course for students who require additional time to master the objectives. The text used in both Algebra and Algebra Block is *Algebra 1, Prentice Hall © 2009*.⁹

Eighth grade teachers recommended a student for the Algebra Block course when the rate at which the student processed information was slower than his or her peers. The recommendation was not to be based upon behavior or attendance. An Algebra Block student could be identified as an English Language Learner (ELL) or have an Individualized Educational Plan (IEP), yet students without either label were also in the class. High school department chairs suggested that middle school teachers recommend the following students to Algebra Block:

- Students who need more examples or time to understand ideas.
- Students who “get it” one day but can seem to forget “it” all the next day.
- Students who work hard but still continue to struggle in math.

The decision to focus the study on Algebra by conducting my research with the participating teacher’s Algebra Block class was for a few different reasons. First, I had several years personal experience teaching Algebra and been solely coaching teachers within the Algebra curriculum for the year and a half prior to this research. Secondly an Algebra Block class was two periods, or 100 minutes, in length, which provided a greater

English Language Learners (ELLs) who have little to no prior schooling. The class is not exclusive to ELLs.

⁹ Bellman, A. E., Bragg, S. C., Charles, R. I., Hall, B., Handlin, W. G. Sr. (2009). *Pearson Hall Mathematics: Algebra 1*. Boston, MA: Pearson Education, Inc.

amount of time to experiment with various instructional strategies focused on reasoning and sense making. Finally, I chose the Algebra Block because the course always included students who struggled in mathematics. If I was conducting this research to find how coaching could be implemented to improve teachers' instruction, with the ultimate goal of increasing student achievement in Algebra, it was important to me to target the class which contained students whose achievement most needed to improve.

Quadratics

For the school district, second semester Algebra curriculum contained six chapters. Each of the six chapters was divided into big ideas and the corresponding textbook sections were listed below each big idea (See Appendix G). The 11 big ideas taught during second semester Algebra were:

- Solve systems of linear equations (Chapter 7)
- Apply systems of linear equations (Chapter 7)
- Simplify exponential expressions (Chapter 8)
- Evaluate and graph exponential functions (Chapter 8)
- Perform operations with polynomials (Chapter 9)
- Factor polynomials (Chapter 9)
- Explore and graph quadratic functions (Chapter 10)
- Solve quadratic equations (Chapter 10)
- Model linear, quadratic, and exponential functions (Chapter 10)
- Simplify, solve, and graph with radicals (Chapter 11)
- Find and use probability (Chapter 2, 12)

Besides the big ideas, the Algebra syllabus also noted the state standards being taught and which objectives were assessed on the state-wide math assessment (see Appendix G).

Teachers throughout the district were required to use the syllabus as a guideline for their instruction. Common chapter assessments were created according to the syllabus and were also a requirement. The curriculum department, including instructional coaches, worked with committees of teachers to create and continually revise the syllabus and summative assessments.

Out of all the second semester Algebra concepts, quadratics was the focus of our consecutive coaching. Quadratics is a logical concept to use for the context of a study about coaching reasoning and sense making since the symbolic manipulation and functionality of quadratics are key components of the algebra curriculum. Chapter 10, which was the textbook chapter devoted to quadratic equations and functions, included three mathematical big ideas (as determined by the school district): 1) explore and graph quadratic functions; 2) solve quadratic equations and; 3) model linear, quadratic, and exponential functions.

Washington High School

Washington High School is a midwestern high school that served a little fewer than 2000 students. At the time of the study, close to 40% of the student population were minority students and approximately 50% qualify for free or reduced lunch. The building staffed 136 teachers, with 209 total staff members, and the principal had over 30 years of educational experience.¹⁰ The math department was composed of 16 math teachers and

¹⁰ Statistics came from the district 2010-2011 Statistical Handbook and from www.nde.state.ne.us.

one math interventionist. The math interventionist was a new position in the spring semester. The grant-funded interventionist worked closely with students who were at high risk of failing. The position included but was not limited to teaching an intervention-type course for students repeating Algebra, meeting with suspended students prior to reentering a classroom, and working with students one-on-one. Second semester there were three Algebra Block classes being taught by three different teachers. Each Algebra Block had between 18 and 24 students.

The math department teachers could be characterized as young. The math teachers were on average 32 years old and had on average eight years teaching experience. They were overall very open to new ideas and willing to try various instructional strategies if it would help their students learn mathematics. Per district guidelines, the teachers met in Professional Learning Communities (PLC) once a week for an hour. The PLC time at Washington High School was spent on planning for a course. Teachers discussed planning in terms of scheduling, such as the pacing of the chapters and the order concepts were taught. They created common rubrics to grade summative assessments and discussed the common conceptual and computation errors students made when completing problems. The PLC also shared formative assessments and talked about how to help keep their students engaged. Much of the PLC work overflowed into the everyday conversations and email correspondence among teachers of the same course. The math department was highly collaborative with teachers who often worked together during plan periods, before school, or after school.

Although I was an instructional coach employed by the district office, I was seen as a member of Washington's math department from early on. I was invited to the

department's beginning of the year dinner, my birthday was added to the celebration list for treats, and I was provided with the annual Washington High School t-shirt in August. Teachers would also just stop me in the hall to see how my day was or how my children were doing. They would invite me to join them at a nearby restaurant on Fridays if several teachers were planning to meet and "unwind." And at department meetings the chair asked if there was anything I would like to add to the agenda or conversation. Unlike some of the other teachers I worked with at other schools, the teachers at Washington High School did not view me as an extension of the district office placed in their building to spy on them.

The leaders at Washington High School trusted me and were supportive of my work as an instructional coaching. Both the math department chair and the principal believed instructional coaching helped teachers improve their instruction, especially over time. This belief stemmed from their observations of me coaching mathematics teachers in the building. Both school leaders had made comments to the math curriculum specialist and me about how coaching impacted the teachers at their school. When it was announced that the district received the grant and each PLAS school would be assigned their own math coach, the principal and department chair personally found me in a classroom and asked me to come to Washington High full-time. They argued that I was the perfect fit for their teachers, the staff already knew and trusted me, and they wanted me to be the coach at Washington.

Much of the trust and relationships I had with teachers and administrators at Washington High School stemmed from my prior experience with the school. I was an Algebra teacher at Washington High for two years (2004-2006) before moving to a

different city. During those two years I taught with seven of the current 16 math teachers, including a close teaching partnership with the current department chair. The current principal was the same principal that hired me as a teacher. When I was looking to return to the district in the fall of 2008, the leaders at Washington High School created a part-time teaching position specifically for me. During the 2008-2009 school year I taught Algebra part-time at Washington and coached Algebra part-time at Washington High and another high school. Through these experiences, I had fostered deep connections to the school and many of the staff prior to coaching there full-time.

Washington was a good school to choose for this research for several reasons. The highly collaborative culture within the math department made coaching more widely accepted. Teachers did not question why I planned with teachers or co-taught in their classrooms. Collaboration was a “way of life” in contrast to some other schools where teachers work in isolation, making coaching seem more natural. Since the department and administration had already welcomed and accepted me as a colleague. I could simply go in and work with teachers. I was able to more quickly get to the actual act of coaching rather than navigate the other complications of coaching relationships.

Nathaniel

Nathaniel was a 28 year old teacher at Washington High School who was in his third year of teaching. His specific teaching schedule for the 2010-2011 school year included Algebra Block, two classes of Geometry, and Accelerated Math. He was also a member of the ninth grade academy and met weekly with the ninth grade team to discuss students and instruction. (The ninth grade academy was a group of approximately 24 teachers whose classes were filled with only ninth grade students. The ninth grade

academy had two interdisciplinary teams of teachers that functioned similar to middle school teams. Each team included two math teachers.)

In order to minimize the coaching complexities that I was experiencing in my practice and to focus on one teacher's learning, I needed a teacher who was open and excited about our coaching interactions. He had to be someone who was interested in communicating with me on a regular basis and intrigued with Algebra instruction.

Nathaniel embodied these characteristics.

Since our very first coaching cycle in November 2009, Nathaniel continuously welcomed me into his classroom. In fact, he would ask me to come in certain days or request me to plan for particular concepts. He was open and excited about our coaching interactions. This help-seeking behavior could have been due to the fact that he was a new teacher in search of resources (Burke, 1987). When Nathaniel and I met during a coaching cycle or when we saw one another in the building, he asked for help or approached me with instructional questions. For example, he would ask me if I thought his students needed more instruction on a concept, if I had ideas on how to introduce an algebraic idea, or if I had resources for an objective. Nathaniel was a very positive person, continually encouraging his students and his colleagues. Many of these teaching traits could be found in several other math teachers at Washington High School, but were not as widely apparent when looking at math teachers across the school district.

Conclusion

Approaching my study as a teacher researcher allowed me to continue as a practitioner while using research to systematically inquire about my practice. Every choice I made in this study (coaching changes, focus of research, data collection tools,

the context of my study) was done from a dual perspective. And all of my research decisions were made with the goal of better understanding how coaching can be used to improve teacher instruction, with an emphasis on learning more about what and how a teacher learns within the coaching process.

Chapters five, six, and seven offer the story of Nathaniel and my coaching interactions. Chapter five explains what coaching Nathaniel was like prior to Spring 2011 by providing a more thorough description of why Nathaniel was a good participant, details of the coaching relationship between Nathaniel and me, examples of his Algebra instruction prior to Spring 2011, and patterns in our initial coaching. What Nathaniel learned during the quadratics phase of my research will be more noticeable if a clear initial depiction of Nathaniel and his teaching are described. Chapter six provides details of the intense, consecutive coaching in quadratics in Spring 2011, as well as how the mathematical concepts were taught and why Nathaniel and I made various instructional decisions. Included in this chapter is an analysis of the data collected on what and how Nathaniel learned. Finally, chapter seven presents concluding thoughts about instructional coaching and future research ideas prompted by my study.

CHAPTER 5: EARLY COACHING WITH NATHANIEL

This chapter focuses on Nathaniel's teaching and coaching prior to the spring of 2011. Doing so will later help to illustrate what and how Nathaniel learned about teaching quadratics with reasoning and sense making. After initially introducing Nathaniel, I travel back in time to Spring 2009 and my initial coaching interactions with Nathaniel. Determining what he learned or did not learn during the later consecutive days of coaching is made possible by understanding where his teaching and my coaching started.

Nathaniel earned his Bachelors of Science in Education and Human Science, majoring in both secondary mathematics education and athletic coaching. Upon graduation, Nathaniel immediately began earning his Masters of Science in Kinesiology from another prominent mid-western university. His decision to postpone teaching and earn a higher degree was two-fold. First, he was intrigued and passionate about exercise science and felt like he could not pass up the opportunity to learn more about something he loved. Secondly, Nathaniel had received a significant scholarship that covered the cost of a Masters degree program if entered into immediately following graduation with a Bachelors degree. Therefore, he had earned two degrees from two separate colleges prior to beginning his career as a teacher in the fall of 2008. In June of 2009, following his first year of teaching, Nathaniel was accepted into a mathematics education graduate program aimed at helping secondary mathematics teachers improve practice. Since then, he had been participating in graduate courses part-time.

Nathaniel began teaching high school mathematics in 2008. He originally became a teacher because he wanted to have an impact on people's lives and have a job where he

was not sitting down all day (Written Correspondence, July 6, 2011). According to Nathaniel, education and health are the two careers that best fit his personality and interests. He explains that one thing he loves about these fields is that “you are pushing people to do what they never thought they could – so you help transform self-confidence and belief in one’s self” (Written Correspondence, July 6, 2011). In other words, he became a teacher because he just wanted to “touch a life or two” (Written Correspondence, July 6, 2011).

He started his career teaching Algebra Block and Geometry part-time (0.8) at another local high school. Due to staffing reduction, he was transferred to a Washington High School at the beginning of his second year of teaching. Nathaniel had been at Washington High School for one and a half years teaching Algebra Block and Geometry in the ninth grade academy prior to our research timeframe, meaning he had a total of two and a half years of teaching experience entering into the data collection.

Background – Our Coaching Relationship

A trusting relationship is “necessary for all change initiatives” (Hull, Balka, & Miles, 2009, p. 56). Towards the end of my first year of full-time coaching, I began to view the goal of coaching as helping teachers change and improve their instructional practices. Therefore trust between a teacher and myself was essential if I wanted to help him change his instruction. Nathaniel and my coaching relationship began in spring of 2009. The coaching continued on a weekly basis throughout the following year and a half when he transferred to Washington High School.

First Impressions

Nathaniel and I first met in the spring of 2009 at a Professional Learning Community (PLC) meeting at his previous high school. It was his first year of teaching and my first year as a coach. I was a part-time coach, but I was not assigned to coach teachers at his school. The school district's curriculum specialist had asked me to attend Nathaniel's Algebra PLC to help the teachers plan. Our school district was committed to a PLC model, providing high school teachers one hour each Tuesday to meet and discuss instruction. These weekly collaborative meetings were meant to provide teachers the opportunity to openly study and share teaching practices.

I entered this particular PLC meeting at a new high school for me, not knowing what to expect. Who would be at the meeting? What would their idea of planning entail? How did they feel about a coach being asked to come to their PLC? I had not previously met Nathaniel, or any of the other three Algebra teachers present at the meeting. We met in Nathaniel's classroom, which was a portable trailer behind the school. (The school was in the midst of renovations, so several classes were held in portables.) Nathaniel was fairly quiet, especially at the beginning of the PLC. He listened intently as the other teachers talked about plans for the next week.

The three other teachers were using their collection of worksheets to determine what skill to teach next and what formative assessments (worksheets) to assign. It was evident through the teachers' interactions that collaborating during PLC for them largely meant talking about what they had done last year (i.e., what worksheets they had assigned, number of days spent on an objective) and deciding on the pacing and homework for the upcoming week. No one was talking about actions that would increase

student understanding such as how to teach a topic conceptually (Hiebert et al., 1997) or ways to organize and engage students during instruction (Nebesniak & Heaton, 2009).

These behaviors exhibited by Nathaniel's colleagues could have stemmed from a number of issues common in education, such as educators' reluctance to change (Guskey, 1986), pressures to conform to the school's culture (Stigler & Hiebert, 1999), or the influenced of educational setting they experienced as students (Lortie, 2002). A few of the characteristics exhibited by the three teachers in the PLC can be used to describe an "unaware teacher" (Hall & Simeral, 2008, p. 57). The term "unaware teacher" refers to the teachers' states of mind and levels of self-awareness as they teach and then reflect on their teaching. A few of these "unaware" characteristics included planning lessons that were vague and lacked direction or rationale, exhibiting little effort to make mathematics meaningful, creating lessons built on direct instruction and homework, and collaborating that appeared to be on a superficial level. These teachers were not "bad" teachers, unwilling to improve their instruction. If they were indeed unaware teachers, they were merely not aware of effective teaching practices (Hall & Simeral, 2008) and therefore gravitated toward the traditional teaching strategies they were more accustomed to. The way the teachers chose to collaborate during PLC time could have been reflective of an unaware state of mind. If the teachers were not aware of effective teaching strategies, they could not successfully use the PLC meeting to improve their instruction.

To push the PLC towards improving instruction and to start bringing awareness to effective teaching strategies, I began asking the teachers if they could think of another way to use their worksheets to engage students. I chose one instructional issue (student engagement) to build the teachers' awareness of effective instruction. Then I introduced

the teachers to a different instructional strategy (cooperative learning), which I was familiar with from my own teaching experiences. By doing this, my hope was to demonstrate to the teachers that I could provide some knowledge and skill in teaching Algebra and cause them to become more open to the coaching process. This process of demonstrating knowledge and showing teachers you can be a valuable resource is a strategy often employed by novice coaches (Killion, 2009). I was pulling from my cooperative learning background at that moment and was hoping I could provide the teachers with a formative assessment strategy more engaging than individuals completing worksheets. It was at that moment that I saw Nathaniel sit up in his chair. I continued by suggesting a few different cooperative learning techniques such as having pairs of students work together on two problems and then checking with another pair before continuing to the next problem (Kagan, 1994). One teacher hesitantly took an idea I suggested and started talking about how she could use a particular worksheet in a more cooperative learning fashion the next day. As she talked, Nathaniel jumped in and gave his ideas and suggestions about what could be done. He offered to type up this cooperative learning idea, create instructional materials for students, and send cooperative learning task to everyone in the PLC.

I returned to the PLC a couple of weeks later. This time Nathaniel excitedly shared a few ideas at the beginning of the meeting. In the middle of the group, he put a few examples of cooperative learning tasks he had created. Nathaniel asked for the other teachers' opinions. He wanted to know if they thought the task would be helpful or if they had other ideas on how to make it better. His colleagues did not reciprocate his excitement, but instead answered his questions with "sure" and "maybe." Then they

focused their attention back on their large notebook of worksheets to guide their lessons. Nathaniel tried to engage his colleagues in a discussion about creating cooperative learning tasks for a little while longer before retreating. He put away the cooperative learning tasks he created and opened up his lesson plan book to record which worksheet would be assigned on which day.

Striving for something more. I was impressed by the way Nathaniel tried to lead his PLC and by the risk he took by putting himself and his ideas out there. I remember thinking that he encompassed a real passion for teaching and that his hunger to improve his teaching was not being fed by his colleagues. I was worried that Nathaniel would be swallowed up by the negative attitudes that surrounded him if someone did not help him. It was evident to me from these two initial interactions that Nathaniel expected more out of the PLC collaboration than his colleagues did. He radiated a desire to improve his instruction and search for a better way to do things. I was not sure where the desire came from, or what exactly he was looking for, but Nathaniel was definitely searching for something more in his teaching.

Nathaniel also remembered the first time we met. He recalled me coming to a couple PLC meetings during his first year of teaching. During these meetings, he remembers trying to get the other teachers to talk about more than looking “assignment by assignment” (Initial Interview, January 28, 2011). When I asked Nathaniel for his perspective on how our coach-teacher relationship began, his initial response was, “Well, to begin with I was pretty intimidated by you” (Initial Interview, January 28, 2011). He explained that I was more direct than the other secondary coach who had previously met with their PLC. Nathaniel continued to describe how he was initially not sure how to

respond to our straight-forward interactions. From his perspective, I knew where I wanted to go and I was determined to get everyone else to that goal.

Nathaniel explained that intimidation turned to trust when I wrote him a note after one of these first PLC meetings. The note praised him for trying to be a leader and encouraging his colleagues to improve their instruction, as well as his own. Nathaniel said those words were “kind of good to hear because they (PLC colleagues) were hard people to budge. Kind of stuck in their ways. I think that is the first thing I really remember (about our relationship)” (Initial Interview, January 28, 2011). Trust, which “paves the way for coaches to work directly with teachers to improve their use of instructional strategies” (Hull et al., 2009, p. 56) developed early in Nathaniel and my coaching relationship through these initial PLC interactions.

First Year and a Half of Coaching: August 2009 – December 2010

Nathaniel transferred to Washington High School in the fall of 2009 when his previous building was forced to reduce staff. I explained to the math department chair and the administration at Washington High School what I experienced with Nathaniel in those PLC meetings. I shared with them the spark and fire I saw Nathaniel had to become a better teacher. Yet he was only in his second year of teaching and, therefore, demonstrated struggles often seen in new teachers. A few of these struggles included pacing instruction within a lesson (Freiberg, 2002), being able to effectively manage student behaviors within the classroom (Ewing & Manuel, 2005), and combating the isolated nature of teaching (Little & McLaughlin, 1993). Due to the fact that Nathaniel was a new teacher and also had a strong desire to improve his practice, the department chair, principal, and I all agreed that he would be a great teacher for me to coach. Plus

Nathaniel, along with one other Algebra teacher, was piloting a new Algebra textbook during the 2009-2010 year. Since part of my responsibilities as a secondary instructional coach is to implement new curriculum and gather feedback from teachers regarding assessments, pacing, and instruction, I needed to be in close contact with all teachers piloting the new textbook.

The coaching interactions Nathaniel and I had during the first year and a half helped me better understand who Nathaniel was as a teacher and as a person. The structure of our coaching largely resembled the coaching that occurred with Sarah, the teacher I described in Chapter 2, as well as all of the Algebra teachers I coached on a regular basis. However, there was also something different about Nathaniel and my coaching interactions that made him stand out from the other teachers. These distinctions are brought to light through the story of our first year and a half of coaching.

Impressionable stage. Although I casually talked with Nathaniel at the beginning of the school year, we did not begin formal coaching until I returned from maternity leave at the beginning of November. Nathaniel was one of the six Algebra teachers at Washington High School that I initiated coaching interactions with upon my return. We began working together on a weekly basis on November 12, 2009. The structure of Nathaniel and my coaching interactions resembled how I worked with Sarah. I emailed Nathaniel a few days prior to the twelfth to see if that day would work in his schedule to engage in the basic coaching cycle (pre-lesson planning session, the lesson, post conference debrief meeting). During his third period plan time, Nathaniel and I met for our planning session. Similar to Sarah, Nathaniel walked me through the lesson he had already laid out for his Algebra Block class. He showed me the materials he had

prepared, explained the mathematics he planned to teach, and provided me with information about specific students within the classroom. Prior to the planning element of the coaching cycle, both Nathaniel and Sarah had already determined what to teach and how to teach the lesson.

During the planning session on November 12, 2009, Nathaniel was open to trying new strategies I suggested. These suggestions were largely for formatively assessing the students' understanding of the math concept. Nathaniel even asked if I could model a cooperative learning activity in his classroom. During the actual lesson, I largely observed or answered individual students' questions until it was my turn to lead the cooperative learning activity. My role during the lesson was again similar to the role I played in Sarah's classroom; I helped students individually and modeled cooperative learning strategies. In our post lesson debrief, Nathaniel and I reflected on the lesson together. We discussed the instructional strategies used during class, his students' understanding of the mathematics, and the behavior and mathematical expectations we should hold for students. After this first coaching cycle with Nathaniel, I wrote, "Nathaniel seems to be in a very impressionable stage right now. I would like to spend as much time with him and in his classroom as possible. He is my main focus at Washington High" (Personal Journal, November 12, 2009). Nathaniel demonstrated a desire to gain knowledge from his colleagues with more teaching experience, a trait of many new teachers (Bullough, 1989).

I worked with Nathaniel 16 more times during the 2009-2010 school year (his first year at Washington High School) and three times in the fall of 2010. This equated to approximately one coaching cycle every seven to ten days. The planning and debrief

discussions we had varied over the year and a half. Some days we were largely focused on how to improve classroom management, an area Nathaniel self-identified as a weakness in one of our conversations (Personal Journal, November 18, 2009). Our classroom management discussions were largely about getting all students to participate in the instruction, decreasing student disruptions, and using organization techniques (both classroom and instructional organization) to foster a more controlled learning environment in which negative behaviors would decrease. Other days we concentrated on how we could help students understand the math concepts. At times the two areas of classroom management and teaching math concepts overlapped. The instructional approach to teaching a concept influenced the students' behaviors and the classroom management strategies impacted instruction (Brophy, 1999).

Through our year and a half of coaching, Nathaniel came to think of me as someone to help lead him in the right direction. He explained, "I kind of just wanted to keep reaching out to you to get more help. And I think you were just like, 'Oh wow, I think he wants to keep learning.' I think you didn't mind taking me under your wing" (Initial Interview, January 28, 2011). At the end of the 2009-2010 school year he gave me a thank you note that said, "Your guidance has helped me immensely" and that he was "looking forward to revolutionizing math" with me (Written Communication, May 2010). These comments by Nathaniel made me think that he saw me as an expert, or as the person telling him how to best teach Algebra and get his students to behave in class. In fact, in our day-to-day interactions and more in the initial interview on January 28, 2011, Nathaniel refers to me sharing my "expertise." I felt like Nathaniel viewed me as someone who could tell him how to fix things and offer guidance.

At the same time, Nathaniel also viewed our work together as a partnership. This belief was demonstrated by his comment “looking forward to revolutionizing math with you” (Written Communication, May 2010). And when he referred to his Algebra class throughout a coaching cycle, he talked about “our” students and “our” lessons, as opposed to “my.” The way he referenced his class highlighted his view of us as partners.

A different kind of collaboration. To get Nathaniel’s perspective on our first year of work together, I asked Nathaniel in an interview if he had any memories about our coaching that stood out for him. He immediately recalled the planning session we had following our first coaching cycle. (This planning session also serves as an example of how coaching Nathaniel was different from coaching Sarah and other teachers.)

During the post conference debrief of the November 12, 2009 lesson, Nathaniel asked for my ideas about teaching the first few days of the next chapter. He was going to start the chapter on slope and linear functions soon and was asking me for ideas on how to teach slope and slope-intercept conceptually, without forcing students to memorize the formula

$\left(m = \frac{y_2 - y_1}{x_2 - x_1} \right)$. We set aside time that day after school to discuss ways to teach slope.

Since slope was a critical component of linear functions and functions were a central focus of school algebra (Dossey, 1998; National Council of Teachers of Mathematics (NCTM), 2000), Nathaniel and I knew that determining how to teach slope was important. In order to have our students engage in “creative and original thinking” as opposed to a “procedural and formula-based study” (Burke et al., 2008, p. 18), Nathaniel and I decided to focus on the mathematical reasoning underlying the concept of slope. Representing the relationship of slope as the change in y-values divided by change

in x-values, rather than memorizing the formula, helped students see algebra as a tool for problem solving (Smith & Thompson, 2007) and helped connect the symbols with the meaning (Graham, Cuoco, & Zimmerman, 2010).

Our planning took place at a long table in the middle of the teachers' planning center. For over four hours, we discussed how to help students discover slope and connect it to what they already knew about rate of change and tables of values. Nathaniel and I both threw out ideas about how to build the concept of slope on what students already knew, what real-life applications we could use, what mathematical tasks could be created for students, and how slope was represented. Nathaniel's approach to teaching and my discussion about slope were rooted in getting students' to reason and make sense of the concept. We worked together to create ways to connect the slope concept to prior knowledge and help students draw conclusions based on real-life on how to find slope (Graham, Cuoco, & Zimmerman, 2010; NCTM, 2009).

In true collaboration, Nathaniel and I worked together to think and find solutions to how to teach slope (Wenger, McDermott, & Snyder, 2002). Nathaniel described this planning session as an exciting, productive time. In our first formal interview he told me what our slope discussion meant to him.

For me that was the first time I really got to plan the way I always wanted to plan. A lot of times when you are planning it is like, "Here is the homework assignment." That is not what I want to do when I plan with people. Everyone can figure out good problems and stuff to do. After I got done working with that I thought, "This is what planning should be. This is what planning is." We were just bouncing ideas back and forth and trying to get better things. So for me when

we got done with that session, I was like, “This is what I want to do when I plan.” It felt productive. I was really excited about that...I want to teach the concepts and the big idea. I really felt like that is what we were getting at when we were planning together. So that was the most exciting thing for me. (Initial Interview, January 28, 2011)

Nathaniel found this exchange of slope ideas to be more helpful than discussing what homework to assign, which is what he had previously experienced in his PLC meetings. He appreciated being able to talk about the mathematics at a deeper level and to investigate the concept of slope collaboratively with a colleague.

I also found our discussion of how to teach slope exciting and invigorating. The collaborative brainstorming we engaged in reminded me of the times I collaborated with my own colleagues as a teacher in my fourth or fifth year of teaching. Those deep, problem-solving discussions I had with other Algebra teachers as we tried to find a meaningful way to introduce math concepts in our classrooms were very similar to Nathaniel and my slope conversation. Nathaniel’s desire to teach for conceptual understanding also paralleled my own desires as a teacher. He wanted his students to see the big picture and make meaning of the mathematics in order to better understand the concepts (NCTM, 2009), which was similar to what I had learned to strive for in my own instruction as a teacher.

When comparing the discussion Nathaniel and I had about slope with the conversation we had during the pre-conference planning session earlier the same day, it was evident that planning conversations were very different. Earlier on November 12, 2009, Nathaniel had already created a lesson and the materials that would be used in

class. So when we met to “plan” during third period, our time was spent having Nathaniel explain to me his lesson outline. He had already established how he was going to be teaching the concept. And with such little time, the lesson could not easily be modified. Therefore, as the coach, I was restricted to discussing teaching strategies, such as cooperative learning, that could quickly be implemented into the formative assessment part of class. This situation put me into the role of quickly fixing his lesson and trying to make it better rather than in the role of helping him think deeply about how to teach the mathematical concept.

The planning Nathaniel and I did after school on November 12, 2009 had a completely different look and feel to it. We took a more collaborative approach to the discussion since we had time to brainstorm how to teach the concept of slope. My role was collaborator rather than expert largely due to the fact that this planning was focused on brainstorming ideas rather than me modifying Nathaniel’s lesson. Entering into the discussion as more of a brainstorming session allowed both Nathaniel and I to generate reasonable ideas to use when teaching slope from which he could choose. We also had conversations about the types of questions that could be asked to build understanding (William, 2007) and the possible responses students might have for a given task (Smith & Stein, 2011). The biggest difference, when compared to the earlier pre-conference planning, was our focus on conceptual understanding of the mathematics rather than the procedures of a teaching strategy.

The “x-factor.” Nathaniel made tremendous progress during his second year of teaching (first year at Washington High). After our last coaching session in May 2010, I wrote:

He has made such remarkable progress since the beginning of the year. He has always had the amazing teaching qualities and ideas, but now he has the classroom management skills to effectively implement his great ideas. He has come a long way and I am very proud of him. (Personal Journal, May 26, 2010)

The biggest changes for Nathaniel that year occurred in the area of classroom management, which is an area that Nathaniel noted as a weakness (Personal Journal, November 18, 2009) and is an area of concern for the majority of new teachers (Ewing & Manuel, 2005; Freiberg, 2002). By the end of the year Nathaniel had established classroom routines and modified his lesson plans to include more engaging tasks, specifically implementing cooperative learning to formatively assess students, both of which helped him improve the overall effectiveness of his teaching.

During his first year and a half (August 2009 – December 2010) at Washington High School, administrators and the department chair also noticed how much Nathaniel had grown in his classroom management abilities. When they observed his class, they noticed that his students were on task more often and the students were given more opportunities to learn the mathematics. One administrator told me how impressed he was with Nathaniel's implementation of numerous activities that kept students engaged.

In April of 2010, I nominated Nathaniel for a statewide award that recognized a teacher in his first three years of teaching who excelled in the profession. I chose to nominate him because at his core he possessed the qualities an excellent educator. Here is an excerpt from my recommendation letter:

He simply radiates a passion for learning and a belief in children. His teaching is based upon students building their own understanding of the mathematics. This

teacher is an incredibly reflective educator, continually reflecting on his instruction and how he can improve. He challenges himself to find the best way to reach each one of his students. He has that “It” factor, with a capital I! He was meant to be a teacher. He was meant to be an inspiration to his students. He was meant to be an inspiration to all educators who interact with him. (Written Communication, April 2010)

I tried to explain the qualities Nathaniel possessed that made him a good candidate for the award. The “It” factor, or the teacher “X-Factor” (Hall & Simeral, 2008, p. 12) is that quality that is difficult to name, but makes someone stand out. In the case of Nathaniel, he stands out from other teachers due to his enthusiasm for student learning and dedication to improving himself as an educator. Teacher efficacy, or a teacher’s belief that he could positively impact student learning in all situations, may be a better way of describing the “It” factor I believed Nathaniel possessed (Cantrell & Hughes, 2008; Schunk, 2008). Teachers who have strong self-efficacy are more likely to implement change in their classrooms (Cantrell & Hughes, 2008). Nathaniel received this award in early Fall 2010.

How I perceived Nathaniel. From the moment I met Nathaniel at the PLC meeting in the spring of 2009, I was impressed with his excitement and dedication to improving his instruction. His desire to improve was most evident during the planning and debriefing phases of the coaching cycle. Nathaniel continually asked for ways to help students better understand the mathematics and solicited my feedback after each lesson. He was continuously open to new ideas and was willing to try any instructional strategy I suggested if he thought it would increase student understanding.

During these initial coaching cycles, my perception was that Nathaniel was simply eager to build up his repertoire of instructional strategies since he was a new teacher. Early career teachers often seek out colleagues as they become familiar with their new environment (Ewing & Manuel, 2005). I assumed Nathaniel was seeking me out and was engaged in our coaching due to the fact that he was as a new teacher. Yet now as I reflect back on our first year and a half of coaching, I realize Nathaniel was in fact demonstrating characteristics of a learner. He persisted in difficult situations, focused on mastering effective instructional strategies, and willingly accepted challenges and took risks, all of which are characteristics of someone who is seeking to improve himself (Bruning, Schraw, & Norby, 2011). He was a new teacher, but more importantly he was a learner.

Since I did not recognize Nathaniel as a learner at the time, I largely treated our coaching interactions as transferring knowledge from me to him. I presented him with an instructional strategy to use and I simply expected him to adopt that practice. If I had recognized that Nathaniel was a learner and approached our coaching as a means for helping him actually learn how to improve his instruction, I would have helped him build meaning behind the instructional strategies by discussion how and why to choose particular instructional techniques (Prawat, 1996).

Initial Coaching with Nathaniel

If the goal of my research is to examine what Nathaniel learned during intense, consecutive coaching in quadratics and how he learned it, it is important to have a clear image of what his initial teaching and our initial coaching entailed. The following vignette is offered as a way to paint a picture of Nathaniel's initial teaching and our initial

coaching. The vignette also allows the reader to compare and contrast coaching Nathaniel with coaching Sarah (see chapter two) and most other teachers with whom I worked.

Vignette: Coaching Nathaniel

Nathaniel and I had completed eight other coaching cycles prior to this February 16, 2010 lesson. My role in his classroom had previously consisted of observing, modeling activities, collecting data, co-teaching concepts, and assisting students. Nathaniel often asked me for cooperative learning techniques to incorporate into the lesson and requested that I model many of these strategies. During all eight of the previous coaching cycles we had discussed classroom management at some point. The mathematics was a secondary focus in our first coaching cycles. Beyond the after school planning session we had on November 12, 2009 where we discussed how to teach slope for understanding, Nathaniel and I had not participated in an in-depth discussion about teaching mathematics. Similar to Sarah, Nathaniel and my coaching primarily focused on classroom management and cooperative learning strategies to engage students.

Planning as telling. I emailed Nathaniel on Friday to see if I could come to his two-period Algebra Block class the following Tuesday. I would not be at Washington High School again before Tuesday, so we decided to meet during his third period to touch base regarding the lesson. He emailed me over the weekend telling me he planned to

teach division properties of exponents (i.e., $\frac{x^3}{x^5} = \frac{1}{x^2}$) and to introduce quotient to a

power (i.e., $\left(\frac{x^5}{y^3}\right)^2 = \frac{x^{10}}{y^6}$) on Tuesday. This gave me the opportunity to think about ways

I wanted to suggest he approach teaching the concepts. We had been meeting during

lunch so I could be in another teacher's classroom during third period, yet Nathaniel had mentioned on several occasions that having more time to talk about introducing the math concept, brainstorm ways to engage students in formative assessments, and determine each person's role prior to class would be helpful. So we met during his third plan period, giving us about 50 minutes to discuss the lesson he had previously created.

Nathaniel regularly typed out his lesson plans. This particular lesson for teaching and practicing division properties of exponents and quotient to a power included a warm-up, discovery activity, formative assessment, introduction to quotient to a power, scavenger hunt, and a book assignment for homework (see Appendix H). We used these ideas and the corresponding materials he had made to guide our discussion. Our planning began with a discussion about how to teach division properties of exponents. He had planned to have students simplify expressions by expanding them out and canceling a number or variable if it appeared in both the numerator and denominator (see Figure 5.1).

Although Nathaniel and I were both aware that saying $\frac{x}{x} = 1$ meant that x could not equal zero, we did make this distinction for students.

$$\frac{3x^4}{6x^2} = \frac{3 \cdot x \cdot x \cdot x \cdot x}{2 \cdot 3 \cdot x \cdot x} = \frac{\cancel{3} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{2 \cdot \cancel{3} \cdot \cancel{x} \cdot \cancel{x}} = \frac{x \cdot x}{2} = \frac{x^2}{2}$$

Figure 5.1: Nathaniel had planned to teach students to simplify exponents by expanding the expression and canceling common factors.

The notion of writing an expression in expanded form as a way of increasing students' understanding of simplifying exponents was a strategy I had recommended to Nathaniel several weeks earlier when he expressed concern with giving students exponential rules. I explained to him that when I taught exponents, I told my students "When in doubt, write it out" to emphasize the understanding. Based on my students' learning, I realized that approaching exponents with laws or rules only confused my students. They struggled to memorize the rules. And the algebraic notations $\left(\frac{a^m}{a^n} = a^{m-n}\right)$ associated with the exponential laws only caused my algebra students to shutdown and refuse to attempt simplification. I had come to realize in my fourth or fifth year of teaching that a more thoughtful approach to teaching the meaning behind mathematical rules and formulas helped students understand the mathematics (Chazan, 2000). After our conversation, Nathaniel chose to use the saying "When in doubt, expand it out" with his students starting the first day of the exponents chapter.

For division properties of exponents, Nathaniel and I discussed how having the students visualize the expanded form was especially important. I forewarned Nathaniel about the common mistake students make when simplifying these types of expressions. Students figure out they can simply subtract the exponents, but they do not remember if the variable should then be placed in the numerator or denominator (see Figure 5.2). For example, for the expression $\frac{x^3}{x^5}$ students attempting to apply the exponential rule tend to subtract three from five ($5 - 3 = 2$) and then write x^2 as the simplified form. This is incorrect. If the rule is applied correctly, students should subtract five from three ($3 - 5 = -2$)

resulting in x^{-2} or $\frac{1}{x^2}$. Nathaniel and I discussed how emphasizing expanded form would help alleviate this common misconception since students would have the visual representation and could see that the x^2 belongs in the denominator.

$$\text{Common Student Mistake : } \frac{x^3}{x^5} = x^{5-3} = x^2$$

$$\text{Correct Simplification using rule : } \frac{x^3}{x^5} = x^{3-5} = x^{-2} = \frac{1}{x^2}$$

$$\text{Expanded Form : } \frac{x^3}{x^5} = \frac{\cancel{x \cdot x \cdot x}}{\cancel{x \cdot x \cdot x} \cdot x \cdot x} = \frac{1}{x \cdot x} = \frac{1}{x^2}$$

Figure 5.2: A common mistake students make is subtracting exponents incorrectly. The correct way to simplify exponential expressions involving division is by using expanded form.

After I cautioned Nathaniel about common misconceptions students have when simplifying exponential expressions, he included two notes in his typed lesson plan. He wrote, “Focus on getting students to visualize their simplification. Do not emphasize subtraction rule...(future – alter form to not even mention it?)” (Lesson Plans, February 16, 2010). Nathaniel made note of both the approach to dividing exponential expressions that I had suggested and the common student mistakes I warned him of. He had already created and made copies of the 8.5 Discovery Form (see Appendix I) since he was teaching the lesson in a few hours. The form, which each student would have completed, included a place for students to write down a rule for dividing exponential expressions

(see Figure 5.3). Through our discussion of how to approach this concept, Nathaniel decided that he did not need to force his students to summarize a formal rule and made note of this change he wanted to make for next year.

BIG IDEA : DIVIDING POWERS WITH THE SAME BASE

When you divide powers with the same base, you _____.

