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FAST TRACK COMMUNICATION

Resonant phenomena in laser-assisted radiative attachment or recombination

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Abstract

Resonant enhancements are predicted in cross sections σ_n for laser-assisted radiative attachment or electron–ion recombination accompanied by absorption of n laser photons. These enhancements occur for incoming electron energies at which the electron can be attached or recombined by emitting μ laser photons followed by emission of a spontaneous photon upon absorbing $n + \mu$ laser photons. The close similarity between rescattering plateaus in spectra of resonant attachment/recombination and of high-order harmonic generation is shown based on a general parametrization for σ_n and on numerical results for e−H attachment.

An electron colliding with an atom in the presence of an intense laser field can efficiently convert the combined energy of a large number of laser photons into the energy of a spontaneously emitted photon [1]. This process occurs, in particular, in laser-assisted radiative attachment (LARA) or recombination (LARR), in which the emission of a spontaneous photon is accompanied by absorption of n laser photons and the formation of a negative ion or neutral atom. Although experimental studies of these processes have only begun recently [2], theoretical studies began more than a decade ago. The first theoretical investigations employed the strong field approximation (SFA) [3–5], in which the effects of the atomic potential $U(r)$ on the scattering state of the incident electron are neglected. These studies show that (even in lowest order in the potential $U(r)$) the LARR cross sections as a function of n (or the energy of the spontaneous photon) exhibit a plateau structure, whose shape and extent can be described by treating the LARR process classically [6]. The first-order correction in the potential $U(r)$ to the SFA LARR amplitude was introduced in [7] (see also [1, 8]) taking into account $U(r)$ perturbatively, using the Born approximation. Inclusion of higher-order corrections in $U(r)$ (or rescattering effects) into the LARA/LARR amplitudes results in a second, high-energy (or rescattering) plateau in the LARA/LARR spectra [7, 9]. However, as for laser-induced processes, such as high-order harmonic generation (HHG) and above-threshold ionization (cf [1]), the cross sections for the high energy rescattering plateau are orders ofmagnitude smaller than those for the low-energy plateau. Hence, mechanisms for increasing the high-energy plateau cross sections are of great interest. For laser-assisted collisions, one way to achieve such an increase is to tune the incoming electron energy so that it can be temporarily captured (by stimulated emission of μ laser photons) to a bound state of the potential $U(r)$. Obviously, such a resonance phenomenon cannot be described in the Born approximation and requires an accurate account of the potential $U(r)$. Significant enhancement of plateau structures in resonant laser-assisted electron–atom scattering (LAES) was predicted recently [10]. However, resonant phenomena in LARA/LARR processes remain unexplored.

In the present communication, we extend the study of laser-induced resonant phenomena in collision problems to the case of LARA/LARR. We present a general parametrization for the resonant LARA/LARR cross sections and show the following at resonant electron energies: (i) the shape of LARA/LARR spectra as a function of n coincides with that for HHG; (ii) the electron energy dependence of the n-photon LARA/

LARR cross section exhibits an asymmetric profile similar to the Fano profile in photoionization cross sections [11]; and (iii) LARA cross sections can be enhanced by more than two orders of magnitude. To describe LARA analytically, we use time-dependent effective range (TDER) theory, which provides a means to account for the short-range atomic potential $U(r)$ in LARA non-perturbatively.

To describe electron–atom collisions in a monochromatic field with electric vector $\mathbf{F}(t) = \mathbf{e}_z F \cos\omega t$ (where F and ω are the field amplitude and frequency) using the electric dipole approximation, the quasienergy (or Floquet) approach [12] is most appropriate. Within this approach, the laser-dressed scattering state of an electron with momentum $\mathbf{p}$ and energy $E = p^2/(2m)$ in the potential $U(r)$ has the form

$$\Psi_{\epsilon,\mathbf{p}}(\mathbf{r},t) = \mathrm{e}^{-\mathrm{i}\epsilon t/\hbar}\Phi_{\epsilon,\mathbf{p}}(\mathbf{r},t),$$
$$\Phi_{\epsilon,\mathbf{p}}(\mathbf{r},t) = \Phi_{\epsilon,\mathbf{p}}(\mathbf{r},t+T), \quad T = 2\pi/\omega, \tag{1}$$

where ϵ is the quasienergy, $\epsilon = E + u_p$, where $u_p = e^2F^2/(4m\omega^2)$ is the mean quiver energy of an electron in the field $\mathbf{F}(t)$. For $\mathbf{F}(t) = 0$, the quasienergy state $\Phi_{\epsilon,\mathbf{p}}(\mathbf{r}, t)$ reduces to the scattering state $\psi_{\mathbf{p}}(\mathbf{r})$ of the recombining electron in the potential $U(r)$. In the field $\mathbf{F}(t)$, the bound (final) state $\psi_0(\mathbf{r})$ with energy E_0 evolves to the quasistationary quasienergy state (QQES), $\Psi_\varepsilon(\mathbf{r}, t)$, which also has the form (1), but with the complex quasienergy $\varepsilon = E_0 + \Delta E_0 - \mathrm{i}\Gamma/2$, where ΔE_0 and Γ are the field-induced Stark-shift and width (or total decay rate $\Gamma/\hbar$) of the state $\psi_0(\mathbf{r})$ [13].

