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Simulation of a Microdosimetry Problem: Behavior of a Pseudorandom Series at a Low Probability

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1 Introduction

The carcinogenic effects for low dose irradiations are not very well known. Estimations usually are made based on the effects observed at high doses that are then extrapolated to low doses. To estimate low dose effects, the ICRP (International Commission on Radiological Protection) uses a linear extrapolation matched with a dose-rate reduction factor equal to two. This proportionality of the effect and dose, even for the lowest doses and dose-rates, leads to two assumptions which must be questioned [1]:

1. the efficiency of DNA repair in cells does not vary with the dose and the dose-rate,
2. when one single particle crosses one single cell, a carcinogenic transformation may occur.

Low doses are frequently generated by fast electrons at low fluence.

We must consider the irradiated medium as an assembly of targets and the cross section as a representation of the probable interaction between the incident particle and the target.[2, 3, 4] Biologically, cells contain internal structures which are the sensitive elements. Physically, a hit is interpreted as a registered event caused by a particle passing through the sensitive site.

The Poisson law describes the statistic behavior of this event:

$$P(x) = (e^{-m}/x!)\, m^x \qquad (1)$$

where $P(x)$ is the realization probability of the event x and m represents the average hits per target (ratio of the number of hits per number of targets). For example, consider that a flux of exactly 10^6 particles/cm^2 reaches a cell population whose sensitive elements have a geometric cross section of 100 μm^2. The average number of particles per cell will be one, but according to the Poisson statistics, about 37% of the cells will survive (0 hits), about 37% of the cells will be hit only once and 26% will be hit twice or more.[5]

This problem can be extended to the response of many radiation detectors—one-hit or multi-hits detectors.[6]

2 Calculations

Microphysical processes have a random character that are ruled by continuous or discrete probability laws. By using numerical simulations with Monte Carlo methods, random experiences are imitated by a sampling of pseudorandom numbers which are generated by algorithms such as the linear congruential generators (LCG) that use the integer recursion [7]:

$$X_{i+1} = (aX_i + c) \bmod M, \tag{2}$$

where the integers a, c, and M are constant. They generate a sequence X_1, X_2, ... of random integers between 0 and M – 1 (into [1, M – 1] if c = 0). Each X_i is then put into the interval [0,1).

We used a class of random empirical test methods (chi-square test, Kolmogorov- Smimov test, gap test, run-up run-down test, serial test) that seem to give valid results.[8] Valid as they may be, these tests are not sufficient; therefore new tests created for specific applications are often advisable. We tested two LCG generators, the subroutine RAND from the Matlab software [9] and the subroutine RND from the Fortran 77 software.[10] As a systematic test, we compared the spectrum of hits obtained with the given pseudorandom numbers generators with the one predicted by Poisson's distribution law.

Calculations were performed at a HP 9000 workstation and probabilities were determined for only one series. Each pseudorandom number generated between [0,1) (original number) is put in the interval corresponding to the number of digits. As an example, for 3 digits each original number is multiplied by 10^3 and we extract the integer part (integer x); and for 4 digits each original number is multiplied by 10^4. Experimental probability Π for integer x (independently of number of digits) to be hit is defined as:

$$\Pi(x) = n_x / N, \tag{3}$$

where n_x is the number of drawings for integer x and N the total number of drawings.

The discrepancy $\xi(x)$ between experimental and theoretical (1) probabilities is calculated from the expression (if $P(x) = 0$ and $\Pi(x) \neq 0$, we put arbitrarily $\xi(x) = -10$):

$$\xi(x) = (|P(x) - \Pi(x)|)/P(x) \times 100 \tag{4}$$

We are interested in the probability of hitting a particular number, the number of hits ranges from 0 to 10 in the case of the presented application. The number m, considered as the average of hits per integer is defined as:

$$m = N/card\{interval\} \tag{5}$$

where $card\{interval\}$ is the number of integers in this interval.

Experimental probabilities for a same value of m (in our study m ranges from 0.1 to 10) were calculated as a function of the number of digits. Let us consider the case where 3 digit numbers are used:

- $m = 1$, if we calculate the hit spectrum of integers in the interval $[0,10^3)$ for 10^3 integers generated in this interval,
- $m = 2$, if we calculate the hit spectrum of integers in the interval $[0,10^3)$ for 2×10^3 integers generated in this interval.

3 Results

We have represented the descrepancies (4) between calculations and theory for the Fortran generator on Figures 2 through 5. We have obtained similar results for the Matlab generator. The Poisson law (1) is represented as a function of m and x on Figure 1. Two areas can be observed: a first one corresponding to non-null probabilities (relief zone) and a second one corresponding to probabilities which tend towards zero (flat zone).