Figure 5.3: Nathaniel provided a place for students to formalize a rule for dividing exponents. (Classroom Materials, February 16, 2010)

As we continued to discuss the remainder of the lesson, I encouraged Nathaniel to include some individualized guided practice after the discovery activity and before having students do the Quiz-Quiz-Trade formative assessment (Kagan, 1994). For the Quiz-Quiz-Trade activity, each student has a card with an exponential expression. Students partner with one another and each person simplifies her partner's expression. The students would be expected to not only simplify the expression correctly for their partner, but to also be able to determine if another person's answer was correct. In essence, the students would be relying on one another to check for understanding. I explained that it would be helpful if we as teachers got a better idea of student understanding before the students participated in a cooperative learning activity that expected them to check one another's work. He agreed that some guided practice would be beneficial and we decided to include some individual whiteboard problems as a way to observe students' thinking.

Nathaniel also asked during this time if I would lead the Quiz-Quiz-Trade cooperative learning activity. He had tried the activity with different mathematics on a day I was not present and he said it did not go as well as he would have liked. When he implemented the activity, some students were engaged in socializing rather than the mathematics. Some students were checking their partners' answers while others were not even listening to what their partner said. And some students were telling the answer but not explaining how they got that answer or how they knew the answer was correct. Nathaniel asked to see me set up the activity and lead it so he could learn how to do the cooperative learning activity better on his own next time. I agreed to model that component of the lesson.

During the 50 minute planning session, I had suggested a way to approach the mathematics for understanding, helped him anticipate common mistakes students make, encouraged him to include an individual check for student understanding, and explained ways to better manage student behaviors during the cooperative learning portion of the lesson. Nathaniel commented that the extra time spent discussing his lesson was helpful. He said he felt better prepared to teach the lesson and was more confident about how to approach the mathematics (Field Notes, February 16, 2010).

A large portion of our coaching was spent on the technical aspects of teaching. How do I get students to stop talking? What is the best way to use Quiz-Quiz-Trade? And like I did with Sarah, I worked hard to "fix" Nathaniel's instruction during the planning session. My goal was to help Nathaniel teach a great lesson rather than help him learn how to teach a great lesson. Yet coaching Nathaniel during the planning component was also different from Sarah and other teachers. First of all, he was highly

involved in the planning. He valued the collaborative work we did together. His enthusiasm for our collaboration reminded me of how much I valued collaboration when I was an Algebra teacher myself. He enjoyed discussing various instructional strategies and often took on the task of creating his own formative assessments that would engage students and force them to think at a higher level.

Nathaniel was also interested in learning how to better teach a concept, as demonstrated through the numerous questions he asked in order to gain a deeper understanding. We talked about common student misconceptions and how to combat those common errors. During several other coaching sessions, we got into conversations about mathematical concepts that caused us to search for answers beyond the two of us. (For example, when solving radical equations and checking for extraneous solutions, what happens if you get something like $\sqrt{-25} = \sqrt{-25}$. Those values are equal, making the equation true. Yet $\sqrt{-25}$ is not a real number. Is the solution being checked extraneous or not, and why?) Sometimes I would raise these types of mathematical questions and other times Nathaniel would.

When I stepped back and personally reflected on the entire coaching cycle later that day in my personal journal, I noted that I would like to have spent more time discussing mathematics instruction with Nathaniel. He was so eager to talk about how to teach a concept and I felt like I was doing him a disservice by planning with him the day of the lesson. Under the current structure, we did not have enough time to talk in depth about mathematical concepts. Since we met just a few hours prior to the lesson, we were restricted on how much we could change his original materials or lesson outline. I did not feel comfortable suggesting major changes that close to the class time. In my

personal journal I wrote about how I would like to meet with him prior to the day I would be in his classroom. I thought planning the day before the lesson would allow Nathaniel and me to have conversations about mathematical concepts and possible ways to teach them to students for understanding. I thought planning the day before would give Nathaniel and me more time to discuss ideas and create a lesson together, rather than tweak the lesson he created a few hours prior to class.

I began to ask myself questions about the planning that was occurring as I coached Nathaniel. Why was I so intent on fixing his teaching? How do I know what Nathaniel is learning about teaching and mathematics? What would happen if I did not tell Nathaniel how he should teach a concept? What would our coaching look like if we truly entered into it as colleagues rather than me as the expert? Is there any better time to plan lessons together rather than the day of the lesson?

Lesson as imitation. When the bell rang, Nathaniel told students to start working on the warm-up problems that they picked up when they entered. Nathaniel, the special education co-teacher, and I walked around the room. We looked at students' work to see if they had the correct simplification of the exponential expressions. If students were stuck, I heard Nathaniel asking the students questions to help them remember what to do. Instead of telling the students how to simplify the expression, both Nathaniel and I encouraged students to write the expression in expanded form so they could think about what the exponents actually meant (see Figure 5.4). We did not discuss or encourage students to use exponential rules or procedures. Students were also encouraged to ask their partner for help if they needed an idea of how to begin.

$$(3a^4)^2 = (3a^4)(3a^4) = 3 \cdot a \cdot a \cdot a \cdot a \cdot 3 \cdot a \cdot a \cdot a \cdot a = 9a^8$$

Figure 5.4: Nathaniel and I encouraged students to write the warm-up problem in expanded form.

After some time, Nathaniel asked a few students to put their answers up on the whiteboard at the front of the classroom. He asked the students questions about how they knew what to do or to explain the mathematics. Nathaniel encouraged student discourse. Various students and Nathaniel used the phrase “When in doubt, expand it out” as the entire class looked at the key features of the warm-up problems. The fact that so many students were expanding out the warm-up problems demonstrated that Nathaniel had emphasized mathematical understanding in previous lessons.

After the warm-up, each student was given a piece of paper titled 8.5 Division of Exponents Discovery (see Appendix I). Students were asked to expand several expressions and simplify in order to find a pattern with each problem and across several problems. Nathaniel modeled the first problem for them as a way to demonstrate how to simplify by expanding out the expression and crossing out a factor if it appears in the numerator and denominator. Students quickly began “canceling” to simplify (see Figure 5.5). They appeared to understand the initial direction Nathaniel had given them. Since I had previously taught exponents in a similar fashion, I noticed that the students did not know why the cancelling was occurring. When I questioned a couple of students about why the “canceling” works mathematically, each student replied with “I don’t know” or “They just do.” The students did not recognize that having the same factor (number or

variable) in both the numerator and denominator is the same as dividing a number by itself. In other words, if a factor in the numerator is being divided by the same factor in the denominator, the resulting value is one. For example, $\frac{5}{5} = 1$ and $\frac{a}{a} = 1$ (assuming $a \neq 0$).

Expanded Power	Simplified Power
$\frac{5^6}{5^2} = \frac{\cancel{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}}{\cancel{5 \cdot 5}}$	5^4
$\frac{3^7}{3^3} = \frac{\cancel{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}}{\cancel{3 \cdot 3 \cdot 3}}$	3^4
$\frac{a^3}{a^5} = \frac{\cancel{a \cdot a \cdot a}}{\cancel{a \cdot a \cdot a} \cdot a \cdot a}$	$\frac{1}{a^2}$

Figure 5.5: Students began canceling out factors without understanding the mathematics.

I knew that we needed to further explain the canceling. While the students were working, I quickly talked to Nathaniel about emphasizing this point to the class. After the majority of the students had found a pattern in the exponent using the first three examples on the page (that you subtract the exponents), Nathaniel brought the class back together. At this time he used the example of $\frac{5}{5} = 1$ to emphasize the reason why mathematically you can cancel the two factors that are the same. He went back to the first example and showed students that five divided by five is one (see Figure 5.6). After quickly demonstrating one problem with canceling, Nathaniel moved to asking students

what they saw as a pattern. A few students offered explanations of a pattern and Nathaniel wrote down the big idea: When dividing powers with the same base, you subtract the exponents.

$$\frac{5^6}{5^2} = \frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}} \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{\cancel{5} \cdot \cancel{5}} = 1 \cdot 1 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^4$$

Figure 5.6: “Canceling” a common factor in the numerator and denominator results in one.

After writing down a formal rule, students began applying the pattern or exponential rule to guided practice problems. Unfortunately, several of the students were applying the rule incorrectly. They were making the common mistake we had discussed prior to class (see Figure 5.7). Many students were not expanding the expressions and were, instead, attempting to use the rule that states one can subtract the exponents. The common misconception happened when students subtracted the smaller exponent from the larger, not taking into account integers. I again approached Nathaniel and suggested that we do something to get the students’ focus on visualizing the expanded form. Together we brought students’ attention back to the front and emphasized the expanded form. We did not mention subtraction at all. After doing a couple examples with the students and putting a focus back on expanded form (see Figure 5.7), many students began to understand what was really happening when they divided and why the exponents were being subtracted. They began to notice that the exponents were being

subtracted because factors that were the same were being “canceled” or were divided to make ones. Again, making ones with variables assumes that the variables cannot equal zero.

$$\text{Common Mistake: } \frac{p^4 c^6}{p^5 c^2} = p^{5-4} c^{6-2} = pc^4$$

$$\text{Expanded Form: } \frac{p^4 c^6}{p^5 c^2} = \frac{\overset{|}{p} \cdot \overset{|}{p} \cdot \overset{|}{p} \cdot \overset{|}{p} \cdot \overset{|}{c} \cdot \overset{|}{c} \cdot \overset{|}{c} \cdot \overset{|}{c} \cdot \overset{|}{c} \cdot \overset{|}{c}}{\overset{|}{p} \cdot \overset{|}{p} \cdot \overset{|}{p} \cdot \overset{|}{p} \cdot \overset{|}{p} \cdot \overset{|}{c} \cdot \overset{|}{c}} = \frac{c \cdot c \cdot c \cdot c}{p} = \frac{c^4}{p}$$

Figure 5.7: Students made a common mistake when simplifying the exponential expression. When Nathaniel emphasized expanded form, more students simplified correctly.

Nathaniel then gave the entire class an expression involving division properties of exponents. He starting with an expression that he was confident they would simplify correctly (i.e., $\frac{x^5}{x^7}$). Students simplified the expression on their own whiteboards and showed one of the teachers to receive immediate feedback. Nathaniel was very particular about the problems he chose. He picked problems based on how students performed on the previous one. For example, when the class did well on the first problem, he gave them the second problem that would highlight students who were still making the common mistake (i.e., $\frac{y^3}{y^6}$). When the majority of the students correctly simplified that

expression, he chose a problem that combined the first two types of problem (i.e., $\frac{a^3b^6}{a^6b^3}$).

This process was repeated several times, allowing students to recognize patterns (visualizing the expanded form and where there would be more factors) and to gain confidence. I noted that the students' rate of correct simplification increased as time passed. During this time, Nathaniel encouraged students to help one another, provided students with positive reinforcement, and repeatedly asked students why they were cancelling or how they knew the simplified answer was correct.

After the whiteboard problems, I led the Quiz-Quiz-Trade formative assessment. I gave the students directions on how to do the cooperative learning activity. I was sure to specifically address the management issues Nathaniel mentioned when he explained to me what did not go well when he previously tried the activity. As previously explained, each student was given an exponential expression. The students paired up and asked their partner to simplify the expression and explain why that was the correct simplification. For example, when Jorge and Alexis were partners, Jorge asked Alexis to simplify the expression $\frac{n^9}{n^{16}}$. She answered $\frac{1}{n^7}$ because there are nine n's in the numerator and 16 n's in the denominator. If you make "ones" and cancel n's that are in both the numerator and denominator (i.e., $\frac{n}{n} = 1$) there would be seven n's left in the denominator. The students then switched roles and Alexis asked Jorge her question. Once both partners had correctly simplified the expression, they traded their problems and the process was repeated with a different partner.

Nathaniel could have taught students the law of exponents $\left(\frac{a^m}{a^n} = a^{m-n}\right)$ and had them practice applying the rule to various expressions. The students would have practiced basic symbol manipulation with no meaning or understanding. Instead, with my guidance, Nathaniel chose to approach teaching simplifying exponential expressions for conceptual understanding. He used expanded form to help students form meaning behind the mathematics (NCTM, 2009). When we originally discussed teaching the concept, I did not specifically emphasize to Nathaniel what occurred mathematically when “canceling.” Nathaniel did not recognize this important omission from the lesson until I noticed students were lacking full understanding of dividing exponents and brought the deficit to Nathaniel’s attention. With my suggestion, Nathaniel explained the reasoning behind the canceling, connecting the explanation of $\frac{5}{5} = 1$ to prior knowledge of simplifying fractions. Connecting new ideas to students’ prior knowledge is an instructional technique shown to increase students’ understanding (Bruning, 2004).

The inclusion of various formative assessments allowed Nathaniel to gather information on students’ understanding and consequently make instructional decisions (Smith & Stein, 2011). He used information from the warm-up, individual whiteboards, and Quiz-Quiz-Trade activity, to determine the next steps he made in instruction. For example, the students continued to simplify expressions on the whiteboards until the problems were at the rigorous level Nathaniel desired. Another strength of the lesson was the inclusion of math discourse among students. Having students talk about mathematics (what they did, how they did it) is a key aspect of improving mathematics

(Hufferd-Ackles, Fuson, & Sherin, 2004). Nathaniel continuously encouraged students to talk with others, as well as talk about mathematics with him.

Asking students to formalize a rule for dividing exponential expressions could be seen as either a strength or limitation of Nathaniel's lesson. The process of having the students draw conclusions based on evidence, or reason, is a NCTM process standard (NCTM, 2000). Yet at the same time, asking his students to find a pattern and apply the pattern so quickly caused some students to use the rule incorrectly. Another limitation of Nathaniel's lesson was that he did not notice when his students were lacking understanding of canceling. His lack of noticing may have been due to the fact that Nathaniel had not experienced teaching this approach to exponents. Due to their lack of experience, novice teachers often do not notice students' misunderstandings in the classroom as quickly as experienced teachers (Jacobs, Lamb, Philipp, & Schappelle, 2011). He also may have not noticed because he was engaged in other aspects of the teaching process. Or it may have been his understanding of the mathematics caused him to overlook the issue.

Overall, Nathaniel kept a strong focus on mathematical conversations as a way to increase student understanding while teaching the lessons (Boaler & Humphreys, 2005). Talking about math was encouraged among students, as well as between teacher and student. He asked students a lot of why questions and he encouraged the students to explain their thinking behind the mathematics they were doing. The activities that occurred in his class, especially during the chapter on exponents, frequently asked students to look for patterns as a way to introduce a concept. Students would complete a specific problem and then look for a pattern. He would then help reinforce the pattern

with a rule, formula, or specific procedure. I rarely observed these characteristics (i.e., mathematical discourse, student discovery, questioning why) in other teachers' math lessons, but these qualities of mathematics teaching did remind me of the shift I was making in my own teaching during the last couple of years I taught my own Algebra students.

If Nathaniel was in fact interested in improving his understanding of teaching mathematical concepts, what could I do as a coach to facilitate that learning? What would need to change about the way I approached coaching? How would coaching for deeper understanding of mathematics look? How would I know if he was learning? Would Ball, Hoover, and Phelps' (2008) ideas about mathematical tasks for teaching help me as a researcher better understand Nathaniel's knowledge of math and teaching? What would be the benefit and limitations to changing the focus of our coaching?

Debrief as reflection. Since Nathaniel had a plan period immediately following his Algebra Block class, we had gotten in the habit of moving directly to the teachers' planning center after class to debrief. Nathaniel usually blocked off the entire period to talk with me and this day was no different. We discussed all of the ways we adjusted the lesson prior to class and during class, and how those decisions impacted student understanding. In advance of the class, Nathaniel and I decided to emphasize writing exponential expressions in expanded form. By expanding expressions and focusing on the meaning of simplify, his students correctly simplified expressions. We also adjusted the lesson in-the-moment twice in order to incorporate more of an emphasis on visualizing the mathematics in order to increase understanding. Reflecting upon this decision, we both agreed that these refocusing moments were critical in building

students' conceptual learning. During our third period conversation, Nathaniel and I also decided to modify his original lesson plan to include individual white board problems after the discovery form and before the cooperative learning activity. My rationale for adding the white board formative assessment was to give us feedback on student understanding, as well as an opportunity to address student errors. Nathaniel commented that the individual white board guided practice activity was essential for gaining information about student understanding. The whiteboard activity provided him with a good picture of where his students were in terms of simplifying expressions of varying difficulty.

To guide our reflection I asked questions such as: How many students do you think could simplify with no error? Do you think the whiteboard activity was helpful? Do students understand why the numbers “cancel out”? Did the students behave better during the Quiz-Quiz-Trade activity than previously? Nathaniel contemplated the questions and responded that he thought approximately 85% of his students could simplify with no errors. He thought the whiteboard activity provided him with valuable information, but he was not convinced that his students fully understood “canceling” was essentially dividing a number by itself. Nathaniel also commented that his students did respond better to my directions in the cooperative learning activity than when he had facilitated the same activity in the past. I found the conversation to be comfortable and relaxed. I did not feel as though I was pulling answers out of him or forcing a discussion.

I also questioned Nathaniel and made suggestions about how to increase the effectiveness of the lesson. In particular, I was concerned about the pressure of formally stating a pattern or exponent rule. Students seemed to begin making mistakes once a rule

was established and they demonstrated that they thought they no longer needed to think about the meaning behind the expression. I asked Nathaniel what he thought about letting students realize this pattern in their own time and if he thought that would help with their understanding. He agreed with the idea and said he would be sure to read his lesson plan note about leaving the big idea box off the discovery form next year.

We then talked about what he could do in class the next day. He wanted us to discuss how to continue teaching operations with exponential expressions based on how the lesson went that day. We talked about key ways to approach specific concepts to encourage students' understanding of the meaning behind what they were doing rather than just the procedure. Although the conversation was not focused on specific lessons, it was evident that Nathaniel appreciated talking through teaching ideas with someone else. He wrote down everything we discussed and continually offered more ideas or questions to continue the collaborative brainstorming. To finish the post conference debrief, we quickly sketched out the pacing for the remainder of the chapter on exponents.

Nathaniel's dedication to reflecting on his teaching was above and beyond any other teachers'. He cleared everything else from his schedule and consistently made our debriefing time a priority within his schedule. On the rare instance when we were not able to debrief after class, he would email me his ideas for moving forward. During the debrief sessions, Nathaniel was very willing to admit weaknesses and his lack of confidence, but he was not as willing to stress what went well. He tended to focus our debriefing conversations on classroom management issues. When we talked about the mathematics, he asked for ways to teach the concept for the next day of instruction (even

though I would not be in the class to observe the lesson) and would often times make notes about changes he wanted to make in his math instruction for the next school year.

The discussions Nathaniel and I had during the debriefing component of our coaching cycles caused me to think about how coaching could better provide continuous support to Nathaniel. He often used our debriefing conversation to begin planning for his next lesson. What would be the benefit of our coaching extending into the next day as well? Or what would be the benefits of working with Nathaniel for several consecutive days? How could I help us reduce the amount of time we spent discussing classroom management?

So, why did I choose Nathaniel to be the participant in my study? He was highly interested in learning and improving his teaching. He fully engaged in the planning and debriefing components of the coaching cycle, making time to fully participate in the coaching process and often requesting extended time. Our lesson planning was more focused on mathematics and how to teach concepts, whereas my planning with other teachers largely centered on formative assessments. Nathaniel's teaching already emphasized mathematical conversations and discovery tasks, and he was already a very reflective teacher continually looking for ways to improve himself. Rather than viewing the coaching process as a series of hoops, he saw it as a way to support his continuous desire to become better. And most importantly, Nathaniel identified himself as a learner (Initial Interview, January 28, 2011). He was eager to learn how to improve his instruction and ultimately his students' understanding of Algebra.

Changing my Approach to Coaching

Although my interactions with Nathaniel were focused more on mathematics than other teachers I coached, I sensed that our coaching still did not place enough emphasis on how to teach mathematics to students for understanding. I was trying to fix Nathaniel's lessons, his instruction, and his classroom management on a weekly basis rather than allowing him to discover and learn these skills on his own. I was expecting Nathaniel to imitate my instructional strategies rather than think about his own instruction. I viewed him as someone who needed me to tell him how to be a good teacher. I was not approaching Nathaniel as a learner. Viewing Nathaniel as a learner would have meant I helped him construct his own knowledge of teaching mathematics rather than tell him how to teach. I would have taken more time to understand what he did know about teaching to inform what we focused on during our coaching interactions.

Nathaniel also noticed that the coaching structure, specifically when and how we planned together, was limiting our coaching process. He commented numerous times during our first one and a half years working together that he wanted more time to plan together. Nathaniel asked if we could plan a few days in advance or even after school the day before the lesson. On a few special occasions I arranged my schedule so I could be at his school two days in a row, allowing us to plan the first day and then execute the lesson the second. When we planned a day in advance, Nathaniel seemed more at ease and confident with our lesson plans and it appeared as though our time together positively impacted his instruction. Therefore, when I was assigned to Washington High School full-time in January 2011, both Nathaniel and I were anxious to have more discussions about mathematics and instruction the day prior to teaching a lesson.

Concepts and the Big Idea

Based on the patterns in my initial coaching of Nathaniel, simply planning the day prior to teaching a lesson was not enough. I also needed to change the focus of my coaching. As I was thinking about a coaching focus, I was reminded of the event Nathaniel highlighted as a significant coaching moment. Before he taught slope, we met for about three hours and collaboratively created a few math lessons that helped students build understanding of the concept. In an interview on January 28, 2011, Nathaniel described his thoughts after this noteworthy planning session.

After I got done working with that (planning session for slope) I thought, “This is what planning should be. This is what planning is.” We were just bouncing ideas back and forth and trying to get better things. So for me when we got done with that session, I was like, ‘This is what I want to do when I plan.’ (Initial Interview, January 28, 2011)

He went on to explain, “I want to teach the concepts and the big idea. I really felt like that is what we were getting at when we were planning together.” Nathaniel’s reflection on this planning session emphasized collaboration and planning how to help students learn concepts rather than individual skills.

When I asked Nathaniel what he would like to work on during the 2011 spring semester with regard to struggling students’ learning, he answered:

I think it goes back to how can we make different kinds of connections to make it really holistic for them so it isn’t just some random abstract ideas. How can we make it more just one whole picture? And they can see the web connects a lot of

places. I really think if we can make connections, I think they can really learn better.

It is evident in this response, as well as his reflection upon our intense planning session for slope, that Nathaniel was primed and ready to focus our coaching cycles on reasoning and sense making. He did not specifically name reasoning and sense making, but that was essentially what he was talking about. He wanted our coaching to be about bouncing ideas back and forth, teaching concepts and big ideas, and forming connections to increase student understanding (Initial Interview, January 28, 2011).

Least Confident in Quadratics

During our formal interview on January 28, 2011, I asked Nathaniel which mathematical topic he would like to work on during the Spring 2011 semester.

Amy: If I could help you anywhere this semester, what mathematical topic do you think would be most beneficial?

Nathaniel: I think quadratics is probably it. Out of everything I have a comfort with I would say that is my least experienced area. I know they do a lot of that in Advanced Algebra and I guess become more comfortable with that so I can become a better teacher at it would probably be good.

Amy: What part of quadratics?

Nathaniel: I guess I understand the relationships in terms of formulas and stuff but just being able to teach those concepts better. Like how we can get students to discover those patterns for themselves and how they can maintain all of that in their repertoire of graphing and solving different things. Being able to apply it.

Nathaniel identified quadratics as the concept he was least confident in. He wanted to find a way to help students better understand the concepts, discover patterns for themselves, connect graphing and solving, and apply the new knowledge.

Interestingly, prior to Nathaniel requesting to focus on quadratics, I was thinking to myself that quadratics would be a good concept to focus our intense coaching on. First, students tend to struggle with the quadratics content. The problems are more time-consuming due to the number of steps and require mastery of numerous skills such as factoring, solving equations, evaluating expressions, and plotting points on a coordinate plane. Quadratics is taught during the fourth quarter when student motivation is rapidly declining. Students are expected to compare their new knowledge of quadratic functions with other functions previously learned (i.e., linear, exponential), which has also proved difficult for students in the past. Secondly, I thought quadratics would be an excellent concept to focus on because my experiences with quadratics had been largely procedural. I, as well as many teachers I had coached, struggled to make mathematical connections while teaching students how to graph and solve quadratics. The process for graphing quadratics was largely introduced as a set of procedures, with the first step being find the vertex using the formula $x = -\frac{b}{2a}$. Solving quadratics was taught using three methods, but each method was often taught in isolation. I felt like learning to teach quadratics with reasoning and sense making would be an interesting and enlightening task that both Nathaniel and I could learn from. I anticipated that teaching quadratics with reasoning and sense making would help us teach students the concept instead of just the procedure and would allow us to make more connections among the various skills.

Moving Forward using Baseline Data

Moving forward, I used the baseline information on Nathaniel's teaching and our coaching. Changing my coaching approach to consecutive days grew out of Nathaniel's desire to discuss the mathematics and instruction the day before the lessons. He was interested in engaging in more in-depth conversations about teaching mathematics for understanding, which was evident in many of our coaching cycles and obvious when we worked collaboratively in November 2009 as a way to introduce slope. And teaching quadratics was a concept Nathaniel self-identified as not being confident in. Thus, using consecutive coaching focused on teaching quadratics with reasoning and sense making served me as a coach trying to implement effective coaching strategies, and served me as a researcher trying to answer bigger questions about coaching and teaching mathematics. Specifically, as a coach I would be able to support Nathaniel in an area he is not confident with and use what I learn from the study in my practice. As a researcher I would be gaining a greater understanding of the coaching process and teacher learning through my practice.

I spent time during the first part of Spring 2011 analyzing our coaching, looking for patterns in his teaching and our interactions as a way to gather baseline data for my research. Yet as I engaged in this process I began to realize something very important as a practitioner. I began to recognize that Nathaniel himself was a learner. He was continually working to improve himself and his instruction. He asked questions and was not afraid to try new instructional strategies. Nathaniel wanted to learn how to be a highly effective teacher. Recognizing Nathaniel as a learner caused a significant shift in

both my coaching perspective and research perspective as I entered into the quadratics phase of data collection.

CHAPTER 6: TEACHER LEARNING IN THE CONTEXT OF COACHING

What does an Algebra teacher learn about teaching reasoning and sense making of quadratics while working with an Algebra coach? How does an Algebra teacher learn about teaching reasoning and sense making of quadratics while working with an Algebra coach? These are the questions being used to further investigate teacher learning in the context of coaching. The chapter has been divided into three parts to better analyze what Nathaniel learned during the intense coaching. Part I discusses how Nathaniel and I mapped the terrain of quadratics and how this process increased his understanding of the mathematical content. In Part II, a detailed explanation of how Nathaniel taught quadratics and an analysis of what he learned about teaching mathematics is offered. And finally, in Part III, I analyze what Nathaniel learned about reasoning through teaching dilemmas as he mapped the terrain and taught quadratics.

Part I: Mapping the Terrain to Increase Understanding of Quadratics

Since the beginning of the second semester, both Nathaniel and I knew that we would be engaging in this new, intense coaching when he taught quadratics. We continued with our traditional coaching cycles January through March, and on Monday, April 4, 2011 Nathaniel told me that he was ready to start planning for quadratics whenever I was. His mannerisms conveyed excitement, almost as if he had been waiting all semester for the chapter on quadratics (Field Notes, April 4, 2011). I honestly did not know exactly how we were going to teach the concept for reasoning and sense making. My own previous experiences teaching quadratics were largely procedural and focused

on the isolated skills involved in quadratics. In the past, the textbook dictated my own instruction as I followed section by section.

I started questioning what exactly was meant when we said, “teach quadratics.” What about quadratics needed to be taught? How would we go about teaching quadratics? In other words, what did the mathematical terrain of quadratics look like? Depending on where I looked (i.e., textbook, syllabus, assessments, colleagues), the map of the terrain was slightly different. In her study, Lampert (2001) recognized that the connections involved in learning mathematics were not sequential and she constructed her own map of the mathematical terrain. I recognized that Nathaniel and I needed to do a similar mapping.

Creating a Map of the Quadratics Terrain

Nathaniel and I figuratively created a map of the quadratics terrain. The first step was defining the context in which we wanted to teach quadratics by investigating the Common Core State Standards for Mathematics (CCSSM), the National Council of Teachers of Mathematics’ (NCTM) standards, and literature on reasoning and sense making. Nathaniel and I then began to form our own map of the quadratics terrain by studying the textbook and course syllabus. Feeling uncertain about the map provided by the curriculum resources, we turned to our colleagues to discuss different options. And using all of the information collected, Nathaniel and I had in depth discussions about the concept of quadratics and how to approach teaching quadratics for understanding.

My purpose in mapping the quadratics terrain with Nathaniel was two-fold. First, Nathaniel and I both wanted to teach quadratics for understanding. We wanted students to be able to not only be proficient in the procedures, but to also conceptually understand

the mathematical ideas within the concept of quadratics. Therefore, Nathaniel and I needed to think deeply about what he would teach and how. Our understanding of quadratics and the mathematics embedded in the concept needed to be strengthened and I thought engaging in this mapping process would help increase Nathaniel's knowledge of quadratics. Secondly, I hoped mapping the quadratics terrain would help Nathaniel learn more about the role the textbook and other curricular resources played in teaching. Through the mapping process, Nathaniel gained a greater understanding of quadratics, as well as the role curriculum plays in teaching mathematics with reasoning and sense making.

Identifying the Foundation

To start our discussion about what about quadratics should be taught and how it should be taught, Nathaniel and I first needed a better understanding of the foundational ideas behind the concept of quadratics. I felt that we needed to begin by identifying the standards related to quadratics. The CCSSM provided Nathaniel and I with information about teaching quadratics. Searching through the mathematics core standards, four of the 27 Algebra standards directly relate to quadratics. Other standards may apply to quadratics in a more generic sense. My interpretations of the standards that relate to quadratics began to define the quadratics terrain (see Figure 6.1).

The National Council of Teachers of Mathematics (2000) offer another broad look at the quadratics terrain, similar but different from CCSSM's standards framework. Algebra is one of NCTM's five mathematical standards for K-12 students. The Algebra standard is applied across all grade levels and does not singularly apply to a high school Algebra course. Upon investigating the Algebra standard as it applies to grades 9-12,

Seeing Structure in Expressions

A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- a. Factor a quadratic expression to reveal the zeros of the function it defines. (CCSSO, 2010, p.64)

Reasoning with Equations and Inequalities

A-REI.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.4. Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b . (CCSSO, 2010, p.65)

Arithmetic with Polynomials and Rational Functions

A-APR.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (CCSSO, 2010, p. 64)

Figure 6.1: CCSSM that apply to the mathematical concept of quadratics.

several NCTM expectations are related to the teaching of quadratics in a first year Algebra course (see Figure 6.2). Although other NCTM Algebra expectations may also be connected to the teaching of quadratics, the two general categories of functions and algebraic symbols contain the six most applicable expectations.

Examining the CCSSM and NCTM standards related to quadratics opened our eyes to the big picture of quadratics. Nathaniel and I identified and discussed the standards related to quadratics and how those standards could be incorporated into instruction. Individual standards and how they applied to our specific map of the quadratics terrain are described in detail in Part II.

NCTM Algebra Standard

Represent and analyze mathematical situations and structures using algebraic symbols

- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency – mentally or with paper and pencil in simple cases and using technology in all cases

Understand patterns, relations, and functions

- Generalize patterns using explicitly defined and recursively defined functions
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions
- Interpret representations of functions of two variables (NCTM, 2000, p. 296)

Figure 6.2: NCTM Algebra Standards that apply to the mathematical concept of quadratics.

In addition to the standards, Nathaniel and I focused much of our initial attention on teaching quadratics with reasoning and sense making. The authors of Focus in High School Mathematics: Reasoning and Sense Making (NCTM, 2009) wrote:

A focus on reasoning and sense making, when developed in the context of important content, will ensure that students can accurately carry out mathematical procedures, understand why those procedures work, and know how they might be used and their results interpreted. (p. 3)

Teaching with reasoning and sense making helps lay a foundation on which students can build an understanding of concepts, as well as knowledge of procedures. Students currently struggle to learn mathematics because they find it meaningless. By teaching mathematics with thinking and analysis, those struggling students suddenly experience

mathematics for themselves rather than merely observing the teacher doing mathematics (NCTM, 2009).

As a broad definition, reasoning is “the process of drawing conclusions on the basis of evidence or stated assumptions” (NCTM, 2009, p. 4). Reasoning can come in the form of formal proofs, but usually begins with exploring a mathematical idea and creating conjectures. Nathaniel and I largely incorporated reasoning into the instruction by informal means such as exploration and pattern recognition. Sense making is defined as “developing understanding of a situation, context, or concept by connecting it with existing knowledge” (NCTM, 2009, p. 4). In our work, connecting newly learned concepts to prior knowledge incorporated sense making. The prior knowledge could have been knowledge gained earlier in the course or in previous mathematics classes. Reasoning and sense making are essentially the core of the NCTM Process Standards, which include Problem Solving, Reasoning and Proof, Connections, Communication, and Representation (NCTM, 2000).

Reasoning and sense making are the terms Nathaniel and I tended to use most often. Yet other phrases such as conceptual understanding and adaptive reasoning could be used to describe the way we hoped to teach quadratics. These phrases are two of the five strands of mathematical proficiency as highlighted by Kilpatrick et al. (2001). Conceptual understanding refers to students’ ability to comprehend, integrate, and relate mathematical ideas (Kilpatrick et al., 2001). Adaptive reasoning is a student’s ability to think logically about conceptual relationships (Kilpatrick et al., 2001). Both components of mathematical proficiency are closely related to reasoning and sense making.

As a visual reminder of our goal to teach quadratics for reasoning and sense making, I posted a half sheet above Nathaniel's desk and above my desk with the definitions of Algebra reasoning and sense making (see Figure 6.3) as defined by NCTM (2009). I chose to post the definitions above Nathaniel's desk since that was the area we usually discussed the lessons, and above my desk since that was where I usually sat to reflect on the lesson and coaching. We found ourselves looking at the definitions or pointing to them as we discussed ways to incorporate more reasoning and sense making into our lesson plans. I personally needed the reminder to help me remember our goal as we discussed how to teach quadratics. Since Nathaniel and I were both trying to change our practice and teach quadratics in a new way, the visual reminder was helpful.

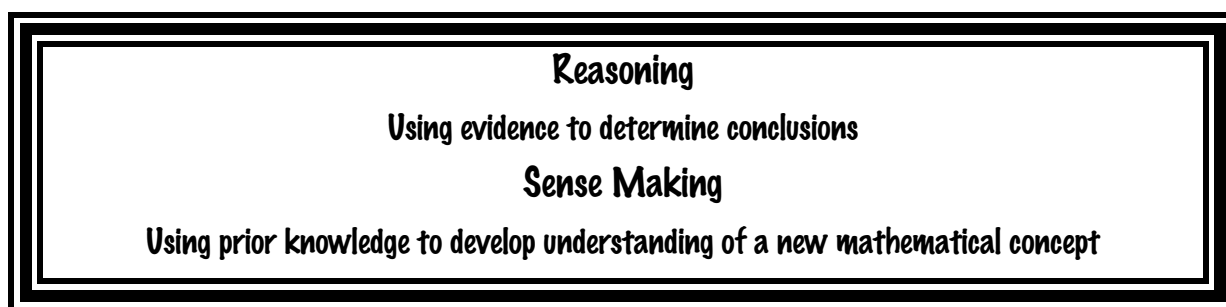


Figure 6.3: The definitions of reasoning and sense making, which Nathaniel and I both posted above our desks.

As a way to gather another mathematician and curriculum developer's perspective on incorporating reasoning and sense making into teaching quadratics, I met with one of my mathematics professors on Monday, April 4 to discuss quadratics and teaching mathematics for understanding. Our conversation prompted me to think about how

Nathaniel and I would approach quadratics. A few of the ideas I wrote in my personal journal after our discussion included:

A student should be prompted to want to learn how to solve quadratics... We (Nathaniel and I) need to find a way to help students make sense of the x-intercepts being the solutions... Graphing is a method to solve quadratics... Teaching quadratics is not about teaching exactly what the book tells you. (Personal Journal, April 4, 2011)

As I reflected on our conversation, I tried to put these thoughts into the context of teaching quadratics. Getting students to want to learn new mathematics, or helping them build a need for new mathematics, are underlying components of thinking mathematically. When students are given a new mathematical situation, they use their existing knowledge to reason through and brainstorm ideas about how to approach the new problem. By asking students questions such as “What’s going on here?” and “Why do you think that?” (NCTM, 2009), students begin to realize that they need to learn some new mathematics. The students can then determine that their prior mathematical knowledge is not enough and they decide more mathematics is needed.

The connections between algebraic and graphical representations were important pieces that I felt needed to be emphasized if we were going to teach quadratics with reasoning and sense making. In our Algebra textbook, and in the conversations I had with Nathaniel, the solutions of a quadratic function referred to the real numbers x , such that $f(x) = 0$. When discussing solutions throughout this entire research study, we continually thought of the x-intercepts as the solution to the function. Yet in other contexts, the solution to a function may be referring to all x 's such that $f(x) = 2$. The

solutions in this case would not be the x-intercepts, but would instead refer to the points on the graph where the function intercepts the graph of $y = 2$. Nathaniel and I understood that quadratic “solutions” had a broader meaning beyond the x-intercepts. Throughout all of our discussions and lessons, we assumed we were finding the values of x such that $f(x) = 0$ when we used term solution.

Getting students to realize the solutions of a function are the x-intercepts (as Nathaniel and I defined them) and that graphing is a method of solving were tied to the key elements of algebraic reasoning and sense making. I had posted a paraphrased list of these elements (Graham, Cuoco, & Zimmerman, 2010), which come in two parts, by Nathaniel’s and my desk as well (see Appendix J). The first part is reasoning and sense making with algebraic symbols. By thinking of graphing as another way to solve quadratics, we would be representing geometric situations (or the graph) algebraically (Graham et al., 2010). The students would see graphing (geometry) and solving equations (algebra) both as viable ways to find a solution (see Figure 6.4). The second part of reasoning and sense making in algebra focuses on functions, with one key element being the use of multiple representations. The idea that x-intercepts are solutions to a quadratic equation can only be deeply understood when functions are represented both graphically and symbolically. For quadratics, the two branches of algebraic reasoning and sense making (i.e., algebraic symbols and functions) begin to blur, especially in the instance in which connections are being made between solving quadratic equations and graphing quadratic functions.