We consider the LARA/LARR process as a dipole transition between initial and final states $\Psi_{\epsilon,\mathbf{p}}(\mathbf{r}, t)$ and $\Psi_\varepsilon(\mathbf{r}, t)$ with emission of a spontaneous photon, whose energy differs from the field-free energy $\hbar\Omega_0 = E - E_0$. Within the QQES approach, the LARA/LARR cross section $\sigma(\Omega)$, integrated over the directions of emission and summed over polarizations of the spontaneous photon, can be written as (cf [7])

$$\sigma(\Omega) = \frac{4m\Omega^3}{3\hbar c^3 p}|\mathbf{d}(\Omega)|^2, \tag{2}$$

$$\mathbf{d}(\Omega) = \frac{1}{T}\int_0^T \mathrm{d}t \int \mathrm{d}\mathbf{r}\tilde{\Psi}_\varepsilon^*(\mathbf{r},t)\mathbf{d}\Psi_{\epsilon,\mathbf{p}}(\mathbf{r},t)\mathrm{e}^{\mathrm{i}\Omega t}, \tag{3}$$

where $\mathbf{d} = e\mathbf{r}$ ($e = -|e|$) and Ω is the frequency of the spontaneously emitted photon:

$$\hbar\Omega = \epsilon + n\hbar\omega - \mathrm{Re}\,\varepsilon.$$

The function $\tilde{\Psi}_\varepsilon(\mathbf{r}, t)$ in (3) is the so-called dual function to $\Psi_\varepsilon(\mathbf{r}, t)$. If $\mathbf{F}(t)$ is linearly polarized and $\psi_0(\mathbf{r})$ is a bound s-state, then $\tilde{\Psi}_\varepsilon(\mathbf{r}, t)$ is defined as [14–16]

$$\tilde{\Psi}_\varepsilon(\mathbf{r},t) = \mathrm{e}^{-\mathrm{i}\varepsilon^* t/\hbar}\Phi_\varepsilon^*(\mathbf{r},-t). \tag{4}$$

Since the QQES wavefunctions $\Phi_\varepsilon(\mathbf{r}, t)$ diverge asymptotically as $r \to \infty$ (since they describe the ionization of a bound state $\psi_0(\mathbf{r})$ in the field $\mathbf{F}(t)$), the use of dual functions as bra-vectors in the QQES approach is necessary to ensure proper normalization of the wavefunctions $\Phi_\varepsilon(\mathbf{r}, t)$ and the regularization of matrix elements involving these functions (cf [14–16] for further details).

To describe resonant LARR or LARA processes, we note first that the wavefunction $\Psi_\varepsilon(\mathbf{r}, t)$ can be obtained as a residue of the scattering state $\Psi_{\epsilon,\mathbf{p}}(\mathbf{r}, t)$ in the complex plane of ϵ at $\epsilon = \varepsilon + \mu\hbar\omega = \mathrm{Re}\,\varepsilon + \mu\hbar\omega - \mathrm{i}\Gamma/2$ [17]:

$$\mathrm{Res}\Psi_{\epsilon,\mathbf{p}}(\mathbf{r},t)\big|_{\epsilon=\varepsilon+\mu\hbar\omega} \sim \mathrm{e}^{\mathrm{i}\mu\omega t}\Psi_{\varepsilon+\mu\hbar\omega}(\mathbf{r},t) = \Psi_\varepsilon(\mathbf{r},t),$$

where μ is an integer. Therefore, for $\epsilon \approx \epsilon_\mu = \mathrm{Re}\,\varepsilon + \mu\hbar\omega$, the scattering state $\Phi_{\epsilon,\mathbf{p}}(\mathbf{r}, t)$ can be approximated by a sum of potential (non-resonant) and resonant parts [17]:

$$\Phi_{\epsilon,\mathbf{p}}(\mathbf{r},t) = \Phi^{(p)}_{\epsilon_\mu,\mathbf{p}_\mu}(\mathbf{r},t) + \mathcal{B}(\mathbf{p}_\mu)\frac{\Phi_\varepsilon(\mathbf{r},t)}{E - E_\mu + \mathrm{i}\Gamma/2}, \tag{5}$$

where $E_\mu = \mathbf{p}^2_\mu/(2m) = \mathrm{Re}\,\varepsilon + \mu\hbar\omega - u_p$ is the resonant electron energy and the coefficient $\mathcal{B}(\mathbf{p}_\mu)$ is proportional to the amplitude for stimulated μ-photon recombination or attachment (cf (30)). Substituting (5) into (3), the amplitude $\mathbf{d}(\Omega)$ can also be presented as a sum of potential and resonant terms:

$$\mathbf{d}(\Omega) = \mathbf{d}^{(p)}(\Omega) + \mathcal{B}(\mathbf{p}_\mu)\frac{\tilde{\mathbf{d}}(\Omega)}{E - E_\mu + \mathrm{i}\Gamma/2}, \tag{6}$$

where the potential term $\mathbf{d}^{(p)}(\Omega)$ is given by (3) (upon substituting there $\Phi^{(p)}_{\mu,\mathbf{p}_\mu}(\mathbf{r}, t)$ for $\Phi_{\epsilon,\mathbf{p}}(\mathbf{r}, t)$), while the resonant term involves the dual dipole moment, $\tilde{\mathbf{d}}(\Omega) = \tilde{d}(\Omega)\mathbf{e}_z$, which determines the rate $\mathcal{R}(\Omega)$ for the generation of a harmonic of the field $\mathbf{F}(t)$ with frequency $\Omega = (n + \mu)\omega$ by a bound electron in the s-state $\psi_0(\mathbf{r})$ [16]:

$$\mathcal{R}(\Omega) = \frac{\Omega^3}{2\pi\hbar c^3}|\tilde{\mathbf{d}}(\Omega)|^2. \tag{7}$$