Discrepancies $\xi(x)$ are localized into two types of areas: a first one which does not move with increasing number of digits and corresponding to n_x (number of hits) $\in [0,2]$ and $m \in [5,10]$; and a second zone, much more important, which seems to be localized at the border of high Poisson probabilities (Figure 1) and moves toward high number of hits with increasing digit numbers. As a matter of fact, we observe that an increase of the number of digits results in a decrease of the total descrepancy, in the studied ranges of n_x and m.

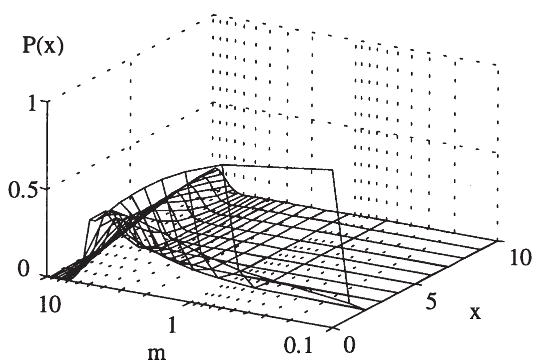

Figure 1. 3-D representation of the Poisson distribution.

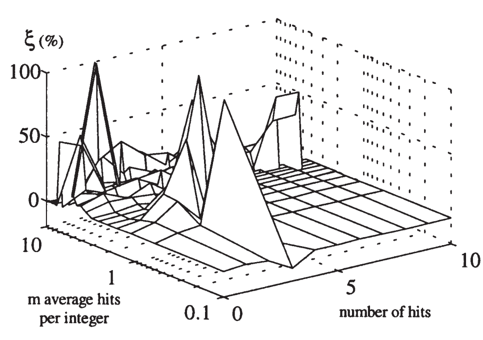

Figure 2. $\xi(x)$ for 3-digit numbers.

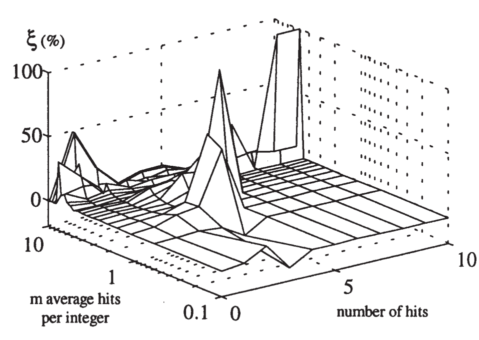

Figure 3. $\xi(x)$ for 4-digit numbers.

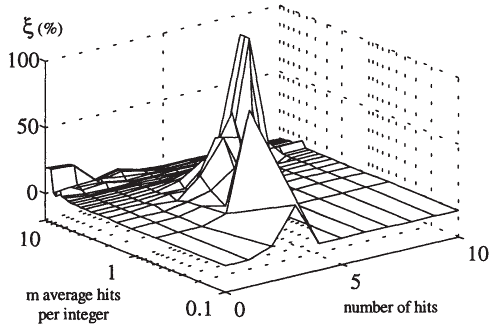

Figure 4. $\xi(x)$ for 5-digit numbers.

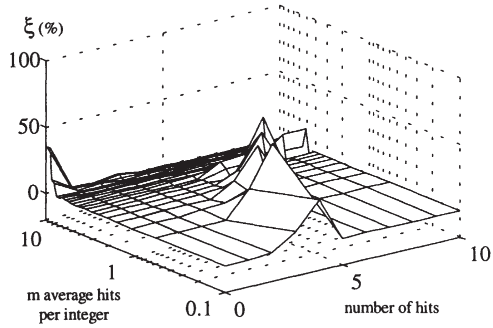

Figure 5. $\xi(x)$ for 6-digit numbers.

The results presented in the 3D graphs (Figures 2 to 5) correspond to only one history. In general, many histories must be used to obtain accurate statistics. We made the calculations (average and standard deviation for $\Pi(x)$) on the largest discrepancies, and even though a large number of histories were used, an important standard deviation remains. For such calculations, we find that the general behavior is a stabilization of the obtained average values of $\Pi(x)$ around the needed value of $P(x)$ when the number of histories increases.

4 Conclusion

We examined the properties of two pseudorandom generators using a test which mimics the properties of the application in which the generators will be used. In this case, these applications are ruled by the Poisson law. We have shown that the two generators fail in the described cases. It also appears that an increase of digits improves the properties of the tested generators about the Poisson law. It should be noticed that in the case of our test, the use of a number of digits greater than 6 was not possible due to the needed memory capacity and time of computation. Such a test seems necessary if a pseudorandom generator can be expected to provide reliable results, we strongly recommend this test if a number of digits lower than 7 is used.

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