NCTM and CCSSM offer an expansive, larger perspective of the quadratics terrain, with many of the individual standards incorporating mathematical skills beyond

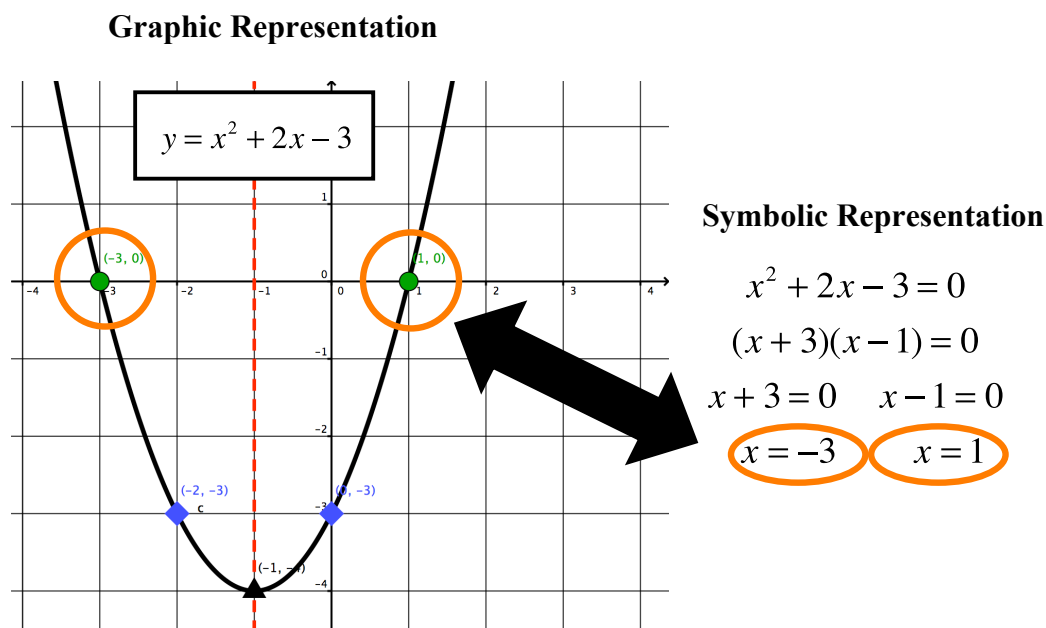


Figure 6.4: The solutions to a quadratic equation, as we defined them, could be show graphically and algebraically.

the narrow scope of quadratics. Both sets of standards continue to generally categorize quadratics into the two big ideas of algebraic symbols (equations) and algebraic functions. I would not consider these standards or NCTM's explanations of reasoning and sense making (2000), to be the map of the quadratics terrain. The standards do not specifically explain what about quadratics should be taught. Instead the information thus far served as a general description of the big mathematical ideas Nathaniel and I needed to include when teaching the topic of quadratics.

Using Curricular Resources to Begin Mapping

By determining the common core standards and NCTM's standards related to quadratics, defining reasoning and sense making, and talking to another mathematician about the algebraic concepts, Nathaniel and I were able to gain necessary background

knowledge. The discussion I had with my mathematics professor reminded us to not let the curriculum dictate what or how we taught quadratics. He meant that we should teach quadratics based on our discussions and ideas rather than simply follow the order of the textbook. Yet mathematics instruction has a long tradition of being driven by the textbook, which can partly be attributed to society's expectations and teachers' level of content knowledge (Remillard, 2005). Although Nathaniel and I were determined to not allow the textbook to control our instruction, Nathaniel and I did initially seek out the curriculum in the textbook to gain ideas and see the mathematical objectives included in the quadratics chapter.

In the most basic sense, our textbook served as the initial map of the mathematical terrain. The textbook was the reference Nathaniel and I used as we began deciding what to teach. For Chapter 10, the sections in our textbook were laid out as follows:

10-1 Exploring Quadratic Graphs

10-2 Quadratic Functions

10-3 Solving Quadratic Equations (by square rooting)

10-4 Factoring to Solve Quadratic Equations

10-5 Completing the Square

10-6 Using the Quadratic Formula

10-7 Using the Discriminant

10-8 Choosing Linear, Quadratic, or Exponential Model

We were not required by the district to teach sections 10-5 (completing the square) or 10-7 (using the discriminant) in first-year Algebra, and were therefore left with six other sections to cover. Whether the six skills were meant to be taught by a teacher in a linear

fashion or not, the sequential organization of the textbook implies the objectives are to be taught in that order. Mapping the terrain solely based on the textbook would be linear (see Figure 6.5). Each section would introduce a different skill or objective, with little to no connection between the concepts.

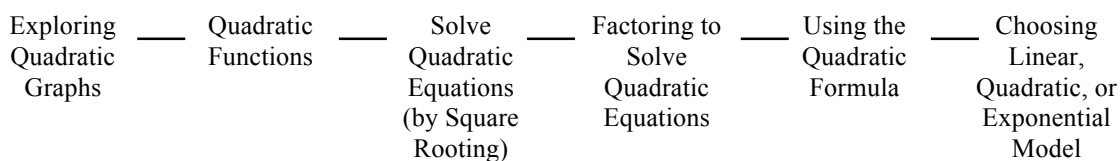


Figure 6.5: Using the textbook, the map of the quadratics terrain appeared linear.

In addition to the textbook, Nathaniel and I used the district's Algebra syllabus to help define what about quadratics should be taught (see Figure 6.6). A syllabus had been created by the district curriculum department for every course taught in middle school and high school across the district. Creating the Algebra syllabus, with the guidance and feedback from teachers, was one of my responsibilities as an instructional coach. The skills included on the syllabus were chosen based on state standards and assessments, which was noted in the last column. The first column on the syllabus listed the skills to be taught and the middle column was the corresponding section in the textbook.

The Algebra syllabus included sections from the textbook, but with a few adjustments. First of all, the skills within the chapter were written as objectives rather than titles for each textbook section. Also, section 3-8 (find and estimate square roots) was added to the syllabus as a pre-requisite skill that should be taught prior to 10-3 (solve quadratic equations using square rooting). Finally, the syllabus grouped related skills

EXPLORE AND GRAPH QUADRATIC FUNCTIONS		
*Explore Different Quadratic Graphs	10-1	[SS12.3.1a, SS12.3.1b, SS12.3.1e] DK2 & DK3
*Graph Quadratic Functions	10-2	[SS12.3.1a, SS12.3.1b, SS12.3.1e] DK2 & DK3
SOLVE QUADRATIC EQUATIONS		
Pre-requisite Skill: Find and Estimate Square Roots	3-8	[SS12.1.1a]
Solve Quadratic Equations	10-3	[SS12.3.2d, SS12.3.3i]
Use Factoring to Solve Quadratic Equations	10-4	[SS12.3.1e, SS12.3.2d, SS12.3.3i]
Use the Quadratic Formula to Solve Quadratic	10-6	[SS12.3.1e, SS12.3.2d, SS12.3.3i]
MODEL LINEAR, QUADRATIC, AND EXPONENTIAL FUNCTIONS		
*Determine type of function given a graph or table	10-8	[SS12.3.1d, SS12.3.2c, SS12.3.2d] DK2 & DK3
Write a function to model data	10-8	[SS12.3.2a, SS12.3.2c, SS12.3.2d]

Figure 6.6: The section of school district's semester two Algebra syllabus that applied to quadratics.

together to form big ideas within the chapter. The 2010-2011 school year was the first year big ideas were included on the course syllabi. As a curriculum department, our goal for grouping objectives into big ideas was to encourage teachers to teach concepts (big ideas) rather than individual skills (objectives or sections in the textbook). A map of the quadratics terrain based on the syllabus grouped several skills together into big ideas, yet the vertical listing of big ideas on the syllabus still suggested a linear progression (see Figure 6.7).

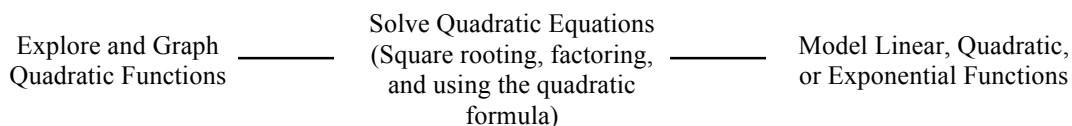


Figure 6.7: A map of the quadratics terrain using the school district's Algebra syllabus.

The textbook and syllabus provided a starting point as Nathaniel and I thought about how to teach quadratics. The reality of our situation was that the chapter summative assessments would include the big ideas and objectives included on the

syllabus. Our major resource, as well as the students' main resource, was the textbook. Although Nathaniel and I used both the textbook and the syllabus to begin our discussion about teaching quadratics, we also believed there was something more to the map. We did not believe quadratics was best learned in a linear way, as suggested by the textbook and syllabus. Nathaniel and I then turned to our colleagues and one another to determine how to map the terrain of quadratics with reasoning and sense making. We were looking for a way to teach quadratics for understanding as opposed to following the sequencing of the textbook.

Conversations with Colleagues to Brainstorm Ideas

On Tuesday, April 5, 2011 the Algebra professional learning community (PLC) at Washington High School met for our weekly hour planning. The PLC included six Algebra teachers, one student teacher, and me. We gathered around two tables on the far side of the classroom near a whiteboard. The first 50 minutes of our time together was spent on department announcements and creating a common rubric for the chapter test our students would be taking on Thursday. We had a lot of discussion about what constitutes a computational mistake and what constitutes a conceptual mistake. Our conversation got at the heart of the textbook's chapter 9 big ideas: Perform operations with polynomials and Factor polynomials.

We finished discussing the rubric with about ten minutes left in our PLC. Julie, one of the Algebra teachers, asked if we could quickly discuss the textbook's chapter 10. A few teachers opened their textbooks to section 10-1: Exploring Quadratic Graphs, while a couple others opened lesson plan books from past years. Since the first two

sections in the book focused on graphing quadratics, our discussion started with graphing. We instinctively used the curriculum to direct our discussion.

Within the PLC, the teachers started comparing how they taught students to set up their table of values for graphing, discussing if each person expects students to compute five or three solutions in the table. A few teachers thought students should calculate all five of the y -values as a way for checking computation. (If a student did not get the same y -value for the reflected point, he would know one of his calculations was incorrect.) Other teachers thought that if a student understood that quadratic functions reflect in both the graph and the table, he could simply compute three y -values (including the vertex) and reflect the values over the vertex (see Figure 6.8). Nathaniel remained silent.

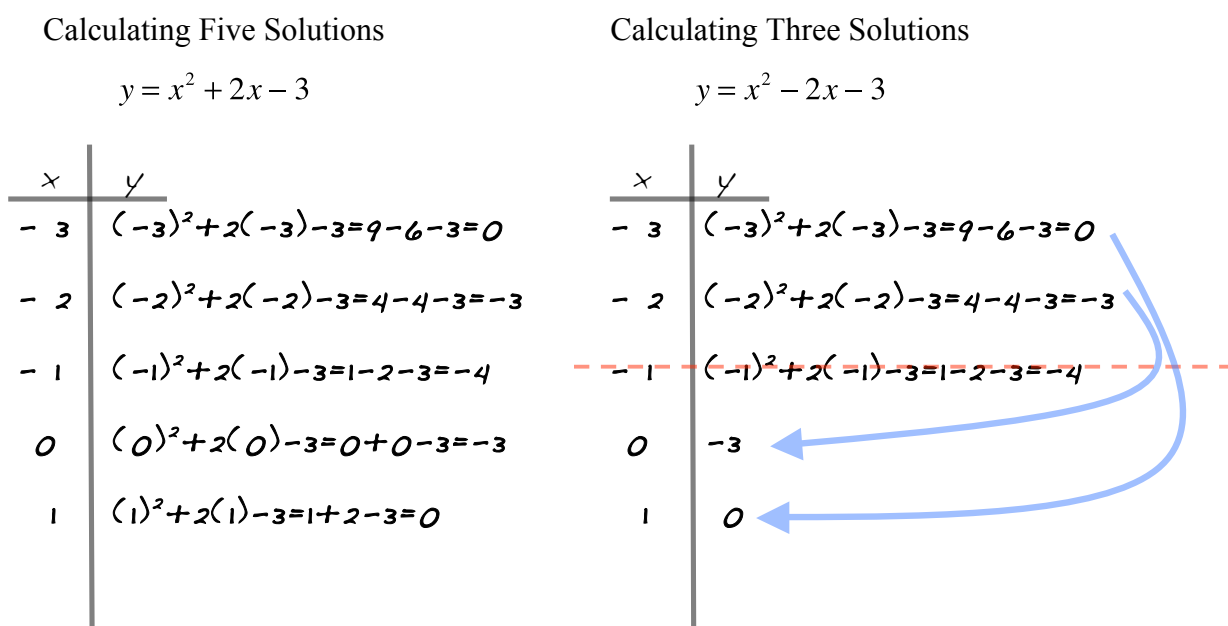


Figure 6.8: Some of the Algebra teachers taught students to calculate five solutions when graphing a quadratic functions, while other teachers taught their students to calculate three solution in a table of values.

Then someone asked how everyone else taught the formula for finding the x-value of the vertex $\left(x = -\frac{b}{2a}\right)$. The question was asked how to mathematically explain to students the origin of the formula $x = -\frac{b}{2a}$. In my field notes I wrote about what happened in the PLC when the question was raised.

We talked about how to teach the meaning behind finding the vertex using $-\frac{b}{2a}$.

All of us seemed to just fumble around, not knowing the best way to explain

where $-\frac{b}{2a}$ comes from. We all agreed that showing students that the x-value of

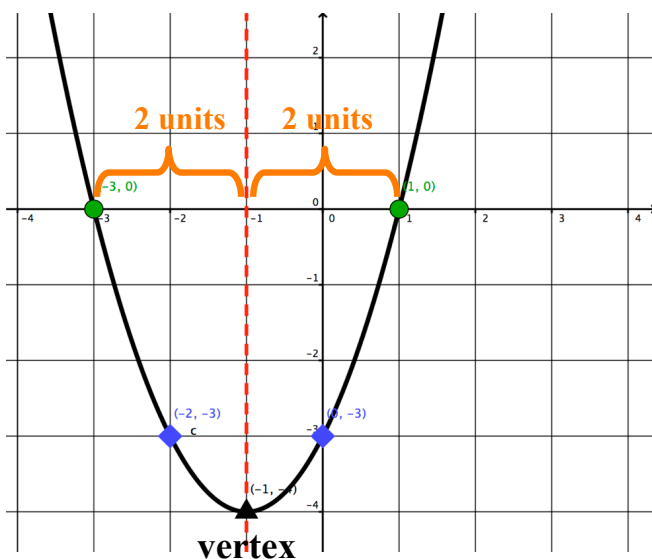
the vertex is the midpoint between the two roots was important. But we all

struggled to explain how to get students to understand why it was $-\frac{b}{2a}$. We

talked about doing several examples of finding the midpoint and tying it to the original function. Yet even with that no one seemed convinced they could fully

explain where $-\frac{b}{2a}$ comes from. (Field Notes, April 4, 2011)

As a PLC, we agreed that one way to begin to understand the vertex formula was by showing that the vertex was directly in the middle of the two roots/solutions/zeros on a graph (see Figure 6.9). And the mean of the two roots produces the x-value of the vertex, since one is finding the middle of the two numbers. We agreed that this approach would give insight into why the formula includes dividing by two. Yet as a PLC, we could not figure out how to explain $-\frac{b}{2a}$ in the vertex formula.



Find the mean.

$$x = \frac{-3 + 1}{2} = \frac{-2}{2} = -1$$

Vertex : $(-1, -4)$

Figure 6.9: As a PLC, we agreed that the vertex is in the middle of the roots and the x-value of the vertex could be found by finding the mean of the two roots.

A couple teachers did bring up the fact that the vertex formula could be found in the quadratic formula if you remove the discriminant $(\sqrt{b^2 - 4ac})$ (see Figure 6.10). Yet students did not yet know the quadratic formula or the origin of the quadratic formula since it was not introduced in the textbook until section 10-6. Therefore that reasoning would not work either. Besides, we did not know how to help students understand why they would remove the discriminant.

$$\begin{array}{rcl} \text{Quadratic Formula} & - & \text{Discriminant} & = & \text{Vertex Formula} \\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & - & \sqrt{b^2 - 4ac} & = & \frac{-b}{2a} \end{array}$$

Figure 6.10: Teachers recognized that the vertex formula was the quadratic formula with the discriminant removed.

At this point we were reaching the end of our PLC time. As teachers began collecting their materials to leave, I quickly told the group that I had been thinking more about using graphing as another method to solve a quadratic equation. We had all taught students that the x-intercepts on a graph, or what are also called the zeros, were the solutions to the equation. But I personally had not made that transition to thinking of graphing as a fourth method of solving. A few teachers nodded in agreement, but we did not have time to discuss this idea any further.

After leaving the PLC I wrote in my personal journal, “I felt like we all left that meeting with little direction or confidence in teaching the graphs of quadratics” (Personal Journal, April 5, 2011). There were more questions and greater confusion than when the quadratics conversation began. I personally was feeling even more anxious and panicked about how Nathaniel and I would teach quadratics with reasoning and sense making. How could we teach students the vertex formula with reasoning? I noted that Nathaniel was particularly quiet during the PLC discussion about explaining $x = -\frac{b}{2a}$. He appeared intently involved with following the conversation, but he did not say much (Field Notes, April 5, 2011). This was typical behavior for Nathaniel in PLC. Nathaniel and I had scheduled to meet the next day (Wednesday, April 6) during his 8th plan period. I was hoping that either Nathaniel had some great idea he did not share with the PLC, or that I came up with a way to explain $x = -\frac{b}{2a}$ to students before 1:30 the next day when we were schedule to meet.

That evening I thought about nothing but teaching quadratics. What were the various methods used to solve quadratic equations? How could we use graphing as a

method to solve equations? What could we do to connect solutions to the roots of the parabola? How do we explain $x = -\frac{b}{2a}$? If our goals were to help students use evidence to draw conclusions (reasoning) and use prior knowledge to build new understanding (sense making), how would we even begin to teach quadratics when the first section is graphing? It was at that moment I first started questioning the map of the quadratics terrain created by our textbook and syllabus. That is when I had an idea. Who said we had to start with graphing just because the first section in the textbook is graphing? If graphing is viewed as another method to solve a quadratic equation, why could we not teach that method last? Although I did not believe changing the order of the big ideas would help explain $-\frac{b}{2a}$, I was intrigued with the idea of solving quadratic equations algebraically prior to graphing.

I thought there had to be a flaw with this plan. If there was not a flaw, why did the book not teach solving equations first? That is why I presented my idea to a few math teachers at lunch the next day. None of the teachers at lunch taught Algebra, meaning they all taught upper level courses. I explained to them how the textbook organizes the quadratics chapter and asked them what would be wrong with first teaching students how to solve quadratic equations algebraically. The first response I got was, “What does the syllabus say?” I explained how the syllabus included three big ideas for the chapter: Explore and graph quadratic function, solve quadratic functions, and model linear, quadratic, and exponential functions.

The teachers and I talked about what they had done in the past when they taught quadratics either in Algebra or Advanced Algebra. One or two teachers had taught both

ways in the past. They had begun with graphing, then taught solving one year. Then the next year they began with solving, then taught graphing. No one could highlight any major flaws of either approach, or explain why one approach would be better than another. The overall advice from the lunch group was to follow the syllabus and textbook if one approach was not better than the other. My colleagues appeared to be comfortable with using the curriculum's map of the terrain to teach quadratics. I was not sure that I was. I did not believe starting by graphing was sufficient if we could not teach students meaning behind the vertex formula. The graphing seemed so disconnected from what the students had previously learned.

Although I appreciated the discussion I had at lunch with my math colleagues, I was quickly realizing that reasoning and sense making had not been a priority when they previously taught quadratics either. For instance, none of the eight math teachers eating lunch had a way to explain why $-\frac{b}{2a}$ results in the x-value of the vertex. When talking about how to initially approach quadratics, little concern was given to how the concepts would build on students' prior knowledge. I decided that I would need to discuss the various approaches with Nathaniel and we would need to make a decision about how to introduce quadratics since we were concentrating on teaching for reasoning and sense making.

Creating A New Map of the Terrain

As scheduled, Nathaniel and I met during 8th period on Wednesday, April 6 to officially begin planning. He did not offer any insight into how to explain the vertex formula like I was hoping. So I presented Nathaniel with the idea of starting to teach quadratics with solving equations rather than graphing. When I suggested teaching

quadratics in a different order than the textbook his initially reaction was that of surprise and apprehension. I think he was assuming we would largely follow the textbook and he was not expecting me to suggest a different approach to quadratics. Nathaniel and I decided to go ahead and sketch out the chapter in two ways: teaching graphing quadratic functions first and teaching solving quadratic equations first.

For over an hour we talked about how each approach (starting with graphing or starting with solving) could be done with reasoning and sense making. We continued to discuss ways to connect students' prior knowledge to new information, how the various approaches could get students to reason about the mathematics, and the pros and cons of teaching graphing first or solving equations first. The pros for beginning with graphing functions were that the curriculum was structured in that way and we both had experience teaching graphing first. We also thought that beginning with the graphical representation would provide students with a visual of solutions for when they solved quadratic equations later. The negative side of starting with the graphs was that the concepts did not connect to anything the students learned earlier in the semester. As far as starting quadratics with solving equations, an obvious pro was the direct connections to the skill of factoring, which the students had recently learned. Another attractive aspect of this approach was that students could first graph the solutions (which they could find by solving the equation) and then use the mean to find the vertex in the middle of the solutions. A con was that students would not see a visual representation of the solutions until later in the chapter. And a con for both approaches was that we still did not know how to help students make sense of the vertex formula.

After we roughly sketched out the two approaches and discussed the pros and cons to each approach, I told Nathaniel the decision was his to make. He would ultimately be the one teaching each day, so I wanted him to be comfortable with the approach we would take to teaching quadratics. I assured Nathaniel that I was not biased and would be okay with whatever he decided. Nathaniel finally determined that teaching solving algebraically first, then graphing, would “allow us to infuse more reasoning and sense making into our instruction” (Field Notes, April 6, 2011). He thought the connections would be clearer and students would create better understanding if we started with solving.

Our planning session was driven by the idea of connecting mathematics. We decided that each method of algebraic solving would be presented to students as an extension of the other methods they already knew. I wrote in my field notes:

As we talked, we immediately felt like a strong connection could be made to solving quadratics by factoring immediately since the students just learned factoring. From there a connection could be made to solving by square rooting (using the difference of squares) and solving using the quadratic formula (having a trinomial that is not factorable). (Field Notes, April 6, 2011)

For each new method taught, a need would be built by giving students a slightly different quadratic equation. By having students attempt to apply a previously learned concept and recognize that the pattern or relationship is not the same, we would be encouraging students to analyze problems and activate their reasoning skills (NCTM, 2009).

For example, Nathaniel and I discussed connecting solving by square rooting to solving by factoring with an equation that could be factored using the difference of

square problem (see Figure 6.11). After the connection was made to the prior knowledge of factoring to solve, we would present students with a quadratic that looked similar (missing an x-term). The students would try to apply the same process as they previously did (factor using the difference of squares) and realize the equation cannot be factored using rational numbers and therefore did not fit the pattern of the previous equations. Since the new equation was not something they could solve by factoring, the students would have created a need to learn a new method for solving equations that are missing the x-term.

Prior Knowledge (Solve by Factoring)	Applying Prior Knowledge to New Kind of Problem	Prior Knowledge Cannot be Applied to ALL of the New Kinds of Problems
$2x^2 + 9x - 5 = 0$	$x^2 - 25 = 0$	$x^2 - 20 = 0$
$(2x - 1)(x + 5) = 0$	$(x - 5)(x + 5) = 0$	$(x - ??)(x + ??) = 0$
$2x - 1 = 0 \quad x + 5 = 0$	$x - 5 = 0 \quad x + 5 = 0$	
$2x = 1 \quad x = -5$	$x = 5 \quad x = -5$	
$x = \frac{1}{2}$		<i>Not Factorable with rational numbers</i>

Figure 6.11: Nathaniel and I discussed how we could encourage mathematical reasoning by connecting new knowledge of solving by square rooting to students' prior knowledge of solving by factoring.

We went through the same process as we worked to connect solving quadratic equations algebraically with graphing quadratics. The basic idea we brainstormed was to initially graph the roots/solutions, which connects to students' understanding of solving quadratics algebraically. With well-designed mathematical tasks, we thought we would

then have students reason that the vertex was in the middle of the roots and could be found by averaging the x-values of those roots (see Figure 6.9). This approach would allow us to at least explain why we divide by two in the vertex formula $x = -\frac{b}{2a}$. From there we would teach students how to graph by determining the vertex first. In our planning notes for April 6, I wrote down our general outline for teaching quadratics. The list below is exactly what and how I wrote our thoughts.

Factor to Solve

$\sqrt{\quad}$ to Solve

Quadratic Formula

→ All methods ←

Graphing to Find Solutions

- Roots
- Vertex/Axis of Symmetry
- Patterns in the Graphs
 - width
 - open up/down
 - shifts

Graph with the Vertex $-\frac{b}{2a}$

Unfortunately, we were still stumped with how to explain the vertex formula to students. We did have a small connection to dividing by two, since two solutions are being averaged to obtain the middle value (i.e., vertex). Yet we both left that planning session fully aware of the big question that still hung over us. How do we explain

$x = -\frac{b}{2a}$? Part of me thought maybe the students simply would not have enough mathematical knowledge to fully understand the formula and we would not have a choice but to give the vertex formula to them with little explanation. We had already decided that was the case with the quadratic formula. But we still wanted to find some way to explain the vertex formula so students could understand rather than memorize. We wanted them to still be able to graph a quadratic using their reasoning skills in case they forgot the vertex formula.

After our planning session that day, Nathaniel went to the textbook (which we had used very little since our initial mapping discussion) looking for ways to explain the vertex formula. He sent me an email Wednesday evening with the subject line “vertex thought attached” (see Figure 6.12). In section 10-6 (Using the Quadratic Formula), the textbook had this problem buried in the independent practice section. In the email, Nathaniel pointed out that our students would already understand that the quadratic formula gives the two solutions or roots. Therefore by building on their knowledge of the quadratic formula and the fact that the vertex is the midpoint between the two roots, we would have a way to explain the vertex formula.

We now had a way to answer the big question Nathaniel and I, as well as the other teachers in our PLC, had been struggling to explain. At the end of the chapter when I asked him why he went looking through his book for this idea, he explained:

I was just like, “Gosh, I’ve got to just browse for some ideas.” And I looked through every page, just looking at problems. And I came across that challenge one and I was like, “Hey this really relates to what we want to do and we could prove it this way.” And so I probably wouldn’t have been browsing (if we

wouldn't had discussed how to explain the formula) because that is what I was trying to do. I was like, "How in the heck do we get $-\frac{b}{2a}$?" (Post Lesson

Debrief, April 28, 2011)

On page 590 there is a critical thinking question (#42).

I was like... oh... this would be perfect to use for proving our point about $-b/2a$ without getting too crazy algebraically.

We have two roots using our quadratic formula, $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

Well, we can take the sum of our two roots and find the midpoint of this general form...

$$\begin{aligned} \text{Sum of our two roots} &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-b + \sqrt{b^2 - 4ac} + -b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2b}{2a} \\ &= \frac{-b}{a} \end{aligned}$$

Then, the half way point is just half of that so our mid point = $\frac{-b}{2a}$

I don't know... just an idea about how to transition to finding a general x coordinate for the vertex.... They'd understand that the quadratic gives us our solutions... so to me seems somewhat easier than some other approaches...

But then again... how to prove that darn quadratic formula nicely lol...

Tell me what you think. And sorry for the slop on this document. Just thought about this before heading to practice and wanted to share before I forgot.

Figure 6.12: The email from Nathaniel used a problem from the textbook to explain the vertex formula. (Written Communication, April 6, 2011)

Nathaniel and I met again the next day (Thursday, April 7) to start selecting specific tasks and problems in more detail. We met during both his plan period and for a couple hours after school. Our focus was on how to start the concept of quadratics and build a need within the students to learn how to solve quadratic equations. Reflecting on everything we went through as we discussed the big picture of quadratics, I wrote in my personal journal:

I am getting very excited about all of this! And the email from Nathaniel about using the quadratic formula to explain where $x = -\frac{b}{2a}$ comes from was just the cherry on top! It was almost as if we were being rewarded for thinking through the best way to approach teaching quadratics. We were being sent a gift – the answer to our biggest obstacle as we thought about teaching quadratics. And it fit perfectly because we had already decided to start with solving quadratics first. Yay! (Personal Journal, April 7, 2011)

The map of our quadratics terrain did start with the textbook and syllabus, but ultimately took on its own shape. In fact, the basic structure of our terrain was the syllabus map. As Nathaniel and I discussed the various elements we wanted to incorporate into teaching quadratics, our thoughts and discussions went from the concept of quadratics being linear in nature (see Figure 6.5 and Figure 6.7) to being a complex network of mathematical connections (see Figure 6.13). The linear map was transformed into a complex, webbed diagram. Not only were the big ideas on our map connected in numerous ways, but our terrain also reached out to several other mathematical concepts. Although we did not physically draw this map of the quadratic terrain prior to teaching, our discussions about what should be taught and how we would connect the mathematical

ideas embedded in quadratics resembled the webbed mapping. It was not until after we taught the entire chapter that we put our map of the terrain of quadratics on paper.

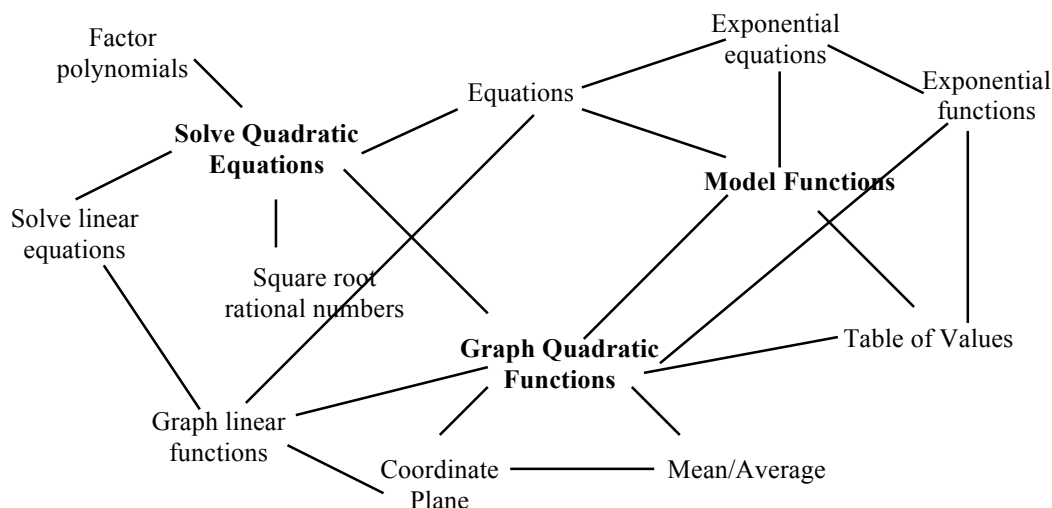


Figure 6.13. The map of Nathaniel and my quadratics terrain.

In our general pacing, we allotted six instructional days to introduce ways to solve quadratic equations and four days for lessons on graphing quadratics concepts. The final four days of instruction would be spent on miscellaneous concepts and connecting knowledge. In all we devoted 14 instructional days, 89-109 minutes each day, to teaching quadratics with reasoning and sense making (see Figure 6.14).

I was feeling more confident towards the end of the week, after talking with my professor and other math teachers, and having lengthy, detailed discussions with Nathaniel as we decided how we were going to approach quadratics. As I reflected back on the first few days of this process, I wrote:

I was honestly scared and apprehensive about how we were going to tackle this concept with reasoning and sense making. I have not worked with quadratics a

lot, so I don't have a large knowledge base to pull from. This was made even more apparent when I talked with (my professor). He talked a lot about some higher math connections, but honestly I don't feel like we are there yet. We are still making some basic connections for ourselves. (Personal Journal, April 7, 2011)

Monday	Tuesday	Wednesday	Thursday	Friday
4 <i>Amy met with math professor to discuss quadratics</i>	5 <i>Algebra PLC discusses quadratics</i>	6 <i>Nathaniel and Amy outline quadratics concept</i>	7 <i>Nathaniel and Amy discuss teaching quadratics in more detail</i>	8 First Day of Quadratics: Solve by factoring
11 Solve by factoring, included GCF <i>Amy gone</i>	12 Solve by square rooting <i>Amy gone</i>	13 Solve by factoring or square rooting; introduce quadratic formula <i>Amy gone</i>	14 Solve using any method	15 Solve using any method; quiz
18 Characteristics of quadratic graphs; vertex as midpoint	19 Graph using roots and the vertex as midpoint	20 Show formula for vertex; graph using $-\frac{b}{2a}$	21 Graph using $-\frac{b}{2a}$; parabola widths	22 No School
25 Parabola widths; connect solving algebraically and graphically	26 Vertical shifts; determining functions by graph and equation	27 Application problems; determining functions by table	28 Review quadratics (solving and graphing)	29 Last Day of Quadratics: Students tested on quadratics

Figure 6.14: Fourteen days in April were spent teaching quadratics.

My knowledge of teaching quadratics with reasoning and sense making was lacking compared to my professor's knowledge and compared to my own knowledge of other algebraic concepts. I was cognizant of that fact. Yet the comment above in my journal shows that I understood this was a learning process. Nathaniel and I were just beginning to embark on learning how to teach with reasoning and sense making as a primary goal.

What Nathaniel Learned while Mapping the Terrain

A map of the quadratics terrain was created as a way to begin our intense, consecutive coaching. Nathaniel and I examined the standards, defined what we meant by teaching for understanding, used the textbook and syllabus to begin our discussions, conversed with colleagues, and participated in meaningful discussions to determine what we wanted to teach about the concept of quadratics and how we wanted to make the mathematical connections. During this process, Nathaniel gained a greater understanding of quadratics and increased his ability to appraise and adapt the textbook.

Deeper Understanding of the Mathematical Content

On January 28, 2011, before we started the semester of intense coaching, I had asked Nathaniel which mathematical topic he would like to work on during the Spring 2011 semester. He identified quadratics.

Nathaniel: Out of everything I have a comfort with I would say that is my least experienced area. I know they do a lot of that in Advanced Algebra and I guess become more comfortable with that so I can become a better teacher at it would probably be good.

Amy: What part of quadratics?

Nathaniel: I guess I understand the relationships in terms of formulas and stuff but just being able to teach those concepts better. Like how we can get students to discover those patterns for themselves and how they can maintain all of that in their repertoire of graphing and solving different things. (Initial Interview, January 28, 2011)

After our coaching collaboration, Nathaniel felt more confident and comfortable with the concept of quadratics.

I didn't have a great understanding of it (quadratics) and I think I definitely feel more comfortable with it. And if I had to share with someone, I would feel really comfortable saying, "Hey, this is what we did and why." I would feel really comfortable giving an alternative approach to what was, what is typically done. And probably before I wouldn't have been able to do that cause we kind of experienced that together. (Final Interview, May 25, 2011)

He felt like he gained a deeper understanding of the quadratics through our work together. Nathaniel typically listens and takes in what others say during PLC. It is not very often where he will speak up and give ideas. Yet towards the end of the quadratics chapter, Nathaniel did start sharing how we approached teaching quadratics with other teachers (Field Notes, April 19, 2011). In addition, Nathaniel wrote an article for a statewide math teachers' organization about teaching quadratics with reasoning and sense making, which again demonstrates his increased confidence with the concept (Field Notes, June, 2011).

Nathaniel's learning about quadratics began with our initial planning discussions. As we talked about what to teach in quadratics, and specifically the order we should

introduce the individual objectives, Nathaniel began to form the larger mathematical picture or the mathematical terrain. This was an important aspect of the learning process for Nathaniel. He knew how to do all of the mathematics he was required to teach. Yet engaging in deep conversations about the concept of quadratics and discussing how to map the terrain of quadratics helped him better understand the mathematical ideas embedded in the concept.

Prior to our work on quadratics, Nathaniel said he wanted to make the concept of quadratics more holistic. He thought that by making quadratics “one whole picture”, like “a web that connects a lot of places” then students would learn better (Initial Interview, January 28, 2011). At the end of the semester, as he reflected on this goal, he felt like the numerous connections that were made as we planned quadratics helped facilitate this. He commented, “But in terms of making it holistic, I really did think it all flowed into just one, one...quadratics. That’s what it was. Just one nice beautiful piece of that” (Final Interview, May 25, 2011). Through our mapping of the quadratics terrain, Nathaniel was able to conceptualize quadratics as a number of mathematical connections.

A prominent example of Nathaniel increasing his knowledge of quadratics during our initial discussions of how to teach quadratics was the vertex formula. Nathaniel knew how to graph a quadratic function using $x = -\frac{b}{2a}$ and could quickly graph a quadratic if asked. Yet both Nathaniel and I needed to gain deeper understanding of the vertex formula and learn the rationale behind the procedure in order to fully understand the mathematical reasoning for the formula. Nathaniel found a way to use the quadratic formula and students’ knowledge of solutions (or roots) to explain the formula. This new knowledge helped Nathaniel think more deeply about the meaning of the vertex formula,

understand how the formula connected to solving quadratics equations, and more clearly explain this mathematical concept to students.

Appraising and Adapting the Textbook

Prior to our intense, consecutive coaching in quadratics, Nathaniel's instructional choices were largely determined by the textbook. He told me during a post lesson debrief that if we had not chosen to collaborate to teach quadratics, he would have taught the concept based on the sections of the textbook just as he had done the previous year (Post Lesson Debrief, April 28, 3011). A teacher's reliance on the textbook, especially in mathematics instruction, is an element of traditional instruction (Remillard, 2005). Nathaniel had also explained to me that he lacked confidence in his knowledge of quadratics (Initial Interview, January 28, 2011), which also could have led to a greater dependence on the written curriculum (Remillard, 2005).

As we mapped the quadratics terrain, Nathaniel and I found that some textbooks are not written to foster reasoning and sense making. We decided to reorganize the sections from the textbook and supplement the curriculum with other resources in order to teach quadratics for understanding. Learning how to undergo this mathematical task of analyzing and modifying a textbook demonstrates how Nathaniel gained specialized content knowledge (Ball et al., 2008). Specialized content knowledge is the unique knowledge educators use to unpack mathematics for their students (Ball et al., 2008). Nathaniel initially viewed the textbook as the map of the terrain; one that he later realized could not make the mathematical connections he wanted to make with students. Learning to appraise and adapt the textbook was fostered by a focus on mathematical content.

Specifically, Nathaniel became more analytical of the curricular materials as we discussed how to make connections between mathematical ideas.

When we initially began discussing how to approach quadratics, Nathaniel was visibly surprised and somewhat apprehensive when I suggested teaching the skills in a different order from the textbook (Field Notes, April 6, 2011). As we worked together to map the terrain of quadratics, Nathaniel began to realize that the textbook did not present the mathematical ideas in a conceptual manner. Nathaniel began to view the textbook as a resource rather than the map, which can be seen in his explanation of making connections within the curriculum.

I would just say that if you are persistent enough you really can make connections where it seems like it is impossible, because it has been impossible for a lot of people for a lot of years, you know...Just don't settle for the way the book has it laid out... Just because the book flows 11.1 11.2 11.3 doesn't mean that is really the best way to make those connections, you know? And so if you look at ours, gosh we were like 10.6, 10.4...We were bouncing all over the place, you know? And so you don't have to settle for a way it's planned out (in the textbook) if you can think of a better way to connect it. (Post Lesson Debrief, April 28, 2011)

Nathaniel began to appraise the textbook and realized that he (and other teachers) should not feel obliged to follow the textbook. As he tried to make connections between concepts, he recognized that the order of the textbook is not always best. Sometimes rearranging the sections in the textbook, similar to what we did with quadratics, is the best way to help students make connections.