Since the problem involves only two vectors, $\mathbf{e}_z$ and $\hat{\mathbf{p}}$ (mutually oriented at an angle θ), the vector $\mathbf{d}^{(p)}(\Omega)$ lies in the plane $(\mathbf{e}_z, \hat{\mathbf{p}})$ and can be presented as

$$\mathbf{d}^{(p)}(\Omega) = d_\parallel^{(p)}(\Omega)\mathbf{e}_z + d_\perp^{(p)}(\Omega)\mathbf{e}_\perp, \tag{8}$$

where $\mathbf{e}_\perp = [\mathbf{e}_z \times [\hat{\mathbf{p}} \times \mathbf{e}_z]]$. Using (6) and (8), we obtain the *general parametrization* for the LARA/LARR cross section (2) near a μ-photon resonance

$$\sigma(\Omega) = \sigma^{(p)}(\Omega) + \sigma_\parallel^{(p)}(\Omega)\frac{2(\mathrm{Re}\,q - \delta\,\mathrm{Im}\,q) + |q|^2}{\delta^2 + 1}, \tag{9}$$

where $\delta = 2(E - E_\mu)/\Gamma$, $q = -2\mathrm{i}(\mathbf{p}_\mu)\,\tilde{d}(\Omega)/(\Gamma d_\parallel^{(p)}(\Omega)$, and $\sigma^{(p)}(\Omega)$ and $\sigma_\parallel^{(p)}(\Omega)$ are given by (2) upon substituting there $\mathbf{d}(\Omega) \to \mathbf{d}^{(p)}(\Omega)$ or $\mathbf{d}(\Omega) \to d_\parallel^{(p)}(\Omega)\mathbf{e}_z$. The parametrization (9) simplifies for parallel geometry, $\mathbf{p} \parallel \mathbf{e}_z$, in which case $\sigma_\parallel^{(p)} = \sigma^{(p)}$:

$$\sigma(\Omega) = \sigma^{(p)}(\Omega)\frac{(\mathrm{Re}\,q + 1)^2 + (\mathrm{Im}\,q - \delta)^2}{\delta^2 + 1}. \tag{10}$$

Results (9) and (10) showthat (for a given n) $\sigma(\Omega)$ as a function of E is asymmetric with respect to the resonance energy E_μ. For small Γ (i.e. taking into account only terms $\sim 1/\Gamma^2$), the result for the cross section $\sigma(\Omega)$ at the resonance, $\delta = 0$, is

$$\sigma(\Omega) \approx \frac{32\pi m|\mathcal{B}(\mathbf{p}_\mu)|^2}{3p_\mu\Gamma^2}\mathcal{R}(\Omega). \tag{11}$$

Since $\mathcal{B}(\mathbf{p}_\mu)$ does not depend on the number of absorbed photons, the shapes of resonant LARA/LARR spectra as functions of n replicate the shapes of the corresponding bound state HHG spectra.

To present quantitative results for laser-induced resonance phenomena in LARA, we use TDER theory to describe both the incident continuum ($\Phi_{\epsilon,\mathbf{p}}$) [18] and final bound ($\Phi_{\varepsilon}$) [19] field-dressed states of the active electron. This theory assumes that the interaction of an electron with a short-range potential $U(r)$ (having only a single bound state $\psi_{E0lm}(\mathbf{r})$ with angular momentum l) is described by the l-wave scattering phase $\delta_l(E)$ that is parameterized by the scattering length a_l and the effective range r_l, which are parameters of the problem. For simplicity, we consider the case of a bound s-state $\psi_0(\mathbf{r})$ of energy $E_0 = -(\hbar\kappa)^2/(2m)$. For this case, the TDER wavefunctions $\Phi_{\epsilon,\mathbf{p}}(\mathbf{r}, t)$ and $\Phi_{\varepsilon}(\mathbf{r}, t)$ are expressed in terms of one-dimensional integrals [18, 19]:

$$\Phi_{\epsilon,\mathbf{p}}(\mathbf{r},t) = \mathrm{e}^{\mathrm{i}[\mathbf{P}(t)\cdot\mathbf{r}-\mathcal{S}(\mathbf{p},t)]/\hbar} - \frac{2\pi\hbar^2}{m\kappa}\int G(\mathbf{r},t,0,t')f_\epsilon(\mathbf{p},t')\,\mathrm{e}^{\mathrm{i}\epsilon(t-t')/\hbar}\,\mathrm{d}t', \tag{12}$$

$$\Phi_\varepsilon(\mathbf{r},t) = -\frac{2\pi\hbar^2\sqrt{\kappa}}{m}\int G(\mathbf{r},t,0,t')g_\varepsilon(t')\,\mathrm{e}^{\mathrm{i}\varepsilon(t-t')/\hbar}\,\mathrm{d}t', \tag{13}$$

where $G(\mathbf{r}, t, \mathbf{r}', t')$ is the retarded Green function and $S(\mathbf{p}, t)$ is the classical action of an electron in the field $\mathbf{F}(t)$,

$$\mathcal{S}(\mathbf{p},t) = \int^t [\mathbf{P}^2(t')/(2m) - \epsilon]\,\mathrm{d}t',$$