Nathaniel had numerous discussions with me, as well as his colleagues, about rearranging the content in the textbook's chapter. This collaboration, which helped him decide to break away from the textbook, assisted in helping Nathaniel appraise the textbook. He saw the short-comings in the textbook, which is noted when he explained that with our approach to graphing, his students "...had come full circle, connecting all the ways to solve quadratics algebraically with how to graph. The text could not make such a connection because they decided to start solving quadratics with the vertex formula $\left(x = -\frac{b}{2a}\right)$ " (Written Artifact, June, 2011). Nathaniel learned that the textbook was a resource to supplement his reasoning and sense making instruction instead of the resource that determined his instruction.

An example of how Nathaniel adapted the mathematical content of the textbook was demonstrated when we struggled to explain the vertex formula $\left(x = -\frac{b}{2a}\right)$. After trying for two days to think of a way to explain the formula so students could remember it and find it meaningful, Nathaniel went looking in the textbook.

I was just like, "Gosh, I've got to just browse for some ideas." And I looked through every page, just looking at problems. And I came across that challenge one and I was like, "Hey this really relates to what we want to do and we could prove it this way." And so I probably wouldn't have been browsing (if we wouldn't had discussed how to explain the formula) because that is what I was trying to do. I was like, "How in the heck do we get $-\frac{b}{2a}$?" (Post Lesson

Debrief, April 28, 2011)

Nathaniel was not using the textbook to make instructional decisions, but instead was using it as a resource for teaching quadratics with understanding. The textbook no longer determined how Nathaniel taught. As an alternative, the curriculum was being used to support the reasoning and sense making instruction he was planning to implement.

Although Nathaniel did increase his ability to appraise and adapt the mathematical ideas in his textbook, he continued to refer to the textbook in our planning. For example, Nathaniel continued to organize and title his materials by the textbook's corresponding section number for each skill rather than titling his materials with the objective or concept. The lesson opener for the first day of quadratics, Nathaniel had "10.4 Day 1" at the top, indicating this skill was related to section 10.4 in the textbook (see Figure 6.15). So even though he adapted the curricular materials to teach in a more conceptual manner, Nathaniel still felt it necessary to associate his instructional materials with the sections in the textbook.

10.4 Day 1 Lesson Opener			Name: _____
1. A sum of a number and three is eleven. What is the number?	2. Two times a number plus seven is 11. What is the number?	3. A number and its square is 6. What is the number?	

Figure 6.15: Nathaniel continued to label his lesson opener with the corresponding textbook section number. (Artifact, April 8, 2011)

Conclusion

Towards the end of the quadratics unit, Nathaniel commented that he would like to have conversations about teaching other topics (i.e., Geometry), as well as other algebra concepts (i.e., linear functions) in the future (Field Notes, April 25, 2011). He felt that the in-depth discussions we had surrounding what about quadratics should be taught and how to approach teaching the concepts was beneficial to him as a mathematics teacher and to his students learning the math. Nathaniel felt as though the collaboration we engaged in as we mapped the quadratics terrain increased his own understanding of the mathematical ideas involved in quadratics and increased his confidence in teaching the concepts (Final Interview, May 25, 2011). In other words, Nathaniel gained a deeper understanding of the mathematical concepts included in quadratics. His knowledge of the mathematical content increased.

Nathaniel also indicated that he wanted to apply his new understanding of quadratic functions to how he thought about and taught linear functions. Specifically, he talked about the conceptual idea of connecting solving equations with graphing functions. The mathematical connection through the solution (or x-intercept) was a way of thinking about functions Nathaniel had not previously considered. He explained that helping students understand the mathematical connection with linear functions would positively impact a similar connection with quadratic functions (Field Notes, April 25, 2011).

Increasing his own knowledge of quadratics through the mapping process also equipped Nathaniel with a greater capacity to assess and adapt the textbook. Since he gained a deeper understanding of quadratics through our discussions, Nathaniel was better prepared to thoroughly examine how the textbook suggested content be taught. He

searched the textbook for ways to make connections between the mathematical ideas. If Nathaniel could not find what he was looking, he modified the textbook in order to teach the connections we discussed when mapping the terrain. Nathaniel's increased content knowledge of quadratics gave him the confidence and in-depth mathematical understanding to use the textbook to supplement rather than dictate his instruction.

Part II: Expanding on Mathematical Tasks of Teaching

Nathaniel and I participated in the intense, consecutive coaching structure focused on quadratics from April 8 through April 29, 2011. Our planning, as described in Part I, began as early as April 5. During the 16 instructional days I was in Nathaniel's Algebra block class with him the first two periods of the day, with the exception of three days¹¹. This was approximately 105 minutes of class time each day, except for Tuesdays, which were about 90 minutes long due to early release. We also met daily to debrief about the lesson and plan instruction for the next day. The debrief and planning sessions occurred during one of Nathaniel's two plan times and during extra hours after school. We spent anywhere from one hour to four hours each day discussing the previous lesson, identifying what students did and did not understand, talking about ideas for introducing new concepts, determining ways to connect the mathematics, deciding on appropriate mathematical tasks and formative assessments, and creating materials needed to facilitate learning. All of this took place in Nathaniel's teacher planning center (TPC) either at his desk or at a large middle table located in the middle of the room.

¹¹ I attended the National Council of Supervisors of Mathematics (NCSM) conference April 11-13, 2011 in Indianapolis, IN and therefore was not in Nathaniel's class or able to plan with him in person those days.

Part II provides a detailed explanation of what and how Nathaniel taught the concept of quadratics. The day-to-day curricular decisions that were made through our planning and debriefing discussions are included for the three phases of his instruction: solving quadratic equations, graphing quadratic functions, and connecting concepts. Through the collaboration and conversations Nathaniel and I had, Nathaniel gained a greater understanding of the mathematical tasks (Ball et al., 2008) involved in teaching mathematics. At the same time, I increased my knowledge of what several of the mathematical tasks include and expanded my understanding of how the tasks are used by mathematics teachers.

Solve Quadratic Equations

(Friday, April 8, 2011 – Friday, April 15, 2011)

Nathaniel's students completed a summative assessment on chapter 9 material (operations with polynomials and factoring polynomials) on April 7, so we started teaching quadratics on Friday, April 8. The majority of our discussions on how to teach solving quadratics took place during both of Nathaniel's plan periods and for three to four hours after school on that Thursday and Friday. We compacted our discussion of solving quadratics and planning of instructional tasks into these two days since we were starting a new concept and we wanted to be sure we thought through the big idea of quadratics as a whole. Overall, Nathaniel devoted five and a half instructional days to lessons on solving quadratic equations and gave students a quiz during the second half of class on Friday, April 15 (see Figure 6.16). During this time, we made daily changes to our original lesson plans based on students' performance in class by continually reflecting and

planning together. These conversations occurred in person on April 8, 14, and 15 and via email or phone communication April 11-13.

Monday	Tuesday	Wednesday	Thursday	Friday
4	5	6	7 <i>Tested Ch. 9 material (simplify & factor polynomials)</i>	8 First Day of Quadratics: Solve by factoring
11 Solve by factoring, included GCF <i>Amy gone</i>	12 Solve by square rooting <i>Amy gone</i>	13 Solve by factoring or square rooting; introduce quadratic formula <i>Amy gone</i>	14 Solve using any method	15 Solve using any method; quiz

Figure 6.16: Nathaniel devoted six days in April to solving quadratic equations.

As a way to teach quadratics with reasoning and sense making, Nathaniel and I approached the instruction of solving quadratics using two major strategies. Our first priority was using prior knowledge to develop understanding of a new mathematical concept (Graham et al., 2010). We worked to make as many connections to prior mathematical understanding as possible. As we talked through different instructional ideas, we often made reference to helping students open a certain math file (i.e., concept) in their brains so that we could help them construct more knowledge. In essence, we were referring to the theory of learning in which prior knowledge or schemata are brought to working memory in order to develop connections and larger schemata (Shell et al., 2010). Since the new information is stored in larger schemata with a greater

number of connected knowledge, students are able to more easily retrieve this newly learned information (Shell et al., 2010).

Secondly, we wanted students to activate their reasoning skills each time a new skill was introduced. One strategy that helped Nathaniel and I think about these analytical habits was to get students to see a reason for learning the new skill. If students attempt to use prior knowledge to solve a problem and their usual patterns or procedures do not work, the students move into a reasoning phase in which they try to determine how the problem is different, why the previous skill does not work, and what patterns can be seen in this new data. Using this strategy also helped students form a greater understanding of when and how to use various mathematical procedures (NCTM, 2009). As we worked to incorporate a need or purpose for each new skill taught, we also found that building a need to learn a new skill was closely tied to connecting new learning to prior knowledge (Graham et al., 2010).

Mapping the Terrain of Solving Quadratic Equations

Within the big idea of solving quadratic equations are three major objectives: 1) solve quadratic equations by factoring; 2) solve quadratic equations by square rooting; and 3) solve quadratic equations using the quadratic formula. An equation can be solved using factoring when the polynomial (from an equation in standard form) can be factored. The square root method can only be used for equations that do not have an x-term (i.e., $3x^2 - 4 = 0$) since square rooting with an x-term would result in \sqrt{x} . The quadratic formula can be used to find the solutions to any quadratic equation, but it tends to take a long time and provides ample opportunities for computational mistakes. Therefore if an equation can be solved by factoring or by square rooting, these two methods are

preferred. Nathaniel and I discussed all three methods (see Figure 6.17) at length as we planned how to teach solving quadratic equations. (Completing the square, a fourth method that can be used to solve a quadratic, is not taught in the first year Algebra course.)

Solve by Factoring (Factor & apply zero product property)	Solve by Square Rooting (Use inverse operations)	Solve by Quadratic Formula
$2x^2 + 9x - 5 = 0$ $(2x - 1)(x + 5) = 0$ $2x - 1 = 0 \quad x + 5 = 0$ $2x = 1$ $x = \frac{1}{2} \quad x = -5$	$3x^2 - 4 = 8$ $\begin{array}{r} +4 \quad +4 \\ \hline 3x^2 = 12 \\ \hline 3 \quad 3 \\ \hline x^2 = 4 \\ x = \pm 2 \end{array}$	$x^2 - 5x + 3 = 0$ $x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)}$ $x = \frac{5 \pm \sqrt{25 - 12}}{2}$ $x = \frac{5 \pm \sqrt{13}}{2}$ $x \approx 4.3, 0.7$

Figure 6.17: The three methods of solving quadratic equations that Nathaniel and I discussed.

In addition to the procedural skills of solving, Nathaniel and I also discussed how to embed other CCSSM and NCTM standards into learning how to solve using all three methods. Students are expected to use the initial form of an equation to determine the best method to use and be able to justify choosing that method (Council of Chief State School Officers (CCSSO), 2010). Based on the three methods, the types of equations each method can be used to solve, and the assumption that a student does not want to use the quadratic formula for every problem, a general thought process can be enacted to determining the best method (see Figure 6.18). This was the thought process Nathaniel

and I shared when solving a quadratic equation, and consequently the way we hoped the students would approach problems.

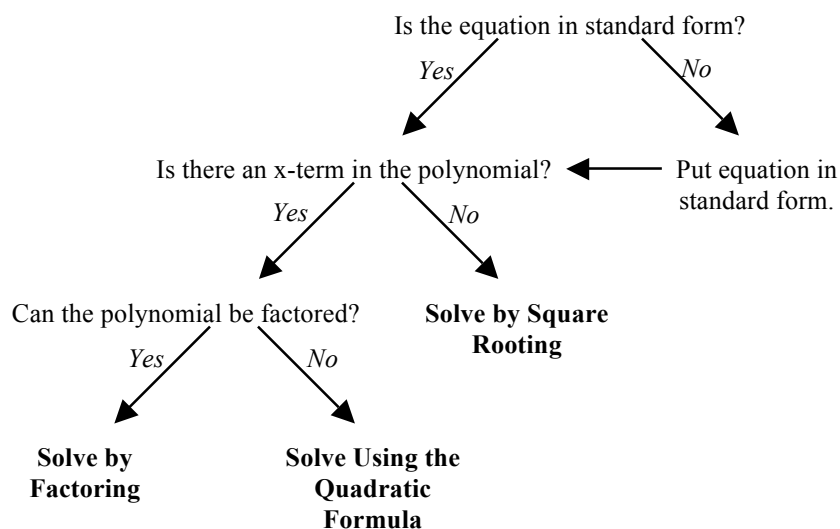


Figure 6.18: A flow chart showing the general thought process for choosing the best method to solve a quadratic equation.

Students also need to be able to rewrite equivalent forms of equations when necessary (NCTM, 2000). This skill can initially be applied when a quadratic equation must be put into standard form in order to factor or use the quadratic formula to solve (see Figure 6.19). Finally, students also should be able to explain steps when solving an equation (CCSSO, 2010). This means that as students solve quadratic equations, they need to be able to identify what mathematical procedures they chose and why that step was a good mathematical choice.

As Nathaniel and I discussed all of the ideas and standards involved in solving quadratics (i.e., solving using all three methods, choosing best method, putting equations



Original Equation		Standard Form
$4x^2 + 1 = 7x$		$ \begin{array}{r} 4x^2 + 1 = 7x \\ \underline{-7x \quad -7x} \\ 4x^2 - 7x + 1 = 0 \end{array} $
		$4x^2 - 7x + 1 = 0$

Figure 6.19: Writing an equivalent quadratic equation in standard form is useful when solving by factoring or using the quadratic formula.

in standard form, explaining steps), we talked about what ties all of the ideas together. We determined that solving a quadratic equation serves the same basic purpose as solving any other type of equation. An equation is solved in order to find the value of the unknown. In a quadratic equation, there is only one unknown or variable. Through one of the three methods of solving, the value or values of that variable can be determined.

Through discussing the mathematical idea of solving quadratic equations, planning how to teach the concept, and reflecting on the lessons we taught, Nathaniel and I essentially created a map of solving quadratic equations (see Figure 6.20). Our map was a subset of the larger map of our entire quadratics terrain (see Figure 6.13). The solution was placed at the center of the solving quadratic equations mapping since it was what brought the three methods together. Also included were some of the key pieces of prior knowledge that we employed to increase the level of sense making in this big idea, and other ways the various concepts were connected.

Factor to Solve Quadratic Equations

As previously explained, we chose to start the chapter on quadratics with the concept of solving by factoring. Students were just assessed on factoring polynomials in the previous chapter and we thought this would be a direct connection to what they most

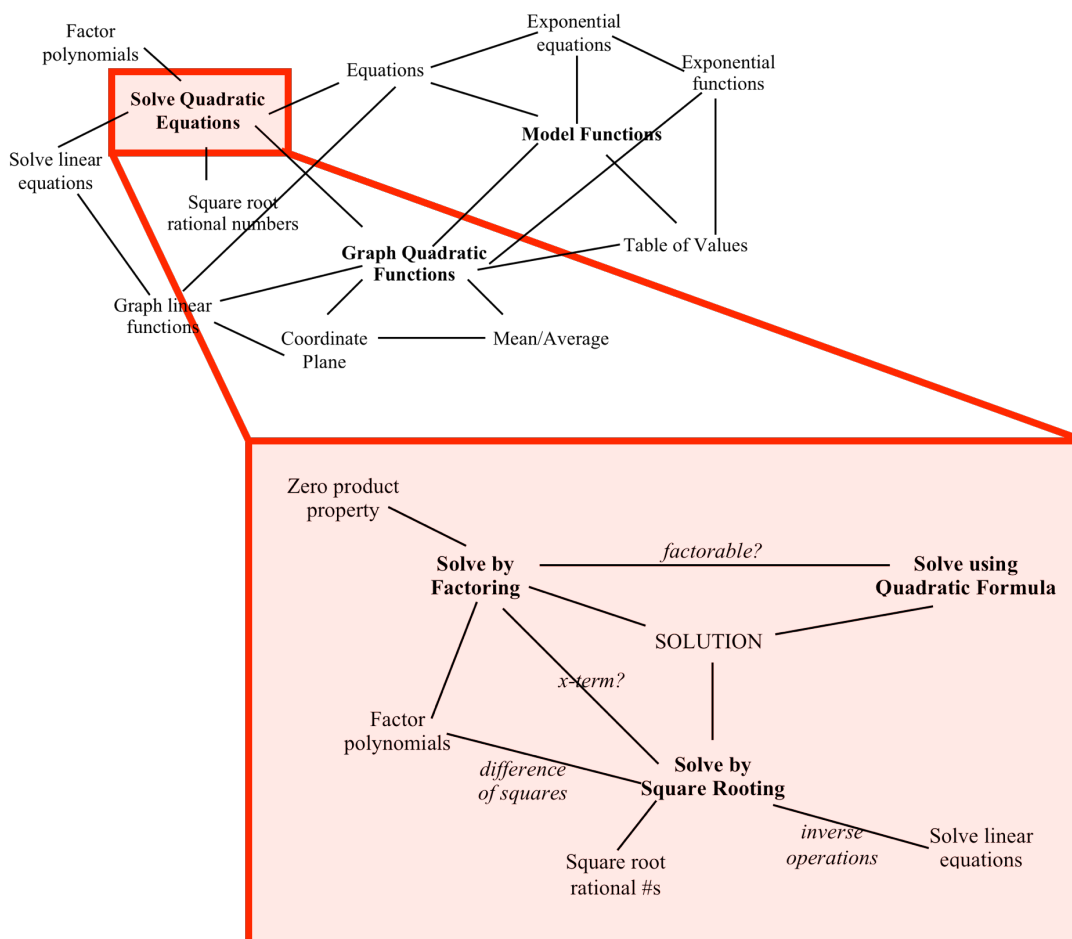


Figure 6.20: The map of our solving quadratics terrain was a subset of our quadratics terrain.

recently learned. The process of factoring was included in the warm-up at the beginning of class. Other connections were made in this lesson to students' prior knowledge, including their understanding of the zero product property (although they did not specifically name the property) and solving linear equations.

To access students' understanding of the zero product property, short "fill-in-the-blank" equations were used (see Figure 6.21). Students recalled that any number

multiplied by zero equals zero. And then they also determined that if any two numbers are multiplied to equal zero, then at least one of the numbers must be 0. Or in more general terms, if two factors are being multiplied and the resulting product is zero, you know that at least one of the factors must be zero. Nathaniel later helped students apply their knowledge of the zero product property to two factors of a polynomial, such as $(x - 2)(x + 3) = 0$. Using the same visual representation as he did with integers ($\square \cdot \square = 0$), Nathaniel explained that one of the factors must be zero since the product of the two factors is zero. If the factor $(x - 2) = 0$, that would mean that x equal 2 since $(2 - 2) = 0$. The same applies to the second factor. If $(x + 3) = 0$, that would mean that x equals -3 since $(-3 + 3) = 0$. Therefore the solutions for x would be 2 and -3.

Warm-Up	Name: _____
Find a value for each box that satisfies the equation.	
1. $8 \cdot \square = 0$	
2. $\square \cdot -11 = 0$	
3. What must be true about the boxes below? Explain.	
$\square \cdot \square = 0$	

Figure 6.21: Short fill-in-the-blank questions were used on the warm-up to access students' prior knowledge of the zero product property. (Classroom Materials, April 8, 2011)

The visual representation of the zero product property ($\square \cdot \square = 0$) was a tool continually reinforced throughout the five days of learning how to solve quadratics. Students understood that whatever was in each box needed to equal zero in order for the product to be zero. This visual helped students apply what they knew about integers to multiplying binomials. Nathaniel constantly highlighted the zero product property as students solved equations. An example of this can be seen in these notes taken on the April 11 classroom video.

- Nathaniel went back to $\square \cdot \square = 0$ to help students visualize zero product property.

- Student asks if you just do the opposite to see what x equals. Nathaniel went back to emphasizing the factor must be zero. He did not go with the procedure, which he easily could have.

- Nathaniel later comes back to that issue and says that we need that piece

$\square \cdot \square \cdot \square = 0$ to tell us how many solutions we have. He then wrote a 0 above each factor and talked about thinking through what x would be if that factor was 0 and also talked about the possibility of writing an equation to solve. (Videotape Notes, April 11, 2011)

Although it may have been easier to tell students a procedural method of solving once the polynomial was factored, Nathaniel did not do that until day four when encouraging a few students to set each factor equal to zero. At the time when he told students to set each factor equal to zero and solve, he did not explain why or mention the zero product property. Once they finished solving, he returned to the group of students and asked why

each factor could equal zero (Videotape Notes, April 13, 2011). Besides this one incident, Nathaniel continued to emphasize the understanding of the zero product property rather than a procedure.

During the first lesson, a connection to students' understanding of solving linear equations was also made. Students were asked to find an unknown number given a clue (see Figure 6.22). We decided to allow students to use any method they chose, but planned to encourage all students to write an equation if they struggled or even if they had already found the number using guess and check (Lesson Plan, April 8, 2011). Nathaniel used some students' work to show the equations for numbers one and two, pointing out the way they used inverse operations to find the solution. He emphasized that both guessing and checking and solving an equation were methods of finding the number or solution.

10.4 Day 1 Lesson Opener		Name: _____
<p>1. A sum of a number and three is eleven. What is the number?</p> $x + 3 = 11$ $x = 8$	<p>2. Two times a number plus seven is 11. What is the number?</p> $2x + 7 = 11$ $2x = 4$ $x = 2$	<p>3. A number and its square is 6. What is the number?</p>

Figure 6.22: This “guess my number” lesson opener was used to help students connect solving quadratic equations to solving linear equations. (Classroom Materials, April 8, 2011)

The third problem was purposely more difficult. All students used the guess and check strategy as they struggled to find the number that satisfied the problem. When Nathaniel asked the class what the number was, all of the teams answered, “Two.” One team quickly chimed in and said that it could also be -3. (This team had determined the solution of 2 fairly quickly. While all of the other teams were still working, I asked this team if that was the only number that worked. They thought about it, discussing that they knew any number bigger than two would not work since the sum would be too large. After they tried one and zero, they declared two the only solution, to which I responded, “There aren’t any other numbers?” They worked together a little longer and then found the second solution of -3.)

Once the solutions of 2 and -3 were offered, Nathaniel asked if any teams had written an equation for the last question. Since no one had written an equation, Nathaniel and the class wrote the equation $x + x^2 = 6$ together. Nathaniel connected this new type of problem to their previous knowledge of solving linear equations by doing the following:

- Nathaniel made it a point (with the class) that the first two problems only had one solution while the last problem had two solutions.
- The students helped him write an equation for #3 ($x + x^2 = 6$). He asked them what was different about this equation compared to the first equation. A student said there was an x^2 that was different. (Field Notes, April 8, 2011)

After making connections to the previous two linear equations, Nathaniel proceeded into the ‘build a need’ phase of the lesson opener. As we were discussing this lesson a few days earlier, we talked about making the point that students could now apply

their skills of factoring to solving equations. Prior to this, students did not know of a productive or mathematical reason to factor a polynomial. We wrote in our planning notes, “Encourage students to connect to factoring – ‘Is there anything you have the urge to do?’” (Lesson Plan, April 8, 2011). Nathaniel did just that. He asked students what they thought they could do with the equation so that they would not need to just guess and check. He made the point that guessing and checking could potentially result in forgetting a solution, which many of the teams did. Nathaniel and the class worked together to rearrange the equation so it was in standard form ($x^2 + x - 6 = 0$). He drew students’ attention to just the polynomial ($x^2 + x - 6$), encouraging them to use prior knowledge of factoring. Nathaniel showed the students how they could use their knowledge of factoring to help them solve the equation and find the number.

At this point in the lesson, students were beginning to lose focus since the discussion about the three problems had taken some significant time. They were struggling to pay attention or stay engaged in the class discussion. Nathaniel talked very quickly and made a decision to abandon the next example we had ($2x^2 + 3x = -1$) and pushed the instruction forward. We thought it would be best to change activities in order to keep students engaged. The next example we had originally planned to use was a problem in which guessing and checking would be very difficult. Students would have been forced to start reasoning through the mathematics by looking for relationships between factoring and solving. Nathaniel and I had created a way for students to build a need and to look at the mathematics to identify relevant information and its solution, all of which are reasoning and sense making skills (NCTM, 2009). Unfortunately our plan

had to change in order to keep management of the classroom. Nathaniel and I discussed this after class.

Nathaniel: I heard some people say, “Well, why are we doing this? Because I have the answer already. We have the answers.” And that’s like, true I mean. And that’s what they want to do. I go, “How can we get x out of that?” I remember asking that and they are like, “We know x . It’s two.”

Amy: Yep, Yep, so it may not have built the need as much as we needed it to.

Nathaniel: Yeah. Yeah. So maybe just say all right, we got the solutions, we guessed and checked it and then kind of say, “Can you do this one?”

Amy: Yeah, guess and check on this one then. I think that would have been a smarter approach. Because we were building this up, and there were connections to what’s this look like? It looks like factoring. You know, all those connections were made but it was kind of just building on it, and I think some kids tuned us out when they figured out x is 2 and -3. They were done. They found the answer.

They didn’t care about all the other stuff. (Post Lesson Debrief, April 8, 2011)

Our attempt to simultaneously make connections and get students to reason about a new type of problem actually caused students to disengage. The students already knew the solution to the problem and therefore did not feel a “need” to learn any other method. Their previous method worked, so there was no need to apply reasoning habits such as seeking new patterns or drawing conclusions (NCTM, 2009). The students’ old patterns and conclusions already worked for them.

Our lesson continued by discussing how the zero product property also applied to quadratic equations and students then practiced solving by factoring in various formative

assessments. As Nathaniel taught factoring to solve quadratic equations, he did not stop instruction until all students had mastered the content. This was a teaching strategy Nathaniel and I had discussed in our planning session to combat Nathaniel's concern about classroom management. I had observed in prior lessons that most negative student behaviors arose during the direct instruction, and I hypothesized this may be due to the wide range of ability levels in the class. Therefore, we talked about how to keep the instruction moving, or pushing the instruction. We needed to keep engaging the students who understood the concepts while simultaneously providing extra support to the students who struggled. In our planning notes we wrote, "Push through these examples" (Lesson Plans, April 8, 2011) or simply "Push" next to various tasks (Lesson Plans, April 12, 2011). Nathaniel and I defined "pushing instructions" as keeping a quick pace to the lesson. We worked to plan enough mathematical tasks so that Nathaniel could continually transition to a different formative assessment or task as a method of pushing the instructional pace high. As we continued the lesson on factoring to solve equations, Nathaniel used the strategy of pushing instruction as he moved to various formative assessments.

The next instructional day (Monday, April 11) the students spent more time solidifying the concepts introduced the previous Friday. Those students who had not mastered the skill were given further individual instruction. The students who were demonstrating understanding were engaged in tasks that strengthened their knowledge. Nathaniel also increased the difficulty of the factoring by adding greatest common factors (GCF) to the trinomials. Students engaged in multiple formative assessments as a way to

practice solving by factoring. During these assessments students worked in teams, partners, and individually.

When we were deciding what problems to have students practice during the final whiteboards formative assessment, Nathaniel and I intentionally included a few problems that required students to factor using the difference of squares. We discussed how incorporating difference of squares during the formative assessment this day would help them for what they would be introduced to the next day. Nathaniel and I also decided we should engage students in a discussion about perfect squares and square roots at the end of class on Monday. A common mistake students make when solving quadratics by square rooting is taking the square root of a negative number. We decided to address the idea about why one cannot square root a negative number on Monday so the students would remember when square rooting to solve quadratic equations the next day.

Square Root to Solve Quadratic Equations

The third day of instruction began with activating students' prior knowledge of solving quadratic equations by factoring and solving a linear equation using inverse operations. We wanted students to be able to compare the two equations and recognize that a different method would need to be used to solve each one. By including a linear equation, our idea was also to bring students' linear equations schemata into the working memory since solving quadratics by square rooting also utilizes inverse operations (see Figure 6.23). That way we could connect the new concept being learned to the old concept they had mastered the previous semester.

Besides connecting to knowledge of solving linear equations, Nathaniel and I also wanted to connect the concept to what students recently learned about solving quadratic

Linear Equation

$$\begin{array}{r}
 3x - 4 = 8 \\
 \underline{\quad +4 \quad +4} \\
 3x = 12 \\
 \underline{\quad 3 \quad 3} \\
 x = 4
 \end{array}$$

Quadratic Equation

$$\begin{array}{r}
 3x^2 - 4 = 8 \\
 \underline{\quad +4 \quad +4} \\
 3x^2 = 12 \\
 \underline{\quad 3 \quad 3} \\
 x^2 = 4 \\
 x = \pm 2
 \end{array}$$

Figure 6.23: Nathaniel and I had students solve a linear equation using inverse operations to make a connection to solving quadratic equations using inverse operations.

equations by factoring. Nathaniel first had students solve the equation $x^2 - 25 = 0$. They used the difference of squares pattern to factor into $(x - 5)(x + 5) = 0$, meaning the solutions would be $x = 5, -5$. Then Nathaniel wrote the same problem right next to the first. In our planning notes we wrote that at this point we wanted to solicit ideas for a quicker way of solving the problem. Our goal was to get kids to see how similar the equation was to solving linear equations, which they just did in the daily warm-up. We wrote in our notes, “Get kids to suggest adding 25. If no suggestions, push” (Lesson Plan, April 12, 2011). Watching the videotape of class, Nathaniel did not ask students for suggestions on how to solve the quadratic another way like we had originally planned. He did still connect the new method of solving (solving by square rooting) to solving linear equations by telling them to use opposite operations. Yet Nathaniel made this connection was made simply telling the students. Through the problems we chose to use, a connection was made to students’ prior knowledge in two ways: First by factoring to solve $x^2 - 25 = 0$ and secondly by drawing links to inverse operations used to solve linear equations (see Figure 6.24).

Solve by Factoring

$$x^2 - 25 = 0$$

$$(x - 5)(x + 5) = 0$$

$$x = 5 \quad x = -5$$

Solve by Square Rooting

$$x^2 - 25 = 0$$

$$\frac{+25 \quad +25}{x^2 = 25}$$

$$x = \pm\sqrt{25}$$

$$x = 5, -5$$

Figure 6.24: These problems were chosen to make a connection between solving quadratic equations by factoring and square rooting.

Nathaniel continued with instruction and gave students a few more problems to solve by square rooting. He was able to get his students to think and reason with his second example ($5x^2 + 5 = 20$). This problem was similar to the first problem ($x^2 - 25 = 0$) in that it was missing an x-term. Yet the second example was different enough from the first example that students needed to use their own reasoning skills to determine which method could be used to solve the equation. Nathaniel was able to use the first example ($x^2 - 25 = 0$) to make connections to prior knowledge and the second example ($5x^2 + 5 = 20$) to emphasize the need to use the new square rooting method to solve. This was the instructional piece that we missed two days earlier when we introduced factoring to solve. Since we chose to skip our second example when teaching factoring to solve quadratic equations, students did not have the second example to help them see the need for a new method of solving.

That evening Nathaniel and I debriefed via the phone. He reflected on how the lesson on solving by square rooting went and I listened and asked questions. I noted in my field notes something interesting Nathaniel pointed out.

As we were talking on the phone Tuesday night, one of the things Nathaniel said was that students were largely remembering to include both the positive and negative solutions when solving with square roots. This is typically something students forget, but Nathaniel said his students were remembering. When I asked him why he thought that was, he thought that showing the difference of squares problem at first really helped solidify that idea of a positive and negative answer. (Field Notes, April 12, 2011)

By connecting solving by square rooting to factoring difference of squares, we unintentionally addressed a common mistake we both knew students make when solving by square rooting. The difference of squares example helped students draw connections to the idea that there will be two solutions when you square root to solve a quadratic (see Figure 6.24). We were both very excited that we unknowingly helped combat a common student misconception.

The remainder of Tuesday was spent practicing the new skill of solving quadratics by square rooting. Nathaniel also led a class discussion about equations with no solution, focusing on why it is impossible to square root a negative number (if we are working in the real numbers) (see Figure 6.25). His conversation about equations with no solution tied back to the discussion he had with students at the end of class on Monday about square roots.

During the first half of class on Wednesday, April 13, students practiced choosing a method (factoring or square rooting) to solve equations. The students solved the two almost identical equations and then formalized a way to remember when to use each

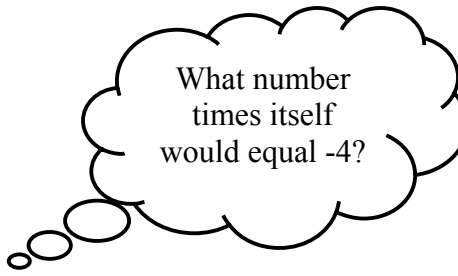
$$\begin{array}{r}
 2x^2 + 12 = 4 \\
 -12 \quad -12 \\
 \hline
 2x^2 = -8 \\
 x^2 = -4 \\
 x = \sqrt{-4} \\
 \text{No Solution}
 \end{array}$$


Figure 6.25: Nathaniel used this example of quadratic equation to demonstrate when and why there would be no solution.

method (see Figure 6.26). This mathematical task not only got students to apply previously learned concepts to a new situation (where they had determine the correct method) and solidify mathematical relationships (NCTM, 2009), but students were also asked to justify in writing why they chose each method (CCSSO, 2010).

You be the judge...

Solve.

1. $2x^2 + 3x = 5$

2. $2x^2 + 3 = 5$

You solve a quadratic equation by factoring when _____.

You solve a quadratic equation by square rooting when _____.

Figure 6.26: Students solved the two almost identical quadratic equations and summarized when to use the two different methods for solving. (Classroom Materials, April 13, 2011)

Quadratic Formula to Solve Quadratic Equations

Students had spent about 45 minutes at the beginning of class on Wednesday, April 13 solving equations by factoring or square rooting. They had to determine which method to use, explain why they chose that method, and then solve the equation.

Halfway through class, Nathaniel switched gears and asked the students to solve the equation $2x^2 + 4x + 3 = 0$. Students knew that square rooting would not work to solve the equation since there was an x -term, so they tried to solve the quadratic by factoring. Students quickly began to complain that the problem did not work (Videotape Notes, April 13, 2011). The polynomial was not factorable. The students were out of options. They were out of methods to try. Several of them determined that since it could not be solved with either method, the equation simply had no solution.

Nathaniel explained that there are solutions to the equation and the quadratic formula (which he wrote on the board) would help them find those solutions. He explained to the students that they would need to develop some more mathematical skills in order to fully understand where the quadratic formula came from. For now they would simply be using it to find the solutions. After solving the equation with the quadratic formula, Nathaniel put the original equation in front of the class again. He explained that even when a polynomial cannot be factored, using the quadratic formula does lead to the solutions (Videotape Notes, April 13, 2011).

Interestingly, the next day students were given an equation that required use of the quadratic formula ($-8x^2 + 3 = 6x$). All of the students put the equation into standard form ($-8x^2 - 6x + 3 = 0$) and began trying to factor the polynomial. Most students realized the trinomial was not factorable and began using the quadratic formula. A few

students who were gone the previous day (when the quadratic formula was introduced) also attempted to solve by factoring. They realized the polynomial was not factorable, but they quickly became frustrated. They did not understand how they were supposed to solve if they could not factor or square root. Nathaniel noted that having a discussion about why we would want to use the quadratic formula was beneficial to the students who were absent, as well as a “good point to still bring back up” (Post Lesson Debrief, April 14, 2011) to emphasize the reason for the quadratic formula for all students.

After addressing the need for the quadratic formula, the students practiced solving equations in a very structured task that allowed Nathaniel to address the common computation errors. Each student had his own paper (see Figure 6.27) and everyone in the team wrote the equation in standard form. Once all of the students in the team were finished, they all passed their papers one person to the right and checked the previous student’s work. Then everyone substituted the numbers into the quadratic formula. Once finished, the team rotated the papers and checked. This continued until the problem was completed. Most teams did not finish the assessment, so the activity was carried over into the first part of class on Thursday. Nathaniel used the formative assessment the next day as a way to check students’ understanding of the concept.

Choose a Method to Solve Quadratic Equations

Starting the second half of class on Thursday, April 14 and continuing into the first half of class on Friday, students spent time choosing a method and solving a variety of quadratic equations. Asking themselves questions such as “What is going on here?” and “Why do I think that?” are two ways the students began to apply their reasoning and sense making skills to each new problem they encountered (NCTM, 2000; NCTM, 2009).

2. $6x^2 - 2 = 3x$

a. Set equal to zero and identify a, b, & c. Check _____	b. Plug a, b & c into the quadratic formula. Check _____
c. Simplify. (One number in the radical) Check _____	d. Find the two solutions. Check _____

Figure 6.27: Students used this formative assessment format to practice using the quadratic formula to solve equations. (Classroom Materials, April 13, 2011)

To initially model these skills, Nathaniel had the students list the three methods of solving quadratic equations and three examples (see Figure 6.28). He then solicited student input as to which method would be best to use for each model. The students wrote in their own words which method would be best and why (Field Notes, April 14, 2011), while Nathaniel summarized their thoughts on the overhead. So although the questions were used to elicit reasoning and sense making, the students did not fully engage in these skills until they used their understanding of mathematics to explain their mathematical choices.

Students had an opportunity to logically think through which method to use when solving equations in a variety of mathematical tasks. During all of these tasks, students were repeatedly asked to explain why they were using a certain method. For example, Nathaniel wrote in an email regarding a formative assessment on Thursday, “They would first have to write a sentence stating which method they are choosing and why” prior to

Best Method Mini Notes

We have discussed 3 methods to solve quadratics.

A. Square rooting

B. Factoring

C. Quadratic formula

Analyze the following. Which method is best! (sic)

1. $8x^2 + 4 = 36$

2. $5x^2 + 18x = -3$

3. $3x^2 + 4 = 13x$

Figure 6.28: Students listed the three methods to solve quadratics and then determined which would be the best methods to solve the three equations. (Classroom Materials, April 14, 2011)

completing the problem (Written Communication, April 14, 2011). During class he facilitated student reasoning by asking “why” when students suggested a certain method. He also told one student in particular, “You told me exactly what to do. Now explain why. You know it, but explain it now” (Videotape Notes, April 14, 2011). Nathaniel was listening to see if students were analyzing the problem and choosing a method based on mathematical reasons. He was checking to see if students were using a similar thought process for solving equations as Nathaniel and I had discussed in our initial planning (see Figure 6.18). If students could explain why they chose a certain method and then successfully perform those procedures, Nathaniel knew that they understood the mathematics. Nathaniel worked more closely with students who either struggled to successfully determine the correct method or correctly apply procedures. As he worked

with them one-on-one, he determined where their understanding was lacking and provided instruction on deficient skills.

Then on Friday, students created posters explaining when and why each method was used. When they finished writing their explanations, pairs of students were given three equations to determine the appropriate method to apply and then solve (Field Notes, April 15, 2011). The final 30 minutes of the week were spent having the students individually complete a quiz that included several procedural and higher-level questions on solving quadratic equations.