$\mathbf{P}(t)$ is the canonical momentum,

$$\mathbf{P}(t) = \mathbf{p} - \mathbf{e}_z\frac{eF}{\omega}\sin\omega t,$$

and $f_\epsilon(\mathbf{p}, t)$ and $g_\varepsilon(t)$ are dimensionless periodic functions,

$$f_\epsilon(\mathbf{p},t) = \sum_k f_k(\mathbf{p})\mathrm{e}^{-\mathrm{i}k\omega t}, \quad g_\varepsilon(t) = \sum_k g_k\mathrm{e}^{-\mathrm{i}k\omega t}.$$

The Fourier-coefficients $f_k(\mathbf{p})$ and g_k as well as the complex quasienergy ε can be found from a system of inhomogeneous (for $f_k(\mathbf{p})$) or homogeneous (for g_k and ε) linear equations:

$$\sum_{k'}\mathcal{M}_{k,k'}(\epsilon)f_{k'}(\mathbf{p}) = c_k(\mathbf{p}), \tag{14}$$

$$\sum_{k'}\mathcal{M}_{k,k'}(\varepsilon)g_{k'} = 0, \tag{15}$$

$$\mathcal{M}_{k,k'}(\mathcal{E}) = R(\mathcal{E}+k\hbar\omega)\delta_{k,k'} + M_{k,k'}(\mathcal{E}), \tag{16}$$

$$R(\mathcal{E}) = \frac{1}{a_0\kappa} - \frac{mr_0\mathcal{E}}{\hbar^2\kappa} + \mathrm{i}\sqrt{\frac{\mathcal{E}}{|E_0|}}, \tag{17}$$

$$c_k(\mathbf{p}) = \mathrm{i}^{-k}\sum_s J_{k+2s}\left(\frac{2eFp_\parallel}{m\hbar\omega^2}\right)J_{-s}\left(\frac{u_p}{2\hbar\omega}\right), \tag{18}$$

where a_0 and r_0 are the scattering length and the effective range, $p_\parallel = (\mathbf{e}_z \cdot \mathbf{p}) = p\cos\theta$, and $J_n(x)$ is a Bessel function. The matrix elements $M_{k,k'}(\mathcal{E})$ are nonzero only if the difference $k - k'$ is *even* and have the form

$$M_{k,k'}(\mathcal{E}) = \sqrt{\frac{\mathrm{i}^{k-k'}\hbar\omega}{8\pi\mathrm{i}|E_0|}}\int_0^\infty\frac{\mathrm{d}\tau}{\tau^{3/2}}\,\mathrm{e}^{\mathrm{i}[2\mathcal{E}/(\hbar\omega)+(k+k')]\tau} \times\left\{\mathrm{e}^{\mathrm{i}\lambda(\tau)}J_{(k-k')/2}\left[z(\tau)\right]-\delta_{k,k'}\right\}, \tag{19}$$

where

$$\lambda(\tau) = \frac{2u_p}{\hbar\omega}\left(\frac{\sin^2\tau}{\tau}-\tau\right),$$

$$z(\tau) = \frac{u_p}{\hbar\omega}\left(\sin 2\tau - \frac{2\sin^2\tau}{\tau}\right).$$

From the explicit form of $M_{k,k'}(\mathcal{E})$ follow the symmetry relations

$$\mathcal{M}_{k,k'}(\mathcal{E}) = \mathcal{M}_{k',k}(\mathcal{E}), \quad \mathcal{M}_{k,k'}(\mathcal{E}+p\hbar\omega) = \mathcal{M}_{k+p,k'+p}(\mathcal{E}). \tag{20}$$

As shown in [19], the function $g_\varepsilon(t)$ (as well as the system of equations (15)) includes only coefficients g_k with even k. The complex quasienergy ε is given by that root of the transcendental equation, Det $\|\mathcal{M}_{k',k}(\varepsilon)\| = 0$, which becomes E_0 when $F \to 0$. For $\mathbf{F}(t) = 0$, the matrix elements $M_{k,k'}(\mathcal{E})$ are zero and coefficients $f_k(\mathbf{p})$ and g_k reduce to

$$f_k(\mathbf{p}) = R^{-1}(E)\delta_{k,0}, \quad g_k = C_\kappa\delta_{k,0}, \tag{21}$$

where C_κ is a dimensionless coefficient in the asymptotic form of $\psi_0(\mathbf{r})$ for $r \gg \kappa^{-1}$:

$$\psi_0(\mathbf{r}) \approx C_\kappa\sqrt{\kappa/(4\pi)}\,r^{-1}\exp(-\kappa r).$$

With the use of (12) and (13), the analytic evaluation of $\mathbf{d}(\Omega)$ in (3) involves the spatial integration of two Green functions and a threefold integration over time. The spatial integration and two of the temporal integrations can be performed analytically (as done in [16]) and the final result for $\mathbf{d}(\Omega)$ can be presented as

$$\mathbf{d}(\Omega) = \mathbf{d}_{pw}(\Omega) + \mathbf{d}_{\mathrm{resc}}(\Omega), \tag{22}$$

$$\mathbf{d}_{pw}(\Omega) = \mathcal{C}\left[\mathbf{e}_z\sum_{p=\pm1}\frac{\mathcal{L}_{n+p}}{\omega/\Omega+p} - 2\mathrm{i}\mathcal{L}_n\frac{\mathbf{p}\omega}{|e|F}\right], \tag{23}$$

$$\mathcal{L}_n(\mathbf{p}) = \sum_{k=-\infty}^{\infty}g_k c_{k+n}(\mathbf{p}), \quad \mathcal{C} = \frac{\pi e^2\hbar F\kappa^{1/2}}{m^2\omega\Omega^2}, \tag{24}$$

$$\mathbf{d}_{\mathrm{resc}}(\Omega) = \mathbf{e}_z\mathcal{C}\sum_{k,k'}f_k(\mathbf{p})W^n_{k,k'}g_{k'}, \tag{25}$$

where the matrix elements $W^n_{k,k'}$ are non-zero only if the difference $n - k$ is *odd*:

$$W^n_{k,k'} = \mathcal{D}\int_0^\infty\frac{\mathrm{d}\tau}{\tau^{3/2}}\mathrm{e}^{\mathrm{i}[(\epsilon+\varepsilon)/(\hbar\omega)+k+k']\tau+\mathrm{i}\lambda(\tau)} \times\{j_-(\tau)J_s[z(\tau)] - \mathrm{i}j_+(\tau)J_{s+1}[z(\tau)]\},$$
$$j_\pm(\tau) = \frac{\sin\tau\sin(\Omega\tau/\omega)}{\tau} - \frac{\Omega\sin(\Omega/\omega\pm1)\tau}{1\pm\omega/\Omega},$$
$$s = \frac{n-k+k'-1}{2}, \quad \mathcal{D} = \mathrm{i}^{s+1/2}\sqrt{\frac{\hbar\omega}{2\pi|E_0|}}. \tag{26}$$

Result (22) for $\mathbf{d}(\Omega)$ is exact within the TDER theory and valid for both resonant and non-resonant electron energies E. The term $\mathbf{d}_{pw}(\Omega)$ originates from the first term in (12) and corresponds to the first-Born (or plane wave) approximation in the potential $U(r)$ for the scattering state. This term is smooth at the resonant energy $E = E_\mu$ and contributes only to the potential part of the

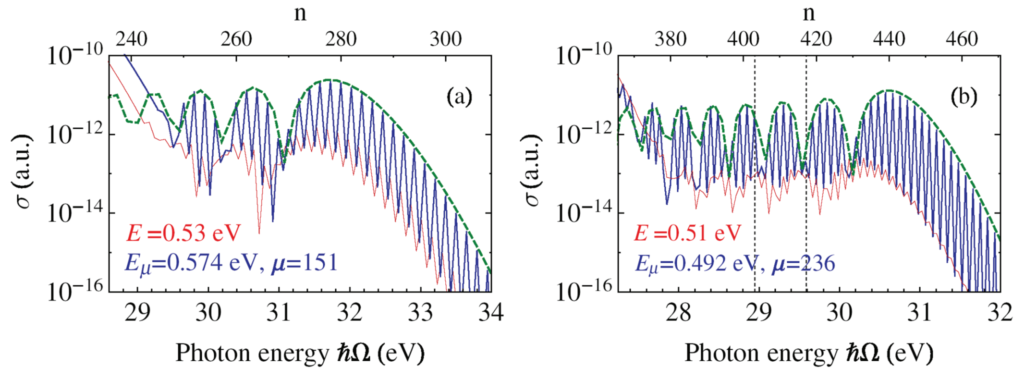

Figure 1. LARA spectra for $e - H$ attachment in a linearly polarized field $\mathbf{F}(t)$ (with $\mathbf{e}_z \parallel \mathbf{p}$) having (a) intensity $I = 3.75 \times 10^{11}$ W cm^{-2}, $\hbar\omega = 0.098\,|E_0| = 0.074$ eV or (b) $I = 1.35 \times 10^{11}$ W cm^{-2}, $\hbar\omega = 0.06\,|E_0| = 0.0453$ eV. Thick (blue) and thin (red) solid lines: exact TDER results for the resonant (E_μ) and non-resonant (E) electron energies shown in each panel. Dashed (green) lines: HHG spectra of the H$^-$ ion (in arbitrary units) for the same field parameters I, ω as in (a) and (b). Vertical lines in (b) mark photon numbers $n = 403, 417$ (cf figure 2).