Reflecting on Solving a Quadratic Equation

Overall, both Nathaniel and I were pleased with the way we structured solving quadratic equations. We felt like we had created opportunities for students to use their reasoning skills and made connections to prior knowledge. As we reflected on our teaching of the solving quadratic equations big idea, we began to think about how rearranging the introduction of some of the skills would have potentially benefitted our students. The following is an excerpt from our post lesson debrief on April 14, 2011:

Amy: And I also wonder if, in the sequencing, if we would have done the quadratic formula right after factoring. I wonder if that would have helped with some of that deciphering more, you know? Put it together more clearly that those two (factoring and quadratic formula) are very tied. If you can't factor you have to do the quadratic formula.

Nathaniel: Yea. And that's what Cheryl (another Algebra teacher) did. She did it that way. And it makes sense. It would help with that aspect. I don't know.

Maybe that would have been the better way to go about it, just in terms of keeping...

Amy: ...those two connected. Because then you would have been working with three terms consistently. And then we go to these with two terms. For consistency...

Nathaniel: Yea. And then we could have just went, "Alright, factor this." It is a perfect square. And then go on to...probably, might have been all right doing it that way.

Amy: That might have helped scaffold their connections a little better.

Nathaniel: Yea. You are probably right.

Some students took longer to realize that the quadratic formula was the "back-up" method if factoring was not possible. Nathaniel and I discussed how teaching solving by factoring and solving using the quadratic formula back-to-back might have helped students make a stronger connection between those two methods.

Nathaniel came back to the discussion about the order of teaching the various methods later in our reflective conversation.

Nathaniel: Uh-huh. The more I think about it, going from factoring to quadratic (formula) would branch that thinking a lot easier. I think it would be less difficult for the kids to make that jump to no x 's in deciphering.

Amy: It would be more of a shock; I don't know if that's the right word. It would be a shock to their system because they have been using three terms consistently.

Nathaniel: Yea.

Amy: They have been seeing trinomials consistently. And now ... there is a major need. This looks different. There is a need. (Post Lesson Debrief, April 14, 2011)

After thinking about this idea for several minutes, Nathaniel agreed that teaching factoring, quadratic formula, and then square rooting would have facilitated a better scaffolding of understanding. We thought that having students reason through factoring or the quadratic formula first would have helped solidify that connection. Then we could have incorporated an equation that looked different from the others. Nathaniel also reasoned that incorporating quadratics without an x-term in between methods that have all three terms might have caused some difficulty for a few students. Through our reflective conversations, we determined that students would likely benefit in several ways if next time we were to teach methods of solving by factoring first, then go to the quadratic formula, and finally to square roots.

As we reflected on what and how we taught solving quadratic equations, Nathaniel and I noted a major connection we struggled to make. What is the meaning of a solution? Nathaniel and I talked about connecting the solution back to the original equation on several different occasions during our planning and debriefs. We thought that having students put the solutions back in for the variable would help students construct understanding of the solution (see Figure 6.29). Finding the solutions is an important aspect of solving, yet knowing what those solutions mean in terms of the equation is important as well. The meaning of a solution in terms of the equation was a concept we struggled to incorporate throughout our lessons on solving quadratic equations. One reason this was difficult was because once students solve a problem, they believe they are finished. The students were not routinely asked what the solution meant

and requiring them to substitute the solutions back into the equation would become another procedure rather than a way to show the meaning of the solution.

<u>Solve the Equation</u>	<u>Check the solutions</u>	
$3x^2 - 4 = 8$	$3(2)^2 - 4 = 8$	$3(-2)^2 - 4 = 8$
$\begin{array}{r} +4 \quad +4 \\ \hline 3x^2 = 12 \end{array}$	$3(4) - 4 = 8$	$3(4) - 4 = 8$
$\begin{array}{r} \underline{\quad} \quad \underline{\quad} \\ 3 \quad 3 \end{array}$	$12 - 4 = 8$	$12 - 4 = 8$
$x^2 = 4$	$8 = 8 \checkmark$	$8 = 8 \checkmark$
$x = \pm 2$		

Figure 6.29: Nathaniel and I discussed the idea of having students check the solutions of an equation in order to strengthen the students' understanding of a solution.

In the afternoon of April 8, after the first day of teaching quadratics, Nathaniel and I shared concerns about the meaning of a solution.

Amy: Now I do want to make sure when we come back and double check do they understand that, for example if they get x is 2, do they understand...are they sure that 2 is what works when I put 2 into that equation. I'm not sure we came back around to that like we should have.

Nathaniel: Yeah, that's right. We do probably need to come on Monday and be able to incorporate that in there because I think initially they got the idea, "Hey I found the solutions by guessing and checking. I know those work." But just because I solved them by factoring they might not have made that connection because we didn't really come back to it all. (Post Lesson Debrief, April 8, 2011)

When students were solving the word problems in order to find the missing number on the first day of quadratics, they understood that they were determining the one (or two) numbers that satisfied the problem. Yet when we transitioned to algebra equations, we did not make the connection back to our original goal of determining the number that satisfies the problem. Students did not connect the process of solving as the method for finding the value of the variable.

To compensate for our oversight on Friday, Nathaniel and I planned to incorporate a larger focus on the meaning of a solution during class on Monday. Nathaniel did mention solutions during class on Monday, April 11, all instances being noted here:

- Nathaniel asks the class, “What does $x = 5$ mean?”
- “How many solutions do I have?” Nathaniel does not emphasize what that means exactly. He does not explain that means all three numbers can be substituted in for the variable and make the equation equal.
- “How many solutions do I have here? That means 2 numbers should work when I plug it in.” Nathaniel then plugged in the two solutions to show that both work.

(Videotape Notes, April 11, 2011)

Monday evening Nathaniel sent me an email describing how class went. On the issue of developing understanding of what a solution is, he wrote:

We talked about what it means to have a solution - but not sure everyone was really on page with what it really meant. I tried to incorporate it as an extra part to our exit ticket but most just had enough time to get the initial solving part of it done. (Written Communication, April 11, 2011)

Nathaniel did attempt to emphasize the meaning of a solution during class, although the evidence is not clear as to whether the students gained a deeper understanding through these attempts or not.

Once we put all of the methods together, Nathaniel and I felt like the students were missing the big picture. They were reasoning through which method would be best to solve various quadratic equations, but we were not sure they understood that all of the methods were ways to help them determine the solution.

Nathaniel: I think they are able to determine (the correct method), but then they are like, “What do we do now?” Like today on the last two problems, a lot of people were like, “I square root this, but how do I solve it?” They just blanked out how to finish it off and stuff. So I don’t know. I just think we need another day of just wrapping up the ideas and concepts. I think they kind-of get the idea a little bit, in terms of differentiating how to use the methods. But then, I don’t know, maybe it just a lack of motivation a little bit to finish those or...I don’t know.

Amy: So you are talking about specifically at the end of class today where you were talking about how to decipher which method to use and then we gave them the two problems to just decipher and solve.

Nathaniel: Yep. And some people deciphered it and wrote it down. In their mind they were done even though we said solve it. It seemed like I had to keep pushing people. “Hey, I know you said factoring but why. Keep going. Solve it.” Well not factoring, but square rooting.

Amy: So they are missing the big, overall what-are-we-doing-this-for?

Nathaniel: Yea, maybe. They got all these tools, now use them.

Amy: Right. Ok

Nathaniel: Didn't you kind of feel like at the end like...I don't know...like yesterday I thought the radicals went fine with square rooting and stuff. And the day before. Now it just seems like they forgot it.

Amy: I think it is probably because they are missing that connection that all of these are ways to get to this one goal.

Nathaniel: Yea.

Amy: But we need to figure out which tool to use then do it. They are not just doing the same thing over and over again. (Post Lesson Debrief, April 14, 2011)

Nathaniel and I reasoned that one rationale for students struggling to continue solving when asked to first identify a method to solve might have been their lack of understanding for the purpose of solving quadratic equations.

As we continued to look back on everything the students learned about solving quadratic equations, we both were bothered by this gap in their understanding. In our debrief, we continued to discuss where the mis-step may have taken place.

Amy: I am not completely convinced we really got the goal of all of this to them.

The goal of everything - we are doing it to ...

Nathaniel: Find the x .

Amy: ...find the x . To find the solutions and what works for x . I think they are seeing them as these individual things. But all of these are getting us to the same goal, helping us meet the same goal. So I think if we would have...

Nathaniel: I think initially with factoring we did a good job with that zero product concept. But then afterwards maybe it started fading away a little bit.

Amy: Yea. We maybe got into too much procedure?

Nathaniel: Yea.

Amy: It is almost like, and we didn't talk about this, it was almost like we need to emphasis from the get-go everyday, all the time, our goal is to find what values work for x . That is what we are doing. Now we are going to try a different way to find them. You know? Like when we go to quadratic formula in our language maybe we should have said, "Well, we have one that isn't factorable. That isn't going to help us solve for x . We are not going to be able to find the values of x that way. So we need another way to find the values of x ." Really...

Nathaniel: Kind of reiterating that term.

Amy: Yea. Because I think some kids are...I am still not convinced that they know what, so they got x equals this, do they understand that as well as they understand when they are solving a linear and getting x by itself. I don't know if they are correlating it to linears. They may be. I am just not sure. I think that is something we could have done better. (Post Lesson Debrief, April 14, 2011)

Nathaniel and I agreed that emphasizing the meaning of a solution (the value of x) throughout the instruction could increase students' understanding and would be a priority when we teach solving quadratics again.

What Nathaniel Learned While Teaching Solving Quadratic Equations

While teaching students how to solve quadratic equations, Nathaniel learned a different way of introducing and connecting the various methods. He had previously

taught the skills in the order of the textbook and largely introduced the three methods independently. Through our consecutive coaching cycles, Nathaniel gained a new way to approach solving equations that emphasized connections and understanding. Besides gaining his own reasoning and sense making of quadratics, Nathaniel also gained a deeper knowledge of choosing specific problems to make a mathematical point.

Finding and using an example to make a specific mathematical point is one of Ball et al.'s (2008) mathematical tasks of teaching and is an element of teachers' specialized content knowledge (Ball et al., 2008). My work with Nathaniel suggests that specific mathematical examples are chosen to make various mathematical points. These mathematical points could include making connections to prior knowledge, showing students the necessity to learn a new mathematical skill, and addressing common student misconceptions. Many of Nathaniel and my collaborative conversations included determining these problems. We discussed the exact problems we wanted to include on warm-ups and the precise problems we wanted to use during direct instruction. In each of these situations, the examples we chose had a specific purpose and were chosen for a reason.

Nathaniel and I discussed how to introduce each method of solving quadratic equations during our initial planning. These discussions started with an idea about the concept and quickly produced a problem or example that would allow us to make the point we wanted to make. The initial mathematical point we focused on making was how the new content connected to prior knowledge. We did this when we introduced each of the methods of solving. When we initially introduced solving quadratic equations, students saw two linear equations being solved (see Figure 6.22). These problems helped

students see how solving quadratic equations compared to solving linear equations.

When we transitioned to solving by square root, we chose the problem $x^2 - 25 = 0$ so students could solve it by both factoring and square rooting. And as we introduced the quadratic formula, we chose a problem that helped students first determine the trinomial was not factorable. This problem was chosen to help students recognize how using the quadratic formula as a strategy to solve a problem was related to using factoring.

Occasionally we also chose problems to include in formative assessments that would help students make connections between previous knowledge and new knowledge. An example of this occurred on April 11 when we purposefully included a few difference of squares problems as students used dry erase board to practice solving by factoring (see Figure 6.30). By incorporating these problems into their formative assessment, we hoped students would be reminded of the difference of squares. Then the next day when we introduced solving by square rooting with a difference of squares problem, we thought the previous exposure would help students make connections between factoring and square rooting to solve.

5. Dry Erase Boards – Lettered Heads Together

$x^2 + 5x + 6 = 0$	$x = -3, x = -2$	<i>*DO*</i> $3x^2 - 300 = 0$	$x = 10, x = -10$
$b^2 - 7b = 18$	$x = 9, x = -2$	$2x^2 + 20x = 0$	$x = 0, x = -10$
<i>*DO*</i> $r^2 - 4 = 0$	$x = -2, x = 2$	$2x^2 + 4x - 30 = 0$	$x = -5, x = 3$
$x^2 - 8x = 0$	$x = 0, x = 8$	$x^2 + 7x = 8$	$x = 1, x = -8$
$2x^2 + 5 = 7x$	$x = 1, x = 2.5$	$3x^3 - 39x^2 + 36x = 0$	$x = 1, 12, 0$

Figure 6.30: Nathaniel and I intentionally included difference of squares problems in this formative assessment to make connects to solving quadratic equations by square rooting. (Lesson Plans, April 11, 2011)

In addition to choosing specific problems to emphasize the mathematical point of connected concepts, Nathaniel and I also chose examples to make the point that new knowledge needed to be learned in order to solve the problem. To introduce factoring to solve quadratics, students could solve the first example by using the guess and check method. The question read, “A number and its square is six. What is the number?” (Classroom Materials, April 8, 2011). Our follow-up question ($2x^2 + 3x = -1$) was the problem that we chose to make the point that guessing and checking was not a viable method and new mathematics would need to be learned to solve the equation. A similar problem choice was made when introducing square rooting to solve. The second problem we selected ($5x^2 + 5 = 20$) could not be factored into the difference of squares like the first example. This example allowed Nathaniel to then make the point that a new method needed to be learned in order to solve the quadratic equation. And when we introduced the quadratic formula, we intentionally chose the problem $2x^2 + 4x + 3 = 0$ because the students would quickly be able to recognize that it could not be factored (over the real numbers).

As mentioned, Nathaniel and I collaborated about this idea of choosing special examples to make mathematical points frequently during our initial planning for the chapter. These were conversations in which both of us brainstormed ideas and chose special problem. After numerous discussions, Nathaniel began to think about how to incorporate specific problems on his own. As Nathaniel began choosing problems to make a specific mathematical point, he started by using examples that addressed a common student misconception.

For example, Nathaniel and I discussed using a formative assessment on April 14 to help students remember when to use each method. Together we decided to show students an equation and they would need to vote on the best method to solve the equation: factor, square root, or quadratic formula. Due to lack of planning time, Nathaniel offered to come up with the problems for the activity. The next morning before class he showed me the problems he came wrote (see Figure 6.31). Students usually struggle to decide what method to choose when seeing a variety of quadratics. To help combat the common misconception that factoring should be used to solve all equations, Nathaniel explained that he first wanted to do just square rooting and factoring to emphasize that one type of equation will have an x-term and one will not. Then he said he wanted to start bringing in some equations that could only be solved using the quadratic formula to address the common student misconception that a quadratic with an x-term can always be solved by factoring.

f = factoring, r = square rooting, q = quadratic formula

1. $4x^2 - 12 = 34$	r	5. $8x^2 + 3x - 2 = 0$	q	*9. $3x^2 + 2x - 12 = x^2$	f
2. $2x^2 - 7x = -3$	f	6. $5x^2 - 2 = 3x$	f	10. $-12x^2 + 32 = -232$	r
3. $18 - 3x^2 = 25$	r	7. $2x^2 + 6 = -21x$	q	11. $3x^2 + 4x = -5$	q
4. $10x^2 + 12x = -2$	f	8. $2x^2 - 4 = 10$	r	12. $x^2 - 15x + 7 = 0$	q

Figure 6.31: Nathaniel chose these problems to have students vote on the best method to solve. He purposefully chose to emphasize factoring and square rooting initially.

(Lesson Plans, April 14, 2011)

Another example that demonstrates how Nathaniel learned how to choose specific examples to make a mathematical point was the problem he chose to include in the April 15 warm-up. Nathaniel explained to me that a common student mistake he witnessed on Tuesday was that students continued to square root a number even though there was no longer an x^2 . He wanted to incorporate a “find the mistake” problem into the warm-up to help students recognize their own errors (see Figure 6.32). The problem he chose to include had a specific purpose and was used to make the point that it is incorrect to square root only one side of the equation. Students quickly identified the error in the problem. After incorporating this warm-up and practicing more problems, Nathaniel and I no longer saw students making this mistake.

Warm-up	Name _____
1. Ted solved the following by square rooting. <u>Explain Ted's error.</u>	
<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">$\frac{2x^2}{2} = \frac{32}{2}$$x^2 = 16$$x = \pm 4$$x = \pm 2$</div>	

Figure 6.32: Nathaniel suggested using this problem for the warm-up because he noticed this was a common error among the class. (Classroom Materials, April 15, 2011)

Graphing Quadratic Functions

(Monday, April 18, 2011 – Tuesday, April 26, 2011)

During our initial planning, Nathaniel and I decided to teach graphing quadratic functions after students learned the various methods of solving quadratic equations. As we discussed how to approach graphing, we initially struggled with how to explain the vertex formula $\left(x = -\frac{b}{2a}\right)$. Even when collaborating with our colleagues, we could not figure out a way to describe why the vertex formula found the x-value of the vertex.

When searching in the textbook one day, Nathaniel found a way to explain the vertex formula by averaging two solutions using the quadratic formula (see Figure 6.12). This discovery opened numerous possibilities for teaching quadratic functions with reasoning and sense making. We had a way to explain the concept that we were most unsure about.

As previously explained, we taught solving quadratic equations from April 8, 2011 through Friday, April 15. We were ready to introduce graphs of quadratic functions the Monday following the six days of solving equations instruction. Graphing lessons occupied four full instructional days and a portion of two more classes (see Figure 6.33). Since we did not have school on Friday, April 22, the two partial days spent teaching graphing concepts were April 25 and 26. The introduction of graphing quadratic functions occupied approximately five instructional days total.

Monday	Tuesday	Wednesday	Thursday	Friday
18 Characteristics of quadratic graphs; vertex as midpoint	19 Graph using roots and the vertex as midpoint	20 Show formula for vertex; graph using $-\frac{b}{2a}$	21 Graph using $-\frac{b}{2a}$; parabola widths	22 No School
25 Parabola widths; <i>connect solving algebraically and graphically</i>	26 Vertical shifts; <i>determining functions by graph and equation</i>	27	28	29

Figure 6.33: Approximately five instructional days were spent teaching students how to graph quadratic functions.

Mapping the Terrain of Graphing Quadratic Functions

When Nathaniel and I began discussing how we wanted students to learn graphing quadratic functions, we once again started with the textbook and curricular materials we had. The objectives listed on the syllabus for the graphing big idea were explore different quadratic graphs and graph quadratic functions. Our approach to teaching the big idea of graph quadratic functions was similar to the way we approached solving quadratic equations. Nathaniel and I continued to place a large emphasis on sense making by continually discussing ways to connect new knowledge to mathematics the students already knew. And since we just finished teaching students how to solve quadratic equations, those concepts were now prior knowledge upon which new skills could be connected. Whenever possible, we also encouraged students to think, seek patterns, and

draw conclusions concerning the mathematics. Having students look at graphs, tables of values, or equations and make conjectures about what was happening mathematically facilitated much of the reasoning.

When determining how to teach graphing quadratic functions, Nathaniel and I agreed that it was important for students to recognize and interpret the key characteristics of the graph (NCTM, 2000). The actual graph, which is called a parabola, is a u-shaped curve that is symmetric (see Figure 6.34). The parabola can be folded on the axis of symmetry and the two halves will match. The highest or lowest point on the parabola is called the vertex and lies on the axis of symmetry. When a parabola opens up, the vertex is a minimum since it is the lowest point of the parabola. When a parabola opens down, the vertex is a maximum, or the highest point of the parabola. These were all elements of quadratic graphs that we wanted to recognize and understand ourselves and wanted students to understand.

Nathaniel and I also had numerous discussions about the zeros, or x-intercepts, of a parabola. We recognized that they are key elements of graphing quadratics and are connected to the solutions that are found by solving a quadratic equation when the value of the y-coordinate is zero. In the Algebra classroom, the terms zeros, x-intercepts, and solutions to the equation all refer to the same point(s) on graph (see Figure 6.35) and can be found by using factoring, square rooting, or the quadratic formula to solve the equation. Students should be able to not only use the zeros to investigate a function, but should also be able to create a graph using the zeros. (CCSSO, 2010; NCTM, 2000). Therefore Nathaniel and I wanted to be sure that students understood the importance of the solutions (zeros, x-intercepts) to a quadratic equation as they appear on the graph.

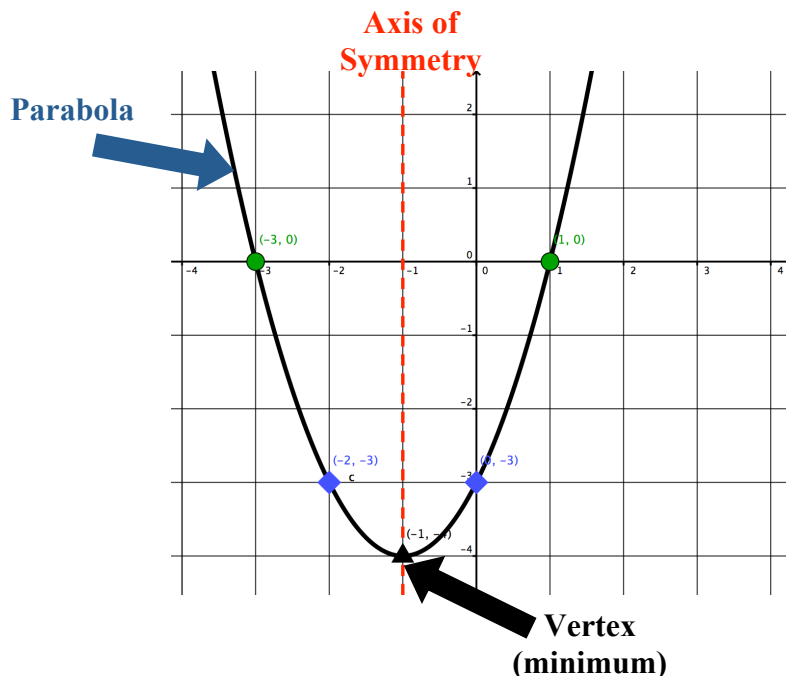


Figure 6.34: The graph of a quadratic function is called a parabola. A parabola reflects across the axis of symmetry. The vertex lies on the axis of symmetry and is the minimum or maximum y-value of the parabola.

Finally, Nathaniel and I recognized in our planning together that teaching the graphs of a quadratic function included using a variety of representations. Students should be able to connect the graph, equation, and table of values of a function, recognizing that all representations offer more information about the function (NCTM, 2000; Graham et al., 2010). By using the various representations of a quadratic function, students can use their reasoning and sense making skills as they investigate the various patterns and details.

Nathaniel and I contemplated all of the mathematical ideas included in the big idea as we planned and discussed the instruction devoted to graphing quadratic functions. We worked together to connect the new instruction to students' prior knowledge.

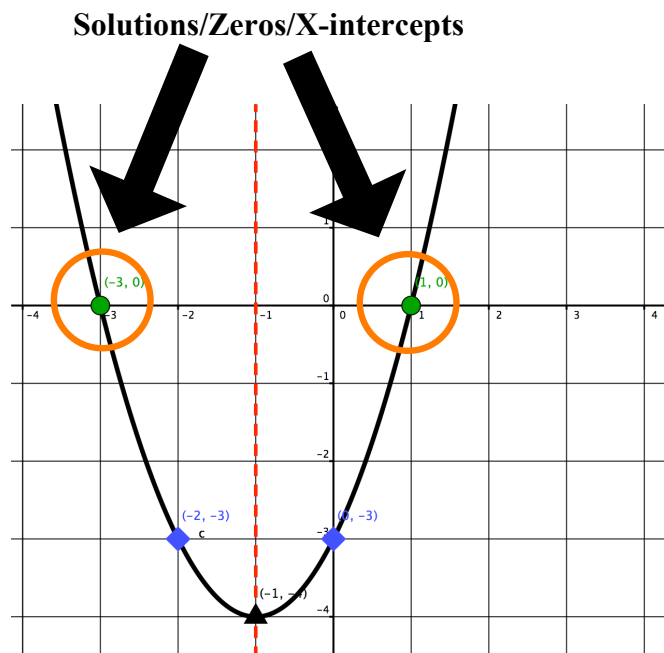


Figure 6.35: The x-intercepts of a quadratic function are called the solutions, or zeros, of the function.

Nathaniel and I recognized the important role the solutions would play in making the connection between solving and graphing quadratic equations. The characteristics of parabolas and the various representations of a function were also aspects of our instruction that we felt needed to be emphasized. After discussing our instructional plan, delivering the lessons, and then reflecting on what we taught and what the students learned, a map of our graphing quadratics terrain was created (see Figure 6.36). This map was a subset of our original quadratics map.

Explore Characteristics of Quadratic Graphs

On Monday, April 18 the students were introduced to quadratic graphs and tables of values. Rather than telling the students that quadratic functions produce u-shaped, symmetric graphs called parabolas and that the highest or lowest point is called the

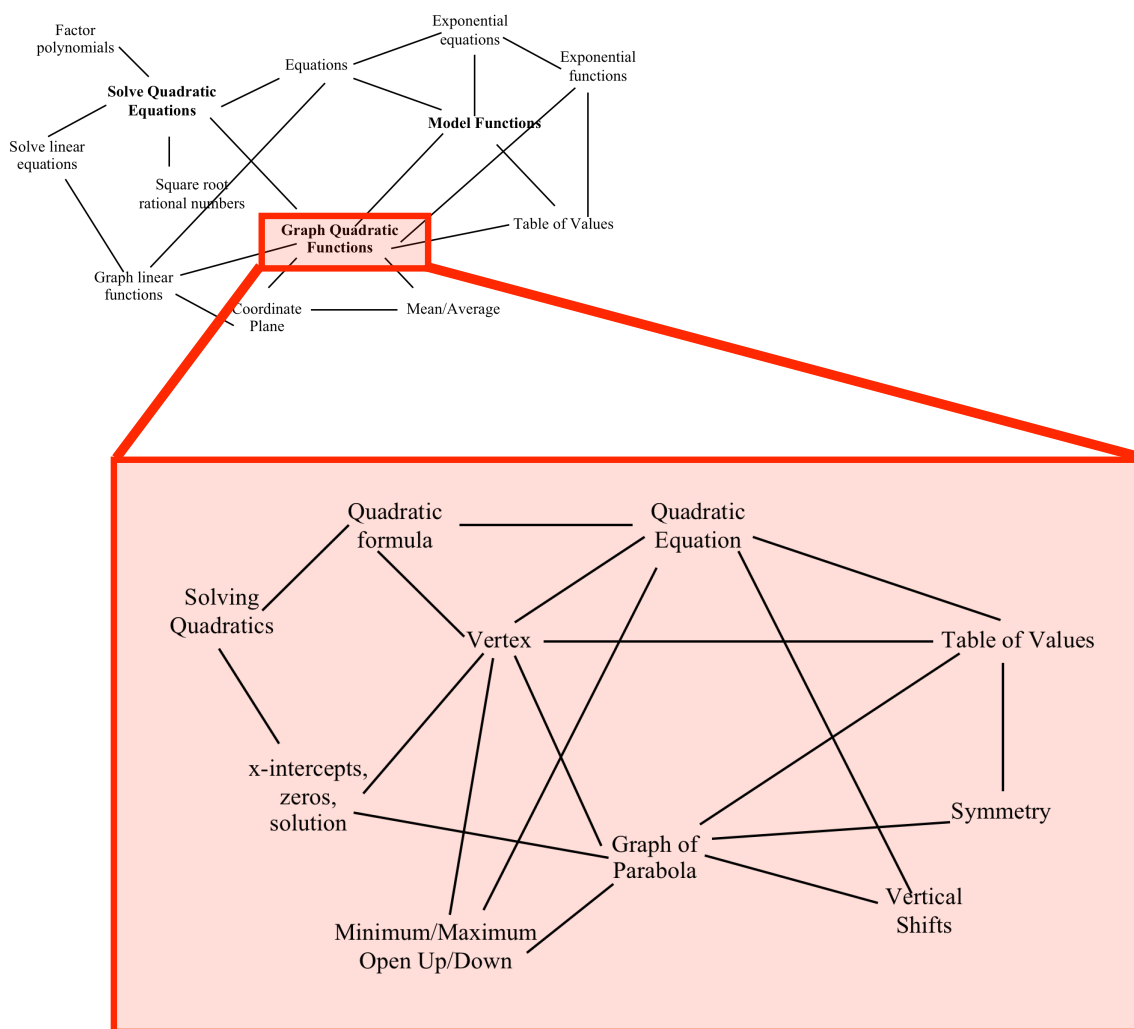


Figure 6.36: The map of our graphing quadratics terrain was a subset of our quadratics terrain.

vertex, Nathaniel and I thought this would be a good opportunity for students to make their own conjectures about quadratic graphs, equations, and table of values based on evidence. This was a way for the students to reason about the new types of functions (NCTM, 2009). Given a graph of a parabola, students were asked to place ordered pairs into a table of values. They did this with three different graphs. Then using a table of values, students plotted the points on a graph to create a parabola (see Figure 6.37). The

students created three graphs using a completed table of values and then answered the following questions: What do you notice about the quadratic graphs? What do you notice about the quadratic tables? (Classroom Materials, April 18, 2011)

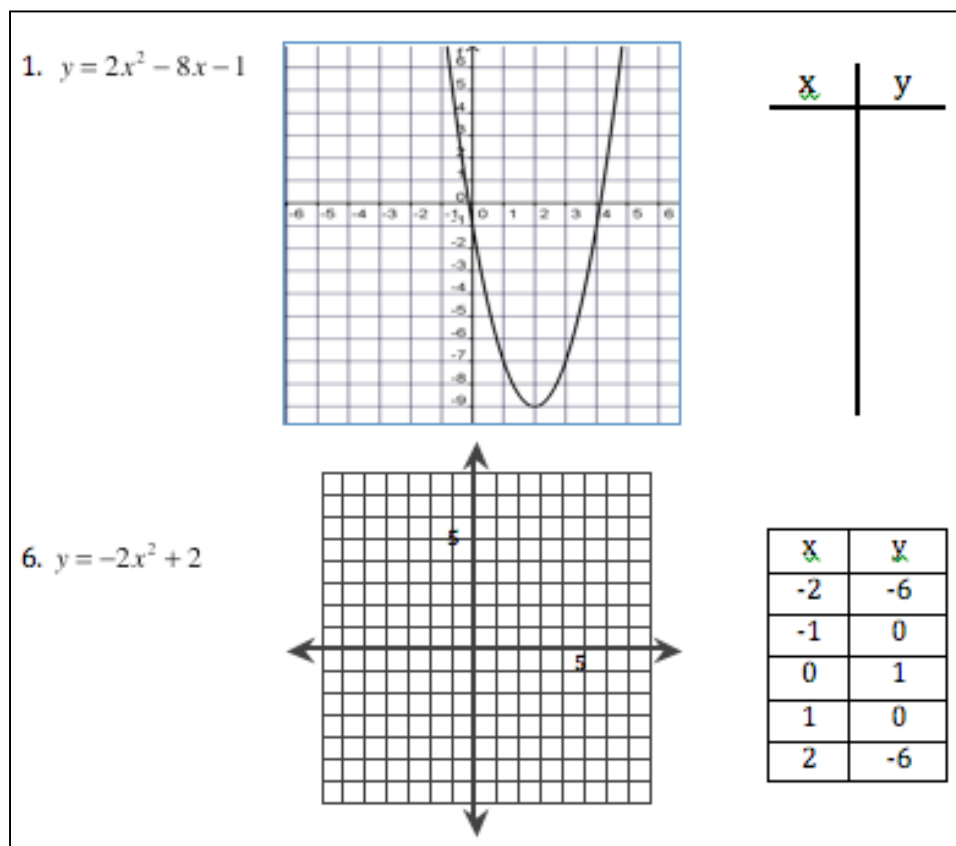


Figure 6.37: These example of graphs and tables were two problems completed by students to help them understand the characteristics of quadratic functions. (Classroom Materials, April 18, 2011)

Students wrote down the patterns they noticed. Phrases such as “pairs except for where it curves”, “u-shaped”, and “all have matches except the middle one” were used to describe the symmetry in both the graph and table. Since the students were finishing this

task at different times, I thought it would be a good idea to have students put their ideas up on the whiteboard. Having students' responses visibly displayed would also allow Nathaniel to pull together students ideas as he led a class discussion. I approached Nathaniel with this idea, explaining my reasoning for wanting to make the teaching move. Nathaniel agreed it would be a good idea. As students finished making conjectures about quadratic graphs and table of values, they recorded their observations on the whiteboard. Nathaniel then led a class discussion and highlighted the major characteristics of quadratic functions. Through this reasoning task, students came to their own conclusions about the symmetry of quadratic functions and determined that the point in the middle of the u-shape was significant.

During this activity, we also attempted to have students find patterns in the parabola opening up or down. It was another way to connect the equation with the graph, as well as an objective in our curriculum. The following questions were used to prompt student discovery.

9. Sarah can **look at the table** and determine if the graph will open up or down.

What does she see in the table that helps her make this conclusion?

10. Habib can **look at the equation** and determine if the graph will open up or down. What does he see in the equation that helps him make this conclusion?

(Classroom Materials, April 18, 2011)

When Nathaniel got to this point in the class discussion, student engagement was very low. No students had made a conjecture concerning the opening up or down of a parabola. Nathaniel tried to help students see the patterns within the table, getting them to visualize where the vertex would be in relation to the other points. Yet students

struggled to follow the discussion. Nathaniel pushed students to see that when the lead coefficient of a quadratic function is positive, the parabola opens up. And when the lead coefficient of the quadratic function is negative, the parabola opens down (see Figure 6.38)

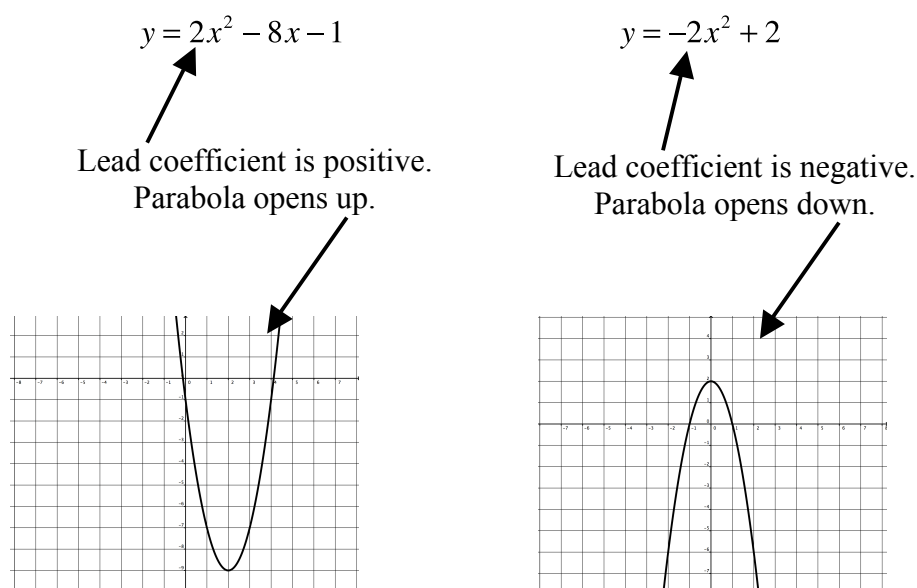


Figure 6.38: When the lead coefficient of a quadratic function (in standard form) is positive, then the parabola opens upward. When the lead coefficient is negative, the parabola opens downward.

The idea of a parabola opening up and down was again touched upon in the next two days in class. Nathaniel and I tried to continually ask students if an equation or table of values represented a graph opening up or down. On Wednesday, we used a formative assessment to help solidify the idea of having students determine if a function would open up or down, or given a graph determine if the lead coefficient would be positive or negative. It was not until Thursday, April 21, when Nathaniel and I incorporated the

terms minimum and maximum for the vertex that we realized we should have connected these two words to a parabola opening up or down much earlier. Maybe students would have made a better connection between a vertex being a minimum and the parabola opening up, or vice versa.

As a way to help students' connect what they understood about the characteristics of quadratic functions and to emphasize the connection between the various ways functions are represented, we gave students the three representations on Wednesday, April 20 (see Figure 6.39). Students were asked to explain their answer to the following question for each representation: "How do you know the table/equation/graph represents a quadratic function?" (Lesson Plan, April 20, 2011). Nathaniel and I thought having students write about the three forms of quadratics would increase students' reasoning habits while helping connect their own knowledge (NCTM, 2009). Students used words such as "reflect", "mirror", "symmetric", "parabola", "u-shape", " x^2 ", and "vertex" as they explained their understanding of quadratics.

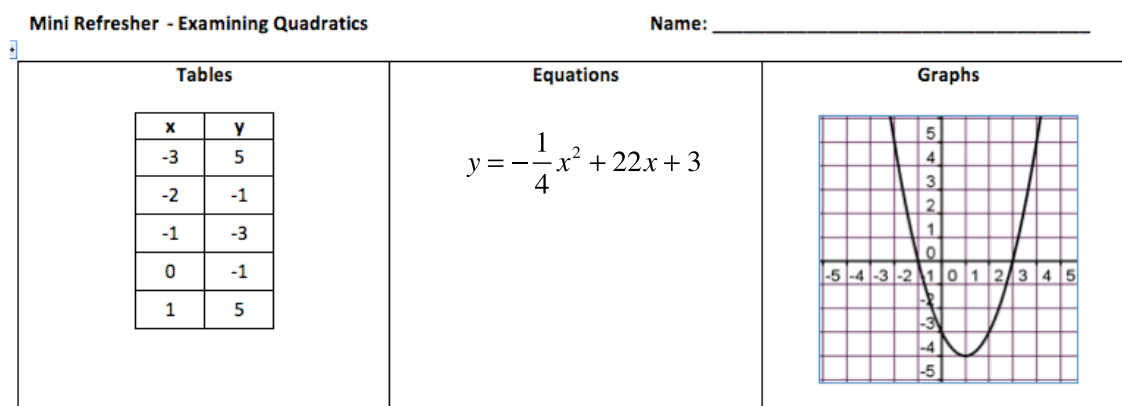


Figure 6.39: Students were asked to explain how they knew each of the three examples represented a quadratic function. (Classroom Materials, April 20, 2011)

When asked to do so, the students were able to draw connections between the three representations of quadratic functions. Nathaniel viewed this task as not only a way to check for student understanding, but also as a means for solidifying the relationship between a table, equation, and graph.

Nathaniel: Well I think it really gives them a good connection, like they can say they all have a vertex, they all go up and down like it just really makes it one thing ...It's just different forms of it and so just being able to build that concept and connection that it is all interchangeable - it is the same. I think that goes pretty far in terms of understanding and being able to retrieve something later in the future if you have a lot more schema they can connect to.

Amy: Why did you (discuss) the graph first?

Nathaniel: Because you can see the vertex there, and they can see the symmetry and being able to relate that... And I just went to the table after that. And it's the same thing, we had symmetry here, we had that vertex in the graph so we have it over here also and so I was just trying to make that connection - it's everywhere.

(Post Lesson Debrief, April 20, 2011)

Nathaniel felt that having students write about how they knew the table, equation, and graph represented a quadratic function helped students connect the knowledge of the three representations.