LARA amplitude (6). Moreover, for an intense low-frequency ($\hbar\omega \ll |E_0|$) field $\mathbf{F}(t)$, the coefficients g_k with $k = 0$ are small compared to g_0 (cf [19]). Approximating in (24) $g_k = g_0\delta_{k,0}$ and $g_0 \approx C_\kappa$ (cf (21)), $\mathbf{d}_{pw}(\Omega)$ yields an exact (i.e. without using saddle-point methods for its evaluation) TDER result for the LARA amplitude in the SFA [3, 4]. Resonant phenomena are described by the second (rescattering) term, $\mathbf{d}_{\rm resc}(\Omega)$, in (22). This term originates from the integral term in (12) and, within the TDER theory, ensures an exact account of the effects of the potential $U(r)$ on the field-dressed initial and final states of the attaching electron (through the coefficients $f_k(\mathbf{p})$ and g_k in (25)).

To extract from (25) the resonant part of the amplitude (6) in an explicit form, we solve the system (14) for $f_k(\mathbf{p})$ near the resonance, i.e. for $\epsilon \sim (\varepsilon + \mu\hbar\omega)$. Expanding matrix elements in (14) up to the linear term in $\Delta\epsilon = \epsilon - \varepsilon - \mu\hbar\omega = E - E_\mu + {\rm i}\Gamma/2$, approximating $c_k(\mathbf{p}) \approx c_k(\mathbf{p}_\mu)$ and employing the symmetry relations (20), we obtain

$$\sum_{k'} \mathcal{M}_{k,k'}(\varepsilon) f_{k'-\mu}(\mathbf{p}) + \Delta\epsilon \sum_{k'} \mathcal{M}'_{k,k'}(\varepsilon) f_{k'-\mu}(\mathbf{p}) = c_{k-\mu}(\mathbf{p}_\mu), \quad (27)$$

where $M'_{k,k'}(\varepsilon) = \partial M'_{k,k'}(\varepsilon)/\partial\varepsilon$. In the lowest resonant approximation ($\Delta\epsilon \to 0$), the coefficients $f_{k'-\mu}(\mathbf{p})$ in (27) are proportional to $g_{k'}$: $f_{k'-\mu} = \alpha(\mathbf{p}) g_{k'}$. To find $\alpha(\mathbf{p})$, we multiply the system (27) by g_k and then sum over k. Taking into account the symmetry relations (20) and the equality $\Sigma_k\, g_k M_{k,k'}(\varepsilon) = 0$ (cf (15)), the system (27) reduces to a single equation

$$\Delta\epsilon \sum_{k,k'} g_k \mathcal{M}'_{k,k'}(\varepsilon) f_{k'-\mu}(\mathbf{p}) = \sum_k g_k c_{k-\mu}(\mathbf{p}_\mu), \quad (28)$$

from which $\alpha(\mathbf{p})$ is easily obtained upon substituting $f_{k'-\mu}(\mathbf{p}) = \alpha(\mathbf{p}) g_{k'}$. The resulting resonant approximation for $f_{k-\mu}(\mathbf{p})$ is

$$f^{(r)}_{k-\mu}(\mathbf{p}) = \frac{g_k}{E - E_\mu + {\rm i}\Gamma/2} \frac{\sum_s g_s c_{s-\mu}(\mathbf{p}_\mu)}{\sum_{s,s'} g_s \mathcal{M}'_{s,s'}(\varepsilon) g_{s'}}. \quad (29)$$

Changing in (25) the summation index k to $k - \mu$ and substituting there the result (29) for $f_{k-\mu}(\mathbf{p})$, the resonant term in $\mathbf{d}_{\rm resc}(\Omega)$ can be presented in the same form as in (6), where the explicit form for the dual dipolemoment $\tilde{\mathbf{d}}(\Omega)$ in the TDER theory (in terms of g_k, $g_{k'}$ and the matrix elements $W_{n\;k-\mu,k'}$) is given in [20]. The coefficient $\mathcal{B}(\mathbf{p}_\mu)$ in (6) is related to the amplitude $A_\mu(\mathbf{p}_\mu)$ for μ-photon laser-stimulated attachment:

$$A_\mu(\mathbf{p}_\mu) = \sum_k g_k^* c^*_{k-\mu}(\mathbf{p}_\mu),$$

$$\mathcal{B}(\mathbf{p}_\mu) = \frac{2\pi\hbar^2\sqrt{\kappa}}{m} A^*_\mu(\mathbf{p}_\mu). \quad (30)$$

The potential part $\mathbf{d}^{(p)}(\Omega)$ of $\mathbf{d}(\Omega)$ in (6) is given within TDER theory by (22)–(25) in which we set $\mathbf{p} = \mathbf{p}_\mu$ and replace $f_k(\mathbf{p}_\mu)$ in (25) by $f_k(\mathbf{p}_\mu) - f^{(r)}_k(\mathbf{p}_\mu)$.