On Monday (April 25) and Tuesday (April 26), a mathematical task similar to the initial introduction to quadratic functions was used to help students recognize patterns in parabola widths and vertical shifts (see Appendices K and L). Rather than tell students that the width of a parabola depends on the lead coefficient of the equation or that the

constant term of a function determines the shift of a parabola in relation to the parent graph, we created mathematical tasks that encouraged students to reason about these patterns. In both instances the students worked with their team to complete the task, write about the patterns they noticed, and continue to reason through the connections between a quadratic function and graph. Nathaniel then brought the class together to discuss the students' findings and formative assessments were used to practice the newly discovered pattern. This was another way Nathaniel helped students explore the characteristics of quadratic functions.

Graph Using the Solutions (X-intercepts) and Vertex as Midpoint

As Nathaniel and I discussed how to teach students to graph quadratics using reasoning and sense making, we kept coming back to the idea of introducing graphing based on what students already knew and understood. We had spent the previous six days of class solving quadratic equations; so one major connection we wanted to make was that the solutions to an equation were the x-intercepts on the graph. Since students began Monday, April 18 noticing patterns in quadratic graphs and tables, they recognized that quadratic graphs are symmetric and the vertex is in the middle of the u-shaped graph (parabola). We used these two pieces of prior knowledge, along with their reasoning of distance between two points, to teach students how to graph a quadratic function.

Nathaniel and I chose to begin by viewing the vertex as the midpoint between the two x-intercepts since our goal was to teach graphing for understanding and to hopefully give them a way to reason through graphing a quadratic if they later did not remember the vertex formula. As a way to build that understanding, Nathaniel posed a question in the warm-up about distance (see Figure 6.40). We thought we could use this example to help

students see that in order to find the middle of two points you can use the mean. Then we would be able to make the connection between logically finding the midpoint to finding the vertex as the midpoint between two solutions. Most students found the distance between Angel's House and Carla's House (eight blocks) and some of those students took it a step further to say Bryce's house was four blocks away from both Angel and Carla. It was not until we questioned students about how far Bryce's house was from the school that a few students came up with the answer of seven blocks by adding the three blocks to Angel's house and then four more blocks to Bryce. As Nathaniel was discussing this problem with the class, and students were giving suggestions on how to figure out how far Bryce's house was from school, a couple students began suggesting finding the mean or average. We used this distance example to build future work of finding the vertex as the midpoint between the x-intercepts.

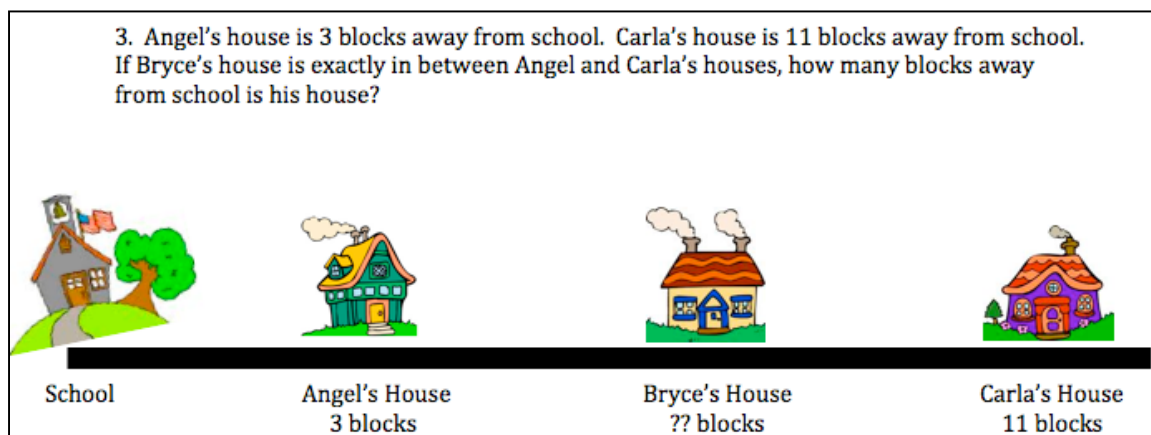


Figure 6.40: Students used their understanding of distance to find how far Bryce's house was from school. This thinking process paralleled the thinking used to find the vertex of a parabola. (Classroom Materials, April 18, 2011)

During our development of the graphing lessons, we decided students' prior knowledge of graphing linear functions and determining the x-intercept of a linear function would also be an important connection. After we verbally created a plan for bringing all of these ideas together, Nathaniel typed up a more formal outline of how we proposed to initially introduce graphing quadratics.

4) A Look At The Past - Linears

- a. Introduce by reminding we can graph this using the x and y-intercept.
- b. Emphasize that with linears, all we need are two points to graph.
- c. BIG IDEA
 - i. The line represents all solutions as ordered pairs that satisfy the equation $y=2x - 6$.
 - ii. When we found the x-intercept using $0=2x - 6$, we found the only solution to the equation.

5) A Look To The Future – Quadratics

- d. $y = x^2 - 6x + 5$
 - i. “Here’s a function. When you graphed the previous function we found the x-intercept. Can we do that here? How?”
 - ii. So $0 = x^2 - 6x + 5$

$$0 = (x - 1)(x - 5), \text{ therefore } x = 1 \text{ and } x = 5.$$
 - iii. Stress that these intercepts are points. “Is this enough? We need a third point. What might be a good point to have? The VERTEX!”
 - iv. “Where is the vertex located? Any ideas about how we find the middle?” (Lesson Plan, April 18, 2011)

Nathaniel and the students first found the x-intercept of the linear equation by plugging zero in for y (see Figure 6.41). (This came after a discussion about the y-value of the x-intercept always being zero since the x-intercept is on the x-axis.) He then asked the students how many more points they needed to graph the linear function and the students suggested finding one more point. They specifically suggested finding the y-intercept. Nathaniel emphasized the fact that only two points are needed since they knew a linear function produces a straight line.

Linear Function:

$$y = 2x - 6$$

x-intercept $(\quad, 0)$
 $\quad x \quad y$

$$0 = 2x - 6$$

$$\begin{array}{r} +6 \\ \hline \end{array} \quad \begin{array}{r} +6 \\ \hline \end{array}$$

$$6 = 2x$$

$$\frac{6}{2} = \frac{2x}{2}$$

$$3 = x \quad (3, 0)$$

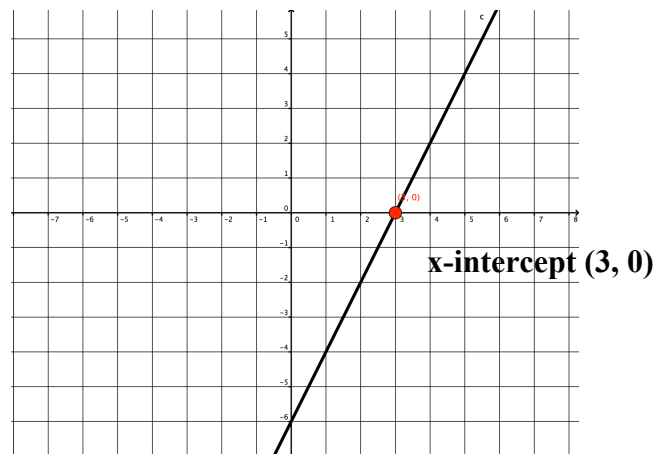


Figure 6.41: Students found the x-intercept of a linear function and discussed why the y-value of the x-intercept is zero.

Moving to quadratics, Nathaniel asked the class if they could begin graphing $y = x^2 - 6x + 5$ by finding the x-intercept(s) like they did with the linear function. Students were okay with that approach, so they put zero in for y in the equation and solved using their skills acquired the previous week (see Figure 6.42). Graphing using

the x-intercepts was an intentional way to connect students' prior knowledge of solving quadratics to the new knowledge of graphing. They produced two points, (5, 0) and (1, 0), which were the x-intercepts.

Quadratic Function:

$$y = x^2 - 6x + 5$$

x-intercepts (, 0)
x y

$$0 = x^2 - 6x + 5$$

$$0 = (x - 5)(x - 1)$$

$$x - 5 = 0 \quad x - 1 = 0$$

$$x = 5 \quad x = 1$$

$$(5, 0) \quad (1, 0)$$

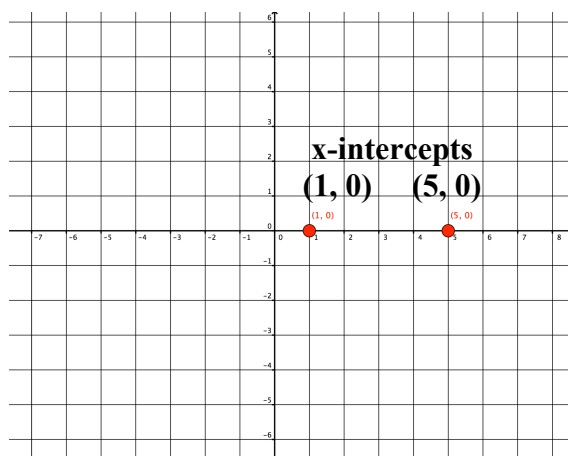


Figure 6.42: Students found the x-intercept of the quadratic function by substituting zero in for y and solving.

After finding the two x-intercepts, the student plotted them on the coordinate grid.

Then the remainder of graphing the quadratic function unfolded as described:

- Then Nathaniel posed the question, “Can we graph the function now that we know these 2 points?” A couple students immediately said no since it will be a u-shape. This led them to discuss what other point(s) would be necessary to graph. A couple students immediately shouted out the middle point (aka vertex).
- Nathaniel named this middle point as the vertex and reasoned through how to find the vertex with the students. (Finding the mean of the two roots - which

connected to the warm-up problem with the houses.) (Field Notes, April 18, 2011)

Nathaniel and the students used the same quadratic function to find the vertex by finding the mean (see Figure 6.43). Once the x-intercepts and the vertex were graphed, the students sketched in the remainder of the parabola.

Quadratic Function:

$$y = x^2 - 6x + 5$$

Mean and Vertex

$$x = \frac{1 + 5}{2} = \frac{6}{2} = 3$$

$$y = (3)^2 - 6(3) + 5$$

$$y = 9 - 18 + 5$$

$$y = -4$$

Vertex: (3, -4)

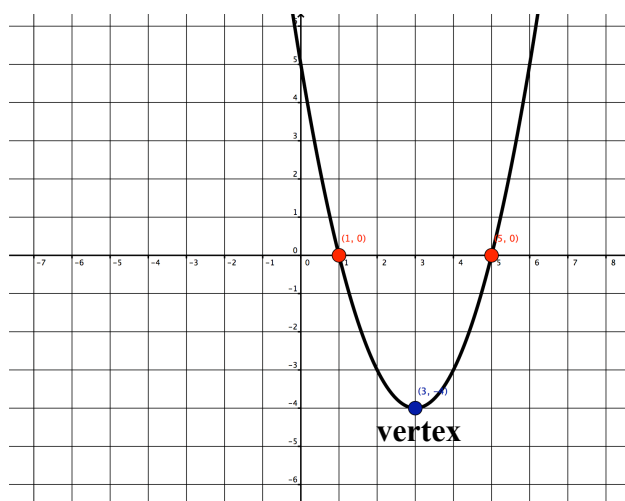


Figure 6.43: Nathaniel and the students reasoned that the vertex could be found by finding the mean of the two x-intercepts.

After class on Monday, Nathaniel and I were not confident students saw the connection that solving quadratics is the same thing as finding the x-intercepts on the graph. The students' actions indicated that they understood we were finding the x-intercepts, yet this connection was largely made with finding the x-intercepts of a linear function. They did not seem to be as intrigued by the fact that when we put zero in for y, the equation resembled all of the problems they had solved the previous week. We felt

like the students perceived graphing as a new skill rather than an idea being built upon solving. We did not see or hear anything from the students that indicated to us that students saw graphing as an extension of solving. To help strengthen this connection, we used the warm-up problems on Tuesday to address x-intercepts (see Figure 6.44). The students went back to basic knowledge of x-intercepts, which they learned first semester, by circling the x-intercepts on the graph and identifying the ordered pair. Students then wrote what is always true about x-intercepts (the y-value is zero) and the follow-up question to that asked them to then find the x-intercepts of a quadratic function (i.e., put zero in for y and solve). Student completed the scaffold tasks with little trouble.

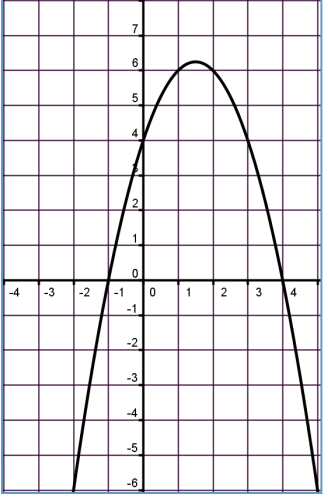
<p>Warm-Up</p> <p>1. Use the graph to the right. Circle and identify the x-intercepts.</p> <p>x-intercept: (_____ , _____)</p> <p>x-intercept: (_____ , _____)</p> <p>2. What is ALWAYS true about the x-intercept?</p> <p>3. Find the x-intercepts of the function $y = x^2 + 3x - 4$</p>	<p>Name: _____</p> 
--	--

Figure 6.44: Nathaniel used these questions to help students connect the idea that the solutions were the x-intercepts of the function. (Classroom Materials, April 19, 2011)

After briefly discussing the importance of x-intercepts, Nathaniel had the students turn their warm-ups over and graph the quadratic function they just solved. This was

another piece we included in the lesson to emphasize that by solving a quadratic equation, you have already found your solutions or x-intercepts. Nathaniel had the students put the x-intercepts into a table and graph (see Figure 6.45). Then through questioning of the class, Nathaniel and the students found and graphed the vertex by finding the mean of the two solutions (see Figure 6.46).

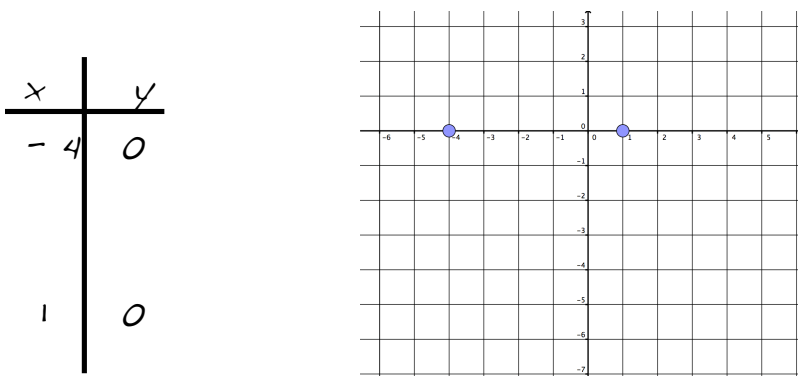


Figure 6.45: Once the students found the x-intercepts by solving the equation, they placed these x-intercepts in the table and on the graph.

$$-4 + 1 = -3$$

$$-3/2 = -1.5$$

$$y = (-1.5)^2 + 3(-1.5) - 4$$

$$y = 2.25 - 4.5 - 4$$

$$y = -6.25$$

x	y
-4	0
-1.5	-6.25
1	0

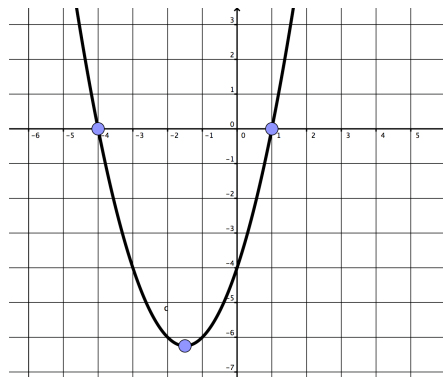


Figure 6.46: Students then found the vertex by calculating the mean of the two solutions and then substituting that x-value to find the y-value.

Nathaniel and I agreed that the x-intercepts warm-up that led into a refresher about graphing quadratics was instrumental in solidifying the connection between solving a quadratic equation and graphing the x-intercepts. The students could see how the solutions that were found by solving the quadratic equation were simply the x-intercepts of the parabola. When I asked Nathaniel in the post lesson debrief where he thought his students were at in terms of this connection, he responded:

Yeah I think most kids are good with it. I asked all the groups, “Why did you write this as zero equals (during a formative assessment)?” And people told me why and I heard that from a lot of students. And so, I know I made that a point to myself when I was checking around asking that question. Um, and so I think that’s pretty solid amongst most of the students. (Post Lesson Debrief, April 19, 2011)

Graphing quadratics by first finding the x-intercepts (by solving the equation) and then using the mean to determine the x-value of the vertex is not a mainstream or straight-forward approach to graphing. In fact, this method is impossible to use if the function has no solution (no x-intercepts) or only one solution (one x-intercept). Nathaniel and I were aware of these limitations, yet felt this initial approach to graphing was appropriate since our goal was reasoning and sense making. We did not want to give students a procedure for graphing. We did not want students to see graphing as something different to learn from what they already knew. We wanted to help build students’ understanding based on what they already knew. Therefore we chose to begin graphing using the solutions (x-intercepts) and vertex as the midpoint.

Graph Using the Vertex Formula

During our discussions about how we wanted to teach students how to graph quadratics, Nathaniel and I toyed with the idea of only focusing on graphing by finding the x-intercepts (by solving the equation) and then finding the vertex by figuring the mean of the two x-intercepts. The idea of connecting skills to solve quadratic equations, understanding of mean and distance, and applying what students reasoned about the characteristics of quadratic graphs was appealing to us as we decided how to teach graphing with reasoning and sense making. Yet we concluded that the vertex formula would eventually need to be taught since some functions have one or no x-intercepts and could not be graphed using the previous method.

The connection between finding the vertex using the average of the two x-intercepts and finding the vertex using the vertex formula was made on Wednesday, April 20. The students had already spent two instructional days determining the characteristics of quadratic graphs and graphing using the solutions and vertex (as the midpoint). To begin teaching students the vertex formula, we asked students to finish graphing the function $y = 5x^2 - 20x + 12$ (see Figure 6.47). We decided to do the majority of the work for them so we could focus students' attention on how they used the quadratic formula to find the two solutions. The students individually determined the solutions (x-intercepts) to be $x = \frac{20 + \sqrt{160}}{10} \approx 3.26$ and $x = \frac{20 - \sqrt{160}}{10} \approx 0.74$.

As the students found the solutions I went to the whiteboard next to the work Nathaniel was doing with the students. I constructed a coordinate plane with the two solutions plotted. Next to the solutions I wrote the generic quadratic formula (one with a

Introduction to $-b/2a$ Name: _____

A student began graphing $y = 5x^2 - 20x + 12$.
 Their work is shown below. Finish graphing the function.

$$y = 5x^2 - 20x + 12$$

$$0 = 5x^2 - 20x + 12 \quad a = 5 \quad b = -20 \quad c = 12$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{20 \pm \sqrt{400 - 4(5)(12)}}{2(5)}$$

$$x = \frac{20 \pm \sqrt{400 - 240}}{10}$$

$$x = \frac{20 \pm \sqrt{160}}{10}$$

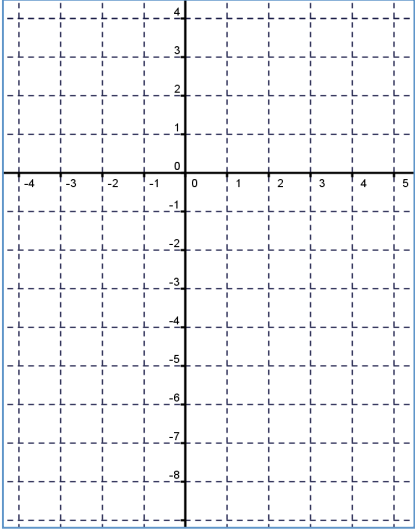


Figure 6.47: Students were asked to graph this quadratic function as a means of introducing the vertex formula. (Classroom Materials, April 20, 2011)

plus and one with a minus) that students used to obtain the two solutions (see Figure 6.48). Since I knew we would be using the quadratic formula to find the vertex formula, I wanted to be sure students saw the abstract connection to the concrete solutions they had just computed for the example. I was helping set the framework for Nathaniel's explanation of the vertex formula.

Once the students found the solutions, Nathaniel encouraged them to continue graphing by finding the mean of these two solutions, or the x-value of the vertex. The class found the mean to be two, and once they plugged that value back in for x they obtained a y-value of -8. Nathaniel then launched into showing students how to find the mean with the two generic quadratic formula solutions. As he was showing how to find

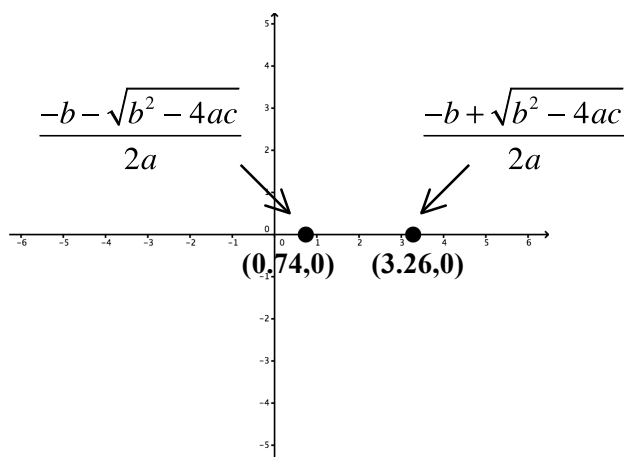


Figure 6.48: I made a sketch of the solutions, along with the generic quadratic formula corresponding to each solution, on the whiteboard.

the mean of the two algebraic solutions, I wrote the concrete example above Nathaniel's work to help students make the connection between what they just did with numbers and what Nathaniel was doing with variables (see Figure 6.49).

$$0.74 + 3.26 = 4 \qquad 4 \div 2 = 2$$

$$\frac{-b - \sqrt{b^2 - 4ac}}{2a} + \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-b + -b}{2a} = \frac{-2b}{2a} = \frac{-b}{a} \qquad \frac{-b}{a} \div 2 = \frac{-b}{2a}$$

Figure 6.49: With the numeric example above, Nathaniel showed the algebraic work to find the vertex formula.

Nathaniel finished showing students how the vertex formula $\left(x = \frac{-b}{2a}\right)$ was

ultimately what they had been doing all along by finding the mean of the two solutions.

He then plugged in the values for the original equation to show that the formula got them

the same vertex they had previously found using the other method (see Figure 6.50). The students then completed a sentence at the bottom of their notes sheet that said, “Big Idea:

We can find the vertex first with... $x = \frac{-b}{2a}$ ” and used the new formula to graph a new function.

$$y = 5x^2 - 20x + 12 \quad \begin{array}{l} a = 5 \\ b = -20 \\ c = 12 \end{array} \quad x = \frac{-(-20)}{2(5)} = \frac{20}{10} = 2 \quad \checkmark$$

Figure 6.50: Nathaniel used the vertex formula to show how he obtained the same x-value as when he calculated the mean.

After the lesson, I asked Nathaniel how he thought the students did with connecting their previous method of finding the vertex with the vertex formula. He responded:

Um, I think a lot of it made sense to what we were doing especially when we kind of started more concrete. We got our roots and then you going up to the board drawing arrows to what each part represented. I think that was nice. Um, I think once we got into the discussion, well when it became a little more (difficult), I think some people got lost. But there were people that were following along with what we were doing and it made sense. So yeah, adding those together and averaging them - that's something we can do. And so, I think that connection wasn't just “voila here it is”. I think it made sense to those that could follow

along with the algebra. And the algebra wasn't that complex. "Opposite (negative and) positive - oh cancel them out." And so I kind of had to guide them a little through that but overall...Add this one with this one, divide it by two and yeah that should be where our vertex is, our axis of symmetry...if you just went abstractly right away without something to connect it to, it would be confusing. Even when we're learning things it's nice to see something like what you know.

(Post Lesson Debrief, April 21, 2011)

He was positive about the connections that were made to the concrete example, yet noted that some students did not follow along with the algebra manipulation.

The next day, Thursday April 22, Nathaniel helped students restate the two ways they had learned to graph quadratics by having them complete two flow charts (see Figure 6.51). A student first offered the vertex formula as a method. Nathaniel asked students questions about how to find the vertex and the axis of symmetry using this method. He then inquired about the first way students learned to graph. After listing the progression of that method (x-intercepts, vertex, then graph), Nathaniel led a discussion about the benefits of each method. He discussed the fact that a function may not always have solutions, therefore graphing using a method that asks one to find the solutions first would be impossible. A few students asked him why he even taught them the first way if the second method (vertex, two points, then graph) always worked. Nathaniel talked to the class about the importance of building understanding of the axis of symmetry, the solutions, and the reasoning behind $x = \frac{-b}{2a}$.

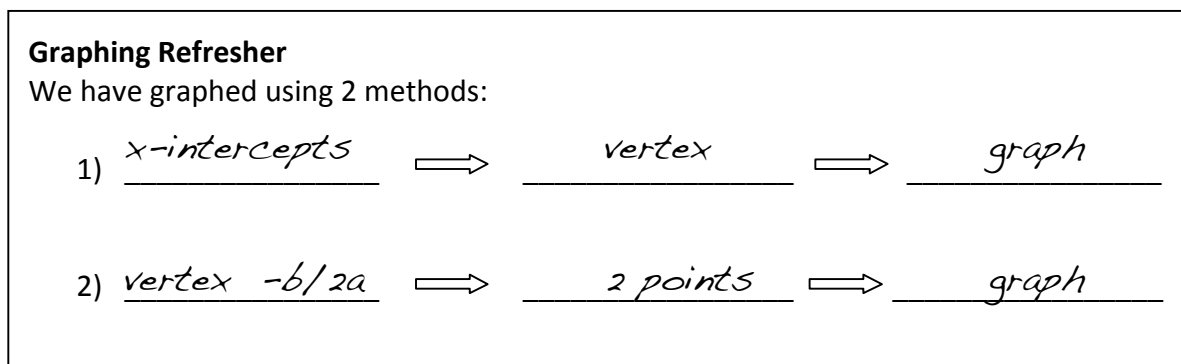


Figure 6.51: Students used the flow chart to restate the two methods for graphing quadratic functions. (Classroom Materials, April 21, 2011)

After determining and discussing the two methods of graphing, Nathaniel asked students to graph a quadratic function by finding the vertex first. Students graphed the function. When they were finished, Nathaniel asked the students to identify the solutions on the graph. Students quickly responded that there are not solutions since the parabola does not cross the x-axis. Nathaniel made the point that they could not have graphed this function by first finding the x-intercepts because there are no x-intercepts. He had specifically chose a no solution graph to reiterate why learning the new method of graphing (using the vertex formula) was necessary.

Reflecting on Graphing Quadratic Functions

As Nathaniel and I reflected on teaching graphs of quadratic functions, overall we were pleased with the decisions we made about how to introduce and connect the concepts of graphing. The fact that we had a connection and explanation for the vertex formula seemed to bring graphing all together, which Nathaniel thought would not have happened if we would have followed the textbook. In an article for a statewide

mathematics teachers' organization, Nathaniel wrote about our experience teaching quadratics with reasoning and sense making. He explained that with our approach to graphing, his students "...had come full circle, connecting all the ways to solve quadratics algebraically with how to graph. The text could not make such a connection because they decided to start solving quadratics with the vertex formula $x = \frac{-b}{2a}$,"

(Written Communication, June, 2011).

We felt like the students built a lot of reasoning skills, which Nathaniel explains in a post lesson debrief.

Amy: Compare this (approach to teaching quadratic functions) to maybe a teacher who came in first day and said, "We're going learn how to graph a quadratic. First thing you have to do is find this vertex. $\frac{-b}{2a}$ is how you do that.

Then you find these..." What would be the benefits of that? And what would be the limitations of that compared to what we're doing?

Nathaniel: Um, I guess the benefits of it would be to get into the process faster I guess. Um, in terms of practicing your algebra more readily, you got to fine tune (computation and substitution skills). But the limitation is like...where did that come from? Where did that $\frac{-b}{2a}$ come from? And I think that is something we are

going to be able to, in the past we weren't able to address at all and now you can.

And if anything we give them a sure-fire plan (unless there's no solution) of how to find the vertex. And so if they forgot $\frac{-b}{2a}$...uhh you know you might be out of

luck to find the vertex unless you can somehow reason. And our kids can reason

how to find the vertex, which is pretty nice. So when you think about your ACTs or tests in the long term, are you going to remember $\frac{-b}{2a}$ or are you going to remember “Hey vertex in the middle. I can average that. I know how to find the middle”? So, we are giving them skills that they can piece things together I think a lot more easily.

Amy: As opposed to memorizing a formula that is only applied in this situation

Nathaniel: Yeah, so some reasoning skills are built! (Post Lesson Debrief, April 19, 2011)

At times throughout the approximately five days spent on graphing quadratic functions, Nathaniel seemed frustrated with students’ computational mistakes when substituting in a value for x to get a y -value. Nathaniel voiced these frustrations in a post lesson debrief on April 21. That day, students used the vertex formula to graph and they were computing several y -values.

I think the big thing with graphing is the computation stuff is so hurting (us). I think that’s kind of one of the things with working backwards is you initially start graphing right away, you have to be able to plug stuff in and practice. And like here we haven’t had to practice that a lot. And so I think that’s hurting us in that respect. But conceptually I really like what we are doing overall. It’s just that the computation stuff itself - we just have to practice a lot more or with this stuff.

(Post Lesson Debrief, April 21, 2011)

Nathaniel clearly explained that he would rather have deep understanding of the concepts along with computational mistakes as opposed to the alternative. For future teaching of

graphing quadratics, we discussed ways to help students with the computation, including more mental math to avoid computation errors.

As I reflected on the mathematics students in Nathaniel's class were doing as compared to my own classes in the past or other Algebra classes, it was apparent that Nathaniel's students had a deeper understanding of core graphing concepts. I explained this to Nathaniel.

Yeah I think, I don't think I've seen a group of kids conceptually understand a graph of a quadratic as much as these kids. They know the axis of symmetry. They know what it is. They know the purpose. They see why we have it. The vertex – they see why opening up and down, um... That's really solid. (Post Lesson Debrief, April 21, 2011)

Nathaniel agreed that his students had a rich conceptual understanding, and correlated their understanding of axis of symmetry to why they are not doing as much substituting to find values as they have in the past. Since his students developed a deeper understanding of symmetry within quadratic graphs, they also understood how to find one point and reflect that across the axis of symmetry to find another point on the graph (see Figure 6.52). In Nathaniel's words, "They have enough math power where they don't need to solve both" (Post Lesson Debrief, April 21, 2011).

What Nathaniel Learned While Teaching Graphing Quadratic Functions

The collaboration and teaching of quadratic functions fostered numerous learning opportunities for Nathaniel. Most prevalent was Nathaniel's growth in his knowledge of

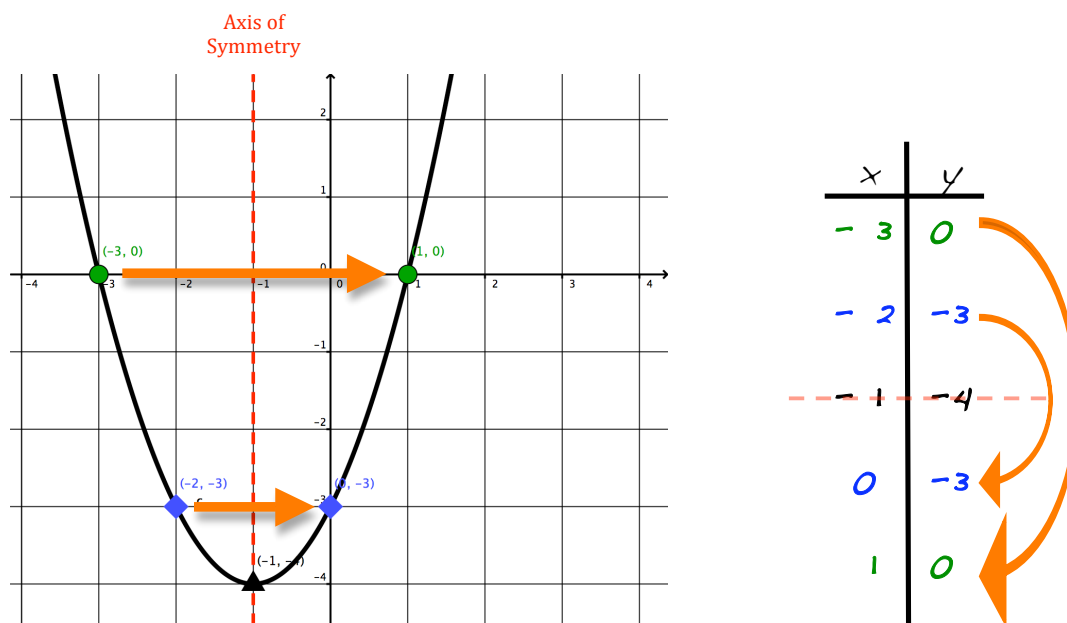


Figure 6.52: Nathaniel's students were able to use their understanding of symmetry to reflect solutions over the axis of symmetry in the graph and the vertex in the table of values.

presenting mathematical ideas. Nathaniel's learning about presenting a mathematical idea was not isolated to the week spent on graphing quadratic functions, yet is best demonstrated during this timeframe.

While helping his students learn about the graphs of quadratic functions, Nathaniel was also learning. One of the teaching skills Nathaniel improved over the course of this week of instruction was his ability to teach mathematical ideas. Ball et al. (2008) categorizes this skill (presenting mathematical ideas) as one of the sixteen mathematical tasks of teaching that are part of a teacher's specialized content knowledge. The previous year Nathaniel presented graphing a function to students by giving them the vertex formula immediately. He did not explain how the formula was mathematically constructed or the immediate connection of solutions, but instead presented graphing

quadratic functions as a process that began with using $x = -\frac{b}{2a}$. Nathaniel explained that if we had not engaged in the intense, consecutive coaching, he would have continued to present graphing by going “with the flow of the book” (Post Lesson Debrief, April 28, 2011). He would have continued to teach graphing through the formulas and procedures outlined in the textbook and would not have made an effort to present the mathematical idea of graphing quadratics.

The way Nathaniel taught quadratics this year began with his own learning about the mathematical idea of graphing. The ways we discussed the mathematics and planned instruction were centered on the big idea. Nathaniel and I discussed ways we could present the idea of graphing so students could understand the vertex formula. Neither of us wanted to present students with the formula and a list of procedures. Nathaniel described these conversations:

We did a lot of... we had a lot of dialogue - a lot of conversation in terms of how to go about teaching it. Um, trying to answer the whys.... And just the approach of how we are going to do it. (Final Interview, May 25, 2011)

Nathaniel explained how our dialogue was focused on the whys of the mathematics. We spent time discussing why the formula $x = -\frac{b}{2a}$ produces the x-value of the vertex and how that mathematical idea could be taught to students. By having a different focus than the textbook, Nathaniel and I were able to help him learn how to teach the idea of graphing quadratics rather than teach the mathematical procedure as he did in the past.

As Nathaniel and I had these discussions about the mathematical idea of graphing quadratic functions, we decided the students would build greater understanding if we

initially presented graphing as an extension of solving quadratic equations. Graphing would be a representation of the solutions to an equation. Presenting graphing in this fashion was a significant shift from presenting graphing through the vertex formula and procedures as Nathaniel had done the previous year. Nathaniel was able to make this change because we participated in thorough discussion about how graphing could be presented as a mathematical idea. In order to enhance his special content knowledge of presenting mathematical ideas, Nathaniel first needed to recognize what was involved in representing graphing as an extension of solving equations. This process evoked a second mathematical task of teaching: Recognizing what is involved in using a particular representation (Ball et al., 2008).

Presenting mathematical ideas (rather than a mathematical formula or procedure) entails a teacher first recognizing what is involved in using a particular representation. Nathaniel was able to modify how he presented graphing quadratics once we discussed what was involved in graphing as a representation of solving quadratic equations. Nathaniel realized that teaching students to first graph the solutions and then find the vertex by averaging the two solutions drew a strong connection to students' prior understanding of solving quadratic equations and x-intercepts of linear functions. Our planning helped Nathaniel decide that students would need a strong understanding about the characteristics of parabolas in order to understand the representation we were choosing to use. The students would need to recognize the symmetry of parabolas and the important role of the vertex in order to graph using the x-intercepts and vertex.

Through our discussions, Nathaniel also recognized that a limitation to graphing the x-intercepts first was that not all functions have solutions (Field Notes, April 18,

2011). Some parabolas do not cross the x-axis and therefore finding the mean of the solutions to determine the vertex would be impossible. And other functions may only have one solution, meaning the vertex was on the x-axis. Nathaniel recognized that in these instances, our initial graphing approach would not be useful. He used this understanding to choose specific problems to use during instruction to not only practice graphing, but then to show students that a mathematical way to calculate the vertex (i.e., vertex formula) is necessary.

After planning and recognizing what may be involved with initially presenting graphing as a representation of solving, Nathaniel taught graphing quadratic functions. He introduced graphing as an extension of solving equations, which allowed his students to not only connect the new knowledge to prior knowledge, but to also build the idea of what it means to graph. Nathaniel organized his instruction so that the vertex formula could be taught with meaning. Students used their understanding of x-intercepts, solving quadratic functions, finding the mean of two numbers, and linear functions to graph quadratics. He also allowed his students to make their own conjectures about patterns within quadratic tables, graphs, and equation. All of these smaller elements came together to create how Nathaniel presented the big picture mathematical idea of graphing quadratic functions.

Since this graphing representation was a new approach for both of us, we did not anticipate the issues we would experience with transitioning students to using the vertex formula. Nathaniel had found a way to help students understand the meaning behind the vertex formula. He used the quadratics formula, a concrete example, and students' prior knowledge of finding the vertex to show why the vertex formula was $x = -\frac{b}{2a}$. Yet

when we planned to use this representation we did not recognize that students would struggle to change their thinking about graphing. The students were somewhat confused with the idea of graphing by first determining the vertex since they had previously found the vertex last. After teaching graphing with this representation, Nathaniel recognized that students may need help seeing the vertex formula. He also recognized that he needed to clearly communicate with students why determining the vertex using the vertex formula is a useful and necessary tool. This new knowledge adds to Nathaniel's understanding about what is involved in using this particular representation. Nathaniel will be better prepared to present the mathematical idea of graphing quadratic functions next time since he now has an even better understanding of what presenting graphing as an extension of solving entails.

Connecting Concepts

(Monday, April 25, 2011 – Friday, April 29, 2011)

The final four instructional days leading up to the quadratics summative assessment were spent practicing the mathematical skills and connecting concepts (see Figure 6.53). Nathaniel and I decided to construct lessons that allowed students to get the repetition they needed to automatize the mathematics. Formative assessments were implemented that allowed students to demonstrate their understanding and to give us teachers the opportunities to address individuals' misunderstandings. Yet we also wanted to be sure we were tying up loose ends and making connections among new skills and between new and prior knowledge. By making these connections between algebraic and geometry representations, as well as the connections between various representations of

linear, quadratic, and exponential functions, we hoped to keep calling attention to the big picture of quadratics.

Monday	Tuesday	Wednesday	Thursday	Friday
25 Parabola widths; Connect solving algebraically and graphically	26 Vertical shifts; Determining functions by graph and equation	27 Application problems; determining functions by table	28 Review quadratics (solving and graphing)	29 Last Day of Quadratics: Students tested on quadratics

Figure 6.53: The final four instructional days were spent connecting concepts within the quadratics chapter.