Key features of resonant LARA cross sections are shown in figure 1. (Qualitatively, resonant LARR features are similar.) Results for $e - H$ attachment with the formation of the H$^-$ ion are shown for both non-resonant (E) and resonant (E_μ) incident electron energies for the cases of odd μ in (a) and even μ in (b). (TDER parameters for this case are (cf, e.g., [19]) $|E_0| = 0.755$ eV, $C_\kappa = 2.304$, $a_0 = 6.16\, a_B$ and $r_0 = 2.64\, a_B$, where a_B is the Bohr radius.) In Figure 1(a), the laser parameters and energy E are the same as in a recent analysis of nonresonant LARA processes [9]. Resonant effects are more pronounced in figure 1(b) for $I = 1.35 \times 10^{11}$ W cm^{-2} and $\hbar\omega = 0.0453$ eV. Figure 1 exhibits several qualitative features: (i) a two orders of magnitude resonant increase of $\sigma(\Omega)$; (ii) perfect coincidence of the resonant attachment spectrum shape with that for high harmonics generated by the H$^-$ ion for the same laser parameters; (iii) the extent of the high-energy plateau in the resonant process ($\approx |E_0| + 3.17 u_p$ as in HHG) exceeds that for the non-resonant case; and (iv) enhancements occur only for those numbers n of absorbed photons whose parity is opposite to that of μ (cf also Figure 2). This last result is a simple consequence of the fact that the resonant cross section (11) involves the rate for emission of the $(n + \mu)$th harmonic of the field $\mathbf{F}(t)$. As is well known, an atom can emit only *odd* harmonics of a monochromatic field (owing to electric dipole selection rules), so that $n + \mu$ must be odd.

Figure 2 shows the energy dependence of the partial (n-photon) LARA cross sections in the resonance region (as well as our general parametrization for $\sigma_n(E)$) for the

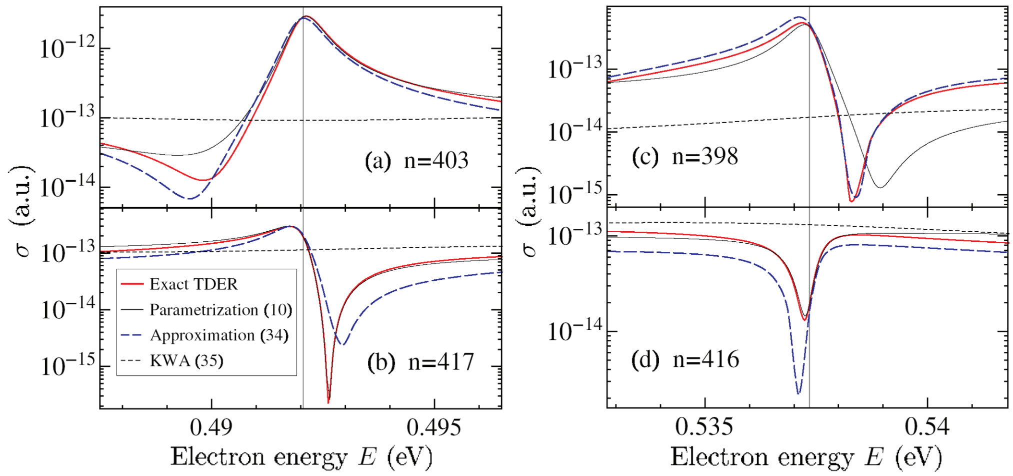

Figure 2. Dependence of LARA cross sections on electron energy for fixed n and the same laser parameters as in Figure 1(b). Thick solid lines: exact TDER results; thin solid lines: parametrization (10); dashed lines: results with coefficients f_k given by (34) and $g_k = C_\kappa \delta_{k',0}$; dotted lines: results with Fourier-coefficients of the KWA function (35). Vertical lines mark the resonance energies E_μ ($\delta = 0$).

laser parameters of Figure 1(b), giving a Stark-shift and width of the H^- ground state $\psi_0(\mathbf{r})$ of $E_0 = \mathrm{Re}\ \varepsilon - E_0 = -6.250 \times 10^{-3}$ eV and $\Gamma = 8.335 \times 10^{-4}$ eV. Owing to the complexity of the parameter q (i.e. the ratio of the resonant and potential parts of the amplitude (6) at $E = E_\mu$), the asymmetric resonance profile shape as well as the positions of the maxima and minima in $\sigma_n(E)$ (for given n and laser parameters) are sensitive to both the absolute value of q and the relation between Re q and Im q. Figure 2 shows examples of resonance profiles for both odd ($\mu = 236$) and even ($\mu = 237$) values of n. In terms of δ, the positions of the maxima (δ_+) and minima (δ_-) of $\sigma_n(E)$ can be obtained from (9) or (10) by equating to zero the derivative of σ (Ω) with respect to δ:

$$\delta_\pm = \frac{|q+1|^2 \mp |q(q+2)| - 1}{2\mathrm{Im}\, q}. \tag{31}$$

Since $\delta_\pm$ are the roots of a quadratic equation, the following relations are valid:

$$\delta_+\delta_- = -1, \quad \delta_+ - \delta_- = -|q(q+2)|/\mathrm{Im}\, q. \tag{32}$$