A large focus during the last portion of our quadratics lessons was on connecting the algebraic solving and the graphing of quadratic functions. The connections between those two representations are habits of reasoning and are standards suggested by NCTM (NCTM, 2009, Graham et al, 2010). Nathaniel and I did not need to remap the terrain for the last four days of instruction. Instead, we focused on the connections within our map of the entire quadratics concept (See Figure 6.13), specifically the connections between solving quadratic equations, graphing quadratic functions, and modeling functions. The connections were emphasized either directly with tasks that emphasized the relationship or implicitly with application story problems.

A second focus during the last week of quadratics was given to the last big idea on the syllabus: Model linear, quadratic, and exponential functions. Nathaniel and my goals were to get students to generalize patterns in functions, use a variety of representations (equation, table of values, graph), and compare properties of functions the

students had learned throughout Algebra. All of these ambitions were also subsets of NCTM standards and included in NCTM's habits of reasoning (NCTM, 2000; 2009).

Solving Quadratics Algebraically and Graphically

On Thursday, April 21 Nathaniel and I sat together and listed the concepts in which students could strengthen their understanding. Although there were a few ideas that we had not yet addressed, such as vertical shifts of parabola, we were more concerned with the big picture connections that students were still missing. One of those concepts was the idea of solutions. After we taught students how to solve quadratic equations, Nathaniel and I thought the students' understanding of what a solution meant was weak. We thought this because only a couple students could identify the solutions on a graph when asked. When Nathaniel originally introduced graphing quadratic functions, he emphasized the meaning of a solution and connected it to the x-intercept on the graph. Yet Nathaniel and I had not put emphasis on solutions since the previous Monday (April 18). Therefore we decided focusing on the solutions of both equations and functions would help strengthen students' understanding of solutions as well as built a stronger connection between the algebraic and graphic representations of quadratics.

To strengthen this connection and give students more practice applying their skills, we had students participate in a pairs compare activity on Monday, April 25. The students were put into pairs; one partner solved the problem algebraically while the other partner graphed. When both students were finished, they compared their solutions with one another (see Figure 6.54). Nathaniel and I traveled throughout the room asking students questions to check for students' understanding of the concept of solutions. We asked questions such as: How do you know that is the solution? What does the solution

mean? How do you know the solutions are the x-intercepts? This mathematical task also emphasized graphing as a method for solving a quadratics. The use of graphing as a method for solving was one of the new ideas we discussed prior to teaching quadratics. Students gained better understanding about how a solution can be found. By the end of this activity, all of the students knew that graphing and identifying the x-intercepts would elicit the same solutions as solving the equation algebraically. Nathaniel and I were both pleased with the students' understanding of the solutions concepts and were happy that students got extra practice graphing and solving.

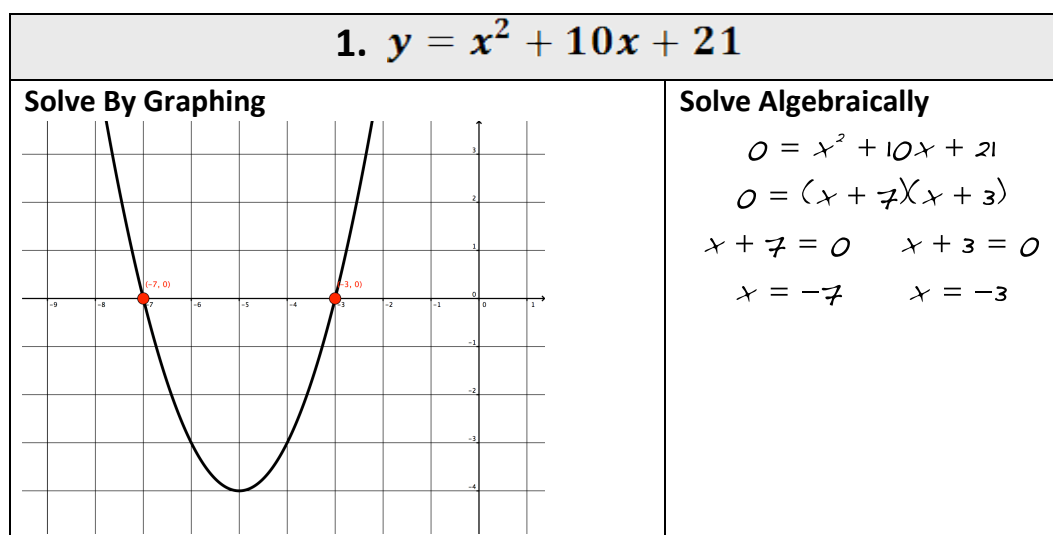


Figure 6.54: One student solved the function by graphing, while another students solved the equation algebraically. The mathematical task was used to emphasize the connection between graphical and algebraic representations of quadratic functions. (Classroom Materials, April 25, 2011)

Comparing the Various Types of Functions

Quadratics was the third type of function students had learned in Algebra. Linear functions were taught towards the end of the first semester and exponential functions were taught in February. The final big idea on the Algebra syllabus for chapter 10 was Model Linear, Exponential, and Quadratic Functions, so Nathaniel and I were expected to have students compare the three types of functions. As we discussed what function concepts students already knew and what skills still needed to be taught, we concluded that the students did not need to be retaught each type of function. We felt the best instruction we could give the students would be to pull all three types together to compare and contrast them.

Since we were both confident students could recall the differences between the graphs and equations of the three functions, we started with those two representations. A significant amount of time was spent on defining characteristics of the graphs and equations when the functions were initially taught. To connect to the mathematics they were most recently immersed in, we used a warm-up question on Tuesday, April 26 (see Figure 6.55). By asking which graph represented a quadratic function, Nathaniel was able to launch the class into a discussion about what students knew about linear and exponential functions. Students offered phrases such as “constant rate of change”, “straight line”, “parabola”, “u-shaped”, and “sudden increase/decrease” (Field Notes, April 26, 2011).

Nathaniel and I thought comparing the tables of values of the three types of functions would require a little more attention. The constant rate of change in a linear table was deeply embedded in students’ knowledge due to the large amount of time and

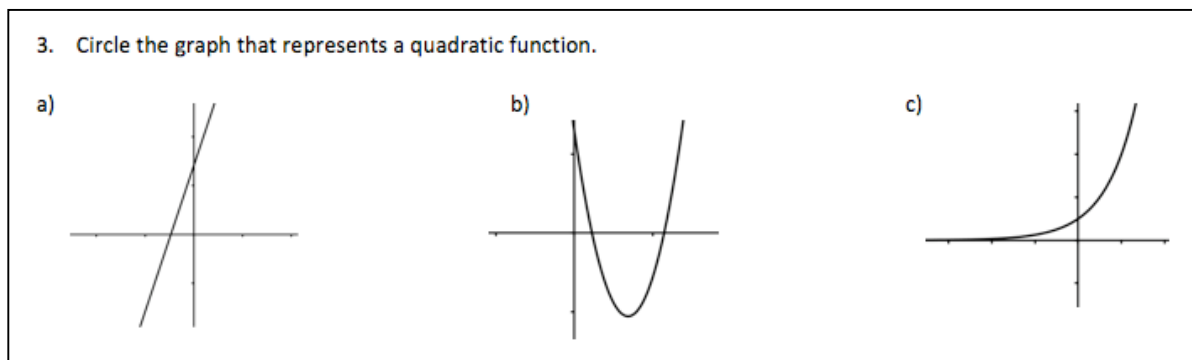


Figure 6.55: Students were asked to identify the quadratic function and then compare the other two functions. (Classroom Materials, April 26, 2011)

emphasis put on this pattern first semester. Yet the patterns for quadratic and exponential tables had not been explicitly taught. On Wednesday, April 27 students used their reasoning skills to find patterns in the tables of linear, quadratic, and exponential functions (see Figure 6.56). Student completed the tables and found patterns in the tables. Then Nathaniel facilitated a class discussion about the patterns in the tables of values for linear, quadratic, and exponential functions.

While comparing linear, exponential, and quadratic functions, we continually asked students to explain how they knew what type of function the representation modeled. Students were rarely asked to simply identify the correct function. Nathaniel continually asked students to explain their reasoning either verbally or in writing. For example, students were asked to write responses to the following questions at the end of class on April 26:

2. Explain how you can determine if an equation represents a linear, quadratic, or exponential function.

Mini Notes: Examining Tables

1.

$y = 2x$	
x	y
-2	
-1	
0	
1	
2	

2.

$y = x^2$	
x	y
-2	
-1	
0	
1	
2	

3.

$y = 2^x$	
x	y
-2	
-1	
0	
1	
2	

Pattern: _____ Pattern: _____ Pattern: _____

Student Practice:
Determine if the table represents a linear, quadratic, or exponential function.

x	y
9	6
10	10
11	15
12	21
13	28

The table represents a _____ function
because _____

Figure 6.56: Students found patterns in the tables of values for the three different types of functions. (Classroom Materials, April 27, 2011)

3. Radliff claims he can determine if a function is linear, quadratic, or exponential just by looking at the graph. Explain how he can decide the type of function from a graph. (Classroom Materials, April 26, 2011)

If students were not explaining their reasoning, they were creating their own example using the knowledge they had for each the three representations for each type of function (see Figure 6.57). Asking students to create an example of functions that fit specific characteristic is another high-level questioning technique.

Application of Learning – Story Problems

The final mathematical task Nathaniel and I used to connect all of the concepts from the chapter, specifically solving and graphing quadratic functions, were application

	Linear Function	Quadratic Function	Exponential Function
Equation			
Graph			
Table			

Figure 6.57: Students created their own examples of the three representations for linear, quadratic, and exponential functions. (Classroom Materials, April 28, 2011).

problems. The school district’s summative assessment expected students to apply knowledge of quadratics using a throwing object function and dropping object function. We wanted to put some meaning behind the story problems, so Nathaniel found a video titled ‘Punkin’ Chunkin’ that accompanied our textbook materials. The video showed teams of people using large machines to catapult pumpkins across fields. The teams were competing for the farthest pumpkin flight. We liked that the video gave students the function and explained what the numbers in the function meant. When the video commentator gave the function used to represent the flight of a pumpkin, Nathaniel paused the video and reinforced the meaning of the function (see Figure 6.58). Our thought was that by emphasizing the components of the function, the students would find more meaning as opposed to following procedures (see Appendix M).

The students then solved the equation to find how much time it would take for the pumpkin to hit the ground when catapulted from 20 feet off the ground. As students finished solving, Nathaniel asked students what the solution meant and if both solutions

$$\begin{array}{l}
 t = \text{time} \\
 h = \text{height}
 \end{array}
 \quad
 h = \underbrace{-16t^2}_{\text{gravity}} + \underbrace{76t}_{\text{velocity}} + \underbrace{20}_{\substack{\text{starting} \\ \text{point off} \\ \text{the ground}}}$$

Figure 6.58: Nathaniel reinforced the meaning of each components of the Punkin' Chunkin' function.

made sense. (One of the solutions was negative, which would not make sense for time.) The graphical representation (see Figure 6.59) was also used to discuss the validity of both solutions for this situation. Interestingly, one student found the vertex after solving the equation. Using the graph, the class discussed what the vertex meant in terms of the pumpkin.

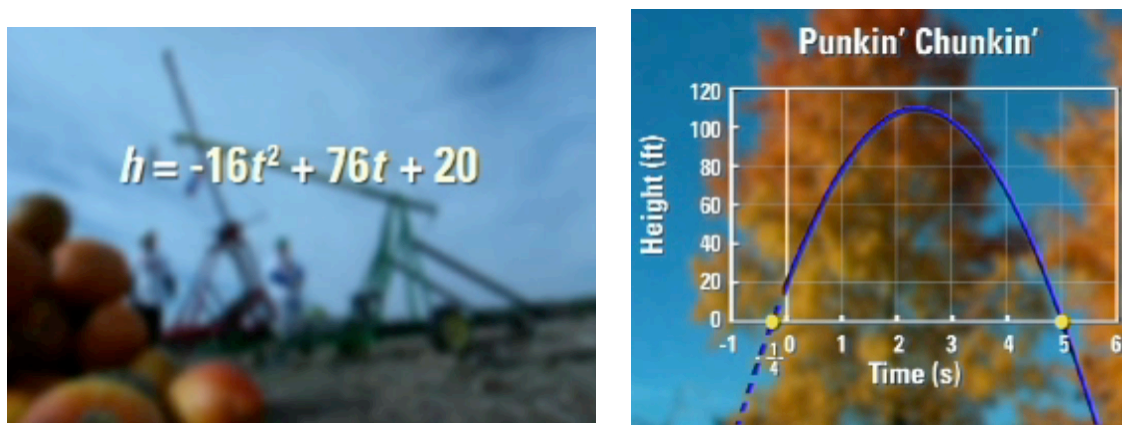


Figure 6.59: The Punkin' Chunkin' function and graphical representation fostered a discussion about applying knowledge of quadratic equations to real-life situations. These screen shots were taken from Bellman et al. (2009). (Classroom Materials, April 27, 2011)

Nathaniel then transitioned to dropping objects, connecting it to the throwing objects example the students had just completed (see Appendix M). He explained to students that the function for dropping objects is only concerned with the downward action of the object and therefore focuses on half of the parabola. Nathaniel also explained how the middle term, which represented velocity in the throwing objects functions such as punkin' chunkin' problem, would be absent since the object would be dropped and would not have any extra force besides gravity.

Reflecting on the application lesson, Nathaniel was not satisfied with students' overall understanding of story problems focused on throwing objects and dropping objects (Post Lesson Debrief, April 27, 2011). He did not think the students really conceptualized what was happening, but instead were just following a procedure to solve. The students continued to put zero in for the wrong variable, a common mistake made by students who follow a procedure rather than think about the situation. Nathaniel also expressed his desire to have more time to go deeper into the application problems (Post Lesson Debrief, April 27, 2011; Post Lesson Debrief, April 28, 2011), making the problems more relevant to the students.

What Nathaniel Learned While Connecting Concepts

The last several days of our consecutive coaching were spent emphasizing connections. Nathaniel and my discussions were focused on how to help students connect their current understanding of quadratics and how to help them connect their knowledge of quadratics to other types of functions. Through the instruction and our collaboration, Nathaniel's understanding of how to link mathematical concepts and representations increased.

Being able to link representations to underlying concepts and to other representations is another one of Ball et al.'s (2008) mathematical tasks of teaching. In our work together, this mathematical task was two-fold. Nathaniel began to link the various concepts involved in teaching quadratics as we discussed, planned, and reflected on his instruction. For his own understanding, he made connections between the big ideas of quadratics. Not only did Nathaniel link the representations for himself, but he also created lessons to help his students make those links or connections. Ball et al.'s (2008) mathematical task of linking representations to underlying ideas and to other representations includes the teacher making the links for himself, as well as the teacher helping the students make the connections among representations.

Making connections among mathematical ideas was something Nathaniel did for his own understanding and his students' understanding throughout the entire chapter on quadratics. Yet during the last five days of instruction, the connections between big ideas were even more apparent. One link Nathaniel made for himself and then helped his students make was the connection between the algebraic and graphic representations of a quadratic solution. Nathaniel and I discussed the importance of understanding solutions when solving equations, as well as knowing that the solutions were the x-intercepts when graphing functions. He learned how to link the graphic and algebraic representations to the underlying idea of a solution. Nathaniel then addressed the solutions link between graphic and algebraic representations with the pairs compare mathematical task that had students comparing the solutions of the two representations of the same function. The students were better able to identify and understand how solutions are represented since

Nathaniel and I had focused much of our discussions on the concept of solutions in order to facilitate the students learning.

Comparing the three types of functions (linear, exponential, quadratic) fostered numerous opportunities for Nathaniel and his students to link mathematical representations. All of the representations of the same function (equation, table of values, graph) and the three types of functions (linear, exponential, quadratic) provided Nathaniel with the task of connecting various mathematical ideas. In general, Nathaniel had already established these connections for himself in previous years. Yet how he went about helping his students link all of these representations emphasized more reasoning. For example, when examining the table of values for each type of function, Nathaniel could have told students the pattern in each table. Instead, he increased the level of the connection by providing students with three equations that looked very similar ($y = 2x$, $y = 2^x$, $y = x^2$) and asked students to create the three tables of values. This required students to begin thinking about what each equation was mathematically asking them to do. Then Nathaniel had students make observations about each table of values. Nathaniel's mathematical task such as this one encouraged students to think about the links between the three different functions.

Finally, the link between real-life situations and symbolic representations was the weakest connection Nathaniel and I tried to help the students make. After teaching the application problems, Nathaniel told me he felt like he could improve his connections. "I probably wish we could have done some more things with applications, getting deeper into that. Just making it more relevant to them" (Post Lesson Debrief, April 28, 2011). He tried to bring all of the students' learning about quadratics together and have them

apply it to flying pumpkins. Yet Nathaniel tried to make this link the second to last day of instruction and the students were not very intrigued with the real-life application (Post Lesson Debrief, April 28, 2011). I think this link was not as strong for the students because Nathaniel and I did not spend as much time discussing the mathematical connections to application problems. We had postponed teaching application problem twice in our planning of the chapter. Therefore Nathaniel did not have as many opportunities to link the real-life problems with symbolic representations for himself. The result was that Nathaniel's students were not able to make a strong link between the two representations. Nathaniel believed he could learn how to better connect students' learning to real-life problems in the future.

Conclusion

What did Nathaniel need to learn in order to teach mathematics effectively? Ball et al. (2008) created a list of mathematical tasks of teaching to describe the specialized content knowledge teachers need for teaching, and thus need to learn to teach mathematics effectively. I used this list to more closely examine what Nathaniel learned during our consecutive coaching in quadratics. And in turn, my work with Nathaniel further explained the following mathematical tasks for teaching: Finding an example to make a specific mathematical point; Presenting mathematical ideas; and Linking representations to underlying ideas and to other representations. Nathaniel's learning was not limited to the three tasks listed above, but was spread across the sixteen tasks. These three tasks of teaching were chosen to focus on in this analysis due to the numerous data sources that highlighted Nathaniel's learning in these areas and the contribution the study could make to further understanding the tasks.

The daily discussions Nathaniel and I had about the mathematics, students' understanding of quadratics, and teaching the concepts within quadratics provided insight into Ball et al.'s (2008) mathematical tasks of teaching. This study adds to the explanation of what a task could entail or further details the application of a task in a mathematics classroom (see Figure 6.60). When a teacher is finding an example to make a mathematical point, he is choosing his example based on what mathematical point he wants to make. These points may include connecting an idea to prior knowledge, helping students see a need for new mathematics, or attending to misconceptions students exhibit. When a teacher is presenting a mathematical idea, the teacher must understand what is involved in that particular idea. And finally, in order for a teacher to link mathematical representations for students, he must first connect the representations for himself and link them to his own understanding.

Ball et al.'s (2008) Mathematical Tasks of Teaching	Expanding on Ball et al.'s (2008) Mathematical Tasks of Teaching
Finding an example to make a specific mathematical point	Teachers choose examples to make a variety of different mathematical points (i.e., connecting prior knowledge, showing a need to learn new mathematics, addressing common student misconceptions).
Presenting mathematical ideas.	Presenting mathematical ideas (rather than a mathematical formula or procedure) requires that a teacher recognize what is involved in using a particular representation. Recognizing what is involved in using a particular representation is also a mathematical task of teaching.
Linking representations to underlying ideas and to other representations	Linking representations of mathematical ideas and representations is important for the teacher, as well as the students.

Figure 6.60: Using this research, I expanded upon Ball et al.'s (2008) mathematical tasks of teaching.

Part III: Reasoning through Pedagogical Dilemmas

Nathaniel increased his knowledge of mathematical content largely through the process of mapping the quadratics terrain. Through planning and teaching quadratics with reasoning and sense making, he learned more about the special content knowledge necessary to teach mathematics. Yet through the entire process, mapping the terrain and then teaching quadratics, Nathaniel also learned some things not directly related to mathematics. He increased his ability to reason through pedagogical decisions by discussing teaching problems in practice and by having pedagogical thinking modeled for him.

Learning to “Think Things Through”

When I asked Nathaniel to explain to me what he had gained from the quadratics coaching experience, he replied, “Um, I mean the big thing.... I don’t think you could gain just one thing, but like...it just felt like I became a better teacher. I really felt better overall in terms of thinking things through” (Final Interview, May 25, 2011). Nathaniel thought that our coaching interactions helped him become a more thoughtful teacher. What helped Nathaniel “think things through” in order to become a better teacher? And what were some of the “things” that needed to be thought through? Lampert (1985) argues that teachers are faced with pedagogical teaching dilemmas in their practice in which there is no right answer. A teacher becomes a dilemma manager and argues with oneself over the various possible ways to approach the dilemma (Lampert, 1985). Each approach provides both positive and negative consequences and neither lead to a completely winning solution to the dilemma. The process of thinking about, analyzing,

and making a choice in the midst of a practical teaching problem is a large portion of a teacher's daily life.

Nathaniel and I had the opportunity to experience and discuss a variety of teaching dilemmas while teaching quadratics. These interactions around teaching dilemmas were not planned and were not an intended focus of our coaching. One example of a dilemma Nathaniel and I discussed occurred during our mapping of the quadratic terrain. The dilemma was how to introduce quadratics. We had to choose between starting by graphing quadratic functions and starting by solving quadratic equations. Nathaniel and I made a detailed list of pros and cons for each approach as we weighed the possible consequences for each solution. Both of us used our previous experiences teaching quadratics and high school students who struggle to inform our ideas for introducing quadratics. As our colleagues had suggested, and as we quickly realized through our dialogue, there were no right answers as to how quadratics should be introduced. Neither approach would be perfect. Both approaches would lead to other problems. After thoroughly discussing the options, Nathaniel had the opportunity to determine which approach he wanted to take since he as the classroom teacher ultimately had to manage the dilemma.

This example early on in our work together demonstrated to Nathaniel a couple important characteristics of teacher decision-making in the midst of dilemmas. First, there is no "right way" to teach. Teaching is full of choices and dilemmas. There is not one way to introduce quadratics. Nathaniel understood that either graphing first or solving first was a valid option. Secondly, thoroughly discussing a dilemma and the consequences of possible approaches that address the dilemma are beneficial to a

teacher's instruction. A teacher should view a dilemma as common and useful rather than burdensome or something that needs to be eliminated (Lampert, 1985). As we discussed the pros and cons of each approach to quadratics, Nathaniel was simultaneously learning ways to teach the content. The ideas surrounding which approach to use when introducing quadratics included ways to connect concepts that impacted his instruction. Finally, the dilemma Nathaniel and I faced demonstrated how teaching decisions are made using pedagogical reasoning. Pedagogical choices should be made through thinking and reasoning, which includes a teacher's prior experiences. Nathaniel and I thoroughly discussed why one approach would be better than another to introduce quadratics. We gave a rationale behind the final choice we made and were able to explain why we chose that approach.

As Nathaniel and I discussed pedagogical problems, the value in talking about various "solutions" and their potential consequences became apparent. As Nathaniel noted in his final interview, he felt like he became a better teacher in terms of "thinking things through." Reasoning and sense making are another way to describe the process of thinking through a teaching dilemma. When contemplating what approach to take to address a problem, a teacher is actually using prior knowledge of teaching experiences, mathematics, and students, and drawing conclusions using evidence. Reasoning through a pedagogical dilemma requires the teacher to think logically about numerous educational factors, consider all alternative approaches, and justify the final decision that is made. Nathaniel was able to improve his pedagogical reasoning and sense making skills through our quadratics coaching interactions.

Learning to Reason Pedagogically

The dilemmas a teacher experiences can range in size and impact. There are dilemmas that occur over time and others that arise suddenly. And teaching problems can take place inside or outside the classroom. Therefore, as Nathaniel and I collaborated during the quadratics chapter, we discussed a variety of teaching dilemmas when they arose. I found myself modeling my thinking as we made instructional decisions in the beginning. Then Nathaniel and my collaborative discussions began to move towards Nathaniel and I both adding reasons for making certain teaching moves. And after some time, Nathaniel began to make his own instructional decisions based on his students' understanding and his previous teaching experiences.

During the first part of our coaching in quadratics, I modeled reasoning and sense making when I made specific instructional decisions. When I suggested to Nathaniel that he use a specific teaching strategy, I often explained why I thought that was a good idea and how that choice could influence student understanding. An example of this occurred on April 18 during class when students were using graphs and table of values to make their own conjectures about quadratic functions. I noticed that students were finishing at different times. In our planning together, Nathaniel and I had decided to have all students finish making conjectures before moving to a class discussion about the students' ideas. I knew from prior experience that when students finish early and are waiting for their peers to complete the task, behavior issues begin to arise. A dilemma quickly arose. Do we expect students who finish early to wait for their peers to complete the task so all students have the opportunity to answer the questions? Or do we stop the entire class after a few students complete the questions in order to avoid student misbehavior? It was not

apparent that Nathaniel sensed the dilemma at that moment, so I quickly considered the options and made a suggestion to Nathaniel.

I recommended to Nathaniel that we put the questions from the mathematical task up on the whiteboards and have students who finish early write their conjectures on the board. Through my explanation, I modeled the pedagogical reasoning I processed through to help me make that recommendation. I explained to Nathaniel that by allowing students to put their answers up, we would be keeping students engaged in the mathematics even though they were finished answering the questions on their sheet. And at the same time, other students who were not finished had little more time to complete the problems. I also described how this teaching move would provide a visual for students to focus on when Nathaniel led the class discussion. The downfall of this solution was that students who were finishing could potentially copy down what was written on the board without thinking for themselves.

Over time, I stopped modeling as much of the reasoning about teaching and purposefully began engaging Nathaniel in more discussions about the way of thinking about approaches to instructional problems. I asked Nathaniel more questions about his thinking behind his ideas. Although I did continue to model some pedagogical reasoning, the conversations about teaching decisions became more collaborative between Nathaniel and I.

An example of our collaboration that included reasoning about a teaching dilemma occurred on April 21 after Nathaniel had taught students how to graph using the vertex formula. The teaching dilemma we were facing was that students did not yet

connect the idea of graphic and algebraic solutions. The students were viewing graphing and solving as two different, unrelated concepts.

Nathaniel: I think when we talked about it, all solutions occur when? When they cross the x intercept. But it's not concrete in everyone yet.

Amy: Right, ok so that might be something we need to add Monday to get that solid. And we may need to...and how we do that we may need to have them going back and graphing something and then going back and solving it. You know, graph it opposite b over $2a$ and then solve it. You know we may need to do something like that to keep practicing our solving, but again make that connection a little bit more.

Nathaniel: Yeah. And maybe you could even do that where you can have your partner, where one person graphing the equation and one person is solving it by whatever method to see the connection.

Amy: Yeah. Yeah.

Nathaniel: And flip-flopping. That would be good. (Post Lesson Debrief, April 21, 2011)

Nathaniel initially identified the dilemma students were having with seeing the connection between solutions of equations and solutions on graphs. The possibilities for how to address the problem were numerous. I made the suggestion of using a mathematical task that included solving equations and graphing functions. My reasoning for thinking that would be a practical solution was that students could get extra practice solving, which we had not done for several days, and at the same time could be directed towards connecting the two representations. Nathaniel agreed with my thinking and then

added the idea that one partner graphs the function while the other partner solves the equation. His reasoning for this suggestion was so pairs of students could see the same solution in both representations (i.e., graph and equation). Through our collaboration, Nathaniel understood that having students simultaneously graph and solve a quadratic would help make the connection of solutions.

Over time, Nathaniel began making instructional decisions on his own and explained to me why he made those choices. As Nathaniel explained his pedagogical reasoning, it was evident that he was learning to logically think about his teaching ideas. An example of his learning happened on April 21. Nathaniel and I had discussed a general outline for the instruction that day. Since he had just taught students how to graph using the vertex formula the day before, we decided a quick lesson to refresh the students' memory was necessary. From the previous day's instruction, Nathaniel and I both knew that several students did not understand why it was now necessary for them to learn how to graph using the vertex formula $\left(x = \frac{-b}{2a}\right)$. The dilemma was that students could graph using the vertex formula, yet they did not see purpose for using the formula when they could graph by finding the x-intercepts and then the vertex.

On his own, Nathaniel decided to approach the dilemma by addressing the problem directly. He chose to highlight the differences between the two methods of graphing quadratics by using two flow charts (see Figure 6.51). He then decided to use a function that did not have a solution so that he could "retouch why $\frac{-b}{2a}$ is going to be a necessary tool" (Written Communication, April 20, 2011). He wanted to be sure students understood that they "can't just find the roots first and then the vertex" (Post Lesson

Debrief, April 21, 2011). Nathaniel included the flow charts and the extra problem into his instruction because he wanted to address the dilemma. He had a reason behind including these instructional pieces, which he explained to me later.

How does a teacher learn to reason through a pedagogical dilemma? For Nathaniel, hearing another educator model reasoning and sense making helped him recognize the various components that could be included in the decision-making process. The modeling also helped him realize that no “solution” would perfect, but the process would be beneficial. Collaborating and discussing possible approaches to a problem further enhanced Nathaniel’s learning. He was able to brainstorm ways to address dilemmas while receiving the support from another person. Being urged to explain to his peer why he felt an approach would be valuable provided an environment where Nathaniel could practice reasoning pedagogically.

Managing Dilemmas

Dilemmas are an on-going part of the teaching profession. Teachers must admit that some conflicts cannot be resolved and cope with the fact that managing the dilemma is the best “solution” to the problem (Lampert, 1985). Discussions with colleagues and learning from personal experiences can help teachers gain understanding of how to best manage teaching dilemmas. Nathaniel and I collaborated to manage various dilemmas while teaching quadratics. One dilemma in particular stood out. The primary dilemma Nathaniel faced in his classroom instruction was student engagement during direct instruction. Before our quadratics coaching, Nathaniel tended to wait for all students to master the content before moving on to formative assessments or any other instruction. At times, this approach resulted in student disengagement.

On the one hand, one way to address the dilemma of student engagement during direct instruction was to “push” instruction. The reasoning I offered to Nathaniel for this approach was that moving into a formative assessment more quickly would allow the students who understand the material to practice rather than be bored and become a behavior problem. And the students that did not understand initially could receive more individualized help during class, would not be singled out during direct instruction as not understanding, and would not hinder the other students learning. A potential problem with the approach would be that students do not receive enough direct instruction and feel rushed during that portion of the class.

Nathaniel described how this approach to the dilemma of student engagement played out in his classroom:

Nathaniel: I would say overall, I felt like the kids were engaged with a lot of different activities, changing it up quite a bit. Um, for myself, I pushed a lot more. In my mind I am trying to be more firm in terms of going forward because I know sometimes in the past I have gotten hung up on my why questions and I lose some kids in the class just because I am virtually having a conversation with just a couple people it seems like. So pushing you get overall more attention.

Amy: And you’re pushing because.....

Nathaniel: Well because I know there is time when there are people that are hung up on certain parts we still have time to practice some of those things. Some of those are issues with individual kids that you have to work more one-on-one with anyway. (Post Lesson Debrief, April 14, 2011).

Nathaniel told me he was seeing the importance of allowing kids to struggle a little bit and how moving on to other mathematical tasks actually helped all of his students (Field Notes, April 15, 2011). He thought increasing the tempo of the instruction kept more students engaged throughout the chapter (Post Lesson Debrief, April 28, 2011).

On the other hand, as Nathaniel alluded to on April 14, students need to be asked why questions. Asking students to think about and explain the mathematics being learned is a second approach to engaging students during direct instruction. The rationale thinking that supports asking why questions, as a solution to the engagement dilemma, is that students could then learn the mathematical concepts at a deeper level. The focus of the instruction could then be on conceptual understanding rather than procedures. A potential problem with this approach is that not all students will become engaged in the mathematical idea. Also, asking the class questions about the deeper reasons underlying the mathematics directly engages the one student answer the question while other students would be expected to engage indirectly.

Increasing the pace of all direct instruction was not the right answer. And using only why questions to engage students in direct instruction was not the right answer either. Nathaniel and I had to work together to manage the engagement dilemma. We recognized that there was no perfect solution that could be applied in every instructional situation. Nathaniel acknowledged the need to balance the two approaches in order to manage the student engagement dilemma.

Trying to find that balance and being able to see that a little bit clearer I think helped me out in terms of my approach to teaching. I think that was a big aspect of it. And sometimes it was just kind of like, “Hey this has definitely gone too

long. We need to switch it up.” But even on the notes, I know for myself I always would ask why (questions). Why? Why? Why? And I think I still need to do that, but I think there is a point where you got to be able to just move on knowing that somewhere throughout the formatives in that day we will answer some of those questions that a couple kids were stuck on. So you don’t lose the whole class trying to answer a couple questions here or there. (Final Interview, May 25, 2011)

Through our discussions, Nathaniel learned to handle the student engagement dilemma in his classroom by balancing the two different approaches.

Conclusion

Pedagogical dilemmas are an ongoing component of teaching. Resolving these dilemmas is neither neat nor easy. Nathaniel and I found that thinking through the various alternatives with logic and reasoning can provide a teacher with a greater sense of how to approach a particular dilemma. Nathaniel learned how to think through various solutions when faced with teaching dilemmas by having another educator model pedagogical reasoning and by discussing various pedagogical solutions to dilemmas with colleagues. Using an analytical mindset helped Nathaniel approach and manage the numerous dilemmas he encountered as he mapped the quadratics terrain, planned instruction, interacted with students, and then reflected on his instruction.

CHAPTER 7: CONCLUSION

The entire research process has been a valuable learning experience. By revising my definition of instructional coaching, I now appreciate how coaching closely resembles teaching. In hindsight, some of the most valuable lessons I learned about instructional coaching are messages that a few of the most respected coaches in college athletics highlight as being key components of effective coaching. With my new understanding and view of coaching as teacher learning, I prepare for my future coaching practice and research endeavors.

Coaching is Teaching

My definition of coaching has continued to evolve over time. My initial experiences as a teacher being coached caused me to view coaching as an appraisal process. I saw coaching as an extension of the form of teaching evaluation I experienced and as a method for administrators to gain information about me as a teacher. When I became an instructional coach, I focused on providing teachers with resources and instructional materials. Transitioning into a full-time coaching position helped me gain a broader perspective of coaching as a process to improve teachers' instruction. Yet it was not until I engaged in this inquiry process that I began to view coaching as a context for teacher learning.

My coaching actions prior to this research demonstrated my belief that teachers' instruction needed to be fixed. I was working to fix teachers by telling them how to teach mathematical concepts and by giving them specific instructional strategies (such as cooperative learning) to use. I was approaching coaching in this manner for a variety of reasons. I found that if I gave teachers a specific procedure to try in their classroom, they

were more likely to try the new strategy. I saw more immediate results since the teacher mimicked the instruction I modeled for him. This approach to coaching seemed to remove the teacher's need to think and therefore also bypassed many of the frustrations and struggles that accompany teaching, learning, and coaching.

Through my intense coaching with Nathaniel, I came to realize that coaching is not about fixing teachers by telling them how to teach. Coaching is meant to help teachers learn. If I continued to give teachers "the answers," they would not learn how to make the sort of teaching decisions that improve instruction on their own. As a coach, I needed to stop doing the thinking for the teacher and instead use collaborative conversations to help teachers think about their own teaching.

My definition of coaching has changed to focus more on teacher learning. I specifically view coaching as the context through which a teacher learns to think and reason about teaching. The coaching cycle (planning session, lesson, debrief meeting) is a way a coach can model thinking through pedagogical decisions, as well as mathematical content. Coaching for me is no longer about fixing teachers or their instruction. Instead, coaching is my way of assessing teachers' current knowledge and helping them construct a deeper understanding of mathematics and teaching. My work as a coach is to help a teacher think about his instruction and help him develop a greater capacity to reason about teaching on his own.

Lessons from My Coaching Experience with Nathaniel

By defining coaching as a context for teacher learning, I also view myself as a teacher. But instead of teaching high school algebra students, I teach mathematics teachers. Approaching coaching with this mentality provides me with a boost of

confidence. I feel as though I can apply many of the pedagogical strategies I have found to be successful in my math classrooms to the teachers with whom I work. Interestingly, several parallels can be highlighted between my journey as an instructional coach and my journey as a mathematics teacher.

Whose learning matters? When I began teaching, I was very self-absorbed in my own teaching. Over time I realized that the students' learning was much more important and began to shift my focus to students' learning and understanding of the material. A similar transition took place during this study. Prior to the spring of 2011, I was overly focused on myself as a coach. I was worried about my actions and what I was learning.

It was not until my intense work with Nathaniel that I realized I was concentrating on the wrong person. I took time through this study to observe Nathaniel and get to know him as a teacher and as a learner. I started listening to Nathaniel more and offering quick fixes less. After each conversation with Nathaniel, I reflected on what he had learned or still needed to learn. I began thinking about what I needed to do as a coach in order to help Nathaniel learn. Through this study, I have realized that my coaching is dependent upon what a teacher needs to learn or has learned. My practice is no longer about me, but instead is about the teacher's learning.

Content knowledge. As a mathematics teacher, I collaborated with others on a regular basis. My initial collaborations with other math teachers centered on instructional pacing and homework assignments. The substance of my collaboration with my colleagues evolved over time and eventually included discussions about how to teach mathematical concepts. My collaboration with my colleagues moved to be focused more

on mathematical ideas. Student learning and students' mathematical understanding also became a driving force behind the conversations in which we engaged.

Similar changes occurred in my coaching collaboration through this study. The first, and probably most obvious change in my coaching interactions, was the increased emphasis on mathematics. During the first couple years as an instructional coach, I spent a significant amount of time discussing pacing and formative assessments with teachers. In fact, the majority of my coaching was spent talking to teachers about cooperative learning strategies. It was not until my intense coaching with Nathaniel that I began focusing my collaboration on deeply understanding mathematics. It was during this study that Nathaniel and I began focusing our collaboration on thoroughly understanding the mathematical content, which was largely done by mapping the terrain of quadratics.

The evolution in my collaborations as a coach also corresponds to my collaborative experiences as a teacher. As a teacher, my conversations were initially focused on me as a teacher. Over time, my colleagues and I began to focus our discussions on student learning and understanding. A similar transformation happened in the way I approach collaboration with the teachers I coach. My coaching collaboration is now based on what the teacher is learning. I ask questions and engage the teacher in conversations meant to help the teacher learn.

From fixing instruction to encouraging pedagogical thinking. The most significant change that occurred in my teaching was in how I taught mathematics. For the majority of my first four years of teaching, I told students exactly how to perform the various mathematical procedures. I gave the students steps to follow to get to the correct answer and did not require them to think or reason about the mathematics. And if a

student was struggling to get an answer, I was quick to fix his mistake by pointing out the error in the procedure or by telling them how to execute the mathematics correctly. In my fifth year of teaching I realized I was doing the thinking for my students. To combat this, I started teaching mathematics more conceptually. I asked my students to make more connections between concepts and to reason through the mathematics.

How I coached mathematics teachers was also the most significant change in my practice. Prior to this study, I spent the majority of my coaching time telling teachers how to teach. I provided teachers with steps to implement various teaching strategies and ideas on how to teach a successful lesson on a topic. My coaching days were spent discussing cooperative learning strategies and classroom management techniques. If I noticed a problem in the teacher's lesson, I quickly told her how to fix it by pointing out the error or providing a solution. I was helping teachers in the short-term, but was not providing them the opportunity to think and reason about teaching.

Similar to my teaching, my coaching is moving past fixing errors and into fostering pedagogical thinking. My work with Nathaniel has taught me the importance of helping teachers think through teaching decisions. Rather than me telling a teacher what he should do, I want teachers to understand teaching and contemplate possible moves and their consequences. I now see my role as a coach to be someone who helps teachers become pedagogical thinkers. By helping teachers learn to reason about teaching, they will be better prepared to make successful teaching decisions in the face of future pedagogical dilemmas.

Coaching as a Context for Teacher Learning

If instructional coaching is viewed as a way to help teachers learn, then what aspects of coaching make it a useful context for teacher learning? How is coaching different from teacher learning in other contexts such as whole-group professional development sessions or educational conferences? What makes coaching a successful approach to improving teachers' instruction? Why is the context of coaching conducive to individual teacher learning? I believe athletic coaching can shed some light on the influence of the coaching context. For example, to highlight the difference between learning to play basketball and learning basketball skills, John Wooden said, "If you keep too busy learning the tricks of the trade, you may never learn the trade" (Wooden, 2004, p. 109).

One of the most successful athletic coaches in the University of Nebraska-Lincoln's history is Coach Tom Osborne. Dr. Tom Osborne was the Husker's head football coach from 1973 to 1997 and is regarded as one of the greatest collegial coaches of our lifetime. Coach Osborne has received numerous awards for his excellence in coaching, including being named Entertainment and Sports Programming Network's (ESPN) Coach of the Decade in 1999 and being inducted in the National Football Foundation's Hall of Fame in 1998.

I had the privilege of interviewing Coach Osborne in 2010 as I prepared to conduct my instructional coaching research. The questions I asked him focused on coaching and what he found to be the key behind his success as a head football coach. At the time of the interview, a large portion of what Coach Osborne shared did not seem to directly connect to my practice as an instructional coach. I think my perception of the

coaching disconnect was largely due to my personal definition of coaching at the time. Coach Osborne spoke about the larger picture of coaching and the qualities of a successful coaching process. I was still consumed with figuring out how to fix teachers.

As I collected and analyzed my Spring 2011 data, Coach Osborne's words began to make more sense. The successful coaching characteristics he emphasized in the interview now seem to directly connect to my coaching practice. I think my new appreciation and understanding of Coach Osborne's coaching insights grew out of my new view of coaching as a context for teacher learning. In the following section I use quotes from Coach Tom Osborne to explain how instructional coaching fosters learning for mathematics teachers.

Learning through Practice

You know, you can talk about being national champions in football. But that's pie in the sky, by and by. It is a long ways off. And it is not nearly as effective as talking about what has to be done today in practice...The things that we do today that move you forward are more important than simply setting some long-term goals that you feel excited about (or) feel good about. You are not going to get there unless you do the daily activities. (That is) the process that will get you there.

- Coach Tom Osborne, August 26, 2010

Although it is important to have long-term goals as a teacher, the everyday work to progress towards those goals is a critical piece of the learning process. Instructional coaching provides teachers a context in which daily activities can be used as a means towards the larger goal. Similar to an athletic coach who uses daily practice to help

athletes improve their skills, academic coaching focuses on everyday components of teaching such as planning, teaching, and reflecting on instruction. The long-term goal of increasing students' understanding of content by improving instruction is still an important aspect of coaching. Yet daily work within a teacher's practice is the process by which the larger goal can be achieved. And coaching is a context that fosters learning within a teacher's daily activities.

Since an instructional coach works with current in-service teachers, a teacher's learning process is directly built into the daily practice. Unlike other professional development approaches that remove teachers from their classroom to receive training, coaching brings the learning process to the teachers in their classrooms. Coaching provides an environment in which teachers can immediately apply their learning to their own teaching. This situation helps teachers connect educational theory or teaching strategies to their practice. A parallel can again be drawn to athletics. A basketball player does not learn how to effectively shoot a basket by sitting in a training session and being told how to shoot. A player needs to be able to immediately try shooting the basketball in order to fully understand the theory behind effectively making a basket. And he needs to be able to do so with a coach by his side to help him learn how to improve his shot. Similarly, a teacher needs to be able to try a teaching strategy and adjust based on what happens. Instructional coaching provides teachers with the context through which learning and practice can occur, while simultaneously providing teachers with support and encouragement from a coach.

Differentiated Teaching

You have to be sensitive to individual differences and try to detect what will work best with each individual. What works for one may not work for everyone else.

- Coach Tom Osborne, August 26, 2010

On an athletic team, no two players are exactly alike. Each player has her own individual strengths and weaknesses. An effective coach will notice what skills that player needs to more fully develop and will tailor his coaching to each individual on the team. Teachers are like players on a team. Each teacher has her own strengths and weaknesses. There are certain aspects of teaching she excels in and other skills that could be more fully developed. The coaching process allows a teacher to customize her learn in order to expand up her teaching strengths and progress in teaching areas that are not as strong. A coach can identify individual teachers' needs and thus develop learn opportunities specifically for that teacher.

Instructional coaching also allows coaches to approach individual teachers based on who they are as learners. Unlike other large-scale professional development techniques that provide all teachers with the same instruction, the context of coaching gives teachers an opportunity to engage in instruction tailored to them. A coach has the opportunity to get to know a teacher as a learner and as a teacher. This allows the coach to determine the best way to approach coaching interactions with the teacher. Does the teacher learn better by observing others? Does he willingly try new strategies? What does he tend to focus on in his instruction and reflections? An effective coach essentially creates an individualized learning plan for each teacher in order to maximize learning.

Beyond Specific Skills

A coach should display good values and be a person of principle. Because usually when you're coaching, you're teaching other things besides specific skills such as blocking and tackling or Algebra or whatever it may be.

- Coach Tom Osborne, August 26, 2010

Lead by example. Not do as I say, but do as I do.

- Coach Tom Osborne, August 26, 2010

When I think back to all of the athletic coaches I had over the years, I do not initially remember the specific skills they taught me. Instead I remember their approach to life, their passion for the sport, the way they interacted with the players on the team, and their emphasis on good sportsmanship. Although I did improve my skills in the activities, I also learned how to analyze and think. I learned how to think through my offensive attack based on the other team's set-up. I learned how to look and listen to others around me in order to make a quality defensive decision. I was taught how to think like an athlete.

During the coaching process, the teaching values of a coach are inadvertently being taught. The way a coach views content or talks about instruction is continually modeled. A teacher has the opportunity to learn how an effective educator approaches and thinks about students and learning simply by interacting and planning with a coach. For example, if a coach is continually reflecting on her own instruction to determine what led to increased student understanding and what could be modified to improve the instruction, a teacher will be indirectly learning how to become a reflective practitioner.

There is more to being an educator than having a set of instructional skills. The context of coaching also provides teachers the space to learn how to think through an educational situation. It allows teacher learning to move beyond training teachers in instructional strategies or basic skills and instead fosters pedagogical reasoning and sense making. This is an important aspect of teacher learning that is not available in other professional development situations.

What's Next?

Everything I experienced and learned throughout the research process has left me pondering what is next in my coaching practice and research. In terms of my practice, I cannot return to the style of coaching I had done prior to Spring 2011. Yet some may argue that the intense coaching Nathaniel and I experienced is not feasible. I explore the possible ways the coaching I experienced in this research could be expanded upon. My research also left me with questions about instructional coaching and teaching mathematics. The final section discusses some of the questions I now have and areas I am interested in studying in the future.

Expanding to other Courses and Teachers

Not all people may be convinced of the value of the type of coaching used in my research with Nathaniel. Some educators may say that a weakness or argument against the intense coaching Nathaniel and I engaged in for four weeks is that the coaching structure is not realistic for all instructional coaches. The argument may be that the time and monetary restraints placed on school districts would not support this structure of coaching. As previously mentioned, the amount of time and effort required of the teacher in this type of coaching is high. Also, a coach's time is largely focused on a single

teacher during the intense coaching, which means other teachers do not receive as much coaching. Yet there are ways the learning that occurs during the intense coaching can be expanded to other courses and teachers.

First of all, the discussions Nathaniel and I had focused on teaching quadratics in his Algebra class affected his other courses as well. Although our intense coaching was set in the context of an algebra course, many of the things he learned could be applied to other courses he taught. The process of mapping the terrain of a mathematical concept in order to better understand the mathematics and curriculum is a practice he could apply to other math topics. The specialized mathematical content knowledge Nathaniel learned and improved, specifically the mathematical tasks (Ball et al., 2008), can be utilized in all mathematical courses he teaches. And finally, the pedagogical thinking and reasoning cultivated during the intense coaching process are thinking skills that will benefit the teacher in all pedagogy. Therefore intense coaching focused on a single course and mathematical concept fosters teacher learning that can be applied to other aspects of the teacher's practice.

In addition to being able to apply learning to other courses, a teacher involved with this coaching structure can also act as a model for other teachers. When a teacher is involved in such intense coaching, the teacher's colleagues begin to notice and ask questions about what the teacher and coach are doing. The teacher can share instructional materials, as well as the thinking and reasoning behind the materials. Professional learning communities can breed this type of sharing, yet a formal meeting is not necessary. Learning between colleagues can be done through daily interactions. Even though the intense coaching with a single teacher may seem to be a limited use of

the coach's time when several teachers are waiting for coaching, what one teacher learns during the intense coaching structure can actually be expanded to other teachers through teachers.

Aspects of the intense coaching I engaged in with Nathaniel could also be applied on a larger scale. The process of mapping the terrain of a mathematical concept is something that could be done with several teachers. A coach could facilitate discussions among math teachers that engage them in the mathematical content and encourage the teachers to analyze their textbook and curriculum. Having groups of teachers create a physical map of a mathematical concept would not only foster math-focused conversations, but could also lead to discussions about how the concept could be taught with reasoning and sense making. For example, teachers could initially be asked to create a map of a concept based on the textbook or curriculum used in the course. Then the teachers could work with other math teachers to include what students already know about the concept or skills necessary to learn about the concept. To take the conversations to a deeper level based on reasoning and sense making, asking teachers to use their prior experiences to denote what students struggle to understand or explain while learning the concept. Teachers can then work together to create a teaching plan for that concept. The coach could encourage the teachers to address the area students struggle to understand by teaching with connections to prior knowledge and mathematical reasoning. Although a large-scale mapping of the terrain with a small group of teachers is not the same as a single teacher and coaching working together, the benefits from doing this type of content planning have a greater chance of fostering teacher learning than no coaching at all.

Future Research

Conducting research and analyzing my findings has sent me deeper into inquiry surrounding my practice. One topic I have begun to question more is the purpose of professional development initiatives. Coaching is one professional development strategy. The process of researching my practice as a coach, and thus a professional developer, has caused me to view my role as being a teacher educator. Yet prior to my research, I saw professional development as something that is done to teachers to give them new skills. I have begun to rethink the term “professional development” and am asking more questions. What is a professional developer? How does professional development help teachers develop professionally? What does it mean to develop as a professional educator? What does effective professional development look like? If a teacher develops, does that mean he learns? My view of professional development has shifted due to my research and makes me wondering if professional development opportunities all view teachers as learners or should.

My research has also caused me to think more about the role of reasoning and sense making in mathematics, teaching, and coaching. During my research, I learned a great deal about how to think through a mathematical concept. The majority of Nathaniel’s and my discussions focused on logical reasoning and how to best connect mathematical ideas. Yet the research was contained to the subject of quadratics. I am curious as to how reasoning and sense making could be emphasized within other mathematical concepts in algebra, as well as other math courses. Beyond incorporating reasoning into other mathematical areas, I am further intrigued by who does the reasoning and sense making in a mathematics class. Nathaniel and I engaged in mathematical

thinking at length, which was a valuable experience. Yet I question how often Nathaniel's students were engaged in mathematical reasoning for themselves. I would like to further explore how reasoning and sense making are incorporated into a mathematics classroom and what a teacher does to encourage students to do more of the mathematical thinking as opposed to the teacher doing the reasoning for everyone.

My second inquiry in relation to reasoning and sense making is connected to teaching. The idea of reasoning and sense making in teaching is a new concept to me, and a stimulating area of interest. In several of my education courses over the years, I have been engaged in the conversation about how teaching is more of an art than a science (Heaton, 2000). I am beginning to wonder if the phrase "art of teaching" is related to the notion of logically thinking through pedagogical dilemmas. Does a teacher who has learned how to successfully balance various teaching dilemmas, while simultaneously making educationally well-thought out decisions, actually an example of an artful teacher? I am interested in researching how the phrase "art of teaching" is associated with the reasoning processes of teaching.

Literature discussing the use of instructional coaching as a way to help teachers learn higher-level thinking skills has recently begun to emerge. Jackson (2011) highlighted her work with instructional coaches and the importance of teaching coaches how to engage teachers in reasoning about teaching. Jackson points out that teachers will not learn how to make effective educational decisions if the coach does the thinking for them. In a similar fashion, Bearwald (2011) notes the importance of a coach using questioning and listening techniques to increase teachers' higher-level thinking skills. Although a coach's instinct is to tell the teacher how to fix instruction, a teacher will

benefit more long-term if a coach instead asks the teacher questions to help build reasoning skills (Bearwald, 2011).

Between the new literature being published (Bearwald, 2011; Jackson, 2011) and my own research, I have increasingly become intrigued by how a teacher learns to think about teaching. How do teachers learn to reason through pedagogical dilemmas? How do teachers communicate their thinking about teaching? What roles do various educational factors (i.e., college courses, school districts, teaching colleagues, administrators) play in the reasoning and sense making a teacher develops? What are the dilemmas of coaching? At this time, the questions focused on a teacher's reasoning skills for teaching are the most interesting to me.

Finally, I think there is a piece of reasoning and sense making that my research does not directly explore, yet is related. I am curious about the logical thinking that an instructional coach engages in while coaching teachers. What takes place when a coach reasons through and analyzes coaching dilemmas? How does a coach improve her own thinking and reasoning skills? What higher-level thinking processes as a classroom teacher can be applied to coaching situations by a coach? How is thinking through a coaching dilemma the same or different from teaching dilemmas? The possibility of engaging coaches in research to better understand their reasoning and sense making practices could provide valuable information to other teacher educators.

In the Field of Instructional Coaching

This study's findings and analysis bring a new point of view to the field of instructional coaching. The mere shift in thinking of coaching as teacher learning has the potential to change the field. The current coaching books and guides, which highlight

steps a coach takes to help a teacher improve his instruction, could instead focus on how teachers learn and how coaches are, in fact, teacher educators. These books could help coaches by discussing the nature of the teaching and learning process about teaching and coaching as an opportunity for teacher learning.

The education and support of practicing coaches takes on new meaning when teacher learning is seen as the focus of instructional coaching. As highlighted in my research, numerous parallels can be drawn between learning how to be an effective classroom teacher and learning how to be an effective instructional coach. Coaching, therefore, does not require an entirely new set of skills. Education for instructional coaches should build on a coach's knowledge of teaching and learning in the classroom and help the coach transfer that knowledge to adult learners (i.e., teachers). Additional training, support, and learning for coaches could be directed towards teacher learning.

As research continues to be conducted in the field of instructional coaching, a focus on teacher learning could lead to greater insight into this growing educational technique. There is still more to learn about what and how a teacher learns when working with a coach. Yet this shift in the research perspective has the potential to bring greater understanding to instructional coaching. Viewing coaching as teaching teachers could impact the work of coaches and teachers, which ultimately has the potential to improve student achievement.

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Appendix A

Notes Used by Sarah

Notes

**Algebra Ext
10.1 #2***Add or subtract as indicated.*

1. $(12x^3 + x^2) - (18x^3 - 3x^2 + 6)$

*Did #1, 3, 5 together,
asking students questions.*

3. $(3x^2 + 7x - 6) - (3x^2 + 7x)$

5. $(z^3 + z^2 + 1) - z^2$

Individual

7. $(9x^3 + 12x) + (16x^3 - 4x + 2)$

Individual

9. $(-7x^2 + 12) - (6 - 4x^2)$

**Went over #7, #9 as a class*

Appendix B

Simon Says Formative Assessment Used during Sarah's class

Simon
Says Name

2. $(2m - 8m^2 - 3) + (m^2 + 5m)$

$$-7m^2 + 7m - 3$$

4. $(x^2 - 7) + (2x^2 + 2)$

$$3x^2 - 5$$

*How did they do?
1/2 did it well.*

6. $(\underline{3n^2} + \underline{2n} - 7) + (\underline{n^3} + \underline{n} + 2)$

$$-n^3 + 3n^2 + 3n - 5$$

8. $(3x + 2x^2 - 4) + (x^2 + x + 6)$

$$x^2 + 2x + 2$$

10. $(\underline{3x^2} + \underline{4}) + (\underline{6x^2} - \underline{8x} - \underline{4})$

$$9x^2 - 8x$$

Appendix C

Independent Practice – Worksheet Given to Students by Sarah

Algebra Ext
10.1 #3

Name _____ (1)

Add or subtract as indicated.

1. $(a + 3a^2 + 2a^3) - (a^2 - a^3)$

2. $(8y^2 + 2) + (5 - 3y^2)$

3. $(4x^2 - 7x + 2) + (-x^2 + x - 2)$

4. $(-3a^2 + 5) + (-a^2 + 4a - 6)$

5. $12 - (y^3 + 10y + 16)$

6. $(3a^3 - 4a^2 + 3) - (a^3 + 3a^2 - a - 4)$

7. $(-2t^4 + 6t^2 + 5) - (-2t^4 + 5t^2 + 1)$

8. $(u^3 - u) - (u^2 + 5)$

9. $(10x^3 + 2x^2 - 11) + (9x^2 + 2x - 1)$

10. $8x^2 - (5x^2 - 6x)$

Appendix D
Draft of Observation Tool

Mathematical Knowledge Rubric
(Adapted from H. Hill's MQI Video Observation Tool)

Indicators	1	2	3	4	Comment
<p>Richness of the Mathematics</p>	<ul style="list-style-type: none"> ▪ No representations or models of math content. ▪ No solution strategies were provided for a single problem. ▪ No links among symbols, pictures, diagrams, solution strategies, etc. <p style="text-align: center;">- 1 +</p>	<ul style="list-style-type: none"> ▪ Very few representations or models of math content. ▪ At least 2 solution strategies for a single problem. ▪ Few links among symbols, pictures, diagrams, solution strategies, etc. <p style="text-align: center;">- 2 +</p>	<ul style="list-style-type: none"> ▪ Some representations or models of math content. ▪ At least 2 solution strategies for a single problem. ▪ Few links among symbols, pictures, diagrams, solution strategies, etc. <p style="text-align: center;">- 3 +</p>	<ul style="list-style-type: none"> ▪ Multiple representations or models of math content. ▪ Multiple solution strategies for a single problem. ▪ Explicit links among symbols, pictures, diagrams, solution strategies, etc. <p style="text-align: center;">- 4 +</p>	
<p>Positive Mathematical Interactions with Students</p>	<ul style="list-style-type: none"> ▪ Does not identify mathematical insight in specific student questions, comments, work ▪ Does not make productive mathematical use of student errors ▪ Teacher does not understand non-standard student solution methods <p style="text-align: center;">- 1 +</p>	<ul style="list-style-type: none"> ▪ Sometimes identifies mathematical insight in specific student questions, comments, work ▪ Sometimes makes productive mathematical use of student errors ▪ Teacher sometimes understands non-standard student solution methods <p style="text-align: center;">- 2 +</p>	<ul style="list-style-type: none"> ▪ Usually identifies mathematical insight in specific student questions, comments, work ▪ Usually makes productive mathematical use of student errors ▪ Teacher usually understands non-standard student solution methods <p style="text-align: center;">- 3 +</p>	<ul style="list-style-type: none"> ▪ Always identifies mathematical insight in specific student questions, comments, work ▪ Always makes productive mathematical use of student errors ▪ Teacher always understands non-standard student solution methods <p style="text-align: center;">- 4 +</p>	
<p>Errors in the Math (by teacher) Errors made and quickly corrected (within a minute)</p>	<ul style="list-style-type: none"> ▪ Major mathematical errors or serious mathematical oversights ▪ Major errors in notation (mathematical symbols) or mathematical language ▪ Serious lack of clarity in presentation of mathematical content ▪ Misunderstands student production and questions <p style="text-align: center;">- 1 +</p>	<ul style="list-style-type: none"> ▪ Mathematical errors or serious mathematical oversights ▪ Many errors in notation (mathematical symbols) or mathematical language ▪ Lack of clarity in presentation of mathematical content ▪ Sometimes misunderstands student production and questions <p style="text-align: center;">- 2 +</p>	<ul style="list-style-type: none"> ▪ Minor mathematical errors or mathematical oversights ▪ Minor errors in notation (mathematical symbols) or mathematical language ▪ Provides general clarity in presentation of mathematical content ▪ Usually understands student production and questions <p style="text-align: center;">- 3 +</p>	<ul style="list-style-type: none"> ▪ No mathematical errors or serious mathematical oversights ▪ No errors in notation (mathematical symbols) or mathematical language ▪ Complete clarity in presentation of mathematical content ▪ Complete understanding of student production and questions <p style="text-align: center;">- 4 +</p>	

Pedagogical Content Knowledge Rubric

Indicators	1	2	3	4	Comments
<p>Best Practices in Mathematics Lesson Structure</p> <p>Adapted from Horizon Research, Inc. - Inside the Classroom: Observation and Analytic Protocol</p>	<ul style="list-style-type: none"> Structure of the lesson <i>not at all</i> reflective of best practices in math education <p>- 1 +</p>	<ul style="list-style-type: none"> Structure of the lesson <i>somewhat</i> reflective of best practices in math education <p>- 2 +</p>	<ul style="list-style-type: none"> Structure of the lesson <i>reflective</i> of best practices in math education <p>- 3 +</p>	<ul style="list-style-type: none"> Structure of the lesson <i>extremely</i> reflective of best practices in math education Adequate time and structure were provided for "sense-making" Lesson reflected careful planning and organization <p>- 4 +</p>	
<p>Best Practices in Math Lesson Implementation</p> <p>Adapted from Horizon Research, Inc. - Inside the Classroom: Observation and Analytic Protocol</p>	<ul style="list-style-type: none"> Implementation of the lesson <i>not at all</i> reflective of best practices in math education <p>- 1 +</p>	<ul style="list-style-type: none"> Implementation of the lesson <i>somewhat</i> reflective of best practice in mathematics education <p>- 2 +</p>	<ul style="list-style-type: none"> Implementation of the lesson <i>reflective</i> of best practice in mathematics education <p>- 3 +</p>	<ul style="list-style-type: none"> Implementation of the lesson <i>extremely</i> reflective of best practice in mathematics education Used a variety of instructional strategies to reach all students Formative assessment was implemented in a highly effective manner Questioning strategies were used to increase student understanding <p>- 4 +</p>	
<p>Mathematical Cognitive Demand</p> <p>Adapted from the work of Norman L. Webb and Stein & Smith (1998) Mathematical Task Framework</p>	<p><i>Memorization</i></p> <ul style="list-style-type: none"> Major emphasis on the recall of routine facts, knowledge, shapes, symbols, etc. Memorization is the most evident instructional technique. <p>- 1 +</p>	<p><i>Procedures</i></p> <ul style="list-style-type: none"> Emphasis on use of procedures to arrive at solutions or answers. Step by step <p>- 2 +</p>	<p><i>Procedures + Connections</i></p> <ul style="list-style-type: none"> Explain reasoning and processes as they pertain to other math ideas. Communication of procedures and findings. <p>- 3 +</p>	<p><i>Doing Mathematics</i></p> <ul style="list-style-type: none"> Make connections between concepts and representations. Extend and build on knowledge. Emphasize reason and logic to prove and justify ideas. <p>- 4 +</p>	

Appendix E
Formal Interview Questions

January Interview Question –January 28, 2011

- As you think back over our first year and a half together, are there any memories that stand out for you? How did you feel when these experiences took place? What did you learn from them?
- I also would like you to hear your perspective on how our relationship got started and how it has evolved over the last year and a half.
- In one or two sentences, how would you summarize our work together the last 1.5 years?
- I have been trying to create a description that I could share with teachers that says what I do and how I work with teachers. What would you said I do and how would you describe the relationship?
- What would you say is the value of such a relationship for any teacher?
- Have there been any downsides to our work together?
- What would you say to teachers who are not sold on the idea of me working with them?
- What do you feel are your strengths as a teacher?
- Have these always been your strengths?
- Where or how did you develop these as strengths?
- Thinking about our weekly work together this semester, are there particular things that you would want to work on with regard to a particular math topic? With regard to teaching? With regard to understanding struggling students' learning of Algebra?

May Interview Questions –May 25, 2011

- As you think back over our last semester together, are there any memories that stand out for you? How did you feel when these experiences took place? What did you learn from them?
- In one or two sentences, how would you summarize our work together this semester?
- After working closely together this semester, what would you say I do and how would you describe the relationship?
- Have there been any downsides to our work together?
- Here are the things that at the beginning of the semester you said you wanted to work on... (Let Aaron read the transcribed portion of the interview.)
- What are you thinking after reading what you said at the beginning of the semester?
- Can you think of any moments throughout the semester that relate to what you highlighted as things you want to work on?
- What would you say to teachers who are not sold on the idea of me working with me?
- What do you feel are your strengths as a teacher?

Appendix F
Data Collection Template

WEEK OF JANUARY 17-21

Coaching Cycle

Preconference Planning Session Field Notes

Date:

Time:

Insert Field Notes Here

Classroom Lesson

Date:

Coach's Role:

Insert Field Notes Here

Postconference Discussion

Date:

What did you learn?

What helped you learn this?

Insert Transcription Here

Next Steps

Insert Field Notes Here

Artifacts related to Coaching Cycle

Insert List of Related Artifacts Here

Personal Reflection/Connections specific to the Coaching Cycle

What do I think the teacher learned this week?

How do I know that is what he learned?

How did he learn it?

Insert Personal Journal entry here

Field Notes

Professional Learning Community (PLC)

Insert Field Notes Here

Other Teacher or School Interactions

Insert Field Notes Here

Artifacts

Insert List of Related Artifacts Here

Personal Reflection/Connections

How did the teacher influence others this week?

How do I know this?

Insert Personal Journal entry here

Appendix G
Spring 2011 – Algebra Syllabus – Semester Two

Concepts and Skills	Section	State Standards
SOLVE SYSTEMS OF LINEAR EQUATIONS APPLY SYSTEMS OF LINEAR EQUATIONS		
Solve Systems by Graphing	7-1	[SS12.3.2a, SS12.3.3p]
Solve Systems Using Substitution	7-2	[SS12.3.2a, SS12.3.3p]
Solve Systems Using Elimination	7-3	[SS12.3.2a, SS12.3.3p]
SIMPLIFY EXPONENTIAL EXPRESSIONS		
*Simplify and Evaluate Exponential Expressions with and without Zero and Negative Exponents	8-1	[SS12.1.3b, SS12.3.3b] DK1 & DK2
Write and Use Scientific Notation	8-2	[SS12.1.3c]
*Use Multiplication Properties of Exponents to Multiply Powers	8-3	[SS12.1.1b, SS12.1.3b, SS12.1.3c, SS12.3.3b] DK1 & DK2
*Simplify Powers and Products Raised to a Power	8-4	[SS12.1.1a, SS12.1.3b, SS12.1.3c, SS12.3.3b] DK1, DK2, & DK3
*Simplify Exponential Expressions using Division Properties of Exponents	8-5	[SS12.1.1b, SS12.1.3b, SS12.1.3c, SS12.3.3b] DK1, DK2 & DK3
EVALUATE AND GRAPH EXPONENTIAL FUNCTIONS		
*Evaluate and Graph Exponential Functions	8-7	[SS12.3.1a, SS12.3.1b, SS12.3.1e, SS12.3.2d] DK2 & DK3
*Model Exponential Growth and Decay	8-8	[SS12.3.1a, SS12.3.1b, SS12.3.1e, SS12.3.2d] DK2
Cumulative Assessment (7-8)		
PERFORM OPERATIONS WITH POLYNOMIALS		
*Add and Subtract Polynomials	9-1	[SS12.3.3c] DK1
*Multiply and Factor Polynomials	9-2	[SS12.3.3d, SS12.3.3e] DK1
*Multiply Binomials	9-3	[SS12.3.3d] DK1
*Multiply Special Types of Binomials	9-4	[SS12.3.3d] DK1
FACTOR POLYNOMIALS		
*Multiply and Factor Polynomials	9-2	[SS12.3.3d, SS12.3.3e] DK1
Factor Trinomials of the Type x^2+bx+c	9-5	[SS12.3.3e]
Factor Trinomials of the Type ax^2+bx+c	9-6	[SS12.3.3e]
Factor Special Types of Binomials	9-7	[SS12.3.3e]
EXPLORE AND GRAPH QUADRATIC FUNCTIONS		
*Explore Different Quadratic Graphs	10-1	[SS12.3.1a, SS12.3.1b, SS12.3.1e] DK2 & DK3
*Graph Quadratic Functions	10-2	[SS12.3.1a, SS12.3.1b, SS12.3.1e] DK2 & DK3
SOLVE QUADRATIC EQUATIONS		
Pre-requisite Skill: Find and Estimate Square Roots	3-8	[SS12.1.1a]
Solve Quadratic Equations	10-3	[SS12.3.2d, SS12.3.3i]
Use Factoring to Solve Quadratic Equations	10-4	[SS12.3.1e, SS12.3.2d, SS12.3.3i]
Use the Quadratic Formula to Solve Quadratic	10-6	[SS12.3.1e, SS12.3.2d, SS12.3.3i]
MODEL LINEAR, QUADRATIC, AND EXPONENTIAL FUNCTIONS		
*Determine type of function given a graph or table	10-8	[SS12.3.1d, SS12.3.2c, SS12.3.2d] DK2 & DK3
Write a function to model data	10-8	[SS12.3.2a, SS12.3.2c, SS12.3.2d]
Cumulative Assessment (9-10)		
SIMPLIFY, SOLVE AND GRAPH WITH RADICALS		
Simplify Radicals	11-1	[SS12.1.1a, SS12.1.3a, SS12.3.2d]
Simplify Radical Expressions (Eliminate rationalizing using conjugates)	11-2	[SS12.1.1a, SS12.3.2d]
Solve Radical Equations	11-3	[SS12.1.3a, SS12.3.2d]
*Graph Square Root Functions Using a Table of Values	11-4	[SS12.3.1a, SS12.3.1b, SS12.3.1e, SS12.3.2d] DK2 & DK3
FIND AND USE PROBABILITY		
*Find the Theoretical and Experimental Probability	2-6	[SS12.4.3b, SS12.4.3d, SS12.4.3e] DK1 & DK2
*Find the Probability of Compound Events	2-7	[SS12.4.3b] DK1 & DK2
*Use the Multiplication Counting Principle (NOT Permutations)	12-7	[SS12.4.3c] DK1 & DK2
Final Assessment	SS= State Standard	* = State Standard Assessed DK = Depth of Knowledge

Appendix H
Example of Nathaniel's Daily Lesson Plans

Algebra-Tuesday

8.5 Warm – Up

Name: _____

1. You went to the Department of Motor Vehicles to get license plates on your new car. Your license plate is 7 characters long made with both numbers and letters. How many different plates could possibly be made?

2a. Your friend was struggling in deciding whether to add or multiply exponents in the following: $(3a^4)^2$. Instead of telling them what to do you said, "If in doubt, _____ it out."

2b. Now simplify $(3a^4)^2$ using the advice you gave your friend.

Simplify

3. $-4g^0$

4. $(-4g)^0$

5. $3^2x^4 \cdot 4^2x$

8.5 Discovery Form

- Focus on getting students to visualize their simplification
- Do not emphasize subtr. rule... (future – alter form to not even mention it?)

Dry – Erase Boards

1. $\frac{5^8}{5^6}$

2. $\frac{7^{-2}}{7^5}$

3. $\frac{x^{-4}}{x^{-5}}$

4. $\frac{m^2n^4}{m^5n^{-6}}$

5. $\frac{k^{18}n^{-5}}{k^{-8}n^{-6}}$

6. $\frac{5^2x^{-2}}{5^4x^{-50}}$

Quiz – Quiz – Trade

Introduce Quotient to a Power

- Begin with $(ab^2)^3$.
- "How would we simplify this?"
- "What if I were in doubt... 'expand it out...'"
- Now you are being given 3 problems...
- Your job is to come up with a method, rule, or conjecture about what happens when you take a quotient to a power.

8.5 Scavenger Hunt

- Expectations
 - Remain in area with their partner – No wondering
 - Rotate in timely manner

Appendix I
Notes Sheet Nathaniel Created for Dividing Exponents

8.5 Division of Exponents Discovery Part II

Name: _____

1. Expand both the numerator and denominator in table on the right. Simplify.

Expanded Power	Simplified Power
$\frac{5^6}{5^2} = \underline{\hspace{2cm}}$	
$\frac{3^7}{3^3} = \underline{\hspace{2cm}}$	
$\frac{a^3}{a^5} = \underline{\hspace{2cm}}$	

2. Examine the exponents of the original fraction and the simplified power. Do you notice any patterns?
Please describe.

3. Simplify the following using the pattern from above.

a. $\frac{b^{14}}{b^4}$

b. $\frac{x^{10}}{x^{80}}$

c. $\frac{w^{-8}}{w^5}$

BIG IDEA : DIVIDING POWERS WITH THE SAME BASE

When you divide powers with the same base, you _____.

Expanding Our Scope

1a. $\frac{p^4 c^6}{p^5 c^2} =$

1b. $\frac{y^{10} x^4}{y^3 x^8} =$

2a. $\frac{3x^6 y^{10}}{6x^{-5} y} =$

2b. $\frac{12a^4 b^{-1}}{4a^{-2} b^2} =$

Appendix J
Algebraic Reasoning and Sense Making Poster

Reasoning and Sense Making with Algebraic Symbols

- **Mindful use of Symbols:** Choosing variables and constructing expressions and equations in context; interpreting the form of expressions and equations; manipulating expressions so that interesting interpretations can be made.
- **Mindful Manipulation:** Connecting manipulation with the laws of arithmetic; anticipating the results of manipulation; choosing procedures purposefully in context; picturing calculations mentally.
- **Reasoned Solving:** Seeing solution steps as logical deductions about equality; interpreting solutions in context.
- **Connecting Algebra with Geometry:** Representing geometric situations algebraically and algebraic situations geometrically; using connections in solving problems.
- **Linking Expressions and Functions:** Using multiple algebraic representations to understand functions; working with function notation.

Reasoning and Sense Making with Functions

- **Using Multiple Representations of Functions:** Representing functions in various ways, including tabular, graphic, symbolic (explicit and recursive), visual, and verbal; making decisions about which representations are most helpful in problem-solving circumstances; and moving flexibly among those representations.
- **Modeling by using Families of Functions:** Working to develop a reasonable mathematical model for a particular contextual situation by applying knowledge of the characteristic behaviors of different families of functions.
- **Analyzing the Effects of Parameters:** Using a general representation of a function in a given family, to analyze the effects of varying coefficients or other parameters; converting between different forms of functions according to the requirements of the problem solving situation.

Appendix K
Widths Discovery

1. Graph the following quadratics functions onto the same graph. Use different colors for each graph.

Graph A	$y = x^2$	
	Color:	
	x	y

Graph B	$y = 2x^2$	
	Color:	
	x	y

Graph C	$y = \frac{1}{2}x^2$	
	Color:	
	x	y

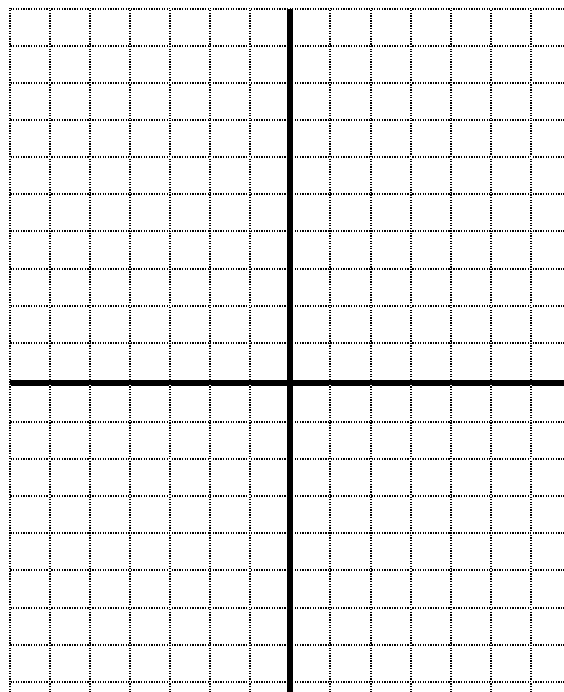
2. What type of patterns are you noticing?
(Hint: Compare the various equations, tables, and graphs)

3. How do you think the function $f(x) = 10x^2$ would look compared to the previously graphed functions?

4. How do you think the function $f(x) = .25x^2$ would look compared to the previously graphed functions?

5. Complete the table and graph the function.

Graph D	$y = -2x^2$	
	Color:	
	x	y

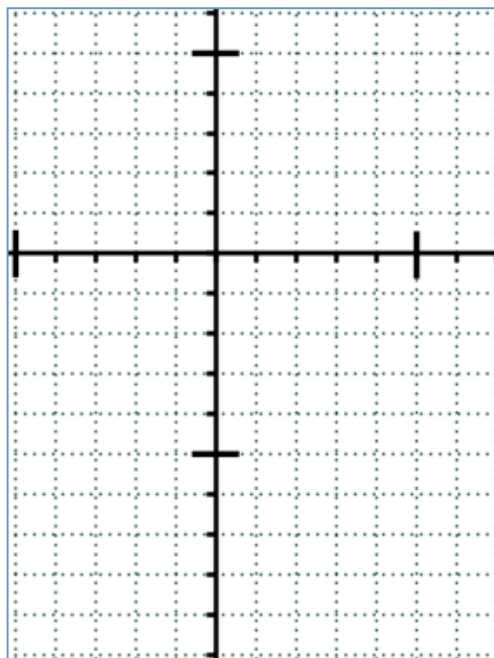


6. Jade examined the equations of Graph B and Graph D. She claims Graph B will be wider since 2 is greater than -2. Do you agree with her conclusion? Why or why not.

Appendix L Shifting Parabolas

1. Graph the Parent Function

Parent Function	$y = 2x^2 - 8x$	
	Color:	
	x	y



2. Graph the assigned quadratic

a) $y = 2x^2 - 8x + 5$

b) $y = 2x^2 - 8x - 2$

c) $y = 2x^2 - 8x + 3$

	Color:	
	x	y

3. How does your graph differ from the parent function? Is this observation true among all your tablemates?

4. Make a conjecture about vertical shifting for all quadratic functions.

5. In the equation $y = 10x^2 - 4x + 12$, explain how the parent graph $y = 10x^2 - 4x$ was shifted.

6. In the equation $y = 3x^2 - 5$, explain how the parent graph $y = 3x^2$ was shifted.