Relations (32) show that for Im$q < 0$ (or Im$q > 0$) the maximum occurs at $E > E_\mu$ (or $E < E_\mu$), while the location of the minimum has the opposite behavior. These results agree with those in Figures 2(a) and (d) (in which Im$q < 0$) and 2(b) and (c) (in which Im$q > 0$). The "window resonance" behavior of $\sigma_n(E)$ (as in Figure 2(d)) occurs whenever $|\delta_-| \to 0$ or, equivalently, the parameter $\Delta \to 1$, where

$$\Delta = \frac{\delta_+ - \delta_-}{\delta_+ + \delta_-} = \frac{|q(q+2)|}{1 - |q+1|^2}. \tag{33}$$

The parameter is positive when $|q+1| < 1$, and this latter inequality is fulfilled for negative Re q (with (Re q)$_{\min} \to -2$ when $|\mathrm{Im}\, q| \to 0$). The limiting case $\Delta = 1$, i.e. $|q(q+2)| = 1 - |q+1|^2$, is realized when Im $q = 0$. Therefore, a "window resonance" in $\sigma_n(E)$ appears for negative Re q when (i) $|\mathrm{Im}\, q| \ll |\mathrm{Re}\, q|$ and (ii) $|q+1| < 1$. (For example, Re $q = -0.612$, Im $q = -0.207$ and $\Delta = 1.124$ for the results in figure 2(d).)

As is clear from our general considerations, the continuum resonance phenomena shown in Figures 1 and 2 disappear in any theory that does not account for the influence of the atomic potential $U(r)$ on the scattering states (such as, e.g., the Born approximation or even an improved SFA [1, 7]). However, an accurate non-perturbative account of the interaction of a recombining electron with both a laser field and an atomic potential leads to complicated results even for a potential $U(r)$ supporting only a single bound state, as in the TDER theory (cf the result (25)). Nevertheless, the results simplify for a low-frequency ($\hbar\omega \ll |E_0|$) field $\mathbf{F}(t)$, in which case the system (14) can be solved iteratively, taking into account nondiagonal matrix elements $\mathcal{M}_{k,k'}(\epsilon)$ perturbatively [21]. In the lowest approximation, neglecting nondiagonal matrix elements, the coefficients $f_k(\mathbf{p})$ take the following form:

$$f_k(\mathbf{p}) \approx f_k^{(1)}(\mathbf{p}) = \frac{c_k(\mathbf{p})}{\mathcal{M}_{k,k}(\epsilon)} = \frac{c_k(\mathbf{p})}{\mathcal{M}_{0,0}(\epsilon + k\hbar\omega)}. \tag{34}$$

Also, as noted above, the coefficients $g_{k\neq 0}$ are small for low frequencies, so that we can make the approximation $g_k \approx C_\kappa \delta_{k,0}$. As is seen in figure 2, the approximations (34) and $g_k = C_\kappa \delta_{k,0}$ reasonably describe the resonant phenomena. The resonant structures originate from the matrix element in the denominator of (34), which has zeros in the complex plane of ϵ at $\epsilon = \tilde{\varepsilon} - k\hbar\omega$, where $\tilde{\varepsilon}$ is the complex quasienergy ε in the low-frequency approximation [19].

Finally, we note that resonant phenomena in LARA/LARR processes cannot be described in the low-frequency Kroll–Watson approximation (KWA) for the scattering state wavefunction [22]. Within the TDER theory, the KWA is formulated in terms of the function $f_\epsilon(\mathbf{p}, t)$ in (12) [23]:

$$f_\epsilon^{\mathrm{KWA}}(\mathbf{p}, t) = \frac{e^{-i\mathcal{S}(\mathbf{p},t)/\hbar}}{R(\mathcal{E}(t))}, \quad \mathcal{E}(t) = \frac{\mathbf{P}^2(t)}{2m}, \tag{35}$$

where $R(\mathcal{E})$ (cf (17)) is related to the partial s-wave amplitude $f_0(E)$ of elastic electron scattering from the potential $U(r)$ in the TDER theory at $\mathbf{F}(t) = 0$: $f_0(E) = [\kappa R(E)]^{-1}$ [19]. The shortcoming of the KWA is that in this semiclassical approximation, the quantization of the photon energy is completely neglected so that both $\mathcal{S}(\mathbf{p}, t)$

and $R(\mathcal{E}(t))$ in (35) depend only on the classical energy $\mathcal{E}(t)$ of an electron in a laser field, which is always positive. Therefore, although the amplitude $f_0(E)$ has a pole at negative energy $E = E_0$ (since $R(E_0) = 0$ [19]), the function $R(\mathcal{E}(t))$ in (35) has no zeros, i.e. the KWA result (35) fails to describe resonant effects (cf Figure 2). (For this reason, the resonant effects disappear also in the KWA for LAES [10]).

In conclusion, we have analyzed the key features of resonant phenomena in LARA/LARR processes that occur for electron energies corresponding to μ-photon laser-stimulated attachment/recombination. For such energies, we find that the spectra of spontaneously emitted photons in the high-energy parts of the LARA/LARR plateaus coincide with the harmonic generation spectra of the bound systems. Owing to the significant enhancement of resonant cross sections versus non-resonant ones, we expect that our findings should facilitate experimental observation of the resonant modification of radiative electron attachment/recombination in a laser field and the emission of high-order harmonics in laser-assisted collision processes.

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